COEN 272

PageRank

Yi Fang

Department of Computer Science and Engineering

Santa Clara University

Motivation and Introduction

- What is PageRank?
  - A method for rating the importance of web pages objectively and mechanically using the link structure of the web.

- Why is Page Importance important?
  - New challenges for information retrieval on the World Wide Web.
  - Huge number of web pages: 150 million by 1998

    1000 billion by 2008
  - Diversity of web pages: different topics, different quality, etc.

The History of PageRank

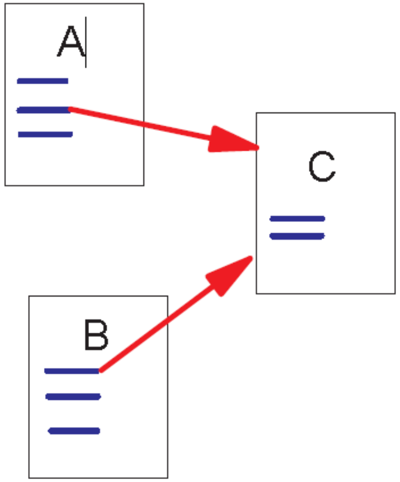
- PageRank was developed by Larry Page (hence the name *Page*-Rank) and Sergey Brin.

- It is first as part of a research project about a new kind of search engine. That project started in 1995 and led to a functional prototype in 1998.

- Shortly after, Page and Brin founded Google.

Link Structure of the Web

- 150 million web pages (in 1998) → 1.7 billion links



In-links and Out-links:

➢A and B are C’s in-links

➢C is A and B’s out-link

Intuitively, a webpage is important if it has a lot of in-links.

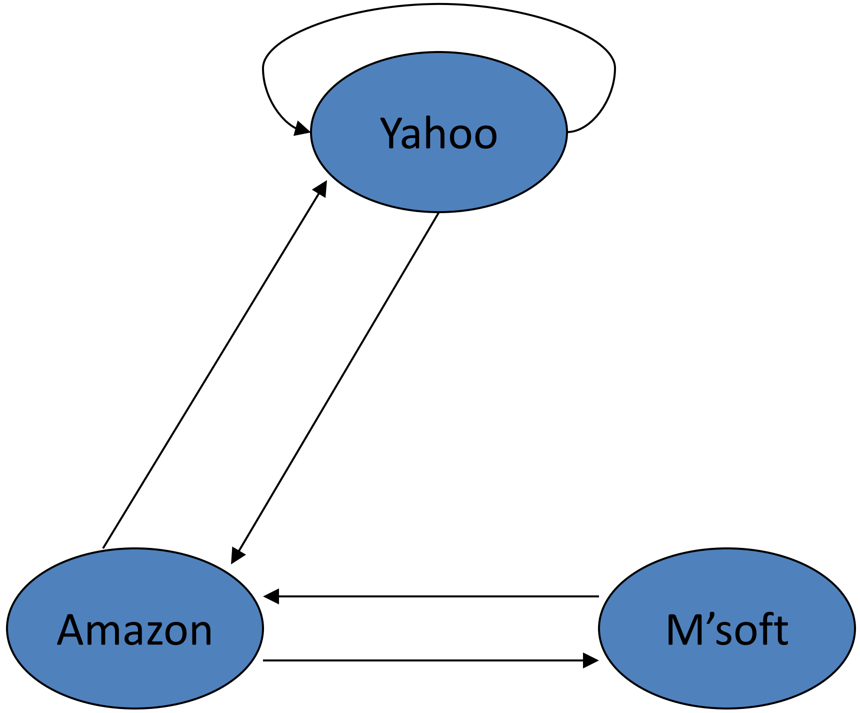
What if a webpage has only one link coming from www.yahoo.com?

Intuition

- Solve the recursive statement:

"a page is important if many important pages exclusively link to it."

Example



Example

- Solving equations:

$PR(y) = PR(y)* 1/2 + PR(a)*1/2+PR(m)*0$

$PR(a) = PR(y) *1/2 + PR(a)*0+PR(m)*1$

$PR(m) = PR(y)*0+PR(a)*1/2+PR(m)*0$

PR(y)		1/3	1/3	5/12	3/8		**2/5**
PR(a)	=	1/3	1/2	1/3	11/24	. . .	**2/5**
PR(m)		1/3	1/6	1/4	1/6		**1/5**

Converge!

Matrix Representation

- $v = \begin{bmatrix} PR(y) \\ PR(a) \\ PR(m) \end{bmatrix}, M = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 1 \\ 0 & 0.5 & 0 \end{bmatrix}$

- $v_{t+1} = M v_t$

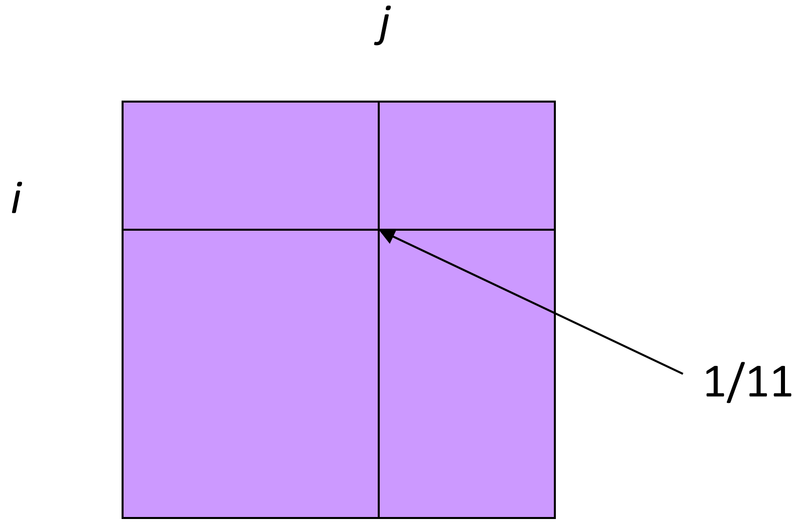
Stochastic Matrix of the Web

- Page *i* corresponds to row and column *I*

- *M* [*i*,*j*] = 1/*n* if page *j* links to *n* pages, including page *i* ; 0 if *j* does not link to *i*.
  - *M* [*i*,*j*] is the probability we'll next be at page *i* if we are now at page *j*.

