Graphing polar curves in a rectangular system

Now that we know how to sketch the graphs of many polar curves, including circles, roses, cardioids and other limaçons, and lemniscates, we want to look at another method we can use to sketch these curves.

This method has us plotting polar points in the rectangular coordinate system (the xy-plane), and then translating those points into the polar coordinate system.

Some people find this method to be easier than sketching the curve directly into the polar system, because we start with the more familiar rectangular system first, before moving the graph into the more foreign polar system.

How to graph in a rectangular system before transferring the graph to a polar system

To use this method, we'll start the same way we did to sketch in the polar system directly.

1. Set the argument of the trigonometric function equal to $\pi/2$, and then solve the equation for θ.

2. Evaluate the polar curve at multiples of the θ-value we solved for in Step 1, starting with $\theta = 0$.

But then, instead of plotting the points we find into the polar system directly, we'll plot them in the rectangular system, treating the horizontal axis as the θ-axis (instead of the x-axis), and the vertical axis as the r-axis (instead of the y-axis).

3. Plot the resulting points in the rectangular system.

4. Connect the points on the rectangular graph with a smooth curve.

Then we'll transfer the graph from the rectangular system into the polar system.

5. Transfer the points from the rectangular system into the polar system.

6. Connect the points on the polar graph with a smooth curve.

Let's re-do an example from the previous lesson about graphing circles, but this time with our new method where we start with the rectangular system.

Example

Sketch the graph of $r = 6\cos\theta$.

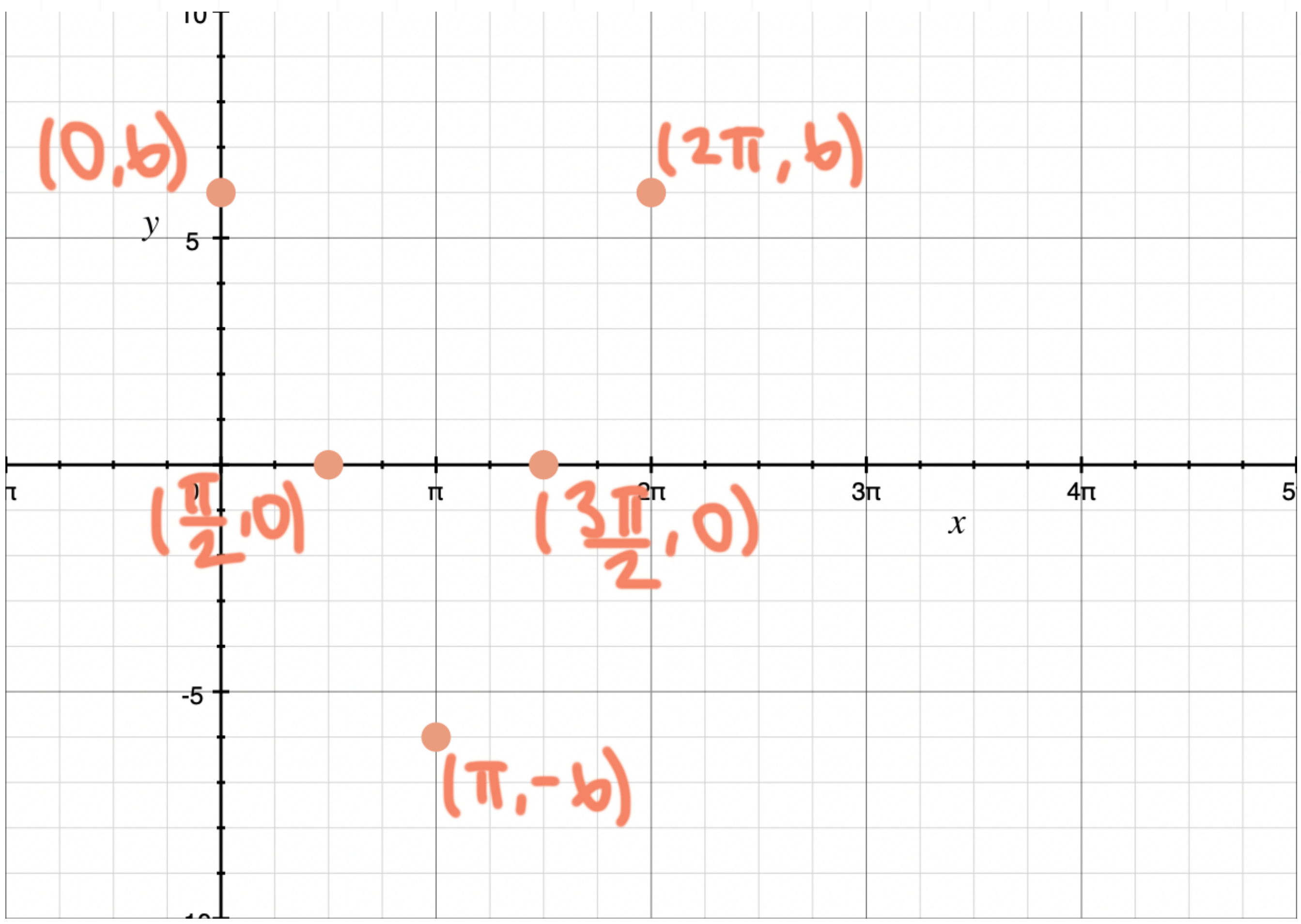
The trigonometric function in this polar equation is $\cos\theta$, and its argument (the angle at which cosine is evaluated) is θ. So we'll set

