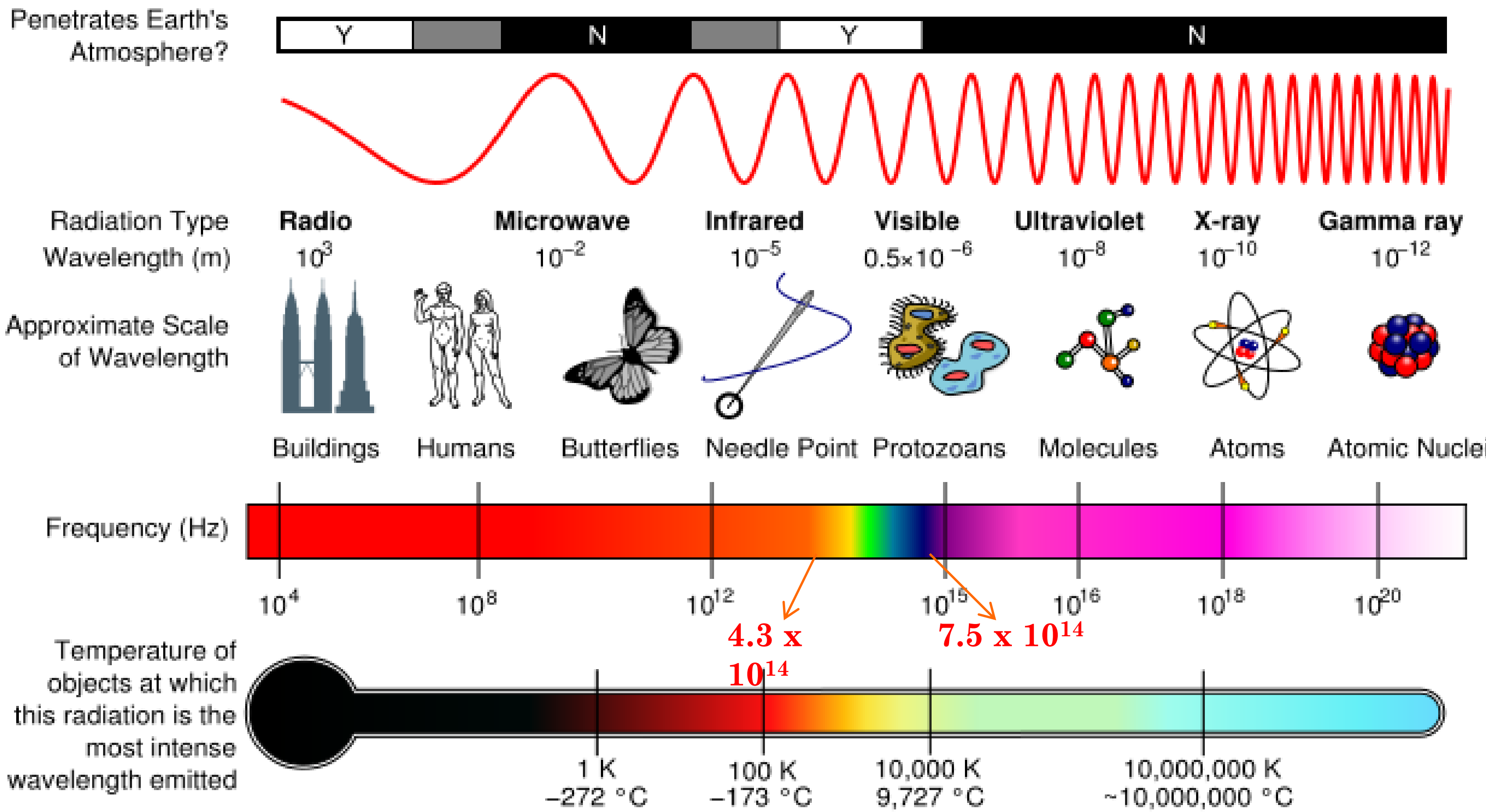
Particle properties of waves

Ch.2 of Arthur Beiser

- Black body radiation
- Photoelectric effect
- X-ray diffraction
- Compton Effect
- Pair production

ELECTROMAGNETIC SPECTRUM



Particle aspect of radiation

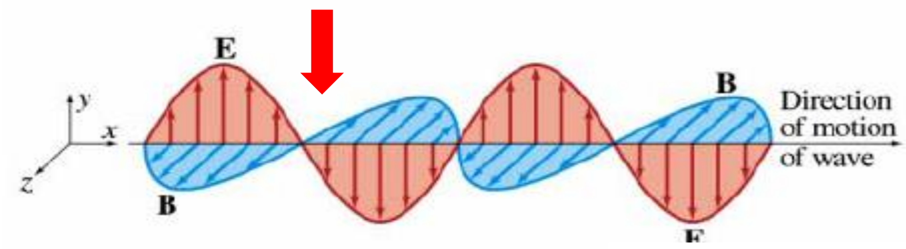
Classical physics: a particle : energy **E** and momentum ***p***

*W*ave : an amplitude and wave vector ***k***

General Consensus:

Particle: e.g e^-: charge, mass : laws of particle mechanics

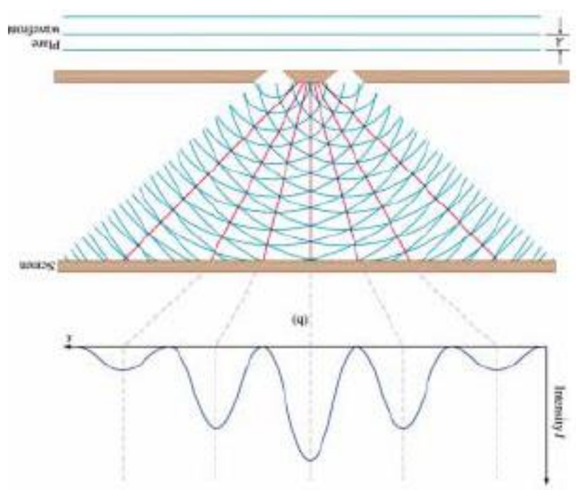
Waves: em waves: diffraction, interference, polarization etc

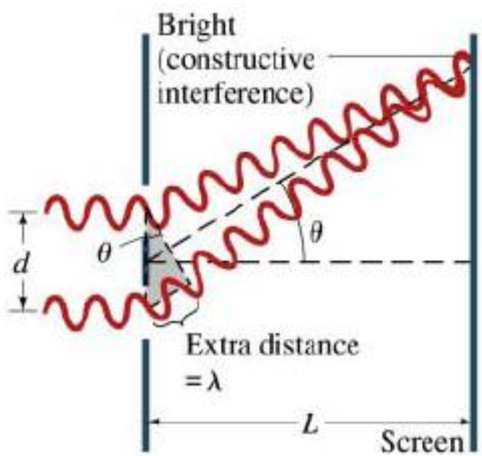


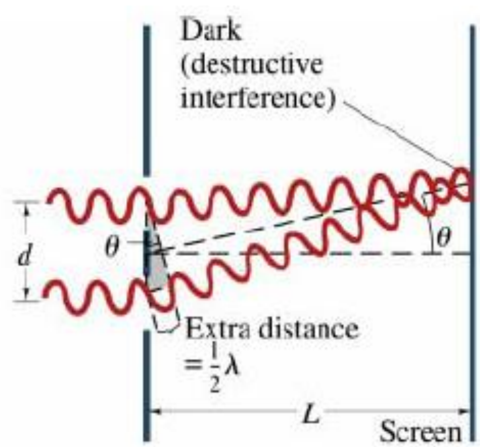
$$c = \frac{1}{\sqrt{\epsilon_o \mu_o}} = 2.998 \times 10^8 \, m/s$$

