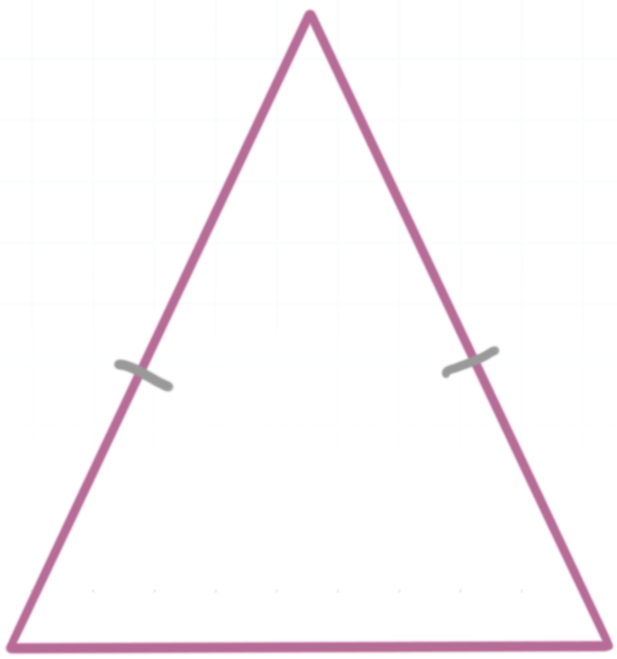
Isosceles triangle theorem

In this lesson we'll look at isosceles triangles and how to use the isosceles triangle theorem to solve problems.

Isosceles triangles

An **isosceles triangle** is a triangle with at least two congruent sides.

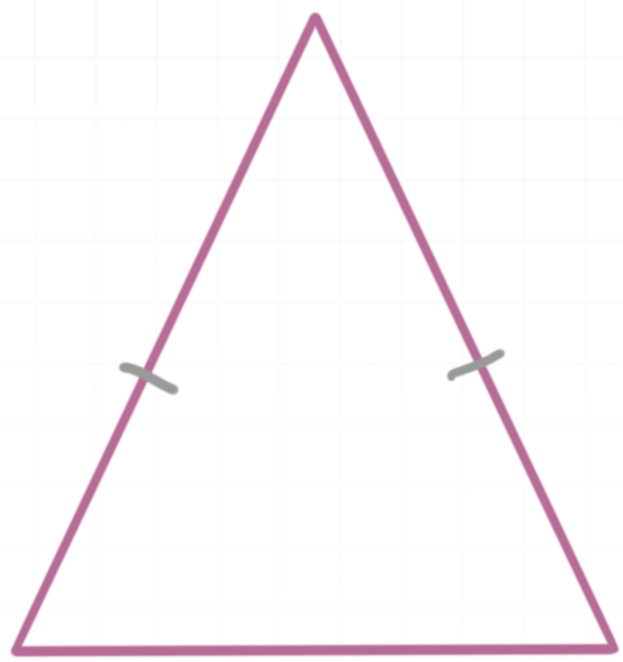


If a triangle is equilateral, then it has three congruent sides, which is therefore considered a special case of an isosceles triangle. The specific case of the equilateral triangle is the reason that the definition of isosceles triangle includes the words "*at least two* congruent sides."

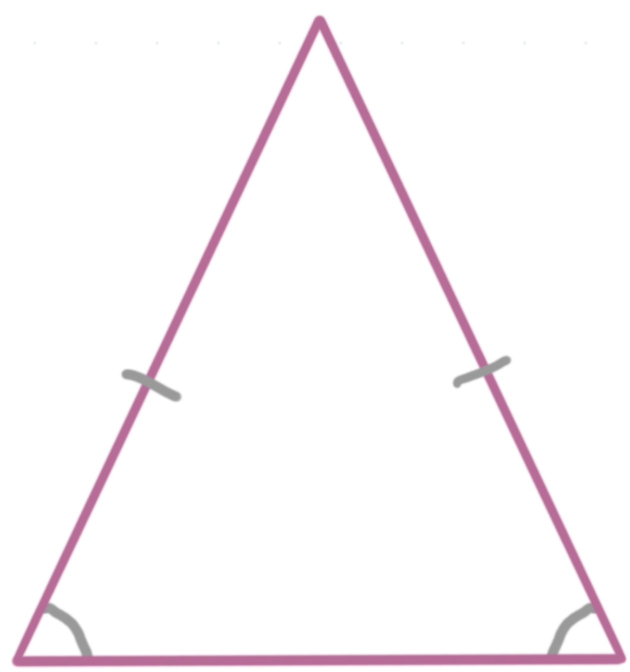
Isosceles triangle theorem

The isosceles triangle theorem says that if two sides of a triangle are congruent, then the angles opposite those sides are congruent.