$$\theta = \frac{\pi}{2}$$

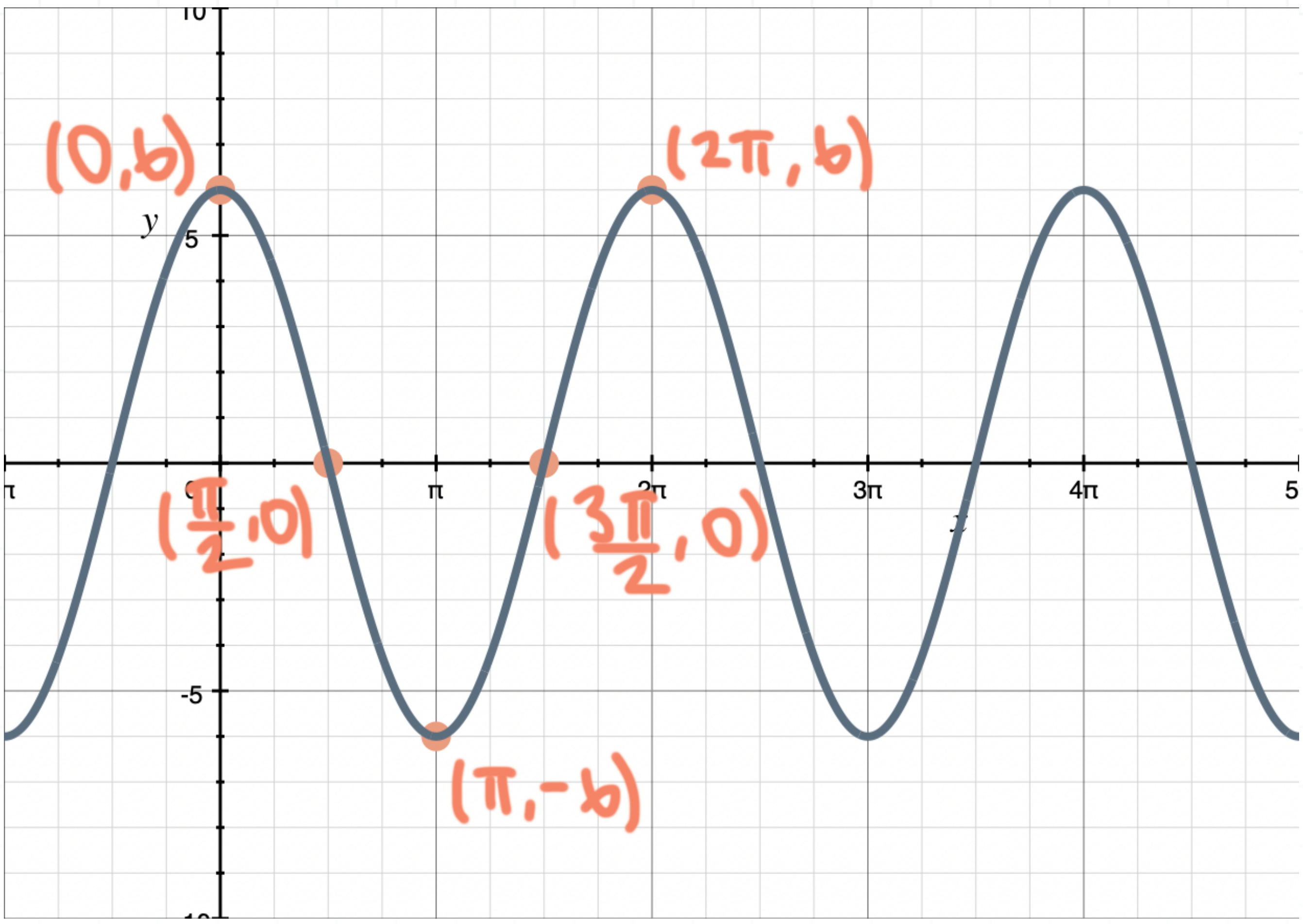
Now we'll make a table with multiples of $\pi/2$, like $\theta = 0$, $\pi/2$, π, $3\pi/2$, 2π, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	6	0	-6	0	6

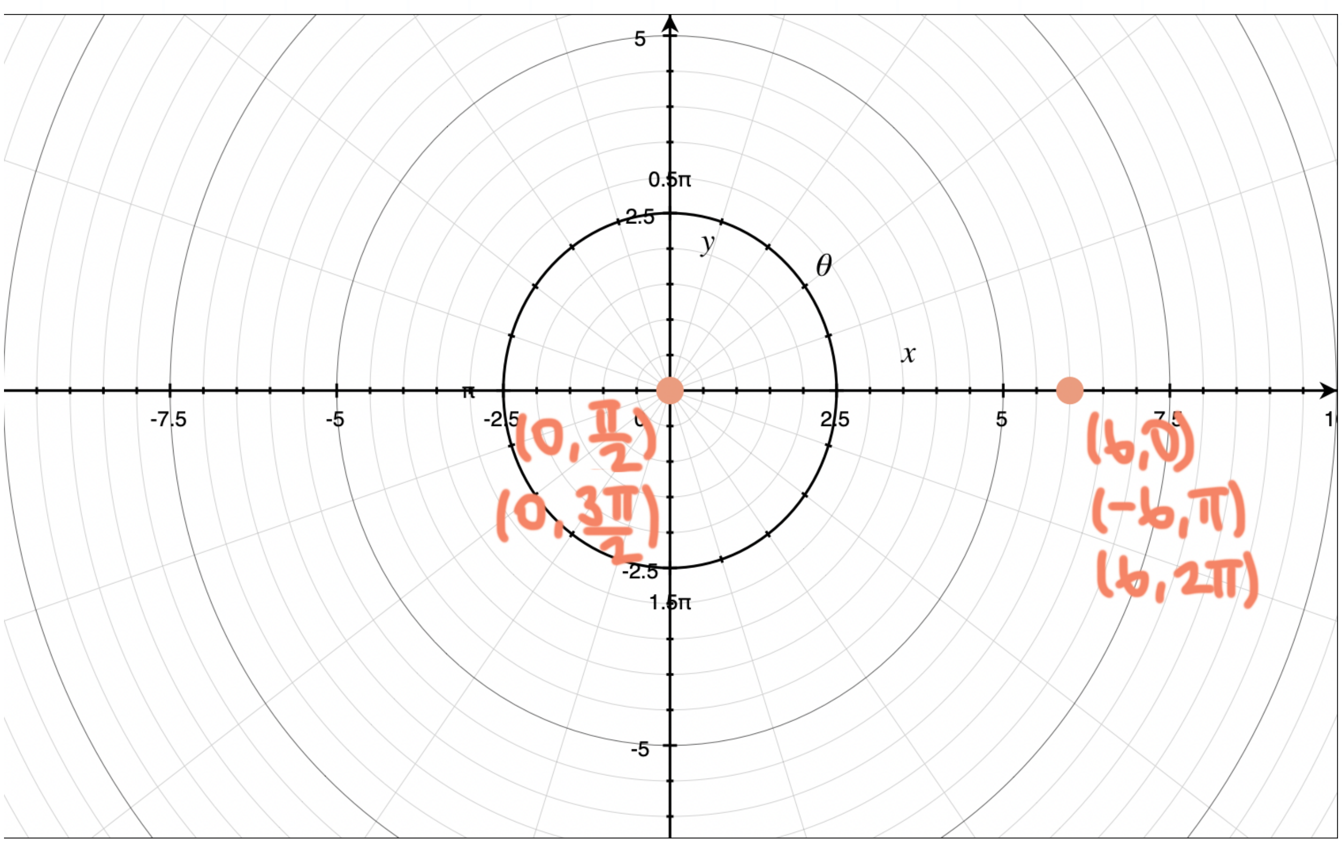
Plotting these points on the rectangular graph gives