Principle of superposition

A characteristic properties of all waves







Young's experiment: Light consists of waves

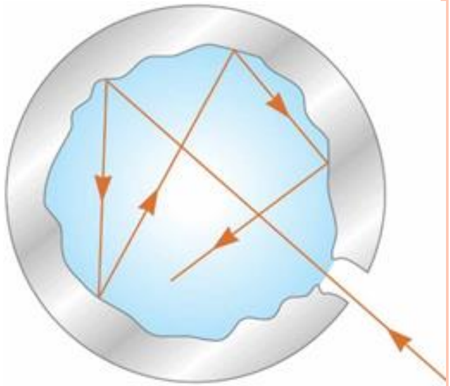
Classical approach

Black Body Radiation

- An object at any temperature is known to emit thermal radiation
- Characteristics depend on the temperature and surface properties
- The thermal radiation consists of a continuous distribution of wavelengths from all portions of the em spectrum
- At room temperature, the wavelengths of the thermal radiation are mainly in the infrared region
- As the surface temperature increases, the wavelength changes
  - It will glow red and eventually white
- The basic problem was in understanding the observed distribution in the radiation emitted by a **black body**
  - Classical physics didn't adequately describe the observed distribution

Black Body

- A **black body** is an ideal system that absorbs all radiation incident on it. When an object is heated, it radiates electromagnetic energy as a result of the thermal agitation of the electrons in its surface. The electromagnetic radiation emitted by a black body is called **blackbody radiation**



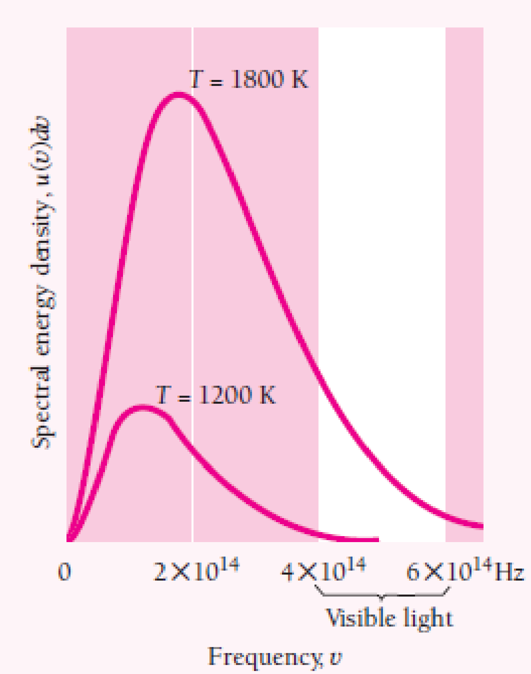
- A good approximation of a black body is a small hole leading to the inside of a hollow object
- The nature of the radiation leaving the cavity through the hole depends only on the temperature of the cavity walls

Blackbody Experiment Results

- The total power of the emitted radiation increases with temperature
  - **Stefan's Law**: $P = \sigma A e T^4$. For a blackbody, $e = 1$
- The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases
  - **Wien's displacement law**: $l_{max} T = 2.898 \times 10^{-3}$ m·K

Intensity of Blackbody Radiation

- The intensity increases with increasing temperature
- The amount of radiation emitted increases with increasing temperature
  - The area under the curve
- The peak wavelength/frequency decreases/increases with increasing temperature

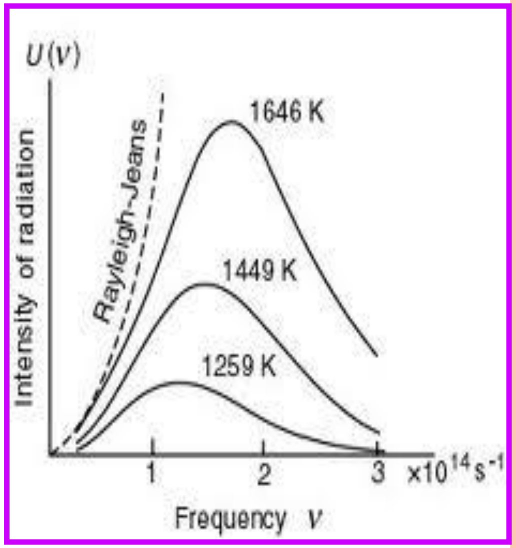


Rayleigh & Jeans

Radiation inside a cavity: series of standing waves. Density of standing waves in cavity:

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3}$$

Classical average energy per standing wave: $\bar{\varepsilon} = kT$



EXPERIMENTAL

Rayleigh-Jeans formula: $u(\nu)d\nu = \varepsilon G(\nu)d\nu = \dfrac{8\pi kT}{c^3}\nu^2 d\nu$

Rayleigh's assumption: a standing wave can exchange any amount (continuum) of energy with matter

Classical: continuous energy distribution

Consequences:

Ultraviolet catastrophe

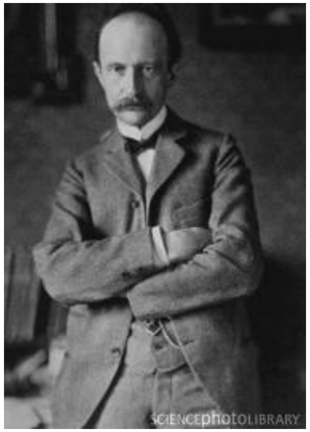
As ν increases toward the ultraviolet end of the spectrum, the energy density should increase as ν^2. In the limit of infinitely high ν, $u(\nu)d\nu$ should also go to ∞. In really, of course, the energy density falls to 0 as $\nu \rightarrow \infty$. This discrepancy called ultraviolet catastrophe.

Classical physics failure ⇒ introduction of Q.M.

