Geometry Workbook Solutions

Circles

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EQUATION OF A CIRCLE

1. A circle has a radius of 4 and center at $(-2,5)$. Write the equation for this circle.

Solution:

The general equation for a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius. $(-2,5)$ is the center, so $h=-2$ and $k=5$. The radius is given as $r=4$. Substitute these values into the general equation and get $(x+2)^2+(y-5)^2=16$.

2. Find the center and diameter of the circle given by $(x-3)^2+(y+2)^2=9$.

Solution:

The general equation for a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius. In this equation, $h=3$ and $k=-2$ making the center $(3,-2)$. The radius is $\sqrt{9}=3$. Therefore, the diameter of the circle is 6.

3. A circle has a diameter with endpoints at $(-3,-1)$ and $(3,7)$. Find the equation of the circle.

Solution:

The diameter of the circle can be found by finding the distance between the points given. Use the distance formula to get

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(7 - (-1))^2 + (3 - (-3)^2}$$

$$d = \sqrt{100}$$

$$d = 10$$

Use the midpoint formula to find the center of the circle.

$$(a, b) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(a, b) = \left(\frac{(-3) + 3}{2}, \frac{(-1) + 7}{2}\right)$$

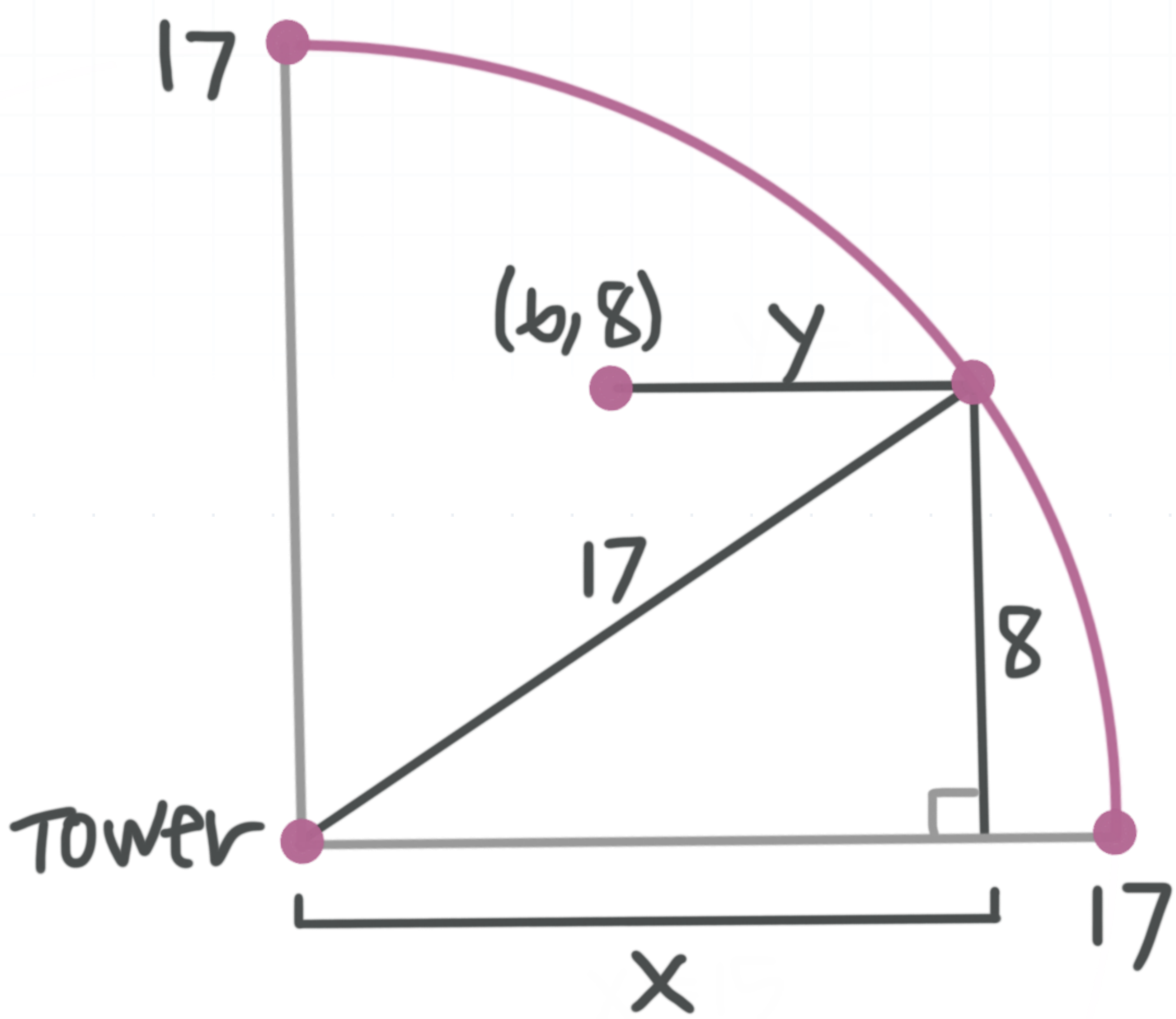
$$(a, b) = (0,3)$$

The general equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius. Substitute $h = 0$, $k = 3$, and $r = 5$ into the equation to get $x^2 + (y - 3)^2 = 25$.

■ 4. A cellphone tower services a 17 mile radius. A rest stop on the highway is 6 miles east and 8 miles north of the tower. If you continue to travel due east from the rest stop, for how many more miles will you be in range of the tower?

Solution:

Sketch the following diagram.



Find the horizontal length of the right triangle shown and get $x^2 + 8^2 = 17^2$. Solve for x.

$$x^2 + 8^2 = 17^2$$

$$x^2 + 64 = 289$$

$$x^2 = 225$$

$$x = 15$$

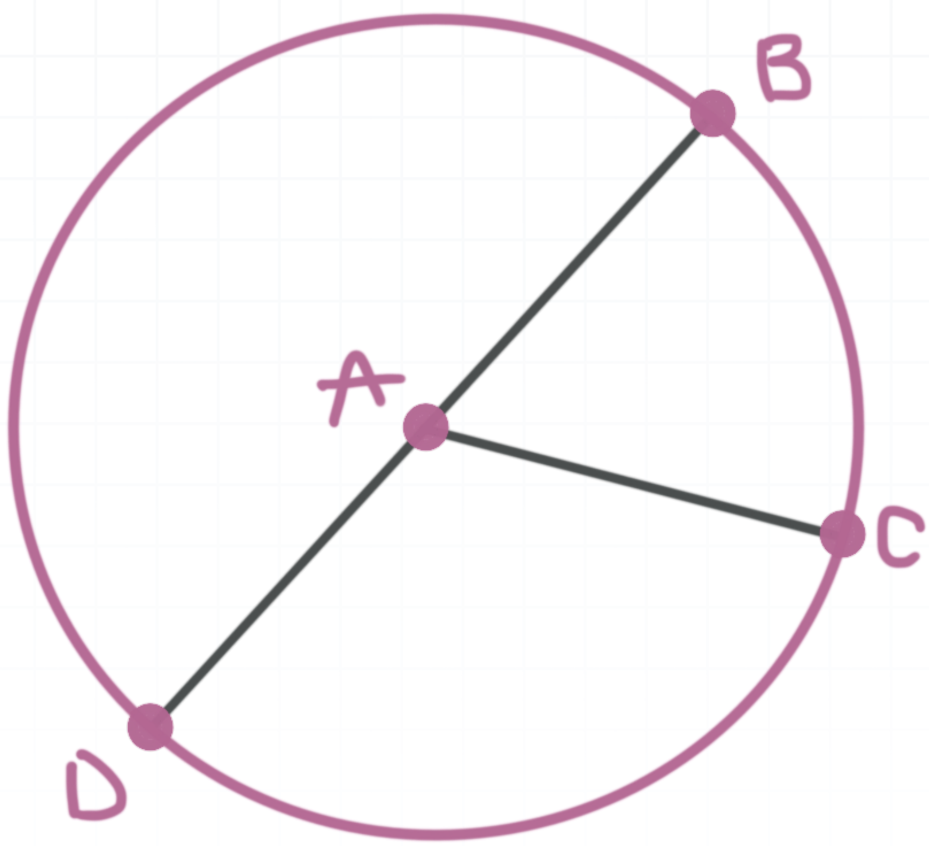
Now find y.

$$y = x - 6$$

$$y = 9$$

DEGREE MEASURE OF AN ARC

■ 1. In $\odot A$, $m\angle BAC = 65^\circ$ and $\overline{BD}$ is a diameter. Find the measure of arc DC.



