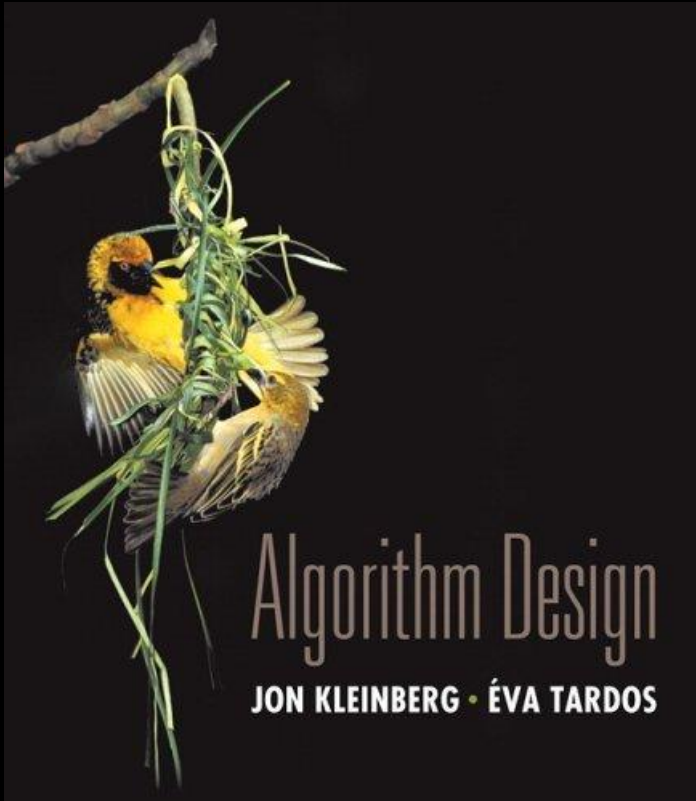


Chapter 7

Network Flow



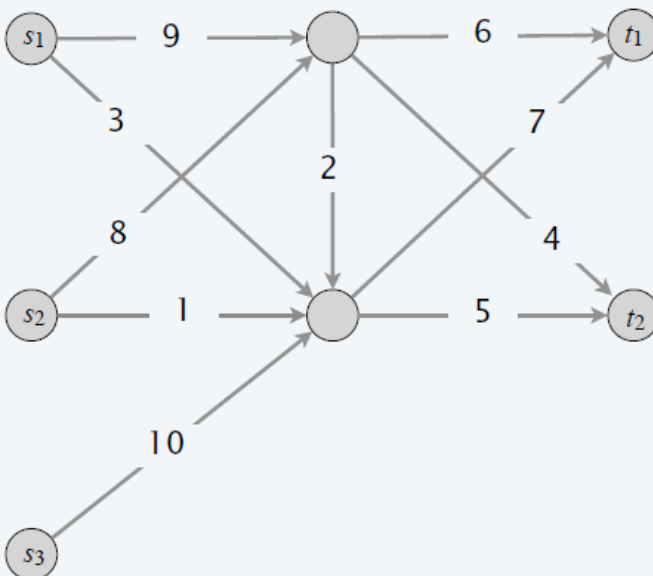
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7.7 Extensions to Maximum-Flow Problem

Multiple sources and sinks

Def. Given a digraph $G = (V, E)$ with edge capacities $c(e) \geq 0$ and multiple source nodes and multiple sink nodes, find max flow that can be sent from the source nodes to the sink nodes.