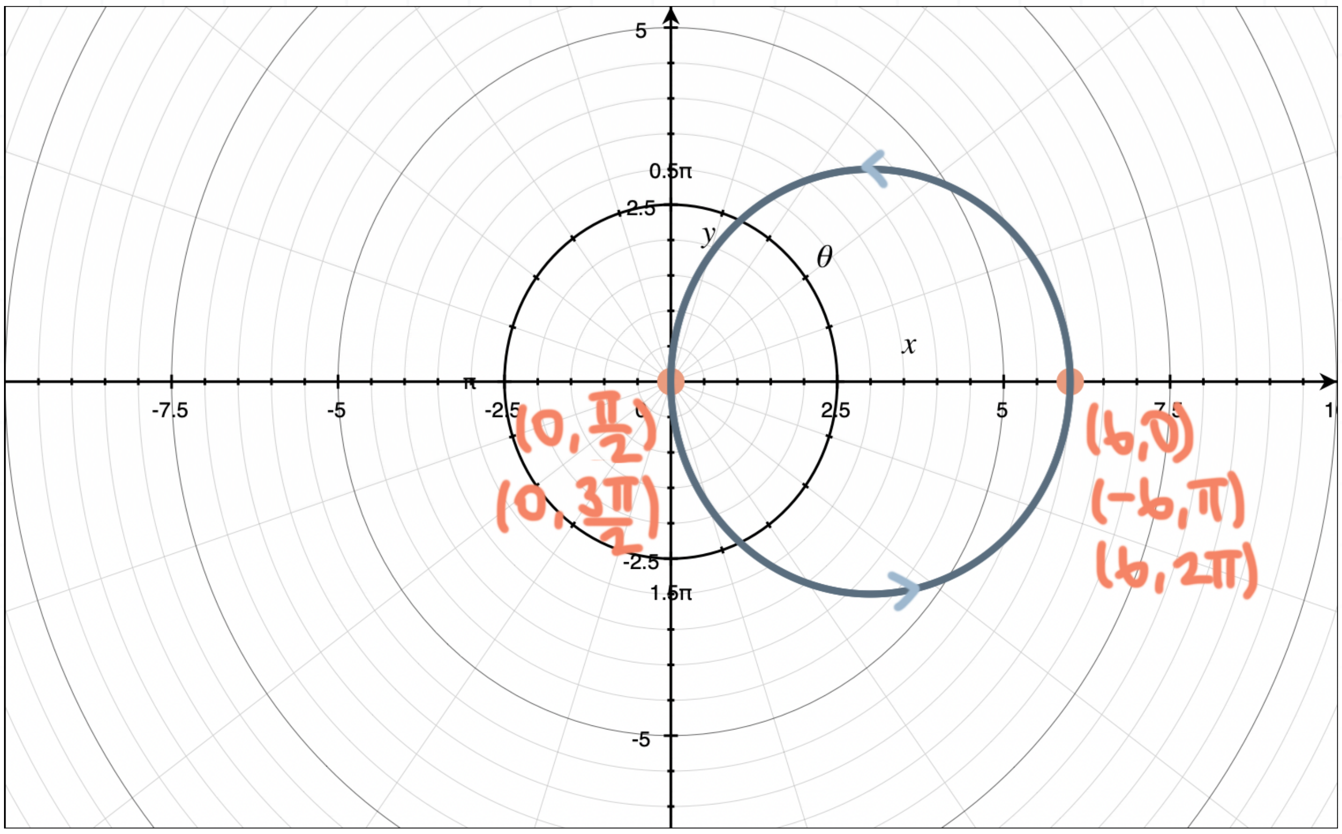
And if we connect these points with a smooth curve, we get



If we then transfer the points from the rectangular system into the polar system, we get



And if we connect these points with a smooth curve, in order, we see the graph of the circle. We start at $(6,0)$, loop up around to the origin at $(0,\pi/2)$, then loop back down around to $(-6,\pi)$, which is actually the same point as $(6,0)$. From there on, we're retracing the same pieces of the circle over and over.