Solution:

$\angle BAC$ is a central angle of $\odot A$, so arc BC is given by the measure of $\angle BAC$, and $m\angle BAC = 65^\circ$. Since $\angle BAC$ and $\angle CAD$ are supplementary,

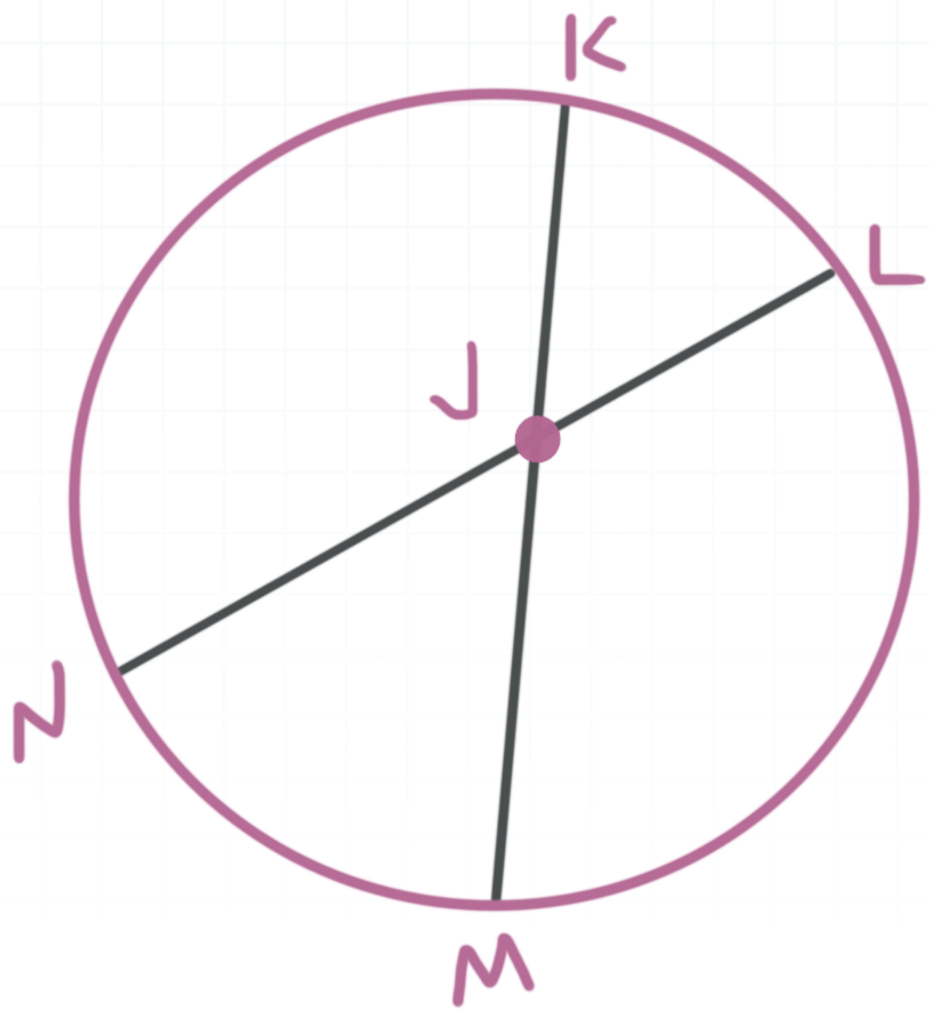
$$m\angle BAC + m\angle CAD = 180^\circ$$

$$65^\circ + m\angle CAD = 180^\circ$$

$$m\angle CAD = 115^\circ$$

Therefore, the measure of arc DC must be 115°.

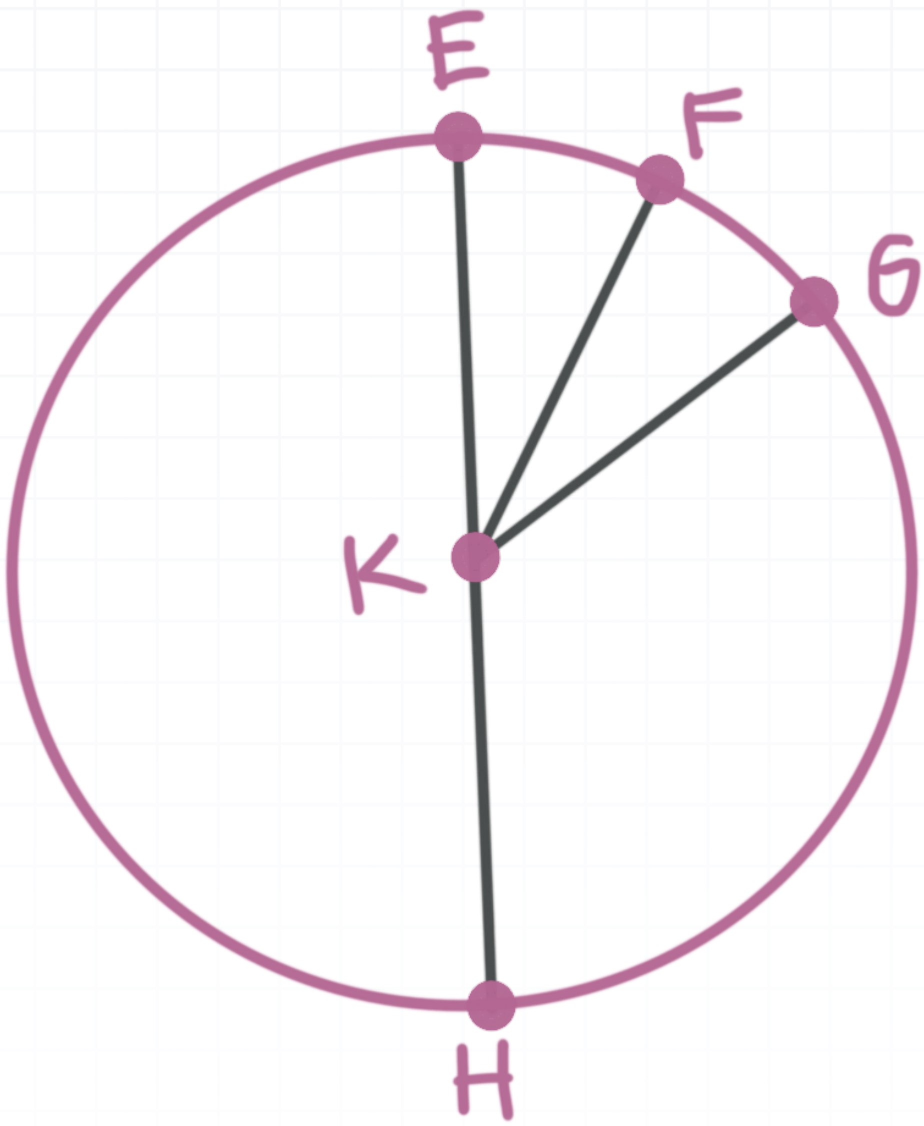
■ 2. In $\odot J$, $m\angle KJL = 54^\circ$ and $\overline{KM}$ and $\overline{LN}$ are diameters. Find the measure of arc MN.



Solution:

$\angle KJL$ is a central angle of $\odot J$. Because they are vertical angles, $m\angle KJL = m\angle NJM$. Because $m\angle KJL = 54^\circ$, that means $m\angle NJM = 54^\circ$, and therefore that the measure of arc MN is also 54°.

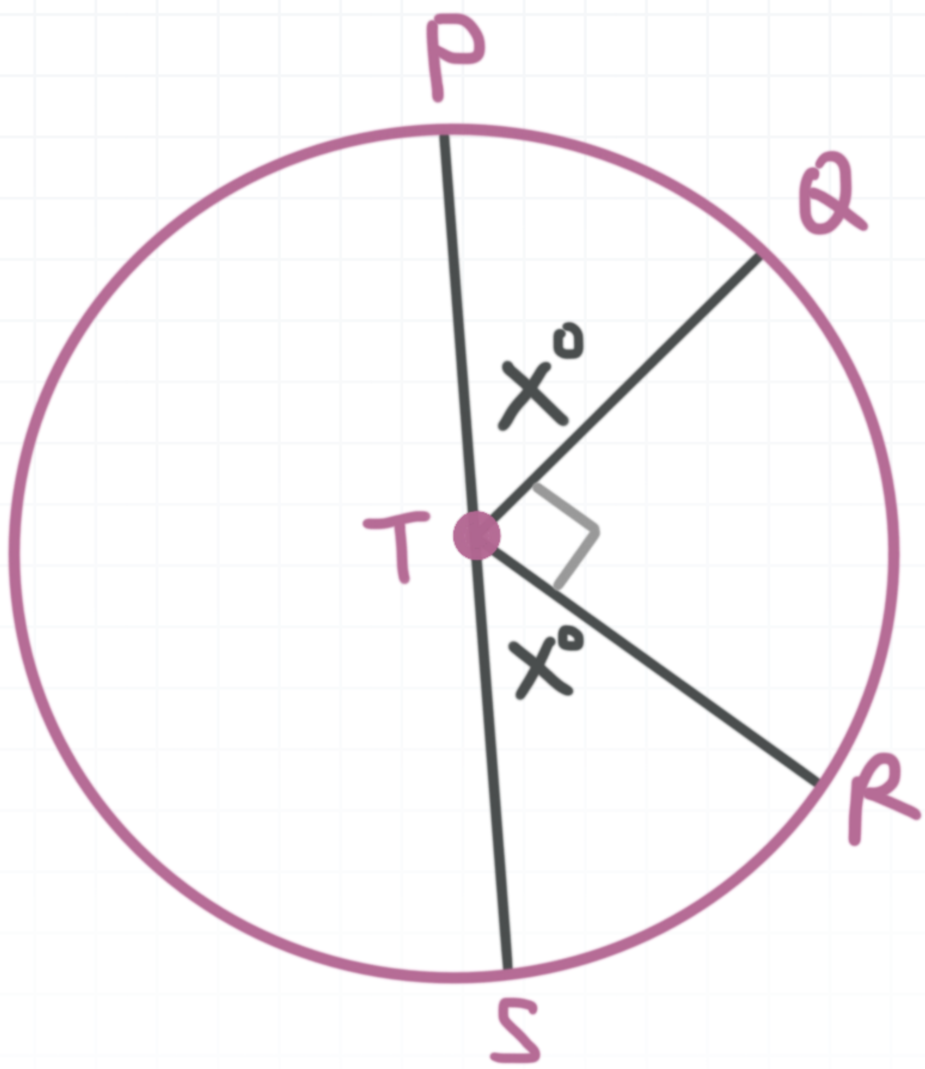
■ 3. In $\odot K$, $m\angle EKG = 70^\circ$, $\overline{EH}$ is a diameter, and $\overline{KF}$ bisects $\angle EKG$. Find the measure of arc FEH.



Solution:

$\angle EKG = 70^\circ$ and is bisected by KF, so $m\angle EKF$ and the measure of arc EF are both equal to 35°. Arc FEH is the sum of arc EF and arc EH, so arc FEH must have a measure of $35^\circ + 180^\circ = 215^\circ$.

■ 4. Find the measure of arc PR, if $\overline{PS}$ is the diameter of $\odot T$.



