Electron Musical Chairs: The Rabi Oscillation

(QuTIP Edition)

by

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# Chapter 1 –

# Introduction

Of the multiple modes proposed for quantum computation; one of the first was quantum optics. It is a natural choice for candidacy due the photon’s high mobility and durability. That is to say that it weakly interacts with its environment. However, despite this, it is because of these strengths that it also lacks. It is difficult to achieve successful photon-photon interactions, when the photons are highly mobile and will withstand external interactions. This has another consequence, in that it is difficult to create nonlinear gates, especially when most optical tools are linear in nature. To remedy this, through the combined efforts of Knill, Laflamme, and Milburn; the KLM/linear optical quantum computing (LOQC) was born; and it seeks to make optical quantum computing a reality.

The KLM/LOQC protocol outlines three distinct criteria that must be attained in order to have a successful optical system:

1. Heralded single photon sources with strict mode and bandwidth characteristics

A heralded photon source is able to generate entangled pairs of photons. This solves the difficulty of creating entanglement reactions between photons.

1. High-efficiency number resolving single photon detectors

The exact method is not outlined, but the detector counts the number of photons incident on the detector. Upon measurement, the photon is destroyed, as such any state information is not discernable.

1. Construction of complicated optical circuits exhibiting both classical and quantum interference effects

We will be addressing the first of three criteria in this thesis. While we have achieved a heralded source in spontaneous parametric down conversion (SPDC). Is it possible for us to do better?

(Picture) Before we continue, I wish to outline the basic premise behind SPDC. This non-linear optical tool takes in a single photon input, incident upon a crystal, and outputs entangled photon pairs. Beta-barium borate, lithium niobate, or potassium dihydrogen phosphate crystals are used as crystals to split the incident photon. The sum of the momenta and energies of the photon pairs total to the initial momentum and energy of the incident photon, which follows in accordance with conservation laws. The resultant output photons are not only identical, but also entangled. While this phenomenon is fascinating, this is not the main focus for this thesis.

SPDC is the most popular entangled photon source, however, is this source deterministic? That is to say, can we produce photons on demand? This will be discussed in detail later on in the thesis but suffice it to say that it is not a deterministic source. While it can be guaranteed if a photon is produced that the entangled pair is present, producing these outputs occurs with an efficiency of 4 outputs for every 106 input photons (source). Alternative deterministic sources are considered, the most promising of which is the quantum dot, details of which will follow in this thesis.

A method for verifying the quantum nature of a system is in the detection of Rabi oscillations. In this thesis, we will confirm the quantum nature of the quantum dot by demonstrating the presence of Rabi oscillations. This is done by contrasting a previous method used in the analysis of the detection of Rabi oscillation against an alternative which will consider environmental effects that may contribute to the damping of the system.

However before this, we must understand the experimental conditions required to replicate this event. An account considering the object measured – the photon, how the object is produced – the quantum dot, and the experimental setup – the double unbalanced Mach-Zehnder interferometer will be provided in chapter two. Along with this, we will consider the theoretical background behind Rabi oscillations as well as the analytic method used to generate results – the Linblad Master Equation. Finally in the fourth and fifth chapters there will be a discussion of what was done with detailed findings and resulting conclusions. Coupled with the information from the thesis for PHYS 437 A, I hope that I will be able to demonstrate the viability of this new model. It is my sincere hope that you will learn, as much as I have, so without further ado, onto chapter two.

(Does this need further elaboration with respect to the quantum dot?)

# Chapter 2 –

# Experimental and Theoretical Context

Before we can delve into the results of the model, it is important to understand the experimental and theoretical context underlying it. In this section, we will cover the experimental parameters along with the theory behind Rabi oscillations, how they are produced in experiment, and the model used to represent them.

2.1 What is Modelled - Rabi Oscillations

2.1.1 General Intuition

When one thinks of a laser pulse exciting an electron, it is easy to consider it as a one and done phenomenon. However, if we consider the time evolution of this system, we’d find something special. The electron does not experience a single excitation and de-excitation, but a cyclical pattern of this. This phenomenon is called the Rabi oscillation and is a purely quantum effect. Rabi oscillations are the focus of this thesis and are elaborated upon in this section.

