Graphing roses

Now we'll look at how to sketch the graphs of polar curves called "roses," which are polar equations in either of these forms:

$$r = c\cos(n\theta)$$

$$r = c\sin(n\theta)$$

where c is a non-zero constant, and n is an integer greater than 1 (or less than -1).

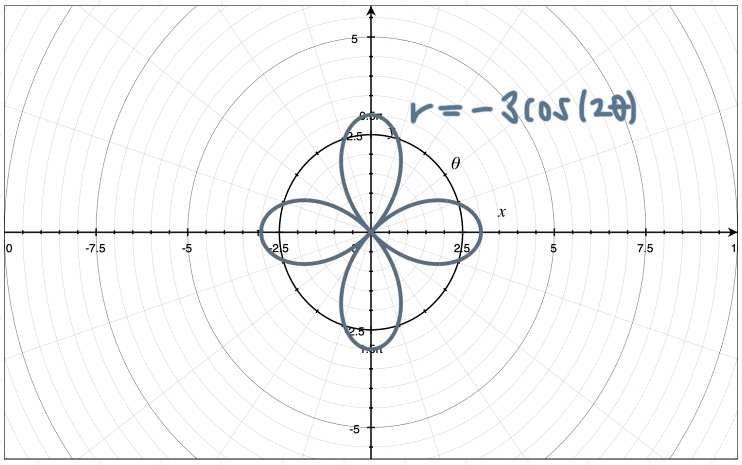
We call these kinds of curves roses because they look a little bit like flowers, with the "petals" of the flower extending out from the origin.

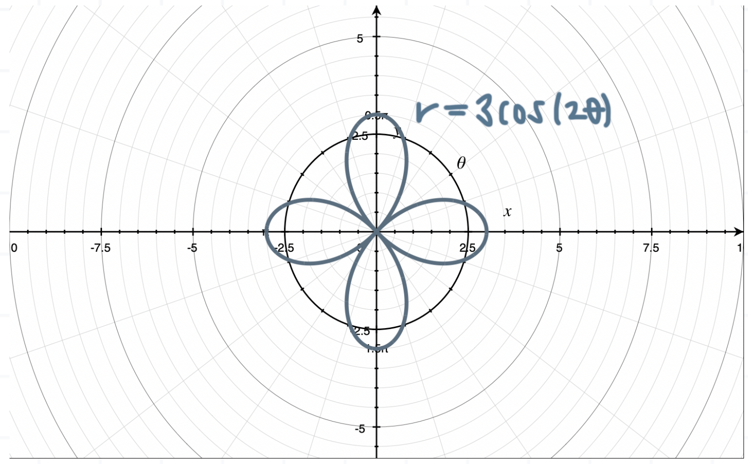
Properties of roses

As we learn to graph these kinds of curves, it's helpful to keep a few properties of roses in mind.

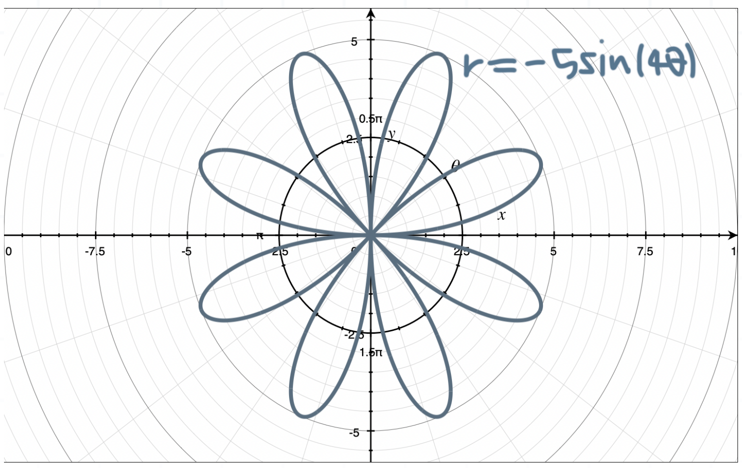
First, the tips of the petals will extend out to a distance of $|c|$ away from the origin. So if $c = \pm 3$, for example, then the petals will extend out to a distance of 3 from the origin.

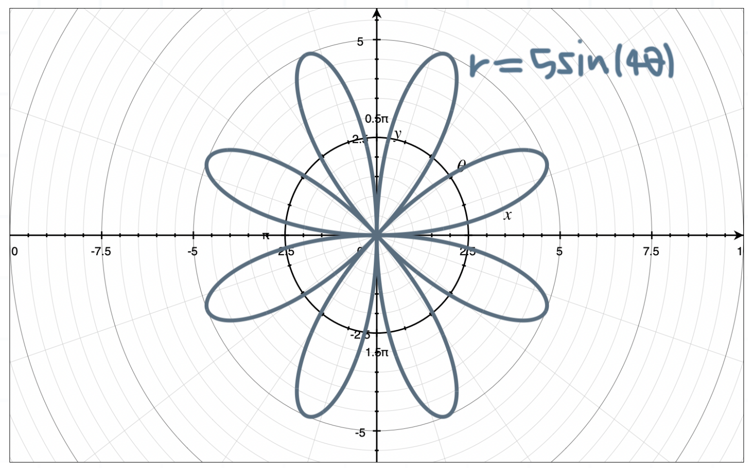
If n is even, the graph of the rose doesn't change when the sign of c changes, so the graph of $r = -3\cos(2\theta)$ will be identical to $r = 3\cos(2\theta)$,