Solution:

Because $\overline{PS}$ is a diameter,

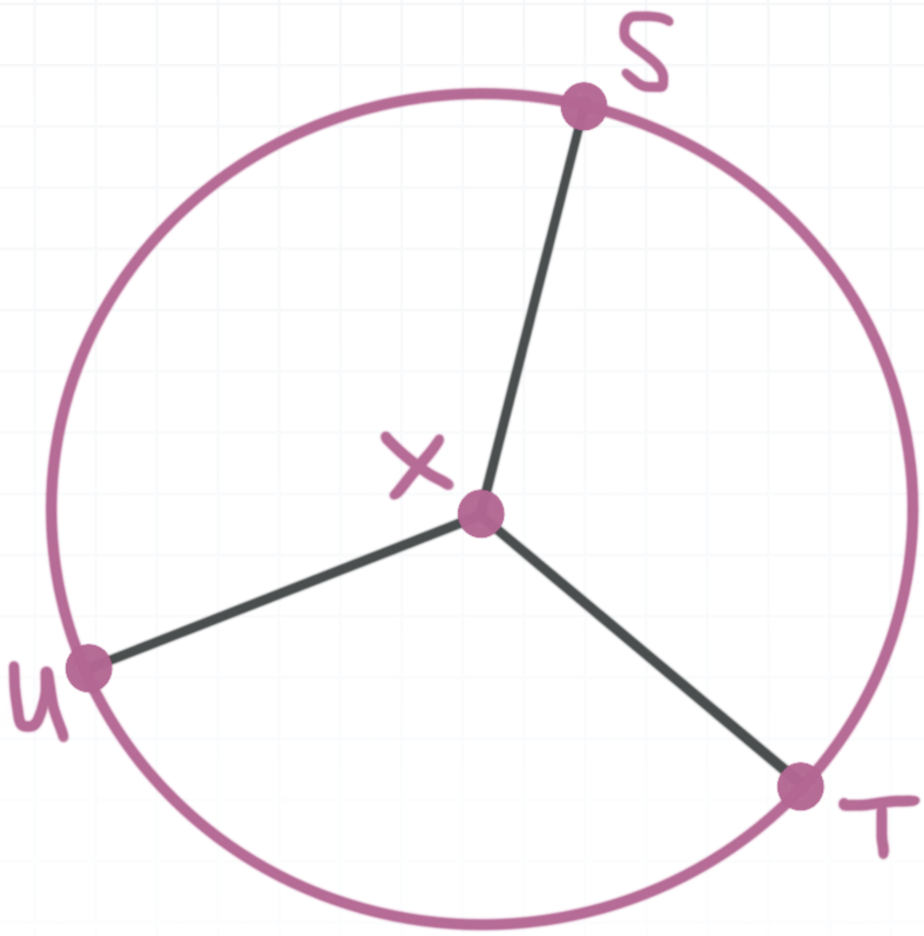
$$m\angle PTQ + m\angle QTR + m\angle RTS = 180^\circ$$

$$x + 90 + x = 180^\circ$$

$$x = 45^\circ$$

Then we know that the measure of arc PQ is 45° and the measure of arc QR is 90°, so the measure of arc PR is $45^\circ + 90^\circ = 135^\circ$.

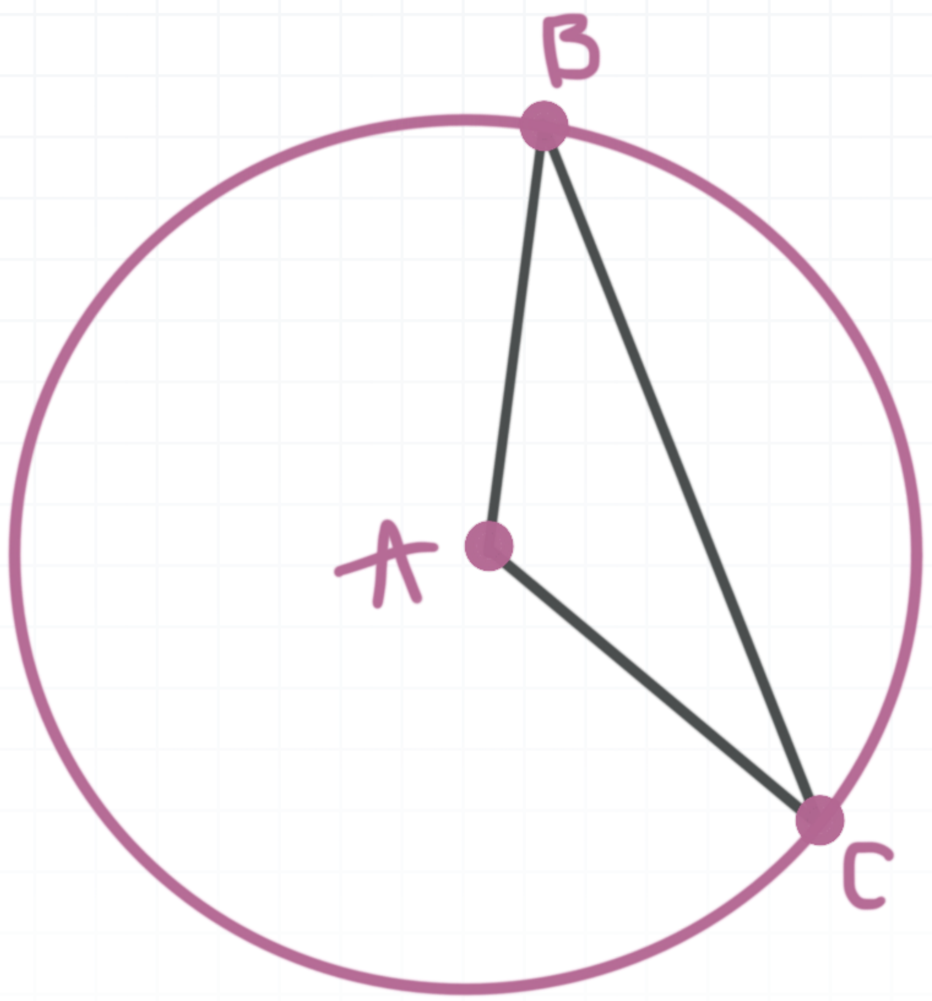
■ 5. In $\odot X$, $\angle UXS \cong \angle SXT \cong \angle UXT$. Find the measure of arc STU.



Solution:

The measures of arcs ST, TU, and US are congruent, and together they form the full circle, so each of those arcs has measure $360^\circ/3 = 120^\circ$. Therefore, arc STU measures $ST + TU = 120^\circ + 120^\circ = 240^\circ$.

■ 6. In $\odot A$, $m\angle ABC = 15^\circ$. Find the measure of arc BC.



Solution:

$\triangle ABC$ is isosceles because $\overline{AB}$ and $\overline{AC}$ are radii of $\odot A$.

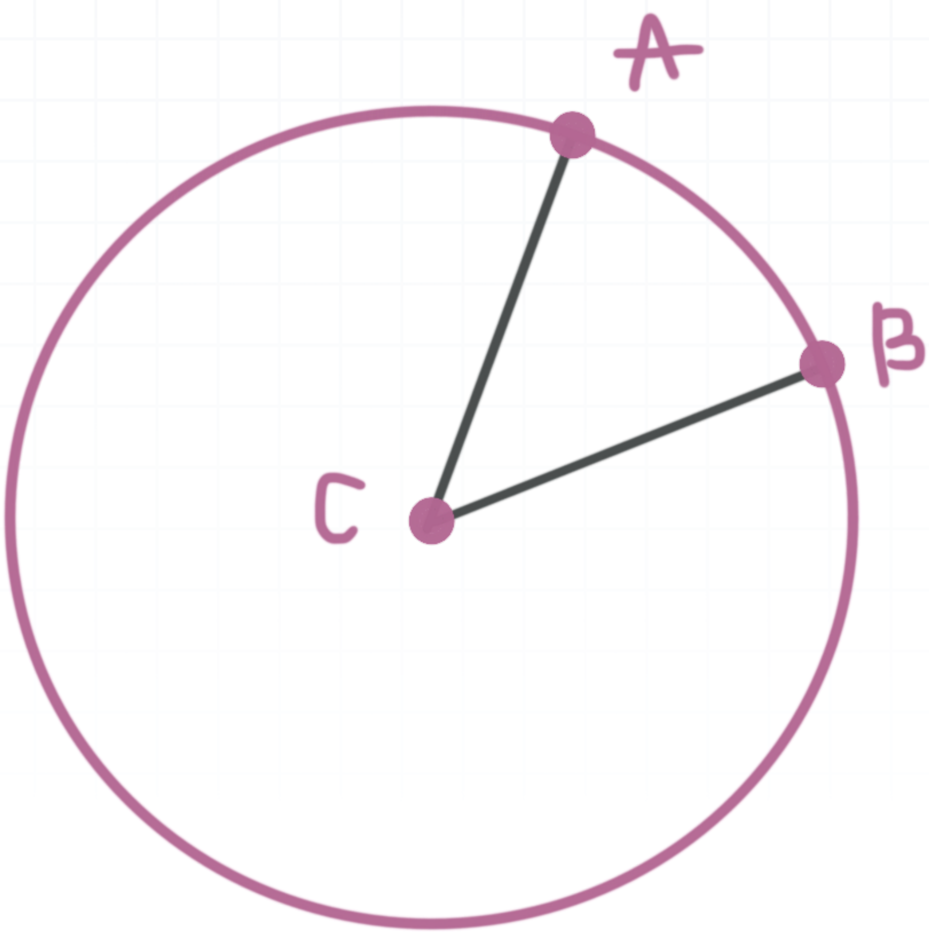
$$m\angle BAC + m\angle ABC + m\angle BCA = 180^\circ$$

$$m\angle BAC + 15^\circ + 15^\circ = 180^\circ$$

The measure of angle BAC is $m\angle BAC = 150^\circ$, which means the measure of arc BC is also 150°.

ARC LENGTH

■ 1. In $\odot C$, $m\angle ACB = 50^\circ$. Find the length of arc AB if $CA = 14$. Round your answer to the nearest hundredth.



