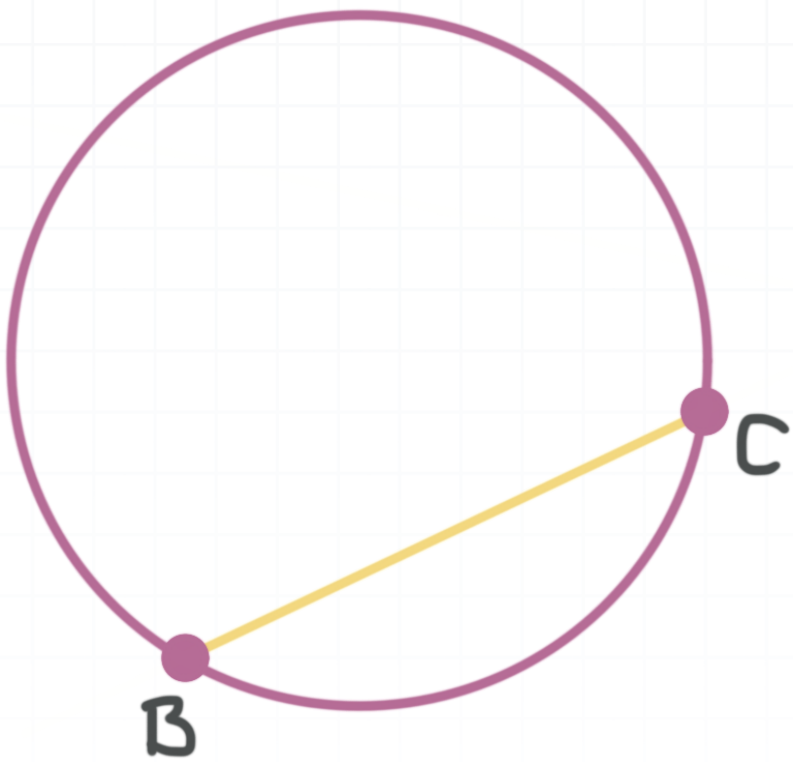
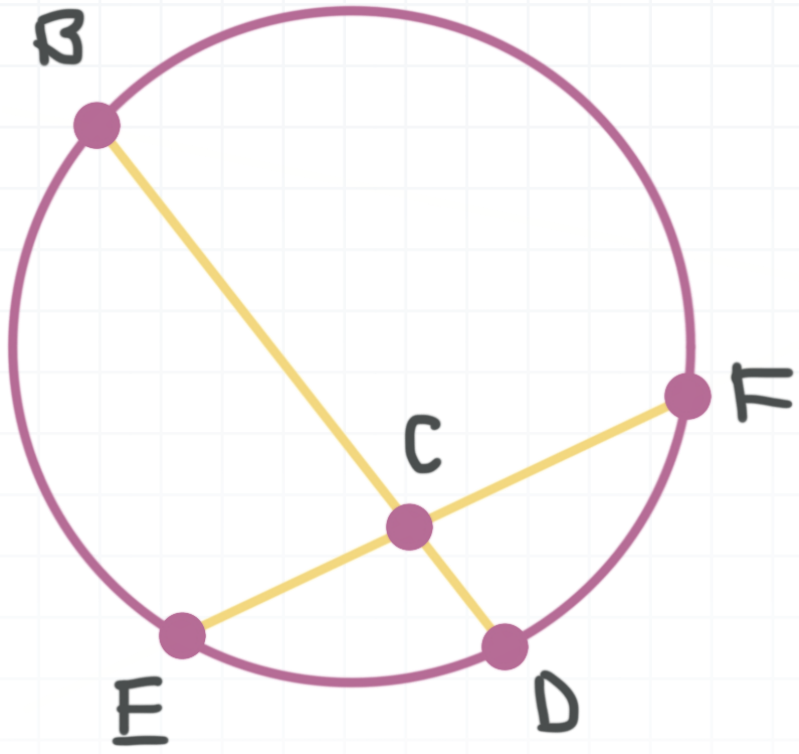
Intersecting chords

A **chord** of a circle is a line segment that has both of its endpoints on the circle. $\overline{BC}$ is an example of a chord.



Intersecting chord theorem

The intersecting chord theorem states that the products of chord segments are always equal. For instance, consider chords $\overline{BD}$ and $\overline{EF}$.



