Electron Musical Chairs: The Rabi Oscillation

(QuTIP Edition)

by

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# Chapter 1 –

# Introduction

Of the multiple modes proposed for quantum computation; one of the first was quantum optics. It is a natural choice for candidacy due the photon’s high mobility and durability. That is to say that it weakly interacts with its environment. However, despite this, it is because of these strengths that it also lacks. It is difficult to achieve successful photon-photon interactions, when the photons are highly mobile and will withstand external interactions. This has another consequence, in that it is difficult to create nonlinear gates, especially when most optical tools are linear in nature. To remedy this, through the combined efforts of Knill, Laflamme, and Milburn; the KLM/linear optical quantum computing (LOQC) was born; and it seeks to make optical quantum computing a reality.

The KLM/LOQC protocol outlines three distinct criteria that must be attained in order to have a successful optical system:

1. Heralded single photon sources with strict mode and bandwidth characteristics

A heralded photon source is able to generate entangled pairs of photons. This solves the difficulty of creating entanglement reactions between photons.

1. High-efficiency number resolving single photon detectors

The exact method is not outlined, but the detector counts the number of photons incident on the detector. Upon measurement, the photon is destroyed, as such any state information is not discernable.

1. Construction of complicated optical circuits exhibiting both classical and quantum interference effects

We will be addressing the first of three criteria in this thesis. While we have achieved a heralded source in spontaneous parametric down conversion (SPDC). Is it possible for us to do better?

(Picture) Before we continue, I wish to outline the basic premise behind SPDC. This non-linear optical tool takes in a single photon input, incident upon a crystal, and outputs entangled photon pairs. Beta-barium borate, lithium niobate, or potassium dihydrogen phosphate crystals are used as crystals to split the incident photon. The sum of the momenta and energies of the photon pairs total to the initial momentum and energy of the incident photon, which follows in accordance with conservation laws. The resultant output photons are not only identical, but also entangled. While this phenomenon is fascinating, this is not the main focus for this thesis.

SPDC is the most popular entangled photon source, however, is this source deterministic? That is to say, can we produce photons on demand? This will be discussed in detail later on in the thesis but suffice it to say that it is not a deterministic source. While it can be guaranteed if a photon is produced that the entangled pair is present, producing these outputs occurs with an efficiency of 4 outputs for every 106 input photons (source). Alternative deterministic sources are considered, the most promising of which is the quantum dot, details of which will follow in this thesis.

A method for verifying the quantum nature of a system is in the detection of Rabi oscillations. In this thesis, we will confirm the quantum nature of the quantum dot by demonstrating the presence of Rabi oscillations. This is done by contrasting a previous method used in the analysis of the detection of Rabi oscillation against an alternative which will consider environmental effects that may contribute to the dampening of the system.

However before this, we must understand the experimental conditions required to replicate this event. An account considering the object measured – the photon, how the object is produced – the quantum dot, and the experimental setup – the double unbalanced Mach-Zehnder interferometer will be provided in chapter two. Along with this, we will consider the theoretical background behind Rabi oscillations as well as the analytic method used to generate results – the Linblad Master Equation. Finally in the fourth and fifth chapters there will be a discussion of what was done with detailed findings and resulting conclusions. Coupled with the information from the thesis for PHYS 437 A, I hope that I will be able to demonstrate the viability of this new model. It is my sincere hope that you will learn, as much as I have, so without further ado, onto chapter two.

(Does this need further elaboration with respect to the quantum dot?)

# Chapter 2 –

# Experimental and Theoretical Context

Before we can delve into the results of the model, it is important to understand the experimental and theoretical context underlying it. In this section, we will cover the experimental parameters along with the theory behind Rabi oscillations, how they are produced in experiment, and the model used to represent them.

2.1 What is Modelled - Rabi Oscillations

When one thinks of a laser pulse exciting an electron, it is easy to consider it as a one and done phenomenon. However, if we consider the time evolution of this system, we’d find something special. The electron does not experience a single excitation and de-excitation, but a cyclical pattern of this. This phenomenon is called the Rabi oscillation and is a purely quantum effect. Rabi oscillations are the focus of this thesis and are elaborated upon in this section.