Let's do another example where we work through this process for graphing a rose.

Example

Sketch the graph of $r = 2.5\cos(3\theta)$.

The trigonometric function in this polar equation is $\cos(3\theta)$, and its argument (the angle at which cosine is evaluated) is 3θ. So we'll set

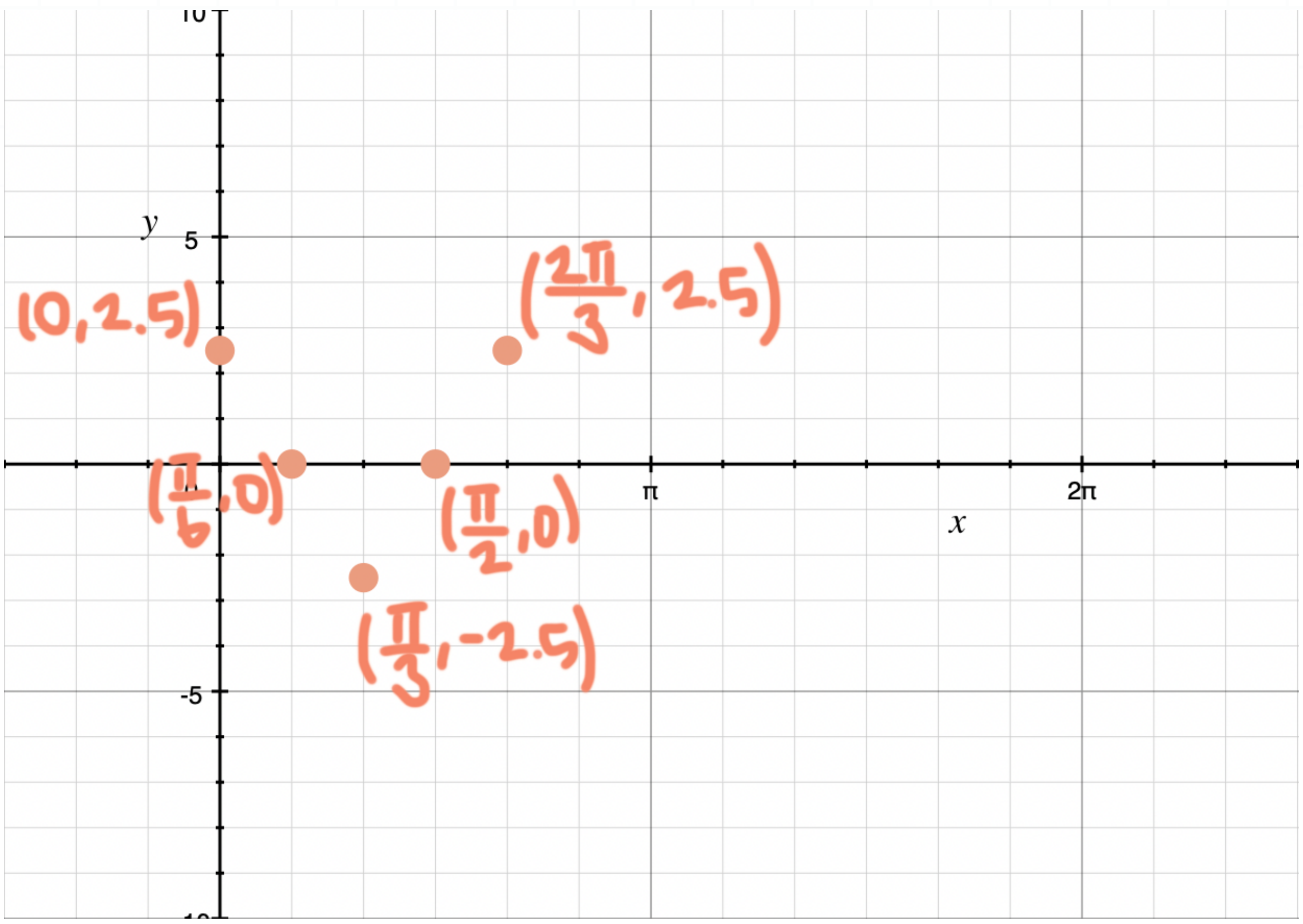
$$3\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{6}$$

