Geometry Workbook Solutions

Angles

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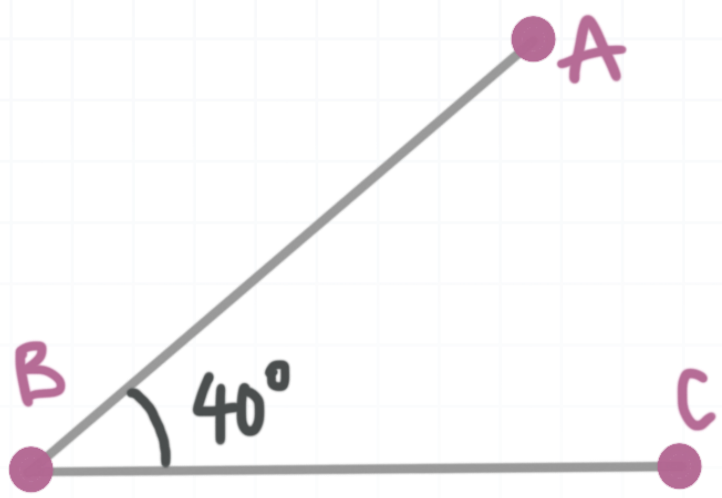
MEASURES OF ANGLES

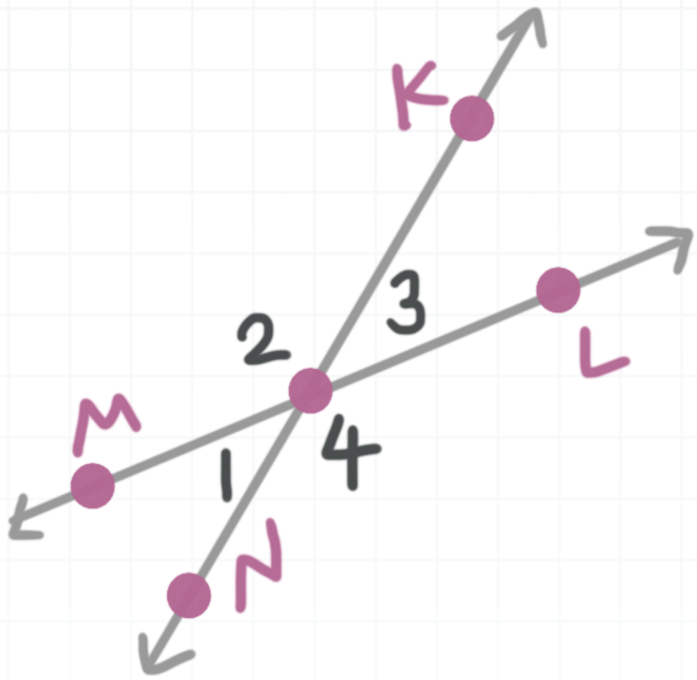
■ 1. Determine whether $\angle ABC$ is obtuse, acute, or right. Then find its supplement.



Solution:

$\angle ABC$ is acute because it has a degree measure less than 90°. Its supplement is 140° degrees.

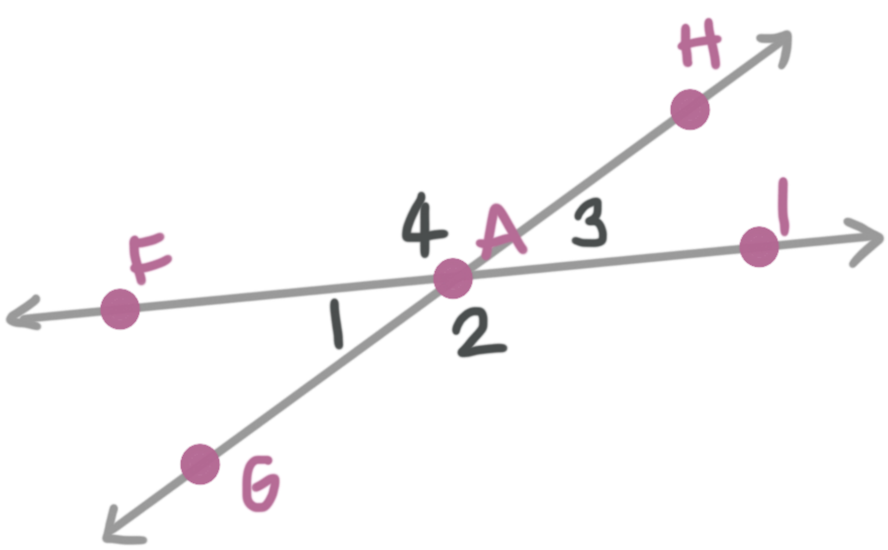
■ 2. $m\angle 1 = 35$. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



Solution:

$m\angle 2 = 145$, $m\angle 3 = 35$, and $m\angle 4 = 145$. Because $\angle 1$ and $\angle 2$ are supplementary angles, $m\angle 2 = 180 - 35 = 145$. $\angle 1 \cong \angle 3$ because they are vertical angles, and $\angle 2 \cong \angle 4$ because they are vertical angles.

