Electron Musical Chairs: The Rabi Oscillation

(QuTIP Edition)

by

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# Chapter 1 –

# Introduction

Of the multiple modes proposed for quantum computation; one of the first was quantum optics. It is a natural choice for candidacy due the photon’s high mobility and durability. That is to say that it weakly interacts with its environment. However, despite this, it is because of these strengths that it also lacks. It is difficult to achieve successful photon-photon interactions, when the photons are highly mobile and will withstand external interactions. This has another consequence, in that it is difficult to create nonlinear gates, especially when most optical tools are linear in nature. To remedy this, through the combined efforts of Knill, Laflamme, and Milburn; the KLM/linear optical quantum computing (LOQC) was born; and it seeks to make optical quantum computing a reality.

The KLM/LOQC protocol outlines three distinct criteria that must be attained in order to have a successful optical system:

1. Heralded single photon sources with strict mode and bandwidth characteristics

A heralded photon source is able to generate entangled pairs of photons. This solves the difficulty of creating entanglement reactions between photons.

1. High-efficiency number resolving single photon detectors

The exact method is not outlined, but the detector counts the number of photons incident on the detector. Upon measurement, the photon is destroyed, as such any state information is not discernable.

1. Construction of complicated optical circuits exhibiting both classical and quantum interference effects

We will be addressing the first of three criteria in this thesis. While we have achieved a heralded source in spontaneous parametric down conversion (SPDC). Is it possible for us to do better?

(Picture) Before we continue, I wish to outline the basic premise behind SPDC. This non-linear optical tool takes in a single photon input, incident upon a crystal, and outputs entangled photon pairs. Beta-barium borate, lithium niobate, or potassium dihydrogen phosphate crystals are used as crystals to split the incident photon. The sum of the momenta and energies of the photon pairs total to the initial momentum and energy of the incident photon, which follows in accordance with conservation laws. The resultant output photons are not only identical, but also entangled. While this phenomenon is fascinating, this is not the main focus for this thesis.

SPDC is the most popular entangled photon source, however, is this source deterministic? That is to say, can we produce photons on demand? This will be discussed in detail later on in the thesis but suffice it to say that it is not a deterministic source. While it can be guaranteed if a photon is produced that the entangled pair is present, producing these outputs occurs with an efficiency of 4 outputs for every 106 input photons (source). Alternative deterministic sources are considered, the most promising of which is the quantum dot, details of which will follow in this thesis.

A method for verifying the quantum nature of a system is in the detection of Rabi oscillations. In this thesis, we will confirm the quantum nature of the quantum dot by demonstrating the presence of Rabi oscillations. This is done by contrasting a previous method used in the analysis of the detection of Rabi oscillation against an alternative which will consider environmental effects that may contribute to the dampening of the system.

However before this, we must understand the experimental conditions required to replicate this event. An account considering the object measured – the photon, how the object is produced – the quantum dot, and the experimental setup – the double unbalanced Mach-Zehnder interferometer will be provided in chapter two. Along with this, we will consider the theoretical background behind Rabi oscillations as well as the analytic method used to generate results – the Linblad Master Equation. Finally in the fourth and fifth chapters there will be a discussion of what was done with detailed findings and resulting conclusions. Coupled with the information from the thesis for PHYS 437 A, I hope that I will be able to demonstrate the viability of this new model. It is my sincere hope that you will learn, as much as I have, so without further ado, onto chapter two.

(Does this need further elaboration with respect to the quantum dot?)

# Chapter 2 –

# Experimental and Theoretical Context

Before we can delve into the results of the model, it is important to understand the experimental and theoretical context underlying it. In this section, we will cover the experimental parameters along with the theory behind Rabi oscillations and the model used to represent them.

* Begin with Rabi oscillations
* Continue with the experiment used to detect them
* Finally, motivate the model

2.1 Rabi Oscillations

* What are Rabi oscillations
  + Start with history
  + What they represent – purely quantum phenomenon
* Mathematical representation

