# The Wade Formula and the oil price fluctuation: An Optimal Control Theory approach

**Abstract.** By considering the Wade Formula, we propose a model to study the evolution of the oil price per barrel. Our model shows that the policy of diversification of the energy is to be supported. This model is proposed to see how it is possible to control parameters so that the oil price should decrease.

**Keywords.** Wade Formula, optimal control, options, volatility, oil prices.

The Wade Formula and the oil price tion: An Optimal Control Theory approaches the evolution of the oil price per barrel. Our model shows that the diversification of the energy is to be supported. This model is propose how it is possible to control parameters so that the oil price should de Keywords. Wade Formula, optimal control, options, volatility, oil price the second half of 2008 and 2014, (see [1], [2]), and the consequence Ukraine conflic on oil price, ([3], [4], [5]), has renewed the interest in the price fluctuation [figure 1, curve(d)], and the effects of energy prices are conomic activity ([6], [11], [7], [8], [9], [10], [13], [15], [14]).

While the oil price shock is asymmetric between oil exporters countries ([12], [16], [17]), this asymmetric oil shock had insported to the oil price per barrel. This formula translates the super-profits generative (all insight proposed a formula related to the oil price per barrel. This formula translates the super-profits generative (all insight proposed and proposed a formula related to the oil price per barrel. This formula translates the super-profits generative (all insight proposed and proposed a formula related to the oil price per barrel. This formula translates the super-profits generative (all insight proposed and proposed a formula related to the oil price per barrel. This formula translates the super-profits generative (all insight proposed and proposed a formula related to the oil price per barrel. This formula translates the super-profits generative (all proposed and The rapid increase in oil prices between 2002 and 2008 and their sharp decline in the second half of 2008 and 2014, (see [1], [2]), and the consequence of covid19 and Ukraine conflic on oil price, ([3], [4], [5]), has renewed the interest in the causes of oil price fluctuation [figure 1, curve(d)], and the effects of energy prices on the macroe-

A large of studies prove that the oil fluctuations have a considerable consequence in

While the oil price shock is asymmetric between oil exporters countries and oil importers countries ([12], [16], [17]), this asymmetric oil shock had inspired the Wade

Between 2005 and 2006, the full professor in Economics Abdoulaye Wade, former President of the Republic of Senegal, proposed a formula related to the evolution of the oil price per barrel. This formula translates the super-profits generated by the oil companies (selling in the world) and countries which produce oil.

At first, from the Wade Formula, we describ in section 2, the model. We resolve and show a necessary optimal condition which could give some hints for the orientations of the energy policies in section 3. We propose some simulations and interpretations of the obtained results, section 4. As conclusion in section 5, we present some variations of the obtained results and politics implications.

This work was completed with the support of the NLAGA project.

## 2. Model description

The formula should be stated as follows: Let be the time  $t \in [T_0, T]$ , where  $T_0$  is the initial state. Since it is possible to find the structure of the oil price/barrel without financial speculation, President Wade studied and proposed a reasonable price (winwin price)  $P_0 = 29$ \$. For more details, see [18].

The Wade Formula for super profit is defined by the equation (2.1)

$$(p_t - 29)q_t = S_t (2.1)$$

where  $p_t$  represents the price which is applied in the world for buying a barrel of oil,  $q_t$  is the quantity of oil bought by the countries, and  $S_t$  is the super profit for the oil companies or the super coast supported by the countries which don't produce oil. It is important to remark that this formula is a conservation law.

# 3. Model resolution: Optimal control approach

In this section, we give a necessary optimal condition to minimize the super coasts, denoted  $^{1}$ , S(t).

A reasonable model to describe the evolution of the reserves of oil in the world can be written as follows:

$$\frac{dR}{dt} = -v(t) + \alpha(t)R(t), t \ge 0 \tag{3.1}$$

where  $\alpha(t) \in [0, 1[$  is the rate of the increase of oil in the world, R(t) are the reserves and v(t) is the demand in the world at time t.

In the following, we are going to take  $\alpha(t) = \alpha$  a real constant. But the theorem below is satisfied even if  $\alpha$  depends on the time t.

**Theorem 3.1.** Let a(t) be the demand of the country which doesn't produce oil, then a necessary optimal condition to minimize the super coasts for a country which doesn't product oil is:

$$a^*(t) = \frac{c_0 \exp(-\alpha t)}{2(p(t) - 29)^2}.$$

*Proof.* Let T be a fixed time.

$$\min \int_0^T S(t)^m dt, \quad m \in \mathbb{N}$$
 (3.2)

$$s.c \begin{cases} \frac{dR}{dt} = -v(t) + \alpha R(t) & on [0, T] \\ R(0) = Q \end{cases}$$
(3.3)

where, Q is the initial stock of the reserves,

$$S(t) = (p(t) - 29)a(t),$$

<sup>&</sup>lt;sup>1</sup>For more details in optimal control theory see [19], [20], [21], [22]

$$v = a + w$$
.

where a is the demand of country which doesn't produce oil. Let's take the Hamiltonian defined by:

$$H(R(t), a(t), \lambda(t), t) = S(t)^m + \lambda(t)(-v(t) + \alpha R(t))$$
(3.4)

and  $\lambda(t)$  being the Pontryagin multiplier.