■ 3. Find x, y, and z if $m\angle 1 = 3x - 2$, $m\angle 2 = 2y$, $m\angle 3 = 2x + 8$, and $m\angle 4 = 4z$.



Solution:

$x = 10$, $y = 76$, and $z = 38$. Since $m\angle 1 = m\angle 3$,

$$3x - 2 = 2x + 8$$

$$x = 10$$

Since $m\angle 1 + m\angle 2 = 180$,

$$(3(10) - 2) + m\angle 2 = 180$$

$$28 + m\angle 2 = 180$$

$$m\angle 2 = 152$$

Because $m\angle 2 = m\angle 4$, $m\angle 4 = 152$. Then

$$2y = 152$$

$$y = 76$$

and

$$4z = 152$$

$$z = 38$$

■ 4. $\angle 5$ and $\angle 6$ are complementary angles. $m\angle 5 = 3x - 6$ and $m\angle 6 = 2x - 14$. Find the measures of $\angle 5$ and $\angle 6$.

Solution:

$m\angle 5 = 60$ and $m\angle 6 = 30$. Because the angles are complementary, we know

$$m\angle 5 + m\angle 6 = 90$$

$$3x - 6 + 2x - 14 = 90$$

$$x = 22$$

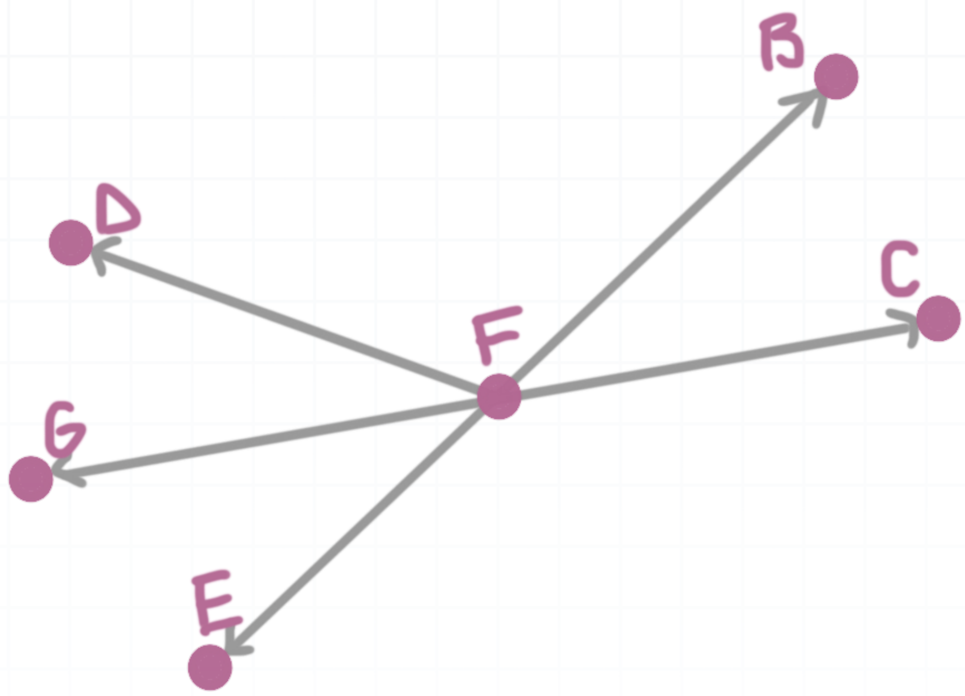
Then we can solve for the measures of each angle.

$$m\angle 5 = 3x - 6 = 3(22) - 6 = 60$$

$$m\angle 6 = 2x - 14 = 2(22) - 14 = 30$$

ADJACENT AND NONADJACENT ANGLES

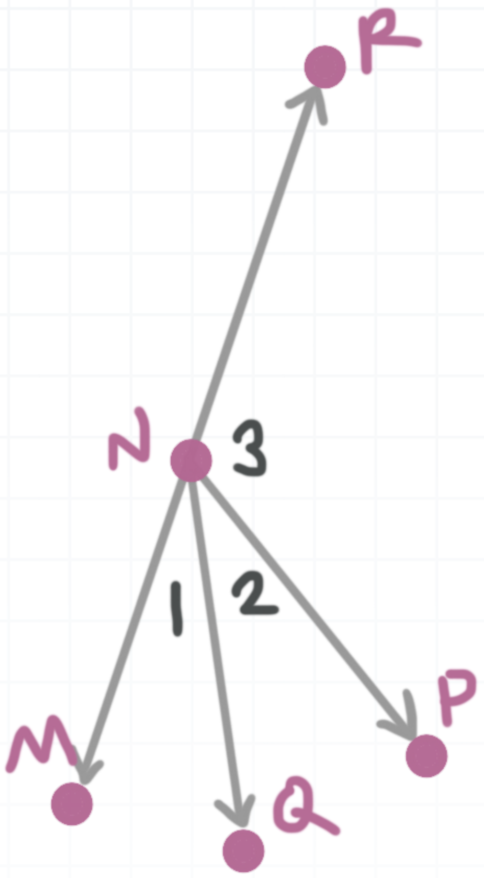
1. Name the angle adjacent to $\angle EFG$.



