National Institute of Technology Rourkela
Department of Mathematics

Course: MA1002 Assignment: I

1. Define the following terms:
   (a) Ordinary Differential Equation (ODE)
   (b) Partial Differential Equation (PDE)
   (c) Order and Degree of an ODE
   (d) Linear and Non-linear ODE

2. Classify the following ODEs as linear or non-linear. Also find their order and degree

   (a) $\frac{dy}{dx} + y^2 = x$ (b) $1 + (y')^2 = c(y'')^{2/3}$ (c) $\frac{d^3y}{dx^3} + \frac{d^2y}{d^2x} \cdot \frac{dy}{dx} + y = x$

3. Find the differential equation for the following family of curves where A, B, C are parameters:
   (a) $y = Ae^{2x} + Be^{-2x}$
   (b) $x^2 + y^2 + 2Ax + 2By + C = 0$
   (c) $y = e^x (A \cos x + B \sin x)$

4. Find the differential equation of the family of parabolas $y^2 = 4ax$.

5. Find the differential equation of all circles of radius a.

6. Solve the following differential equations:

   (a) $\frac{dy}{dx} = e^{y-x} + x^2e^y$ (b) $\sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$

   (c) $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y \cos y}$ (d) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

   (e) $\frac{dy}{dx} = \frac{4x + 6y + 5}{2x + 3y + 4}$ (f) $y - x\frac{dy}{dx} = 3\left(1 + x^2\frac{dy}{dx}\right)$

7. Solve the following differential equations:

   (a) $xdy - ydx = \left(\sqrt{x^2+y^2}\right)dx$ (b) $(4y + 3x)dy + (y - 2x)dx = 0$

   (c) $x\left(\frac{dy}{dx}\right) = y(\log y - \log x + 1)$ (d) $(x^3 - 2y^3)dx + 3xy^2dy = 0$

   (e) $x(x-y)dy = y(x+y)dx$ (f) $(2\sqrt{xy} - x)dy + ydx = 0$

8. Solve the following differential equations:

   (a) $\frac{dy}{dx} = \frac{y - x + 2}{x - 2y - 3}$ (b) $(2x^2 + 3y^2 - 7)xdx - (3x^2 + 2y^2 - 8)ydy = 0$

   (c) $\frac{dy}{dx} = \frac{x + 2y + 3}{2x + 3y + 4}$ (d) $\frac{dy}{dx} = \frac{x + y + 1}{x - y}$

9. Determine which of the following equations are exact and then solve those:
   (a) $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$
   (b) $(xe^{xy} + 2y)\,dy + ye^{xy}dx = 0$

(c) $xdx + ydy = \dfrac{a^2(xdy - ydx)}{x^2 + y^2}$

(d) $y\sin(2x)dx - (1 + y^2 + \cos^2 x)dy = 0$

10. For the following differential equations, find the integrating factors and solve them.

    (a) $x^2(dy/dx) + xy = \sqrt{1 - x^2y^2}$

    (b) $(1 + xy)ydx + (1 - xy)xdy = 0$

    (c) $(x^4 + y^4)dx - xy^3dy = 0$

    (d) $(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$

    (e) $(x^2 + y^2 + 1)dx - 2xydy = 0$

    (f) $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

    (g) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

11. Solve the following differential equations:

    (a) $\sin x(dy/dx) + 3y = \cos x$

    (b) $(x + 3y + 2)\dfrac{dy}{dx} = 1$

    (c) $(1 - x^2)\dfrac{dy}{dx} - xy = 1$

    (d) $\dfrac{dy}{dx} = e^{x-y}(e^x - e^y)$

    (e) $2x^2\dfrac{dy}{dx} = xy + y^2$

    (f) $\dfrac{dy}{dx} + \dfrac{xy}{1 - x^2} = x\sqrt{y}$

12. Find the orthogonal trajectories of the following equations:

    (a) $y = \sqrt{x + c}$ (b) $xy = c$ (c) $y = ce^{-x}$ (d) $x^2 + y^2 = r^2$

13. The population of a city satisfies the logistic law

$$\frac{dx}{dt} = \frac{1}{100}x - \frac{1}{10^8}x^2 .$$

where t is measured in years. Given that the population of this city is 100000 in 2000, answer the following questions:

    (a) what was the population in 2011 ?

    (b) In which year, does the 2000 population shall be doubled ?

14. A tank of 100 gallons capacity is initially full of water. Pure water is allowed to run into the tank at the rate of 1 gallon per minute. At the same time brine containing 0.25 lb of salt flows into the tank at the same rate. The mixture flows out at the rate of 2 gallons per minute. Assuming a perfect mixing, find the amount of salt in the tank after 't' minutes.

15. The IVP governing the current'I' flowing through in a series RL circuit when a voltage $v(t) = t$ is applied, is given by $IR + L\dfrac{dI}{dt} = t, t \geq 0, \quad I(0) = 0$ where R and L are constants. Find the current $I(t)$ at time t.

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