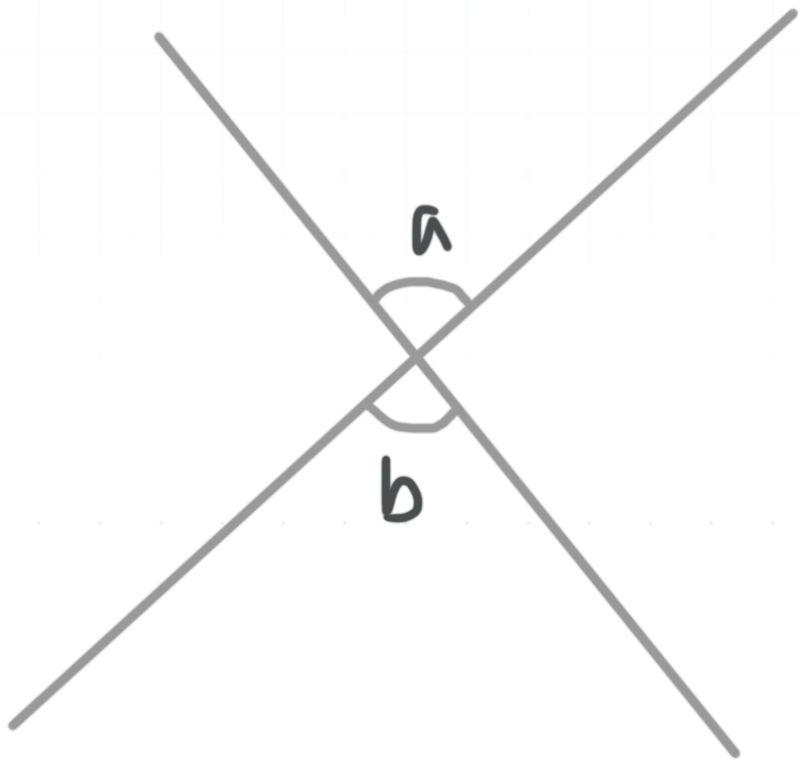
Congruent angles

In this lesson we'll look at how to use vertical angles to solve problems.

Vertical angles

Vertical angles are a pair of angles that share a vertex and whose rays lie on the same pair of straight lines (but point in opposite directions from the common vertex), like these:

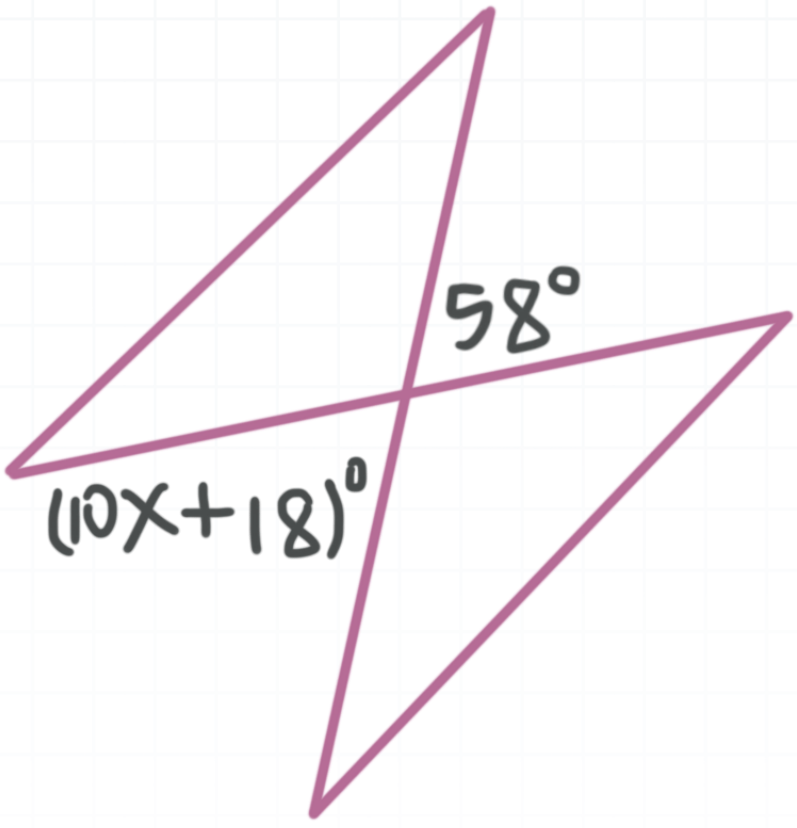


The angles in a vertical angle pair are congruent. Therefore, from the diagram above, $m\angle a = m\angle b$. We can also write this as $\angle a \cong \angle b$. The symbol $\cong$ means "is congruent to." It's used not only to express congruence of angles (any angles, not just vertical angles) but also to express congruence of line segments, triangles, and other geometric figures.