Solution:

$\angle DFG$ and $\angle CFE$ are both adjacent to $\angle EFG$ because they share a common side.

2. $m\angle 1 = 3x - 10$, $m\angle 2 = 2x - 20$, and $m\angle MNP = 60$. Find the value of x and $m\angle 1$, $m\angle 2$, and $m\angle 3$, given that $\overline{NR}$ and $\overline{NM}$ are opposite rays.



Solution:

$x = 18$, $m\angle 1 = 44$, $m\angle 2 = 16$, and $m\angle 3 = 120$. From the figure, we know that

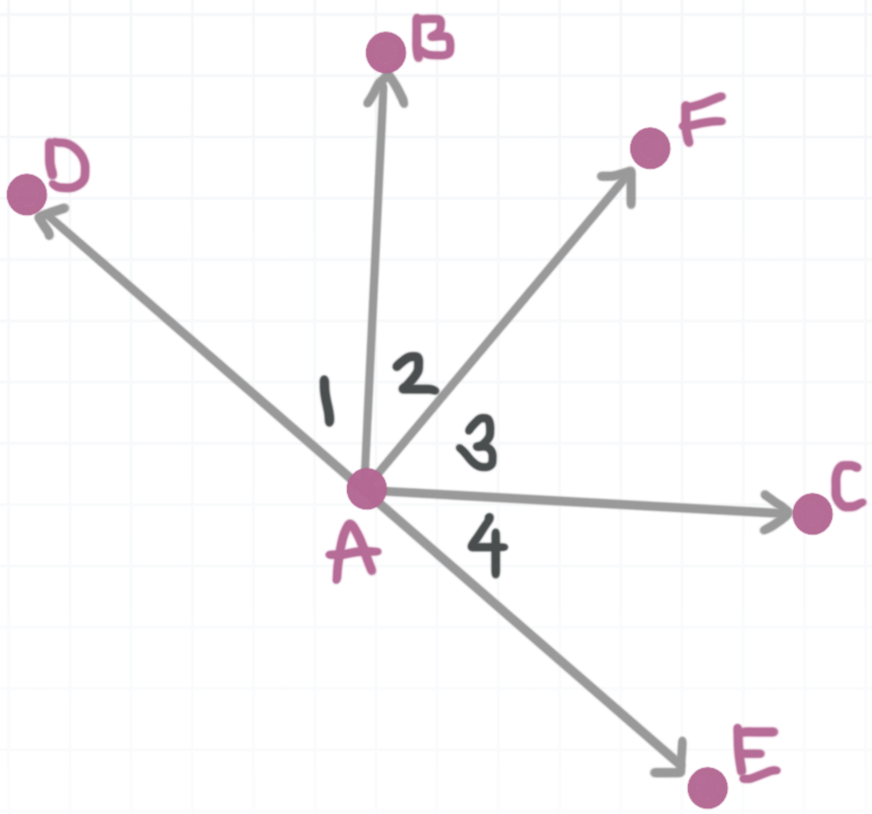
$$m\angle 1 + m\angle 2 = m\angle MNP$$

$$3x - 10 + 2x - 20 = 60$$

$$x = 18$$

Substituting $x = 18$ into each expression gives $m\angle 1 = 44$, $m\angle 2 = 16$, and $m\angle 3 = 120$, because it forms a linear pair with $\angle MNP$.

■ 3. $m\angle 2 = 42$, $\angle 3 \cong \angle 4$, $\angle FAE$ is a right angle, and $\angle DAE$ is a straight angle. Find $m\angle 1$, $m\angle 3$, and $m\angle 4$.



