Course: PHYSICS-I (PH1001)
3 Credits (2-1-0)

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TOPICS

- **A. Beiser, Concept of Modern Physics , Tata-McGraw Hill**

  Ch-1: Relativity

  Ch-2: Particle properties of waves

  Ch-3: Wave properties of particles

  Ch-5: Quantum Mechanics

- **Robert Resnik, Introduction to Special Relativity, John Wiley & Sons(1979)**

Relativity: Galilean relativity and Galilean transformation, Special relativity, Michelson Morley experiment and postulates of relativity, length contraction and time dilatation, Doppler effect, Lorentz transformation & velocity addition, relativistic momentum, mass-energy relation

Quantum Mechanics: INADEQUACIES IN CLASSICAL PHYSICS: Black body radiation, photoelectric effect, X-ray diffraction, Compton Effect, pair production

Davisson-Germer experiment WAVE-PARTICLE DUALITY: Particle nature of wave, Wave nature of particle, de Broglie waves, group waves, phase velocity & group velocity, uncertainty principle and its application.

WAVE FUNCTION: probability & wave equation, linearity and superposition of wave of wave functions, expectation values SCHRÖDINGER EQUATION: time dependent and time independent SE, eigenvalue & eigenfunctions, boundary conditions on wave function,

APPLICATION OF SE: Particle in a box, Finite potential Well (optional).

Special Theory of Relativity

Classical and Modern Physics

Classical Physics -
Larger, slow moving

- Newtonian Mechanics
- EM and Waves
- Thermodynamics

Modern Physics-

- Relativity – Fast moving objects
- Quantum Mechanics – very small

Speed

10% c ↑

Classical	Relativistic

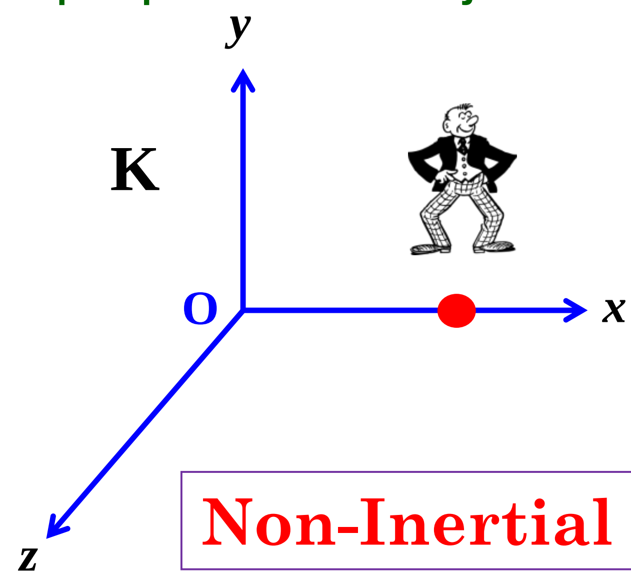
Size

Atomic/molecular size ↓

Quantum	Classical

Reference frames

A **frame of reference** in physics, may refer to a *coordinate system* or set of axes within which to measure the position, orientation, and other properties of objects in it



Inertial frames

- **in which no accelerations are observed in the absence of external forces**
- **that is not accelerating.**
- **Newton's laws hold in all inertial reference frames**

Non-Inertial frames

- **that is accelerating with respect to an inertial reference frame**
- **bodies have acceleration in the absence of applied forces**

Postulates of Special Relativity

The postulates of relativity as stated by Einstein☹1905)

1. <u>Equivalence of Physical Laws</u>

   The laws of physics are the same in all inertial frames of reference.

2. <u>Constancy of the Speed of Light</u>

   The speed of light in a vacuum, $c = 3.00 \times 10^8$ m/s, is the same in all inertial frames of reference, independent of the motion of the source or the receiver.

Before Einstein's work, a conflict had existed between the principles of mechanics, which were then based on Newton's laws of motion, and those of electricity and magnetism, which had been developed into a unified theory by Maxwell.

Einstein showed that Maxwell's theory is consistent with special relativity

At higher speeds Newtonian mechanics fails and must be replaced by the relativistic version

Searching for an absolute Reference System

- Ether was proposed as an absolute reference system in which the speed of light was constant and from which other measurements could be made.

- The Michelson-Morley experiment was an attempt to show the existence of ether

Michelson-Morley Experiment

Experiment designed to measure small changes in the speed of light was performed by A. Michelson (1852-1931, Nobel) and Edward W. Morley(1838-1923)

"Motion of earth through the ether"

"ether": *a hypothetical medium pervading the universe in which light waves were supposed to occur.*

- Used an optical instrument called an interferometer that Michelson invented
- Device was to detect the presence of the ether

Sorry!!!!!