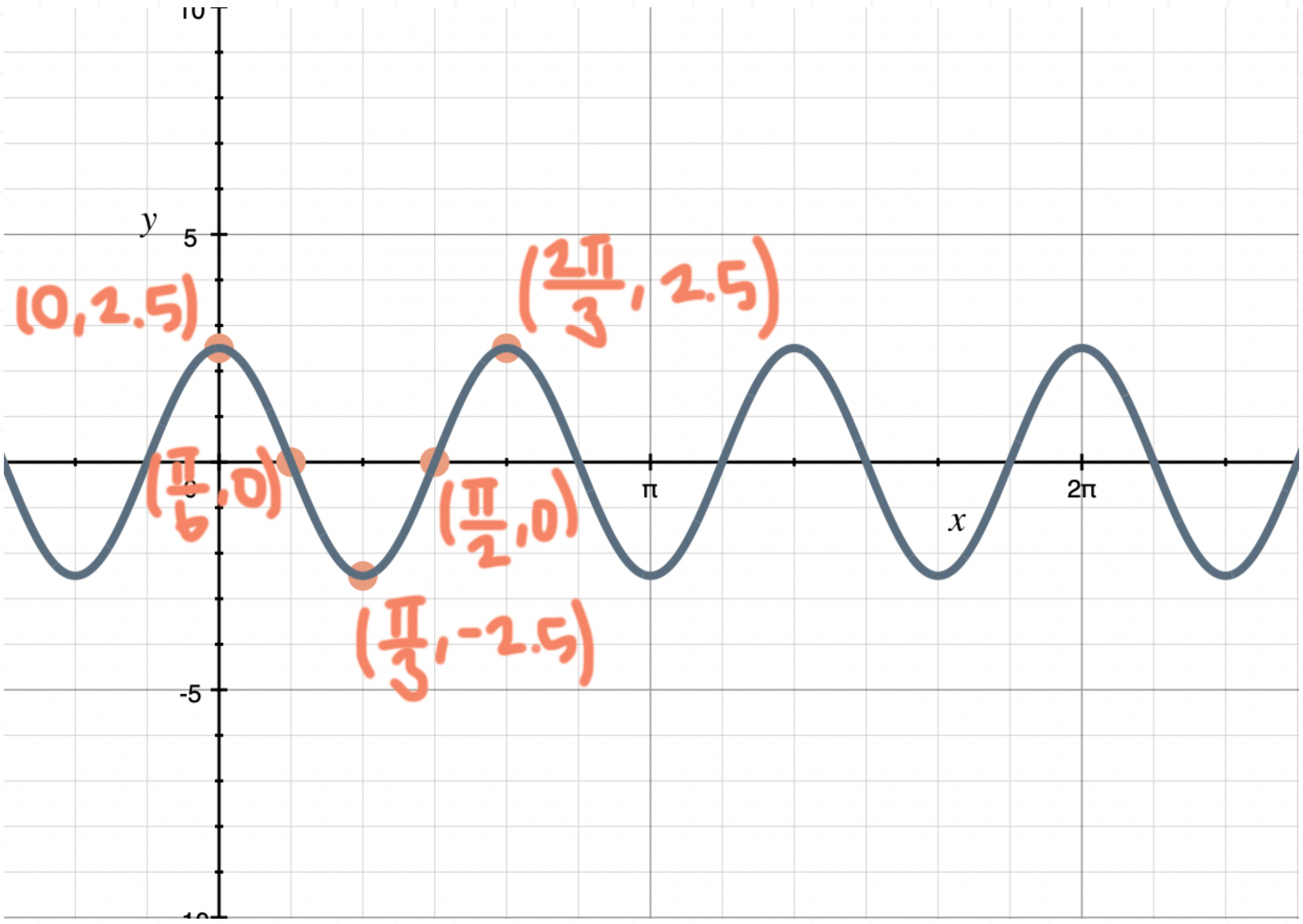
Now we'll make a table with multiples of $\pi/6$, like $\theta = 0, \pi/6, \pi/3, \pi/2, 2\pi/3$, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$
r	2.5	0	-2.5	0	2.5

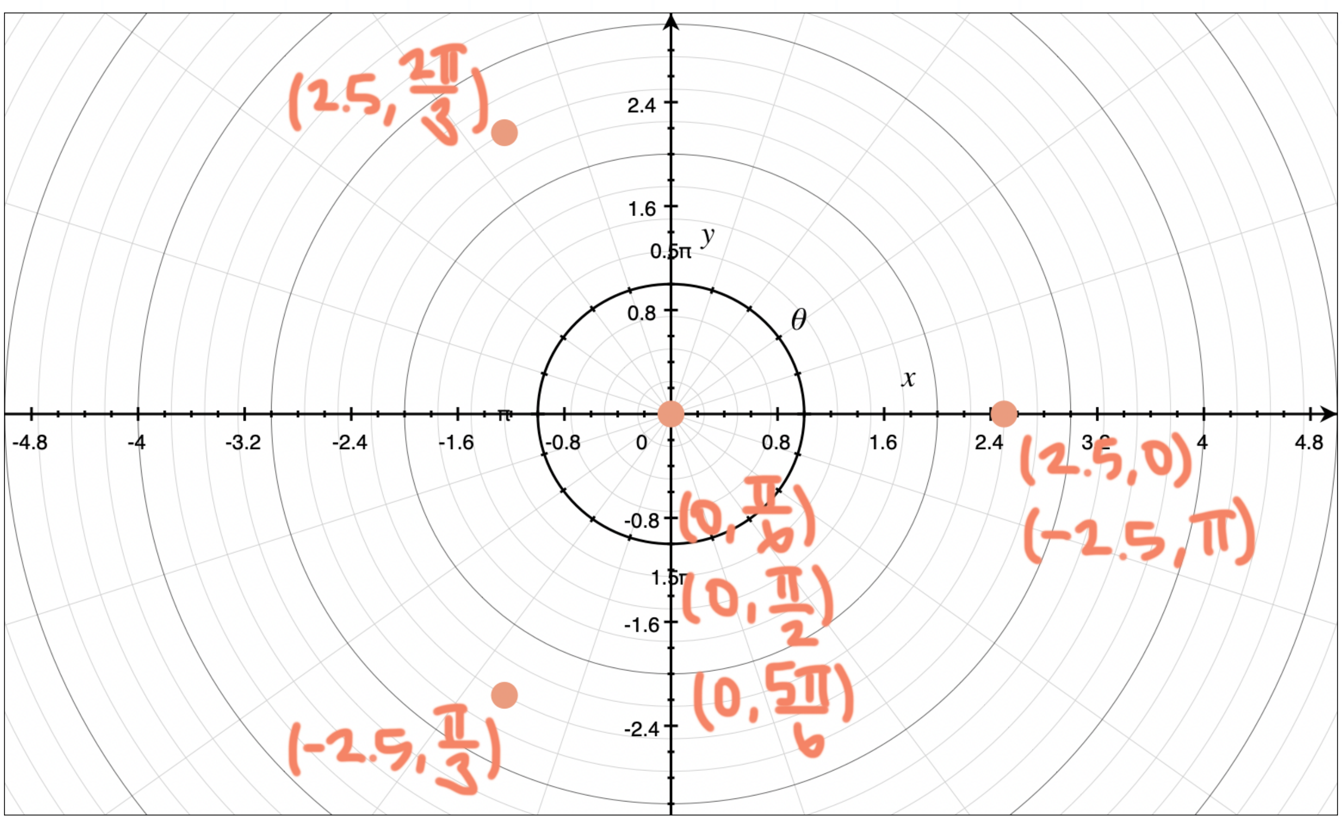
Plotting these points on the rectangular graph gives