Solution:

The circumference of a circle is

$$C = 2\pi r$$

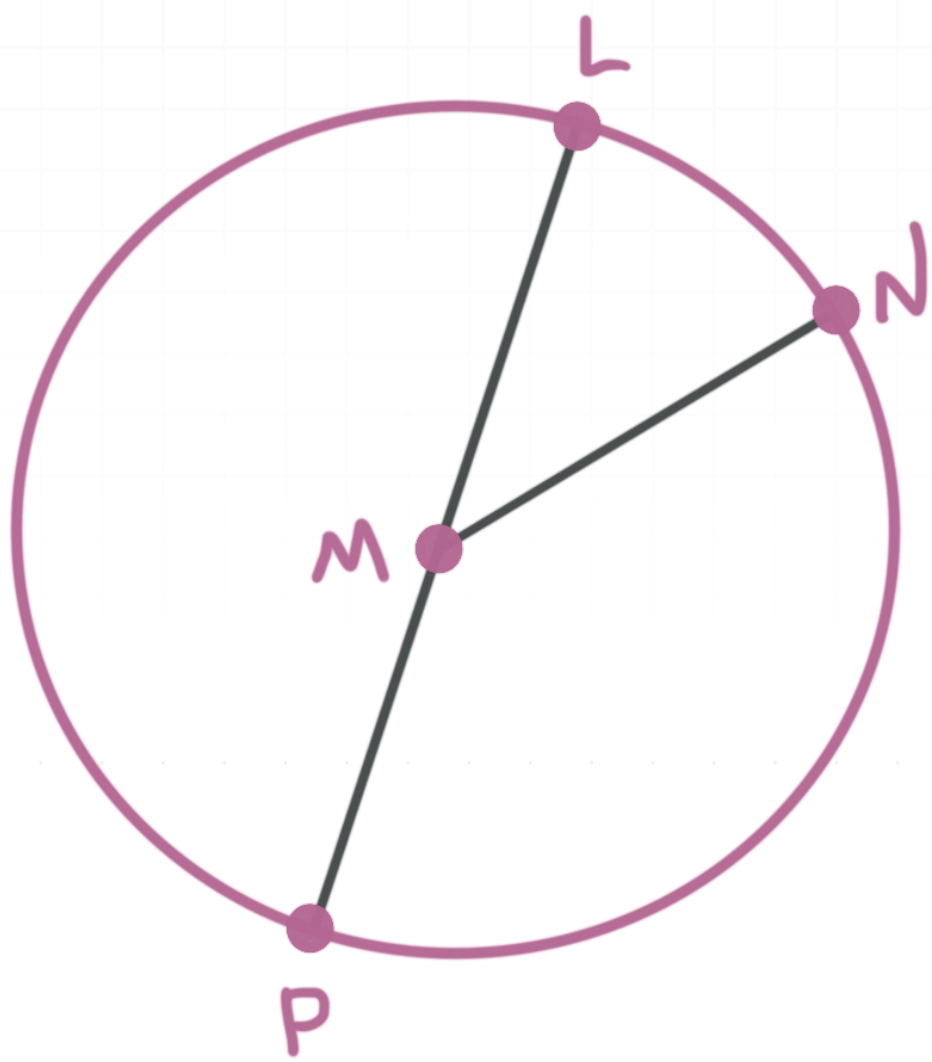
$$C = 2\pi(14)$$

$$C = 87.96$$

Then the measure of arc AB is 50°. The arc length of AB is therefore

$$\frac{50^\circ}{360^\circ}(87.96) = 12.2$$

■ 2. In $\odot M$, $m\angle LMN = 60^\circ$ and $\overline{LP}$ is a diameter. Find the length of arc LPN if $LP = 24$. Round your answer to the nearest hundredth.



Solution:

The measure of arc LN is 60°, so the measure of arc LPN is $360^\circ - 60^\circ = 300^\circ$. The circumference is

$$C = 2\pi r$$

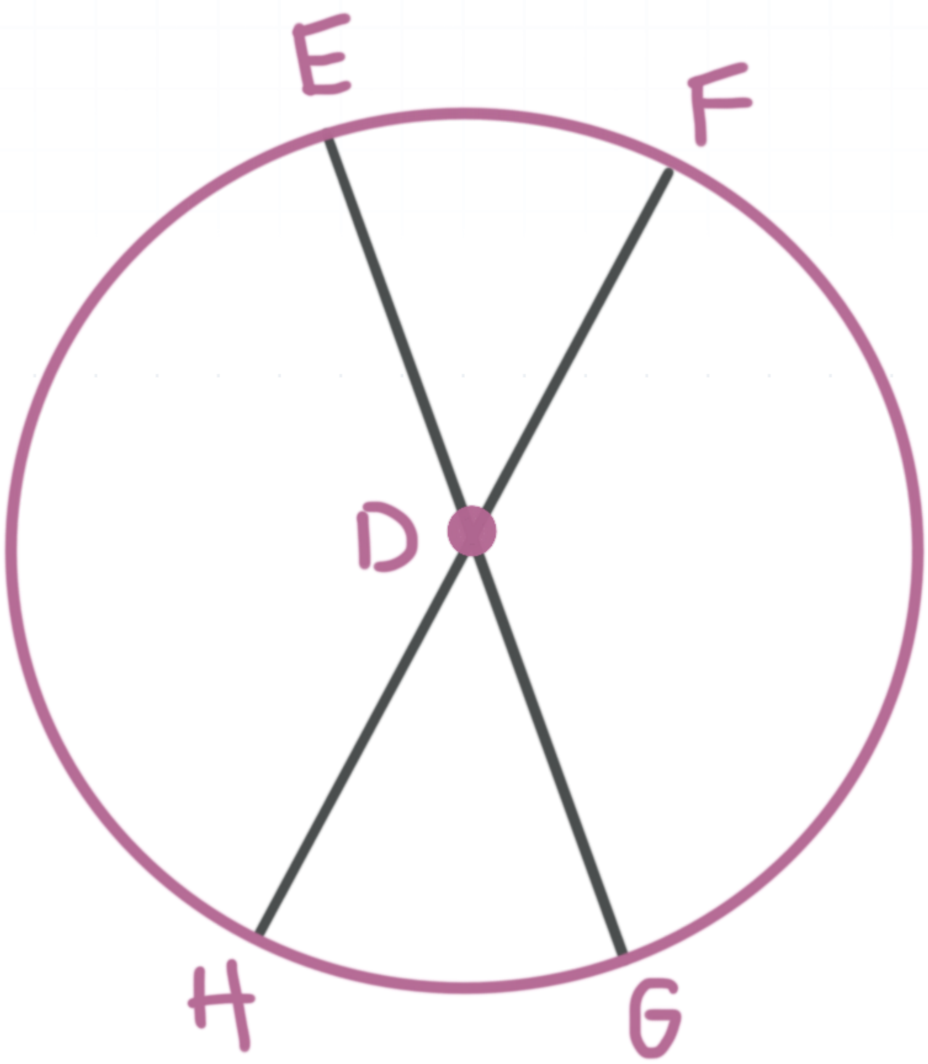
$$C = 24\pi$$

$$C = 75.40$$

So the arc length of LPN is

$$\frac{300^\circ}{360^\circ}(75.40) = 62.83$$

■ 3. $\overline{EG}$ and $\overline{FH}$ are diameters of $\odot D$. Find the length of arc HG if $m\angle EDF = 45^\circ$ and $ED = 16$. Write the exact value.



Solution:

The circumference of a circle is

$$C = 2\pi r$$

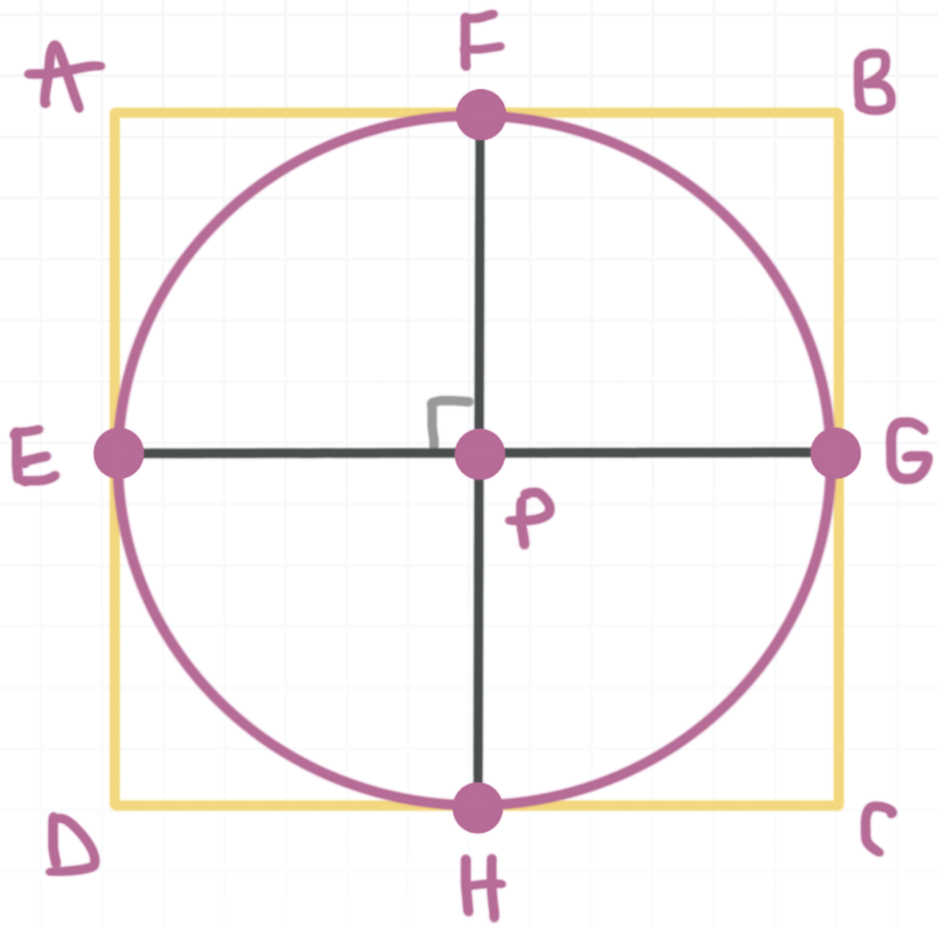
$$C = 2\pi(16)$$

$$C = 32\pi$$

Arcs EF and HG both have a measure of 45° since $\angle EDF$ and $\angle HDG$ are vertical angles. Therefore, the length of arc HG is

$$\frac{45^\circ}{360^\circ}(32)\pi = 4\pi$$

■ 4. The area of square $ABCD$ is 144 cm^2 and circle P is inscribed in the square. $\overline{EG}$ and $\overline{FH}$ are perpendicular to one another, and both are diameters of $\odot P$. E, F, G, and H are midpoints of each side of the square. Find the length of arc EF, rounded to the nearest hundredth.



