Forecasting Tuberculosis Epidemic using Mathematical Model

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Introduction

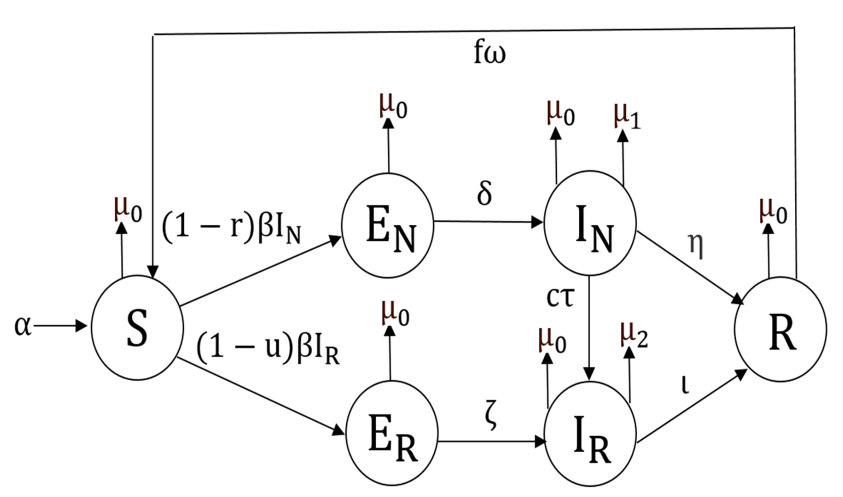
At present, Thailand is ranked 1 in 14 countries that have the most patients in the world. All agencies that involved, whether it is the World Health Organization (WHO), Bureau of Tuberculosis (TB), Department of Disease Control and other organization are taking an investigation to this problem. Now they have defensive measure, which is a National TB Strategy in 2017 – 2021 at which the goal is to treat every patients faster and reduce the number of patients within 2021. Because of the fact that there is no measure to reduce the number of TB patients, this project aims to investigate the factors and to determine the prevention in controlling TB.

Objective

- To forecasting Tuberculosis epidemic.
- To find the most factor that affects the epidemic.
- To suggest the strategy in controlling Tuberculosis

Methodology

Mathematical model of Tuberculosis epidemic



$$\frac{dS}{dt} = \alpha + f\omega R - (\mu_0 + (1-r)\beta I_N + (1-u)\beta I_R)S$$

$$\frac{dE_N}{dt} = (1-r)\beta S I_N - (\mu_0 + \delta)E_N$$

$$\frac{dE_R}{dt} = (1-u)\beta S I_R - (\mu_0 + \zeta)E_R$$

$$\frac{dI_N}{dt} = \delta E_N - (\mu_0 + \mu_1 + \eta + c\tau)I_N$$

$$\frac{dI_R}{dt} = \zeta E_R + c\tau I_N - (\mu_0 + \mu_2 + \iota)I_R$$

$$\frac{dR}{dt} = \eta I_N + \iota I_R - (\mu_0 + f\omega)R$$

Variables of Populations

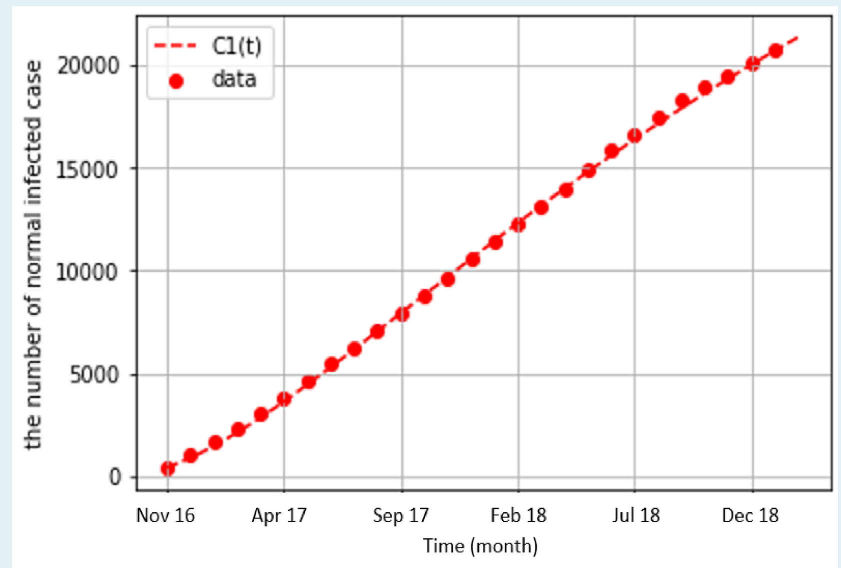
$S(t)$ = susceptible population;
$E_N(t)$ = normal exposed population;
$E_R(t)$ = drug resistance exposed population;
$I_N(t)$ = normal infected population;
$I_R(t)$ = drug resistance infected population;
$R(t)$ = recovered population.