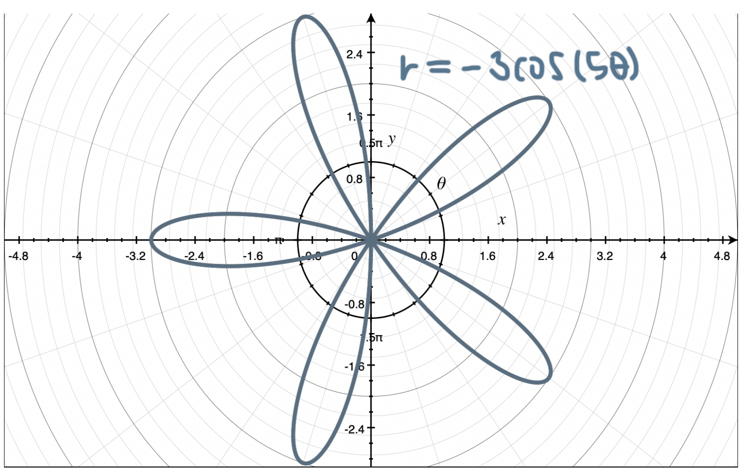


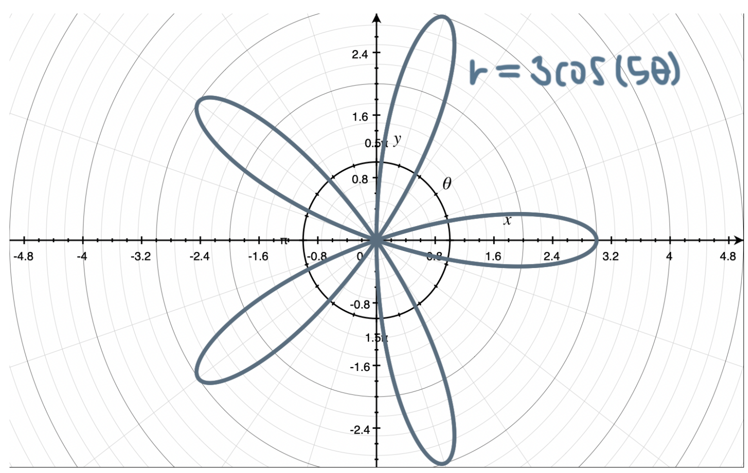
and the graph of $r = -5\sin(4\theta)$ will be identical to $r = 5\sin(4\theta)$.



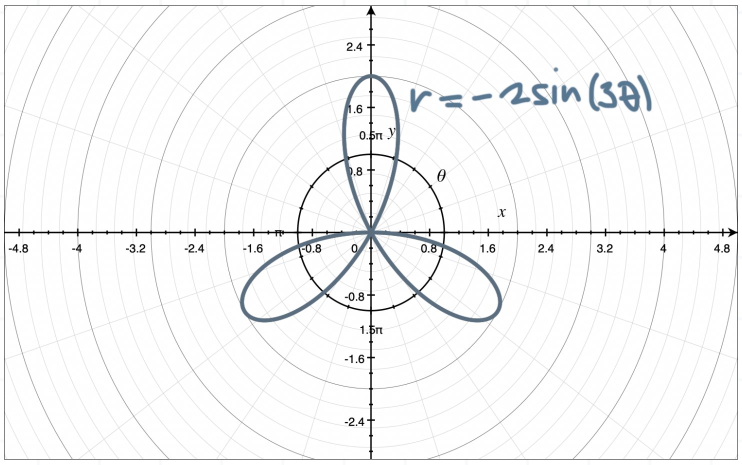


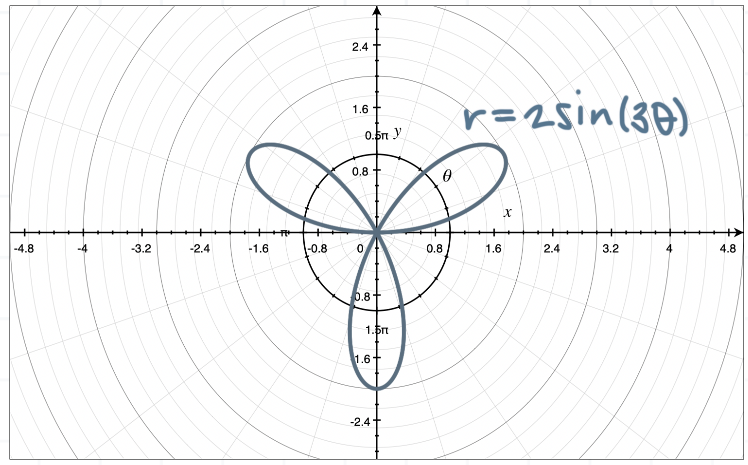
If n is odd, the graph of the rose rotates when the sign of c changes. The graphs of $r = -3\cos(5\theta)$ and $r = 3\cos(5\theta)$ are





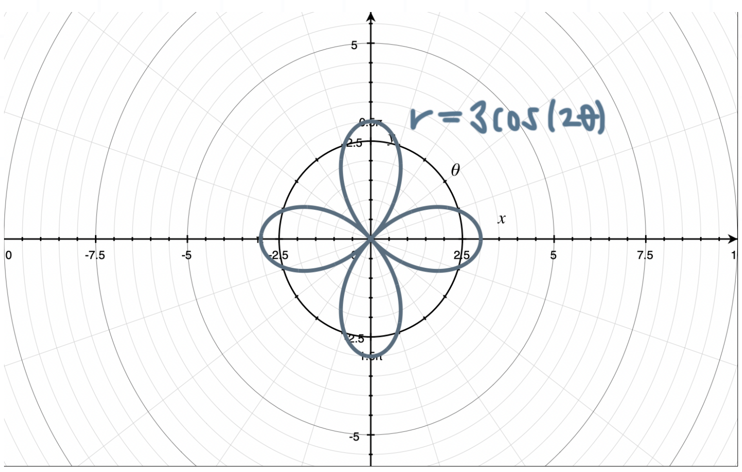
and the graphs of $r = -2\sin(3\theta)$ and $r = 2\sin(3\theta)$ are

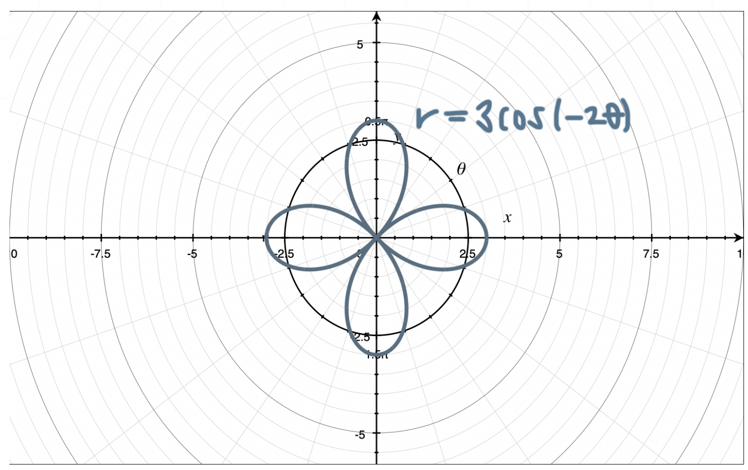




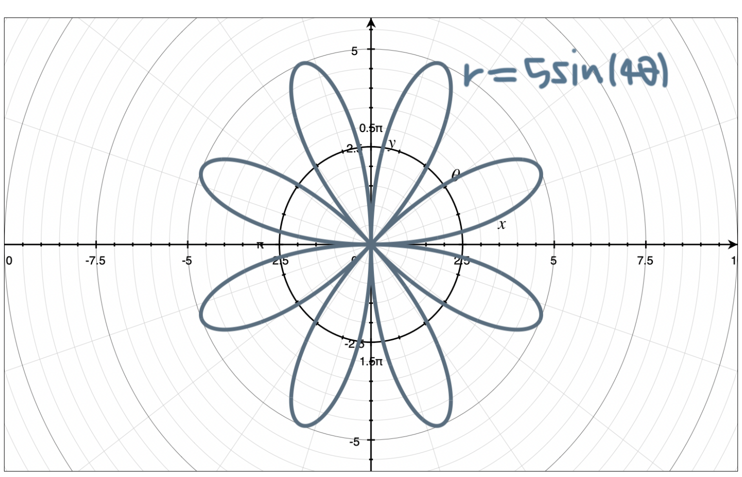
Second, the rose will have $|2n|$ petals when n is an even integer, but $|n|$ petals when n is an odd integer.

