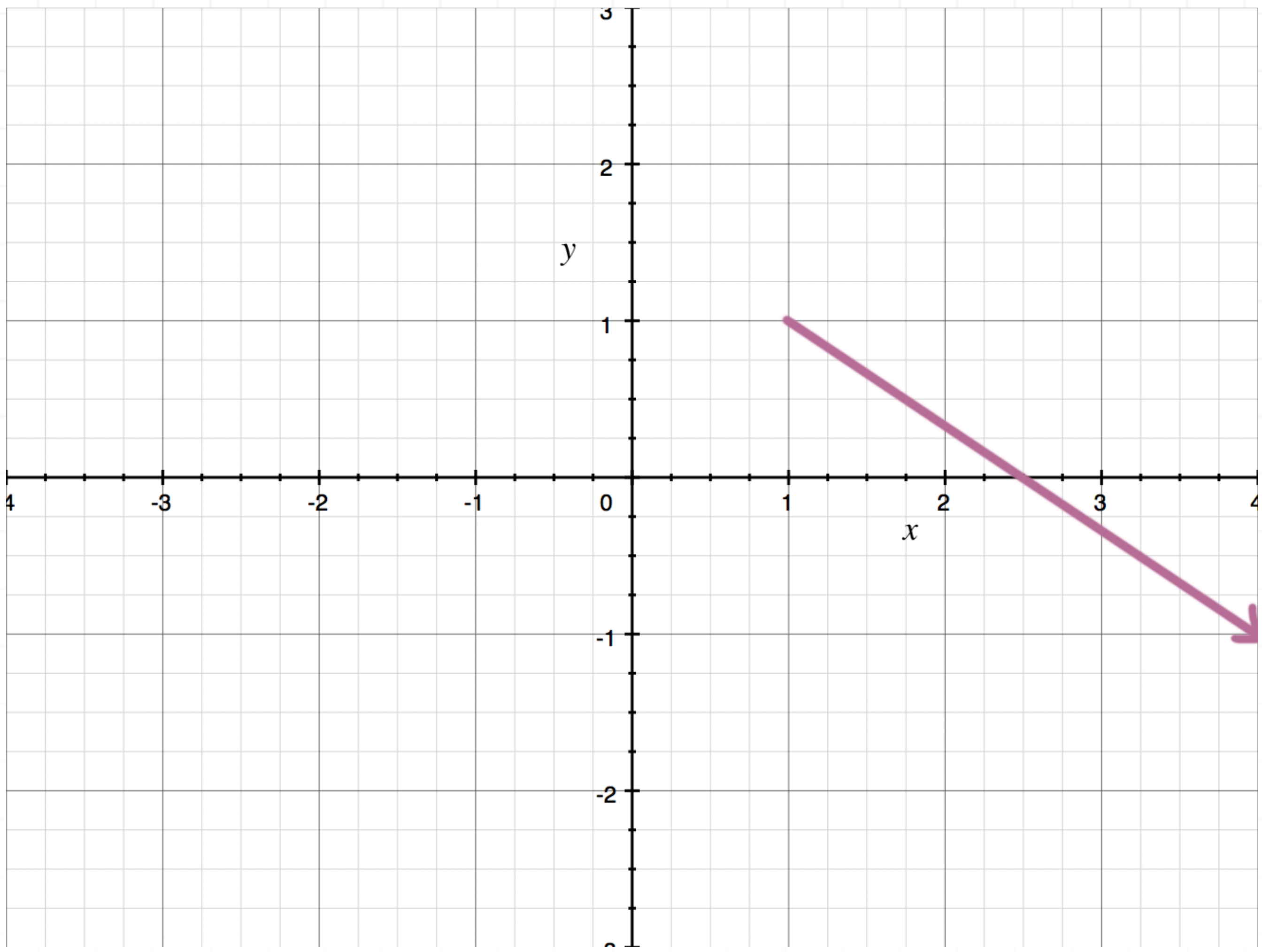
Translation vectors

In this lesson we'll look at how to use translation vectors to indicate a translation of a figure.