Solution:

The length of each side of the square is $\sqrt{144} = 12$. Because $\odot P$ is inscribed in square $ABCD$, the diameter of the circle is 12. The circumference is

$$C = 2\pi r$$

$$C = 12\pi$$

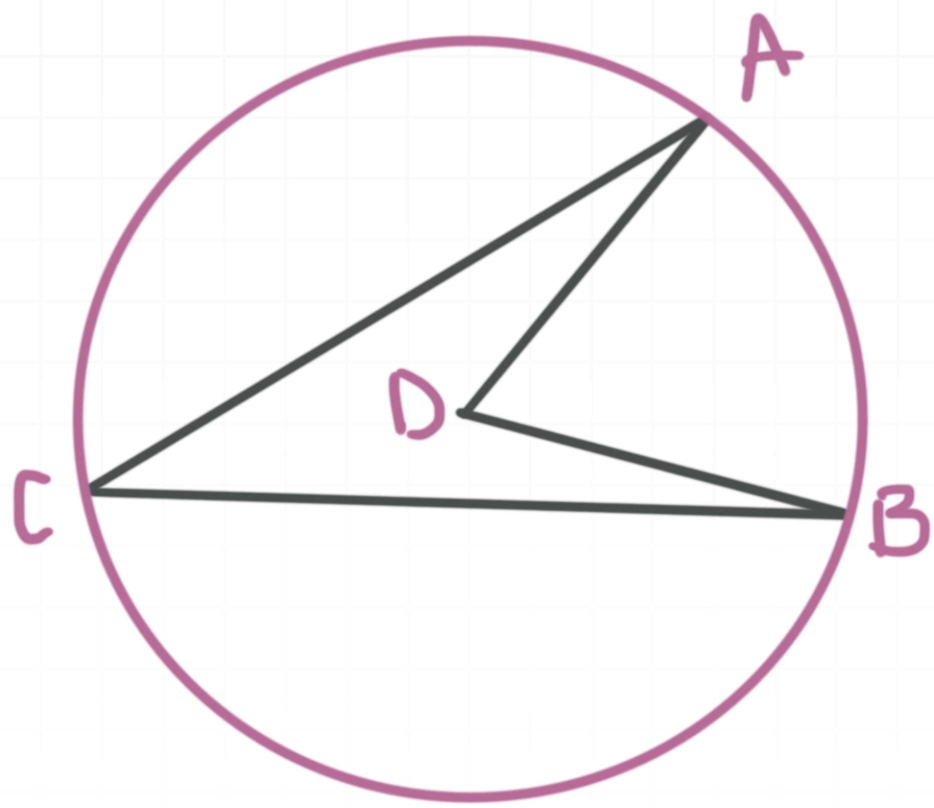
$$C = 37.70$$

$m\angle EPF = 90^\circ$. So the measure of arc EF is also 90°. So the length of arc EF is

$$\frac{90^\circ}{360^\circ}(37.70) = 9.43$$

INSCRIBED ANGLES OF CIRCLES

■ 1. In $\odot D$, $m\angle ADB = 88^\circ$. Find $m\angle ACB$.

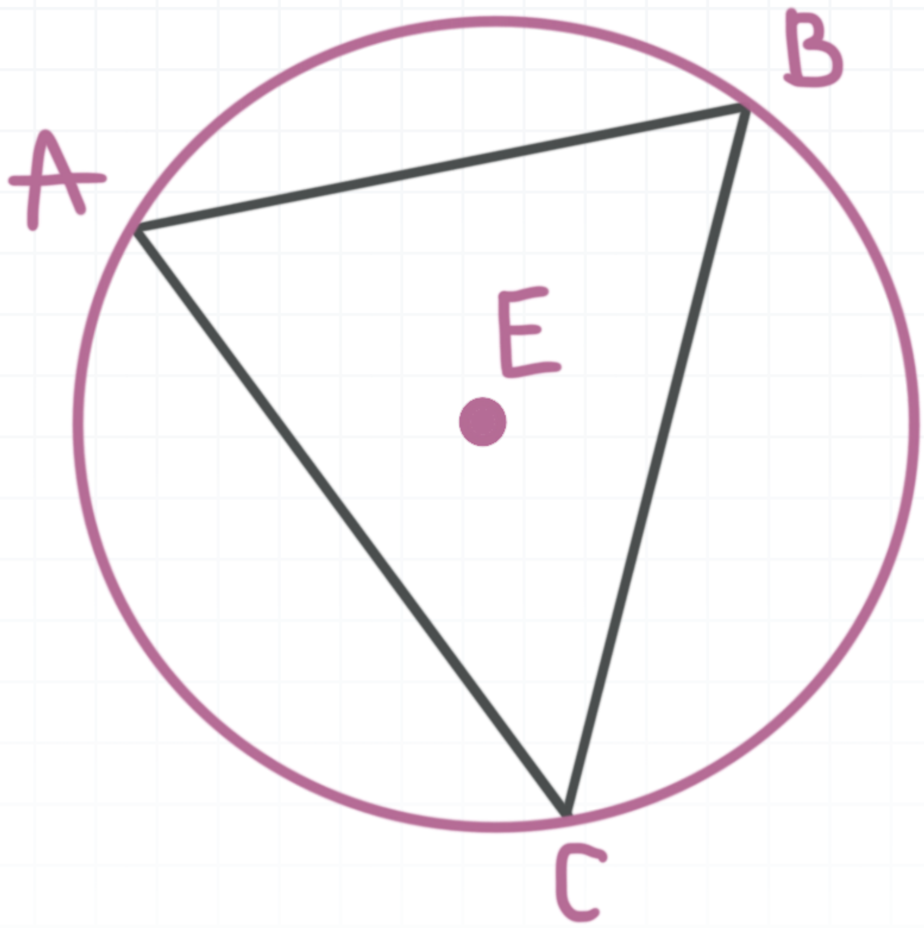


Solution:

$m\angle ADB = 88^\circ$, which means the measure of arc AB is 88°. Which means $m\angle ACB$ is

$$\angle ACB = \frac{1}{2}88^\circ = 44^\circ$$

■ 2. In $\odot E$, $\overline{AC} \cong \overline{CB}$ and $m\angle ABC = 55^\circ$. Find the measure of arc AB.



Solution:

$\triangle ABC$ is isosceles with $m\angle A = m\angle B = 55^\circ$.

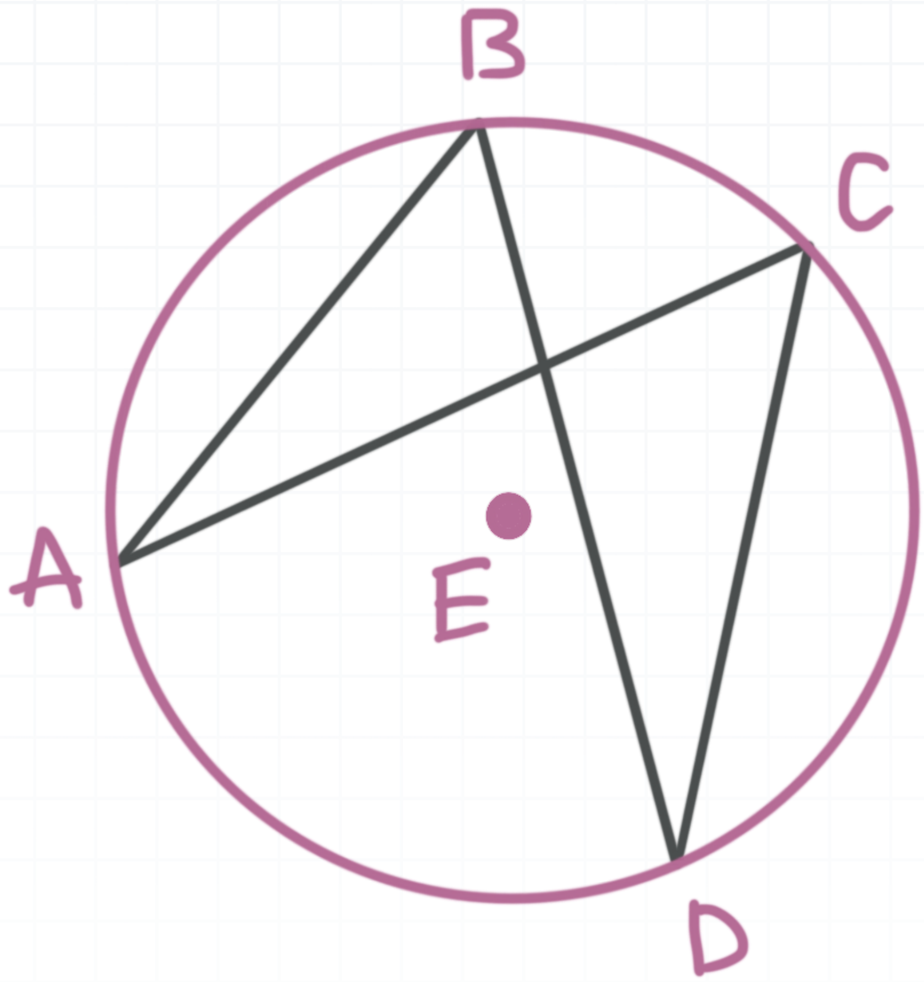
$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$55^\circ + 55^\circ + m\angle C = 180^\circ$$

$$m\angle C = 70^\circ$$

Then the measure of arc AB is $2m\angle C = 2(70^\circ) = 140^\circ$.