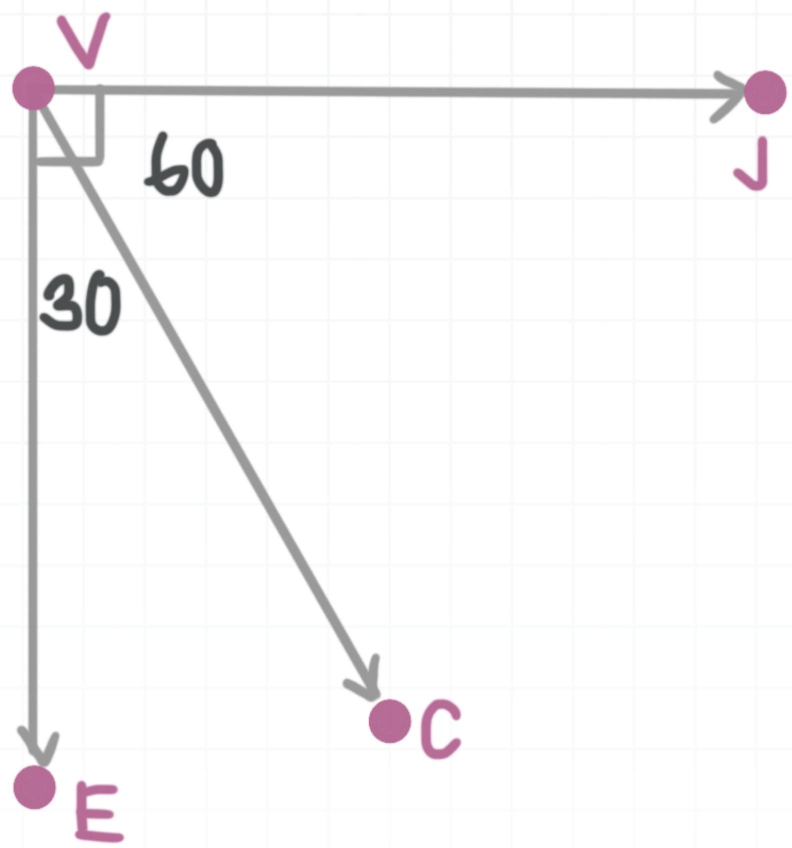
Solution:

$m\angle 1 = 48$, $m\angle 3 = 45$, and $m\angle 4 = 45$. Since $\angle FAE$ is a right angle, $m\angle 3 + m\angle 4 = 90^\circ$. And because $\angle 3$ and $\angle 4$ are congruent, they must both have a measure of 45°. This leaves $m\angle 1 = 48$, so all angles sum to 180°.

■ 4. $\angle JVC$ and $\angle EVC$ are adjacent and complementary. Further, suppose $m\angle JVC = 2m\angle EVC$. Sketch a diagram of this situation and find the measure of each angle.

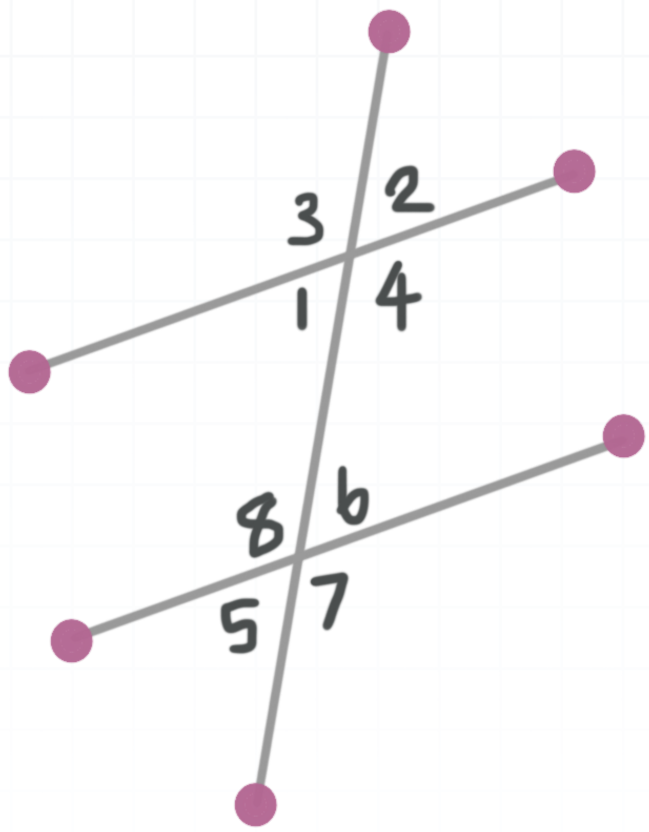
Solution:

$m\angle JVC = 60$ and $m\angle EVC = 30$. A diagram of the figure looks like this:


V
J
60
30
C
E

ANGLES AND TRANSVERSALS

■ 1. Name a pair of corresponding angles.