We have the necessary optimal conditions:

$$\begin{array}{lcl} \frac{\partial H}{\partial R} & = & \alpha \lambda(t) = -\frac{d\lambda}{dt} \\ \frac{\partial H}{\partial a} & = & 0 \end{array}$$

The condition  $\frac{\partial H}{\partial a} = 0$  is equivalent to  $mS(t)^{m-1}(p(t) - 29) - \lambda = 0$ . This implies that:

$$S(t) = \left(\frac{\lambda}{m(p(t) - 29)}\right)^{\frac{1}{m-1}}$$

Since:

$$-\frac{d\lambda}{dt} = \alpha\lambda(t),$$

then  $\lambda(t) = c_0 \exp(-\alpha t)$  where  $c_0$  is a real constant. Finally:

$$S(t) = \left(\frac{c_0 \exp(-\alpha t)}{m(p(t) - 29)}\right)^{\frac{1}{m-1}}$$

If m=2, we obtain:

$$S(t) = \frac{c_0 \exp(-\alpha t)}{2(p(t) - 29)}$$
 and by the following equality:

$$S(t) = (p(t) - 29)a(t)$$
 we have:

$$S(t) = (p(t) - 29)$$

$$S(t) = (p(t) - 29)a(t) \text{ we have:}$$

$$a*(t) = \frac{c_0 \exp(-\alpha t)}{2(p(t) - 29)^2} \square$$

#### 3.1. Initial condition variation

We assume that R(t) is Lipschitzian, so there exists a flow defined by:

$$\begin{cases} X \to X \text{ space of phases} \\ x_0 \to \phi_t(x_0) = x(t) \end{cases}$$

with  $X = \mathbb{R}^n$ , for n = 1,

$$\begin{cases} x(t) \to R(t) \\ x_0 \to Q \end{cases}$$

We consider a period of horizon h on the time interval  $[t_0, t_0 + h]$  and  $Q \in [Q_0, Q^*]$  with  $Q_0 = Q(t_0)$  and  $Q^* = Q(t_{0+h})$ .

We define:

$$Q_k = Q_0 + \frac{k}{t_0 + h}(Q^* - Q_0)$$

The initial conditions vary with the variants of  $k \in [t_0, t_0 + h]$ .

#### 3.2. Terminal condition variation

We still consider equations (3.3) and (3.4) and rewrite them by varying the terminal conditions. Let  $s \in [0, T]$ . Let t = T - s, Y(s) = R(T - s). The optimization problem becomes:

$$\min \int_0^T [S(T-s)]^m ds$$

$$sc \begin{cases} \frac{dY(s)}{ds} = -v(T-s) + \alpha Y(s) \\ Y(0) = R_T \end{cases}$$

where  $R_T$  is a real value sequence on  $[Y_0, ..., Y^*]$  with  $Y_0 = Y(t_0)$  and  $Y^* = Y(t_0 + h)$ .

$$R_T(k) = Y_0 + \frac{k}{t_0 + h} (Y^* - Y_0)$$
$$S(T - s) = (P(T - s) - 29) a(T - s)$$

In the next section we are going to use the curve below. This represents the plot of the result of the necessary optimal condition. The horizontal axis being the values of the prices while the vertical axis is the values of the demand  $a^*$ .

# 4. Numerical resolution and results interpretations

The simulation parameters are as follows. In the graphs of figure 1, we have taken as constant c the average of the percentage rate increase of the world fossil reserves. In figure 2, we consider the evolution of the rate of the resources over the study period in the numerical resolution of our optimal control problem The curves in Figures 1-2 are the results of the optimal control resolution. The result could be interpreted as follows:

1. Between 1980 and 2021, [1d], there were two periods where the average price of oil was above 29 dollars, 1980 to 1983 and 2004 to 2021, and a period where it was below 29 dollars. In the first period (1980 to 1983), the average price of a barrel of oil fluctuated between 30 dollars and 37 dollars. It peaked at 39 dollars in 1981 due to the Reagan cut taxes. Two other causes of the high fluctuation of the average barrel price are the Iran embargo in 1980 and the end of the recession in 1982.

