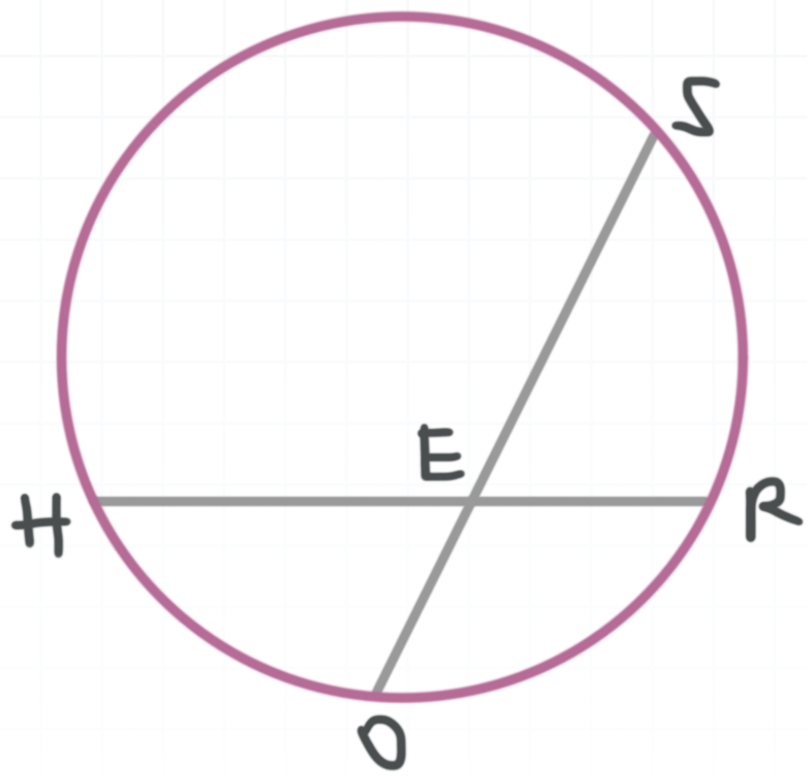
Topic: Vertex on, inside, and outside the circle

Question: In the figure, $m\overset{\frown}{RS} = 90^\circ$ and $m\overset{\frown}{HO} = 50^\circ$. What is $m\angle OER$?



Answer choices:

A 50°

B 70°

C 90°

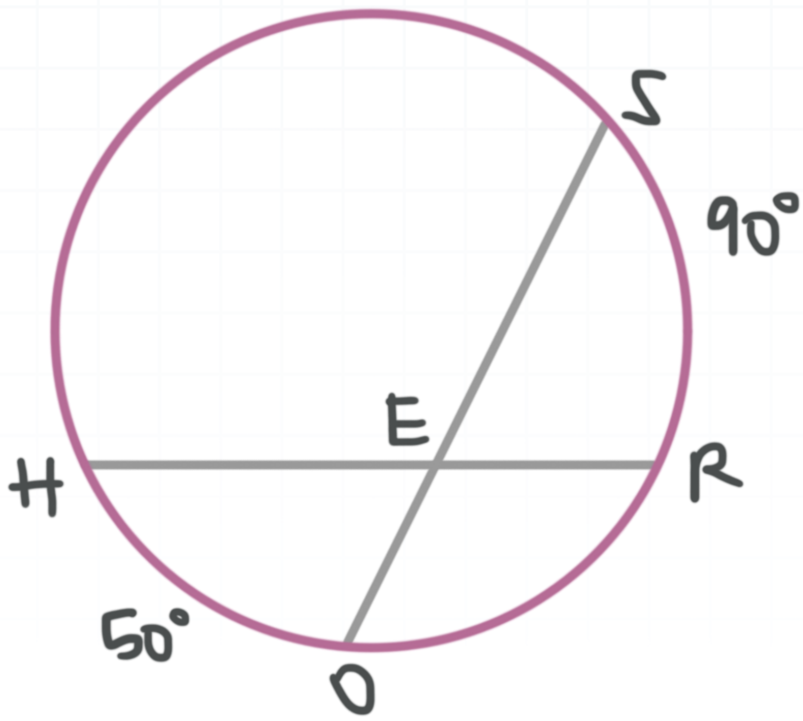
D 110°

Solution: D

We can find the measure of $\angle HEO$ given the arc lengths we already know.

