For a stationary time depending continuous random process the following system input/output expressions are valid

$$S_{xy}(\omega) = \underline{H}(j\omega)\, S_{xx}(\omega) \tag{5.49}$$

$$S_{yx}(\omega) = \underline{H}^*(j\omega)\, S_{xx}(\omega) \tag{5.50}$$

$$S_{yy}(\omega) = \underline{H}(j\omega)\, \underline{H}^*(j\omega)\, S_{xx}(\omega) \tag{5.51}$$

STOCHASTICS

TASK – 5.1

Given:-

$$u(e,t) \longrightarrow \boxed{g(t)} \longrightarrow y(e,t)$$

$$S_{uy} = \frac{S_1}{(1-j\omega b)(1+j\omega T_1)} \quad , \quad S_{yy} = \frac{S_1}{(1+\omega^2 T_1^2)}$$

(a) $$G^*(j\omega) = \frac{S_{yy}}{S_{uy}} = \frac{\dfrac{S_1}{(1+\omega^2 T_1^2)}}{\dfrac{S_1}{(1-j\omega b)(1+j\omega T_1)}} = \frac{(1-j\omega b)(1+j\omega T_1)}{(1+\omega^2 T_1^2)}$$

$$= \frac{(1-j\omega b)(1+j\omega T_1)}{(1+j\omega T_1)(1-j\omega T_1)} = \frac{(1-j\omega b)}{(1-j\omega T_1)}$$

$$\boxed{\therefore G(j\omega) = \frac{1+j\omega b}{1+j\omega T_1}}$$

(b) To identify the causal or non-causal system, we must check where the poles exist

$$\therefore \quad 1 + j\omega T_1 = 0$$

$$j\omega T_1 = -1$$

$$\omega = \frac{-1}{jT_1} = \frac{j}{T_1}$$

T_1 must be greater than zero for the system to be causal

(c) $S_{uu}(\tau) = ?$

$$S_{uu} = \frac{S_{uy}}{G(j\omega)} = \frac{\dfrac{S_1}{(1-j\omega b)(1+j\omega T_1)}}{\dfrac{(1+j\omega b)}{(1+j\omega T_1)}} = \frac{S_1}{(1-j\omega b)(1+j\omega b)}$$

$$= \frac{S_1}{1+\omega^2 b^2}$$

$$\therefore S_{uu}(\tau) = F^{-1}[S_{uu}] = F^{-1}\left[\frac{S_1}{1+\omega^2 b^2}\right]$$

$$= F^{-1}\left[\frac{S_1/b}{b(1/b^2+\omega^2)}\right] = \boxed{\frac{S_1}{2b}\, e^{-\frac{|\tau|}{b}}}$$