Chapter 1: Relativity.

Properties of Frame of Reference:

Here I will discuss and tell you the special theory of relativity in reasonably detail.

First of all, Relativity is a very old theory; it was there even before Einstein came during Newton and Galileo were there.

So, Relativity is not a new idea at all it's an old one and the old ideas can be illustrated as way, that we agree with your own experience.

The standard technique to understand the topic, is to get this high speed train. So here, our hypo-theoretical high speed train.

Things will be moving along the special dimension say X.

You are in this train and all the windows are completely closed (basically you are not allowed to look outside for this experiment), settle down and explore the world around you and you have certain awareness what's happening around you then go for sleep.

Now, so you were sleeping some unseen hand gives to train a large uniform velocity (may be very high). The question is when you wake up, can you tell you are moving or not?

So you might say, I am not moving because this train is not going anywhere and I don't found any different. Claim is nothing is different, you don't know that you are moving or not.

Now, If the train picks up speed or slows down, then you will know right away because you will find yourself being pushed away back of the seat or slam on front of the seat in front of you. No one will say the train is not accelerated, because if you are moving then why I am throwing up. So, once frame accelerate it losses the equal status of accusing each other motion.

Accelerated motion can be detected in closed frame without looking outside but, in case of uniform velocity no matter how high that cannot be detected. So, at the time of Galileo and Newton everybody agreed that you cannot detect it.

Remember, If Newton laws works for the frame of reference, then we call it inertial frame.

So now, if I say everything looks the same I mean the laws of Newton are continued to do the same, because if the laws of Newton remain the same then everything will remain same and continued to do same.

Once the frames are accelerating, then we know imaginary force acts on object and things will become accelerate, that's a non-inertial frame (we are not interested in such frames)

So, claim is the laws of Newton remain unchanged when the uniform velocity is added on to.

Now, we should clear about one thing if there is a train next to you at the beginning and you looked outside and see that train, you would find that train is moving some speed say v

The question is, can you tell that it's you moving or him (who is responsible for such relative motion). And the claim of relativity is that, you cannot tell!!

You could only able to tell is there relative motion between the trains, that wasn't there before (that's very clear if you look outside).

Two uniform relative moving frames cannot tell their actual state of motion (like, you are moving left or I'm moving to right or maybe combination of both) but, only able to say there is relative motion between the frames (That's the word relative)

Acceleration will produce effects that the frames can't get away from it, whereas uniform motion doesn't change the dynamics.

Let me show you that the laws of Newton won't get modify in uniform speed frame…. check out notes.

Galilean transformations... check out notes.

How the existence of Ether medium failed?

After 300 years, people had discovered electricity and magnetism, electromagnetism, Electromagnetic waves which was identified as light travelling in space with certain and that number is $c = 3 \times 10^8$ m/s.

The question was, to whom is this velocity for? For example, you all have studied calculation of wave speed on a string, fluid, etc. Which has to do with its medium elasticity and how dense the medium is.

For instance, waves of sound in room, when I talk to you it delays to get you and beyond, that speed is calculated with respect to air in the room.

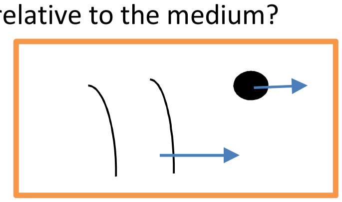
In fact, we all are sitting on the planet with itself is moving with around 1100 miles/hr doesn't matter we still get the same speed of sound, because the air is being carried along with it.

So at that time people wanted to know what is the medium which carries the electromagnetic wave (light).

They realize that, first of all the medium called ether is everywhere, because it travels in space and we can see stars. Then you can sort of ask how dense is the medium, it turns out to be very-very dense to support such wave which travels with incredible velocity!!

Then planets are moving in this medium for years and years and doesn't slows down, it's a very peculiar medium, we all are immersed and drifting around in this medium, because we find light signals from everywhere in our cosmos.

Now the question is, how fast the earth is moving relative to the medium?



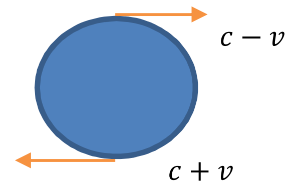
$c' = c - v$; part of the speed gets neutralized.

