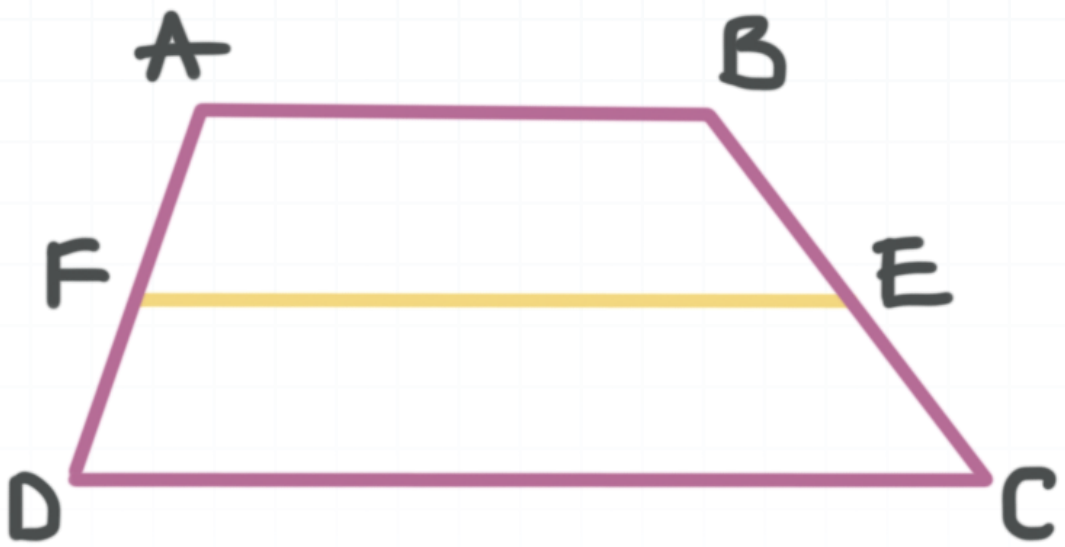
Midsegments of trapezoids

The midsegment of a trapezoid is the segment that connects the midpoints of the opposite non-parallel sides of the trapezoid.

If $\overline{AB} \parallel \overline{DC}$, if F is the midpoint of $\overline{AD}$, and if E is the midpoint of $\overline{BC}$, then $\overline{FE}$ is the midsegment of the trapezoid.



The relationship between the length of the midsegment and the lengths of the parallel sides is

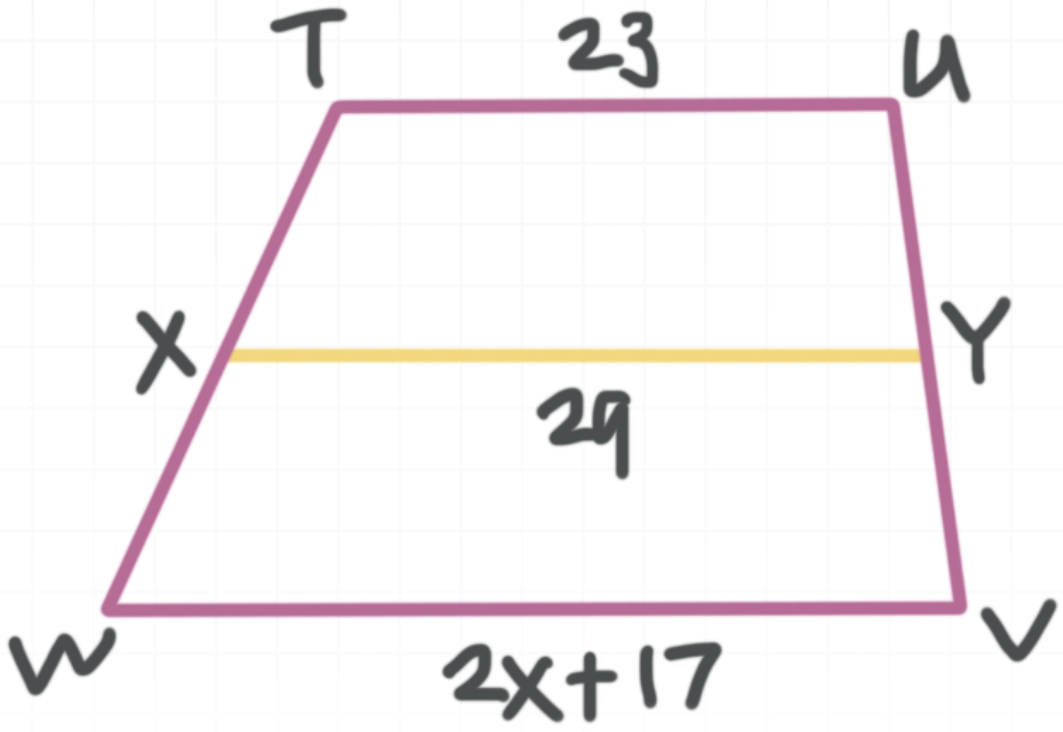
$$\overline{FE} = \frac{1}{2}(\overline{AB} + \overline{DC})$$

The length of the midsegment of a trapezoid is always equal to half of the sum of the lengths of the parallel sides (also called the bases) of the trapezoid.

Let’s work through a couple of examples.

Example

In the trapezoid pictured, $\overline{TU} \parallel \overline{WV}$, X is the midpoint of $\overline{TW}$, and Y is the midpoint of $\overline{UV}$. What is the length of $\overline{WV}$?