Planck's formula

The energy exchange between radiation and matter must be discrete and energy of radiation $E = nh\nu$

Average energy per standing wave $$\varepsilon = \frac{h\nu}{e^{h\nu/kT-1}}$$



Planck's modifications

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/kT-1}}$$

$h = 6.626 \times 10^{-34}$ J.s

Max Planck (1918 Nobel prize)

At low ν

$$h\nu << kT$$

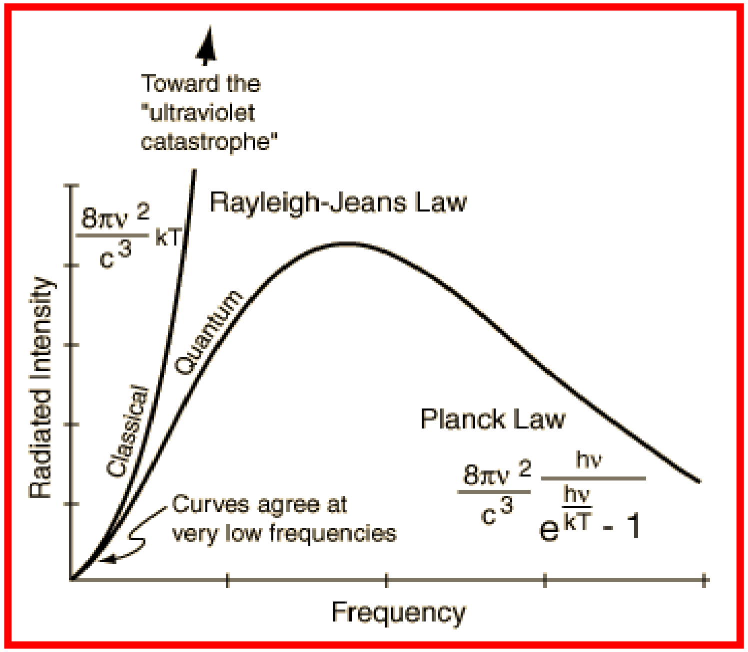
$$\frac{1}{e^{h\nu/kT-1}} \approx \frac{1}{1+h\nu/kT-1}$$

$$\approx kT/h\nu$$

$$u(\nu)d\nu \approx \frac{8\pi h}{c^3}\nu^3\left(\frac{kT}{h\nu}\right)d\nu$$

$$\approx \frac{8\pi kT}{c^3}\nu^2 d\nu$$

Rayleigh & Jeans



At high ν

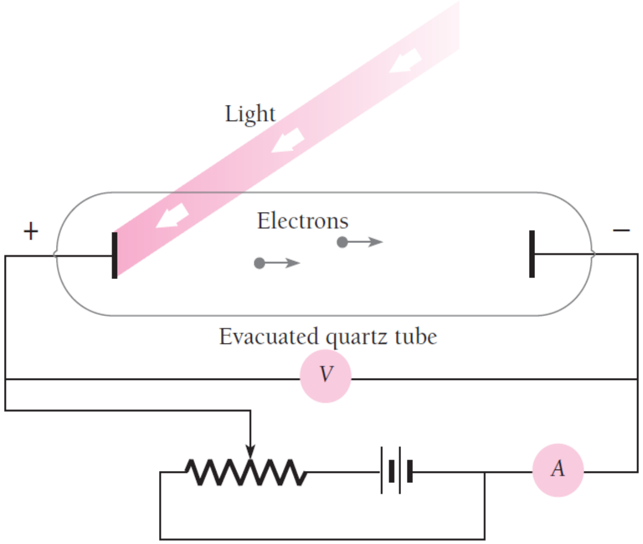
$$h\nu >> kT$$

	# of modes per unit frequency per unit volume	Probability of occupying modes	Average energy per mode
CLASSICAL	$\frac{8\pi\nu^2}{c^3}$	Equal for all modes	kT
QUANTUM	$\frac{8\pi\nu^2}{c^3}$	Quantized modes: require hν energy to excite upper modes, less probable	$\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

Photoelectric effect

Provides a direct confirmation for the energy quantization of light.

The phenomenon of ejection of electron from the surface of a metal when light of a suitable frequency strikes on It, is called photoelectric effect. The emitted electrons are called photo-electrons.



The time between the incidence and emission of a photoelectron is very small, $< 10^{-9}$ s.

When the V is increased to V_0, of the order of several volts, no more photoelectrons arrive, as indicated by the current dropping to zero. This extinction voltage corresponds to the maximum photoelectron KE.

Experimental Findings:

- If the frequency of the incident radiation is smaller than the metal's threshold frequency(a frequency that depends on the properties of metal), no electron can be emitted regardless of the radiation's intensity.

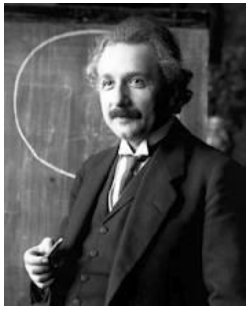
- No matter how low the intensity of the incident radiation, electrons will be ejected *instantly* the moment the frequency of the radiation exceeds the threshold frequency.

- At a fixed frequency, the number of ejected electrons increases with the intensity of the light but doesn't depend on its frequency.

A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same. The em theory of light, on the contrary, predicts that the more intense the light, the greater the energies of the electrons.

- The kinetic energy of the ejected electrons depends on the frequency but not on the intensity of the beam; the kinetic energy of the ejected electron increases *linearly* with the incident frequency.

Einstein interpretation(1905)



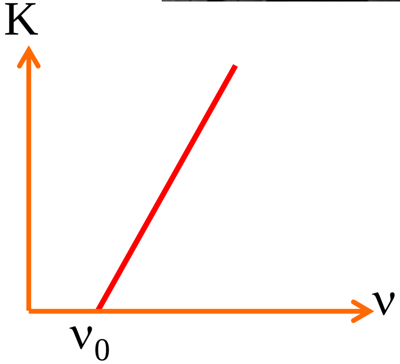
Light comes in packets of energy (photons)

$$E = h\nu$$

An electron absorbs a single photon to leave the material

Work function: $W = h\nu_0$

Larger W needs more energy needed for an electron to leave



ν_0 = *threshold frequency*

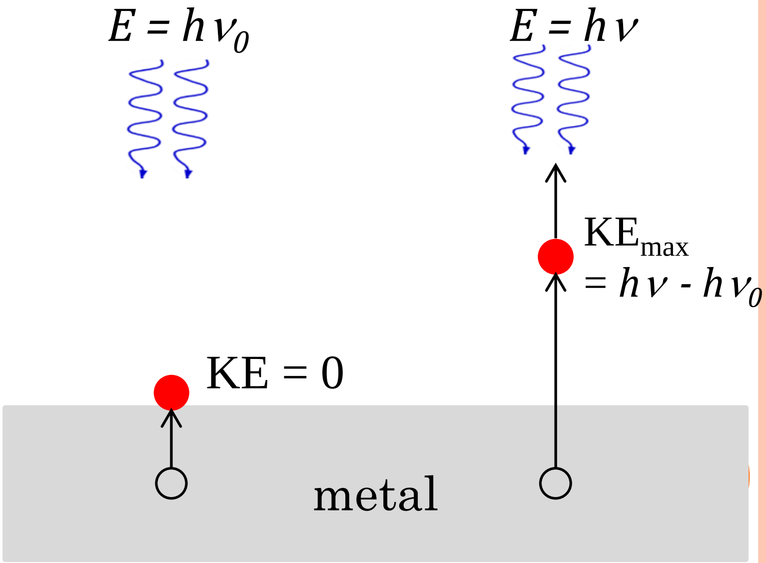
Classical physics fails: for dependence of the effect on the threshold frequency

Photoelectric effect:

$$\boxed{K = h\nu - W = h\nu - h\nu_0}$$

The stopping potential : at which all of the electrons will be turned back before reaching the collector

$$\boxed{V_s = \frac{h\nu}{e} - \frac{W}{e} = \frac{hc}{e\lambda} - \frac{W}{e}}$$



Applications of the Photoelectric Effect

- Detector in the light meter of a camera
- Phototube
  - Used in burglar alarms and soundtrack of motion picture films
  - Largely replaced by semiconductor devices
- Photomultiplier tubes
  - Used in nuclear detectors and astronomy

Ex-1: When two UV beams of wavelengths λ_1 = 280nm and λ_2 = 490nm fall on a lead surface, they produce photo electrons with maximum kinetic energies 8.7eV and 6.67eV, respectively. (a) calculate the value of planck constant. (b) Calculate the work function and the cutoff frequency of lead.