■ 3. In $\odot E$ the measure of arc AB is 100°, the measure of arc BC is 40°, and the measure of CD is 110°. Find $m\angle ABD$.



Solution:

Arcs AB, BC, CD, and DA form the full circle, so they sum to 360°.

$$AB + BC + CD + DA = 360^\circ$$

$$100^\circ + 40^\circ + 110^\circ + DA = 360^\circ$$

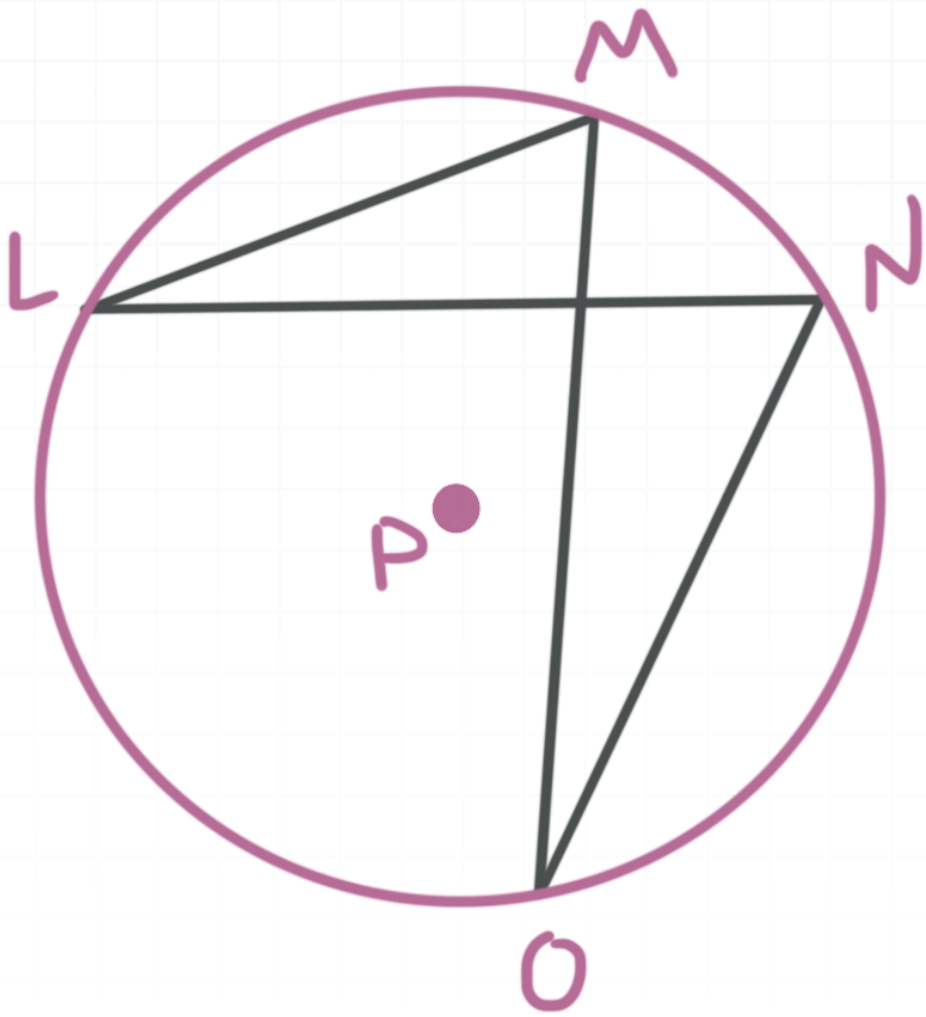
$$250^\circ + DA = 360^\circ$$

$$DA = 110^\circ$$

Therefore, the measure of angle ABD is

$$m\angle ABD = \frac{1}{2}110^\circ = 55^\circ$$

■ 4. In $\odot P$, $m\angle LMO = 2x - 18$ and the measure of arc $LO = 88^\circ$. Find x.



Solution:

The measure of angle LMO, if the measure of arc LO is 88°, is

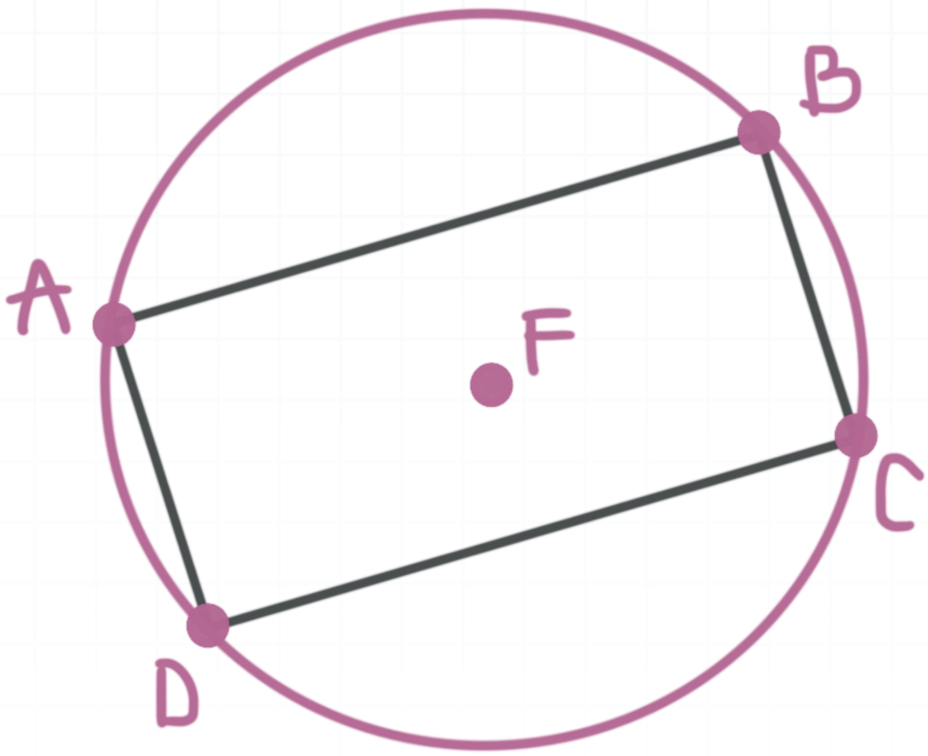
$$\angle LMO = \frac{1}{2}88^\circ$$

$$2x - 18 = \frac{1}{2}88^\circ$$

$$2x - 18 = 44^\circ$$

$$x = 31^\circ$$

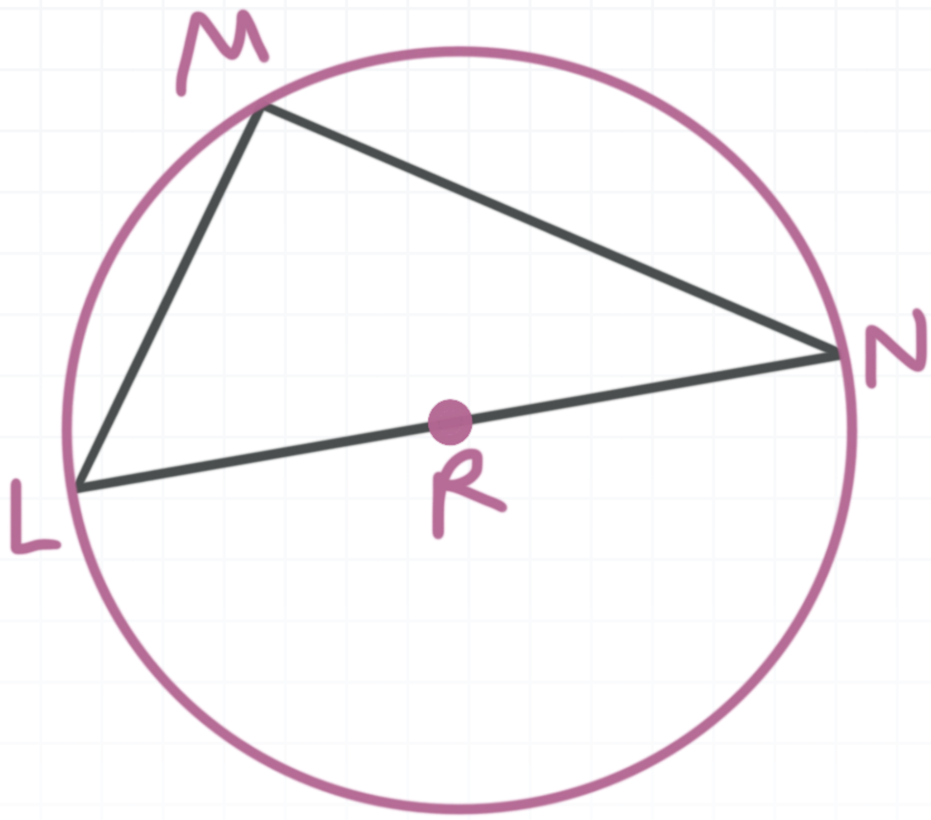
■ 5. Rectangle $ABCD$ is inscribed in $\odot F$ and the measure of arc DAC is 230°. Find the measure of arc AB.



Solution:

All of the angles of the rectangle measure 90°. AC and BD must be diameters of the circle. So the measure of arc DC is $360^\circ - 230^\circ = 130^\circ$. Arcs AB and DC are congruent, so they both have a measure of 130°.

■ 6. In $\odot R$, $\overline{LN}$ is a diameter, $m\angle MLN = 4x + 20$, and $m\angle LNM = 5x - 38$. Find the measure of arc LM.