Example

Suppose page *j* links to 11 pages, including *i*

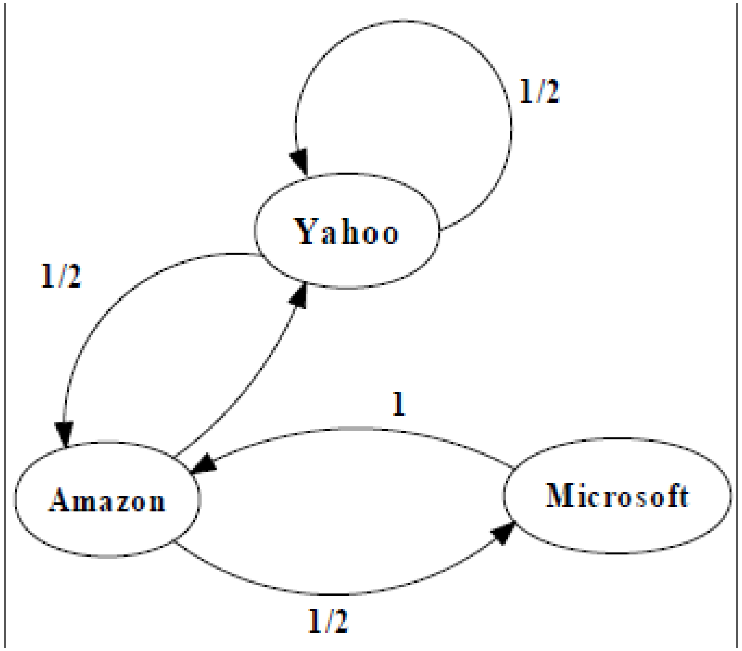


Power Method for PageRank

The iterative method to calculate PageRank

- Pick an initial guess v_0 (v_0 is a vector)
- $v_1 = Mv_0$
- $v_2 = Mv_1 = M^2 v_0$
- $v_3 = Mv_2 = M^3 v_0$
- … …
- Compute v_n until it converges (e.g., $||v_n - v_{n-1}|| < \varepsilon$ where ε is a small value)

An example of PageRank



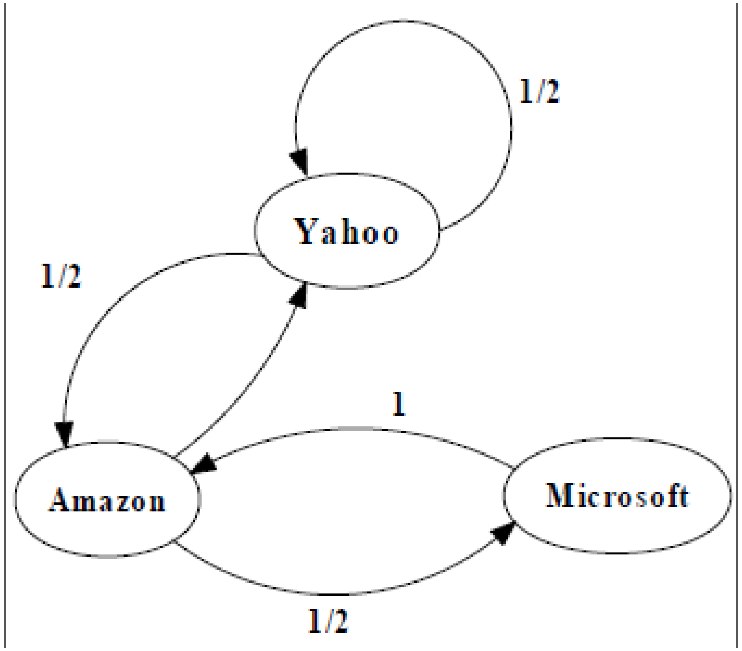
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

PageRank Calculation: first iteration

An example of PageRank



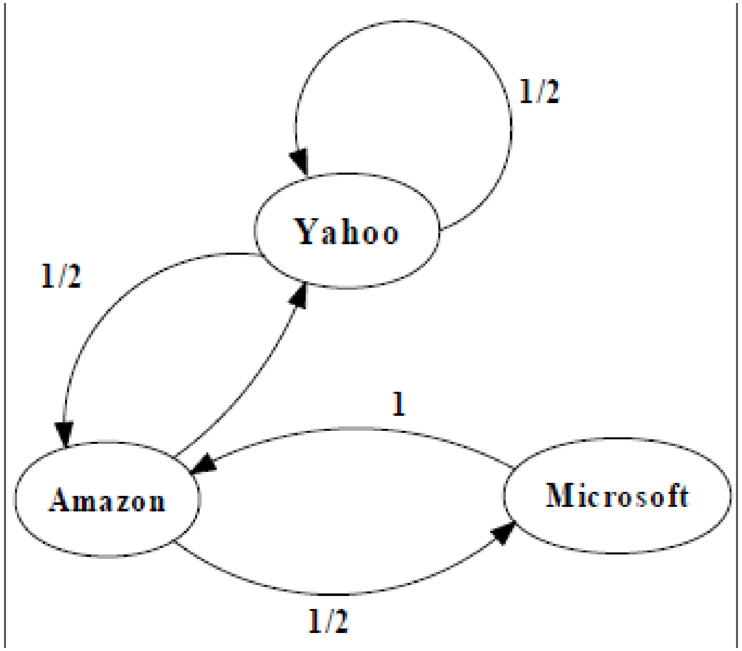
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/12 \\ 1/3 \\ 1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/2 \\ 1/6 \end{bmatrix}$$

PageRank Calculation: second iteration

An example of PageRank



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 3/8 \\ 11/24 \\ 1/6 \end{bmatrix} \begin{bmatrix} 5/12 \\ 17/48 \\ 11/48 \end{bmatrix} \ldots \begin{bmatrix} 2/5 \\ 2/5 \\ 1/5 \end{bmatrix}$$

Convergence after some iterations

PageRank

- PageRank essentially solves the following eigenvector problem:

  Find *v* to satisfy

$$v \quad = \; Mv$$

  Eigenvalue is equal to 1

- The $25,000,000,000 dollar

  eigenvector

FEBRUARY 20, 2006
www.time.com AOL Keyword: TIME
WHO'S BEHIND THE CARTOON MAYHEM? BEING OBAMA
TIME
CAN WE TRUST
Google
WITH OUR SECRETS?
An exclusive inside look at the $100 billion empire that is dominating the Internet
BY ADI IGNATIUS
Google honchos, from top, Larry Page, Eric Schmidt and Sergey Brin

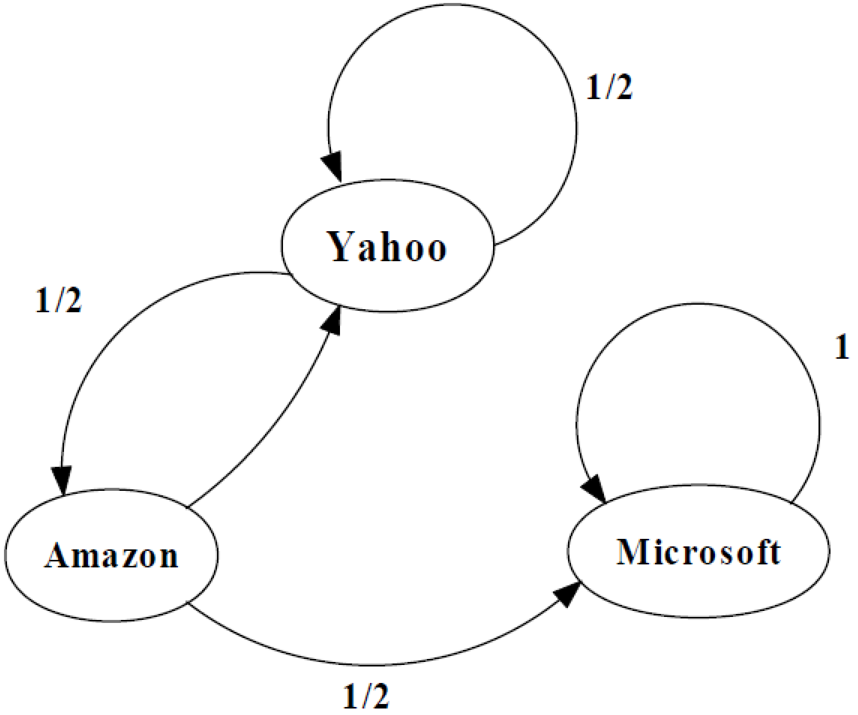
Random Walks on the Web

- Imagine a web surfer doing a random walk on the web
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, with equal probability (the transition probability from page *j* to *i* is M_{ij})

Random Walks on the Web

- After many random walks, the visit rate is the page’s PageRank.
- Intuition: pages are important in proportion to how often a random walker would visit them

An example of the Problem



$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 5/24 \\ 1/8 \\ 2/3 \end{bmatrix} \begin{bmatrix} 1/6 \\ 5/48 \\ 35/48 \end{bmatrix} \quad .... \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Google Solution to Traps: Teleporting

- At a dead end, jump to a random web page.
- At any non-dead end, with probability 20%, jump to a random web page.
  - With remaining probability (80%), go out on a random link.
  - 20% - a parameter.

Result of teleporting

- Now cannot get stuck locally.
- There is a long-term rate at which any page is visited
- How do we compute this visit rate?

Example: Previous with 20% Tax

- Equations **v** = 0.8(*M* **v**) + 0.2*1/3:

  $y = 0.8(y/2 + a/2) + 0.2*1/3$

  $a = 0.8(y/2) + 0.2*1/3$

  $m = 0.8(a/2 + m) + 0.2*1/3$

y		1/3	1.00/3	0.84/3	0.776/3		7/33
a	=	1/3	0.60/3	0.60/3	0.536/3	. . .	5/33
m		1/3	1.40/3	1.56/3	1.688/3		21/33

Modified Version of PageRank

$$PR(p_i) = \frac{1-d}{N} + d \sum_{p_j \in M(p_i)} \frac{PR(p_j)}{L(p_j)}$$

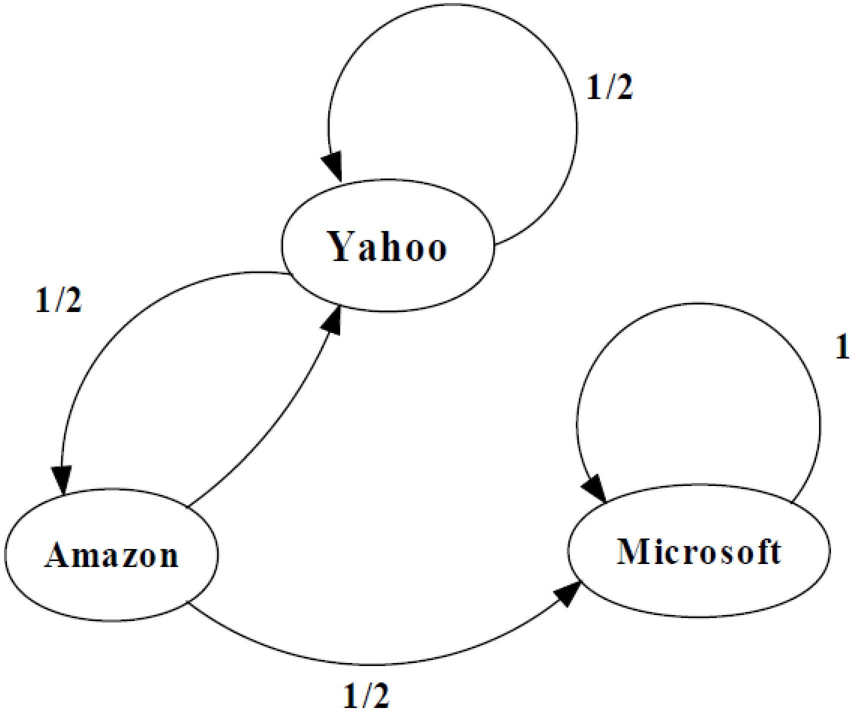
Power Method for PageRank with teleporting

