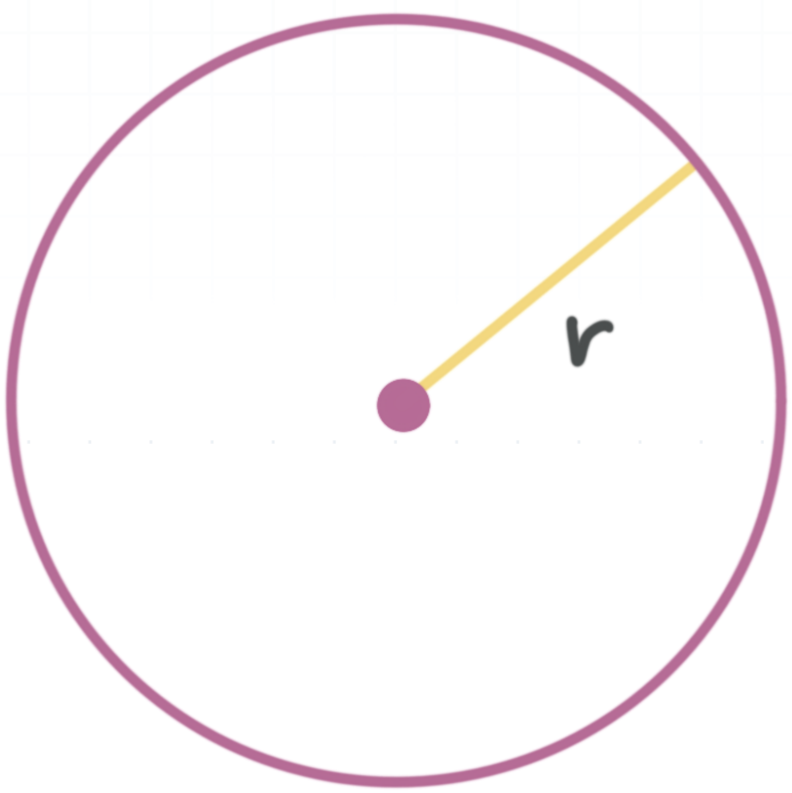
Area of a circle

In this lesson we'll look at how to use the area formula for a circle. In order to do that, we'll need to start by defining the parts of a circle.

Parts of a circle

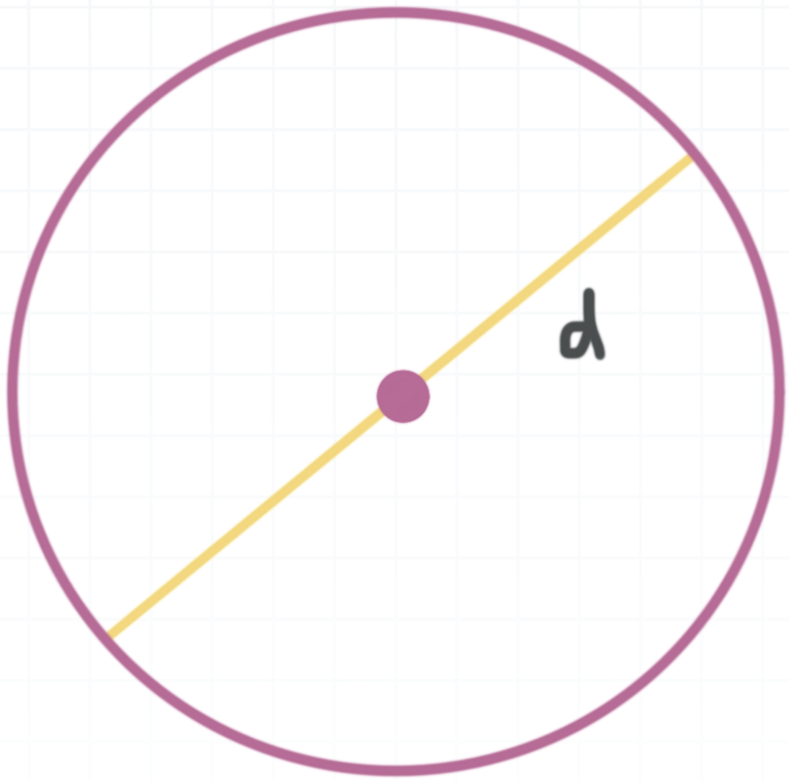
The **radius** of a circle is the length of any line segment from the center of the circle to a point on the circle. We usually use the variable r for the radius.