Solution:

We know that

$$m\angle LMN = \frac{1}{2}(180^\circ) = 90^\circ$$

And since the three interior angles of a triangle always sum to 180°,

$$m\angle LMN + m\angle LNM + m\angle MLN = 180^\circ$$

$$90^\circ + 4x + 20 + 5x - 38 = 180^\circ$$

$$9x + 72 = 180^\circ$$

$$x = 12$$

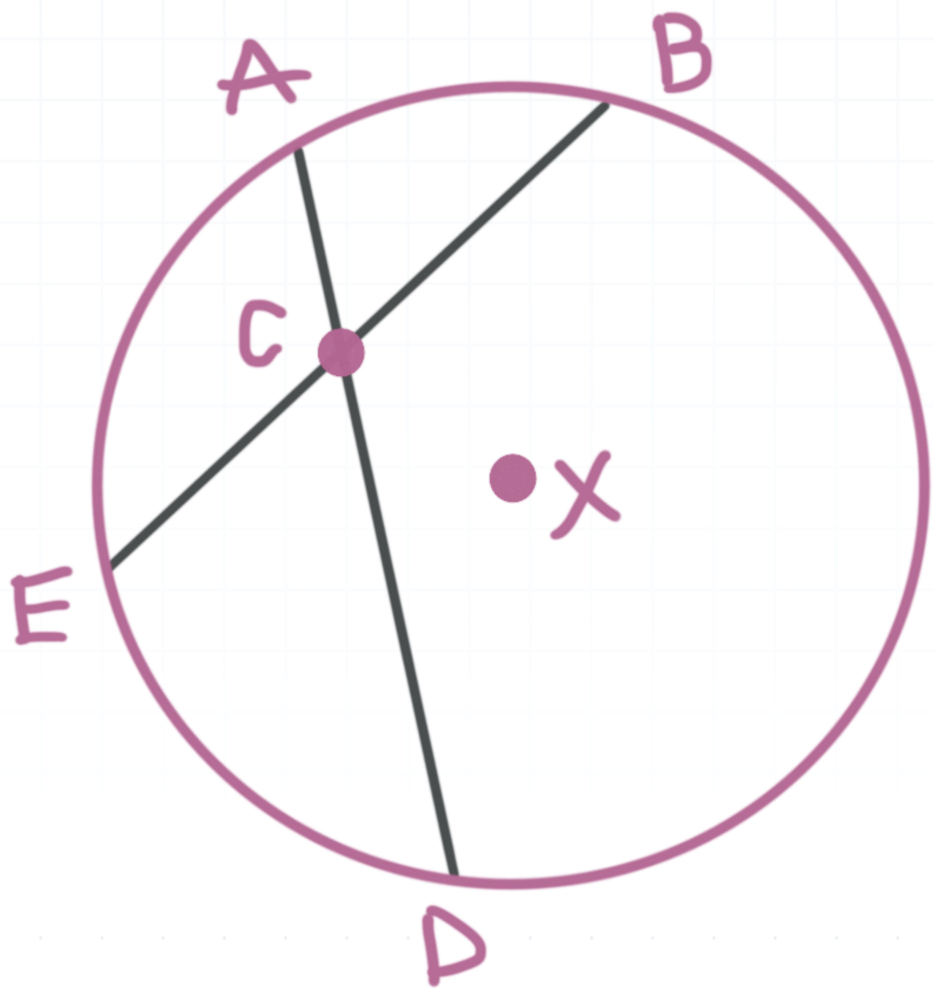
Therefore, $m\angle LNM = 5(12) - 38 = 22^\circ$, so

$$22^\circ = \frac{1}{2}(\text{arc } LM)$$

$$LM = 44^\circ$$

VERTEX ON, INSIDE AND OUTSIDE THE CIRCLE

■ 1. $\overline{AD}$ and $\overline{EB}$ are chords of $\odot X$. The measure of arc AB is 35° and the measure of arc ED is 85°. Find $m\angle ECD$.



Solution:

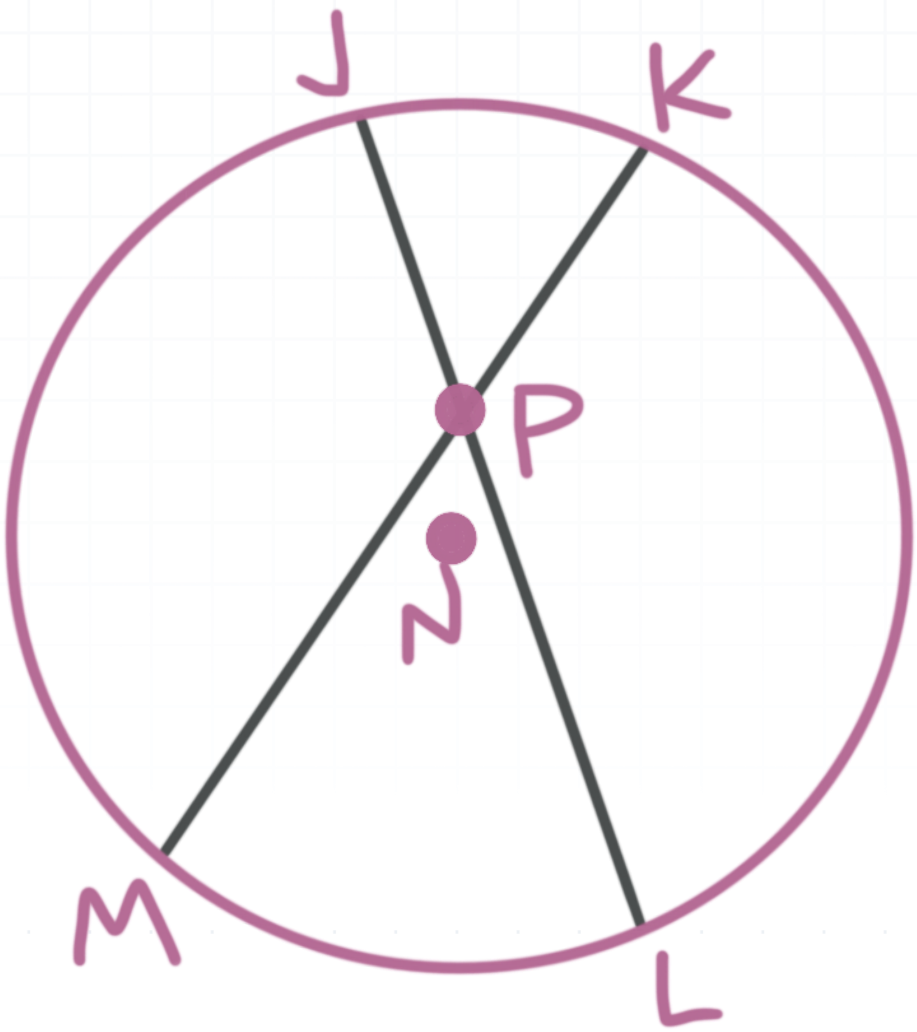
$m\angle ECD$ is given by

$$m\angle ECD = \frac{1}{2}(\text{arc } ED + \text{arc } AB)$$

$$m\angle ECD = \frac{1}{2}(85^\circ + 35^\circ)$$

$$m\angle ECD = 60^\circ$$

■ 2. $\overline{JL}$ and $\overline{KM}$ are chords of $\odot N$. The measure of arc JK is 25° and $m\angle JPK = 40^\circ$. Find the measure of arc ML.



Solution:

The measure of $\angle JPK$ is given by

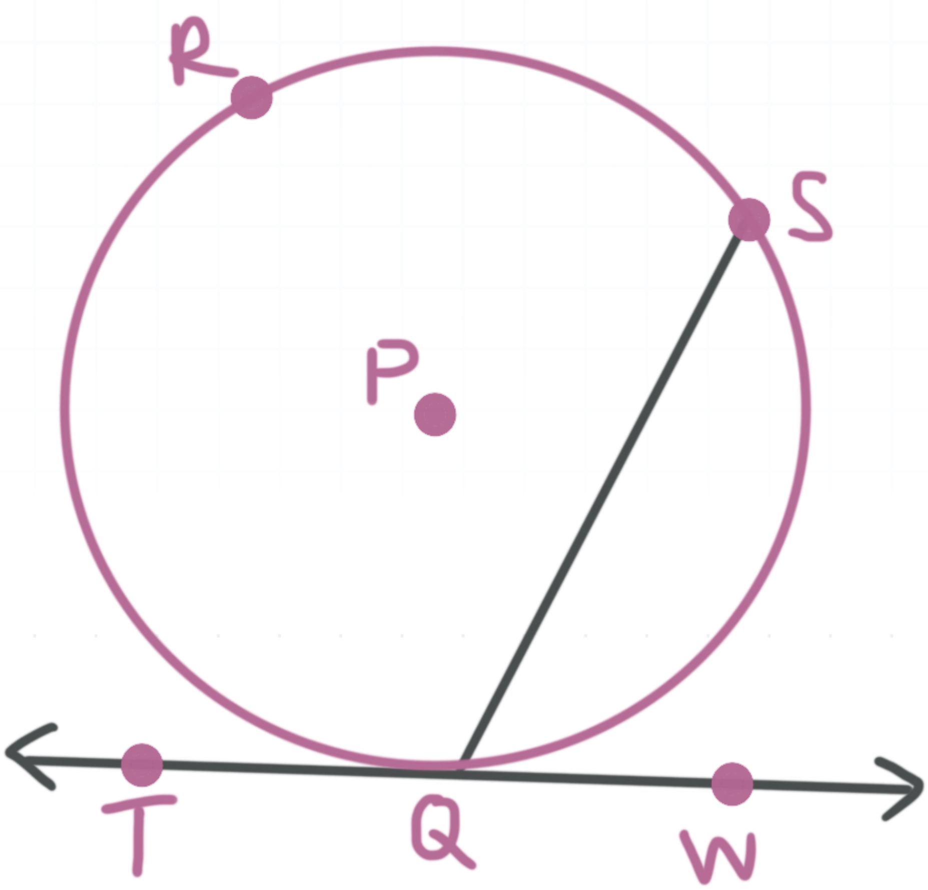
$$m\angle JPK = \frac{1}{2}(\text{arc } ML + \text{arc } JK)$$

$$40^\circ = \frac{1}{2}(\text{arc } ML + 25^\circ)$$

$$80^\circ = \text{arc } ML + 25^\circ$$

$$\text{arc } ML = 55^\circ$$

■ 3. $\overline{SQ}$ is a chord and $\overline{TW}$ is a tangent line of $\odot P$. The measure of arc SRQ is 194°. Find $m\angle SQW$.



