EXERCISE-3

Task 3.2 **Given:-** $X_1 = -1, X_2 = 0, X_3 = 1$

There are three outcomes of a discrete random process, when the process is repeated 3 times. The probability of occurrence of

$$P(X_i | X_j) = \begin{cases} \frac{1}{3}(1 + 2e^{-|2|}) & \text{for } i = j \\ \frac{1}{3}(1 - e^{-|2|}) & \text{for } i \neq j \end{cases} \qquad i, j = 1, 2, 3$$

we will use total probability theorem for finding the probabibity of X_1, X_2, X_3 i.e. $P(X_1), P(X_2), P(X_3)$ which states that -

$$P(A) = \sum_n P(A|B_n) \cdot P(B_n)$$

P = Probability
A = any event
B_n = event

let $P(X_1) = P_1$, $P(X_2) = P_2$ and $P(X_3) = P_3$

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using above theorem, we can write :-

Let :-

$$\& \begin{cases} a, & \text{when } i=j = \frac{1}{3}(1+2e^{-|\tau|}) \\ b, & \text{when } i \neq j = \frac{1}{3}(1-e^{-|\tau|}) \end{cases}$$

$$P_1 = aP_1 + bP_2 + bP_3 \quad \text{—①}$$
$$P_2 = bP_1 + aP_2 + bP_3 \quad \text{—②}$$
$$P_3 = bP_1 + bP_2 + aP_3 \quad \text{—③}$$

$P(X_1|1)\cdot P_1 + P(X_2|1)\cdot P_2 + P(X_3|1)\cdot P_3 = P_1$

{ It means the probability of $X = X_1$ when the process is repeating 3 times }

Solving equation ①, ② & ③ we get -

$$P_1 - P_2 = (a-b)P_1 - (a-b)P_2$$
$$P_1 - P_2 = aP_1 - bP_1 - aP_2 + bP_2$$
$$P_1(1-a+b) = P_2(1-a+b)$$
$$P_1 = P_2$$

Similarly we can calculate from eq^n ② & ③

$\therefore$ $P_1 = P_2$, $P_2 = P_3$ and $P_1 = P_3$

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Now we know that Probability of occurrence of all outcomes of an event is 1.

$$\therefore P_1 + P_2 + P_3 = 1$$

$$P_1 + P_1 + P_1 = 1$$

$$3P_1 = 1$$

$$\Rightarrow \boxed{P_1 = \frac{1}{3} = P_2 = P_3}$$

(b) ACF, $S_{xx}(\tau) = ?$

ACF, $S_{xx}(\tau) = E\{x(\zeta, t+\tau) \cdot x(\zeta, t)\}$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} x_i x_j P(a/b) P_i$$

$$= \{x_1 \cdot x_1 \cdot a P_1 + x_1 \cdot x_2 \cdot b P_2 + x_1 \cdot x_3 \cdot b P_3 +$$
$$x_2 \cdot x_1 \cdot b P_1 + x_2 \cdot x_2 \cdot a P_2 + x_2 \cdot x_3 \cdot a P_3 +$$
$$x_3 \cdot x_1 \cdot b P_1 + x_3 \cdot x_2 \cdot b P_2 + x_3 \cdot x_3 \cdot a P_3 \}$$

$$= \left\{(-1)(-1)\, a\left(\tfrac{1}{3}\right) + 0 + (-1)(1)\, b\left(\tfrac{1}{3}\right) + 0 + 0 + 0 + \right.$$
$$\left. (1)(-1)\, b\left(\tfrac{1}{3}\right) + 0 + (1)(1)\, a\left(\tfrac{1}{3}\right) \right\}$$

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$$= \frac{a}{3} - \frac{b}{3} - \frac{b}{3} + \frac{a}{3}$$

$$= \frac{2}{3}(a-b)$$

$$= \frac{2}{3}\left[\frac{1}{3}(1+2e^{-1|\tau|}) - \frac{1}{3}(1-e^{-1|\tau|})\right]$$

$$= \frac{2}{9} \cdot 3e^{-1|\tau|}$$

$$= \boxed{\frac{2}{3}e^{-1|\tau|}}$$