Topic: Distance between two points in three dimensions

Question: Calculate the distance between P_1 and P_2 .

$$P_1 = (3,6,1)$$

$$P_2 = (0,1,5)$$

Answer choices:

A 5

B $5\sqrt{2}$

C $3\sqrt{5}$

D 7

Solution: B

Plug the coordinates of the given points into the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(3-0)^2 + (6-1)^2 + (1-5)^2}$$

$$d = \sqrt{9 + 25 + 16}$$

$$d = \sqrt{50}$$

$$d = \sqrt{25 \cdot 2}$$

$$d = \sqrt{25}\sqrt{2}$$
$$d = 5\sqrt{2}$$

$$d = 5\sqrt{2}$$

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Question: Calculate the distance between P_1 and P_2 .

$$P_1 = (-5, -2, 6)$$

$$P_2 = (1,2,4)$$

Answer choices:

- **A** $2\sqrt{14}$
- B $2\sqrt{5}$
- **C** 6
- D $2\sqrt{10}$

Solution: A

Plug the coordinates of the given points into the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(-5-1)^2 + (-2-2)^2 + (6-4)^2}$$

$$d = \sqrt{36 + 16 + 4}$$

$$d = \sqrt{56}$$

$$d = \sqrt{4 \cdot 14}$$

$$d = \sqrt{4}\sqrt{14}$$

$$d = 2\sqrt{14}$$

Topic: Distance between two points in three dimensions

Question: Which point is closest to the point P = (2,4,6)?

Answer choices:

A
$$Q_1 = (5, -2,3)$$

B
$$Q_2 = (4,5,10)$$

C
$$Q_3 = (6,1,9)$$

D
$$Q_4 = (1,8,5)$$

Solution: D

We'll use the distance formula to find the distance of each of the four points from P. To make it easier to identify the point that's closest to P, we won't simplify the radicals.

The distance between P and Q_1 is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2 - 5)^2 + (4 - (-2))^2 + (6 - 3)^2}$$

$$d = \sqrt{(-3)^2 + 6^2 + 3^2}$$

$$d = \sqrt{9 + 36 + 9}$$

$$d = \sqrt{54}$$

The distance between P and Q_2 is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2 - 4)^2 + (4 - 5)^2 + (6 - 10)^2}$$

$$d = \sqrt{(-2)^2 + (-1)^2 + (-4)^2}$$

$$d = \sqrt{4 + 1 + 16}$$

$$d = \sqrt{21}$$

The distance between P and Q_3 is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2-6)^2 + (4-1)^2 + (6-9)^2}$$

$$d = \sqrt{(-4)^2 + 3^2 + (-3)^2}$$

$$d = \sqrt{16 + 9 + 9}$$

$$d = \sqrt{34}$$

The distance between P and Q_4 is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2-1)^2 + (4-8)^2 + (6-5)^2}$$

$$d = \sqrt{1^2 + (-4)^2 + 1^2}$$

$$d = \sqrt{1 + 16 + 1}$$

$$d = \sqrt{18}$$

The distances of Q_1 , Q_2 , Q_3 , and Q_4 from P are $\sqrt{54}$, $\sqrt{21}$, $\sqrt{34}$, and $\sqrt{18}$, respectively, so Q_4 is the point closest to P.