A translation vector is a type of quantity that indicates a translation of a figure in the coordinate plane from one location to another. In other words, a translation vector can be thought of as indicating a slide with no rotation. The slide won't change the shape or size of the figure, and because there's no rotation, the orientation won't change either.

A translation vector can be drawn on the coordinate grid or written as $\overrightarrow{v} = \langle a, b \rangle$. For example, a translation vector that indicates translation of a figure 3 units to the right and 2 units down can be represented mathematically as $\overrightarrow{v} = \langle 3, -2 \rangle$, or graphically as an arrow, as shown in the figure.

© Krista King Math



It doesn't matter where the vector is positioned in the plane. In this figure, the vector starts at $(1,1)$, the location of the "tail" of the arrow, and ends at $(4, -1)$, the location of the "head" of the arrow.

Notice that the difference is the x-coordinates of the "head" and "tail" of the vector is $4 - 1 = 3$, and that the difference in their y-coordinates is $(-1) - 1 = -2$.

But the initial point and terminal point of the vector are irrelevant. What matters is the length of the vector and the direction in which it points, so all you have to look at is the difference in the x-coordinates of the head and tail of the translation vector and the difference in the y-coordinates of its head and tail.

A translation vector $\overrightarrow{v} = (a, b)$ has two components: a horizontal component a (which is given by the difference in the x-coordinates of its