Let's do a few problems so you can get the idea.

Example

Find the value of x.



The angles of measure 58° and $(10x + 18)^\circ$ are vertical angles, and are therefore congruent, so we can set their measures equal to each another and solve for the variable.

$$(10x + 18)^\circ = 58^\circ$$

$$10x^\circ = 40^\circ$$

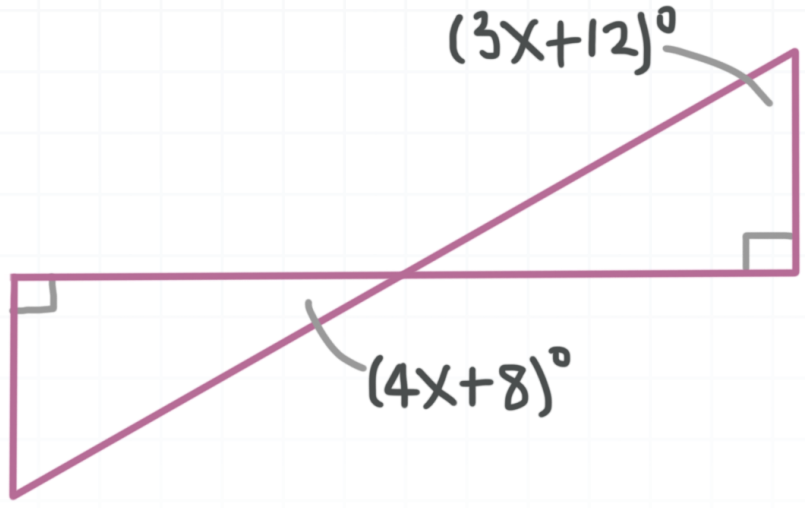
$$x^\circ = 4^\circ$$

$$x = 4$$

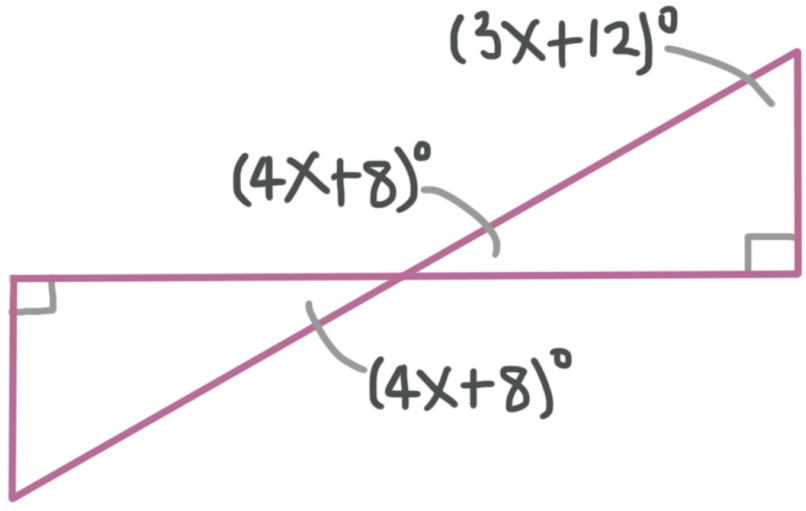
Let’s try another one.

Example

Solve for the variable.



In the triangle on the right, one of the angles is a right angle (and therefore has measure $90°$), and another angle has measure $(3x + 12)°$. The third angle and the angle of measure $(4x + 8)°$ in the triangle on the left are a vertical angle pair, so they’re congruent. Therefore, the measure of the third angle in the triangle on the right is also $(4x + 8)°$.



The measures of the interior angles of a triangle sum to $180°$, so we can set up an equation to solve for the variable. From the triangle on the right, we have

$$(4x + 8)° + (3x + 12)° + 90° = 180°$$

$$7x° + 110° = 180°$$

$$7x° = 70°$$

$$x° = 10°$$

$$x = 10$$