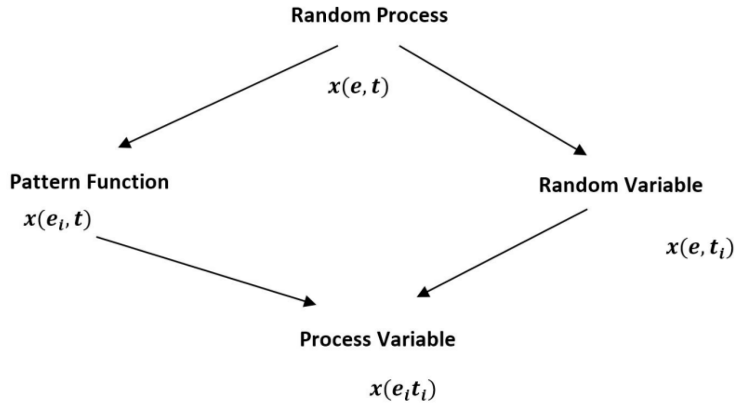
Task 6.3

a) Explain the difference between a random process, a random variable, a pattern function, and a process variable.

Answer:

1) **Random Process:** Random process can be thought of as the collection of random variables. It can also be defined as a time varying function that assign the outcome of a random event or experiment to each time instant. It is denoted with x(e,t).
2) **Random Variable:** A random variable is a variable whose value is the outcome of a random process. In order words, when there is a random event or experiment, then the outcome at any instant of time is assigned to a variable termed as random variable.
3) **Pattern Function:** A random variable x(e) is emerging to a random process x(e,t) in which for any ej the value x is a function of time x(ej ,t). We call these functions pattern functions or realizations of the random process.
4) **Process variable:** It's the value of a pattern function at a fixed point in time, or the value of a random variable at a certain elementary event, i.e. the outcome of one random experiment.



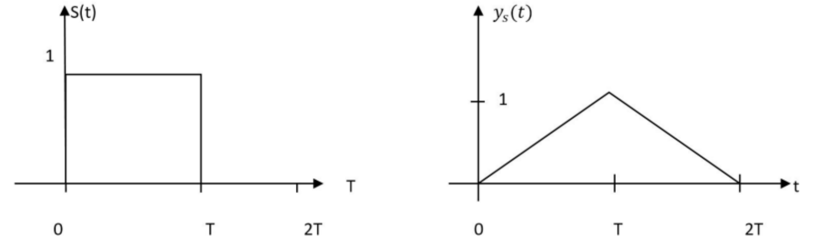
b) What is the fundamental prerequisite for the construction of a Matched Filter?

Answer: The fundamental prerequisite for the construction of a matched filter is the knowledge of the time function (or shape) of the signal that shall be detected.

c) Assume a perfectly working Matched Filter. What is the output of that filter if the shape of the transmitted signal is a rectangular impulse?

Answer: The output of the matched filter is the ACF S_{ss} (τ) of the input rectangular impulse s(t) shifted by $t_0 = T$.

$$s_{ss}(\tau) = \frac{S_0}{T}\int_{-T/2}^{+T/2} s(\tau)\, s(t+\tau)\, dt = \frac{1}{T}\int_{0}^{T-|\tau|} dt = 1 - \frac{|\tau|}{T}$$



a) Input signal and b) Output signal of the matched filter receiver

d) How does the signal-to-noise ratio affect the bit error rate in a Matched Filter receiver? Please answer in only one sentence.

Answer: If the signal-to-noise ratio SNR increases the bit error rate decreases, and vice versa.

e) Write down the Wiener-Hopf equation and explain its parts.

Answer: Wiener-Hopf equation is given by:

$$s_{xd}\left(u\right) - \int_{-\infty}^{+\infty} h_{opt}\left(v\right) \cdot s_{xx}\left(u - v\right) dv = 0; \quad for\ h_{opt}\left(v\right) \neq 0$$

s_{xd} (u) - CCF of filter input signal x(e,t) and the desired output signal d(e,t)

h_{opt} (v) - Input response of the optimum filter

s_{xx} (u − v) - ACF of the filter input signal

f) Assume you should implement a Wiener-Kolmogorov filter. Give a reasoned decision, if a causal Wiener-Kolmogorov filter should be used, or if an acausal one should be applied.

Answer: A causal Wiener filter must be built because it's impossible to build a non-causal filter.