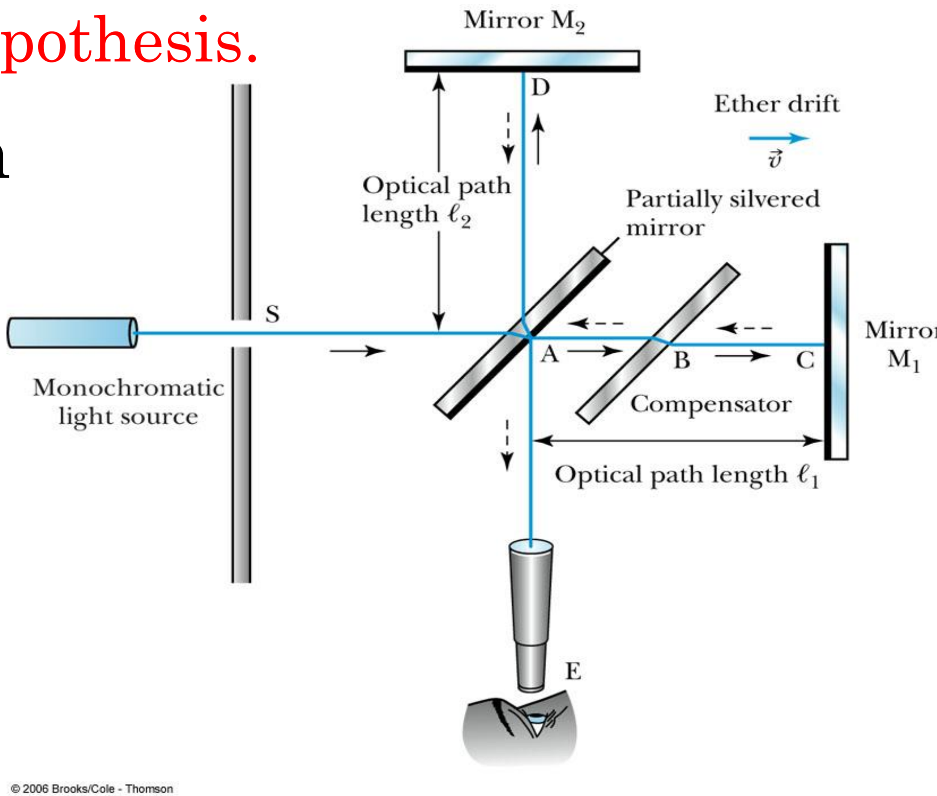
Outcome of the experiment was negative

A. A. Michelson and E. W. Morley, American Journal of Science, 134, 333 (1887)

Experimental setup

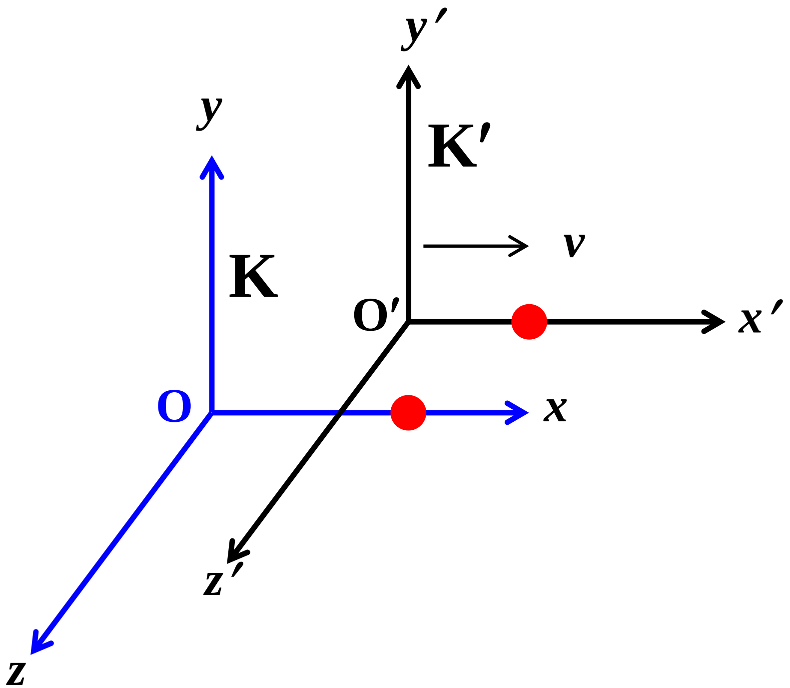
contradicting the "ether" hypothesis.

Michelson's Conclusion



- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He thus concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist!***

Galilean Transformation

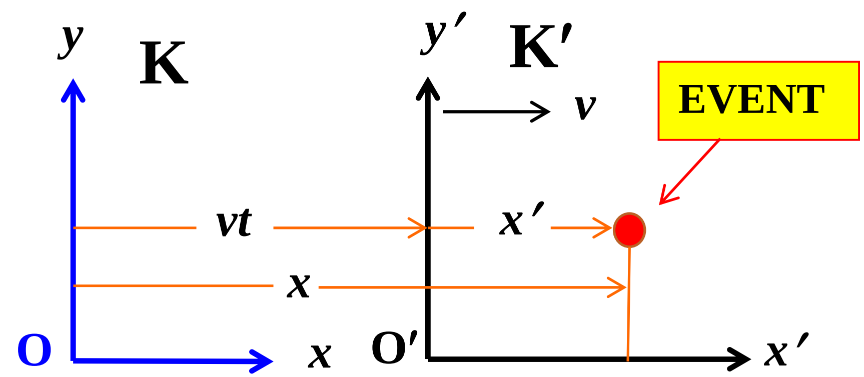


$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t = t$$



Time is absolute

The Galilean transformation is the commonsense relationship which agrees with our everyday experience

Velocity component

$$\boxed{\begin{aligned} v'_x &= \frac{dx'}{dt'} = v_x - v \\ v'_y &= \frac{dy'}{dt'} = v_y \\ v'_z &= \frac{dz'}{dt'} = v_z \end{aligned}}$$

Drawbacks:

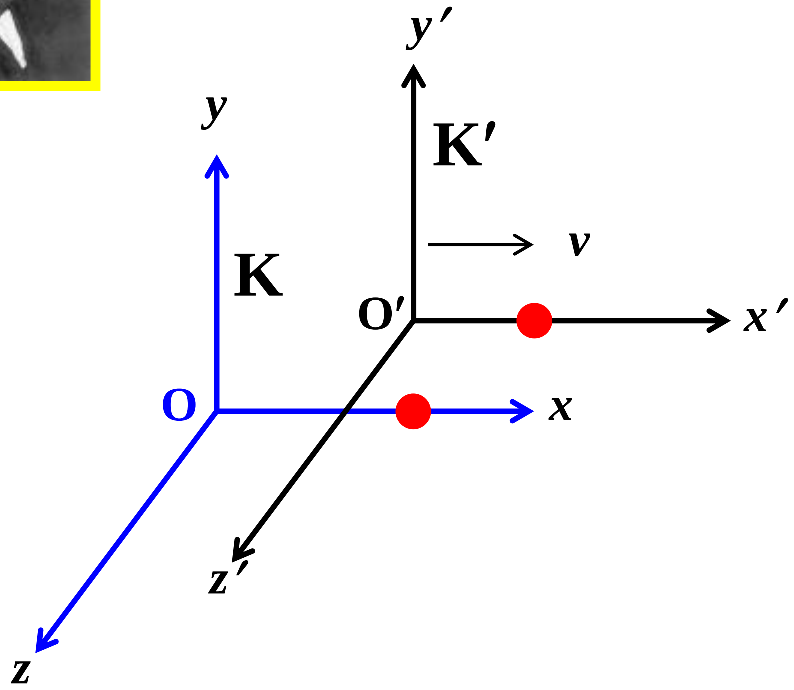
1. Violates both of the postulate of special theory of relativity

[i] Same equations of physics in K and K′, but the equations of electricity and magnetism is entirely different.

[ii] $c' = c - v$

⇒ Go for different transformation

LORENTZ TRANSFORMATION



$$\boxed{x' = k(x - vt)}$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \dfrac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

A more symmetric form:

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x' = \gamma(x - \beta ct)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \beta x/c)$$

- Basic formulas of electromagnetism are the same in all inertial frames

Velocity Addition

The light emitted from the K′ in the direction of its motion relative to another frame K ought to have a speed of c + *v* as measured in K.