- Pick an initial guess v_0 (v_0 is a vector)
- $v_1 = dMv_0 + \frac{1-d}{N} I$ (I is a vector with all ones)
- $v_2 = dMv_1 + \frac{1-d}{N} I$ (***d*** is teleporting parameter with typical value 0.8)
- $v_3 = dMv_2 + \frac{1-d}{N} I$
- … …
- Compute v_n until it converges (e.g., $||v_n - v_{n-1}|| < \varepsilon$ where ε is a small value)

Random Walks in Graphs

- The Random Walk Model
  - The simplified model: the standing probability distribution of a random walk on the graph of the web. Simply keeps clicking successive links at random
- The Modified Model
  - The modified model: the “random surfer” simply keeps clicking successive links at random, but periodically “gets bored” and jumps to a random page based on teleporting

An example of Modified PageRank



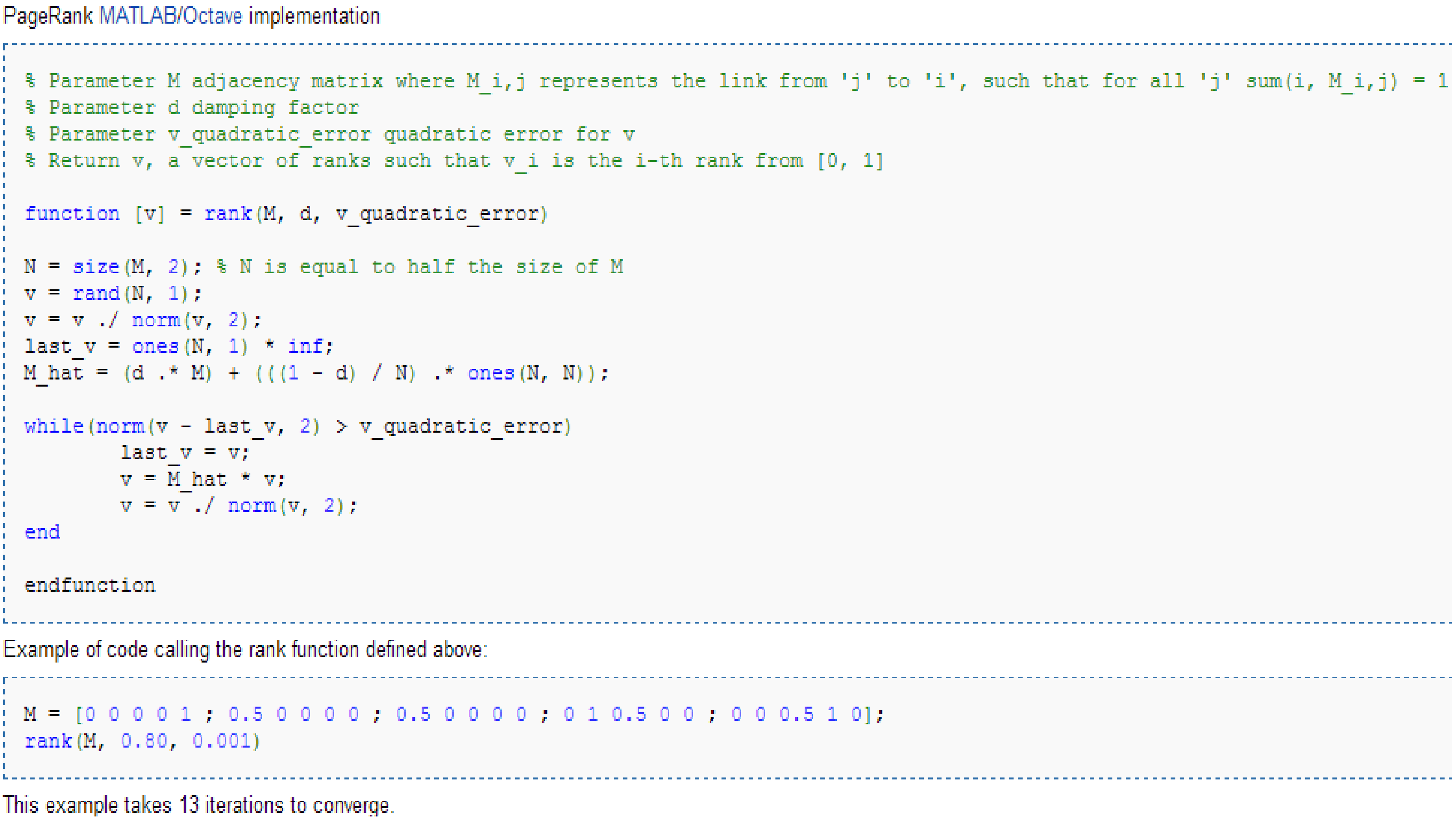
$$M = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \text{yahoo} \\ \text{Amazon} \\ \text{Microsoft} \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$C_1 = 0.8 \qquad C_2 = 0.2$

$$\begin{bmatrix} 0.333 \\ 0.333 \\ 0.333 \end{bmatrix} \begin{bmatrix} 0.333 \\ 0.200 \\ 0.467 \end{bmatrix} \begin{bmatrix} 0.280 \\ 0.200 \\ 0.520 \end{bmatrix} \begin{bmatrix} 0.259 \\ 0.179 \\ 0.563 \end{bmatrix} \ldots \begin{bmatrix} 7/33 \\ 5/33 \\ 21/33 \end{bmatrix}$$

Matlab Implementation



PageRank MATLAB/Octave implementation

```
% Parameter M adjacency matrix where M_i,j represents the link from 'j' to 'i', such that for all 'j' sum(i, M_i,j) = 1
% Parameter d damping factor
% Parameter v_quadratic_error quadratic error for v
% Return v, a vector of ranks such that v_i is the i-th rank from [0, 1]

function [v] = rank(M, d, v_quadratic_error)

N = size(M, 2); % N is equal to half the size of M
v = rand(N, 1);
v = v ./ norm(v, 2);
last_v = ones(N, 1) * inf;
M_hat = (d .* M) + (((1 - d) / N) .* ones(N, N));

while(norm(v - last_v, 2) > v_quadratic_error)
        last_v = v;
        v = M_hat * v;
        v = v ./ norm(v, 2);
end

endfunction
```

