Topic: Radius of the balloon

Question: A thin sheet of ice is in the shape of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of 0.25 m^2/s, at what rate is the radius decreasing when the area of the sheet is 25 m^2?

Answer choices:

A $\quad \dfrac{0.025}{\sqrt{\pi}}$ m/s

B $\quad 400\pi$ cm/s

C $\quad 112\pi$ cm/s

D $\quad \dfrac{1}{400\sqrt{\pi}}$ m/s

Solution: A

The formula for the area of a circle is

$$A = \pi r^2$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dA}{dt} = \pi(2r)\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

From the question, we know that $dA/dt = 0.25$ and that $A = 25$, so first we need to find the radius.

$$A = \pi r^2$$

$$25 = \pi r^2$$

$$r^2 = \frac{25}{\pi}$$

$$r = \frac{5}{\sqrt{\pi}}$$

Now we'll plug in everything we know.

$$0.25 = 2\pi\left(\frac{5}{\sqrt{\pi}}\right)\frac{dr}{dt}$$

$$0.25 = 10\sqrt{\pi}\frac{dr}{dt}$$

Solve for dr/dt, which is the rate we were asked to find.

$$\frac{dr}{dt} = \frac{0.25}{10\sqrt{\pi}}$$

$$\frac{dr}{dt} = \frac{0.025}{\sqrt{\pi}}$$

Topic: Radius of the balloon

Question: Air is being pumped into a spherical balloon at a rate of 192π m^3/hr. How fast is the balloon's surface area changing when $r = 4$ m?

Answer choices:

A $\frac{1}{96\pi}$ m^2/hr

B 3 m^2/hr

C 96π m^2/hr

D $\frac{1}{3\pi}$ m^2/hr

Solution: C

The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

From the question, we know that $dV/dt = 192\pi$ and that $r = 4$, so we'll plug those in.

$$192\pi = 4\pi(4)^2\frac{dr}{dt}$$

$$192\pi = 64\pi\frac{dr}{dt}$$

Solve for dr/dt,

$$\frac{dr}{dt} = \frac{192\pi}{64\pi}$$

$$\frac{dr}{dt} = 3$$

The formula for the surface area of a sphere is

$$S = 4\pi r^2$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dS}{dt} = 4\pi(2r)\frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$$

From the question, we know that $dr/dt = 3$ and that $r = 4$, so we'll plug those in.

$$\frac{dS}{dt} = 8\pi(4)(3)$$

$$\frac{dS}{dt} = 96\pi$$

Topic: Radius of the balloon

Question: Air is being sucked out of a spherical balloon so that its volume is decreasing by 250 cm^3/s. How fast is the radius decreasing when the radius is 5 cm?

Answer choices:

A $\frac{5}{2\pi}$ cm/s

B $-\frac{5}{2\pi}$ cm/s

C $\frac{2}{5\pi}$ cm/s

D $-\frac{2}{5\pi}$ cm/s

Solution: B

The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

From the question, we know that $dV/dt = -250$ and that $r = 5$, so we'll plug those in.

$$-250 = 4\pi(5)^2\frac{dr}{dt}$$

$$-250 = 100\pi\frac{dr}{dt}$$

Solve for dr/dt, which is the rate we were asked to find.

$$\frac{dr}{dt} = -\frac{250}{100\pi}$$

$$\frac{dr}{dt} = -\frac{5}{2\pi}$$