$$m\angle HEO = \frac{1}{2}(m\widehat{HO} + m\widehat{RS})$$

$$m\angle HEO = \frac{1}{2}(50^\circ + 90^\circ)$$



$$m\angle HEO = 70^\circ$$

Because $\angle HEO$ and $\angle OER$ are a pair of adjacent angle that together form a straight line,

$$m\angle HEO + m\angle OER = 180^\circ$$

$$70^\circ + m\angle OER = 180^\circ$$

$$m\angle OER = 180^\circ - 70^\circ$$

$$m\angle OER = 110^\circ$$

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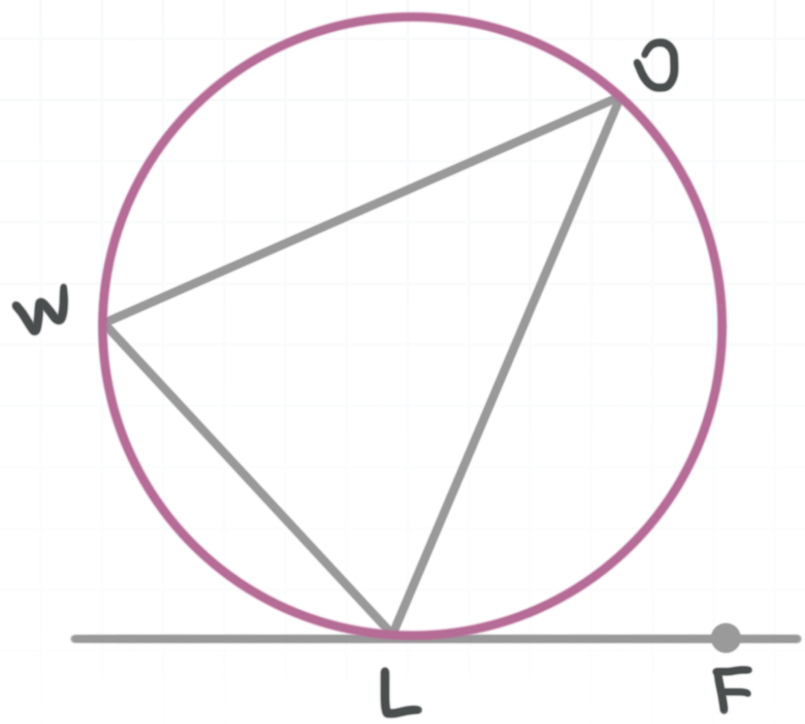
Question: In the figure, $m\angle WOL = 40^\circ$ and $m\angle OLW = 80^\circ$. Also, $\overline{LF}$ is tangent to the circle at L. What is $m\angle FLO$?



Answer choices:

A 30°

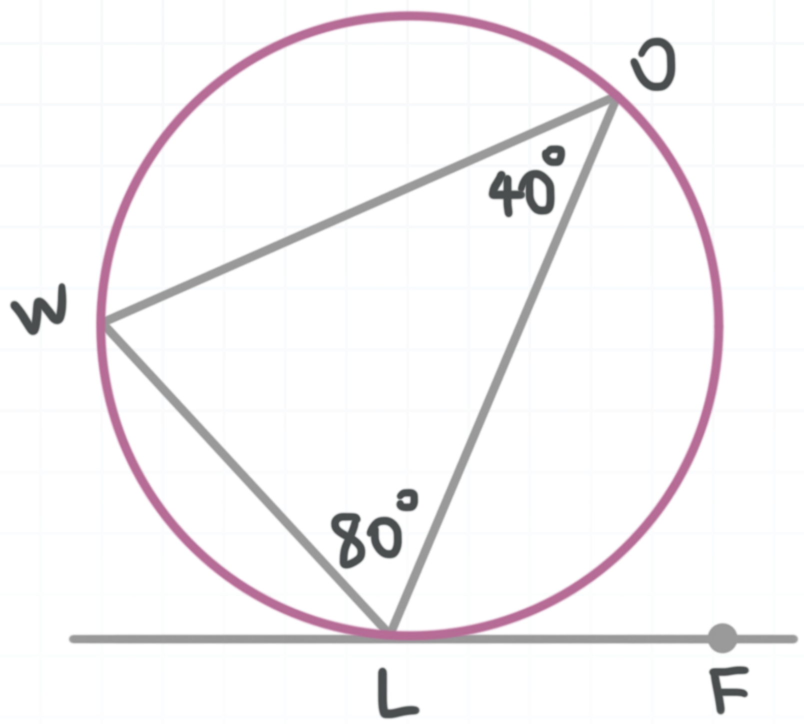
B 40°

C 50°

D 60°

Solution: D

In $\triangle OWL$, the measures of the three interior angles total 180°. We know that two of them are 40° and 80°.



Those two total 120°, which leaves 60° for $m\angle LWO$, which is an inscribed angle, so its intercepted arc $\widehat{LO}$ has measure 120°. $\angle FLO$ has its vertex on the circle, so its measure is half that of its intercepted arc, which is $\widehat{LO}$. Therefore,

$$m\angle FLO = \frac{1}{2}m\widehat{LO} = \frac{1}{2}(120^\circ)$$

$$m\angle FLO = \frac{1}{2}m\widehat{LO} = 60^\circ$$

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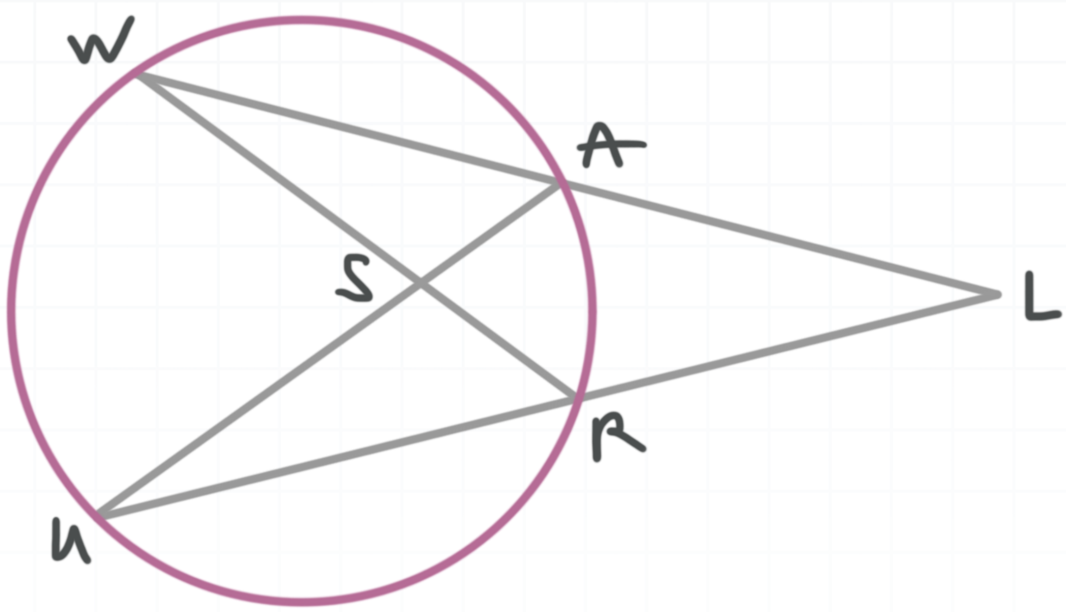
Question: In the figure, $m\angle RWL = 20^\circ$ and $m\angle RSA = 75^\circ$. What is $m\angle WLU$?