Solution:

There are four possible correct answers:

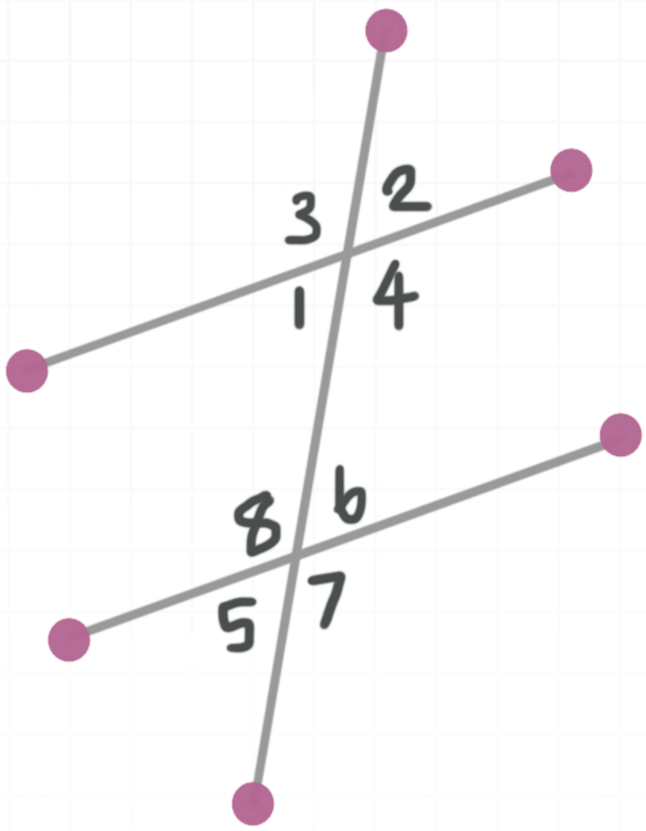
$\angle 7$ and $\angle 4$

$\angle 6$ and $\angle 2$

$\angle 5$ and $\angle 1$

$\angle 8$ and $\angle 3$

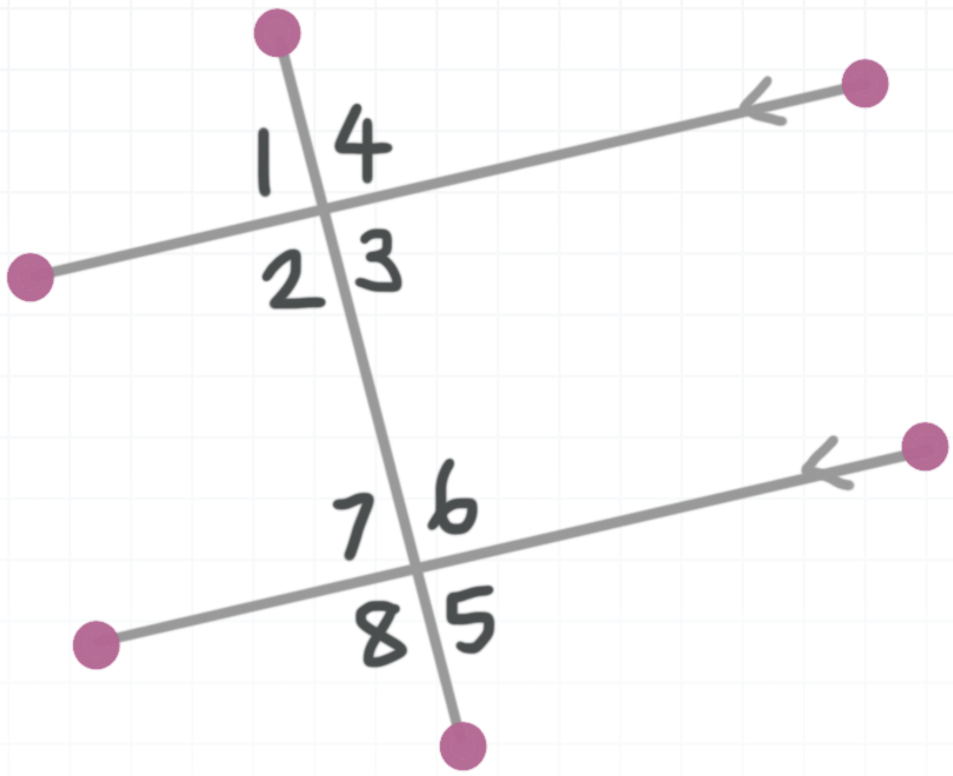
■ 2. Find $m\angle 2$, $m\angle 6$, and $m\angle 5$ if $m\angle 3 = 105$.



Solution:

$m\angle 2 = 75$, $m\angle 6 = 75$, $m\angle 5 = 75$. We know from the figure that $\angle 2$ and $\angle 3$ form a linear pair, making them supplementary. $\angle 2 \cong \angle 6$ because they are corresponding angles, and $\angle 6 \cong \angle 5$ because they are vertical angles.

■ 3. Find x and $m\angle 3$ if $m\angle 2 = 5x + 2$ and $m\angle 7 = 3x + 14$.



