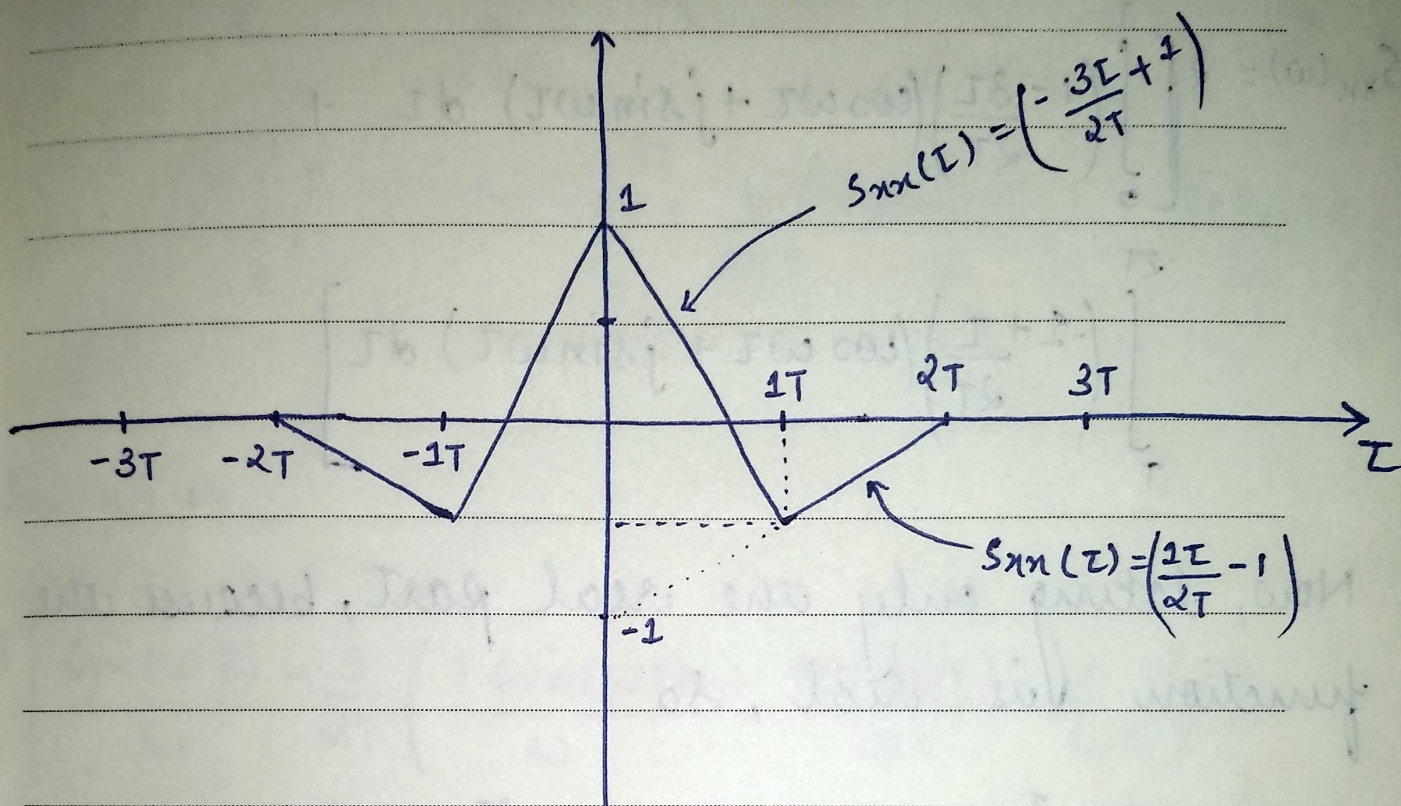


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EXERCISE-4



Power spectral density $S_{xx}(\omega) = ?$

Solution:-

$$\therefore S_{xx}(\omega) = \int_{-\infty}^{\infty} S_{xx}(\tau) e^{-j\omega\tau} d\tau$$

$$S_{xx}(\omega) = 2 \int_0^{\infty} S_{xx}(\tau) e^{-j\omega\tau} d\tau = 2 \int_0^{2T} S_{xx}(\tau) e^{-j\omega\tau} d\tau$$

$$S_{xx}(\omega) = 2 \left[\int_0^T \left(1 - \frac{3\tau}{2T}\right) e^{-j\omega\tau} d\tau + \int_T^{2T} \left(-1 + \frac{\tau}{2T}\right) e^{-j\omega\tau} d\tau \right]$$

$$\left\{ \because e^{-j\omega\tau} = \cos\omega\tau + j\sin\omega\tau \right\}$$

$$S_{xx}(\omega) = 2 \left[\int_0^T \left(1 - \frac{3\tau}{2T}\right) (\cos\omega\tau + j\sin\omega\tau) d\tau + \int_T^{2T} \left(\frac{\tau - T}{2T}\right) (\cos\omega\tau + j\sin\omega\tau) d\tau \right]$$