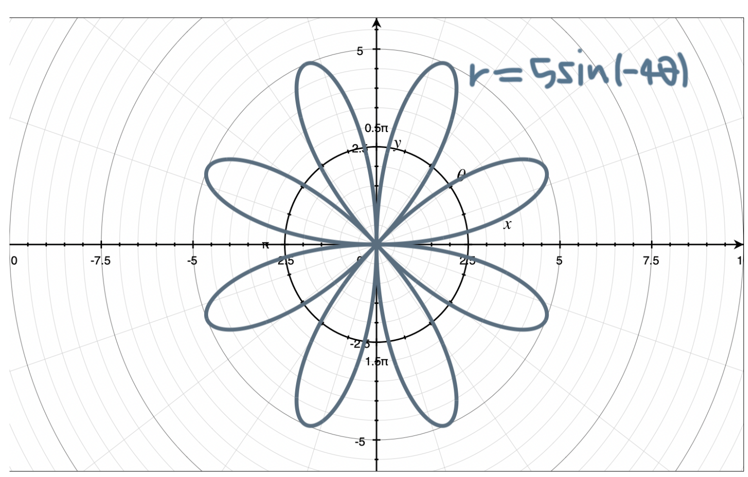
If n is even, the graph of the rose doesn't change when the sign of n changes, so the graphs of $r=3\cos(2\theta)$ and $r=3\cos(-2\theta)$ will be identical with $|2n|=|2(2)|=4$ petals,



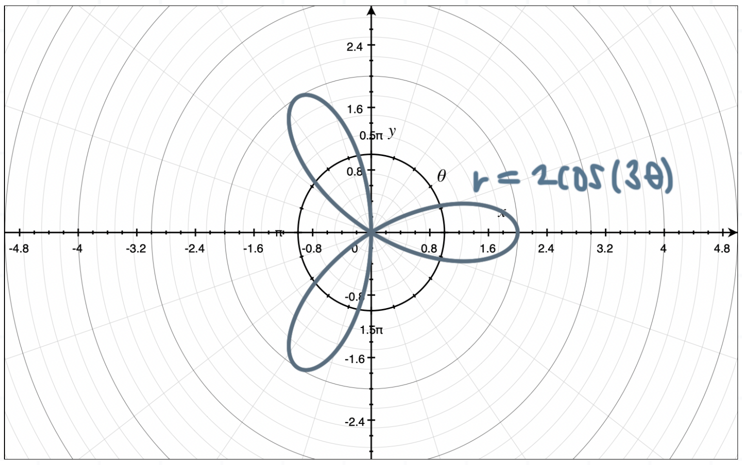


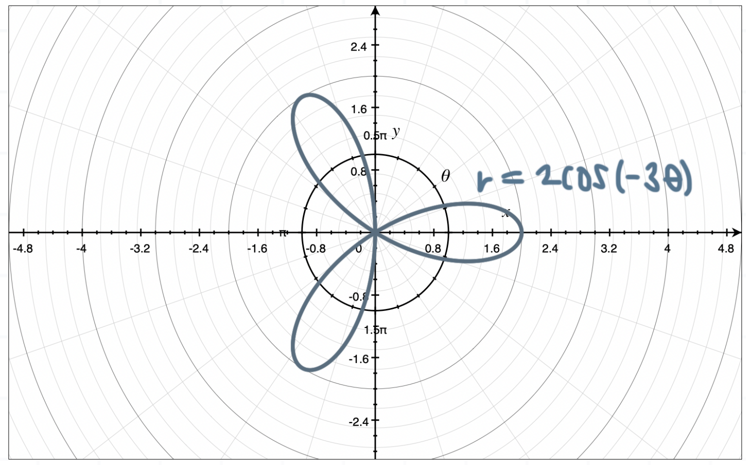
and the graphs of $r=5\sin(4\theta)$ and $r=5\sin(-4\theta)$ will be identical with $|2n|=|2(4)|=8$.



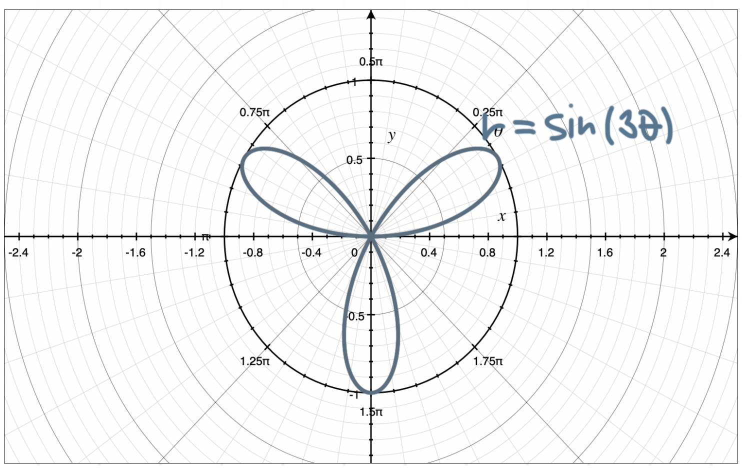


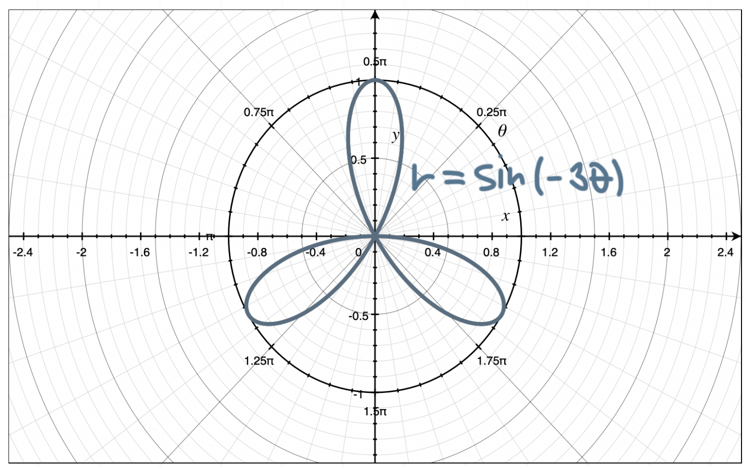
If n is odd, the graphs of cosine roses are identical when the sign of n changes, but the graphs of sine roses rotate. The graphs of $r = 2\cos(3\theta)$ and $r = 2\cos(-3\theta)$ will have $|n| = |3| = 3$ petals,





and the graphs of $r = \sin(3\theta)$ and $r = \sin(-3\theta)$ will have $|n| = |3| = 3$ petals.





