优化 / 数学规划 (Optimization / Mathematical Programming)

从一个可行解的集合中，寻找最优的元素.

$$\text{minimize } f_0(x)$$
$$\text{subject to } f_i(x) \le b_i,\ i=1,\cdots,M$$

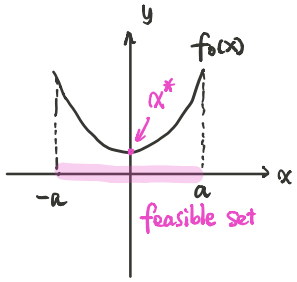
$x=[x_1,\cdots,x_n]^T$ 优化变量 (Optimization Variable)

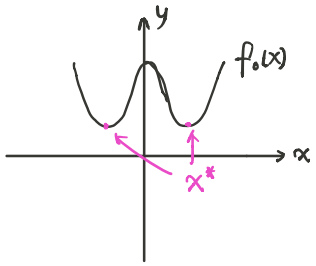
$f_0: \mathbb{R}^n \to \mathbb{R}$ 目标函数 (Objective function).

$f_i: \mathbb{R}^n \to \mathbb{R}$ 不等式约束 (inequality constraint).

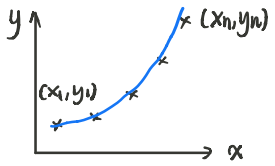
x^* 最优 (optimal) $\Leftrightarrow \forall z,\ z \in \{f_i(z) \le b_i,\ i=1,\cdots,M\}$ 可行解集 (feasible set)

$$f_0(z) \ge f_0(x^*)$$





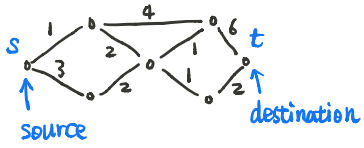
数据拟合问题



$$y = ax^2+bx+c.$$

$$\text{minimize } \varepsilon_1^2+\varepsilon_2^2+\cdots+\varepsilon_n^2$$
$$\varepsilon_i = y_i-(ax_i^2+bx_i+c),\ i=1,\cdots,n$$

最短路径问题



$\{V, E\}$

$$\min \sum_{(i,j)\in E} w_{ij}x_{ij} \qquad x_{ij} \ge 0$$

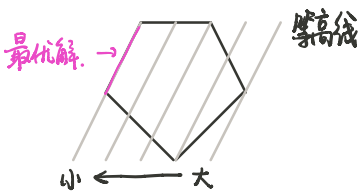
$$\text{s.t } \boxed{x_{ij}=0 \text{ or } 1} \quad (\text{是否选择边}(ij))$$

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1 & i=s \ (\text{源结点}) \\ -1 & i=d \ (\text{目标结点}) \\ 0 & \text{otherwise} \end{cases}$$

$\sum_j x_{ij}$：出边　$\sum_j x_{ji}$：进边

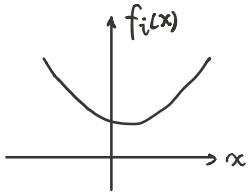
线性规划 / 非线性规划

$$f_i(\alpha x+\beta y) = \alpha f_i(x)+\beta f_i(y)$$
$$\forall i=0,1,\cdots,M \quad (\text{线性函数})$$

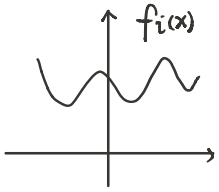


凸规划/非凸规划

$f_i(\alpha x+\beta y) \le \alpha f_i(x)+\beta f_i(y) \quad \forall i=0,1,\cdots,M$

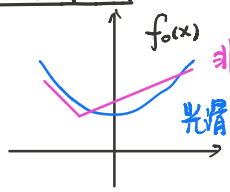


凸函数



非凸函数.

光滑/非光滑



非光滑

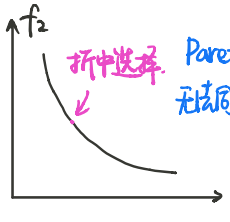
光滑(在每个点都可微)

连续/非连续

可行域

可行域是离散点

单目标/多目标



$\min f_1(x)$
$f_2(x)$

$\Rightarrow \min f_1(x)+\lambda f_2(x)$