If a triangle has only two congruent sides (if a triangle is congruent but not equilateral), the angles opposite the congruent sides are sometimes called the **base angles**. So if you know this:



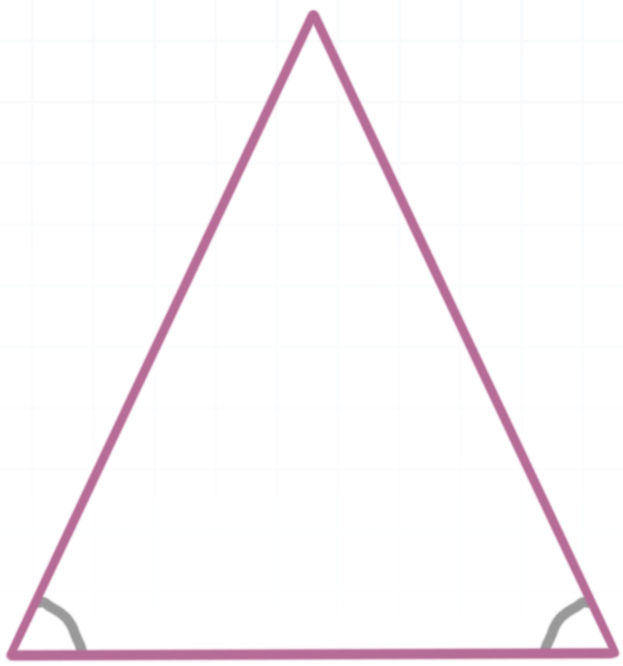
then you'll also know this:



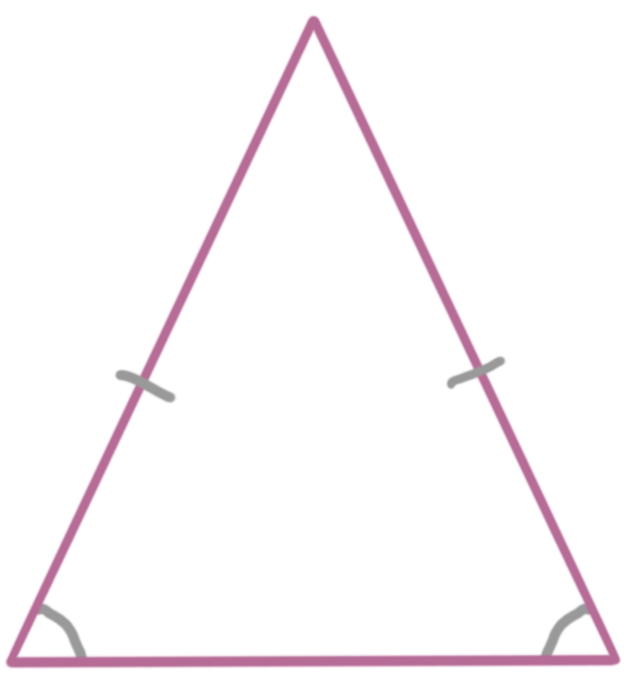
Converse of the isosceles triangle theorem

The converse of the isosceles triangle theorem just turns around the original theorem. It says that, if you know that two angles of a triangle are congruent, then the sides opposite those angles are congruent, which means it's an isosceles triangle.

In other words, if you know this:



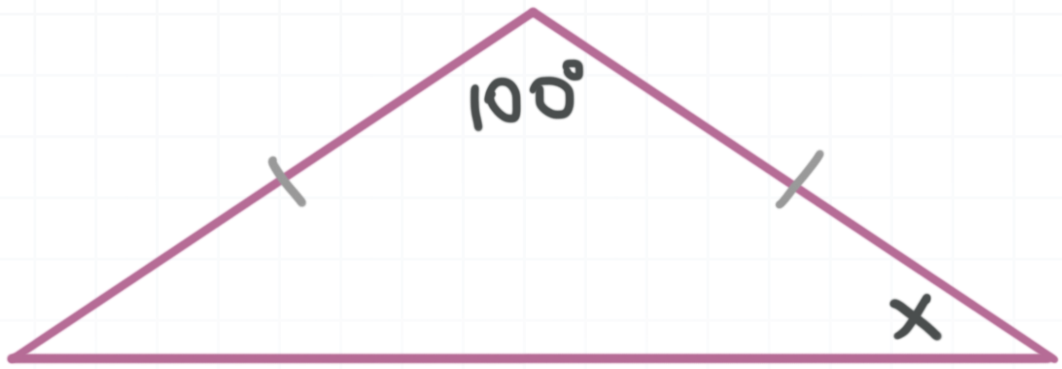
then you also know this:



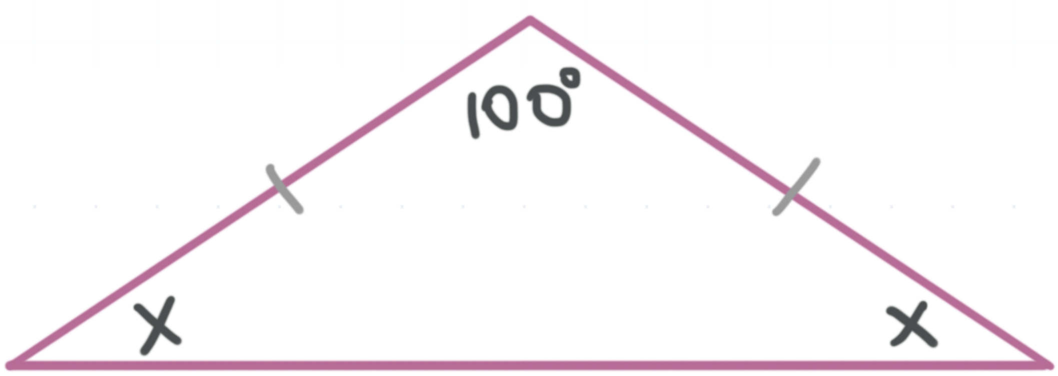
Let's start by working through an example.

Example

What is the value of x?



The triangle is an isosceles triangle, so we know that the angles opposite the congruent sides (the base angles) are congruent.



The measures of the interior angles of a triangle sum to 180°, so we can set up an equation for the sum of the interior angles.