How to sketch roses

We'll use the same approach to sketch roses that we used in the previous lesson for sketching circles:

1. Set the argument of the trigonometric function equal to $\pi/2$, and then solve the equation for θ.

2. Evaluate the polar curve at multiples of the θ-value we solved for in Step 1, starting with $\theta = 0$, and plot the resulting points on the polar graph.

3. Connect the points on the polar graph with a smooth curve.

Let's do an example where we work through these steps in order to sketch a rose in the form $r = c\cos(n\theta)$.

Example

Sketch the graph of $r = 3\cos(2\theta)$.

Because $c = 3$ and $n = 2$, this rose will have $|2n| = |2(2)| = 4$ petals that extend out a distance of 3 from the origin. To sketch the graph, we recognize that the trigonometric function in this polar equation is $\cos(2\theta)$, and its argument (the angle at which cosine is evaluated) is 2θ. So we'll set

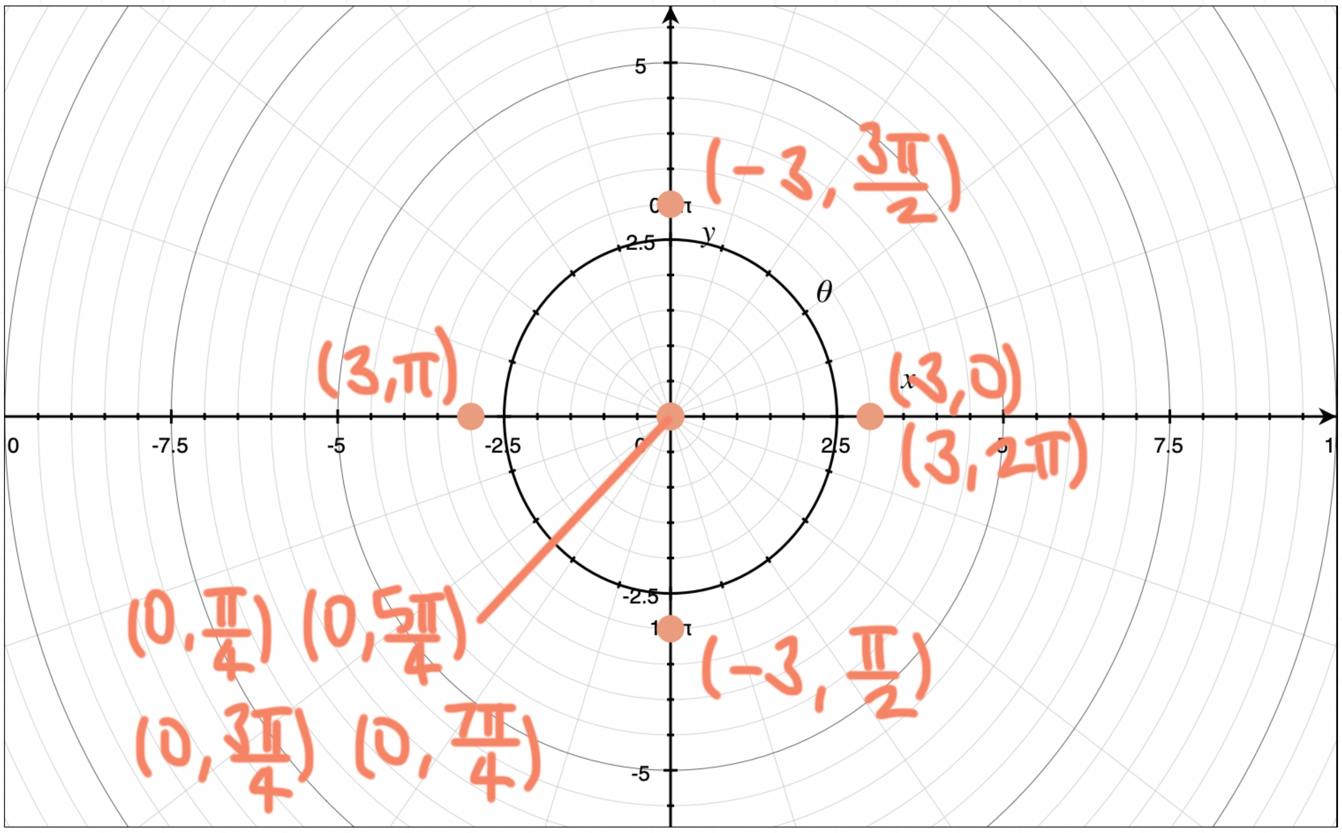
$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

Now we'll make a table with multiples of $\pi/4$, like $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi,$ $5\pi/4, 3\pi,$ etc., and include the values of r that correspond to each of these θ -values.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
r	3	0	-3	0	3