2.2 Object Measured – The Photon

* Statistical nature of light
  + Study light as a stream of photons rather than a classical wave
  + Poissonian, super-Poissonian, and sub-Poissonian (Which one?)
  + Sub-Poissonian not in line with classica description of light
    - Difficult to detect and subject to loss
  + (Motivate with Geiger counter similarity?) – If you do, include detection statistics as well
  + In classical physics we define light as an electromagnetic wave, which has the form:
  + (Equation)
  + With Intensity being the square of the amplitude
  + Given this equation we might assume that a beam of light consists of a stream of photons with photons present at regularly spaced intervals, however that is not the case
* Mathematical representation
  + Consider a coherent beam of light that has an average number of photons within it
  + Explanation of coherent light
  + We can divide this beam into smaller pieces, such that the number of pieces approaches infinite
  + We find that the probability of finding a photon within any given segment is extremely small
  + But let’s expand this to the entire length, what is the probability of detecting n photons in a given beam of length L, with N segments
  + This can be found using the binomial distribution
  + (Equation)
  + Thus the probability of detecting n photons in a given beam of light of length L is given to be:
  + (Equation)
  + This is called a Poisson Distribution
    - Random process that only returns integer values uses Poisson Distribution
    - Characterized by the mean value
  + This has the following properties
    - The mean is equal to the variance
    - Explanation for variance and standard deviation
  + Further characterization of photon statistics using sub and super prefixes have the following properties
    - Standard deviation less than square root of mean – sub-Poissonian
    - Standard deviation greater than square root of mean – super-Poissonian
  + Fluctuations in intensity would cause large fluctuations in photon number, thus resulting in super-Poissonian results
    - Thermal
    - Black-body
  + Sub-Poissonian light has a narrower distribution
    - Perfectly coherent beam is the most stable light in classical optics
    - Thus this is non-classical
    - Here we have the idealized situation that we assumed initially at the beginning of this section - that is a beam of photons with identical time-separation between each photon (delta t)
    - The photocount for total time T would be simply
    - In a perfect situation (nbar=N) and thus a variance would be 0
* Photon source
  + Many sources of sub-Poissonian light
  + 4 broad classes of sub-Poissonian light sources
  + We use quantum dot, an example of the second and third classes, the nature of which is detailed further in the next section

2.3 Production of Object – The Quantum Dot

* History of quantum dot
* Benefit of quantum dot over other photon sources
* General mechanism of the quantum dot
  + 3-Level system
  + Excitation schemes
* Reimer group quantum dot
  + Fabrication
  + General properties

2.4 Experimental Technique Used – The Double Unbalanced Mach-Zehnder Interferometer

* History
* General implementation
* Math (?)
* Why Rabi Oscillations

2.5 Lindblad Master Equation

In physics, there are multiple formalisms used to define the make up of a system, one of which is the Hamiltonian. This takes the Lagrangian formalism, which takes the difference between the kinetic and potential energies, and performs a Legendre transforms. This takes the input configuration space co-ordinates of position and velocity and maps them onto the phase space co-ordinates for position and momentum. Without getting into the minutiae of this transform, we can understand that this transform results in an equation that represents the total energy of the system.

However, this understanding only considers the system itself. While many important findings and results can be derived from this approximation, it can be far from an accurate representation of reality. Enter the Linblad master equation, which is touted as a more generalized version of the Hamiltonian. Not only does this consider the system’s dynamics, but also the environmental effects and whatever mechanism that couples one to the other.

(Picture) So how does one do this? We map the system, the environment, and their coupling onto a space, with careful consideration. In our case, we map our systems onto the Hilbert space. This Hilbert space is a complex vector space that is equipped with an inner product. This space is spanned by a set of basis vectors, from which we can construct the entire space. In the example of the quantum dot, the levels of excitation are used to map the system. One can map the system the environment and the coupling using separate Hilbert spaces and take their sum as the total Hilbert space, which looks as follows:

(Equation)

As we are dealing with a quantum dot system, we will use the same basis states to define the environment and the coupling. This is made possible under the understanding that any state can be written as a linear combination of alternate basis states. This is written mathematically as:

(Equation)

It is key to understand that while these use the same language, i.e the same basis states, the system, environment, and coupling belong to separate Hilbert spaces, and as such the basis states are separate as well. We are just able to translate each system into a single language, to gain a better understanding of the what we have.

We are able to manipulate this Lindblad master equation, similar to how we would a regular Hamiltonian, such that we can extrapolate information of all possible trajectories for a system.

(Picture) Suppose we wanted to understand the progression of the system coupled to a given environment. We can intuit this by considering the following equation:

In plain terms, we can reprsent our system by a so-called density matrix, which contains all the relevant information of the system under the basis of the Hilbert space. Its progression over time, i.e, rho-dot, can be defined under the Heisenberg picture as the commutation relationship between the Hamiltonian and the system’s density matrix. Solving this grants us the equations of motion of the system, and as such gives us information about the progression of the system. This process is the quantum analogue for the Poisson bracket, which is used to derive equations of motion from the Hamiltonian. (Should I include the derivation?)

While the Heisenberg picture is equivalent to the Schrodinger picture under the Stone-von Neumann theorem, and as such realize the same results. However, the inclusion of the environment and coupling is easier under the Heisenberg picture (Should I include the derivation?) and looks as follows:

(Equation)

Wherein are called jump operators and act as ladder operators for the interaction between the system and the environment.

This inclusion of the environment as well as the coupling defining the interaction between the two, is a picture that is more grounded in reality. As such, this has the potential to provide further insight and predictive capabilities to any model using it. It is this consideration that governs the technique used in this thesis.

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Now that we have established context for parts used in the building of this model, it’s now time to put it work for some usable results, which follow in the next chapter.

# Chapter 3 –

# Findings and Analysis

# Chapter 4 –

# Conclusions and Future Considerations