Answer choices:

A 30°

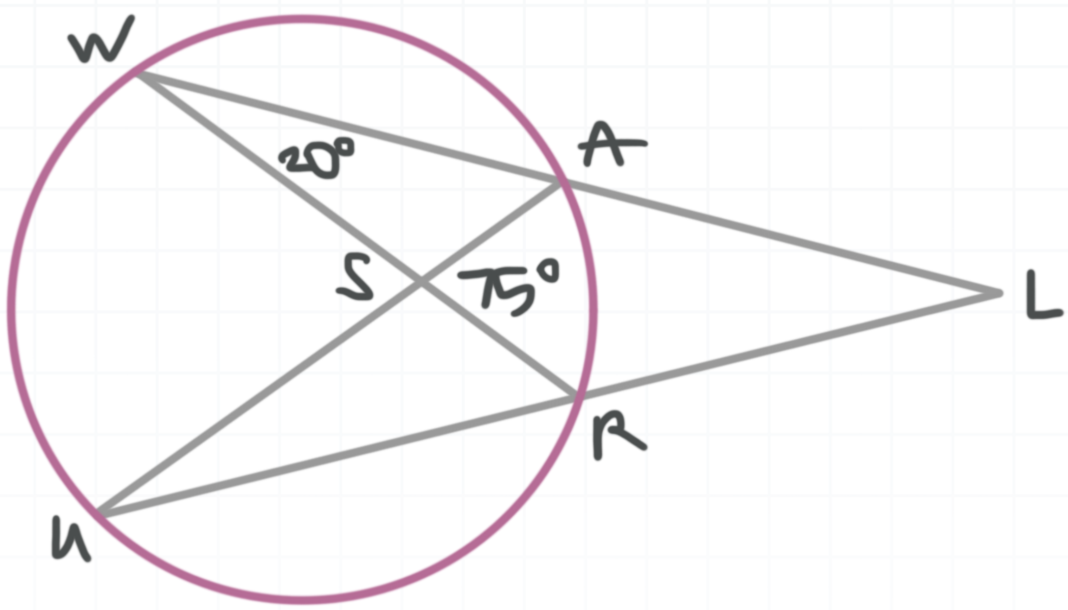
B 35°

C 40°

D 45°

Solution: B

From the information in the problem, we can fill out the figure.



First we'll find the measure of $\overset{\frown}{RA}$, which is the arc intercepted by $\angle RWL$ (an inscribed angle).

$$m\angle RWL = \frac{1}{2}m\overset{\frown}{RA}$$

$$20^\circ = \frac{1}{2}m\overset{\frown}{RA}$$

$$m\overset{\frown}{RA} = 40^\circ$$

Now we'll use this to find the measure of $\overset{\frown}{WU}$. Notice that $\angle RSA$ and $\angle WSU$ are a pair of vertical angles, and that their intercepted arcs are $\overset{\frown}{RA}$ and $\overset{\frown}{WU}$, respectively. Since their common vertex is inside the circle,

$$m\angle RSA = \frac{1}{2}(m\overset{\frown}{RA} + m\overset{\frown}{WU})$$

$$75^\circ = \frac{1}{2}(40^\circ + m\overset{\frown}{WU})$$

$$150^\circ = 40^\circ + m\widehat{WU}$$

$$110^\circ = m\widehat{WU}$$

Finally, we can find $m\angle WLU$. The arcs intercepted by $\angle WLU$ are $\widehat{WU}$ and $\widehat{RA}$. Since the vertex of $\angle WLU$ is outside the circle,

$$m\angle WLU = \frac{1}{2}(m\widehat{WU} - m\widehat{RA})$$

$$m\angle WLU = \frac{1}{2}(110^\circ - 40^\circ)$$

$$m\angle WLU = 35^\circ$$