Plotting these points on the polar graph gives



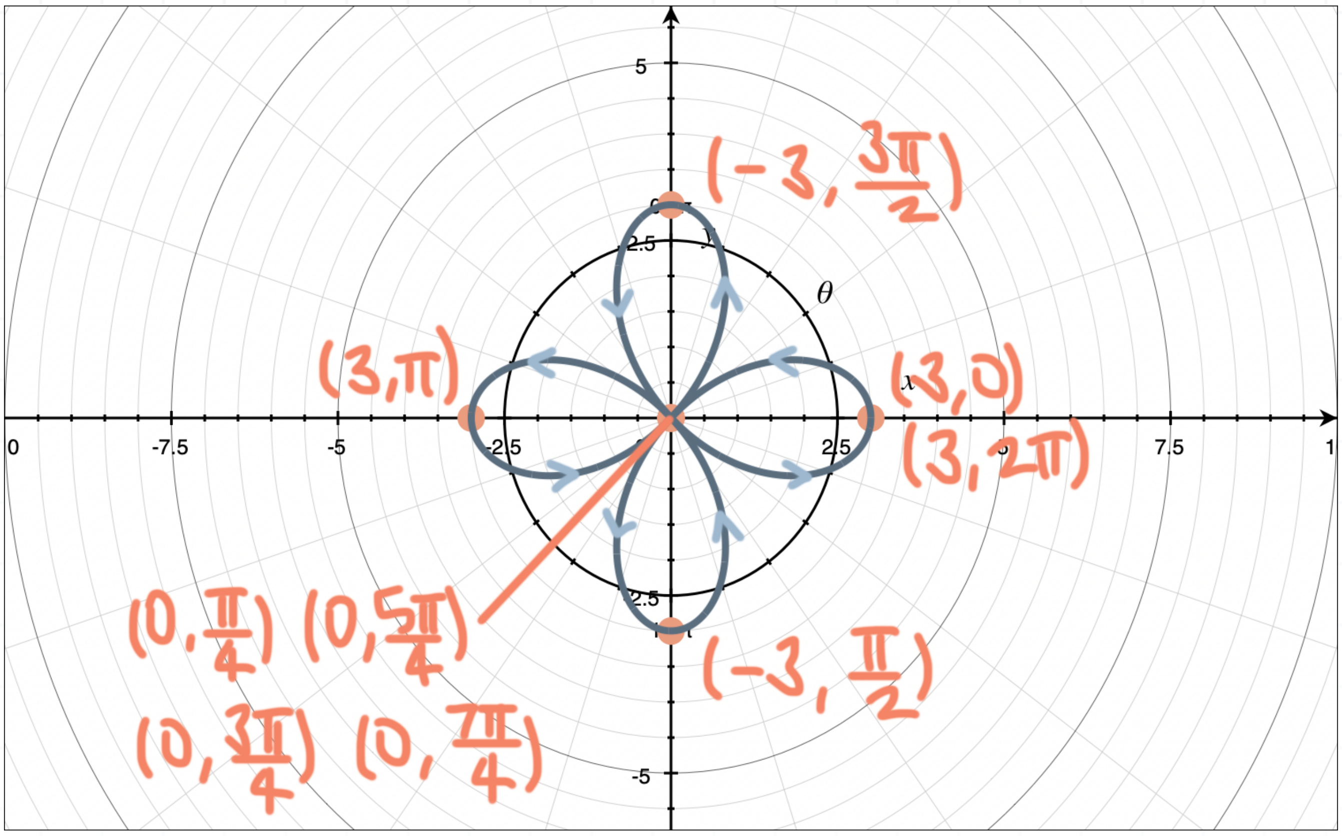
And if we connect these points with a smooth curve, in order, we see the graph of the rose. We start at $(3,0)$, and then loop to the origin at $(0,\pi/4)$,

loop to $(-3,\pi/2)$ then back to the origin at $(0,3\pi/4)$,

loop to $(3,\pi)$ then back to the origin at $(0,5\pi/4)$,

loop to $(-3,3\pi/2)$ then back to the origin at $(0,7\pi/4)$,

then finally loop back to $(3,2\pi)$, which is actually the same point as $(3,0)$. From there on, we're retracing the same pieces of the rose over and over.



Let’s do another example with a cosine rose.

Example

Sketch the graph of $r = -5\cos(4\theta)$.

Because $c = -5$ and $n = 4$, this rose will have $|2n| = |2(4)| = 8$ petals that extend out a distance of 5 from the origin. To sketch the graph, we recognize that the trigonometric function in this polar equation is $\cos(4\theta)$, and its argument (the angle at which cosine is evaluated) is 4θ. So we’ll set

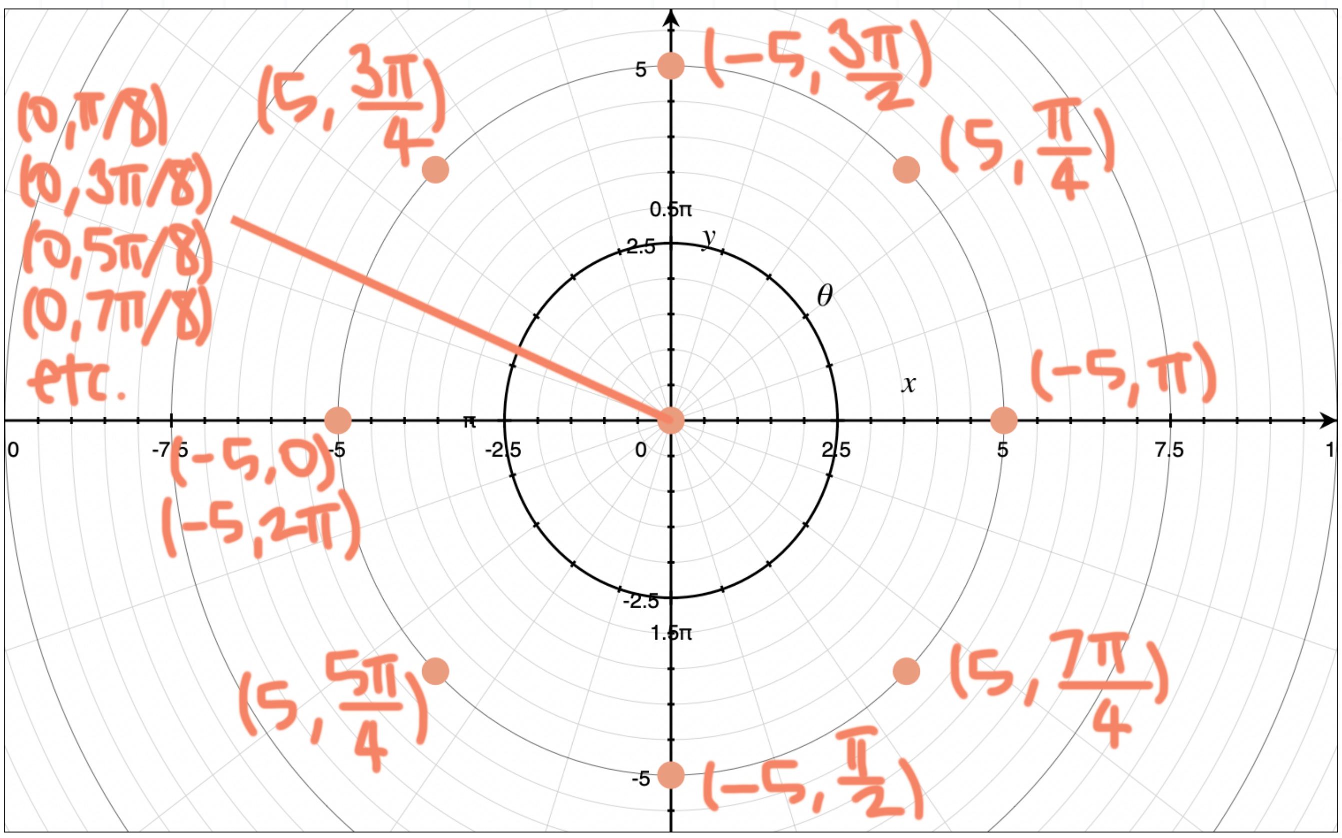
$$4\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{8}$$

