Task 6.2

$$S_{zz}(\tau) = a e^{-\alpha|\tau|} + b$$

$$y(\eta, t) = \begin{cases} 0 & \text{for } t \leq 0 \\ \int_{t_0}^{t} z(\eta, \lambda)\, d\lambda & \text{for } t > t_0 \end{cases}$$

a) Determine the constants a and b

we know, $(m_x^{(1)})^2 = \lim_{\tau \to \infty} S_{zz}(\tau)$

$$\Rightarrow \lim_{\tau \to \infty} a e^{-\alpha|\tau|} + b = 0$$

$$\therefore \quad b = 0$$

also, $m_x^{(2)} = \lim_{\tau \to 0} S_{zz}(\tau)$

$$\Rightarrow \lim_{\tau \to 0} a e^{-\alpha|\tau|} + b = 1$$

$$\therefore \quad a = 1$$

b) Find cross corelation function

$$S_{zy}(t_1, t_2) = E\{z(\eta, t_1), y(\eta, t_2)\}$$

$$= E\left\{z(\eta, t_1) \int_{t_0}^{t} z(\eta, \lambda)\, d\lambda\right\}$$

$$= \int_{t_0}^{t} E\{z(\eta, t_1) \cdot z(\eta, \lambda)\}\, d\lambda$$

$$= \int_{t_0}^{t} e^{-\alpha|\lambda - t_1|}\, d\lambda$$

Case I → $t_0 < t_1 < t$

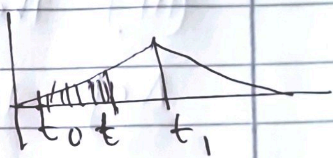
$$\int_{t_0}^{t_1} e^{\alpha(\lambda-t_1)}\, d\lambda + \int_{t_1}^{t} e^{-\alpha(\lambda-t_1)}\, d\lambda$$

$$= \frac{e^{\alpha(\lambda-t_1)}}{\alpha}\Bigg|_{t_0}^{t_1} + \frac{e^{-\alpha(\lambda-t_1)}}{-\alpha}\Bigg|_{t_1}^{t}$$

$$= \frac{1}{\alpha}\left[1 - e^{\alpha(t_0-t_1)} - \left[e^{-\alpha(t-t_1)} - 1\right]\right]$$

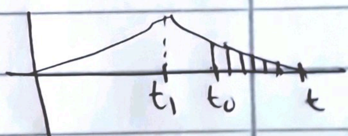
$$= \frac{1}{\alpha}\left[2 - e^{\alpha(t_0-t_1)} - e^{-\alpha(t-t_1)}\right]$$

Case II : $t_0 < t < t_1$



Now, the integral always open (-)ve

$$\int_{t_0}^{t} e^{\alpha(\lambda-t_1)}\, d\lambda = \frac{1}{\alpha}\left[e^{\alpha(t-t_1)} - e^{\alpha(t_0-t_1)}\right]$$



Case III → $t_1 < t_0 < t$

Now, the integral always open (+)ve

$$\int_{t_0}^{t} e^{-\alpha(\lambda-t_1)}\, d\lambda = -\frac{1}{\alpha}\left[e^{-\alpha(t-t_1)} - e^{-\alpha(t_0-t_1)}\right]$$

Case IV

0 elsewhere

c) yes this is weakly stationary random process as its acf depends on τ.

Info:

(I) The process is stationary if the Acf does not depend on t or τ

(II) The process is weakly stationary if its Acf depends on τ.

(III) The process is non-stationary if its Acf depends on t.