

Urban Water Supply Optimization

Cities rely on efficient water distribution systems to ensure every household and facility has access to clean water. In modern urban planning, laying out water pipelines and placing distribution nodes efficiently is crucial for the city.

Typically, cities are connected to a central water grid using a main pipeline. To provide water access across neighborhoods, distribution nodes (like water tanks) are placed throughout the city and connected via pipelines to the main supply. The larger and more complex the city layout, the harder it is to decide where to place nodes and how to lay down the connecting pipes.

Problem

Given a city plan, decide where to build tanks and how to connect them to the main pipeline to maximize coverage and minimize cost

Description

The city is represented as a rectangular grid of cells with H rows and W columns. Each cell is referenced by a pair of 0-based coordinates $[r, c]$, where $[0, 0]$ is the top-left corner.

Example Grid (4 rows × 5 columns):

```
. . . X .  
. . . . .  
. . . . .  
. . . . .
```

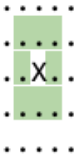
The cell marked “X” is $[0, 3]$.

Cell Types

Each cell in the grid is one of the following:

- $\#$ – **Obstacle cell** (e.g., buildings or terrain that blocks water flow)
- $.$ – **Target cell** (locations that require water supply)
- $-$ – **Void cell** (areas that do not require water supply, such as parks or industrial zones)

Each cell (except those on the edges) has **8 neighboring cells** (including diagonals). For example, the cell marked “X” below has the following neighbors:



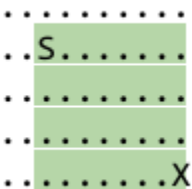
Water Tanks

Each water tank supplies water to a square area of up to $(2R + 1) \times (2R + 1)$ cells around it, unless blocked by an obstacle.

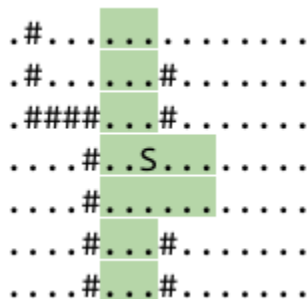
For a tank placed at $[a, b]$, it can supply water to cell $[x, y]$ if:

- $|a - x| \leq R$
- $|b - y| \leq R$
- There is no obstacle cell (#) inside the smallest enclosing rectangle between $[a, b]$ and $[x, y]$. That is, no cell $[w, v]$ exists such that:
 - $\min(a, x) \leq w \leq \max(a, x)$
 - $\min(b, y) \leq v \leq \max(b, y)$

For example the tank placed in cell ‘S’ can cover cell ‘X’ if there are no obstacles, with $R=7$



For $R=3$, with obstacles, this would be the coverage



Tanks can be placed in target cells or void cells but not on obstacles

Main Pipeline Connection

Water tanks must be connected to the main water pipeline, which runs through the city. The placement and connection of tanks must be optimized to:

- Maximize coverage of target cells (.)
- Minimize infrastructure cost, including pipe length and tank count

Any cell (target, void, or obstacle) can be connected to the backbone. To connect a new cell to the backbone, one of its eight neighboring cells must already be connected.

Example

If “b” is the initial cell connected to the backbone, and “n” is a cell where we want to place a tank, one possible way to connect the tank is by linking all marked cells below:

```
--#..r.#  
-b#...#  
--#....#
```

Budget Constraints

- Placing a single distribution tank costs P_t
- Connecting a single cell to the main pipeline costs P_p
- The total budget available is B

Example

If “b” is the initial backbone cell, and we place a tank in cell “n” and connect four intermediate cells to the backbone, the total cost is: $1 \times P_t + 4 \times P_p$

This value must be less than or equal to B .

Input Data Format

The input is provided as a plain text dataset file using only ASCII characters with UNIX-style line endings ($\backslash n$).