* Historical Context
  + Named after Isaac Isidor Rabi
  + Derived in 1937, set the basis for the burgeoning field of Nuclear Magnetic Resonance
  + What they represent – purely quantum phenomenon
* General Situation
  + - We can consider a simple two-state model that is excited via an external light source (Picture)
    - This system can be represented via the following Hamiltonian (Equation)
    - This separates the Hamiltonian into a time independent term (representing the ground state atom) and a time dependent term (representing the excitation process between the incident light and the ground state atom)
    - We then seek to solve the Schrodinger equation for this system (Equation)
    - The solution for which is as follows:
    - We can take this solution and substitute it back into the Schrodinger equation, which we can simplify to get (Equation)
    - We can find the average energy of this system by taking its expectation value which results in the following:
    - This gives us a first order differential equation for the complex amplitude
    - To define the complex amplitude we must consider the energy shift of an atomic dipole in the electric field of the incident light (Equation)
    - We can arbitrarily align the polarization along an axis, we can choose the x axis (Equation)
    - The equation then becomes (Equation)
    - Individual matrix elements are given by (Equation)
    - As such the dipole matrix element is then (Equation)
    - Thus the previous equation can be rewritten as (Equation)
    - Wherein the Rabi frequency is defined as (Equation)
  + Idealized
    - Now consider resonantly exciting the atom (excited with energy that is the same as the transition between the ground and excited states)
    - Our system reduces to: (Equation)
    - This system of equations can be solved to get the following (Equation)
    - This second order ordinary differential equation has the following solutions
    - Which is clearly an oscillatory process
  + Dampening
    - Rabi oscillations are difficult to observe in the lab frame
    - One must consider a careful balance between the oscillation period and the radiative lifetime
    - To reconcile this, one must consider higher powers to achieve observable Rabi oscillations (Relationships between power, period, and/or radiative lifetime)
    - However this isn’t the only effect hampering Rabi oscillations:
      * T\_1 – longitudinal relaxation – population decay
        + Electron may decay spontaneously
        + Breaks the coherence of the electronic wave function
        + Interrupts the Rabi oscillation – relies on coherence of superposition states
        + Upper limit set by radiative lifetime
      * T\_2 – dephasing processes
        + Atom in excited state undergoes elastic or inelastic collisions
        + Changes phase of the wave function, but still keeps the population the same
        + Phase coherence also an important property to observe Rabi oscillations
      * These two dephasing mechanisms can be related as follows:
      * Wherein T2 star is called the pure dephasing caused by population decay
      * If T2star<< T\_1 then decoherence is governed by T2 and as such collisions
      * If T2star>> T\_! then decoherence is governed by the radiative lifetime
      * Given this, if we consider that the dampening is rate is gamma
      * The probability of the electron population is given as follows (Equation)
      * However, this may not be the only reason for dampening, there may be environmental effects not considered here that may contribute. These effects are further studied in this thesis .

* One can verify the quantum nature of a system by the presence of Rabi oscillations, however there is no way, at present, to observe electron dynamics and observe the presence of Rabi oscillations. However, one can observe the emissions from the electrons which will still reflect the Rabi oscillations. To get a better picture of this, it is important to understand the framework used to understand photons, which will be considered in the next section.

2.2 Object Studied – The Photon

While it is understood that Rabi oscillations As electron populations oscillate between the ground and excited states, and in particular as the excited electron relaxes into the ground state, it emits photons. It is with these emissions that we wish to study Rabi oscillations. However, before we can do so, it is important to understand that which we wish to study, so without further ado, here we address the statistical nature of light.

Under a classical lens, we think of light as a wave, a steady stream of photons. That is to say that we can depend on photons being present at regularly spaced time-intervals. However, photon production, is a highly probabilistic process which we will address in depth in this section.

The probabilistic process that governs photon production is said to be Poissonian, which can then be further broken into 3 categories: Super-Poissonian, Poissonian, and Sub-Poissonian. But these words on their own have little meaning, so let’s consider a toy model for some context

Suppose we consider a coherent beam of light with length L which contains ‘n’ photons. With coherence defined as light whose photons oscillate with the same frequency and are in phase with one another. (Picture)

We can divide this beam into smaller segments of the same size, and we can do so until we have infinitely many of these identical segments. (Picture)

What we will find is that the probability of finding a photon within a given segment is extremely small. Now suppose we consider the entire length of the beam instead, what is the likelihood of finding n photons? We can do this by considering a binomial distribution, this looks as follows:

(Equation)

Wherein

* N
* ‘n’
* ‘p’

This can be understood piecewise. The first term, the quotient, represents the number of ways one can select n segments with photons out of N segments. Wherein N-n represents the number of segments with zero photons. We then multiply this by the probability of n successes, i.e p^n, and n-N failures, i.e. (1-p)^N-n. The entirety of this product gives us the probability of finding n segments containing one photon. (Include a better understanding of the binomial distribution?) We can rewrite this by considering the average number of photons within a beam of length L (Include explanation of average photons?):

(Equation)

As N is taken to be infinitely large, we can then use Stirling’s approximation,(The approximation), which we can break into two parts:

Part 1: (Equation)

Part 2: (Equation)

We can take what we have above to rewrite equation \_ a:

The resultant equation is used to define a Poisson distribution. We can take special note of how this equation is characterized by the mean n-bar, this is unique to a Poisson distribution. In fact, the standard deviation is given as follows:

(Equation)

Now that we have established what it means to have a Poissonian distribution, the other classes are simple to understand. They are as follows:

|  |  |
| --- | --- |
| Classification of Light | What that Means |
| Sub-Poissonian |  |
| Poissonian |  |
| Super-Poissonian |  |

We can further understand these classifications by considering them under physical contexts. We have defined Poissonian light as that which is coherent light. This has a standard deviation that is the square-root of the mean. This light is characterized by constant frequency and consistent phase, but also as constant intensity. Suppose we were to vary the intensity over time. We would find that the number of photons falling within one standard deviation of the mean would be larger than that of coherent light. This results in super-Poissonian light, examples of this are thermal light and black body radiation.