$$x + x + 100^\circ = 180^\circ$$

$$2x + 100^\circ = 180^\circ$$

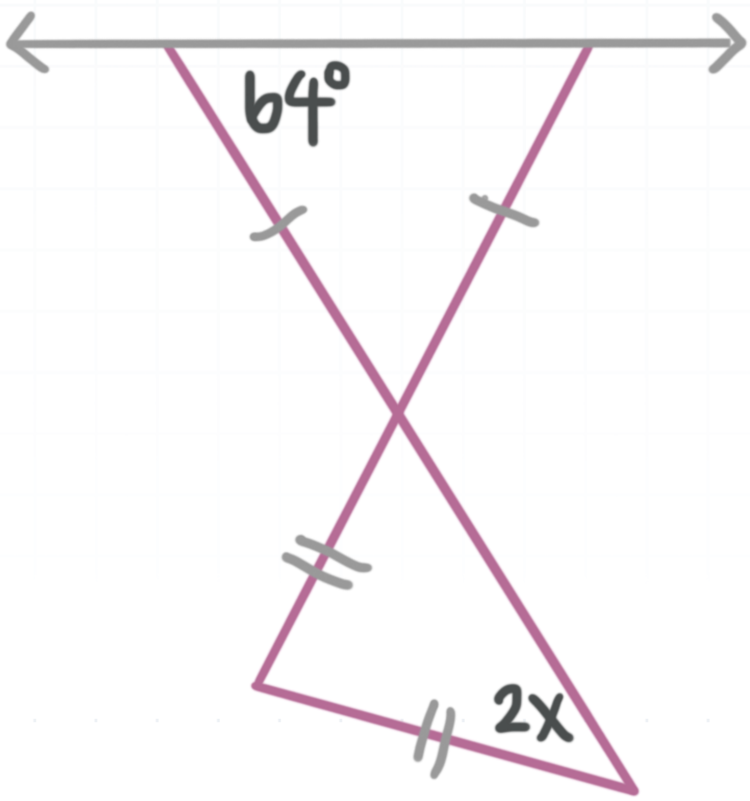
$$2x = 80^\circ$$

$x = 40^\circ$

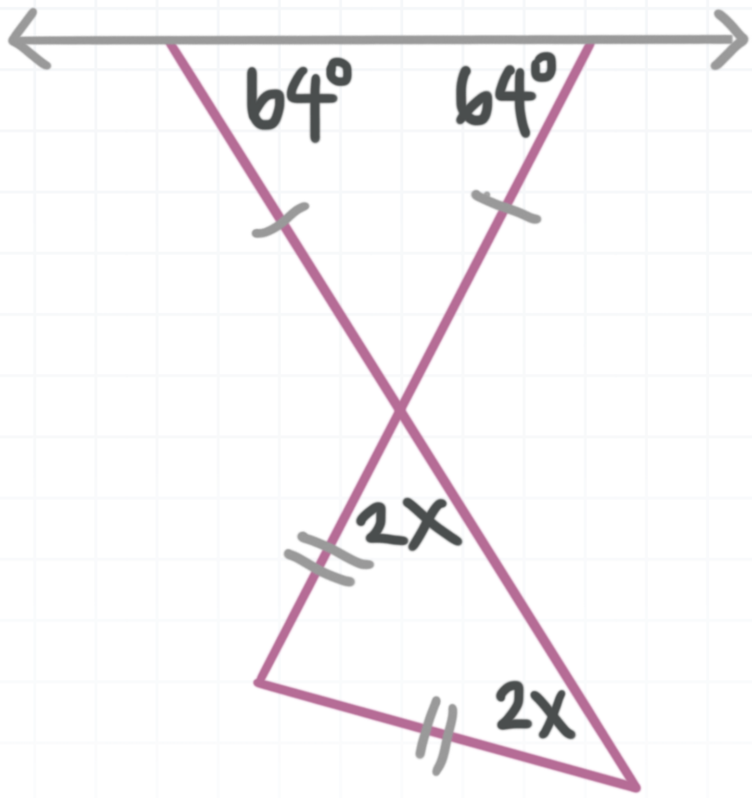
Let's try one more.

Example

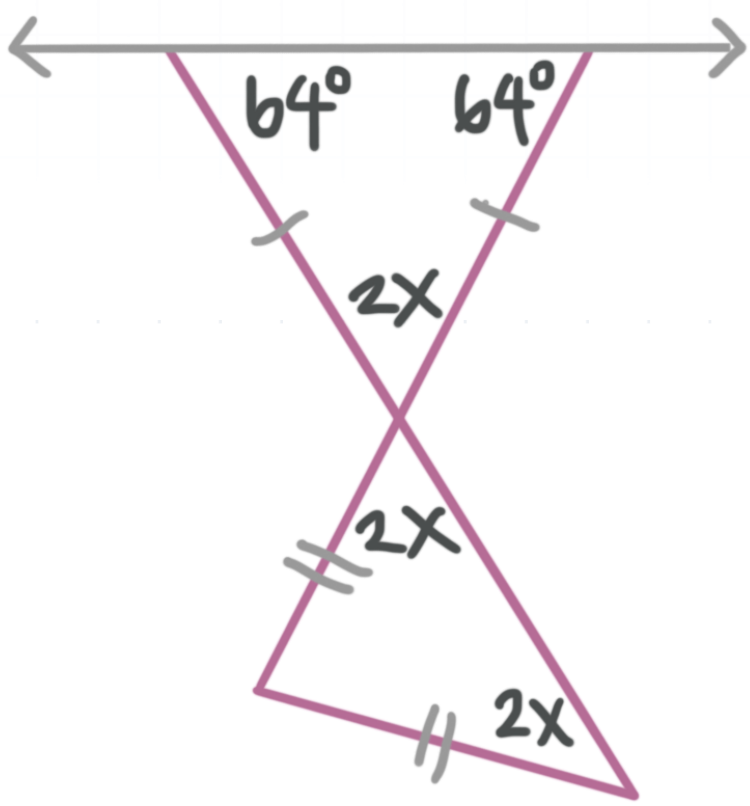
What is the value of x?



Let's use what we know about isosceles triangles to fill in the diagram. Remember that the base angles of an isosceles triangle are congruent.



We also know that vertical angles are congruent, so we can fill in one more angle.



Now we can use the top triangle to solve for the variable by remembering that the measures of the interior angles of a triangle sum to 180°.

$$64^\circ + 64^\circ + 2x = 180^\circ$$

$$128^\circ + 2x = 180^\circ$$

$$2x = 52^\circ$$

$$x = 26^\circ$$