And if we connect these points with a smooth curve, we get



If we then transfer the points from the rectangular system into the polar system, we get

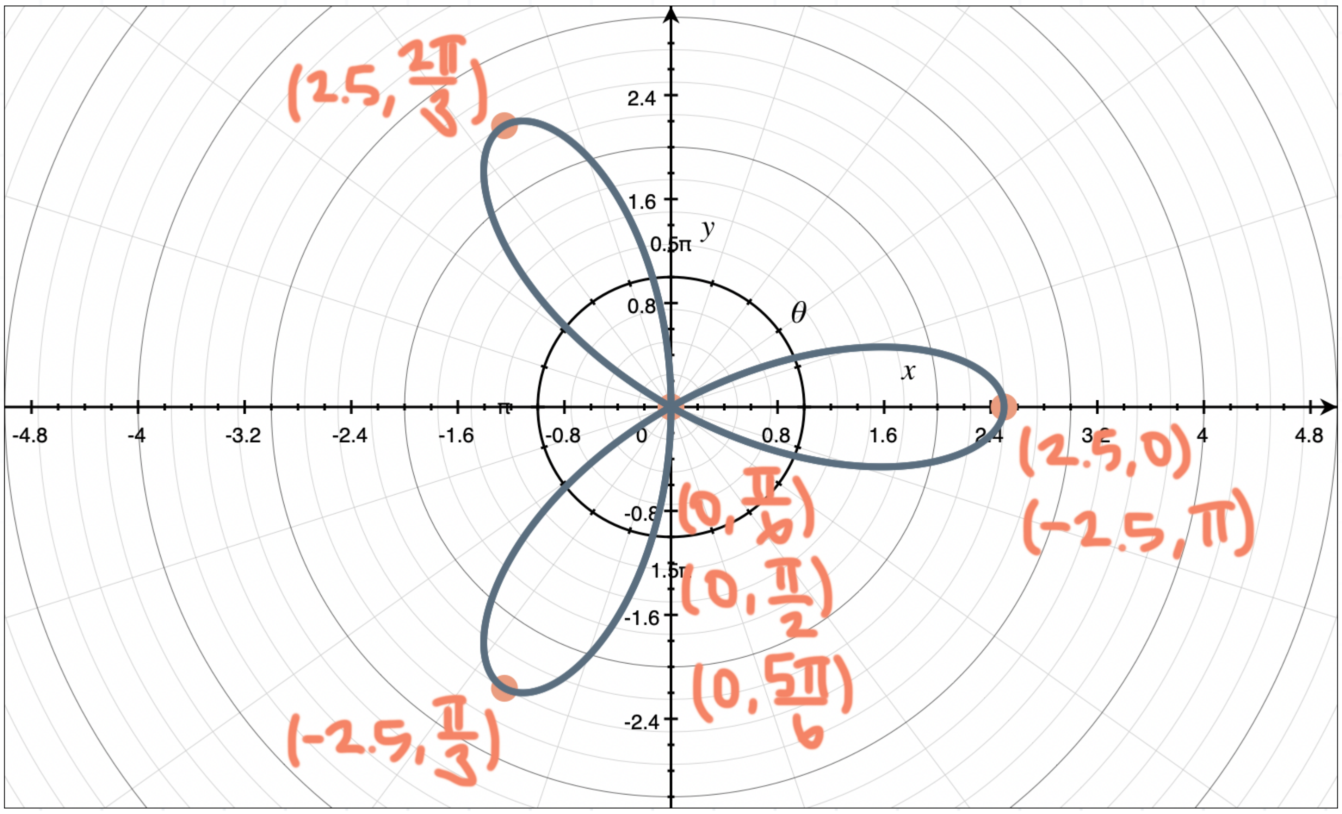


And if we then connect the points, in order, we see the graph of the rose. We start at $(2.5,0)$, and then loop back to the origin at $(0,\pi/6)$ then out to $(-2.5,\pi/3)$,

loop back to the origin at $(0,\pi/2)$ then out to $(2.5,2\pi/3)$,

loop back to the origin at $(0,5\pi/6)$ then out to $(-2.5,2\pi)$,

which is actually the same point as $(2.5,0)$. From there on, we're retracing the same pieces of the rose over and over.



Let's do one more example, this time with a cardioid.

Example

Sketch the graph of $r = 1.8 - 1.8\sin\theta$.