Solution:

$x = 20.5$ and $m\angle 3 = 75.5$. From the figure we know that $\angle 2$ and $\angle 7$ are consecutive interior angles. Consecutive interior angles are supplementary.

$$m\angle 2 + m\angle 7 = 180$$

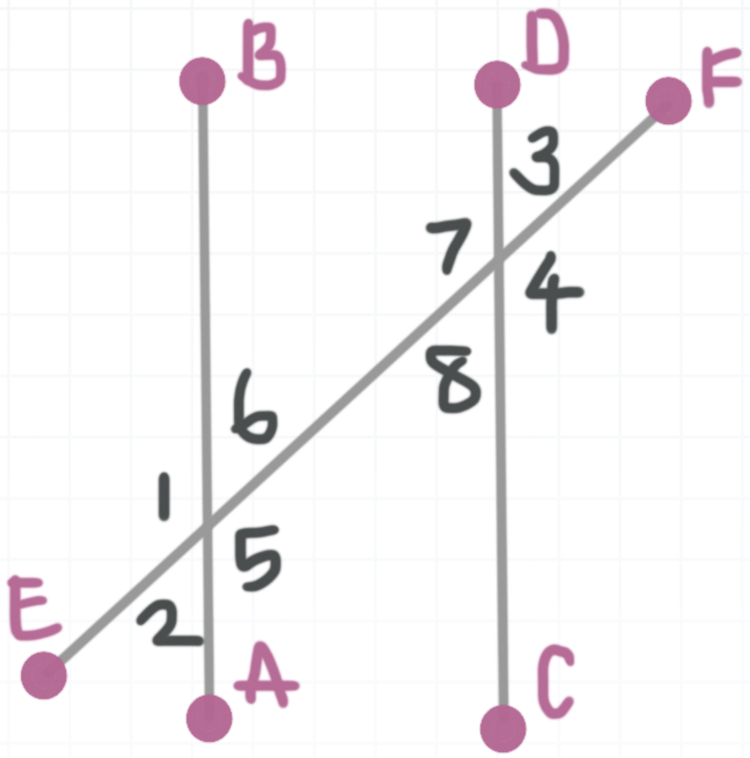
$$5x + 2 + 3x + 14 = 180$$

$$x = 20.5$$

Therefore,

$$m\angle 3 = 75.5$$

■ 4. Find the values of x and y if $\overline{AB}$ and $\overline{DC}$ are parallel lines, and if $m\angle 1 = 2x + y$, $m\angle 2 = 28$, and $m\angle 3 = x + 10$.



Solution:

$x = 18$ and $y = 116$. We know from the figure that $\angle 2 \cong \angle 3$ because they are alternate exterior angles. So we get

$$28 = x + 10$$

$$x = 18$$

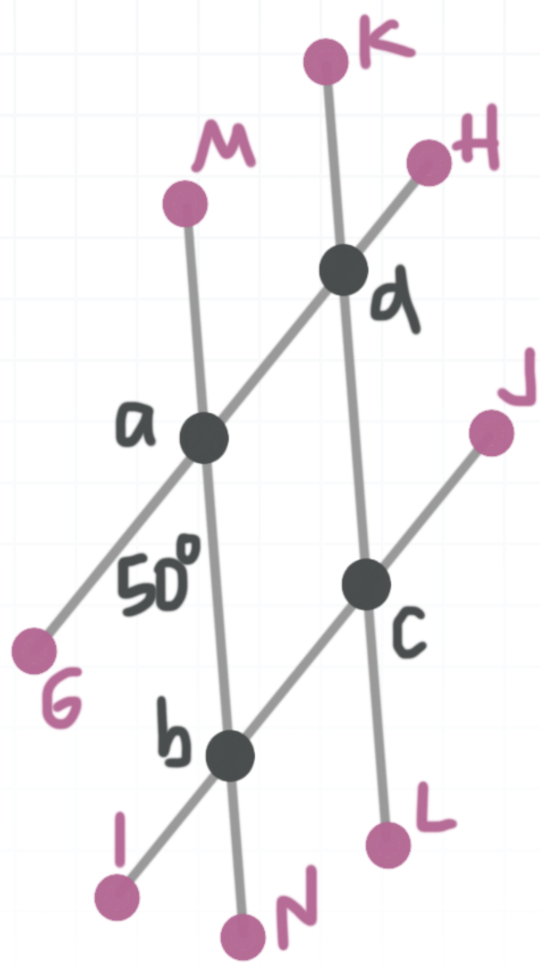
and

$$2x + y = 152$$

$$2(18) + y = 152$$

$$y = 116$$

■ 5. $\overline{MN}$ and $\overline{KL}$ are parallel. $\overline{GH}$ and $\overline{IJ}$ are parallel. Find the values of a, b, c, and d.



Solution:

Given the angle measure of 50°, we know that $m\angle a = 130^\circ$, because angle a is supplementary to the 50° angle. Angles b, c, and d are congruent to angle a, which means that $m\angle a = m\angle b = m\angle c = m\angle d = 130^\circ$.