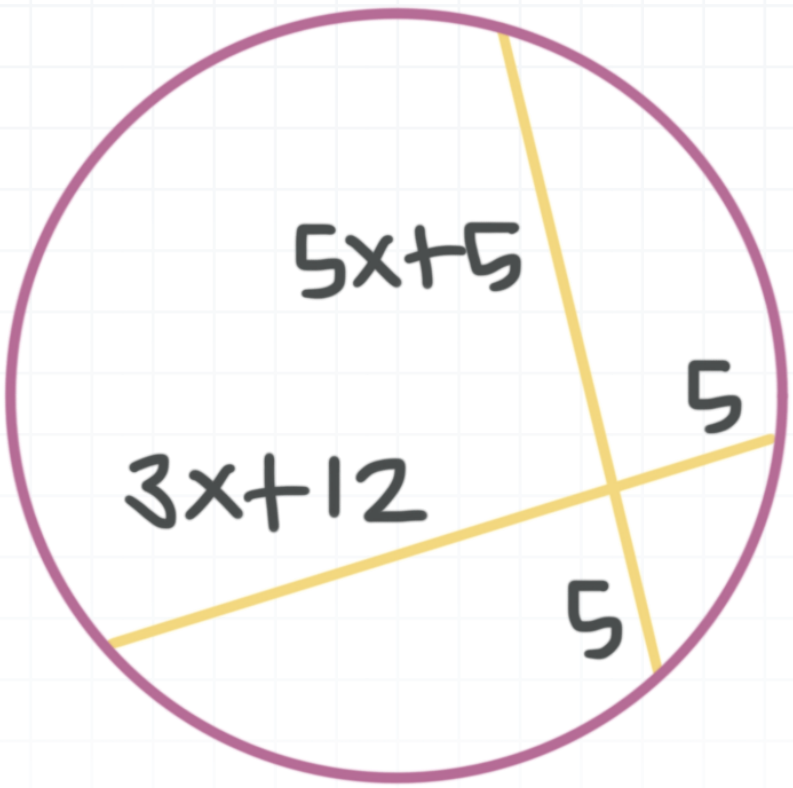
The intersecting chord theorem says that

$$\overline{BC} \cdot \overline{CD} = \overline{EC} \cdot \overline{CF}$$

Let's start by working through an example.

Example

Find the value of x in the figure.



The products of the chord segments are equal. So we can set up an equation.

$$5(5x + 5) = 5(3x + 12)$$

$$25x + 25 = 15x + 60$$

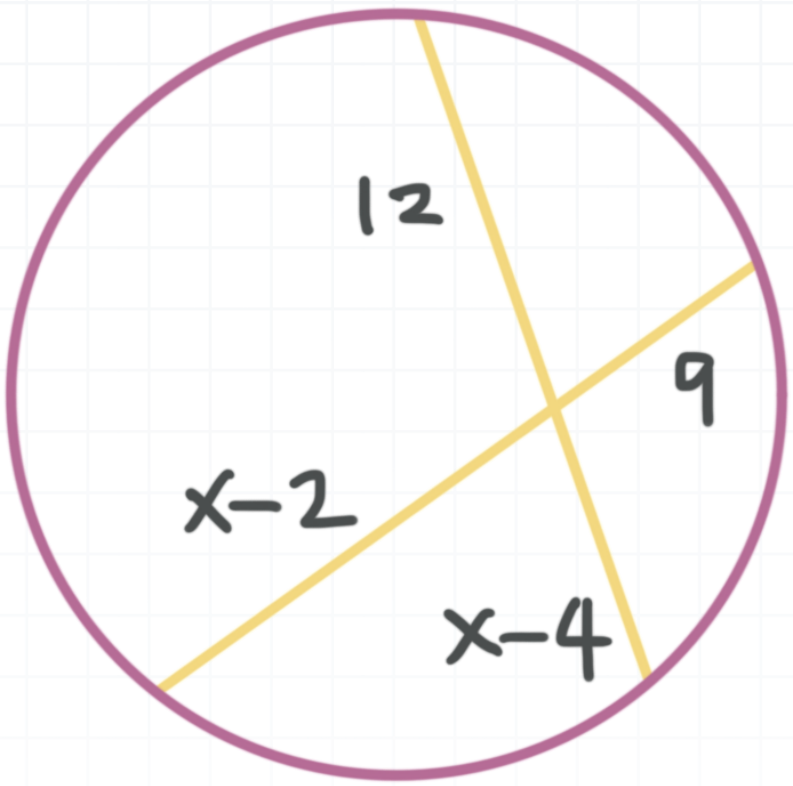
$$10x = 35$$

$$x = 3.5$$

Let's do one more.

Example

Find the length of each chord.



First we need to find the value of x, and then use that to find the lengths of the chords. The products of the chord segments are equal, so

$$12(x-4) = 9(x-2)$$

$$12x - 48 = 9x - 18$$

$$3x = 30$$

$$x = 10$$

Now we can find the length of each chord. One chord has a length of

$$12 + x - 4$$

$$12 + 10 - 4$$

$$18$$

The other chord has a length of

$$x - 2 + 9$$

$$10 - 2 + 9$$

$$17$$

The chords have lengths of 17 and 18.