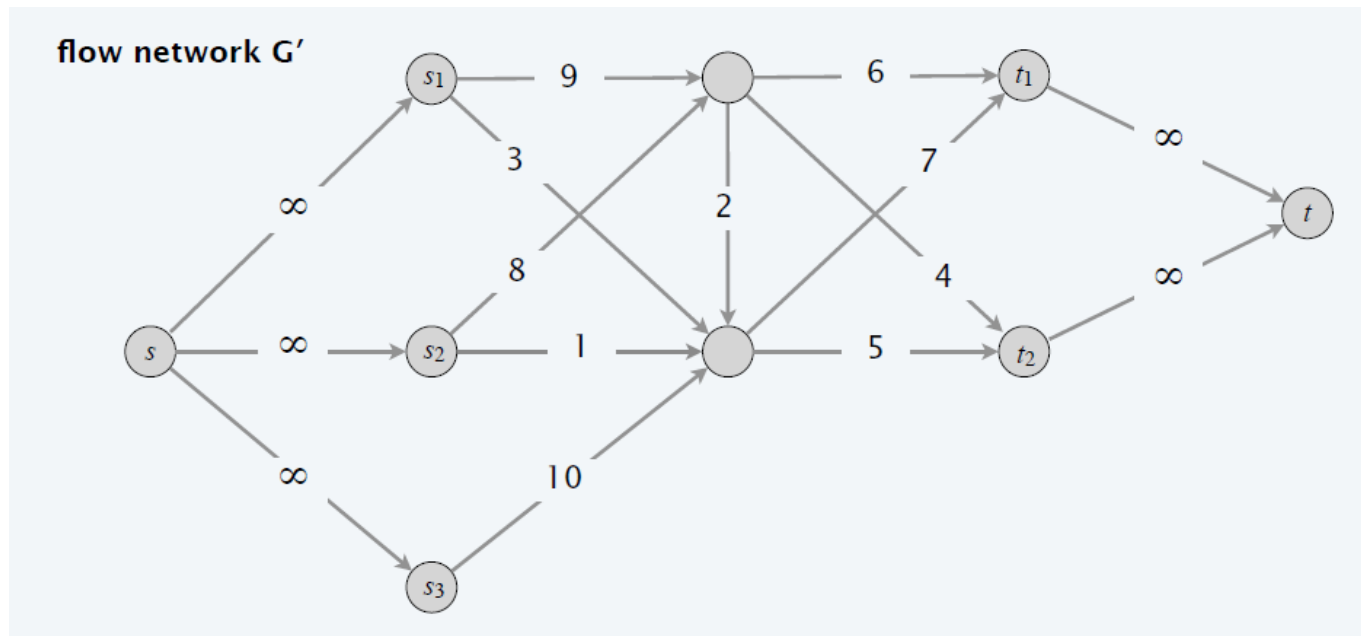
flow network G



Multiple sources and sinks: max-flow formulation

- Add a new source node s and sink node t .
- For each original source node s_i , add edge (s, s_i) with capacity ∞ .
- For each original sink node t_j , add edge (t_j, t) with capacity ∞ .

Claim. 1-1 correspondence between flows in G and G' .