INTERIOR ANGLES OF POLYGONS

■ 1. Find the sum of the interior angles of a hexagon.

Solution:

720. Using the formula for the sum of interior angles, and the fact that there are 6 sides in a hexagon, we get

$$(n-2)180$$

$$(6-2)180$$

$$(4)180$$

$$720$$

■ 2. Find the measure of each interior angle of a regular 15-gon.

Solution:

156. Using the formula for the sum of interior angles, and the fact that there are 15 sides in a 15-gon, we get

$$(n-2)180$$

$$(15 - 2)180$$

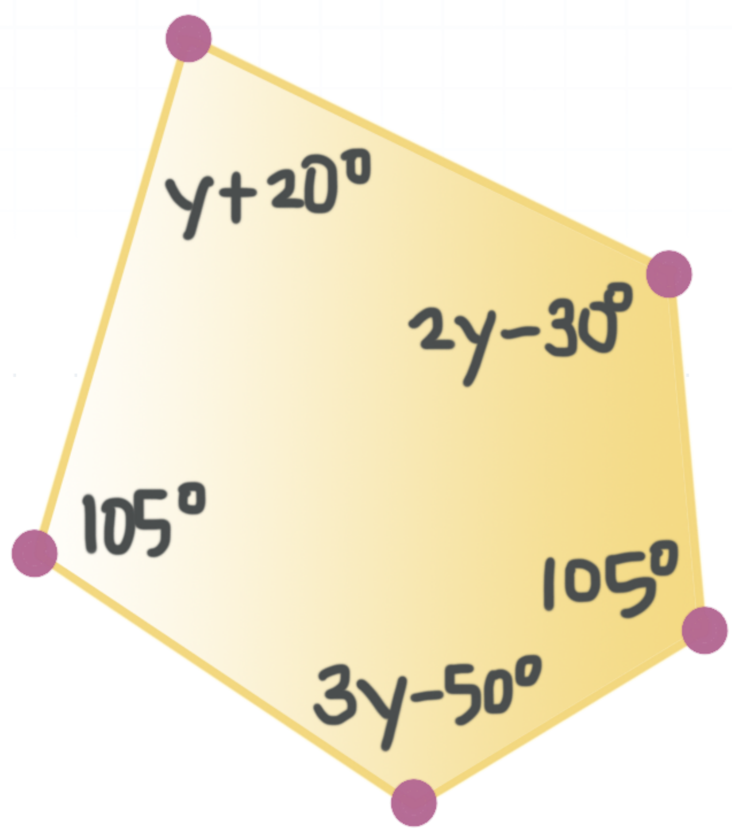
$$(13)180$$

$$2{,}340$$

To find the measure of one interior angle, we divide the sum of all interior angles $2{,}340^\circ$ by the number of interior angles 15.

$$\frac{2{,}340}{15} = 156^\circ$$

■ 3. Find the value of y. Then determine whether this a regular polygon.



Solution:

$y = 65$. This is not a regular polygon because the angle measures are $105, 85, 100, 105, 145$.

The interior angles of a pentagon have a sum of 540°.

$$105 + y + 20 + 2y - 30 + 105 + 3y - 50 = 540$$

$$6y + 150 = 540$$

$$y = 65$$

■ 4. Each interior angle measure of a regular polygon is 160°. Find the number of sides of this polygon.

Solution:

18 sides. From the formula for the measure of a single interior angle of a regular polygon, we get

$$\frac{(n-2)180}{n} = 160$$

$$(n-2)180 = 160n$$

$$180n - 360 = 160n$$

$$20n = 360$$

$$n = 18$$

EXTERIOR ANGLES OF POLYGONS

1. Find the sum of the exterior angles of a decagon.

Solution:

360°. By the Exterior Angle Theorem, we know that the sum of the exterior angles of a polygon is always 360°.