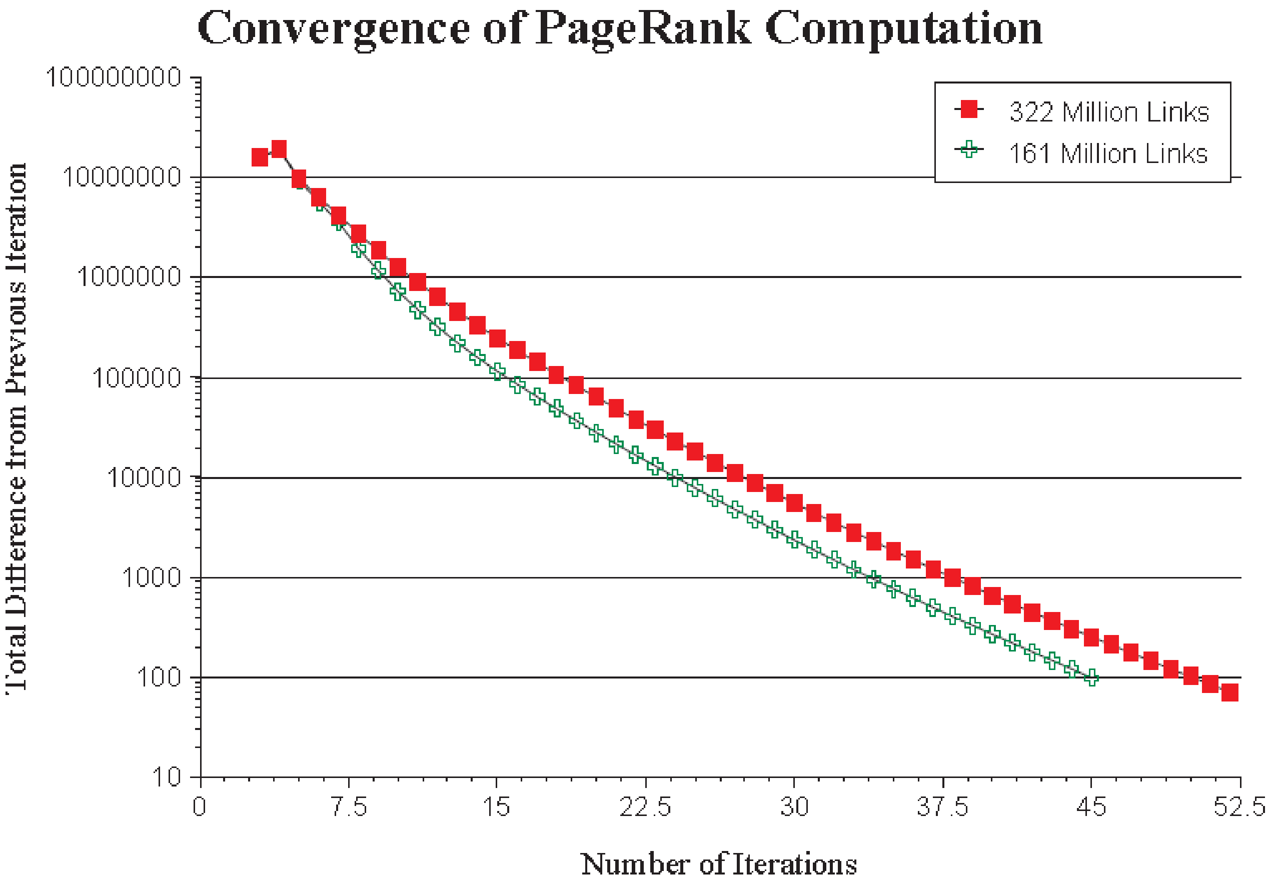
Example of code calling the rank function defined above:

```
M = [0 0 0 0 1 ; 0.5 0 0 0 0 ; 0.5 0 0 0 0 ; 0 1 0.5 0 0 ; 0 0 0.5 1 0];
rank(M, 0.80, 0.001)
```

This example takes 13 iterations to converge.

Convergence Property

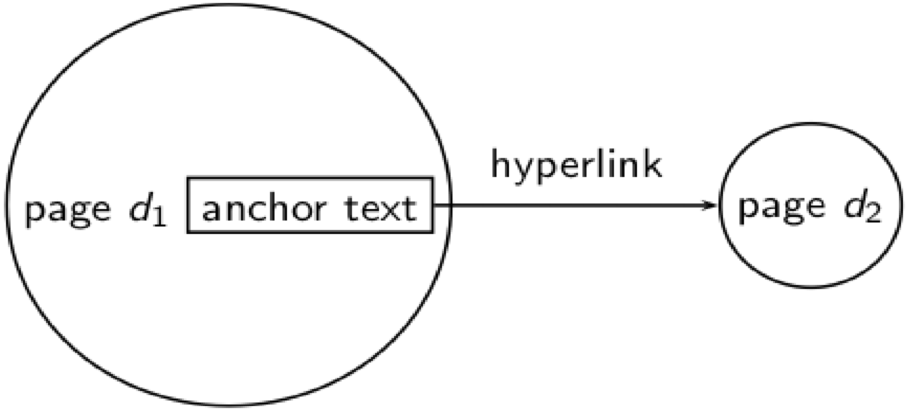
- PR (322 Million Links): 52 iterations
- PR (161 Million Links): 45 iterations
- Scaling factor is roughly linear in *logn*



Questions

- How to find the important persons on Twitter?
- How to find the important persons on Facebook?
- How to find the important persons on Yahoo Answers?