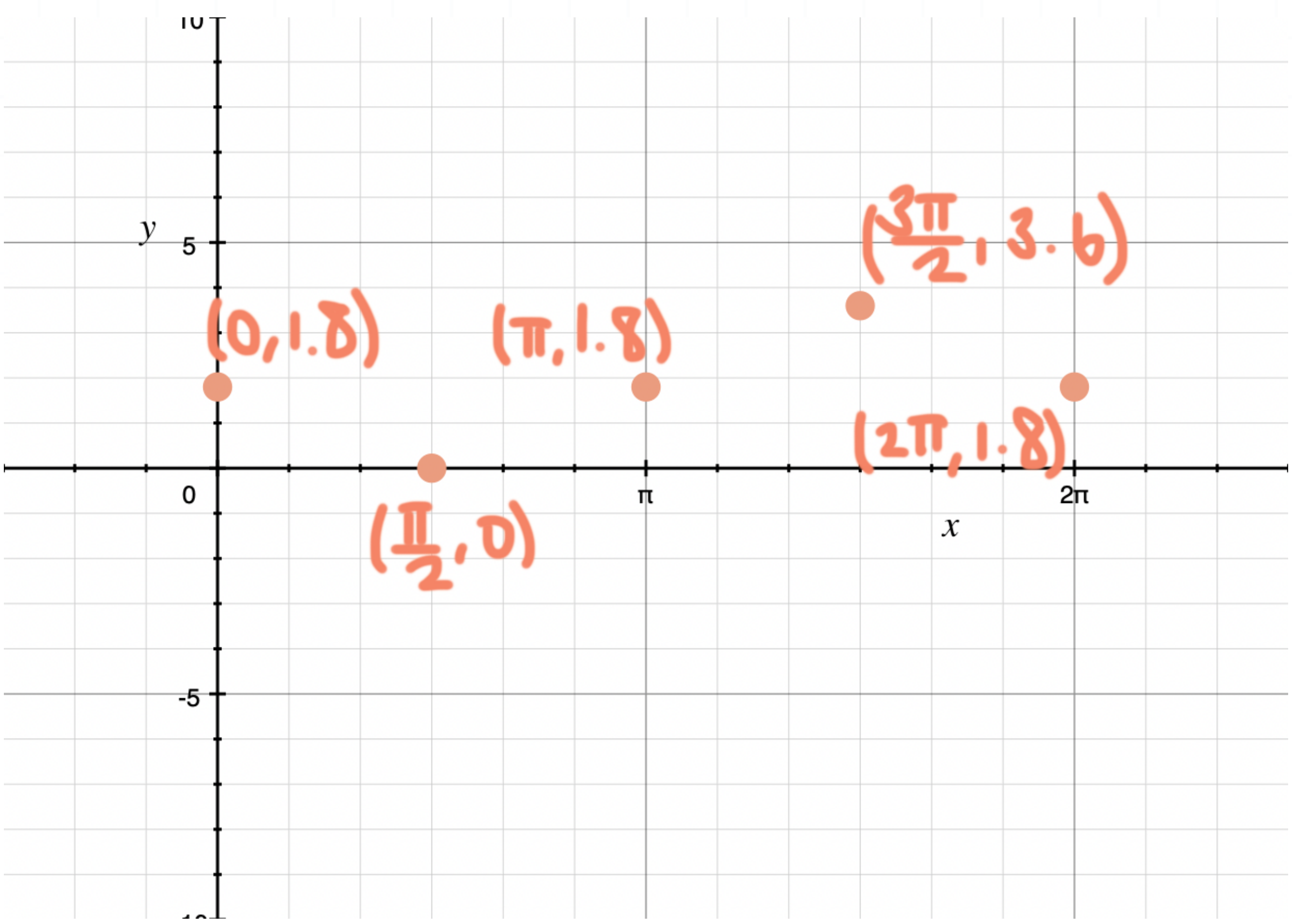
The trigonometric function in this polar equation is $\sin\theta$, and its argument (the angle at which cosine is evaluated) is θ. So we'll set

$$\theta = \frac{\pi}{2}$$

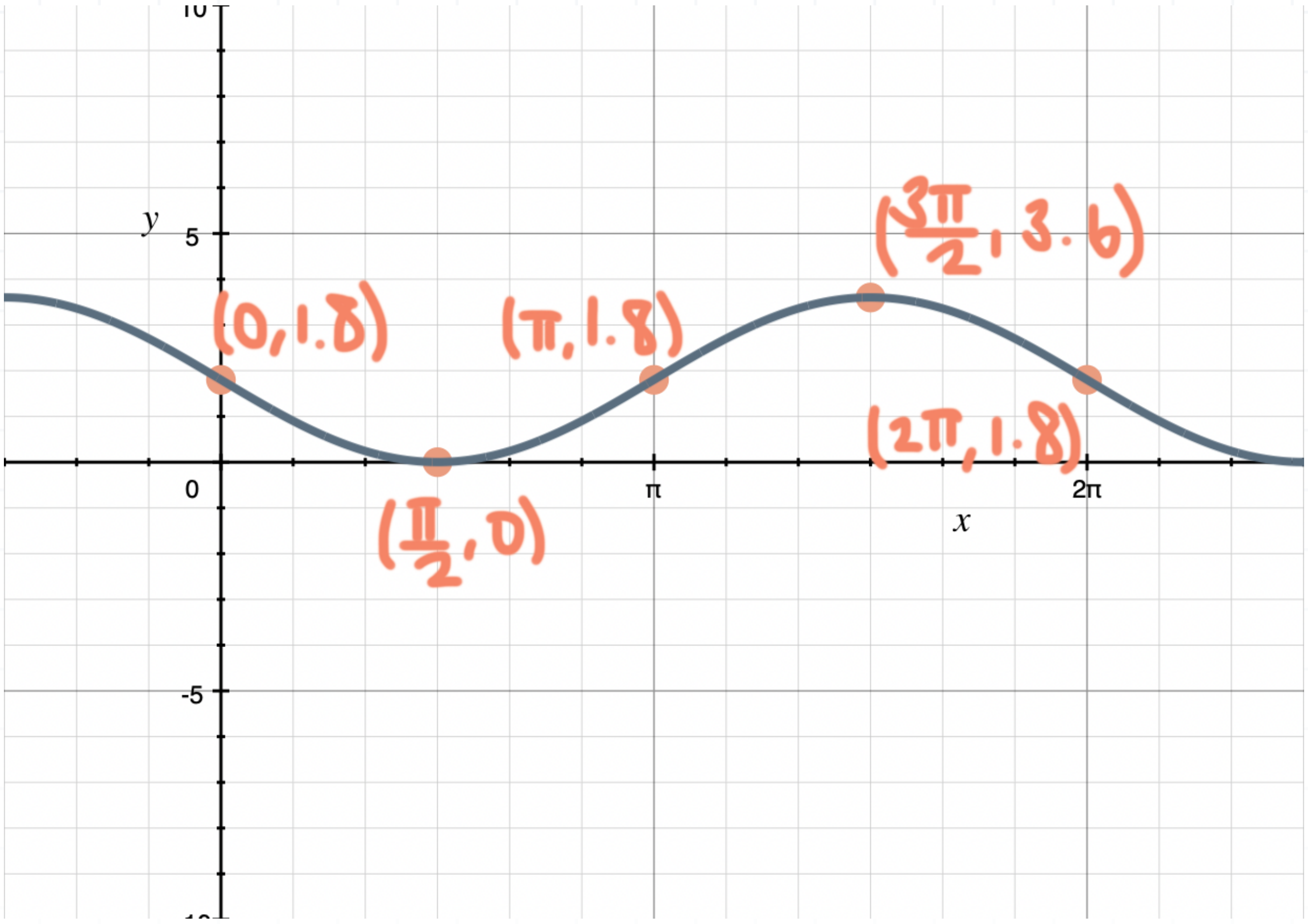
Now we'll make a table with multiples of $\pi/2$, like $\theta = 0,\ \pi/2,\ \pi,\ 3\pi/2,\ 2\pi$, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	1.8	0	1.8	3.6	1.8