By definition, $\overline{XY}$ is the midsegment of the trapezoid. Therefore, we know that

$$\overline{XY} = \frac{1}{2}(\overline{TU} + \overline{WV})$$

Let's plug in what we know and then solve for x.

$$29 = \frac{1}{2}[23 + (2x + 17)]$$

$$29 = \frac{1}{2}(40 + 2x)$$

$$29 = 20 + x$$

$$9 = x$$

Therefore,

$$\overline{WV} = 2x + 17$$

$$\overline{WV} = 2(9) + 17$$

$$\overline{WV} = 18 + 17$$

$$\overline{WV} = 35$$

Let's try one with a few more steps.

Example

In the coordinate plane, a trapezoid $XYWZ$ has vertices at $X = (-2,2)$, $Y = (3,2)$, $Z = (-3, -2)$, and $W = (4, -2)$. What is the length of the midsegment of the trapezoid?

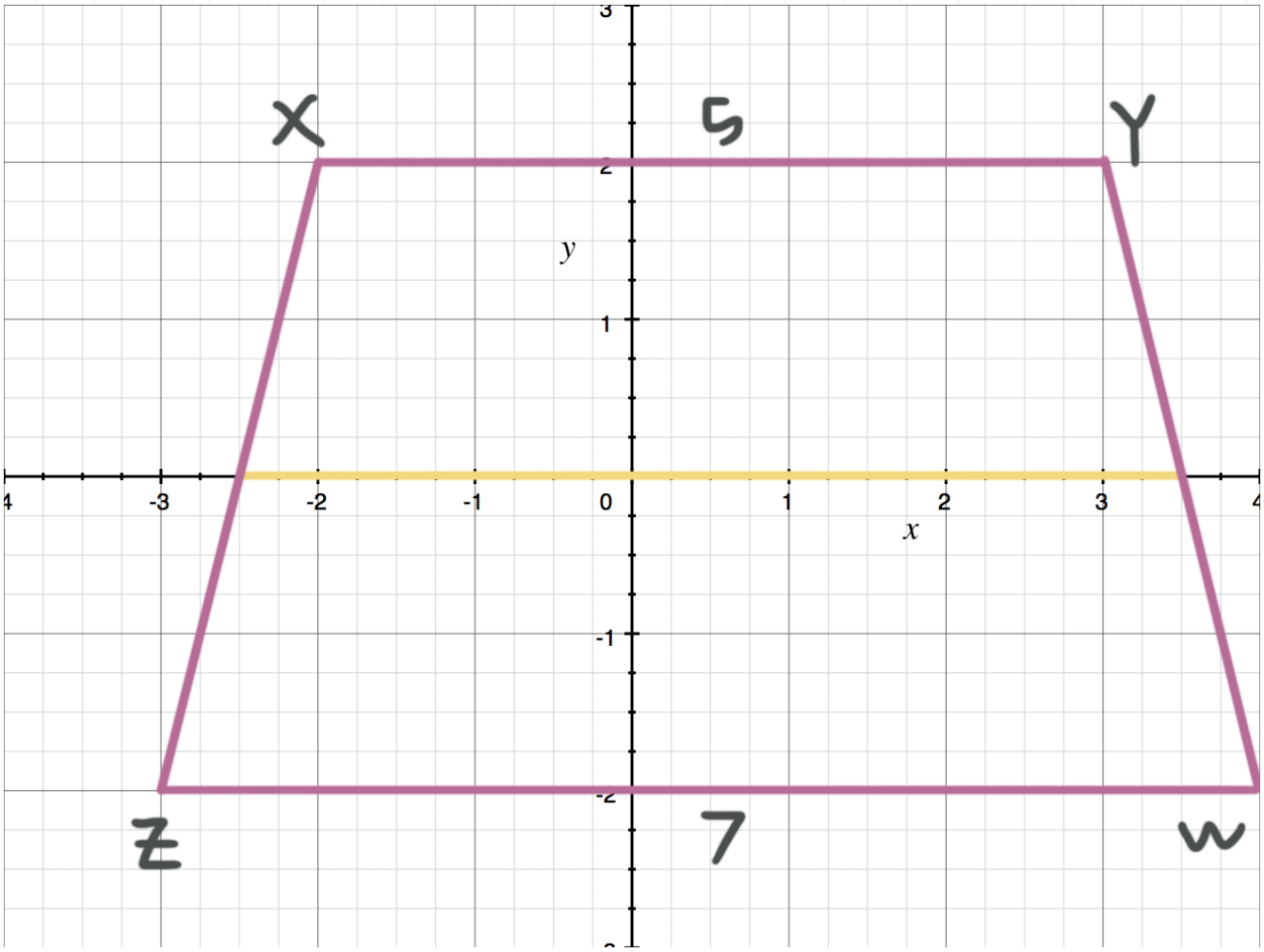
You can plot the trapezoid to identify the parallel sides and find find their lengths.

Notice that the vertices X and Y have the same y-coordinate of 2, and that vertices Z and W have the same y-coordinate of -2. That means that $\overline{XY} \parallel \overline{ZW}$ ($\overline{XY}$ and $\overline{ZW}$ are the parallel sides of the trapezoid). Also, since the y-coordinates of vertices X and Y are equal, the length of $\overline{XY}$ is the difference in their x-coordinates (which are 3 and -2), so

$$\overline{XY} = 3 - (-2) = 5$$

Similarly, since the y-coordinates of vertices Z and W are equal, the length of $\overline{ZW}$ is the difference in their x-coordinates (which are 4 and -3), so

$$\overline{ZW} = 4 - (-3) = 7$$



Remember that the length of the midsegment is equal to half of the sum of the lengths of the parallel sides, so the length of the midsegment is

$$\frac{1}{2}(\overline{XY} + \overline{ZW}) = \frac{1}{2}(5 + 7) = \frac{1}{2}(12) = 6$$