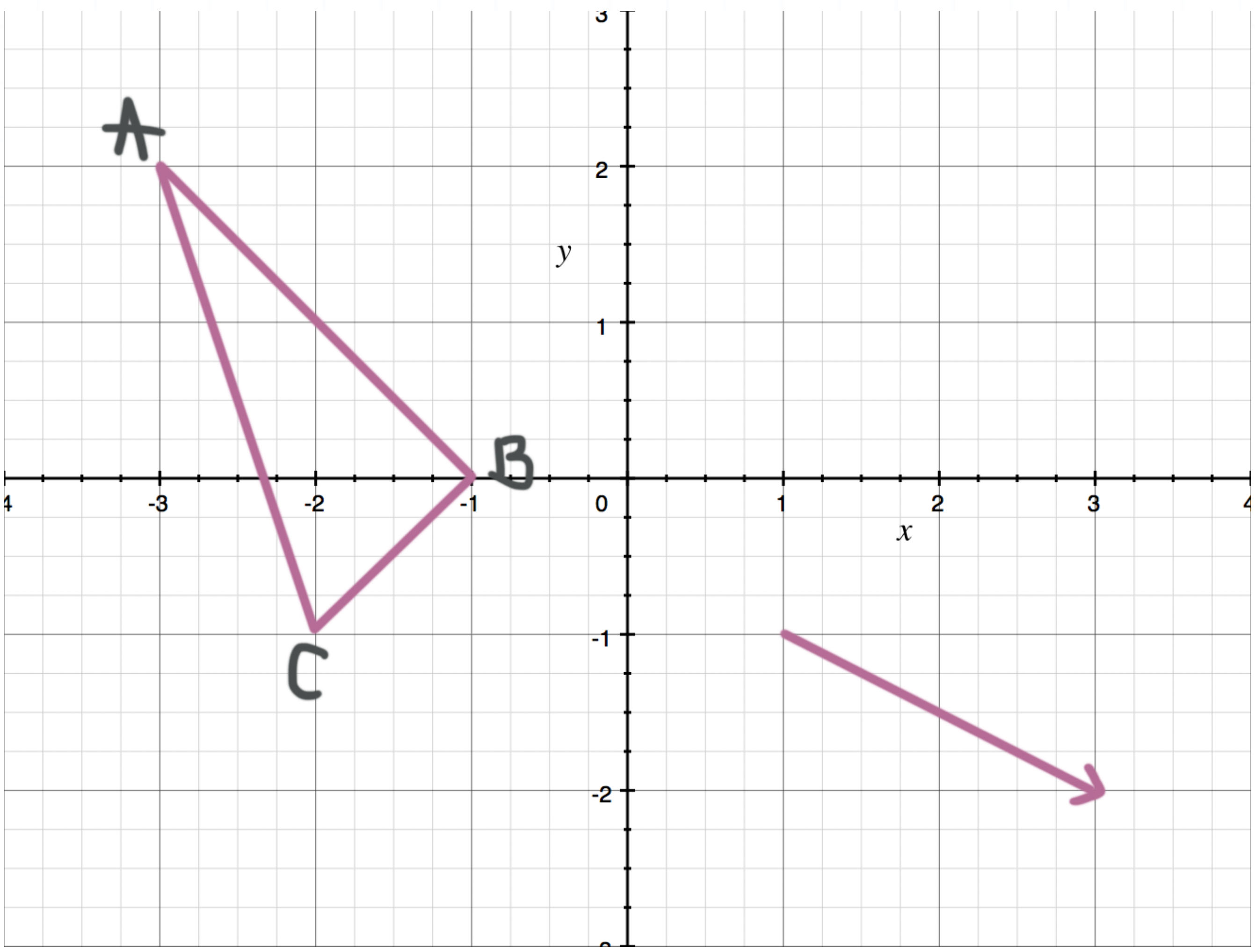
© Krista King Math

head and tail) and a vertical component b (which is given by the difference in the y-coordinates of its head and tail). By the Pythagorean theorem, the length of a translation vector is equal to $\sqrt{a^2 + b^2}$, the square root of the sum of the squares of its horizontal and vertical components. The direction of a translation vectors is given by its slope, which is equal to b/a, the ratio of its vertical component to its horizontal component.

Let's look at some examples.

Example

Use the translation vector shown to find the coordinates of the vertices of triangle $A'B'C'$.



The tail and head of this translation vector are at $(1, -1)$ and $(3, -2)$, respectively, which means that this vector indicates a translation of 2 units to the right (because the difference in the x-coordinates of its head and tail is $3 - 1 = 2$) and 1 unit down (because the difference in the y-coordinates of its head and tail is $(-2) - (-1) = -1$).

We can therefore add 2 to the x-coordinate of each vertex of the triangle in the pre-image, and subtract 1 from the y-coordinate of each vertex of the triangle in the pre-image, to find the coordinates of the vertices of the triangle in the image.

First let's write down the coordinates of the vertices of the triangle in the pre-image ($\triangle ABC$).

$$A = (-3,2)$$

$$B = (-1,0)$$

$$C = (-2, -1)$$

Now we can calculate the coordinates of the vertices of the triangle in the image ($\triangle A'B'C'$).