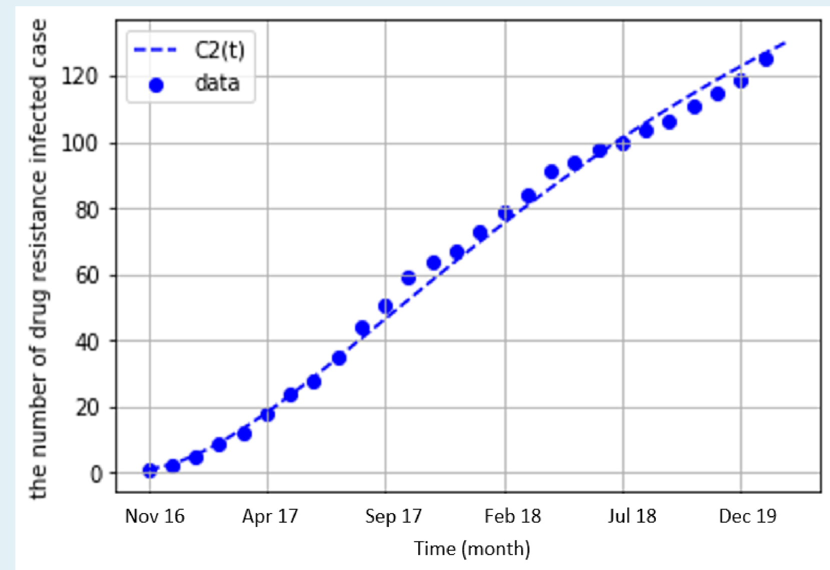
Model parameters

Symbol	Description	Value	Unit
α	Recruitment rate	3,733	person/month
r	Probability that S becomes I_N	0.5	-
u	Probability that S becomes I_R	0.53	-
β	Infection rate	2.08×10^{-7}	month^{-1}
δ	Transfer rate from E_N to I_N	0.03	month^{-1}
ζ	Transfer rate from E_R to I_R	0.06	month^{-1}
c	Probability that I_N becomes I_R	0.0001	-
τ	Transfer rate from I_N to I_R	1.0	month^{-1}
h	Transfer rate from I_N to R	0.14	month^{-1}
ι	Transfer rate from I_R to R	0.071	month^{-1}
f	Probability that R becomes S	0.001	-
ω	Transfer rate from R to S	0.1	month^{-1}
μ_0	Natural mortality rate	6.5×10^{-4}	month^{-1}
μ_1	Death rate due to TB	0.016	month^{-1}
μ_2	Death rate due to infected drug resistance TB	0.083	month^{-1}

Results

Comparison Predicted and Real Data





Reproductive Number (R_0) and Sensitivity Analysis

$$R_0 = \max\{R_1, R_2\}$$

$$R_1 = \frac{(1-r)\alpha\beta\delta}{\mu_0(\mu_0+\delta)(\mu_0+\mu_1+\eta+c\tau)}$$