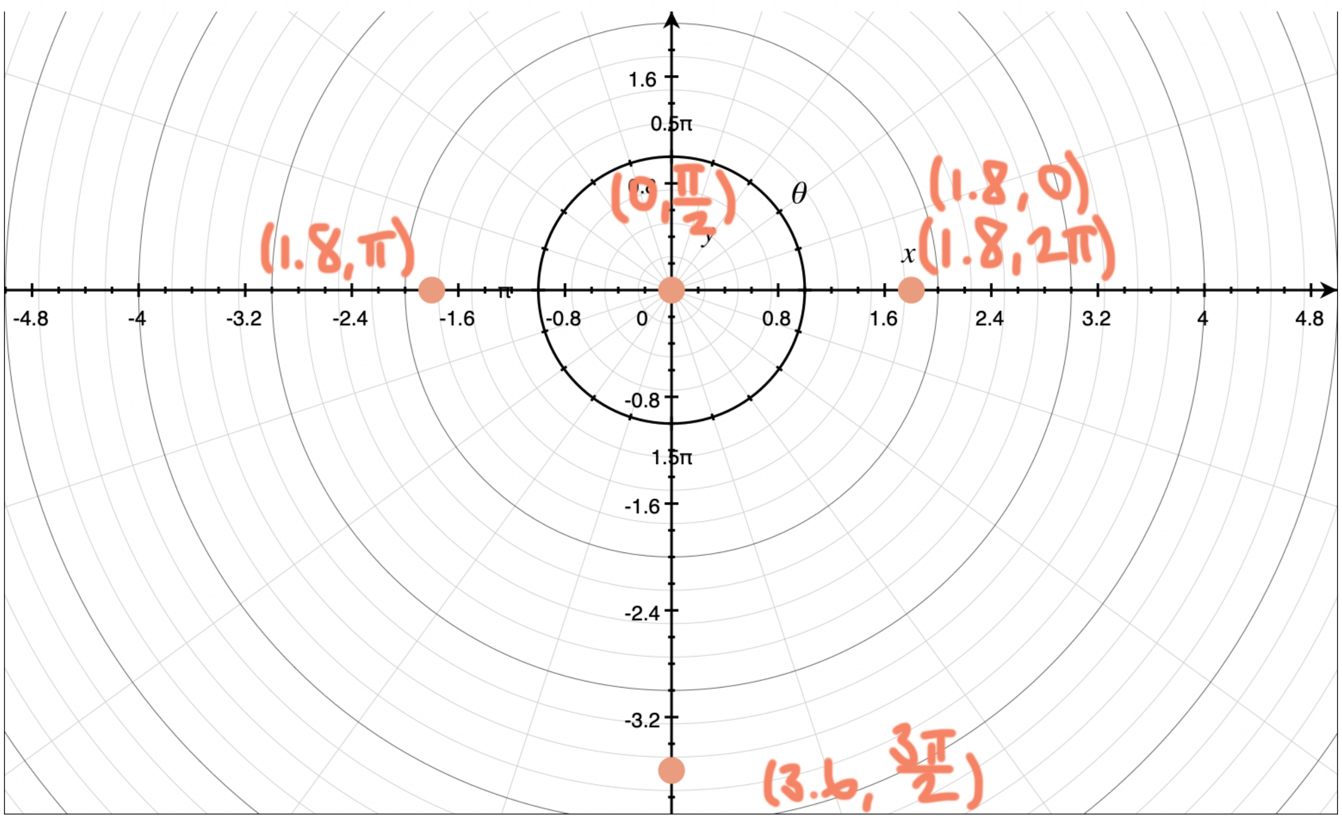
Plotting these points on the rectangular graph gives



And if we connect these points with a smooth curve, we get



If we then transfer the points from the rectangular system into the polar system, we get



And if we then connect the points, in order, we see the graph of the cardioid. We start at $(1.8,0)$, loop up around to the origin at $(0,\pi/2)$, then loop down around to $(1.8,\pi)$, loop down to $(3.6,3\pi/2)$, and then back to $(1.8,2\pi)$, which is actually the same point as $(1.8,0)$. From there on, we're retracing the same pieces of the cardioid over and over.