Named after Isaac Isidor Rabi, in 1937 he derived the effect of oscillating electromagnetic fields that are resonantly tuned to the Zeeman splitting of a spin-½ nucleus. What was found was that the rotating field causes the spin to oscillate, this was the basis for nuclear magnetic resonance (NMR) techniques. As a secondary consequence, this process is not unlike that of Rabi oscillations. However, instead of the oscillation of nuclear spins, it is the oscillation of the electron population. What’s more is that while to oscillation of spin in NMR has a classical description, the repopulation, or revival in Rabi oscillations does not. Thus, this suggests that it is a purely quantum effect.

To understand this process further, we turn to the derivation found in Mark Fox’s Quantum Optics. Let us consider a simplified model: suppose we consider a simple two-state model that is excited via an external light source. This system can be represented via the following Hamiltonian:

(Equation)

This Hamiltonian is clearly split into two separate parts, wherein H\_0 is the time independent term that represents the unilluminated ground state atom and V represents is the time dependent term that represents the interaction between the incident electromagnetic field and the ground state atom. Given this our next step is to solve the Schrodinger equation for this system.

(Equation)

Which has the following solutions:

(Equation)

To determine the constants for the system, we can take the above wavefunction and substitute it back into the Schrodinger equation:

(Equation)

We can then take the expectation value for this system which involves multiplying it by a complex conjugate, integrating over all space, and recalling the orthonormality of states, leaving us with the following:

(Equation)

(Equation)

We have two as the index over ‘i’ is {1,2}. This gives us two first order ordinary differential equations with respect to the complex amplitudes.

We can make this even more concrete by considering the semi-classical understanding of an energy shift on an atomic dipole in the presence of an electric field (in our case, light):

(Equation)

For which we align the polarization of the electric field along the x-axis for simplicity:

(Equation)

This results in the following:

(Equation)

The individual matrix elements can be found as follows:

(Equation)

Wherein the following term is defined as the matrix element mu\_ij:

(Equation)

Thus, we are left with:

(Equation)

Substituting this into the first order ordinary differential equation leaves us with:

(Equation)

Wherein the Rabi frequence is defined as:

(Equation)

2.1.2 The Idealized Scenario

Suppose instead we excited the system resonantly, that is to say it is excited with the same energy as the transition between the ground and excited states. We would find that the system would reduce to:

(Equation)

This simplifies our system. In order to solve this, we can differentiate one of the equations to get a second order ordinary differential equation. Which has the following solution:

This solution is clearly oscillatory. Notice that the frequency of oscillation is omega\_R/2pi, this is the frequency at which the electron populations oscillate from the excited to ground states. This is an astounding result, but suppose we were to consider more realistic parameters. What would that look like?

2.1.3 Approaching the Real World

Rabi oscillations are difficult to observe in the lab frame, due to damping effects within the system as well as the surrounding environment. Here we consider two well defined mechanisms that may contribute damping in Rabi oscillations.

The first of which is called longitudinal relaxation ‘T\_1.’ This is due to the probabilistic nature of electron decay. An electron decay, not under the influence of the Rabi cycle, is probabilistic. This process can be modelled as a Gaussian distribution, with an average number of spontaneous emissions occurring at the radiative lifetime. Thus, even under the influence of Rabi oscillations, spontaneous emission has the potential to break the Rabi cycle. Therefore one must be careful in balancing the radiative lifetime of the electron in question along with the oscillation period of the rabi cycle. In which the radiative lifetime sets the upper limit for the rabi cycle. Despite this consideration, as the radiative process is probabilistic, even with these considerations, spontaneous emission may still occur.