Now we'll make a table with multiples of $\pi/8$, like $\theta = 0$, $\pi/8$, $\pi/4$, $3\pi/8$, $\pi/2$, $5\pi/8$, $3\pi/4$, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
r	-5	0	5	0	-5

Plotting these points on the polar graph gives



And if we connect these points with a smooth curve, in order, we see the graph of the rose. We start at $(-5,0)$ and then loop to the origin at $(0,\pi/8)$,

loop to $(5,\pi/4)$ then back to the origin at $(0,3\pi/8)$,

loop to $(-5,\pi/2)$ then back to the origin at $(0,5\pi/8)$,

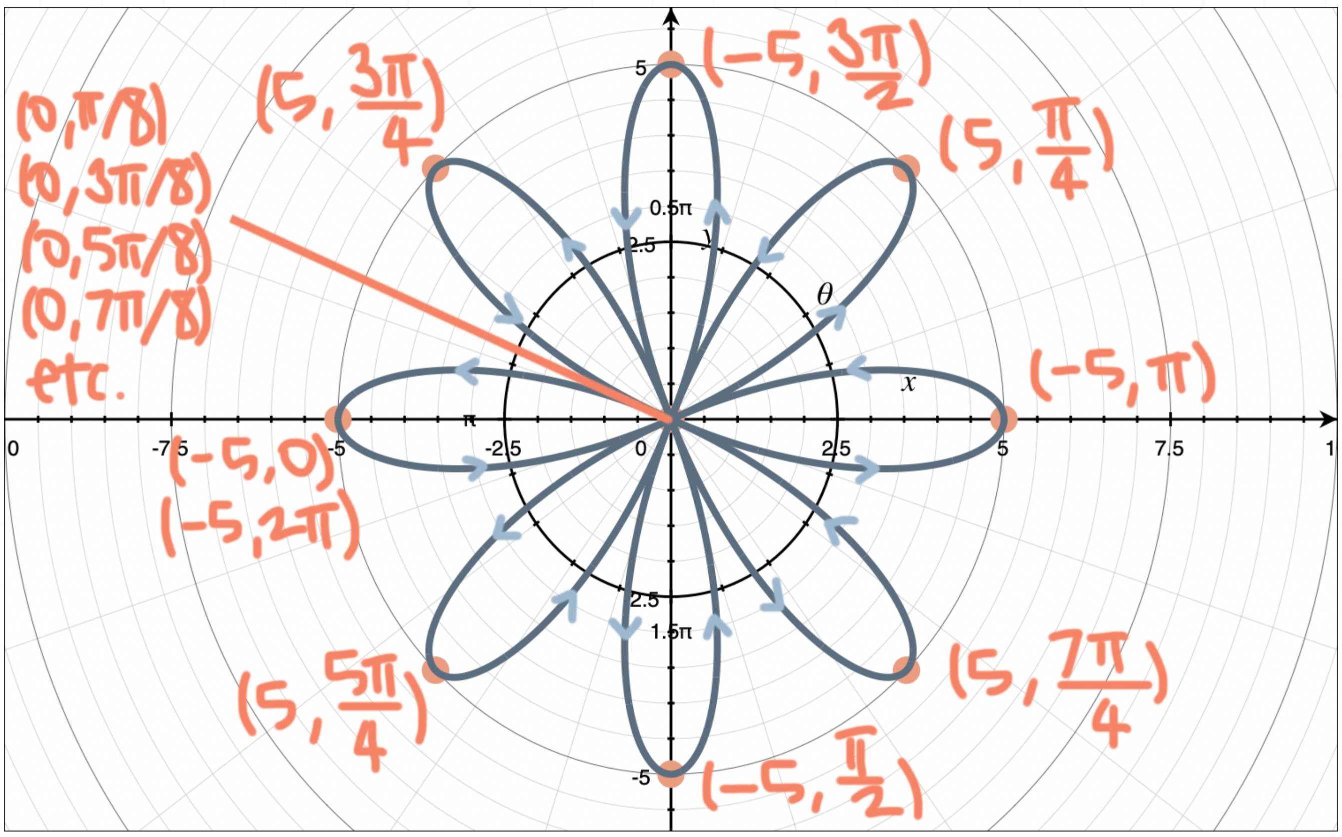
loop to $(5,3\pi/4)$ then back to the origin at $(0,7\pi/8)$,

loop to $(-5,\pi)$ then back to the origin at $(0,9\pi/8)$, loop to $(5,5\pi/4)$ then back to the origin at $(0,11\pi/8)$,

loop to $(-5,3\pi/2)$ then back to the origin at $(0,13\pi/8)$,

loop to $(5,7\pi/4)$ then to the origin at $(0,15\pi/8)$,

then finally loop back to $(-5,2\pi)$, which is actually the same point as $(-5,0)$. From there on, we're retracing the same pieces of the rose over and over.



Let's try an example with a sine rose.

Example

Sketch the graph of $r = 4\sin(6\theta)$.

Because $c = 4$ and $n = 6$, this rose will have $|2n| = |2(6)| = 12$ petals that extend out a distance of 4 from the origin. To sketch the graph, we recognize that the trigonometric function in this polar equation is $\sin(6\theta)$, and its argument (the angle at which sine is evaluated) is 6θ. So we'll set