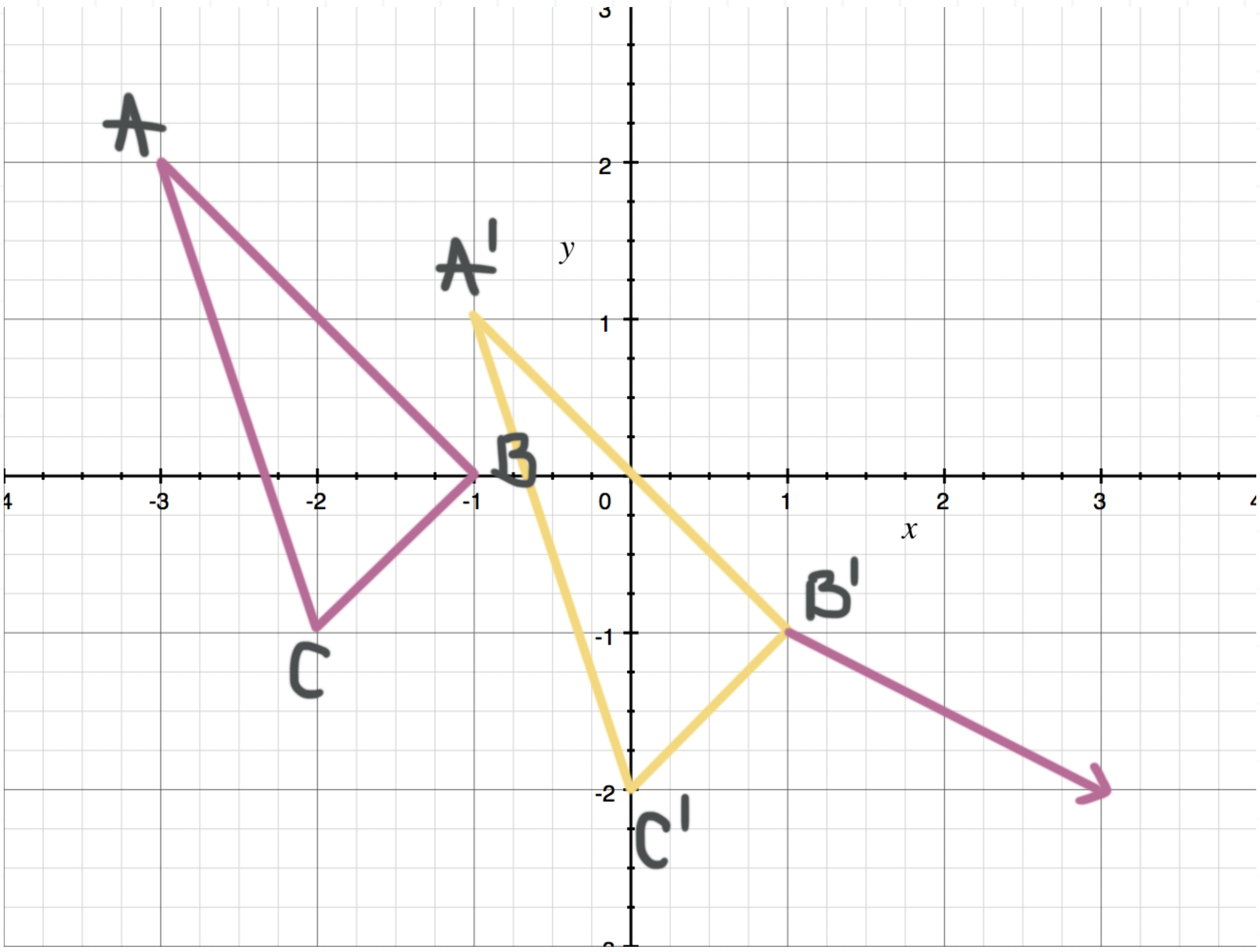
$$A' = (-3 + 2,2 - 1) = (-1,1)$$

$$B' = (-1 + 2,0 - 1) = (1, -1)$$

$$C' = (-2 + 2, -1 - 1) = (0, -2)$$

The pre-image, the translation vector, and the image are shown in the figure below.

© Krista King Math



Let's look at one more example.

Example

The vertices of $\triangle ABC$ are at $A=(-5,5)$, $B=(-2,5)$, and $C=(-3,0)$. If the triangle is translated by $\vec{v}=\langle -5,-6\rangle$, what are the locations of the vertices of the triangle in the image ($\triangle A'B'C'$)?

The translation vector $\vec{v}=\langle -5,-6\rangle$ means each point is being moved 5 units to the left and 6 units down. So for each vertex of $\triangle ABC$, we'll subtract 5 from its x-coordinate and we'll subtract 6 from its y-coordinate.

© Krista King Math

Then the vertices of the triangle in the image are

$$A' = (-5-5, 5-6) = (-10, -1)$$

$$B' = (-2-5, 5-6) = (-7, -1)$$

$$C' = (-3-5, 0-6) = (-8, -6)$$

© Krista King Math