The **diameter** of a circle is the length of any chord that passes through the center of the circle. We usually use the variable d for the diameter of a circle. The diameter is a chord whose length is equal to twice the radius.

$$d = 2r$$

$$r = \frac{1}{2}d$$



The number π (“**pi**”) is a special number - with the lowercase Greek letter pi used as the symbol for it - that describes the relationship between the circumference of a circle (the “length around” a circle) and its diameter:

$$\text{circumference} = \pi d$$

Since the diameter of a circle is twice its radius, we also have

$$\text{circumference} = \pi(2r) = 2\pi r$$

The decimal expansion of π has infinitely many decimal places. Rounded to two decimal places (rounded to the nearest hundredth), its value is 3.14; we write this approximation as $\pi = 3.14$.

Area of a circle

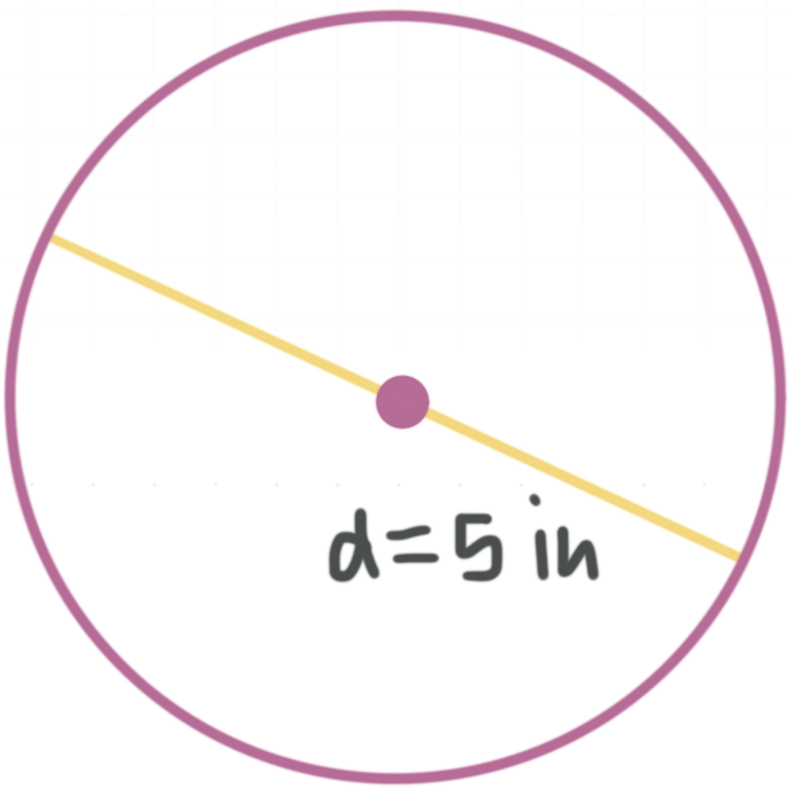
Now that we know the value of π, and the radius of the circle, we can define the area of a circle as the product of π and the square of the radius.

$$A = \pi r^2$$

As with all areas, the area of a circle is in units of length^2 ("length squared"). Let's start by working through an example.

Example

What is the area of the circle? Round your answer to the nearest hundredth.