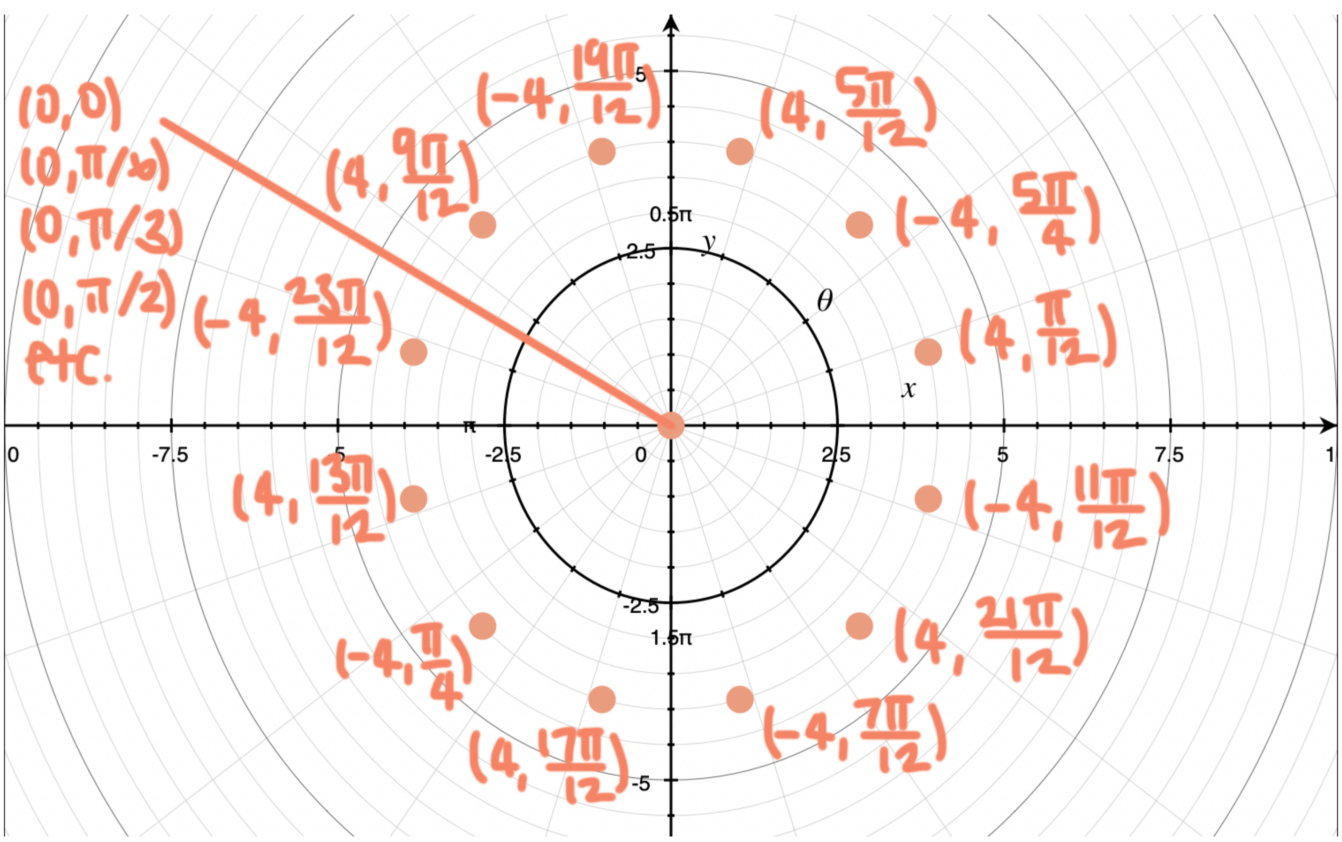
$$6\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{12}$$

Now we'll make a table with multiples of $\pi/12$, like $\theta = 0, \pi/12, \pi/6, \pi/4, \pi/3, 5\pi/12, \pi/2$, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$
r	0	4	0	-4	0

Plotting these points on the polar graph gives



And if we connect these points with a smooth curve, in order, we see the graph of the rose. We start at $(0,0)$ and then loop out to $(4,\pi/12)$,

loop back to the origin at $(0,\pi/6)$ then out to $(-4,\pi/4)$,

loop back to the origin at $(0,\pi/3)$ then out to $(4,5\pi/12)$,

loop back to the origin at $(0,\pi/2)$ then out to $(-4,7\pi/12)$,

loop back to the origin at $(0,8\pi/12)$ then out to $(4,9\pi/12)$,

loop back to the origin at $(0,10\pi/12)$ then out to $(-4,11\pi/12)$,

loop back to the origin at $(0,12\pi/12)$ then out to $(4,13\pi/12)$,

loop back to the origin at $(0,14\pi/12)$ then out to $(-4,5\pi/4)$,

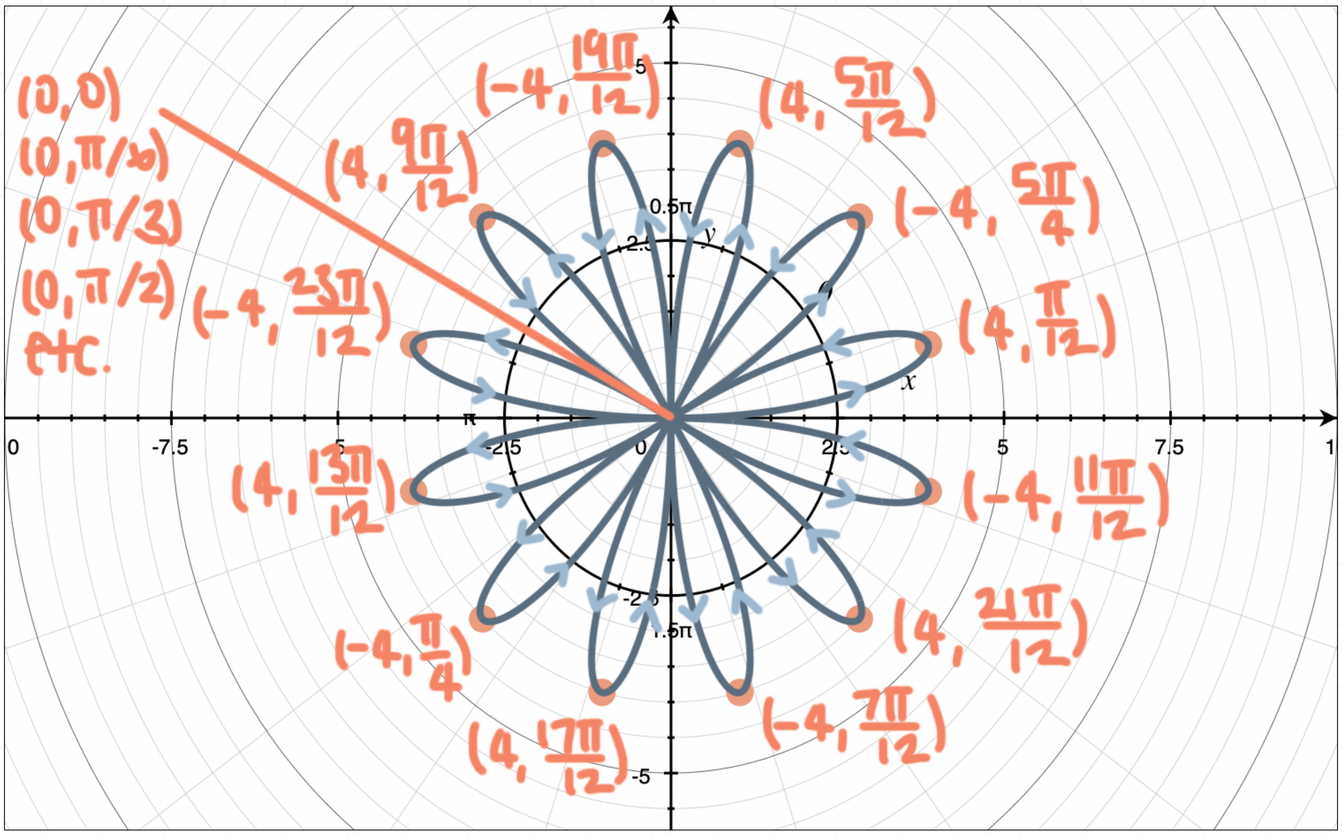
loop back to the origin at $(0,16\pi/12)$ then out to $(4,17\pi/12)$,

