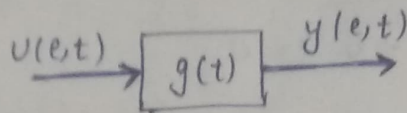


STOCHASTICS

TASK- 5.1

Given:-



$$S_{xy}(\omega) = \underline{H}(\omega) S_{xx}(\omega) \quad (5.49)$$

$$S_{yx}(\omega) = \underline{H}^*(\omega) S_{xx}(\omega) \quad (5.50)$$

$$S_{yy}(\omega) = \underline{H}(\omega) \underline{H}^*(\omega) S_{xx}(\omega) \quad (5.51)$$

$$S_{uy} = \frac{S_1}{(1-j\omega b)(1+j\omega T_1)} \quad , \quad S_{yy} = \frac{S_1}{(1+\omega^2 T_1^2)}$$

$$\begin{aligned} (a) \quad G^*(j\omega) &= \frac{S_{yy}}{S_{uy}} = \frac{\frac{S_1}{(1+\omega^2 T_1^2)}}{\frac{S_1}{(1-j\omega b)(1+j\omega T_1)}} = \frac{(1-j\omega b)(1+j\omega T_1)}{(1+\omega^2 T_1^2)} \\ &= \frac{(1-j\omega b)(1+j\omega T_1)}{(1+j\omega T_1)(1-j\omega T_1)} = \frac{(1-j\omega b)}{(1-j\omega T_1)} \end{aligned}$$

$$\therefore G(j\omega) = \frac{1+j\omega b}{1+j\omega T_1}$$

(b) To identify the causal or non-causal system, we must check where the poles exist

$$\therefore 1+j\omega T_1 = 0$$

$$j\omega T_1 = -1$$

$$\omega = \frac{-1}{jT_1} = \frac{j}{T_1}$$

T_1 must be greater than zero for the system to be causal

(c) $S_{uu}(\tau) = ?$

$$\begin{aligned} S_{uu} &= \frac{S_{uy}}{G(j\omega)} = \frac{\frac{S_1}{(1-j\omega b)(1+j\omega T_1)}}{\frac{(1+j\omega b)}{(1+j\omega T_1)}} = \frac{S_1}{(1-j\omega b)(1+j\omega b)} \\ &= \frac{S_1}{1+\omega^2 b^2} \end{aligned}$$

$$\begin{aligned} \therefore S_{uu}(\tau) &= \mathcal{F}^{-1}[S_{uu}] = \mathcal{F}^{-1}\left[\frac{S_1}{1+\omega^2 b^2}\right] \\ &= \mathcal{F}^{-1}\left[\frac{S_1/b}{b(1/b^2 + \omega^2)}\right] = \boxed{\frac{S_1}{2b} e^{-\frac{|\tau|}{b}}} \end{aligned}$$