Above is there expectation at some instance after all they knew cherish Newtonian physics, we should get different than the original speed.

So later, Michaelson and his assistant Morley they did the experiment and got the answer equal to c. What is that mean?

You might expect you are probably not moving with respect to medium while performing experiment, that's incredible fortunate!! But we know that luck doesn't last forever, we are moving around the sun

6 months later, it's definitely sure you moving relative to the medium.



So people tried other solutions, they tried to argue that earth carries ether medium with it the way it carries the air with it, because then the speed of earth doesn't matter you will keep getting the value c.

But it is very to show from terrestrial experiments with distant star that things cannot carry ether along with it.

So you cannot take ether with you and also cannot leave it, that's impasse the people were in. So they were trying every models, but none of them worked. Nobody know why light is behaving in such a peculiar fashion.

Then, Einstein came and rescues us from that big problem; he stated that *I know why light is behaving in such a peculiar fashion*.

Because, if it doesn't behave in this way then the speed of light will depend on how fast frames are moving, then after I woke up the only thing I need to do is just determine the speed of light in closed uniformly moving inertial frame and take the difference with original value, then you know how fast is the inertial frame is moving even without looking outside. Thus shows that uniformly moving frame shows the observable changes, which in practice they don't

But conversely, the fact we keep getting the same answer means that electric and magnetic fields are the part of the conspiracy to hide the absolute motion of inertial frames. Just like Mechanical phenomenon won't tell you how fast inertial frames are moving, neither will electromagnetic phenomena will tell you.

Because, Einstein was very obvious that nature won't design such system in which mechanical laws will follow relativity, but laws of electromagnetism are don't. So, he postulated.

Postulates of Special Relativity:

If light doesn't behave in this way, then it will violate the principle of relativity.

1. It's just simply a restatement of relativity principle and generalization made by Einstein.
   All inertial observers are equivalent with respect to all natural phenomenon.
   In layman terms: All natural phenomenon would be unaffected from one inertial to another inertial frame. (That's a very brave postulate). Shows complete symmetry between observers in uniform relative motion.
   The intuition for this notion is that we have some faith that the underlining laws and principles have some certain elegance, beauty and uniformity (It's a credit by all scientist and physicist).
2. Velocity of light is *independent* of state of motion of source/observer.
   Example: light is observed inside the rocket or outside the rocket, doesn't matter.

Now, it looks we solved a big problem!! But, it seems we did a terrible bargain.

Because once you take this postulates you restored the relativity principle, you got the mechanical phenomena. But, you find to give-up all the cherish notions of the Newtonian physics.

So, now we need new rules which connects the (x,t) and (x',t') such that postulates are compatible with it. Form of Newton laws will also get modified means it will just different, but the results which you and I measure will be same.

Lorentz Transformation: (new rule)

So, now what it really makes it to show such peculiar behavior? The answer is now we don't agree with our meter sticks and clocks!!

The notion of length is length and time is time in classical physics no longer hold. Certainly at instance our meter sticks and clocks agrees, but changes as we produced in moving inertial frame.

Then, Lorentz Transformations…. Quick minute derivation…. (check out notes)

$$x' = \frac{x-ut}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \qquad t' = \frac{t-\frac{u}{c^2}x}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

Lorentz transformation deduce to the Galilean transformation…. (Check out notes).

You, can see that theory won't admit that velocity greater than light. (we can see that from formula)

Formula also tells you that, one single velocity that you wanted to be same for everyone is also the greatest possible velocity that no observer can move with speed equal or greater than the speed of light with respect to another observer.

So, now what is the meaning that Lorentz transformation tells you or what that means to you? The answer is they connect the co-ordinates of two different observers respect to their own origin monitoring an event happening. Basically, show the relation between the co-ordinates of two observers.