ANS.(a)

$K_1 = hc/\lambda_1 - W \quad and \quad K_2 = hc/\lambda_2 - W$

Use 1eV = 1.6 x 10^{-19} J

$$\Rightarrow h = \frac{K_1 - K_2}{c}\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} \cong 6.62\times 10^{-34}\ Js$$

(b) $W = hc/\lambda_1 - K_1 = -4.14eV$

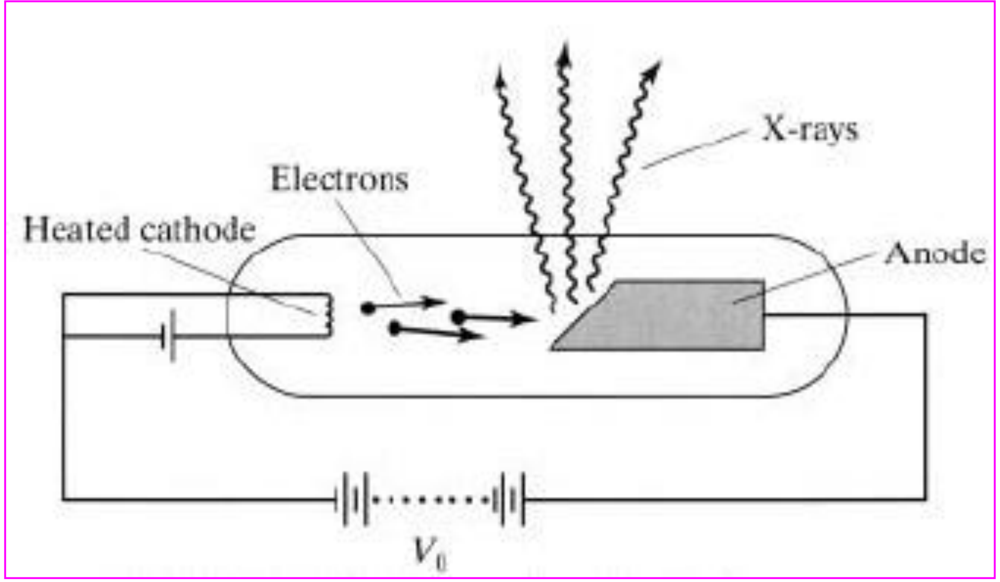
The cutoff frequency of the metal is $\nu_0 = W/h = 10^{15}\ Hz$

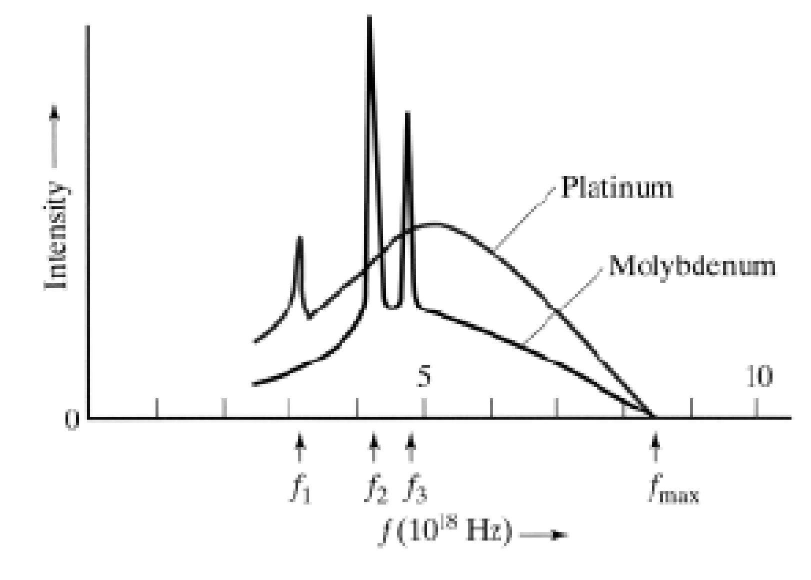
The photoelectric effect provides convincing evidence that photons of light can transfer energy to electrons.

Is the inverse process also possible? That is, can part or all of the kinetic energy of a moving electron be converted into a photon? As it happens, the inverse photoelectric effect not only does occur but had been discovered (though not understood) before the work of Planck and Einstein.

X-rays

EM radiation with 0.01 to 10 nm → x-rays



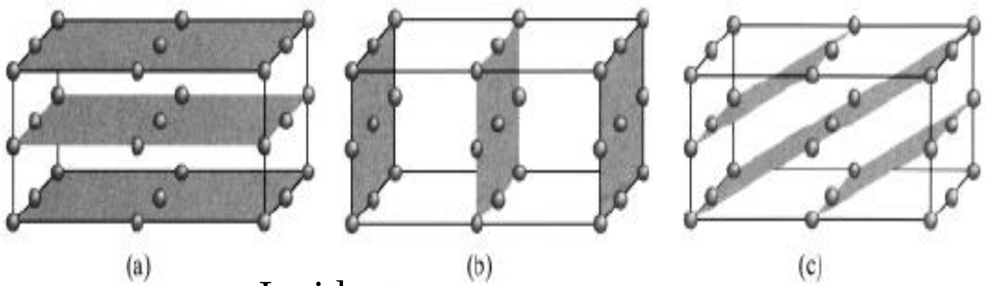


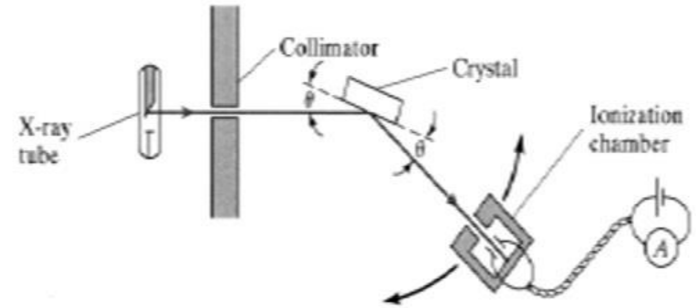
X-ray production:

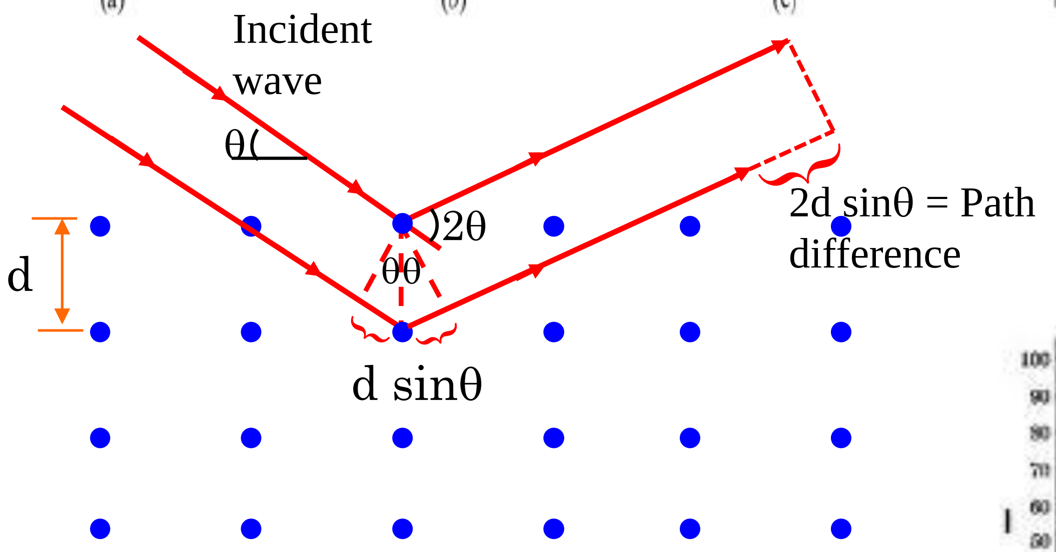
$$\lambda_{\min} = \frac{1.24\times10^{-6}}{V} V.m$$

X-ray diffraction

Crystal structure determination



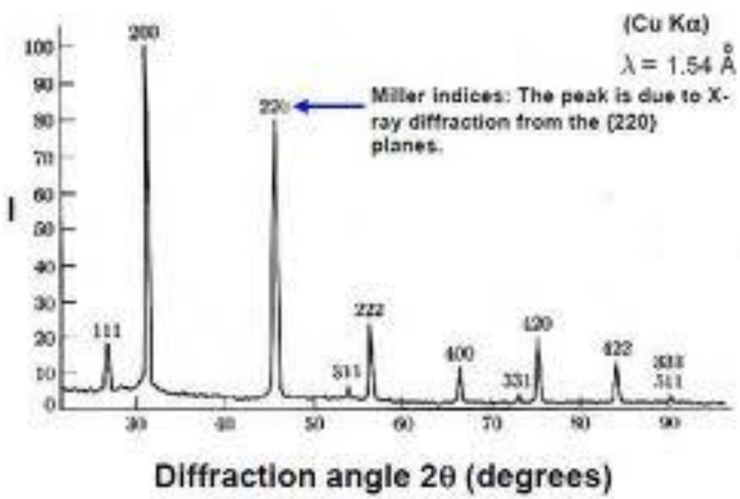




Bragg's Law:

$$2d\sin\theta = n\lambda$$

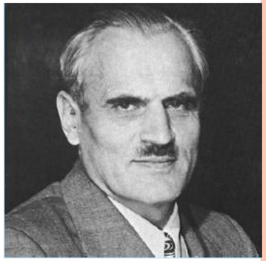
XRD Pattern of NaCl Powder



According to the quantum theory of light, photons behave like particles except for their lack of rest mass.
How far can this analogy be carried?
For instance, can we consider a collision between a photon and an electron as if both were billiard balls?

Compton effect

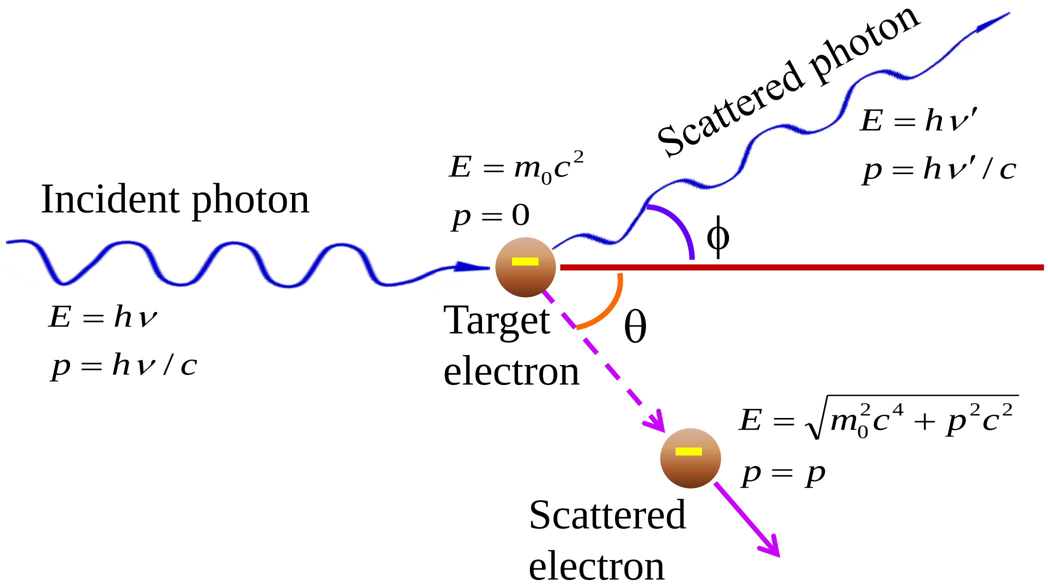
An x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begins to move.

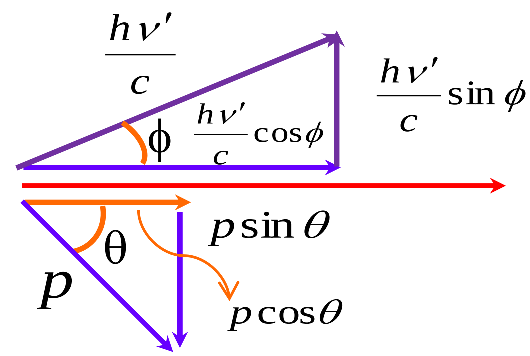


1927 Nobel

We can think of the photon as losing an amount of energy in the collision that is the same as the kinetic energy KE gained by the electron, although actually separate photons are involved

Further confirmation of photon model





Scattering of x-rays from electrons in a carbon target and found scattered x-rays with a longer wavelength than those incident upon the target.

- Compton's experiments showed that, at any given angle, only *one* frequency of radiation is observed

Initial momentum = final

In the original photon direction:

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta$$

in the perpendicular direction:

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta$$

$$pc\cos\theta = h\nu - h\nu'\cos\phi$$

$$pc\sin\theta = h\nu'\sin\phi$$

$$\Rightarrow\ p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2$$

Form the total energy expression we have:

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2m_0c^2(h\nu - h\nu')$$

$$2m_0c^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1-\cos\phi)$$

Compton effect or shift:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)$$

Angle of recoil electron:

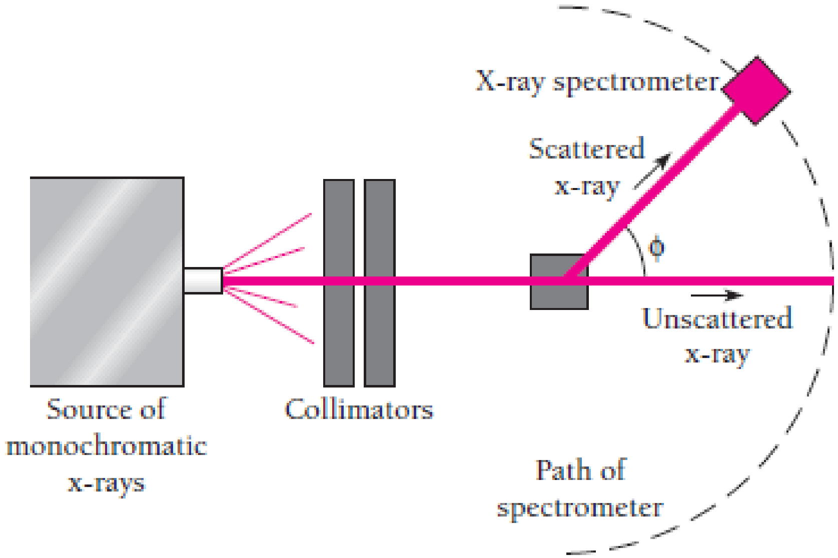
$$\tan\theta = \frac{\sin\phi}{\lambda'/\lambda - \cos\phi}$$

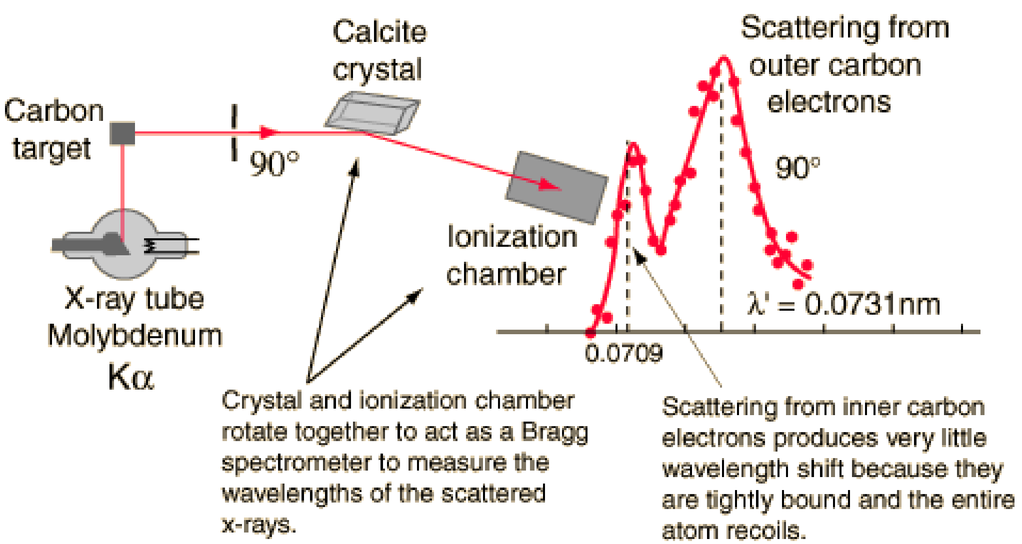
Compton wavelength:

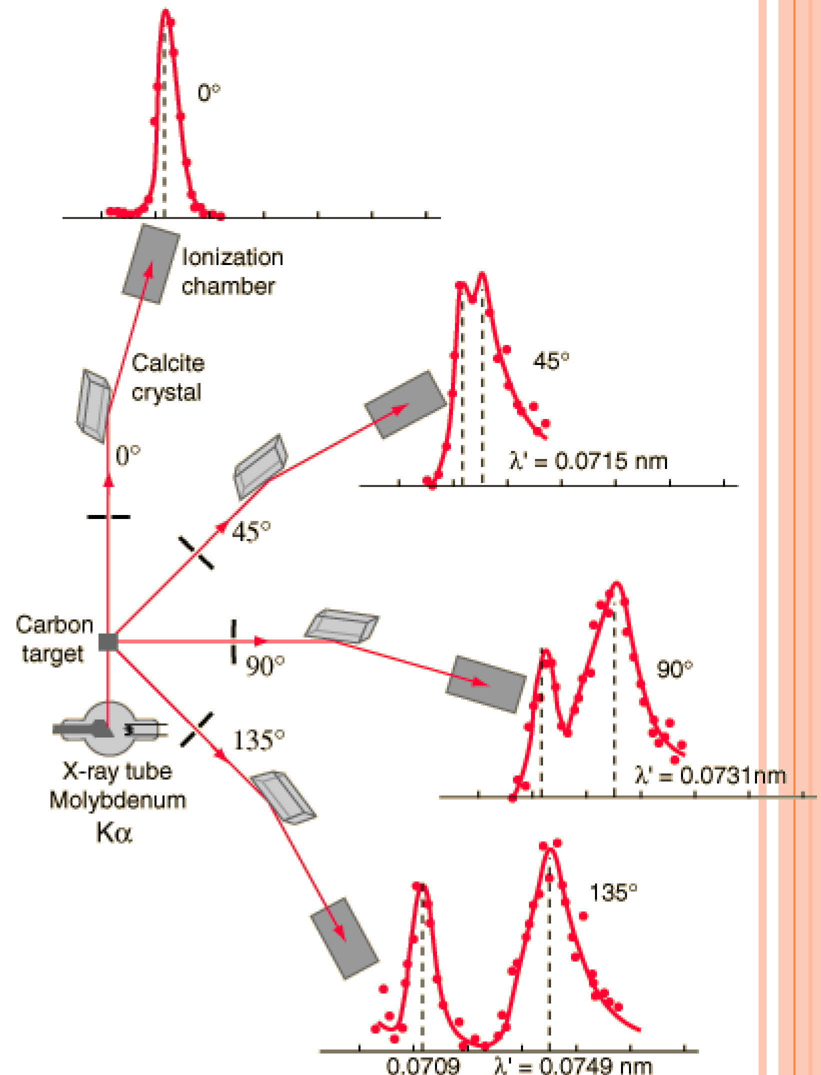
$$\lambda_C = \frac{h}{m_0c} = 0.00243nm$$

gives the scale of the wave length change of the incident photon

Experimental Demonstration







A.H. Compton, *Phys. Rev.* **22** 409 (1923)

Ex-2

High energy photos (γ-rays) are scattered from electrons initially at rest. Assume the photons are backscattered and their energies are much larger than the electron's rest mass energy, $E \gg m_e c^2$.

(a) calculate the wavelength shift,

(b) show that the energy of the scattered photons is half the rest mass energy of the electron, regardless of the energy of the incident photons,

(c) calculate the electrons recoil kinetic energy if the energy of the incident photons is 150 MeV.