The second mode of damping is called transverse relaxation ‘T\_2.’ While the above process is governed by the atom itself but the environment instead. If the atom, while in an excited state, were to undergo elastic or inelastic collisions, this can change the wavefunction of the atom. While the frequency and the intensity are untouched, a more subtle change in the wavefunction’s phase occurs. This change in phase has the potential to destroy the Rabi oscillations. A simple way to understand this is that the Rabi phenomenon is observed under sub-Poissonian conditions, which have very little deviation from the average. Given that Poissonian light is coherent light, which preserves frequency and phase, the more stringent Poissonian light must also preserve the frequency and phase. Thus any change in phase will result in a decrease in coherence, which can result in the destruction of the Rabi cycle.

It is interesting to note that while these phenomenon are (to my knowledge) unrelated, there is a way to relate the two in an equation:

(Equation)

Whereing T\_2star is called the overall decoherence, and is considered the overall dephasing of the system. Given this, we can now modify equation, to include damping effects.

Suppose we consider that overall damping rate is ‘gamma.’ We can make the following considerations:

This results in the following equation:

(Equation)

This result is often used in experimental modelling to confirm the presence of dampened Rabi oscillations. It is my hope in this thesis, that I would be able to improve upon this model so that it would better fit the results found in experiment by the Reimer group

One can verify the quantum nature of a system by the presence of Rabi oscillations, however there is no way, at present, to observe electron dynamics and observe the presence of Rabi oscillations. However, one can observe the emissions from the electrons which will still reflect the Rabi oscillations. To get a better picture of this, it is important to understand the framework used to understand photons, which will be considered in the next section.

2.2 Object Studied – The Photon

While it is understood that Rabi oscillations As electron populations oscillate between the ground and excited states, and in particular as the excited electron relaxes into the ground state, it emits photons. It is with these emissions that we wish to study Rabi oscillations. However, before we can do so, it is important to understand that which we wish to study, so without further ado, here we address the statistical nature of light.

Under a classical lens, we think of light as a wave, a steady stream of photons. That is to say that we can depend on photons being present at regularly spaced time-intervals. However, photon production, is a highly probabilistic process which we will address in depth in this section.

The probabilistic process that governs photon production is said to be Poissonian, which can then be further broken into 3 categories: Super-Poissonian, Poissonian, and Sub-Poissonian. But these words on their own have little meaning, so let’s consider a toy model for some context

Suppose we consider a coherent beam of light with length L which contains ‘n’ photons. With coherence defined as light whose photons oscillate with the same frequency and are in phase with one another. (Picture)

We can divide this beam into smaller segments of the same size, and we can do so until we have infinitely many of these identical segments. (Picture)

What we will find is that the probability of finding a photon within a given segment is extremely small. Now suppose we consider the entire length of the beam instead, what is the likelihood of finding n photons? We can do this by considering a binomial distribution, this looks as follows:

(Equation)

Wherein

* N
* ‘n’
* ‘p’

This can be understood piecewise. The first term, the quotient, represents the number of ways one can select n segments with photons out of N segments. Wherein N-n represents the number of segments with zero photons. We then multiply this by the probability of n successes, i.e p^n, and n-N failures, i.e. (1-p)^N-n. The entirety of this product gives us the probability of finding n segments containing one photon. (Include a better understanding of the binomial distribution?) We can rewrite this by considering the average number of photons within a beam of length L (Include explanation of average photons?):

(Equation)

As N is taken to be infinitely large, we can then use Stirling’s approximation,(The approximation), which we can break into two parts:

Part 1: (Equation)

Part 2: (Equation)

We can take what we have above to rewrite equation \_ a:

The resultant equation is used to define a Poisson distribution. We can take special note of how this equation is characterized by the mean n-bar, this is unique to a Poisson distribution. In fact, the standard deviation is given as follows:

(Equation)

Now that we have established what it means to have a Poissonian distribution, the other classes are simple to understand. They are as follows:

|  |  |
| --- | --- |
| Classification of Light | What that Means |
| Sub-Poissonian |  |
| Poissonian |  |
| Super-Poissonian |  |

We can further understand these classifications by considering them under physical contexts. We have defined Poissonian light as that which is coherent light. This has a standard deviation that is the square-root of the mean. This light is characterized by constant frequency and consistent phase, but also as constant intensity. Suppose we were to vary the intensity over time. We would find that the number of photons falling within one standard deviation of the mean would be larger than that of coherent light. This results in super-Poissonian light, examples of this are thermal light and black body radiation.