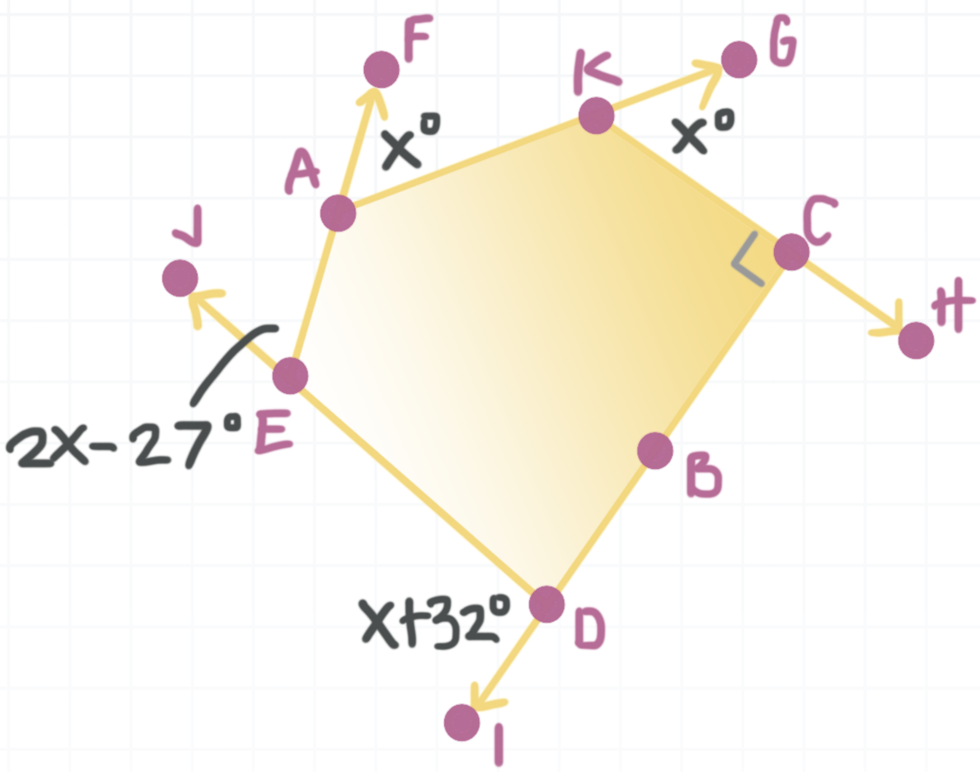
2. Each exterior angle of a regular polygon has measure of 30°. Find the number of sides of this polygon.

Solution:

12 sides. Since the exterior angles of any polygon sum to 360°, the number of sides must be given by

$$\frac{360^\circ}{n} = \frac{360^\circ}{30^\circ} = 12$$

3. Find the value of x.



Solution:

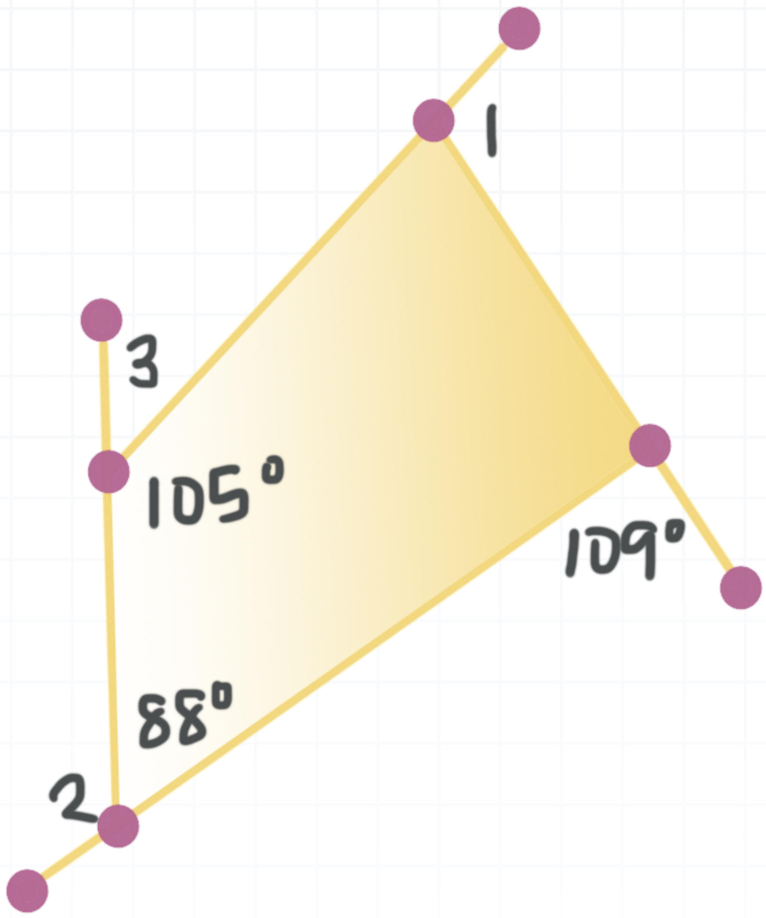
$x = 53$. All exterior angle measures are given, and the exterior angle at C must be 90°, since the interior angle there is also 90°. Because the exterior angles of any polygon always sum to 360°, we get

$$x + x + 90 + x + 32 + 2x - 27 = 360$$

$$5x + 95 = 360$$

$$x = 53$$

■ 4. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ based on the figure.



Solution:

$m\angle 1 = 84$, $m\angle 2 = 92$, and $m\angle 3 = 75$. Find $m\angle 2$ and $m\angle 3$ first because they form a linear pair with their adjacent angle. Then further find $m\angle 1$ by setting the sum of all exterior angles equal to 360°.

W W W . K R I S T A K I N G M A T H . C O M