loop back to the origin at $(0,18\pi/12)$ then out to $(-4,19\pi/12)$,

loop back to the origin at $(0,20\pi/12)$ then out to $(4,21\pi/12)$,

loop back to the origin at $(0,22\pi/12)$ then out to $(-4,23\pi/12)$,

then finally back to $(0,24\pi/12)$, which is actually the same point as $(0,0)$. From there on, we're retracing the same pieces of the rose over and over.



Let's do one more example of a sine rose with a negative c-value and an odd n-value.

Example

Sketch the graph of $r = -6\sin(5\theta)$.

Because $c = -6$ and $n = 5$, this rose will have $|n| = |5| = 5$ petals that extend out a distance of 6 from the origin. To sketch the graph, we recognize that the trigonometric function in this polar equation is $\sin(5\theta)$, and its argument (the angle at which sine is evaluated) is 5θ. So we'll set

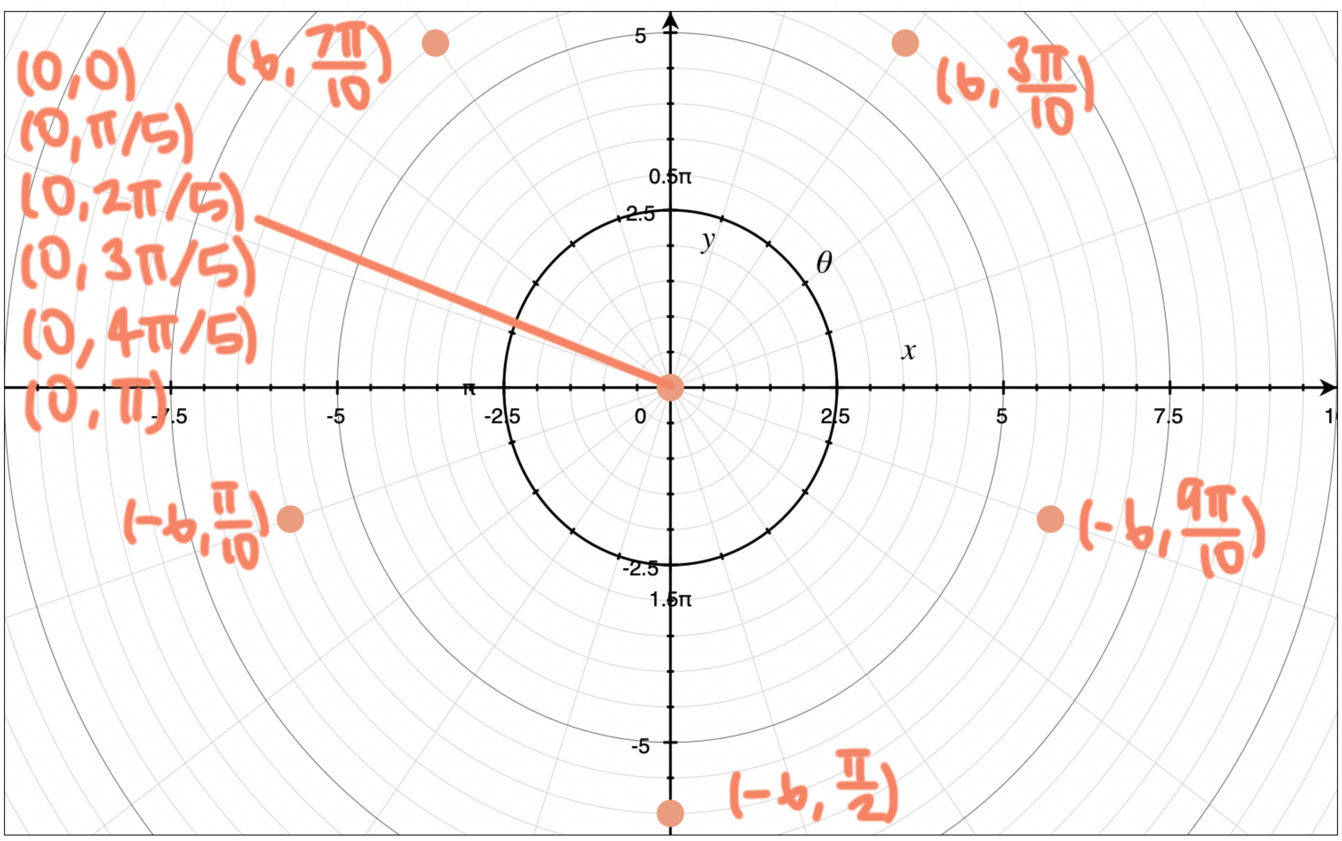
$$5\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{10}$$

Now we'll make a table with multiples of $\pi/10$, like $\theta = 0$, $\pi/10$, $\pi/5$, $3\pi/10$, $2\pi/5$, $\pi/2$, $3\pi/5$, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/10$	$\pi/5$	$3\pi/10$	$2\pi/5$
r	0	-6	0	6	0

Plotting these points on the polar graph gives



And if we connect these points with a smooth curve, in order, we see the graph of the rose. We start at $(0,0)$ and then loop out to $(-6,\pi/10)$,

loop back to the origin at $(0,\pi/5)$ then out to $(6,3\pi/10)$,

loop back to the origin at $(0,2\pi/5)$ then out to $(-6,\pi/2)$, loop back to the origin at $(0,3\pi/5)$ then out to $(6,7\pi/10)$,

loop back to the origin at $(0,4\pi/5)$ then out to $(-6,9\pi/10)$,

then finally back to $(0,\pi)$, which is actually the same point as $(0,0)$. From there on, we're retracing the same pieces of the rose over and over.