Assignment-4

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1 Problem

If three points (h, 0), (a, b) and (0, k) lie on a line, $showthat \frac{a}{h} + \frac{b}{k} = 1$

2 Solution

The input given

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{h} \\ \boldsymbol{0} \end{pmatrix} \tag{1}$$

$$\boldsymbol{B} = \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{pmatrix} \tag{2}$$

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{k} \end{pmatrix} \tag{3}$$

$$\textbf{D=A-B} = \begin{pmatrix} h \\ 0 \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} h-a \\ -b \end{pmatrix} \tag{5}$$

$$\textbf{E=A-C} = \begin{pmatrix} h \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ K \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} h \\ -k \end{pmatrix} \tag{7}$$

Now the matrix is

$$\textbf{F=}\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{E} \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} h-a & -b \\ h & -k \end{pmatrix} \tag{9}$$

In the problem they have given that three points lie on a line, thats means these three points are collinear.
If points on a line are collinear, rank of matrix is "1"then the vectors are in linearlydependent.
For 2×2 matrix Rank =1 means Determinant is 0.
Through pivoting,we obtain

$$= \begin{pmatrix} h-a & -b \\ h & -k \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} h-a & -b \\ h & -k \end{pmatrix} \xrightarrow{\frac{R1}{h-a}} = \begin{pmatrix} 1 & \frac{-b}{h-a} \\ h & -k \end{pmatrix} \xrightarrow{R2 \to R2 - hR1} = \begin{pmatrix} 1 & \frac{-b}{h-a} \\ 0 & -k + \frac{bh}{h-a} \end{pmatrix} \tag{11}$$

if the rank of the matrix is 1 means any one of the row must be zero.So, making the last element in the matrix to 0.

$$-k + \frac{bh}{h-a} = 0 \tag{12}$$

$$(h-a) - k + bh = 0 \tag{13}$$

$$-kh + ak + bh = 0 \tag{14}$$

$$ak + bh = kh \tag{15}$$

Dividing with kh on both sides ,we get

$$\frac{a}{h} + \frac{b}{k} = 1 \tag{16}$$

Hence proved.

3 Construction

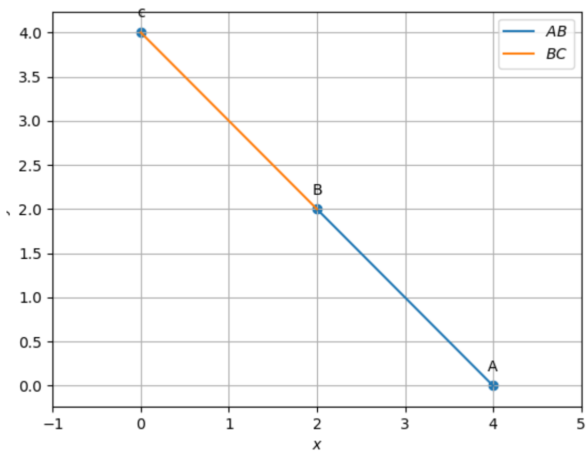

Figure 1:

4 Code

*Verify the above proofs in the following code.
https://github.com/Susi9121/FWC/tree/main/matrix/line