File Format

First Line:

- **H** – Number of rows in the grid ($1 \leq H \leq 1000$)
- **W** – Number of columns in the grid ($1 \leq W \leq 1000$)
- **R** – Radius of a tank's water coverage ($0 \leq R \leq 11$)

Second Line:

- **P_p** – Cost of connecting one cell to the main pipeline ($1 \leq P_p \leq 5$)
- **P_t** – Cost of placing one distribution tank ($5 \leq P_t \leq 100$)
- **B** – Maximum budget ($0 \leq B \leq 10^9$)

Third Line:

- **br**, **bc** – Row and column of the initial cell connected to the backbone ($0 \leq br < H$, $0 \leq bc < W$)

Next **H** Lines:

Each line describes a row of the city grid using **W** characters:

- **#** – Obstacle cell (e.g., building, terrain)
- **.** – Target cell (requires water)
- **-** – Void cell (no water needed)



Example Input File

Code

8 22 3

1 100 220

2 7

```
-----  
#####-----#####  
-#.....#####.....#-  
-#.....#-  
-#.....#-  
-#.....#-  
-#.....#-  
-#####-  
-----
```

- 8 rows, 22 columns
- Tank coverage radius: 3

- Backbone connection cost: 1
- Tank placement cost: 100
- Budget: 220
- Initial backbone cell: [2, 7]

Submission Format

First Line:

- **N** – Number of cells connected to the backbone (excluding the initial cell)

Next **N** Lines:

Each line specifies a cell connected to the backbone:

- **r**, **c** – Row and column of the cell
- Each cell must be a neighbor of a previously connected cell

Next Line:

- **M** – Number of cells where distribution tanks are placed

Next **M** Lines:

Each line specifies a tank placement, where tanks are placed without repetitions. Each of these lines must contain two numbers: **r**, **c** ($0 \leq r < H$, $0 \leq c < W$) respectively the row and the column of each cell where a tank is placed.

```
3
3 6
3 8
3 9
2
3 6
3 9
```

Three cells connected to the pipeline

Cell [3, 6] neighbors the initial pipeline cell [2, 7] so can be connected

Cell [3, 8] also neighbors the initial backbone cell [2, 7]

Cell [3, 9] neighbors the cell [3, 8] already connected to pipeline

Two tanks

[3, 6] is connected to pipeline and not a wall so tank can be put there

[3, 9] is also connected to backbone and not a wall

In the example above, the cells marked below are connected to backbone, 'b' is the initial backbone cell, 'r' are the cells where routers are placed.

```

-----
-#####- -#####-
-#...b#####...#-
-#...r.r...#-
-#...#-
-#...#-
-#...#-
-#####-
-----

```

```

-----
-#####- -#####-
-#...#####...#-
-#...#####...#-
-#...#####...#-
-#...#####...#-
-#####-
-----

```

Validation

The output file is valid if it meets the file format specified above and the following criteria:

- All water tanks are placed in cells that are connected to the water pipeline
- No water tanks are placed in obstacle cells (e.g., walls or terrain)

The **budget is not exceeded**, $N \times P_p + M \times P_t \leq B$

- where:
 - N is the number of additional cells connected to the backbone
 - M is the number of tanks placed
 - P_p is the cost of connecting one cell to the backbone
 - P_t is the cost of placing one tank
 - B is the total budget

Scoring

Each submission earns:

- 1000 points for each target cell successfully covered with water
- 1 point for each unit of remaining budget

If the number of target cells covered is t , the score is calculated as:

$$\text{Score} = 1000 \times t + (B - (N \times P_p + M \times P_t))$$

Example

In the example above:

- Total number of target cells covered: 35
- Budget: 220
- Backbone connections: 3 additional cells
- Tanks placed: 2

Code

$$\text{Score} = 1000 \times 35 + (220 - (3 \times 1 + 2 \times 100))$$

$$= 35000 + 17$$

$$= 35017$$

Note: There are multiple datasets representing separate instances of the problem. The **final score for your team** will be the **sum of your best scores** across all datasets.