Topic: Parallel, perpendicular, or neither

**Question**: Each pair of points in the table below are points that lie on the given line. Which two lines are perpendicular to each other?

| Line    | Point 1 | Point 2 |
|---------|---------|---------|
| Line AB | (-2,2)  | (1,8)   |
| Line CD | (3,6)   | (5,2)   |
| Line EF | (3,0)   | (7,-2)  |

## **Answer choices:**

- A  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$
- B  $\overrightarrow{CD}$  and  $\overrightarrow{EF}$
- C  $\overrightarrow{AB}$  and  $\overrightarrow{EF}$
- D None are perpendicular

Solution: C

Use the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

for each line.

$$\overrightarrow{AB}$$
:  $m = \frac{8-2}{1-(-2)} = 2$ 

$$\overrightarrow{CD}$$
:  $m = \frac{2-6}{5-3} = -2$ 

$$\overrightarrow{EF}$$
:  $m = \frac{-2 - 0}{7 - 3} = -\frac{1}{2}$ 

 $\overrightarrow{AB}$  and  $\overrightarrow{EF}$  have slopes that are negative reciprocals of each other, so they're perpendicular.



Topic: Parallel, perpendicular, or neither

**Question**: Each pair of points in the table below are points that lie on the given line. Which lines are parallel to each other?

| Line    | Point 1 | Point 2 |
|---------|---------|---------|
| Line AB | (0,3)   | (6,7)   |
| Line CD | (5,4)   | (8,6)   |
| Line EF | (1,-2)  | (7,2)   |

## **Answer choices:**

- A  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$
- B  $\overrightarrow{CD}$  and  $\overrightarrow{EF}$
- C  $\overrightarrow{AB}$  and  $\overrightarrow{EF}$
- D All three are parallel

Solution: D

Use the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

for each line.

$$\overrightarrow{AB}$$
:  $m = \frac{7-3}{6-0} = \frac{2}{3}$ 

$$\overrightarrow{CD}$$
:  $m = \frac{6-4}{8-5} = \frac{2}{3}$ 

$$\overrightarrow{EF}$$
:  $m = \frac{2 - (-2)}{7 - 1} = \frac{2}{3}$ 

All three lines have the same slope, so all three are parallel unless two of them (or all three) are actually one and the same line.



Topic: Parallel, perpendicular, or neither

Question: Which statement is true?

| Point | Coordinates |
|-------|-------------|
| А     | (-7,2)      |
| В     | (2,5)       |
| С     | (4,-1)      |
| D     | (-5,-4)     |

## **Answer choices:**

- A Lines  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  are parallel, and lines  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  are perpendicular.
- B Lines  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  are parallel, and lines  $\overrightarrow{AC}$  and  $\overrightarrow{BD}$  are parallel.
- C Lines  $\overrightarrow{AC}$  and  $\overrightarrow{CD}$  are perpendicular, and lines  $\overrightarrow{BC}$  and  $\overrightarrow{BD}$  are perpendicular.
- D Lines  $\overrightarrow{BC}$  and  $\overrightarrow{BD}$  are perpendicular, and lines  $\overrightarrow{AD}$  and  $\overrightarrow{AB}$  are perpendicular.

Solution: A

Use the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

to get the slopes of the lines  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ ,  $\overrightarrow{CD}$ ,  $\overrightarrow{AC}$ ,  $\overrightarrow{BD}$ , and  $\overrightarrow{AD}$ .

$$\overrightarrow{AB}$$
:  $m = \frac{5-2}{2-(-7)} = \frac{1}{3}$ 

$$\overrightarrow{BC}$$
:  $m = \frac{-1-5}{4-2} = -3$ 

$$\overrightarrow{CD}$$
:  $m = \frac{-4 - (-1)}{-5 - 4} = \frac{1}{3}$ 

$$\overrightarrow{AC}$$
:  $m = \frac{-1-2}{4-(-7)} = \frac{-3}{11}$ 

$$\overrightarrow{BD}$$
:  $m = \frac{-4-5}{-5-2} = \frac{9}{7}$ 

$$\overleftrightarrow{AD}$$
:  $m = \frac{-4-2}{-5-(-7)} = -3$ 

Lines  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  have the same slope, so they're parallel, unless they're one and the same line.

Lines  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  have slopes that are negative reciprocals of each other, so they're perpendicular. Combining these two results, we see that answer choice A is correct.

Let's check the other three answer choices.

Answer choice B can't be correct, because lines  $\overrightarrow{AC}$  and  $\overrightarrow{BD}$  have different slopes, so they can't be parallel.

Answer choice C can't be correct, because the slopes of the lines  $\overrightarrow{AC}$  and  $\overrightarrow{CD}$  aren't negative reciprocals of each other, so they can't be perpendicular.

Answer choice D can't be correct, because the slopes of lines  $\overrightarrow{BC}$  and  $\overrightarrow{BD}$  aren't negative reciprocals of each other, so they can't be perpendicular.