- 2. During the second period (2004-2021), the average price of a barrel of oil fluctuated between about 35 dollars in 2004 and 103 dollars in 2011. It reached a peak of 127 dollars in 2008 during the financial crisis. Other factors that explain the high fluctuation during this period are Hurricane Katrina in 2005, Bernanke becomes Fred Chair in 2006, the banking crisis in 2007, the great recession in 2009, Iran threatening the Straits of Hormuz, the increase of the 15 % dollars, the US shale oil increased 2015, the decrease of the dollars in 2016, the OPEC cut oil supply to keep prices stable and demand reduction of the pandemic since 2020.
- 3. From 1984 to 2003, the average price of a barrel oscillated between 14 dollars in 1986 and 27 dollars in 2003. During the Gulf War, the peak was reached with a barrel at almost 33 dollars. Other events during this period allowed this fluctuation are the prices doubled, the recession and the war in Afghanistan in 2002. The barrel reached the lowest price of 10 dollars in 1999.
- 4. The curves [1c] and [1d] have the shape of the sigmoid function. The world's demand for fossil energy cannot grow in an unlimited way. With climate change and the strong fluctuations of the oil price, many countries are diversifying their energy sources toward renewable energies. Fossil fuel reserves are also limited despite the discovery of new deposits.
- 5. The evolution of the Super profit (curve c fig1) is consecutive to the differential between the price 29 and the average price oil with a phase of strong fall between 1984 and 2003
- 6. The curves [2a] and [2b] have the shape of the sigmoid function. The world's demand for fossil energy cannot grow in an unlimited way. With climate change and the strong fluctuations of the oil price, many countries are diversifying their energy sources toward renewable energies. Fossil fuel reserves [2c] are also limited despite the discovery of new deposits.
- 7. The countries which don't product oil have to diversify their energy sources as soon as possible. It would be a good orientation even for the countries which product oil. Because the resources will disappear at a time  $T^*$ .
- 8. The strategies to develop the research in the other type of energy and their production have to be encouraged. This assertion is justified in the following sense: when we plot the necessary optimal condition (see Figure 1 a)<sup>2</sup>, it is easy to see that, if the price of the barrel goes far from 29 dollars,  $a^*$  decreases. And then the super coast S(t) is minimized. But this quantity will not suffice for the consumption in the country because of growth of the needs in energy.

<sup>&</sup>lt;sup>2</sup>The same interpretation can be done for the real data [Figure 1b)]

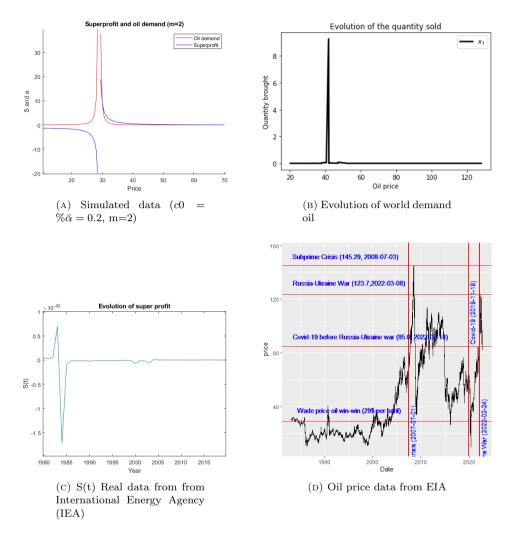
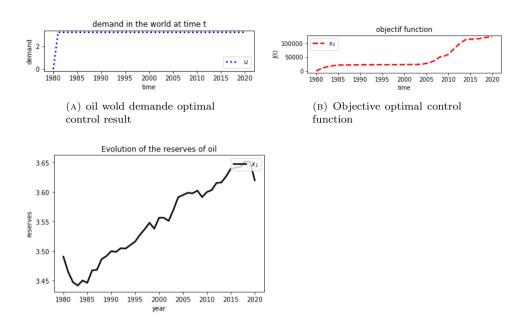


FIGURE 1. Wade Curve

# 5. Concluding and Remarks

The Wade Formula is also satisfied if one day in the future the 29 dollars are not reasonable like a win win price. It suffices to replace 29 by  $P_0(t)$  where  $P_0(t)$  could satisfy the following system of equation and without financial speculation  $\frac{dP_0(t)}{dt} = f(i(t)) - \mu P_0(t)$  and  $P_0(t) = i(t) + pr$ . The index i is the investment per barrel,



(c) Reserves real data from IEA

Notes: The horizontal lines in Fig. 2 represent the maximum price of a barrel of oil
during the different crises (Subprime around \$146, covid19 before the Ukrainian
war, \$85 and Russo-Ukrainian war around \$124). The vertical lines are the beginning
of the different cries

 $\mu \in [0,1[$  is the depreciation rate, pr represents the reasonable profit and f is a function to be determined.