$$R_2 = \frac{(1-r)\alpha\beta\delta}{\mu_0(\mu_0+\zeta)(\mu_0+\mu_2+\iota)}$$

Table 2. Reproductive number associated with normal infection: R_1 = 3.65

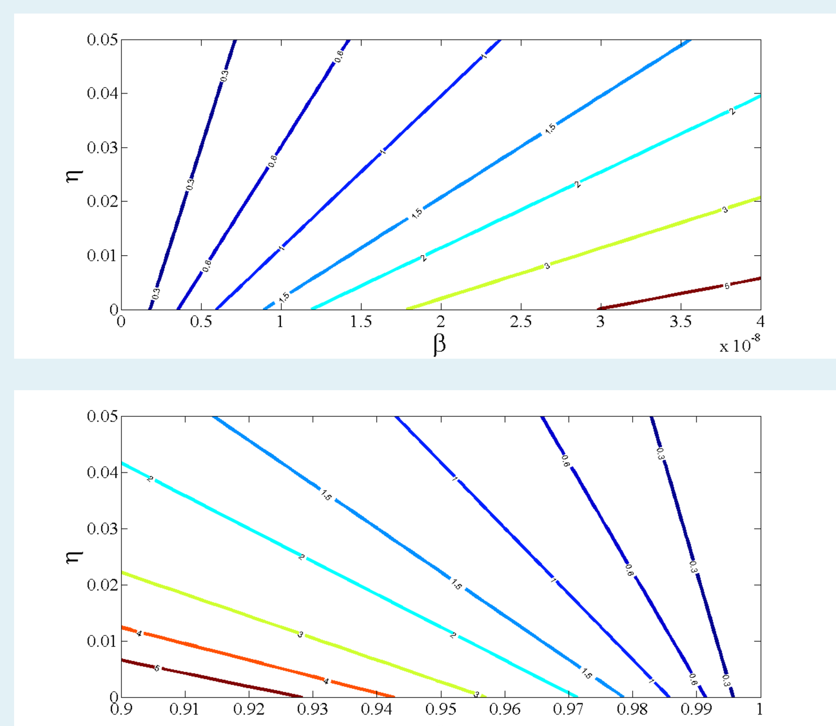
Parameter	sensitivity analysis
β	1.0
α	1.0
r	-1.0
μ_0	-0.98
h	-0.89
μ_1	-0.1
δ	0.019
τ	-0.00062
c	-0.00062

Table 3. Reproductive number associated with drug resistance infected: R_2 = 3.57

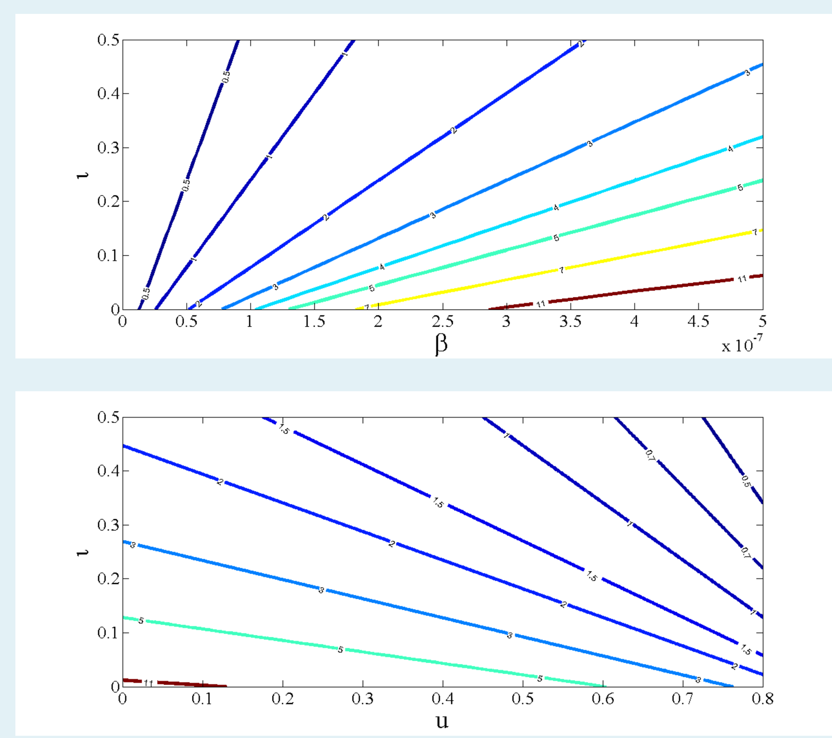
Parameter	sensitivity analysis
u	-1.13
μ_0	-1.015
α	1.0
β	1.0
ι	-0.54
μ_2	-0.46
ζ	0.01

Contour Plot of Reproductive number

Contour Plot of R_1



Contour Plot of R_2



Conclusion

The model is used to predict the cumulative number of infected individuals and they are compared with the real data in the years 2016- 2018. The appropriate values parameter are indicated in Table 1. The reproductive number (R0) of tuberculosis in Thailand is R0 = R1 = 3.6 > 1 that is verifies that the normal infected population is the main cause of TB epidemic.

From sensitivity analysis, it is found that the factors that are influential to TB epidemic are infection rate (β) and probability (r) that S becomes I_N. This study results suggest the way of prevention are to avoid close contact with patients who are coughing and have not been treated with TB drugs, and to give BCG vaccine protection when TB is epidemic in community.

References

BUREAU OF TUBERCULOSIS, 2018, National Tuberculosis control Programme Guidelines 2561, Aksorn Graphic And Design Publishing House LP, Bangkok, pp. 7-29.

BUREAU OF TUBERCULOSIS, 2017, Thailand Operational Plan to End TB 2017-2021, Aksorn Graphic And Design Publishing House LP, Bangkok, pp. 4-5.