

EXERCISE-3

Task 3.2

Given:- $x_1 = -1, x_2 = 0, x_3 = 1$

There are three outcomes of a discrete random process, when the process is repeated 3 times.

The probability of occurrence of

$$P(X_i | X_j) = \begin{cases} \frac{1}{3}(1 + 2e^{-|x_i|}) & \text{for } i=j \\ \frac{1}{3}(1 - e^{-|x_i|}) & \text{for } i \neq j \end{cases} \quad i, j = 1, 2, 3$$

we will use total probability theorem for finding the probability of x_1, x_2, x_3 i.e. $P(x_1), P(x_2), P(x_3)$ which states that -

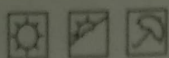
$$P(A) = \sum_n P(A|B_n) \cdot P(B_n)$$

P = Probability

A = any event

B_n = event

Let $P(x_1) = P_1, P(x_2) = P_2$ and $P(x_3) = P_3$



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using above theorem, we can write:-

Let:-

$$a_{ij} = \begin{cases} a, & \text{when } i=j = \frac{1}{3}(1+2e^{-1\tau}) \\ b, & \text{when } i \neq j = \frac{1}{3}(1-e^{-1\tau}) \end{cases}$$

$$P_1 = aP_1 + bP_2 + bP_3 \quad \text{--- (1)}$$

$$P_2 = bP_1 + aP_2 + bP_3 \quad \text{--- (2)}$$

$$P_3 = bP_1 + bP_2 + aP_3 \quad \text{--- (3)}$$

$$\begin{cases} P(X_1|1) \cdot P_1 + \\ P(X_2|1) \cdot P_2 + \\ P(X_3|1) \cdot P_3 = P_1 \end{cases}$$

{ It means the
Probability of $X=X_1$ when
the process is repeating
3 times }

Solving equation (1), (2) & (3) we get -

$$P_1 - P_2 = (a-b)P_1 - (a-b)P_2$$

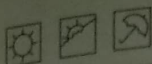
$$P_1 - P_2 = aP_1 - bP_1 - aP_2 + bP_2$$

$$P_1(1-a+b) = P_2(1-a+b)$$

$$P_1 = P_2$$

Similarly we can calculate from eqⁿ (2) & (3)

$$\therefore P_1 = P_2, P_2 = P_3 \text{ and } P_1 = P_3$$



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Now we know that Probability of occurrence of all outcomes of our event is 1.

$$\therefore P_1 + P_2 + P_3 = 1$$

$$P_1 + P_1 + P_1 = 1$$

$$3P_1 = 1$$

$$\Rightarrow \boxed{P_1 = \frac{1}{3} = P_2 = P_3}$$

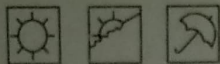
(b) ACF, $S_{xx}(L) = ?$

$$\text{ACF, } S_{xx}(L) = E \{ x(y, t+L) \cdot x(y, t) \}$$

$$= \sum_{i=1}^3 \sum_{j=1}^3 x_i x_j P(a/b) P_i$$

$$= \{ x_1 \cdot x_1 \cdot a P_1 + x_1 \cdot x_2 \cdot b P_2 + x_1 \cdot x_3 \cdot b P_3 + \\ x_2 \cdot x_1 \cdot b P_1 + x_2 \cdot x_2 \cdot a P_2 + x_2 \cdot x_3 \cdot a P_3 + \\ x_3 \cdot x_1 \cdot b P_1 + x_3 \cdot x_2 \cdot b P_2 + x_3 \cdot x_3 \cdot a P_3 \}$$

$$= \{ (-1)(-1) a (\frac{1}{3}) + 0 + (-1)(1) b (\frac{1}{3}) + 0 + 0 + 0 + \\ (1)(-1) b (\frac{1}{3}) + 0 + (1)(1) a (\frac{1}{3}) \}$$



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$$= \frac{a}{3} - \frac{b}{3} - \frac{b}{3} + \frac{a}{3}$$

$$= \frac{2}{3}(a-b)$$

$$= \frac{2}{3} \left[\frac{1}{3}(1+2e^{-1\pi}) - \frac{1}{3}(1-e^{-1\pi}) \right]$$

$$= \frac{2}{9} \cdot 3e^{-1\pi}$$

$$= \boxed{\frac{2}{3}e^{-1\pi}}$$