Based on what we have above, we can that understand that sub-Poissonian light has a narrower distribution. In an idealized situation, we can assume that each segment used in the Poissonian light has a single photon with certitude. Thus the number of photons found in a time T, would be exactly the number of subsegments. Our standard deviation would be zero. This sounds like a deterministic source, rather than the probabilistic model encapsulated by light. But what is characterized by this? A perfectly coherent beam is the most stable light in classical optics; thus we must consider a non-classical source, a quantum source.

This sounds rather mythical, however there are many sources of sub-Poissonian light. There are four broad classes:

|  |  |
| --- | --- |
| Type | Examples |
|  |  |
|  |  |
|  |  |
|  |  |

The source used in this thesis will be the quantum dot, which is an example of the second and third classes. As we will not be dealing with two photon systems, we are dealing purely with the second class. Further description of this is given in the next section, the quantum dot.

2.3 Production of Object – The Quantum Dot

In order to study photons, one needs a source for them, and in particular, it is better for it to be deterministic. As mentioned previously, if we consider an idealized sub-Poissonian system (like the quantum dot) for each section of length delta t, instead of a chance, there would be a guaranteed photon. In essence, giving us a deterministic source. In this section we will consider in depth the source used in this experiment: the quantum dot.

Something that came at quite the surprise is that the history of the quantum dot is quite recent, especially in consideration to my own birth year in 1992. First synthesized by Alexey Ekimov, Allen J. Bard, and Louis E. Brus in 1981, the quantum dot was created in order to address dependence on fossil fuels and shift towards solar energy conversion. This quantum dot was created via the placement of a semiconductor within a glass matrix. The semiconductor used was lead (II) sulfide (PbS), and it was noticed that while bulk PbS is black, the nanocrystal exhibited an array of colours that were dependent on the size of the crystal. Moreso, it was noticed that as the crystal decreased in size, the colour shifted from red to blue – a sort of blue shifting. These findings were surprising and interesting, however the language for understanding this was not given until 1982 by Alexander Efros. We will also consider a theoretical explanation for the quantum dot which will follow below; however, we will first consider the general properties of the quantum dot.

2.3.1 Quantum Dot Structure and Mechanism – Core-Shell Quantum Dot

There are at least three possible structures for quantum dots, for which we will focus on the core-shell structure. The general structure of this quantum dot is a semi-conductor coated with another substance, with the defining characteristic that the bandgap of this material is higher than that of the semiconductor. In effect, this creates a confining potential, and as demonstrated in the previous thesis when finding harmonic oscillator solutions; the presence of this potential creates a spectrum of bound states. (Picture)

Before we continue, I want to further elaborate on the language used above. Any solid can be characterized as having a valence band – wherein the electrons are bound to the atom and a conduction band wherein in the electrons are excited and free and not bound to the atom. The bandgap is an intermediate section within which there are no electronic states. In the quantum dot structure given above, the quantum dot alone would have infinitely (theoretically) many states accessible in the conduction band. Coating this with a second material with a higher bandgap, creates a maximum potential for the electron to achieve, effectively confining it. This confining potential then serves to create a spectrum of bound states. The use of a confining potential to create bound states mimics the mechanisms in an atom and as such the quantum dot is also called an artificial atom.

One of the reasons for the popularity of the quantum dot is in its ability to create entangled pairs. However, I am getting ahead of myself, let us consider a simple ground to excited state model. In order to create a photon in this system, one must excite an electron from the valence band into the conduction band. This creates a hole that the electron leaves behind in the conduction band which can be treated like a particle. When the electron relaxes from the excited state to the hole in the ground state, it emits a photon with the same energy between the excited and ground states. This process is called radiative recombination. An ideal quantum dot will only allow for a select number of electron-hole pairs to exist, thus controlling the number of photons emitted.

