

## Storing the nonlinear expansions

### SSM parametrisation

$$\mathbf{W}_\epsilon(\mathbf{p}, \phi) = \mathbf{W}(\mathbf{p}) + \epsilon \mathbf{X}(\mathbf{p}, \phi) + \mathcal{O}(\epsilon^2)$$

#### Autonomous

Expanded as  $\mathbf{W}(\mathbf{p}) = \sum_{\mathbf{m}} \mathbf{W}_{\mathbf{m}} \mathbf{p}^{\mathbf{m}}$  - stored in struct array W0

W0.coefs(m) - array that contains all order  $m$  coefficients  $[\mathbf{W}_{\mathbf{m}_1}, \dots, \mathbf{W}_{\mathbf{m}_{z_m}}]$

W0.ind(m) - array that contains the corresponding multi-indices  $[\mathbf{m}_1, \dots, \mathbf{m}_{z_m}]^T$

#### Nonautonomous

Expanded as  $\mathbf{X}(z, \phi) = \sum_{\mathbf{m}} \sum_{\kappa} \mathbf{X}_{\mathbf{m}, \kappa} z^{\mathbf{m}} e^{i\langle \phi, \kappa \rangle}$  stored in struct array W1

W1(i).kappa contains the  $i$ -th harmonic  $\kappa_i$

W1(i).W contains the coefficients corresponding to  $\kappa_i$

W1(i).W(m).coefs contains  $[\mathbf{X}_{\mathbf{m}_1, \kappa_i}, \dots, \mathbf{X}_{\mathbf{m}_{z_m}, \kappa_i}]$

W1(i).W(m).ind contains  $[\mathbf{m}_1, \dots, \mathbf{m}_{z_m}]^T$

### Reduced Dynamics

$$\mathbf{R}_\epsilon(\mathbf{p}, \phi) = \mathbf{R}(\mathbf{p}) + \epsilon \mathbf{S}(\mathbf{p}, \phi) + \mathcal{O}(\epsilon^2)$$

#### Autonomous

Expanded as  $\mathbf{R}(\mathbf{p}) = \sum_{\mathbf{m}} \mathbf{R}_{\mathbf{m}} \mathbf{p}^{\mathbf{m}}$  - stored in struct array R0

R0.coefs(m) - array that contains all order  $m$  coefficients  $[\mathbf{R}_{\mathbf{m}_1}, \dots, \mathbf{R}_{\mathbf{m}_{z_m}}]$

R0.ind(m) - array that contains the corresponding multi-indices  $[\mathbf{m}_1, \dots, \mathbf{m}_{z_m}]^T$

#### Nonautonomous

Expanded as  $\mathbf{S}(z, \phi) = \sum_{\mathbf{m}} \sum_{\kappa} \mathbf{S}_{\mathbf{m}, \kappa} z^{\mathbf{m}} e^{i\langle \phi, \kappa \rangle}$  stored in struct array R1

R1(i).kappa contains the  $i$ -th harmonic  $\kappa_i$

R1(i).R contains the coefficients corresponding to  $\kappa_i$

R1(i).R(m).coefs contains  $[\mathbf{S}_{\mathbf{m}_1, \kappa_i}, \dots, \mathbf{S}_{\mathbf{m}_{z_m}, \kappa_i}]$

R1(i).R(m).ind contains  $[\mathbf{m}_1, \dots, \mathbf{m}_{z_m}]^T$