Recipe to solve any problem of Relativity:

Whenever you get a problem in relativity take pair of events, you will get success. They can be totally dependent or independent, doesn't matter.

$$Event\ 1; \qquad x_1' = \frac{x_1 - ut_1}{\sqrt{1-\beta^2}} \ \ ; \ t'_1 = \frac{t_1 - \frac{u}{c^2}x_1}{\sqrt{1-\beta^2}}$$

$$Event\ 2; \qquad x_2' = \frac{x_2 - ut_2}{\sqrt{1-\beta^2}} \ \ ; \ t'_2 = \frac{t_2 - \frac{u}{c^2}x_2}{\sqrt{1-\beta^2}}$$

$$\Delta x' = x'_2 - x'_1 \ \ ; \quad \Delta t' = t'_2 - t'_1$$

$$\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-\beta^2}} \qquad \Delta t' = \frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-\beta^2}}$$

$$\Delta x = \frac{\Delta x' + u\Delta t'}{\sqrt{1-\beta^2}} \qquad \Delta t = \frac{\Delta t' + \frac{u}{c^2}\Delta x'}{\sqrt{1-\beta^2}}$$

These shows the difference in co-ordinates that also obeys the Lorentz Transformations. So using the above four relations get the results what you want?

What are the Implications: Because the equations are dramatic variation in Newton laws? Example, distance and time interval between pair of events are different for every different observer.

Implications of Lorentz transformation:

FIRST IMPLICATION:

Let's, consider two pair of events as follows,

EVENT 1: Fire the gun; EVENT 2: Bullet hits the wall.

$v =$ velocity of bullet for me (S) = $\frac{\Delta x}{\Delta t}$

$w =$ velocity of bullet for you (S') = $\frac{\Delta x'}{\Delta t'}$

$$w = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x - u\Delta t}{\Delta t - \frac{u}{c^2}\Delta x} = \frac{\frac{\Delta x}{\Delta t} - u}{1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}} = \frac{v-u}{1-\frac{uv}{c^2}}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta x' + u\Delta t'}{\Delta t' + \frac{u}{c^2}\Delta x'} = \frac{\frac{\Delta x'}{\Delta t'} + u}{1 + \frac{u}{c^2}\frac{\Delta x'}{\Delta t'}} = \frac{w+u}{1+\frac{uw}{c^2}}$$

The above results are called as Relativistic addition of speeds. Let's check the strength of the result.

Now, we will check the preposition that according to theory of special relativity nothing can move faster than speed of light.

You can try to beat that system as follows, you can fire a bullet with speed $\frac{3}{4}c$ in a moving train also with speed $\frac{3}{4}c$ which is certainly possible.

Our classical expectations are 1.5c!! something dramatic, but we know from above results the value would be $\frac{24}{25}c\ (< c)$. It is built in the formula that denominator will make things won't go crazy.

This show why speed shouldn't add linearly, if they do things can be dramatic, otherwise you can achieve whatever the speed you like. It takes the consideration the fact relative speeds should be less than c.

Now let's apply it to light pulse,

$$w = \frac{c-u}{1-\frac{uc}{c^2}} = \frac{c(c-u)}{(c-u)} = c$$

Rest implications continues in another pdf called derivation of Lorentz Transformation, Time Dilation and Length contraction.... sorry for inconvenience

Simultaneity: (Again very staggering)

Simultaneity is a relative concept, in other words if two pairs events happens at same time for me, but it will be different for you!!

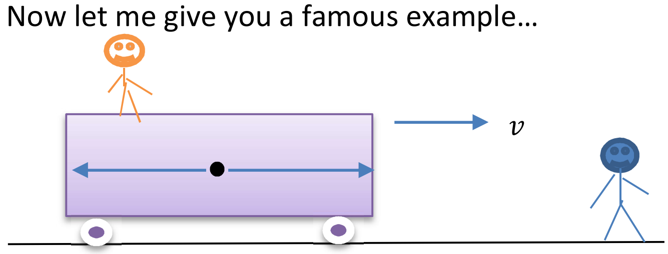
$$\Delta t' = \frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-\frac{u^2}{c^2}}}$$

Since, $\Delta t = 0\ and\ \Delta x \neq 0\ shows\ that\ \Delta t' \neq 0$

If; $u \ll c \Longrightarrow \Delta t' = \Delta t;\ \ Pre-Einstein\ world$

That's why in Relativity space-time mixes to give another space-time, this show why time deserves to be a fourth co-ordinate in Special Relativity unlikely in Newtonian physics where time is time for everyone doesn't deserves to be a fourth co-ordinate.