***violets** the postulate of relativity*

Common sense is no more reliable as a guide in science than it is elsewhere

Suppose something is moving relative to both K and K′ . An observer in K measures its three velocity components to be

$$V_x = \frac{dx}{dt} \qquad V_y = \frac{dy}{dt} \qquad V_z = \frac{dz}{dt}$$

While to an observer

$$V_x' = \frac{dx'}{dt'} \qquad V_y' = \frac{dy'}{dt'} \qquad V_z' = \frac{dz'}{dt'}$$

By differentiating the inverse Lorentz transformation equations for x, y, z and t, we have

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}} \qquad dy = dy' \qquad dz = dz' \qquad dt = \frac{dt' + \frac{v\,dx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

and so $$V_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v\,dx'}{c^2}} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

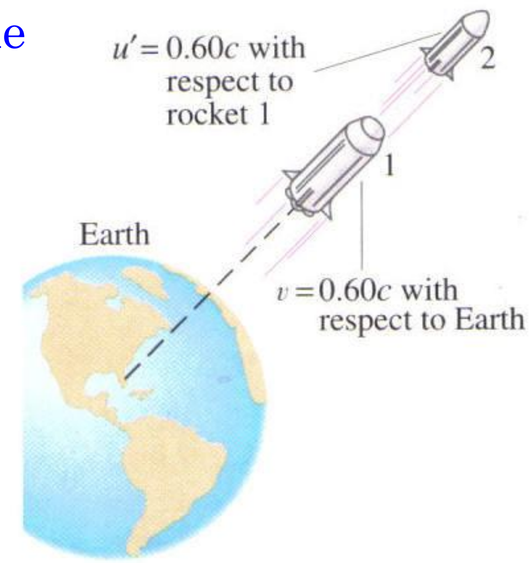
Relativistic velocity transformation

$$\boxed{V_x = \frac{V'_x + v}{1 + \frac{vV'_x}{c^2}}} \qquad \boxed{V_y = \frac{V'_y\sqrt{1 - v^2/c^2}}{1 + \frac{vV'_x}{c^2}}} \qquad \boxed{V_z = \frac{V'_z\sqrt{1 - v^2/c^2}}{1 + \frac{vV'_x}{c^2}}}$$

If $V'_x = c$, if the light is emitted in the moving frame K′ in its direction of motion relative to K, an observer in frame K will measure the speed:

$$\boxed{V_x = \frac{V'_x + v}{1 + \frac{vV'_x}{c^2}} = \frac{c + v}{1 + \frac{vc}{c^2}} = \frac{c(c + v)}{c + v} = c}$$

Example-1. What is the speed of the second stage of the rocket shown with respect to the earth?



Ans. $$\mathbf{u = \frac{v + u'}{1 + vu'/c^2}}$$

$$= \frac{0.60c + 0.60c}{1 + [(0.60c)(0.60c)/c^2]}$$

$$\mathbf{u = 0.88\ c}$$

(classical addition would give you 1.20c, over the speed of light)

2. Suppose a car travelling at 0.60c turns on its headlights. What is the speed of the light travelling out from the car?

$$\mathbf{u = \frac{v + u'}{1 + vu'/c^2}}$$

$$u = \frac{0.60c + c}{1 + [(0.60c)(c)/c^2]} = \frac{1.60c}{1.60}$$

$$\mathbf{u = c}$$

3. Now the car is travelling at c and turns on its headlights.

$$\mathbf{u = \frac{v + u'}{1 + vu'/c^2}}$$

$$u = \frac{c + c}{1 + [(c)(c)/c^2]} = \frac{2c}{2}$$

$$\mathbf{u = c}$$

Example-4

Spacecraft Alpha is moving at 0.90*c* with respect to the earth. If spacecraft Beta is to pass Alpha at a relative speed of 0.50*c* in the same direction, what speed must Beta have with respect to the earth?

Ans.

According to the Galilean transformation, Beta would need a speed relative to the earth of 0.90*c* + 0.50*c* = 1.40*c*, which we know is impossible. According to lorentz, however, with $V_x = 0.50c$ and $v = 0.90c$, the required speed is only

$$V_x = \frac{V_x' + v}{1 + \frac{vV_x'}{c^2}} = \frac{0.50c + 0.90c}{1 + \frac{(0.50c)(0.90c)}{c^2}} = 0.97c$$

which is less than *c*. It is necessary to go less than 10 percent faster than a spacecraft traveling at 0.90*c* in order to pass it at a relative speed of 0.50*c*.

Time Dilation

✓ to dilate is to become larger

☞ **A moving clock ticks more slowly than a clock at rest**

Actual difference of elapsed time between two events as measured by observers moving relative to each other

Time where clock is **moving** relative to the observer

Time where clock is **at rest** relative to the observer. **Proper time**

$$t = \frac{t_0}{\sqrt{1 - v^2 / c^2}}$$

Clocks moving relative to an observer are measured by that observer to run more slowly, as compared to the clock at rest.

- This effect arises neither from technical aspects of the clocks nor from the fact that signals need time to propagate, but from the nature of space-time itself.

❖ every observer finds that clocks in motion relative to him tick more slowly than clocks at rest relative to him