Based on what we have above, we can that understand that sub-Poissonian light has a narrower distribution. In an idealized situation, we can assume that each segment used in the Poissonian light has a single photon with certitude. Thus the number of photons found in a time T, would be exactly the number of subsegments. Our standard deviation would be zero. But what is characterized by this? A perfectly coherent beam is the most stable light in classical optics, thus we must consider a non-classical source, a quantum source.

This sounds rather mythical, however there are many sources of sub-Poissonian light. There are four broad classes:

|  |  |
| --- | --- |
| Type | Examples |
|  |  |
|  |  |
|  |  |
|  |  |

The source used in this thesis will be the quantum dot, which is an example of the second and third classes. As we will not be dealing with two photon systems, we are dealing purely with the second class. Further description of this is given in the next section, the quantum dot.

2.3 Production of Object – The Quantum Dot

In order to study photons, one needs a source for them. As mentioned in the introduction, one of the best source for deterministic photons is the quantum dot.

* History of quantum dot
* Benefit of quantum dot over other photon sources
* General mechanism of the quantum dot
  + 3-Level system
  + Excitation schemes
* Reimer group quantum dot
  + Fabrication
  + General properties

2.4 Experimental Technique Used – The Unbalanced Mach-Zehnder Interferometer

* History
* General implementation
* Math (?)
* Why Rabi Oscillations

2.5 Lindblad Master Equation

In physics, there are multiple formalisms used to define the make up of a system, one of which is the Hamiltonian. This takes the Lagrangian formalism, which takes the difference between the kinetic and potential energies, and performs a Legendre transforms. This takes the input configuration space co-ordinates of position and velocity and maps them onto the phase space co-ordinates for position and momentum. Without getting into the minutiae of this transform, we can understand that this transform results in an equation that represents the total energy of the system.

However, this understanding only considers the system itself. While many important findings and results can be derived from this approximation, it can be far from an accurate representation of reality. Enter the Linblad master equation, which is touted as a more generalized version of the Hamiltonian. Not only does this consider the system’s dynamics, but also the environmental effects and whatever mechanism that couples one to the other.

(Picture) So how does one do this? We map the system, the environment, and their coupling onto a space, with careful consideration. In our case, we map our systems onto the Hilbert space. This Hilbert space is a complex vector space that is equipped with an inner product. This space is spanned by a set of basis vectors, from which we can construct the entire space. In the example of the quantum dot, the levels of excitation are used to map the system. One can map the system the environment and the coupling using separate Hilbert spaces and take their sum as the total Hilbert space, which looks as follows:

(Equation)

As we are dealing with a quantum dot system, we will use the same basis states to define the environment and the coupling. This is made possible under the understanding that any state can be written as a linear combination of alternate basis states. This is written mathematically as:

(Equation)

It is key to understand that while these use the same language, i.e the same basis states, the system, environment, and coupling belong to separate Hilbert spaces, and as such the basis states are separate as well. We are just able to translate each system into a single language, to gain a better understanding of the what we have.

We are able to manipulate this Lindblad master equation, similar to how we would a regular Hamiltonian, such that we can extrapolate information of all possible trajectories for a system.

(Picture) Suppose we wanted to understand the progression of the system coupled to a given environment. We can intuit this by considering the following equation:

In plain terms, we can reprsent our system by a so-called density matrix, which contains all the relevant information of the system under the basis of the Hilbert space. Its progression over time, i.e, rho-dot, can be defined under the Heisenberg picture as the commutation relationship between the Hamiltonian and the system’s density matrix. Solving this grants us the equations of motion of the system, and as such gives us information about the progression of the system. This process is the quantum analogue for the Poisson bracket, which is used to derive equations of motion from the Hamiltonian. (Should I include the derivation?)

While the Heisenberg picture is equivalent to the Schrodinger picture under the Stone-von Neumann theorem, and as such realize the same results. However, the inclusion of the environment and coupling is easier under the Heisenberg picture (Should I include the derivation?) and looks as follows:

(Equation)

Wherein are called jump operators and act as ladder operators for the interaction between the system and the environment.

This inclusion of the environment as well as the coupling defining the interaction between the two, is a picture that is more grounded in reality. As such, this has the potential to provide further insight and predictive capabilities to any model using it. It is this consideration that governs the technique used in this thesis.

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Now that we have established context for parts used in the building of this model, it’s now time to put it work for some usable results, which follow in the next chapter.

# Chapter 3 –

# Findings and Analysis

# Chapter 4 –

# Conclusions and Future Considerations