Task 5.3

$$S_{xx}(\tau) = a e^{-\alpha|\tau|} + b$$

$$y(\eta, t) = \begin{cases} 0 & \text{for } t \le t_0 \\ \int_{t_0}^{t} x(\eta, \lambda)\, d\lambda & \text{for } t > t_0 \end{cases}$$

a) Determine the constant a and b

$$(m_x^{(1)})^2 = \lim_{\tau \to \alpha} S_{xx}(\tau) = 0$$

$$\Rightarrow \lim_{\tau \to \alpha} a e^{-\alpha|\tau|} + b = 0$$

$$\Rightarrow b = 0$$

Also, $m_x^{(2)} = \lim_{\tau \to 0} S_{xx}(\tau) = 1$

$$\lim_{\tau \to 0} a e^{-\alpha|\tau|} + b = 0$$

$$\Rightarrow a = 1$$

b) Determine the cross-correlation function

$$S_{xy}(t_1, t_2) = E\{x(\eta, t_1)\, y(\eta, t_2)\}$$

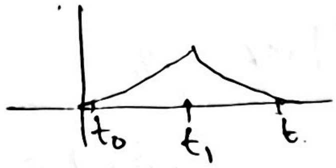
$$\Rightarrow E\{x(\eta, t_1) \int_{t_0}^{t} x(\eta, \lambda)\, d\lambda\}$$

$$\Rightarrow \int_{t_0}^{t} \underline{E\{x(\eta, t_1) \cdot x(\eta, \lambda)\}}\, d\lambda$$

$$\Rightarrow \int_{t_0}^{t} a e^{-\alpha|\tau|} + b \; d\lambda$$

$$\Rightarrow \int_{t_0}^{t} e^{-\alpha|\tau|} \cdot d\lambda = \int_{t_0}^{t} e^{-\alpha|\lambda - t_1|}\, d\lambda$$

Case 1 $t_0 < t_1 < t$



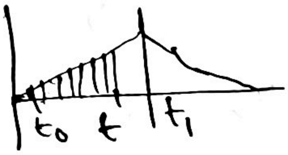
$$\int_{t_0}^{t_1} e^{\alpha(\lambda - t_1)} \, d\lambda + \int_{t_1}^{t} e^{-\alpha(\lambda - t_1)} \, d\lambda$$

$$\Rightarrow \frac{e^{\alpha(\lambda - t_1)}}{\alpha} \Bigg|_{t_0}^{t_1} + \frac{e^{-\alpha(\lambda - t_1)}}{-\alpha} \Bigg|_{t_1}^{t}$$

$$\Rightarrow \frac{1}{\alpha} \left[1 - e^{\alpha(t_0 - t_1)} - \left[e^{-\alpha(t - t_1)} - 1 \right] \right]$$

$$\Rightarrow \frac{1}{\alpha} \quad 2 - e^{\alpha(t_0 - t_1)} - e^{\alpha(t - t_1)}$$

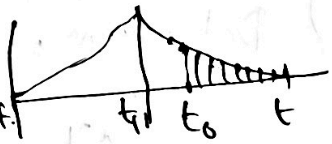
Case 2: $t_0 < t < t_1$



Now the Integral will open only negative part

$$\therefore \int_{t_0}^{t} e^{\alpha(\lambda - t_1)} \, d\lambda = \frac{1}{\alpha} \left[e^{\alpha(t - t_1)} - e^{\alpha(t_0 - t_1)} \right]$$

case 3: $t_1 < t_0 < t$



Now, the integral will open only positive part

$$\int_{t_0}^{t} e^{-\alpha(\lambda - t_1)} \, d\lambda = -\frac{1}{\alpha} \left[e^{-\alpha(t - t_1)} - e^{-\alpha(t_0 - t_1)} \right]$$

case 4: 0 elsewhere

c) As the function depends on time t, the ~~prosse~~ process is non stationary.

Stationary random process → do not depend on τ or t

weakly stationary random process → ~~do not~~ depends on τ

Non-stationary random process → depends on t.