Now let me give you a famous example…



There are two observers one on ground another on moving train, here's the light beam splitter in middle of the train which shoots two opposite light beam simultaneously hits the explosive and boom.

Claim is, the observer on train will see both explosive simultaneously but, the observer on ground things will not be simultaneous no matter how the ideal is the beam splitter. The observer on ground will always see that left one explode first than the right one, because speed of light is same for every observer.

Note: Understand the fact that, with the two events occurs at the same time and same place, meaning

$$\Delta x = \Delta t = 0$$

Then the transformation relation will tell you that;

$$\Delta x' = \Delta t' = 0$$

Example: collisions, accidents, etc. That's why we say that the poor guy was at the wrong place and wrong time. We never say that guy was at the wrong place or wrong time, that doesn't mean anything.

That means something had happened in space-time like collisions than, no any observer can deny the happening of collision each and every observer will agree except few politicians.

Muon Problem:

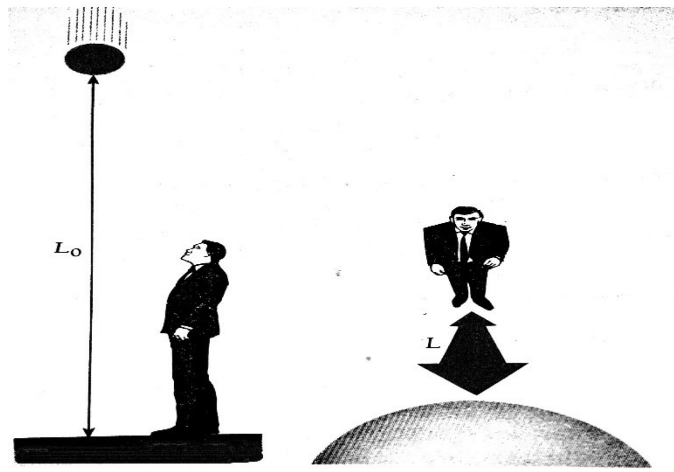
Intro. of Muon: Muon is an elementary particle, sibling of electron with an electric charge $-e$ and spin ½ , but with a mass about 207-times that of an electron and it decays into an electron or a positron (anti-matter of electron) after average lifetime of $2.2\mu s$ after formation of muon.

Problem: Cosmic-ray muons have speed of about 0.998c and one of them reaches earth sea-level passes through each level of atmosphere before getting decay into electron, but the fact that $t_0 = 2.2\mu s$ their average time with given speed of 0.998c it can travel only up to 0.66-km!! and surprisingly they actually travel 6km or more and also width of atmosphere is about 10km. What is that mean? How it could happen? What makes it to travel such distance before getting decay with 0998c speed with respect to earth?

Solution: $t_0 = 2.2\mu s$ is lifetime in muon frame, but for us on earth frame we will see that muon moving with 0.998c speed which indicates for us lifetime for muon will be more than original, *since clocks are faster in own frame*. Therefore for us time dilate to $t = 34.8\mu s$.

The moving muon having lifetime 16-times longer than those at rest. In that time interval it will cover the distance of 10.4km, problem solved.

Now, let's see another way to look at it. If somebody were to accompany a muon in its descent speed 0.998c, so he will found muon at rest and the earth moving toward him at speed 0.998c



As found by observer on the ground, the muon altitude is L_0.

As found by an observer moving with the muon, the ground is L below it, which is a shorter distance than L_0.

For that observer the lifetime of muon will be $t_0 = 2.2\mu s$ and muon can travel up to 0.66km.

So, that can happen because the observer sees the atmosphere about 0.66km due to length contraction, *since lengths are larger in their own frame*.

Twin paradox:

If one of a pair of twins makes a long journey at near the speed of light and then returns, he or she will have aged less than the twin who remains behind!!

So, how could it be possible? Our expectations were that their ages would be same, because they both were moving relative to each other, so each of them has equivalence to accuse each other that your clocks are slower than mine.

Solution: One of a pair of twins specially the one who makes a long journey has actually lost the status of equivalence, because to return back he has to change the inertial frame whereas the one who stayed on earth was on a single inertial frame throughout the whole course of event. In nutshell two different frames was involved in case of twin which a long journey.