Experimental verification

• Time Dilation and Muon Decay

Muon Decay

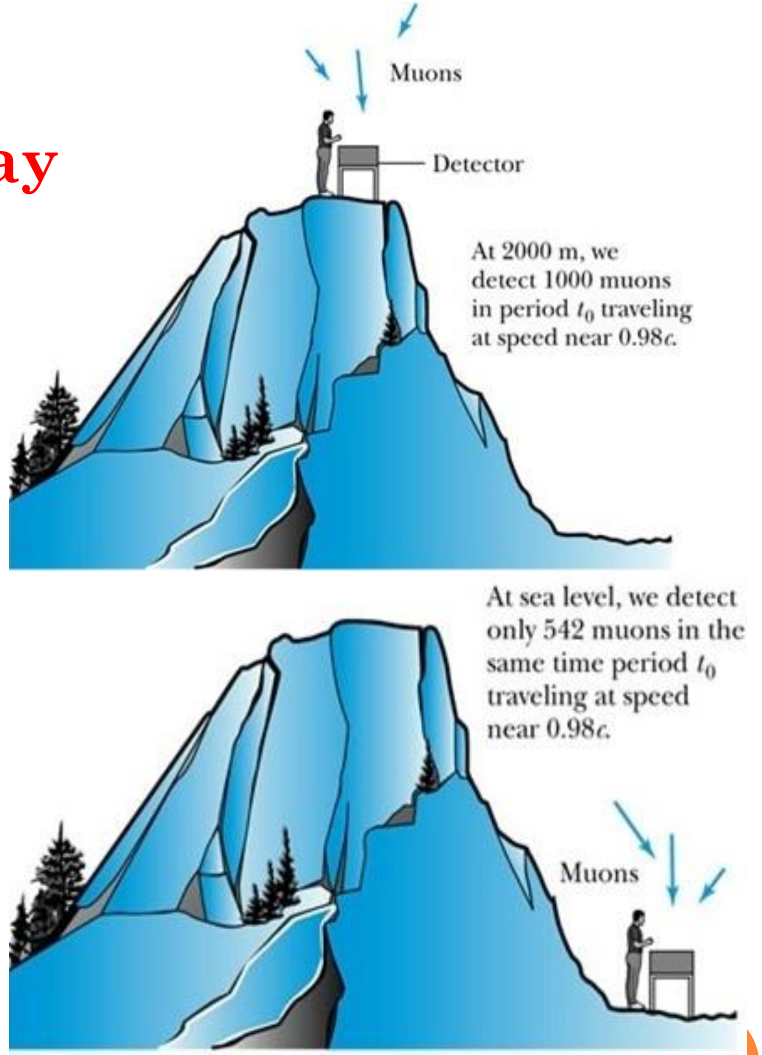
- Cosmic rays enter the upper atmosphere and interact with particles in the upper atmosphere creating π mesons (pions), decay into other particles called muons
- Obey radioactive law:

$$\boxed{N = N_0 e^{-(0.693t/t_{1/2})}}$$

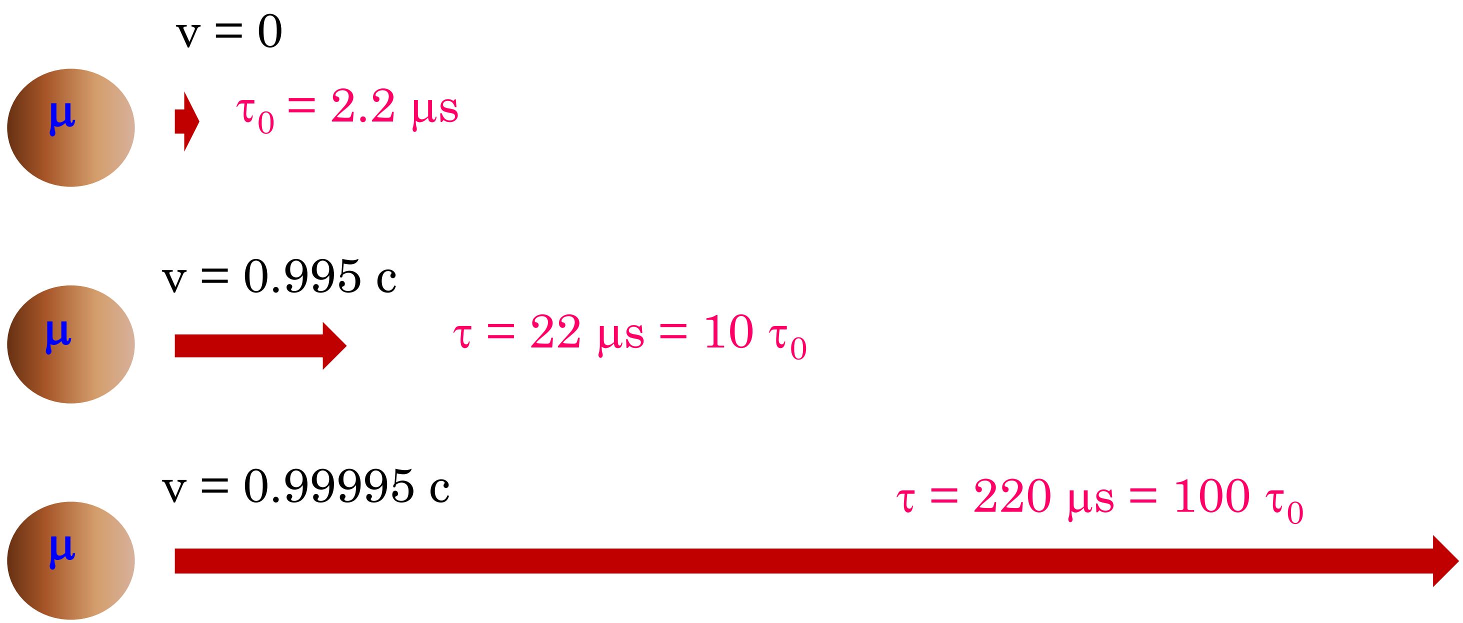
N: No. of muons at t

N_o :No. of muons at t=0

Half life: $t_{1/2}$ =1.5 x 10^{-6} sec



The mean lifetime of a muon in its own reference frame, called the proper life time, is $\tau_0 = 2.2\ \mu s$. In a frame moving at velocity v with respect to that proper frame, the lifetime is $\tau = \gamma\tau_0$, where γ is the time dilation factor.



Mean lifetime τ as measured in laboratory frame

A clock in a moving frame will be seen to be running slow, or "dilated" according to the Lorentz transformation. The time will always be shortest as measured in its rest frame.

Example-5: Derive the formula for time dilation using the inverse Lorentz transformation

Let us consider a clock at the point x' in the moving frame S'. When an observer in S' finds that the time is t'_1, an observer in S will find it to be t_1,

$$t_1 = \frac{t'_1 + \dfrac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

After a time interval of t_0 (to him), the observer in the moving system finds that the time is now t'_2 according to his clock. That is,

$$t_0 = t'_2 - t'_1$$

The observer in S, however, measures the end of the same time interval to be

$$t_2 = \frac{t'_2 + \dfrac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

so to her the duration of the interval t is

$$t = t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1 - v^2/c^2}} = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

Example

6. What is the lifetime of a muon travelling at 0.60 c (1.8 X 10^8 m/s) if its rest lifetime is 2.2 μs?

Ans. $$t = \frac{t_0}{\sqrt{1 - v^2 / c^2}}$$ $$t = \frac{2.2 \times 10^{-6} s}{\sqrt{1 - (0.6c)^2 / c^2}} = 2.8 \times 10^{-6} s$$

7. How long will a 100 year trip (as observed from earth) seem to the astronaut who is travelling at 0.99 c?

Ans. $$t = \frac{t_0}{\sqrt{1 - v^2 / c^2}} \longrightarrow$$ 14 year (check it)

8. A particle travels at 1.90×10^8 m/s and lives 2.10×10^{-8} s when at rest relative to an observer. How long does the particle live as viewed in the laboratory?