(Picture) However as mentioned previously, one of the reasons for the use of the quantum dot is in its ability to produced entangled pairs. It does this via a secondary excited state – the bi-exciton. The biexciton is a collection of two electron and hole pairs, which have the opposite angular momentum and undergo two recombinations. This system is not unlike the filled s-shell of an atom further motivating the name ‘artificial atom.’ Suppose upon the first recombination, the biexciton transition, we generate a left circularly polarized photon (LCPP) and the remaining electron-hole pair. Upon the second, the exciton transition, the remaining pair will recombine and emit a right circularly polarized photon (RCPP). Similarly, if the biexciton transition results in an RCPP and electron hole pair, then the exciton transition will result in a LCPP. This pathway is called the Biexciton-Exciton Cascade and is fundamental in the creation of entangled pairs. In an idealized case, it is impossible to determine which path the transition will take, and as such we end up with a superposition state that is also a Bell state (state of maximal entanglement).

(Equation)

While entanglement will not be an element discussed in this thesis, the need for a secondary excited state, a third state in total, adds an element of difficulty to the project undertaken in this thesis. As mentioned in section 2.1, Rabi oscillations are often modelled in two-state systems. The incorporation of a third adds a level of difficulty which will be considered in chapter 3.

We have spoken at length about radiative recombination and how it is leveraged to create entangled pairs in the biexciton cascade. But how do we excite the electron to begin with? There are several excitation schemes used to excite the quantum dot, but we will pay particular focus to the Two-Photon Resonant Excitation (TPE) scheme.

Prior to the TPE scheme, off-resonant excitation was the main scheme used to excite quantum dots. In this excitation scheme, energy far above the band-gap is applied to the quantum dot. The electron relaxes into the first excited state via phonon (vibrations that can be treated as particles – quasi-particles) interactions and is then able to recombine with its constituent hole and emit light. However, due to the lack of specificity of this technique, the likelihood of multi-photon emission occurring is high which can lead to broadening in the emission lines.

Multi-photon emission is not the only problem caused due to off-resonant excitation, but it is one of the challenges that motivated the development of the TPE scheme. The premise of this scheme is to apply a laser pulse to the quantum dot with an energy equal to:

(Equation)

Wherein:

This energy lies between the biexciton and exciton transition energies. The key to this excitation scheme is in that it does not resonantly excite the biexciton directly. The bi-exciton is like a filled s-shell with two electrons with opposing angular momenta. Therefore an excitation with a photon, which has a spin of 1, would violate the Pauli exclusion principle as the net angular momentum would not sum to 0. To avoid this we can use two photons to excite the biexciton instead. By bombarding the quantum dot with a laser of the energy given above, the likelihood that two photons would be able to excite biexciton is extremely high, resulting in an excitation that does not violate the exclusion principle.

With this we have detailed the structure of the quantum dot as well as the mechanisms governing the production of photons with the quantum dot. We are now able to detail the quantum dot parameters used by the Reimer group as well as in this thesis.

2.3.4 Quantum Dot Used in Thesis

With the appreciable context provided above, we can further delve into the quantum dot parameters used in this thesis. The Reimer group quantum dot is composed of a wurtzite InP nanowire with a tapered edge. The usage of the nanowire is in order to transition the emitted photon from the quantum dot to the external environment adiabatically. This reduces the change for total internal reflection and allows for the emitted photon to transmission smoothly to the external environment without worry for edge effects. Within the nanowire waveguide is a small region of InAsP, the quantum dot itself. As we can see from the use of Indium (Group V) and Phosphide (Group III), thus this quantum dot is composed with a III-V semiconductor. The laser used to excite this quantum dot is the Coherent Mira 900 Ti:Sapphire laser with energy 894.0 nm. The biexciton emission are peaked at 894.6 nm and the excitation emissions are peaked at 893.3 nm.