ANS: (a) Here $\phi = \pi$, wave length shift or Compton shift:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\phi) = \frac{2h}{m_e c} = 4.8\times 10^{-12}\,m$$

(b) Energy of scattered photon E':

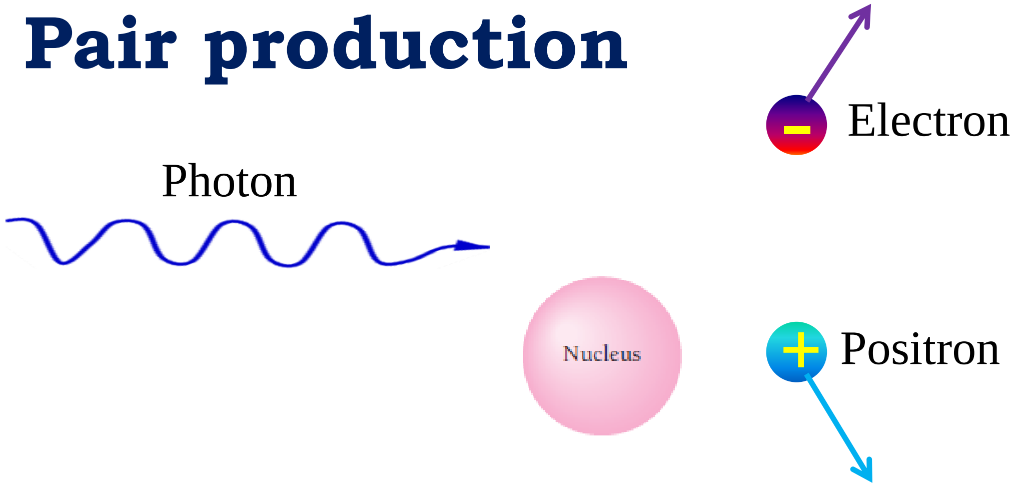
$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda + 2h/(m_e c)} = \frac{m_e c^2}{m_e c^2 \lambda/(hc) + 2} = \frac{m_e c^2}{m_e c^2/E + 2}$$

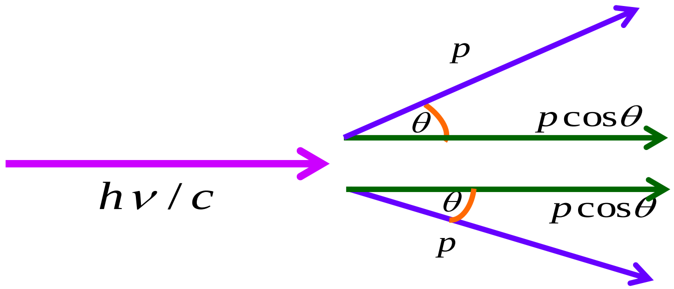
If $E \gg m_e c^2$ we can approximate by

$$E' = \frac{m_e c^2}{2}\left[1 + \frac{2m_e c^2}{E}\right]^{-1} \approx \frac{m_e c^2}{2} - \frac{(m_e c^2)^2}{E} \approx \frac{m_e c^2}{2} = 0.25\,MeV$$

(c) Kinetic energy of recoil electron: $K_e = E - E' \approx$ 150 MeV-0.25 MeV = 149.75 MeV

Pair production





→ In a collision a photon can give an electron all of its energy (the photoelectric effect) or only part (the Compton effect).

→ It is also possible for a photon to materialize into an electron and a positron, which is a positively charged electron.

→ In this process, called **pair production,** electromagnetic energy is converted into matter

Energy and linear momentum could not both be conserved if pair production were to occur in empty space, so it does not occur there

Pair production requires a photon energy of at least 1.02 MeV.

e^- or e^+

$m_0c^2 = 0.51$ MeV (rest mass energy),

$\Rightarrow$ additional photon energy becomes K.E of e^- and e^+.

The pair production : direct consequences of the Einstein mass-energy relation; $E = mc^2$.

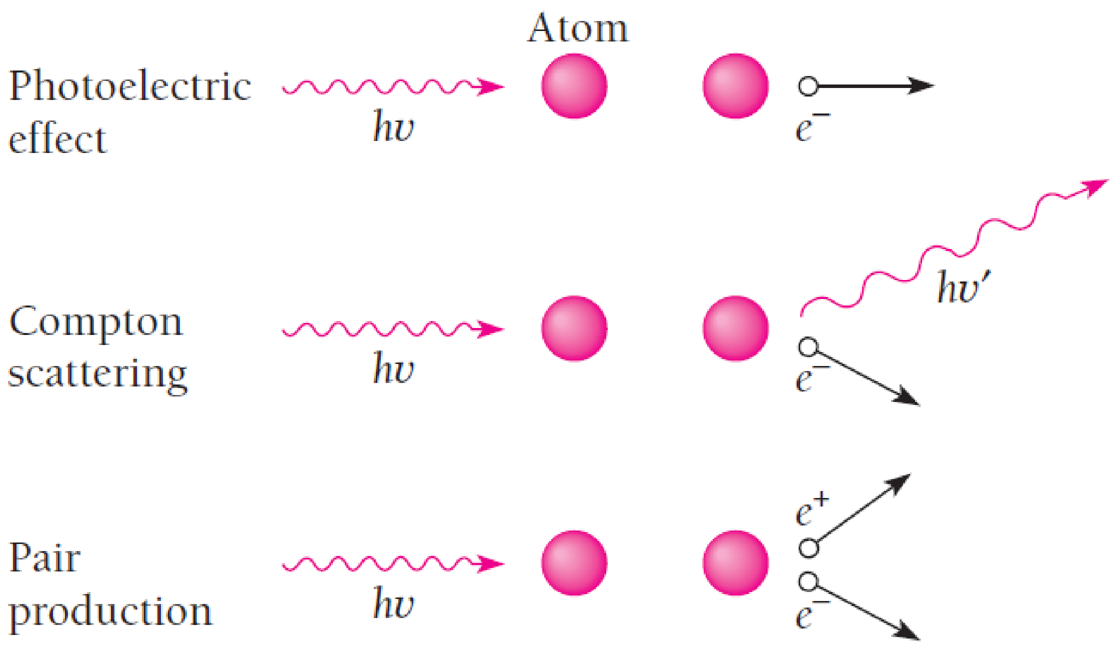
Pair annihilation: $e^- + e^+ \rightarrow \gamma + \gamma$

→ The total mass of the positron and electron is equivalent to 1.02 MeV, and each photon has an energy $h\nu$ of 0.51 MeV plus half the kinetic energy of the particles relative to their center of mass.

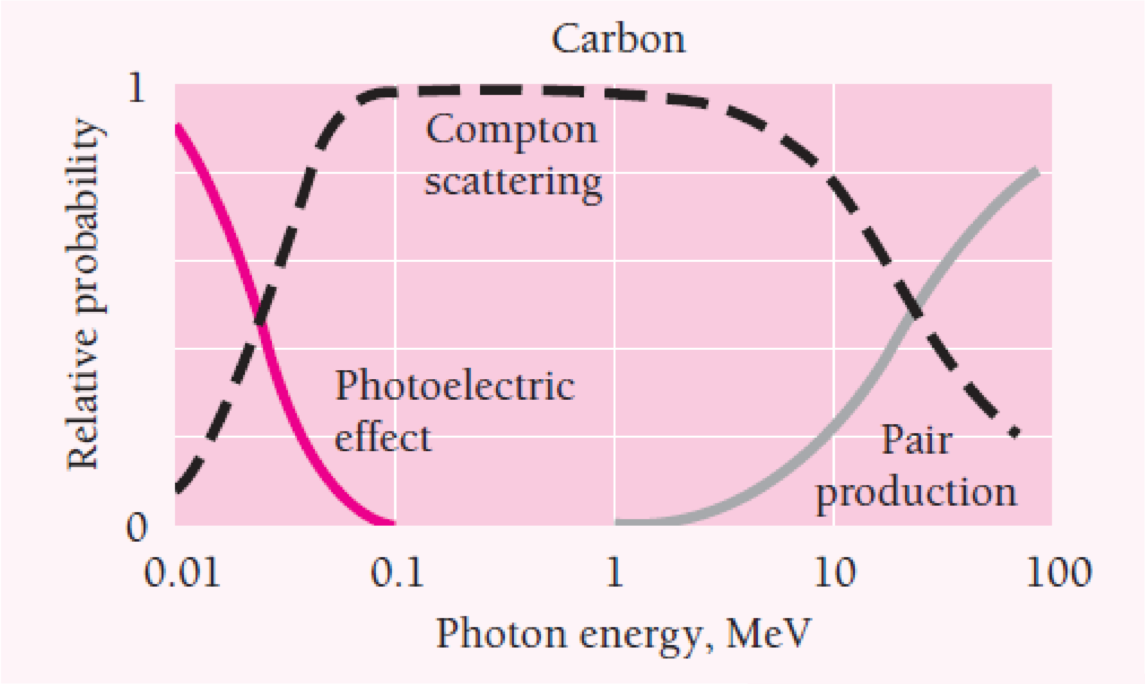
→ The directions of the photons are such as to conserve both energy and linear momentum, and no nucleus or other particle is needed for this **pair annihilation** to take place.