Ans:

$$t = \frac{t_0}{\sqrt{1 - v^2 / c^2}} = \frac{2.10 \times 10^{-8} s}{\sqrt{1 - \frac{\left(1.9 \times 10^8 s\right)^2}{\left(3.0 \times 10^8 s\right)^2}}} = 2.71 \times 10^{-8} s$$

9. A spacecraft is moving relative to the earth. An observer on the earth finds that, between 1 P.M. and 2 P.M. according to her clock, 3601 s elapse on the spacecraft's clock. What is the spacecraft's speed relative to the earth?

Here t_o = 3600 s is the proper time interval on the earth and t = 3601 s is the time interval in the moving frame as measured from the earth. Then

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{t_0}{t}\right)^2$$

$$v = c\sqrt{1 - \left(\frac{t_0}{t}\right)^2} = (2.998 \times 10^8 \text{ m/s})\sqrt{1 - \left(\frac{3600 \text{ s}}{3601 \text{ s}}\right)^2}$$
$$= 7.1 \times 10^6 \text{ m/s}$$

APPOLLO 11

Today's spacecraft are much slower than this. For instance, the highest speed of the Apollo 11 spacecraft that went to the moon was only 10,840 m/s, and its clocks differed from those on the earth by less than one part in 10^9.

Most of the experiments that have confirmed time dilation made use of unstable nuclei and elementary particles which readily attain speeds not far from that of light.

Length Contraction

Faster means shorter

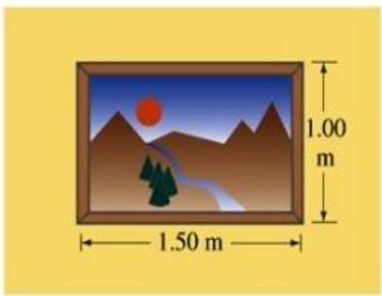
Length where observer is **moving** relative to the length being measured.

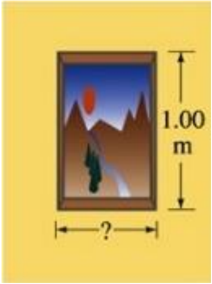
Length where observer is **at rest** relative to the length being measured.

$$L = L_0\sqrt{1-v^2/c^2}$$

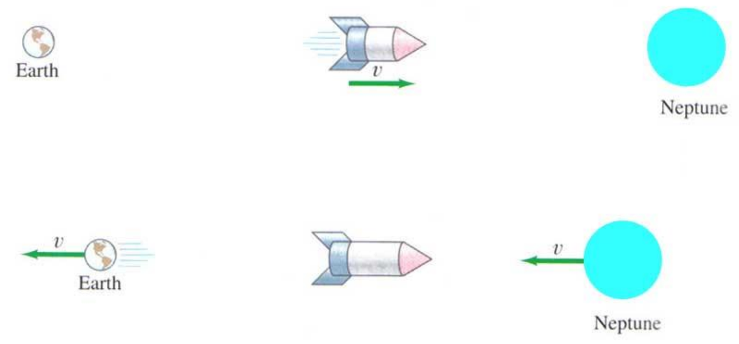
The length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest.

<u>**Contraction occurs only in the direction of the relative motion**</u>





Observers from earth would see a spaceship shorten in the length of travel

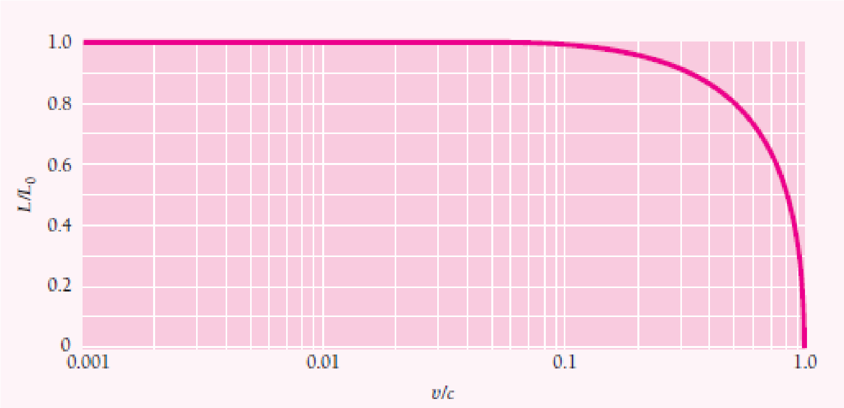


Only shortens in the direction of travel

Length Contraction

$$L = L_0\sqrt{1 - v^2 / c^2}$$

Clearly the length contraction is most significant at speeds near that of light.



A speed of 1000 km/s seems fast to us, but it only results in a shortening in the direction of motion to 99.9994 percent of the proper length of an object moving at this speed.
On the other hand, something traveling at nine-tenths the speed of light is shortened to 44 percent of its proper length, a significant change.

Like time dilation, the length contraction is a reciprocal effect.

To a person in a spacecraft, objects on the earth appear shorter than they did when he or she was on the ground by the same factor of $\sqrt{1 - v^2/c^2}$ that the spacecraft appears shorter to somebody at rest.

The proper length L_0 found in the rest frame is the maximum length any observer will measure.

Example-10: Derive the relativistic length contraction using the Lorentz transformation.

Ans

Let us consider a rod lying along the *x* axis in the moving frame *S*. An observer in this frame determines the coordinates of its ends to be x'_1 and x'_2, and so the proper length of the rod is

$$L_0 = x'_2 - x'_1$$

In order to find $L = x_2 - x_1$, the length of the rod as measured in the stationary frame *S* at the time *t*,

$$x'_1 = \frac{x_1 - vt}{\sqrt{1 - v^2 / c^2}} \; and x'_2 = \frac{x_2 - vt}{\sqrt{1 - v^2 / c^2}}$$

$$L = x_2 - x_1 = (x'_2 - x'_1)\sqrt{1 - v^2 / c^2} = L_0\sqrt{1 - v^2 / c^2}$$

Doppler effect

Why the universe is believed to be expanding

Doppler effect in sound

ν_0: Source frequency

$$\nu = \nu_0 \left(\frac{1 + \boldsymbol{v}/c}{1 - V/c} \right)$$

ν: Observed frequency

c = speed of sound

V = speed of the source

$\boldsymbol{v}$ = speed of observer

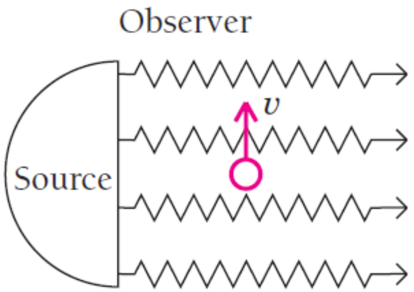
Doppler effect in light differ from that in sound.

