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4.8.3

$$\Pr(Y=k \mid X=x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)}$$

where

$$f_k(x) = \frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

$\Downarrow$

$$P_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}$$

$$\text{Bayes' classifier} = \max P_k(x) = \left(\frac{\pi_k \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)} \right)$$

$$\Longleftrightarrow \max \log P_k(x) = \delta_k(x) = \log(\pi_k) - \frac{\mu_k^2}{2\sigma^2} + x\frac{\mu_k}{\sigma^2} - \frac{x^2}{2\sigma^2}$$

Because the observations within each class are not same ~~the~~ the σ in the previous equation is not a constant.

$$\max \log P_k(x) = \delta_k(x) \log(\pi_k) - \log(\sigma_k) - \frac{\mu_k^2}{2\sigma_k^2} + x\frac{\mu_k}{\sigma_k^2} - \frac{x^2}{2\sigma_k^2}$$

The last term could not be canceled.

So the relationship is not linear.

4.8.7

$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}$$

where $\pi_{yes} = 0.8$ $\quad \mu_{yes} = 10$ $\quad \sigma = 6$

$\pi_{NO} = 0.2$ $\quad \mu_{NO} = 0$

$f_{yes}(4) = 0.0403$

$f_{NO}(4) = 0.0532$

$$P_{yes}(4) = \frac{\pi_{yes} f_{yes}}{\pi_{yes} f_{yes} + \pi_{NO} f_{NO}} = 0.75186.$$