

① Selection sort time complexity:

for $j \leftarrow 1$ to n

$$C_1 \rightarrow n$$

do $\min \leftarrow j$ then $i \leftarrow i+1$

$$C_2 \rightarrow n-1$$

for $i \leftarrow i+1$ to n

$$C_3 \rightarrow n$$

do if $A[i] < A[\min]$

$$C_4 \rightarrow n$$

then $\min \leftarrow i$

$$C_5 \rightarrow n$$

if $j \neq \min$

$$C_6 \rightarrow n-1$$

then $A[j] \leftrightarrow A[\min]$

$$C_7 \rightarrow n-1$$

Time complexity = $Cn^2 + dn + C$

\therefore Time complexity for best case - $O(n^2)$

for worst case - $O(n^2)$

for worst case - $O(n^2)$

Merge sort time complexity:

If the number of element is n then
for best sort its complexity will be

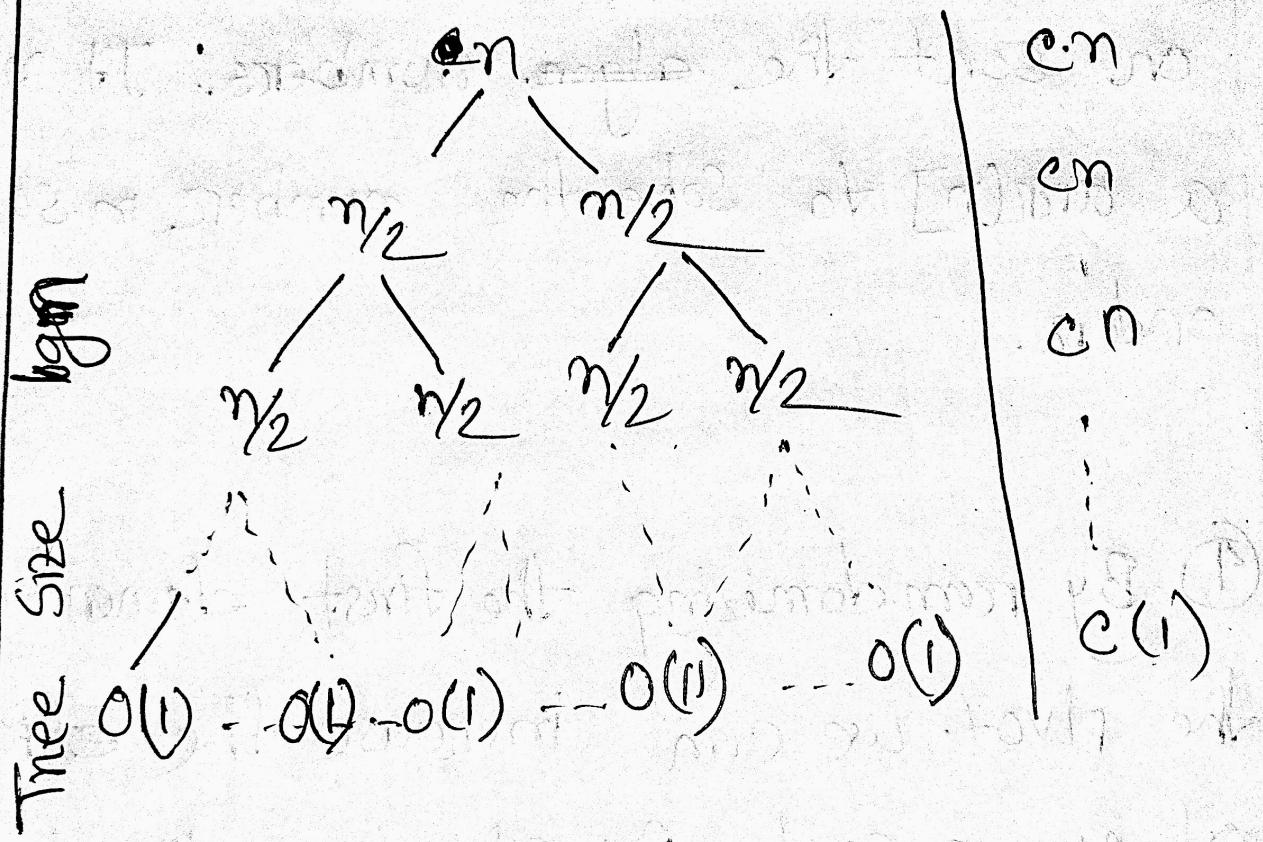
$$O(n \log n)$$

for the worst case it will be same
and also for the average case it remain
same.

Quick sort time complexity:

The time complexity for Quick sort is
 $O(n \log n)$ like merge sort. It also remain
same for worst case and average case.

② Recursion tree for Merge sort Algorithm.



$$\begin{aligned} \text{Total} &= c(n) + c(n/2) + c(n/4) + \dots + c(1) \\ &= \Theta(n \log n) \end{aligned}$$

Size of the tree is $\log n$.

③ The optimal space for the merge sort is $O(n)$ that required to do the task on ~~sort~~ the ~~algo~~ numbers. It need a $\text{arr}[n]$ to solve the numbers in sorting order.

④ By randomizing the first element or the pivot we can increase the efficiency of merge sort. By choosing the pivot randomly we can ignore the worst case for merge sort.

⑤ In question 4 we already said the procedure to increase the efficiency of merge sort algorithm as well as for quick sort.