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We have established the context underlying the production of the photon – the quantum dot, along with the particular parameters that would be considered in this thesis. Knowing this, we can now establish the mathematics used model Rabi oscillations from photon emissions produced by the quantum dot.

2.4 Lindblad Master Equation

In physics, there are multiple formalisms used to define the make up of a system, one of which is the Hamiltonian. This takes the Lagrangian formalism, which takes the difference between the kinetic and potential energies, and performs a Legendre transforms. This takes the input configuration space co-ordinates of position and velocity and maps them onto the phase space co-ordinates for position and momentum. Without getting into the minutiae of this transform, we can understand that this transform results in an equation that represents the total energy of the system.

However, this understanding only considers the system itself. While many important findings and results can be derived from this approximation, it can be far from an accurate representation of reality. Enter the Linblad master equation, which is touted as a more generalized version of the Hamiltonian. Not only does this consider the system’s dynamics, but also the environmental effects and whatever mechanism that couples one to the other.

(Picture) So how does one do this? We map the system, the environment, and their coupling onto a space, with careful consideration. In our case, we map our systems onto the Hilbert space. This Hilbert space is a complex vector space that is equipped with an inner product. This space is spanned by a set of basis vectors, from which we can construct the entire space. In the example of the quantum dot, the levels of excitation are used to map the system. One can map the system the environment and the coupling using separate Hilbert spaces and take their sum as the total Hilbert space, which looks as follows:

(Equation)

As we are dealing with a quantum dot system, we will use the same basis states to define the environment and the coupling. This is made possible under the understanding that any state can be written as a linear combination of alternate basis states. This is written mathematically as:

(Equation)

It is key to understand that while these use the same language, i.e the same basis states, the system, environment, and coupling belong to separate Hilbert spaces, and as such the basis states are separate as well. We are just able to translate each system into a single language, to gain a better understanding of the what we have.

We are able to manipulate this Lindblad master equation, similar to how we would a regular Hamiltonian, such that we can extrapolate information of all possible trajectories for a system.

(Picture) Suppose we wanted to understand the progression of the system coupled to a given environment. We can intuit this by considering the following equation:

In plain terms, we can reprsent our system by a density matrix, a density matrix contains all the relevant information of the system under the basis of the Hilbert space. Its progression over time, i.e, rho-dot, can be defined under the Heisenberg picture as the commutation relationship between the Hamiltonian and the system’s density matrix. Solving this grants us the equations of motion of the system, and as such gives us information about the progression of the system. This process is the quantum analogue for the Poisson bracket, which is used to derive equations of motion from the Hamiltonian. (Should I include the derivation?)

While the Heisenberg picture is equivalent to the Schrodinger picture under the Stone-von Neumann theorem, and as such realize the same results. However, the inclusion of the environment and coupling is easier under the Heisenberg picture (Should I include the derivation?) and looks as follows:

(Equation)

Wherein are called jump operators and act as ladder operators for the interaction between the system and the environment.

This inclusion of the environment as well as the coupling defining the interaction between the two, is a picture that is more grounded in reality. As such, this has the potential to provide further insight and predictive capabilities to any model using it. It is this consideration that governs the technique used in this thesis.

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Now that we have established context for parts used in the building of this model, it’s now time to put it work for some usable results, which follow in the next chapter.

# Chapter 3 –

# Methodology, Findings, and Analysis

3.1 Methodology

3.2 Findings and Analysis

Manipulating 3-state system instead of 2-state

# Chapter 4 –

# Conclusions and Future Considerations

<https://nexdot.fr/en/history-of-quantum-dots/>

<https://www.leica-microsystems.com/science-lab/the-fundamentals-and-history-of-fluorescence-and-quantum-dots/#:~:text=Quantum%20Dots%20(or%20%22Qdots%22,Petersburg>

<https://www.sigmaaldrich.com/CA/en/technical-documents/technical-article/materials-science-and-engineering/biosensors-and-imaging/quantum-dots>

<https://www.benzinga.com/pressreleases/19/02/r13231898/the-history-and-future-of-quantum-dots>