The formula for the area is

$$A = \pi r^2$$

We’re given the diameter, and we need to find the radius. The radius is half of the diameter.

$$r = \frac{1}{2}(5 \text{ in})$$

$$r = 2.5 \text{ in}$$

Now we can use the area formula.

$$A = \pi(2.5 \text{ in})^2$$

$$A = 6.25\pi \text{ in}^2$$

This is the exact answer. We’re asked to round the answer to the nearest hundredth, so we’ll use the fact that π is about 3.14.

$$A \approx 6.25(3.14) \text{ in}^2$$

$$A \approx 19.63 \text{ in}^2$$

Sometimes you’ll be given the area and asked to solve for something else.

Example

What is the radius of a circle with an area of 75π cm^2?

The formula for the area of a circle is $A = \pi r^2$, and the area is 75π cm^2.

$$\pi r^2 = 75\pi \text{ cm}^2$$

$$r^2 = 75 \text{ cm}^2$$

$$r = \sqrt{75 \text{ cm}^2}$$

$$r = \sqrt{75} \text{ cm}$$

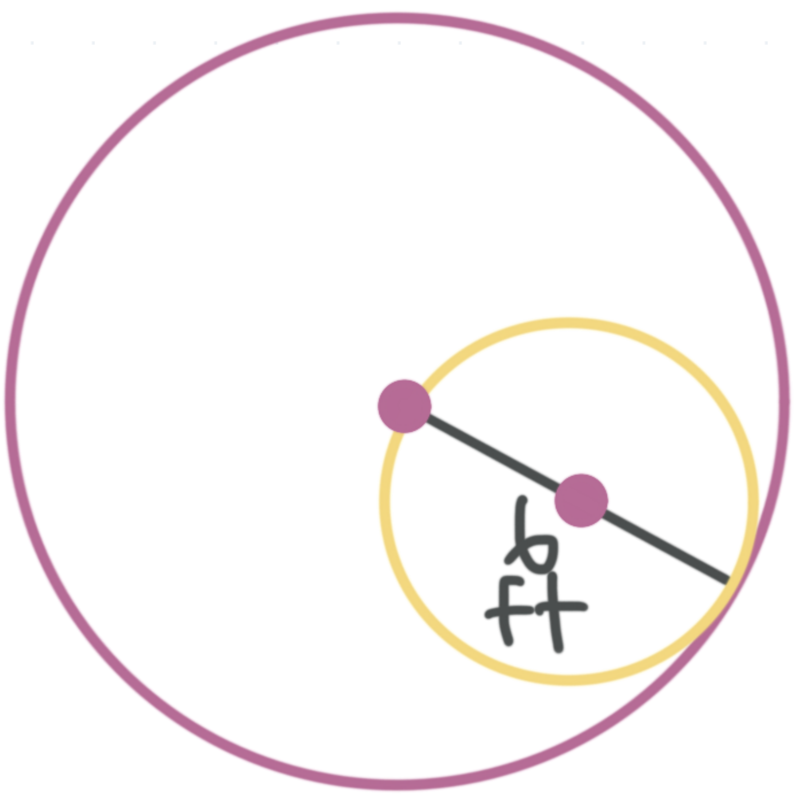
$$r = \sqrt{25} \cdot \sqrt{3} \text{ cm}$$

$$r = 5\sqrt{3} \text{ cm}$$

Sometimes you'll need to find the area of a composite figure that's made of circles.

Example

What is the area of the region that's inside the larger circle, but outside the smaller circle? Leave your answer in terms of π.



We need to find the area of the larger circle and the subtract from it the area of the smaller circle. The formula for the area of a circle is $A = \pi r^2$, so we need to know the radius of each circle.

The radius of the larger circle is 6 feet, so the area of the larger circle is

$$A = \pi \cdot (6 \text{ ft})^2$$

$$A = 36\pi \text{ ft}^2$$

The smaller circle has a diameter of 6 feet, so its radius is $r = 6/2 = 3$ feet. Therefore, the area of the smaller circle is

$$A = \pi \cdot (3 \text{ ft})^2$$

$$A = 9\pi \text{ ft}^2$$

So the area of the region that's inside the larger circle but outside the smaller circle is

$$36\pi \text{ ft}^2 - 9\pi \text{ ft}^2 = 27\pi \text{ ft}^2$$
