BasicHT_Type I & II Errors

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When we need to decide if the evidence is strong enough to reject H_0, we have seen that the key is whether p-value falls below a prespecified significance level α.

We reject H_0 if p-value $\leq \alpha$

We do not reject H_0 if p-value $> \alpha$

The smaller α is, the stronger the evidence must be reject H_0.

Because of sampling variability, decisions in hypothesis testings always have some uncertainty. A decision can be in error.

Hypothesis testing has two types of potential errors

1. Type I error
2. Type II error

Type I error:- Type I error occurs when H_0 is true, but it is rejected.

One can think Type-I error as a false positive, because a positive decision is reached by rejecting H_0, yet the decision is false.

Type II error:- Type II error occurs when H_0 is false, but it is not rejected.

One can think Type-II error as false negative, because a negative decision is reached by not rejecting H_0, yet the decision is false.

Example: Consider a decision in a legal trial.

H_0: defendant is innocent

H_1: defendant is guilty

A Type-I error, rejecting null hypothesis, occurs in convicting a defendant who is actually innocent.

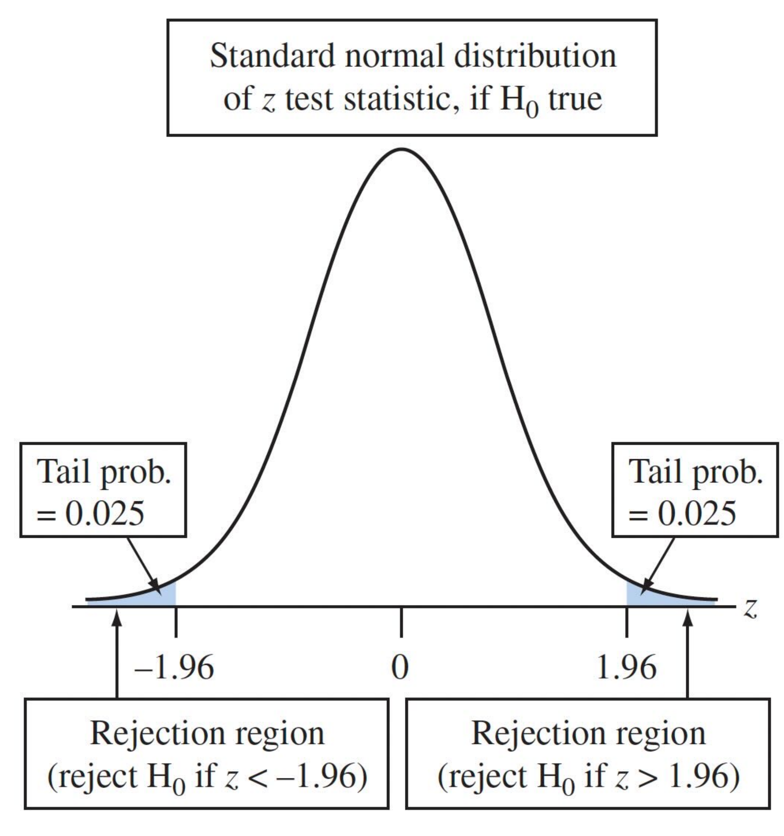
A Type-II error, not rejecting H_0 even though it is false, occurs in acquitting a defendant who is actually guilty

A potential consequence of a Type-I error is sending an innocent person to jail, whereas a potential consequence of a Type-II error is setting free a guilty person.

$\boxed{\text{The significance level is the probability of Type-I error.}}$

Let us look at a two-sided test about a proportion to see how we get this result.

Recall for a two-sided test, the two-tail probability that forms p-value ≤ 0.05 corresponds to test statistics z satisfies $|z| \geq 1.96$, which forms rejection region.



The probability of rejecting H_0 is the probability of of observing the z-statistics in the rejecting region. This prob would be

$$P(Z < -1.96) + P(Z > 1.96) = 0.05$$

We see that the summation of probability is equal to the significance level.

$$\text{Prob (Type I error)} = \text{significance level } (\alpha)$$

The good news is that we can control the prob of Type-I error by our choice of the significance level. The more serious the consequences of a Type-I error, the smaller α should be.

Trade-off between Type-I and Type-II errors:-

Why don't we make extreamly small prob of Type-I error by setting α to be small such as $\alpha = 0.000001$. For example, why don't we make it almost impossible to convict someone who is really innocent.

When we make α smaller, we need a smaller p-value to reject H_0. It then becomes harder to reject H_0. Thus, it will also be harder to reject H_0 even if H_0 is false.

In other words, the smaller we make the prob of Type-I error, the larger the prob of Type-II error becomes, that is failing to reject H_0 even though it is false

$\boxed{\text{As } P(\text{Type-I error}) \text{ goes down, } P(\text{Type-II error}) \text{ goes up}}$

Calculation of P(Type-II error) can be complex. In practice, to make decision, we need to set only P(Type-I error), which is the significance level.