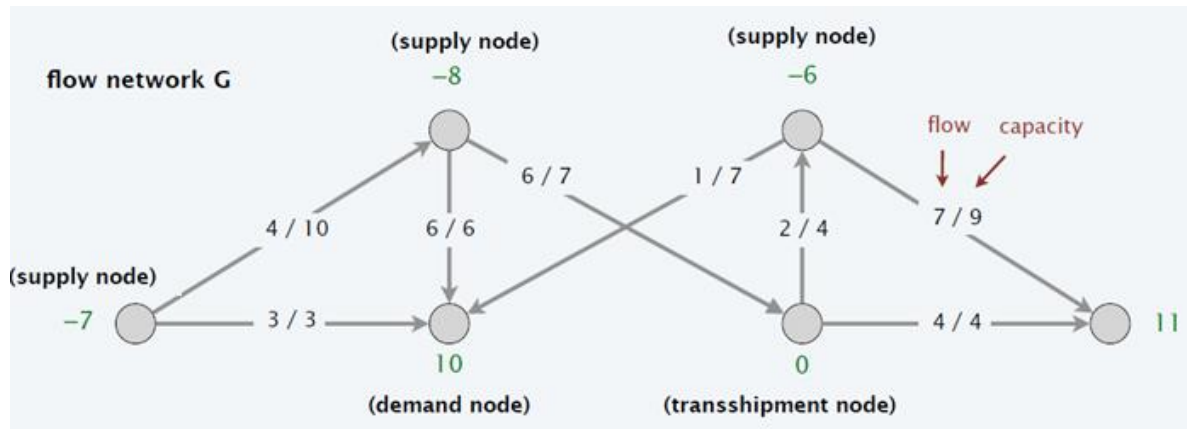


Circulation with supplies and demands

Def. Given a digraph $G = (V, E)$ with edge capacities $c(e) \geq 0$ and node demands $d(v)$, a **circulation** is a function $f(e)$ that satisfies:

For each $e \in E$: $0 \leq f(e) \leq c(e)$ (capacity)

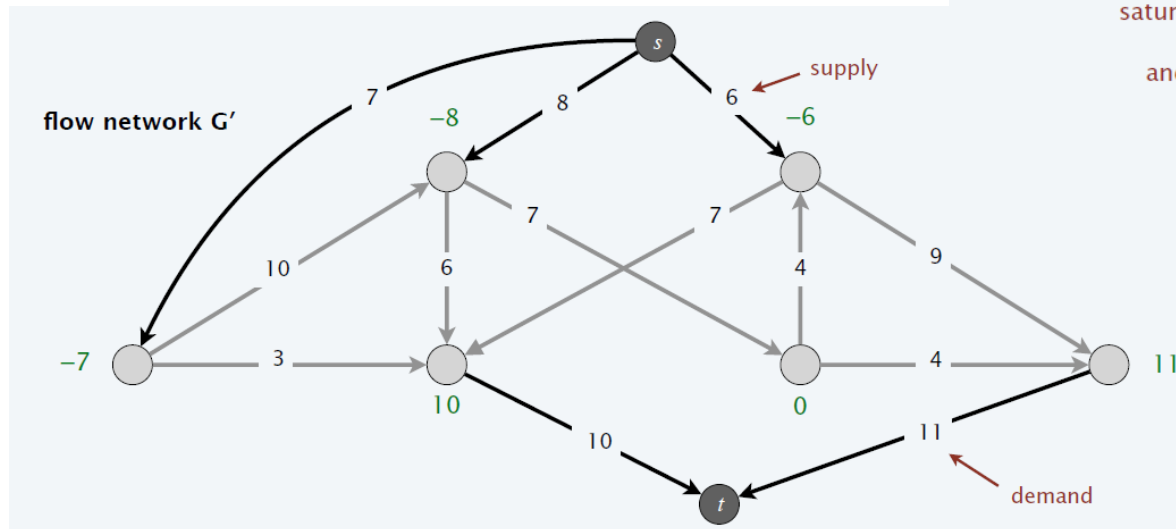
For each $v \in V$: $\sum_{e \text{ in to } v} f(e) - \sum_{e \text{ out of } v} f(e) = d(v)$ (flow conservation)



Circulation with supplies and demands: max-flow formulation

- Add new source s and sink t .
- For each v with $d(v) < 0$, add edge (s, v) with capacity $-d(v)$.
- For each v with $d(v) > 0$, add edge (v, t) with capacity $d(v)$.

Claim. G has circulation iff G' has max flow of value $D = \sum_{v: d(v) > 0} d(v) = \sum_{v: d(v) < 0} -d(v)$



saturates all edges
leaving s
and entering t

Circulation with supplies and demands

Integrality theorem. If all capacities and demands are integers, and there exists a circulation, then there exists one that is integer-valued.

Pf. Follows from max-flow formulation + integrality theorem for max flow.

Theorem. Given (V, E, c, d) , there does **not** exist a circulation iff there exists a node partition (A, B) such that $\sum_{v \in B} d(v) > \text{cap}(A, B)$.

Pf sketch. Look at min cut in G' .

↑
demand by nodes in B exceeds
supply of nodes in B plus
max capacity of edges going from A to B