The web as a directed graph

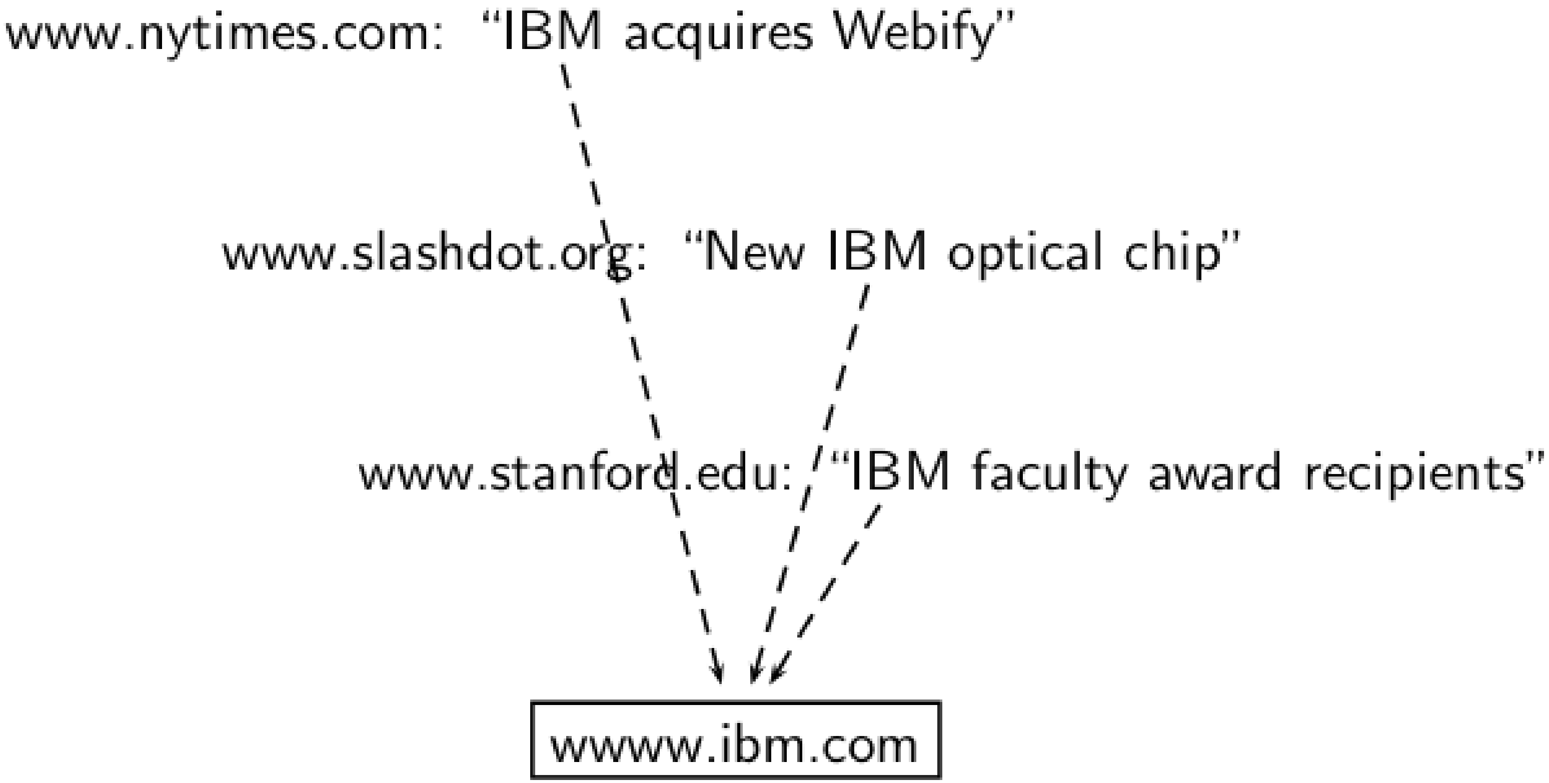


- A hyperlink is a quality signal.
  - The hyperlink d_1 → d_2 indicates that d_1's author deems d_2 high-quality and relevant.
- The anchor text describes the content of d_2.
  - We use anchor text somewhat loosely here for: the text surrounding the hyperlink .
  - Example: “You can find cheap cars <a href =http://…>here </a >. ”
  - Anchor text: “You can find cheap here”

[text of d_2] only vs. [text of d_2] + [anchor text → d_2]

- Searching on [text of d_2] + [anchor text → d_2] is often more effective than searching on [text of d_2] only.
- Example: Query *Bing*
  - Matches Bing's Legal page
  - Matches many spam pages
  - Matches Bing's wikipedia article
  - May not match Bing home page!
  - … if Bing home page is mostly graphics
- Searching on [anchor text → d_2] is better for the query *Bing*.
  - In this representation, the page with most occurrences of *Bing* is www.bing.com

Anchor text containing *IBM* pointing to www.ibm.com



www.nytimes.com: "IBM acquires Webify"

www.slashdot.org: "New IBM optical chip"

www.stanford.edu: "IBM faculty award recipients"