### References

- [1] C. Baumeister and L.Kilian *Understanding the Decline in the Price of Oil since June* 2014, Journal of the Association of Environmental and Resource Economists, 2016 3(1), 131–158.doi:10.1086/684160.
- [2] R. Arezki and O. Blanchard (2015), The 2014 Oil Price Slump: Seven Key Questions, VoxEU.org, 13 January, https://voxeu.org/article/2014-oil-price-slump-seven-key-questions.
- [3] J. Worland, Answers to Six Key Questions About the Oil Price Collapse, https://time.com/5800452/oil-price-questions-coronavirus/March, 11/2020.
- [4] F. Taghizadeh-Hesary., Economic impacts of the COVID-19 pandemic and oil price collapse, Economics health MAY 18, 2020.

- [5] The BBC news. Avalaible on http://www.bbc.com/news/business-60642786.
- [6] F. Taghizadeh-Hesary and N.Yoshino, Macroeconomic Effects of Oil Price Fluctuations on Emerging and Developed Economies in a Model Incorporating Monetary Variables ADBI Working Paper Series, October 2015 No. 546.
- [7] S.Oludiran Akinleye and S.Ekpo Oil Price Shocks and Macroeconomic Performance in Nigeria, economía mexicana nueva época, 2, pp.565-624, 2013.
- [8] T.M. Fullerton and T.Soto Oil Shock Impacts on the Borderplex Regional Economy, (1), 5, pp. 14-26, 2015.
- [9] Ftiti Z. and K. Guesmi and F.Teulon Oil Shocks and Economic Growth in OPEC countries, IPAG Business School Working Paper 2014-064, 2014.
- [10] B.DePratto and C.De Resende and P. Maier, How Changes in Oil Prices Affect the Macroeconomy, Bank of Canada Working Paper 33, 2009.
- [11] R. Jiminez-Rodríguez and M.Sanchez, Oil price shocks empirical evidence for some OECD countries European Central Bank Working Paper Series, NO. 362 / MAY 2004.
- [12] P.Sgro and N.Apergis, Asymmetric oil shocks and external balances of major oil exporting and importing countries, Energy Economics, 2016, vol. 56, issue C, pages 42-50.
- [13] F. Taghizadeh-Hesary, N. Yoshino, E. Rasoulinezhad, and Y. Chang. Trade Linkages and Transmission of Oil Price Fluctuations, Energy Policy, 2019, 133: 110872.
- [14] F. Taghizadeh-Hesary, N. Yoshino, M. M. H. Abadi, and R. Farboudmanesh. The Response of Macro Variables of Emerging and Developed Oil Importers to Oil Price Movements. Journal of the Asia Pacific Economy, 2016, 21(1): 91–102.
- [15] Taghizadeh-Hesary, F., N. Yoshino, G. Abdoli, and A. Farzinvash. 2013. An Estimation of the Impact of Oil Shocks on Crude Oil Exporting Economies and Their Trade Partners., Frontiers of Economics in China, 8(4): 571–591.
- [16] N.S. Balke and S.P. A. Brown and M.K. Yücel Crude Oil and Gasoline Prices: An Asymmetric Relationship?, Economic Review, pp. 1-10, 198
- [17] F. Taghizadeh-Hesary and Rasoulinezhad, Ehsan and Kobayashi, Yoshikazu, Oil Price Fluctuations and Oil Consuming Sectors: An Empirical Analysis of Japan, Economics and policy of energy and the environment, 19 P. 33-51,2016, DOI: 10.3280/EFE2016-002003
- [18] The Wade Formula. Avalaible at http://www.thewadeformula.com.
- [19] J. Sokolowski and A. Zokowski On the topological derivative in shape optimisation, SIAM J. Cont. Optim;, 37, pp.1251-1272, 1251-1272, 1999.
- [20] J.Rustagi Optimization Techniques in Statistics, Academic Press, 1994.
- [21] M.Masmoudi, H. Kawarada and J. Periaux, The topological asymptotic, in computational methods for control applications, 2002 eds, GAKUTO Internat. Ser. Math. Sci. Appli. Gakkotōsho, Tokyo.
- [22] P. Faurre, Analyse Numérique, Notes D'optimisation, 1988, Ellipses.

Pierre Mendy, Babacar M. Ndiaye, Diaraf Seck, Idrissa Ly Laboratory of Mathematics of Decision and Numerical Analysis. Cheikh Anta Diop University. BP 45087, 10700. Dakar, Senegal. e-mail: pierre.mendy@ucad.edu.sn, babacarm.ndiaye@ucad.edu.sn, diaraf.seck@ucad.edu.sn, idrissa.ly@ucad.edu.sn