Doppler effect in light by considering a light source as a clock that ticks 0 times per second and emits a wave of light with each tick

1 *Observer moving perpendicular to a line between him and the light source.*

The proper time between ticks is $t_0 = 1/\nu_0$, so between one tick and the next the time $t = t_o/\sqrt{1 - v^2/c^2}$ elapses in the reference frame of the observer. The frequency he finds is accordingly

$$\nu(transverse) = \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0}$$



Transverse Doppler effect in light

$$\nu = \nu_0\sqrt{1 - v^2/c^2}$$

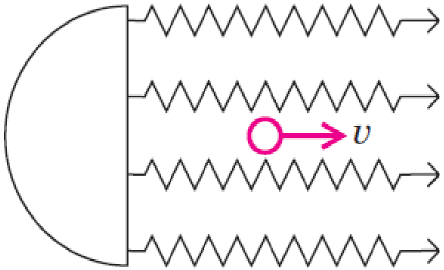
The observed frequency *is always lower than the source frequency* .

2. Observer receding from the light source

Now the observer travels the distance v*t* away from the source between ticks, which means that the light wave from a given tick takes v*t/c* longer to reach him than the previous one.

$$T = t + \frac{vt}{c} = t_0 \frac{1+v/c}{\sqrt{1-v^2/c^2}} = t_0 \frac{\sqrt{1+v/c}\sqrt{1+v/c}}{\sqrt{1+v/c}\sqrt{1-v/c}} = t_0 \sqrt{\frac{1+v/c}{1-v/c}}$$

And the observed frequency is

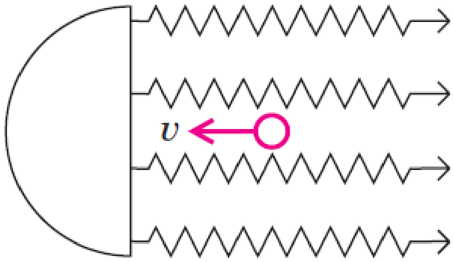


$$\nu(receding) = \frac{1}{T} = \frac{1}{t_0}\sqrt{\frac{1-v/c}{1+v/c}} = \nu_0 \sqrt{\frac{1-v/c}{1+v/c}}$$

The observed frequency ν is lower than the source frequency ν_0. Unlike the case of sound waves, which propagate relative to a material medium it makes no difference whether the observer is moving away from the source or the source is moving away from the observer.

3. Observer approaching the light source.

The observer here travels the distance v*t* toward the source between ticks, so each light wave takes v*t/c* less time to arrive than the previous one. In this case *T* = *t* - *vt/c* and the result is



Longitudinal Doppler effect in light

$$v(approaching) = v_0 \sqrt{\frac{1+v/c}{1-v/c}}$$

Doppler shifts in radar waves are used by police to measure vehicle speeds,

Doppler shifts in the radio waves emitted by a set of earth satellites formed the basis of the highly accurate Transit system of marine navigation.

Stars emit light of certain characteristic frequencies called spectral lines, and motion of a star toward or away from the earth shows up as a Doppler shift in these frequencies.

The spectral lines of distant galaxies of stars are all shifted toward the low-frequency (red) end of the spectrum and hence are called "red shifts."

Such shifts indicate that the galaxies are receding from us and from one another.

The speeds of recession are observed to be proportional to distance, which suggests that the entire universe is expanding. This proportionality is called **Hubble's law.**

Example-11

A driver is caught going through a red light. The driver claims to the judge that the color she actually saw was green ($\nu = 5.60 \times 10^{14}$ Hz) and not red ($\nu_0 = 4.80 \times 10^{14}$ Hz) because of the doppler effect. The judge accepts this explanation and instead fines her for speeding at the rate of $1 for each km/h she exceeded the speed limit of 80 km/h. What was the fine?

Ans

$$v = c\left(\frac{\nu^2 - \nu_0^2}{\nu^2 + \nu_0^2}\right) = (3.00 \times 10^8 \text{ m/s})\left[\frac{(5.60)^2 - (4.80)^2}{(5.60)^2 + (4.80)^2}\right]$$

$$= 4.59 \times 10^7 \text{ m/s} = 1.65 \times 10^8 \text{ km/h}$$

since 1 m/s = 3.6 km/h. The fine is therefore $\$(1.65 \times 10^8 - 80) = \$164{,}999{,}920$.

Example-12

A distant galaxy in the constellation Hydra is receding from the earth at 6.12×10^7 m/s. By how much is a green spectral line of wavelength 500 nm (1 nm = 10^{-9} m) emitted by this galaxy shifted toward the red end of the spectrum?

Ans:

Since $\lambda = c/\nu$ and $\lambda_0 = c/\nu_0$, from Eq.

$$\lambda = \lambda_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

Here $v = 0.204c$ and $\lambda_0 = 500$ nm, so

$$\lambda = 500 \text{ nm} \sqrt{\frac{1 + 0.204}{1 - 0.204}} = 615 \text{ nm}$$

which is in the orange part of the spectrum. The shift is $\lambda - \lambda_0 = 115$ nm. This galaxy is believed to be 2.9 billion light-years away.

Relativistic Mass, Momentum and Energy

Classical mechanics: Linear momentum: *p = mv*,

v <<<<<<<<<<<<<<<<<<<<<<< c

Whether this formula is valid in relativistic inertial frames ?