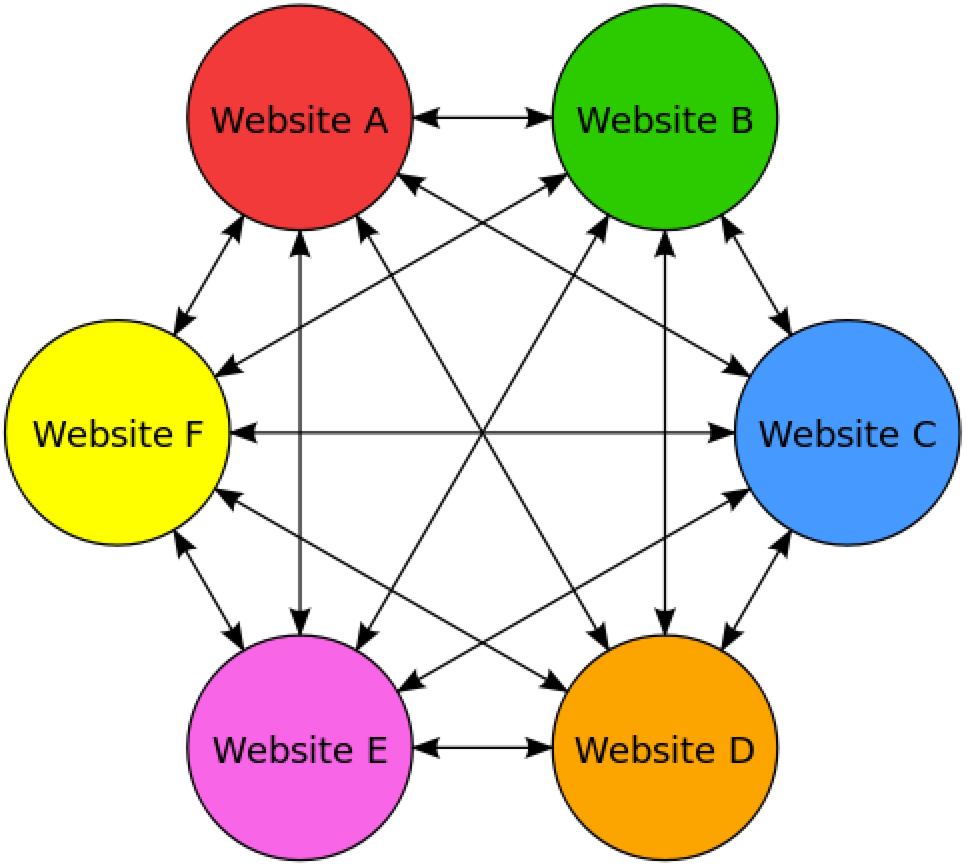
wwww.ibm.com

Exercise: Assumptions

- Assumption 1: A link on the web is a quality signal – the author of the link thinks that the linked-to page is high-quality.
- Assumption 2: The anchor text describes the content of the linked-to page.
- Is assumption 1 true in general?
- Is assumption 2 true in general?

Link Farm

A form of spamming trying to increasing the PageRank of member pages



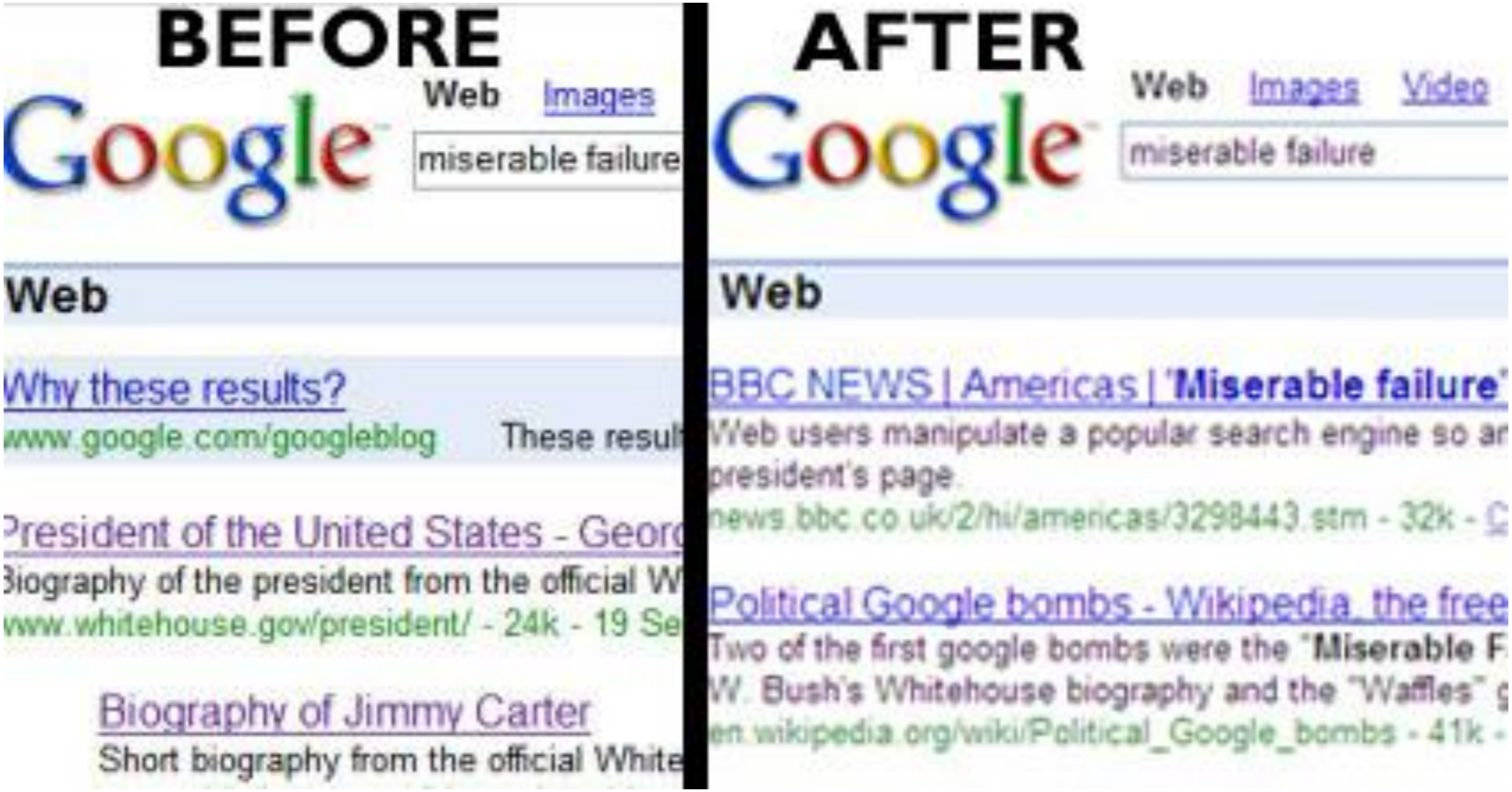
Google bombs

- A Google bomb is a search with “bad” results due to maliciously manipulated anchor text.

- Google introduced a new weighting function in January 2007 that fixed many Google bombs.

