

Topic: Surface area to volume ratio of prisms

Question: Find the surface area to volume ratio of a right rectangular prism that's 2 cm high, 3 cm wide, and 5 cm long.

Answer choices:

- A 2.1 cm^{-1}
- B 2.6 cm^{-1}
- C 3.5 cm^{-1}
- D 3.8 cm^{-1}



Solution: A

Plug the measurements we've been given into the surface area formula.

$$S = 2lw + 2wh + 2lh$$

$$S = 2(5 \text{ cm} \cdot 3 \text{ cm}) + 2(3 \text{ cm} \cdot 2 \text{ cm}) + 2(5 \text{ cm} \cdot 2 \text{ cm})$$

$$S = 2(15 \text{ cm}^2) + 2(6 \text{ cm}^2) + 2(10 \text{ cm}^2)$$

$$S = 30 \text{ cm}^2 + 12 \text{ cm}^2 + 20 \text{ cm}^2$$

$$S = 62 \text{ cm}^2$$

Use $V = lwh$ to find the volume.

$$V = (5 \text{ cm}) \cdot (3 \text{ cm}) \cdot (2 \text{ cm})$$

$$V = 30 \text{ cm}^3$$

Now it's easy to find the ratio.

$$\frac{S}{V} = \frac{62 \text{ cm}^2}{30 \text{ cm}^3} = \frac{31}{15} \text{ cm}^{-1} \approx 2.1 \text{ cm}^{-1}$$



Topic: Surface area to volume ratio of prisms

Question: Find the surface area to volume ratio of a cube with dimensions $1 \times 1 \times 1$. Then calculate the same ratio for a $2 \times 2 \times 2$ cube. When we double the length of each side, the surface area to volume ratio...

Answer choices:

- A stays the same.
- B doubles.
- C quadruples.
- D gets cut in half.



Solution: D

The surface area and volume of the $1 \times 1 \times 1$ cube are

$$S = 2lw + 2wh + 2lh$$

$$S = 2(1 \text{ cm} \cdot 1 \text{ cm}) + 2(1 \text{ cm} \cdot 1 \text{ cm}) + 2(1 \text{ cm} \cdot 1 \text{ cm})$$

$$S = 2(1 \text{ cm}^2) + 2(1 \text{ cm}^2) + 2(1 \text{ cm}^2)$$

$$S = 2 \text{ cm}^2 + 2 \text{ cm}^2 + 2 \text{ cm}^2$$

$$S = 6 \text{ cm}^2$$

and

$$V = lwh$$

$$V = 1 \text{ cm} \cdot 1 \text{ cm} \cdot 1 \text{ cm}$$

$$V = 1 \text{ cm}^3$$

So the surface area to volume ratio of the $1 \times 1 \times 1$ is

$$\frac{S}{V} = \frac{6 \text{ cm}^2}{1 \text{ cm}^3} = 6 \text{ cm}^{-1}$$

The surface area and volume of the $2 \times 2 \times 2$ cube are

$$S = 2lw + 2wh + 2lh$$

$$S = 2(2 \text{ cm} \cdot 2 \text{ cm}) + 2(2 \text{ cm} \cdot 2 \text{ cm}) + 2(2 \text{ cm} \cdot 2 \text{ cm})$$

$$S = 2(4 \text{ cm}^2) + 2(4 \text{ cm}^2) + 2(4 \text{ cm}^2)$$



$$S = 8 \text{ cm}^2 + 8 \text{ cm}^2 + 8 \text{ cm}^2$$

$$S = 24 \text{ cm}^2$$

and

$$V = lwh$$

$$V = 2 \text{ cm} \cdot 2 \text{ cm} \cdot 2 \text{ cm}$$

$$V = 8 \text{ cm}^3$$

So the surface area to volume ratio of the $2 \times 2 \times 2$ is

$$\frac{S}{V} = \frac{24 \text{ cm}^2}{8 \text{ cm}^3} = 3 \text{ cm}^{-1}$$

Comparing these, we can see that the ratio has been cut in half, from 6 to 3.



Topic: Surface area to volume ratio of prisms

Question: Calculate the surface area to volume ratio of a right rectangular prism with dimensions $x \times x \times 2x$.

Answer choices:

A $\frac{5}{x}$

B $\frac{5}{1}$

C $\frac{4x}{1}$

D $\frac{1}{10x}$



Solution: A

Find the surface area.

$$S = 2lw + 2wh + 2lh$$

$$S = 2(2x \cdot x) + 2(x \cdot x) + 2(2x \cdot x)$$

$$S = 2(2x^2) + 2(x^2) + 2(2x^2)$$

$$S = 4x^2 + 2x^2 + 4x^2$$

$$S = 10x^2$$

Find the volume.

$$V = lwh$$

$$V = 2x \cdot x \cdot x$$

$$V = 2x^3$$

The ratio is of the surface area to volume is

$$\frac{S}{V} = \frac{10x^2}{2x^3} = \frac{5}{x}$$