Photon Absorption

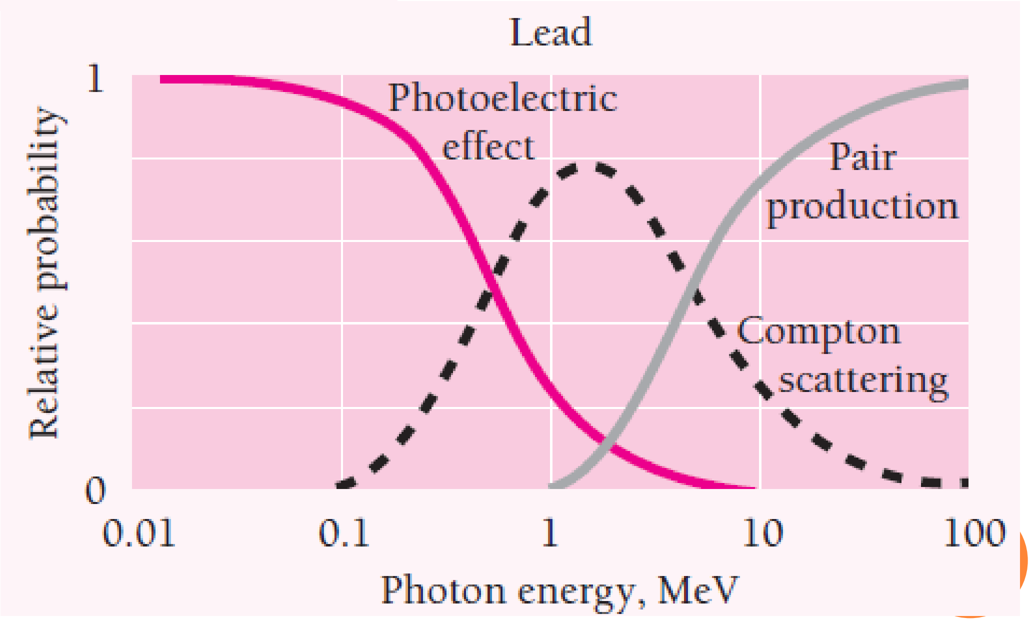
The three chief ways in which photons of light, x-rays, and gamma rays interact with matter. In all cases photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material.



At low photon energies: the photoelectric effect The greater the atomic number of the absorber, the higher the energy at which the photoelectric effect remains significant.



In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of keV, whereas in the heavier ones this does not happen until photon energies of nearly 1 MeV are reached



The intensity I of an x- or gamma-ray beam is equal to the rate at which it transports energy per unit cross-sectional area of the beam. The fractional energy $-dI/I$ lost by the beam in passing through a thickness dx of a certain absorber is found to be proportional to dx:

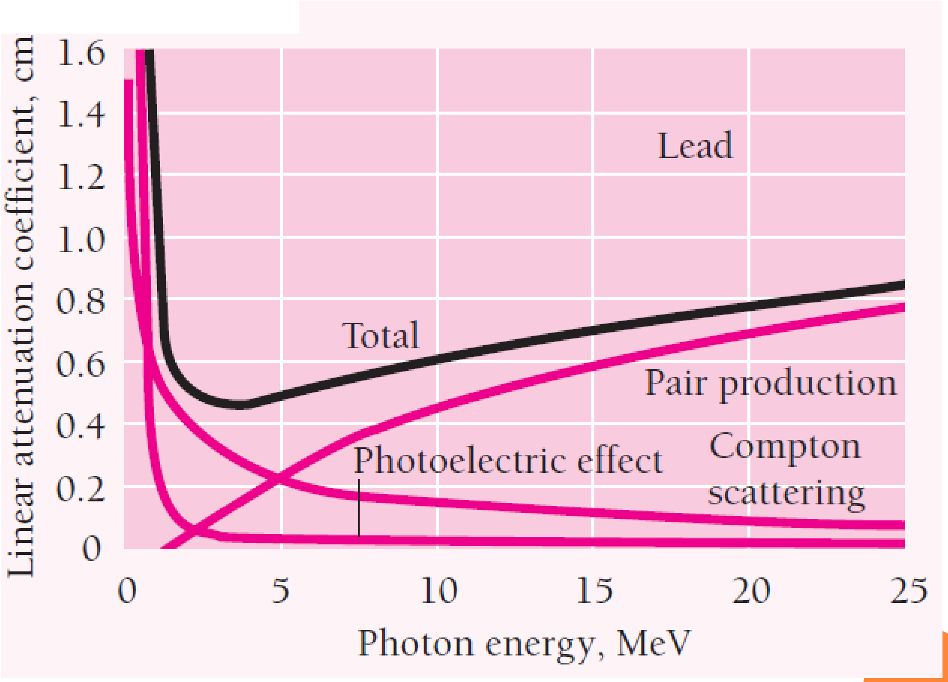
$$-\frac{dI}{I} = \mu\, dx$$

Radiation intensity $$I = I_0 e^{-\mu x}$$

$$\frac{I}{I_0} = e^{-\mu x} \qquad \frac{I_0}{I} = e^{\mu x} \qquad \ln\frac{I_0}{I} = \mu x$$

Absorber thickness

$$x = \frac{\ln (I_0/I)}{\mu}$$



Linear attentuation coefficients for photons in lead.

Example-3

The linear attenuation coefficient for 2.0-MeV gamma rays in water is 4.9 m^{-1}. (*a*) Find the relative intensity of a beam of 2.0-MeV gamma rays after it has passed through 10 cm of water. (*b*) How far must such a beam travel in water before its intensity is reduced to 1 percent of its original value?

(a)

$$\frac{I}{I_0} = e^{-\mu x} = e^{-0.49} = 0.61$$

The intensity of the beam is reduced to 61 percent of its original value after passing through 10 cm of water.

(b)

$$x = \frac{\ln(I_0/I)}{\mu} = \frac{\ln 100}{4.9\ \mathrm{m}^{-1}} = 0.94\ \mathrm{m}$$