Relativistic momentum:

$$\boxed{p = \gamma m u}$$

$$\boxed{where : \gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}}$$

Derivations in separate class

In this form the conservation of momentum is valid in special relativity

Effect on mass:

Mass

Mass observed by an observer **moving** relative to the mass

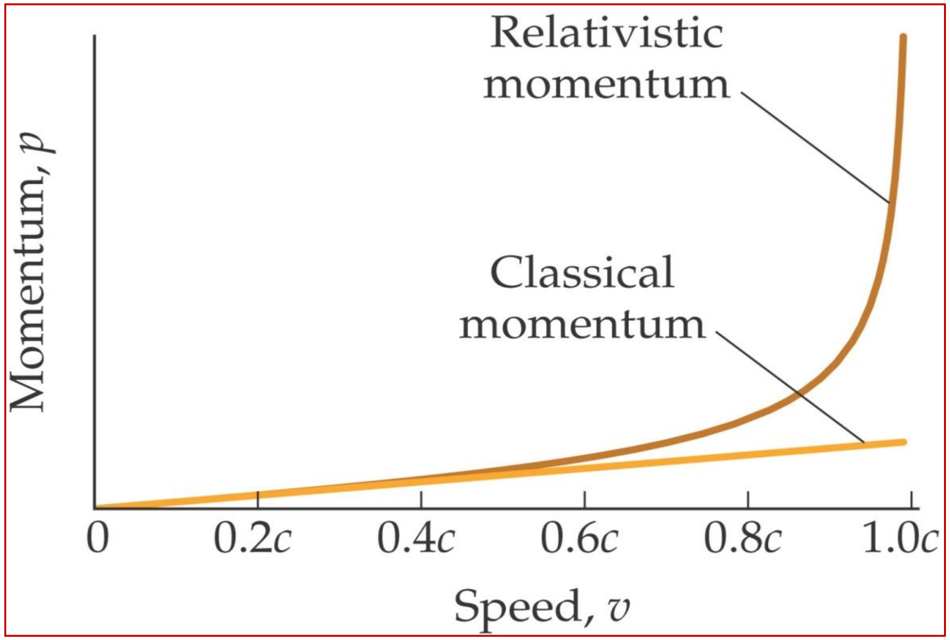
Mass measured when object is **at rest** relative to the observer-**rest mass**

$$\boxed{m' = \frac{m_0}{\sqrt{1 - v^2 / c^2}}}$$

The mass of an object is measured to increase as its speed increases.

As the speed gets closer and closer to *c*, the momentum increases without limit; note that the speed must be close to the speed of light before

Difference between classical and relativistic momentum is noticeable:



The object's velocity can never reach *c* because its momentum would then be infinite, which is impossible.

The relativistic momentum γmv is always correct;

The classical momentum *mv* is valid for velocities much smaller than *c*.

Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.

We modify Newton's second law to include our new definition of linear momentum, and force becomes:

Relativistic second Law: $$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m\mathbf{v})$$

Find the acceleration of a particle of mass *m* and velocity **v** when it is acted upon by the constant force **F,** where **F** is parallel to **v**.

since $a = dv/dt$,

$$F = \frac{d}{dt}(\gamma m v) = m\frac{d}{dt}\left(\frac{v}{\sqrt{1 - v^2/c^2}}\right)$$

$$= m\left[\frac{1}{\sqrt{1 - v^2/c^2}} + \frac{v^2/c^2}{(1 - v^2/c^2)^{3/2}}\right]\frac{dv}{dt}$$

$$= \frac{ma}{(1 - v^2/c^2)^{3/2}}$$

$$a = \frac{F}{m}(1 - v^2/c^2)^{3/2}$$

Even though the force is constant, the acceleration of the particle decreases as its velocity increases. As v $\rightarrow$ c, $a \rightarrow 0$, so the particle can never reach the speed of light

Mass and Energy.

Where $E_o = \mathrm{mc}^2$ comes from ?

For simplicity, let the particle start from rest under the influence of the force and calculate the kinetic energy *K* after the work is done.

$$\mathrm{KE} = \int_0^s F\, ds$$

To find the correct relativistic formula for KE we start from the relativistic form of the second law of motion

$$\mathrm{KE} = \int_0^s \frac{d(\gamma m\boldsymbol{v})}{dt}\, ds = \int_0^{m\boldsymbol{v}} \boldsymbol{v}\, d(\gamma m\boldsymbol{v}) = \int_0^{\boldsymbol{v}} \boldsymbol{v}\, d\left(\frac{m\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2/c^2}}\right)$$

Integrating by parts ($\int x\, dy = xy - \int y\, dx$),

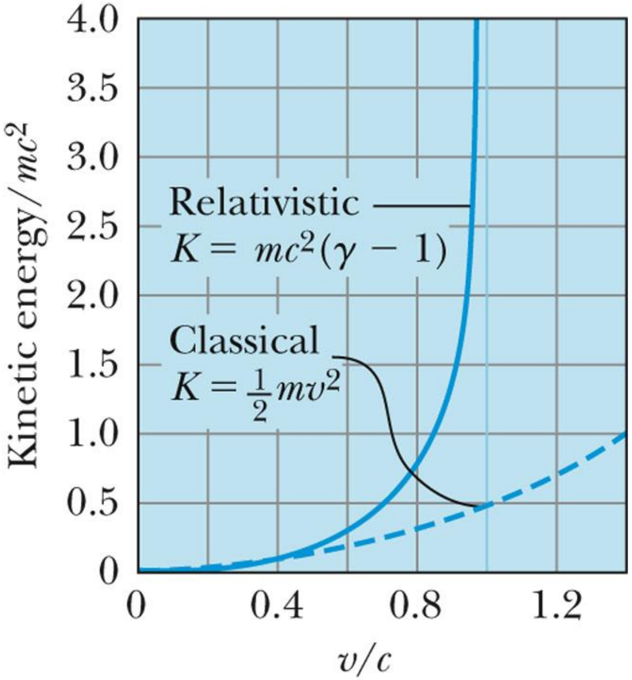
$$\mathrm{KE} = \frac{m\boldsymbol{v}^2}{\sqrt{1-\boldsymbol{v}^2/c^2}} - m\int_0^{\boldsymbol{v}} \frac{\boldsymbol{v}\, d\boldsymbol{v}}{\sqrt{1-\boldsymbol{v}^2/c^2}}$$

$$= \frac{m\boldsymbol{v}^2}{\sqrt{1-\boldsymbol{v}^2/c^2}} + \left[mc^2\sqrt{1-\boldsymbol{v}^2/c^2}\right]_0^{\boldsymbol{v}}$$

$$= \frac{mc^2}{\sqrt{1-\boldsymbol{v}^2/c^2}} - mc^2$$

$$\mathrm{KE} = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

Kinetic energy of an object is equal to the difference between γmc^2 and mc^2



Total Energy and Rest Energy

Total energy

$$E = \gamma mc^2 = mc^2 + \mathrm{KE}$$

If we interpret γmc^2 as the **total energy** E of the object, we see that when it is at rest and KE = 0, it nevertheless possesses the energy mc^2. Accordingly mc^2 is called the **rest energy** E_0 of something whose mass is m.

$$E = E_0 + \mathrm{KE}$$

the rest energy $\boxed{E_0 = mc^2}$

If the object is moving, the **total energy** is denoted by ***E*** and is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

• Even when a particle has no velocity and therefore no kinetic energy, it still has energy by virtue of its mass.