Now, taking only the real part, because the function is real, so

$$S_{xx}(\omega) = 2 \left[\int_0^T \left(1 - \frac{3\tau}{2T}\right) \cos\omega\tau d\tau + \int_T^{2T} \left(\frac{\tau - T}{2T}\right) \cos\omega\tau d\tau \right]$$

$$S_{xx}(\omega) = 2 \left[\int_0^T \cos\omega\tau d\tau - \frac{3}{2T} \int_0^T \tau \cos(\omega\tau) d\tau + \frac{1}{2T} \int_T^{2T} \tau \cos(\omega\tau) d\tau - \int_T^{2T} \cos(\omega\tau) d\tau \right]$$

using integration by parts :-

$$\because \int u v dx = u \int v dx - \int \left[\frac{d(u)}{dx} \int v dx \right] dx$$



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$$S_{xx}(\omega) = 2 \left[\frac{\sin(\omega T)}{\omega} \right]_0^T - \frac{3}{2T} \left(\frac{T \sin(\omega T)}{\omega} + \frac{\cos(\omega T)}{\omega^2} \right) \Big|_0^T - \left(\frac{1}{\omega} \sin(\omega T) \right) \Big|_T^{2T} + \frac{1}{2T} \left(\frac{T \sin(\omega T)}{\omega} + \frac{\cos(\omega T)}{\omega^2} \right) \Big|_T^{2T}$$

$S_{xx}(\omega) \Rightarrow$

$$2 \left[\frac{\sin(\omega T)}{\omega} - \frac{3}{2T} \left(\frac{T \sin(\omega T)}{\omega} + \frac{\cos(\omega T)}{\omega^2} - \frac{1}{\omega^2} \right) - \left(\frac{\sin(2\omega T)}{\omega} - \frac{\sin(\omega T)}{\omega} \right) + \frac{1}{2T} \left(\frac{2T \sin(2\omega T)}{\omega} - \frac{T \sin(\omega T)}{\omega} + \frac{\cos(2\omega T)}{\omega^2} \right) - \frac{\cos(\omega T)}{\omega^2} \right]$$

$S_{xx}(\omega) \Rightarrow$

$$\frac{2}{\omega} \left[\sin(\omega T) - \frac{3}{2} \sin(\omega T) + \frac{3}{2\omega T} - \frac{3 \cos \omega T}{2\omega T} + \sin \omega T + \frac{\cos(2\omega T)}{2\omega T} + \frac{\cos(\omega T)}{2\omega T} \right]$$

$$S_{xx}(\omega) \Rightarrow$$

$$\frac{2}{\omega} \left[2 \sin \omega T - 2 \sin \omega T + \frac{3}{2\omega T} - \frac{3 \cos \omega T}{2\omega T} + \frac{\cos 2\omega T}{2\omega T} - \frac{\cos \omega T}{2\omega T} \right]$$

$$S_{xx}(\omega) \Rightarrow \frac{2}{\omega^2 T} \left[-2 \cos \omega T + \frac{3}{2} + \frac{\cos 2\omega T}{2} \right]$$

$$\left\{ \begin{array}{l} \because \cos 2A = \cos^2 A - \sin^2 A \\ \& \sin^2 A = 1 - \cos^2 A \\ \Rightarrow \cos 2A = 2 \cos^2 A - 1 \end{array} \right.$$

$$\therefore S_{xx}(\omega) = \frac{2}{\omega^2 T} \left[-2 \cos \omega T + \frac{3}{2} + \cos^2 \omega T - \frac{1}{2} \right]$$

$$S_{xx}(\omega) = \frac{2}{\omega^2 T} \left[-2 \cos \omega T + 1 + \cos^2 \omega T \right]$$

$$S_{xx}(\omega) = \frac{2}{\omega^2 T} \left[1 - \cos \omega T \right]^2 \quad \text{--- (1)}$$

∴ $S_{xx}(\omega)$ is

ω	$S_{xx}(\omega)$
0	0
π/T	$8T/\pi^2$
$2\pi/T$	0
$3\pi/T$	$8T/9\pi^2$
$4\pi/T$	0
$5\pi/T$	$8T/25\pi^2$