Google bombs

Coordinated link creation by those who dislike George W. Bush



BEFORE

miserable failure

Web

Why these results?
www.google.com/googleblog These resul

President of the United States - Georg
Biography of the president from the official W
www.whitehouse.gov/president/ - 24k - 19 Se

Biography of Jimmy Carter
Short biography from the official White

AFTER

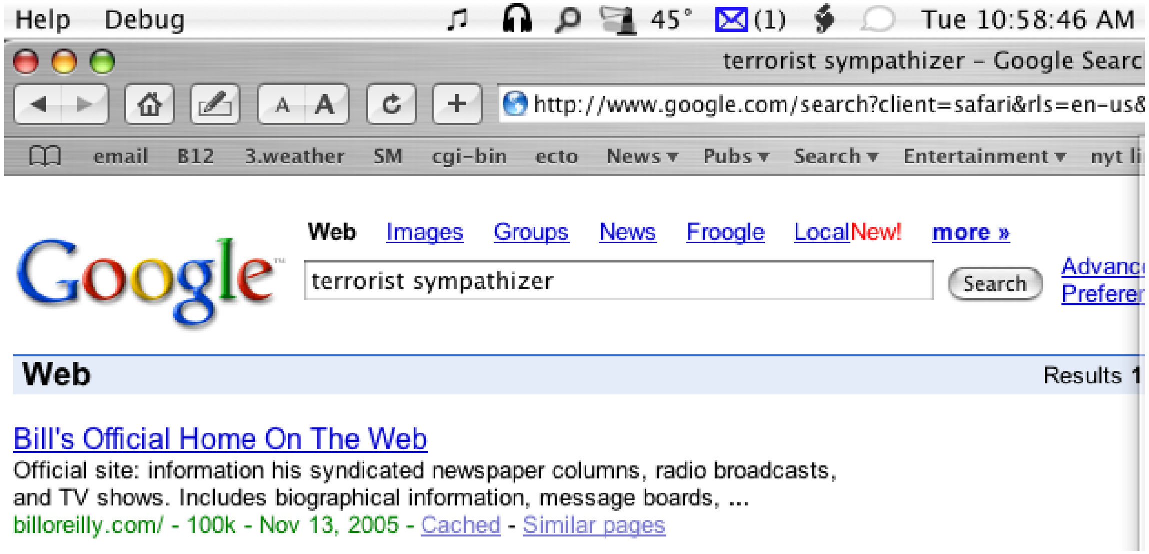
miserable failure

Web

BBC NEWS | Americas | '**Miserable failure**'
Web users manipulate a popular search engine so ar
president's page
news.bbc.co.uk/2/hi/americas/3298443.stm - 32k - C

Political Google bombs - Wikipedia, the free
Two of the first google bombs were the "**Miserable F**
W. Bush's Whitehouse biography and the "Waffles" g
en.wikipedia.org/wiki/Political_Google_bombs - 41k -

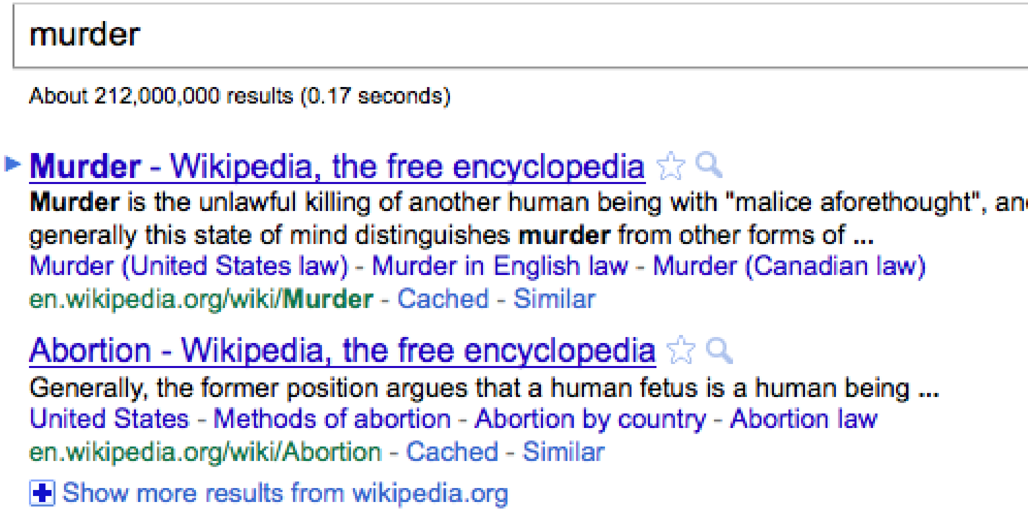
More Google bombs


Help Debug
45° (1) Tue 10:58:46 AM
terrorist sympathizer – Google Searc
http://www.google.com/search?client=safari&rls=en-us&
email B12 3.weather SM cgi-bin ecto News ▾ Pubs ▾ Search ▾ Entertainment ▾ nyt li
Web Images Groups News Froogle LocalNew! more »
Search
Advanc
Preferer

terrorist sympathizer

Web Results 1

Bill's Official Home On The Web
Official site: information his syndicated newspaper columns, radio broadcasts, and TV shows. Includes biographical information, message boards, ...
billoreilly.com/ - 100k - Nov 13, 2005 - Cached - Similar pages



murder

About 212,000,000 results (0.17 seconds)

Murder - Wikipedia, the free encyclopedia
Murder is the unlawful killing of another human being with "malice aforethought", an
generally this state of mind distinguishes **murder** from other forms of ...
Murder (United States law) - Murder in English law - Murder (Canadian law)
en.wikipedia.org/wiki/**Murder** - Cached - Similar

Abortion - Wikipedia, the free encyclopedia
Generally, the former position argues that a human fetus is a human being ...
United States - Methods of abortion - Abortion by country - Abortion law
en.wikipedia.org/wiki/Abortion - Cached - Similar

Show more results from wikipedia.org

Origins of PageRank: Citation analysis

- Citation analysis: analysis of citations in the scientific literature

- PageRank was invented in the context of citation analysis by Pinsker and Narin in the 1960s

- Weighted citation frequency

Question

- How to measure the similarities between webpages just based on the link structure?

- Measure the similarity of two pages by the overlap of other pages linking to them

- Google’s “find pages like this” or “Similar” feature

Origins of PageRank: Summary

- We can use the same formal representation for
  - hyperlinks on the web
  - citations in the scientific literature
  - social networks

  …