Example: 13

A stationary body explodes into two fragments each of mass 1.0 kg that move apart at speeds of 0.6c relative to the original body. Find the mass of the original body.

The rest energy of the original body must equal the sum of the total energies of the fragments. Hence

$$E_0 = mc^2 = \gamma m_1c^2 + \gamma m_2c^2 = \frac{m_1c^2}{\sqrt{1 - v_1^2/c^2}} + \frac{m_2c^2}{\sqrt{1 - v_2^2/c^2}}$$

and

$$m = \frac{E_0}{c^2} = \frac{(2)(1.0 \text{ kg})}{\sqrt{1 - (0.60)^2}} = 2.5 \text{ kg}$$

Since mass and energy are not independent entities, their separate conservation principles are properly a single one—the principle of conservation of mass energy. Mass *can be created or destroyed, but when this happens, an equivalent amount of energy* simultaneously vanishes or comes into being, and vice versa. Mass and energy are different aspects of the same thing.

Kinetic Energy at Low Speeds

When the relative speed v is small compared with *c*, the formula for kinetic energy must reduce to the familiar 1/2mv^2, which has been verified by experiment at such speeds.

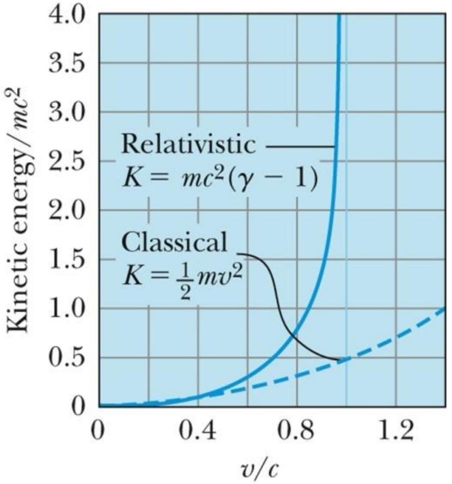
Let us see if this is true. The relativistic formula for kinetic energy is

Kinetic Energy $$\mathrm{KE} = \gamma mc^2 - mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

Since $v^2/c^2 \ll 1$, we can use the binomial approximation $(1 + x)^n \approx 1 + nx$, valid for $|x| \ll 1$, to obtain

$$\frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \qquad v \ll c$$

$$\mathrm{KE} \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)mc^2 - mc^2 \approx \frac{1}{2}mv^2 \qquad v \ll c$$



At low speeds the relativistic expression for the KE of a moving object does indeed reduce to the classical one. So far as is known, the correct formulation of mechanics has its basis in relativity, with classical mechanics representing an approximation that is valid only when v<<*c.*

Energy and Momentum

Total energy and momentum are conserved in an isolated system, and the rest energy of a particle is invariant. Hence these quantities are in some sense more fundamental than velocity or kinetic energy, which are neither.

Total Energy $E = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} \Longrightarrow E^2 = \dfrac{m^2c^4}{1 - v^2/c^2}$

Momentum $p = \dfrac{mv}{\sqrt{1 - v^2/c^2}} \Longrightarrow p^2c^2 = \dfrac{m^2v^2c^2}{1 - v^2/c^2}$

$$\Longrightarrow E^2 - p^2c^2 = \frac{m^2c^4 - m^2v^2c^2}{1 - v^2/c^2} = \frac{m^2c^4(1 - v^2/c^2)}{1 - v^2/c^2}$$

$$= (mc^2)^2$$

Energy and Momentum $\boxed{E^2 = (mc^2)^2 + p^2c^2}$

because mc^2 *is invariant, so is* $E^2 - p^2c^2$: this quantity for a particle has the same value in all frames of reference

Massless Particles:

Can a massless particle exist? Or can a particle exist which has no rest mass but which nevertheless exhibits such particle like properties as energy and momentum? In classical mechanics, a particle must have rest mass in order to have energy and momentum, but in relativistic mechanics this requirement does not hold.

- **For a particle having no mass:**

$$\boxed{E = pc}$$

- **For a massless particle:**

$$\boxed{v = c}$$

Rest energy of a particle: Example: E_0 (proton)

$$E_0(\text{proton}) = (1.67\times10^{-27}\text{ kg})(3\times10^{8}\text{ m/s})^2 = 1.50\times10^{-10}\text{ J}$$

$$= 1.50\times10^{-10}\text{ J}\,\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}} = 9.38\times10^{8}\text{ eV}$$

Example-14: An electron (m = 0.511 MeV/c^2) and a photon (m = 0) both have momenta of 2.000 MeV/c. Find the total energy of each.

(*a*) Electron

$$E = \sqrt{m^2c^4 + p^2c^2} = \sqrt{(0.511\text{ MeV}/c^2)^2c^4 + (2.000\text{ MeV}/c)^2c^2}$$

$$= \sqrt{(0.511\text{ MeV})^2 + (2.000\text{ MeV})^2} = 2.064\text{ MeV}$$

(*b*) Photon $E = pc = (2.000\text{ MeV}/c)c = 2.000\text{ MeV}$

15. Calculate the mass of an electron moving at 0.98 c in an accelerator for cancer therapy.

Ans. $$m = \frac{9.11\times10^{-31}kg}{\sqrt{1-(0.98c)^2/c^2}} = 4.58\times10^{-30}kg(5m_o)$$

16. How much energy would be released if a π^0 meson (m_o=2.4 X 10^{-28} kg) decays at rest.

Ans.

$E = mc^2$

$E = m_oc^2$ (particle is at rest)

$E = (2.4 \text{ X } 10^{-28} \text{ kg})(3.0 \text{ X } 10^8 \text{ m/s})^2$

$E = 2.16 \text{ X } 10^{-11}$ J

17. An electron is moving at 0.999c in the CERN accelerator.

a. Calculate the rest energy
b. Calculate the relativistic momentum
c. Calculate the relativistic energy
d. Calculate the Kinetic energy

Do this.