Solution:

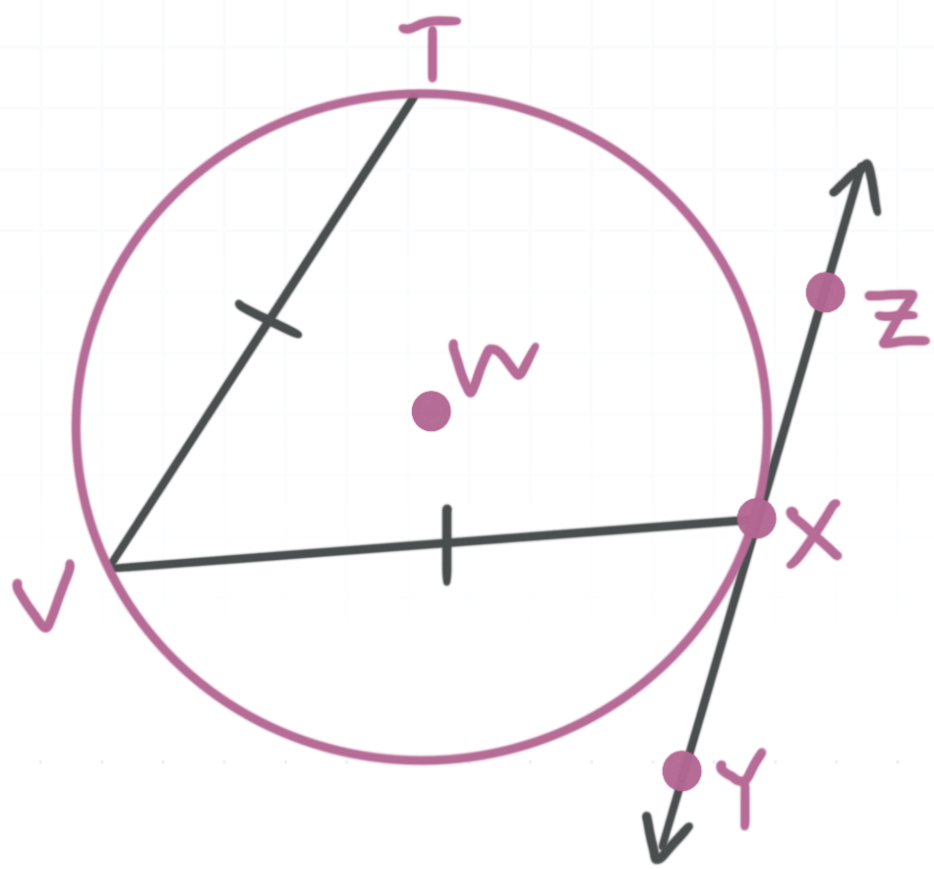
We can find $m\angle SQW$ as

$$m\angle SQW = \frac{1}{2}(\text{arc } SQ)$$

arc $SQ = 360^\circ -$ arc $SRQ = 360^\circ - 194^\circ = 166^\circ$

$$m\angle SQW = \frac{1}{2}(166^\circ) = 83^\circ$$

■ 4. $\overline{TV}$ and $\overline{VX}$ are congruent chords, and $\overline{ZY}$ is a tangent line of $\odot W$. If $m\angle TVX = 48^\circ$, find $m\angle VXY$.



Solution:

Find the length of different arcs in the circle.

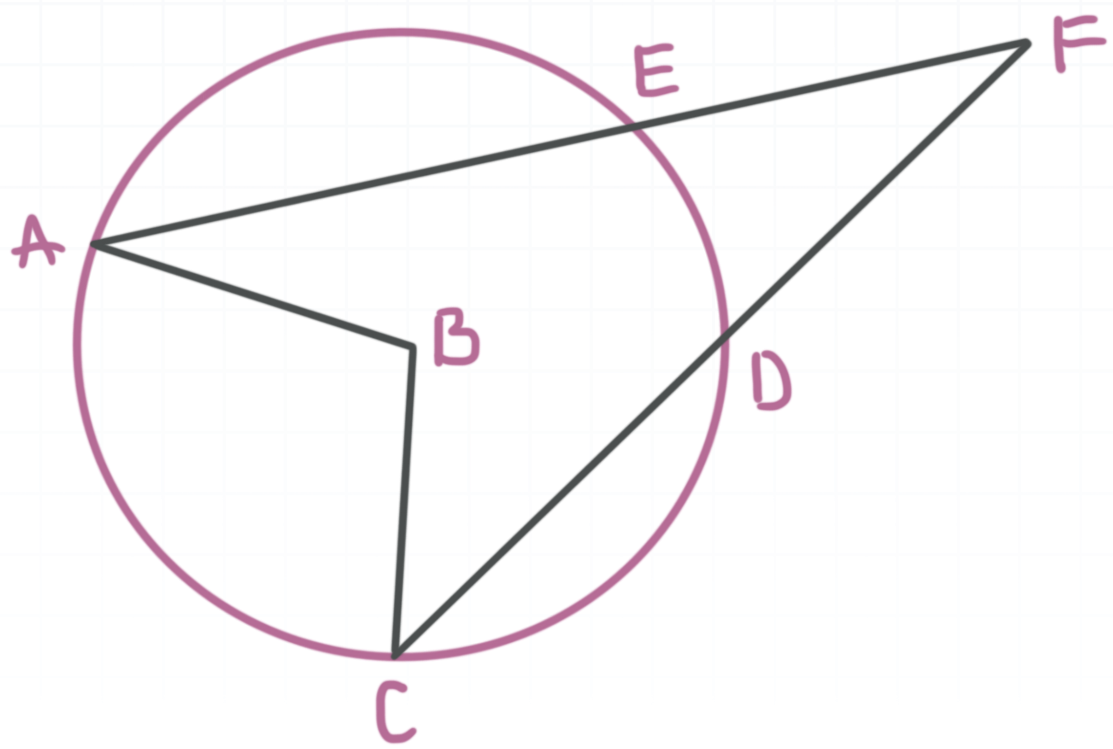
arc $TX = 2(m\angle TVX) = 2(48) = 96$

arc $VX = \frac{1}{2}(360^\circ -$ arc $TX) = \frac{1}{2}(360^\circ - 96^\circ) = 132^\circ$

Then $m\angle VXY$ is given by

$$m\angle VXY = \frac{1}{2}(132^\circ) = 66^\circ$$

■ 5. arc $AC = 98^\circ$ and arc $ED = 54^\circ$ in $\odot B$. Find $m\angle AFC$.



Solution:

We can find $m\angle AFC$ as

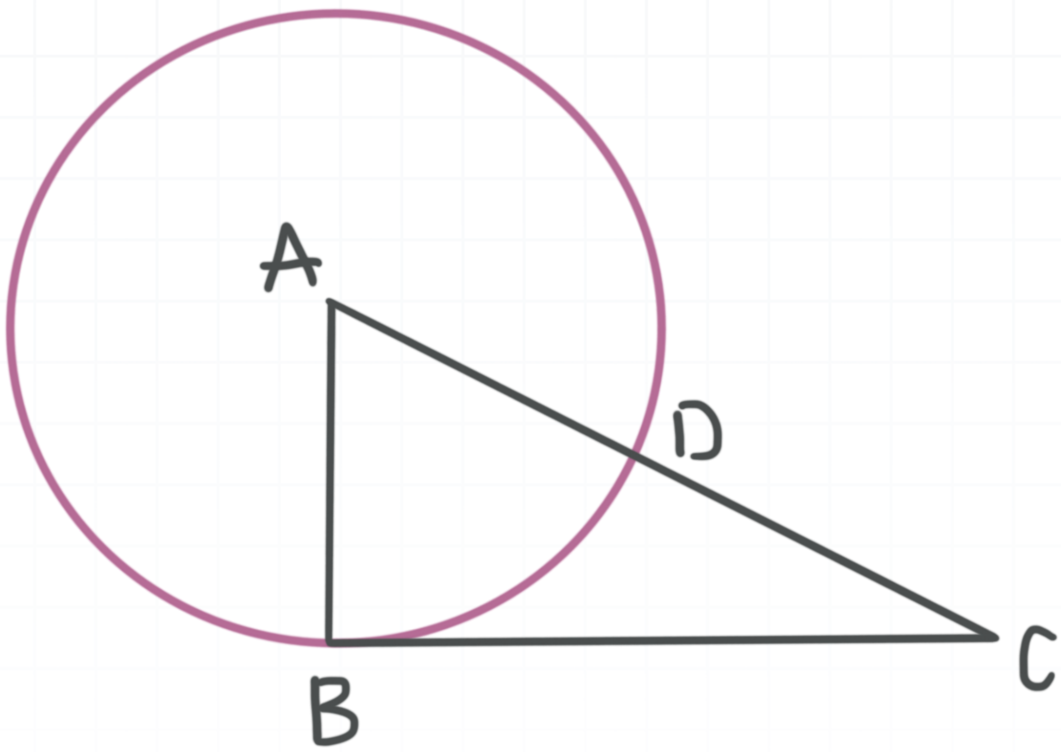
$$m\angle AFC = \frac{1}{2}(\text{arc } AC - \text{arc } ED)$$

$$m\angle AFC = \frac{1}{2}(98^\circ - 54^\circ)$$

$$m\angle AFC = 22^\circ$$

TANGENT LINES OF CIRCLES

1. $\odot A$ has radius AB and tangent line $\overline{BC}$. If $AB = 6$ and $BC = 8$, find DC.



Solution:

We can plug some lengths from the triangle into the Pythagorean Theorem.

$$AB^2 + BC^2 = AC^2$$

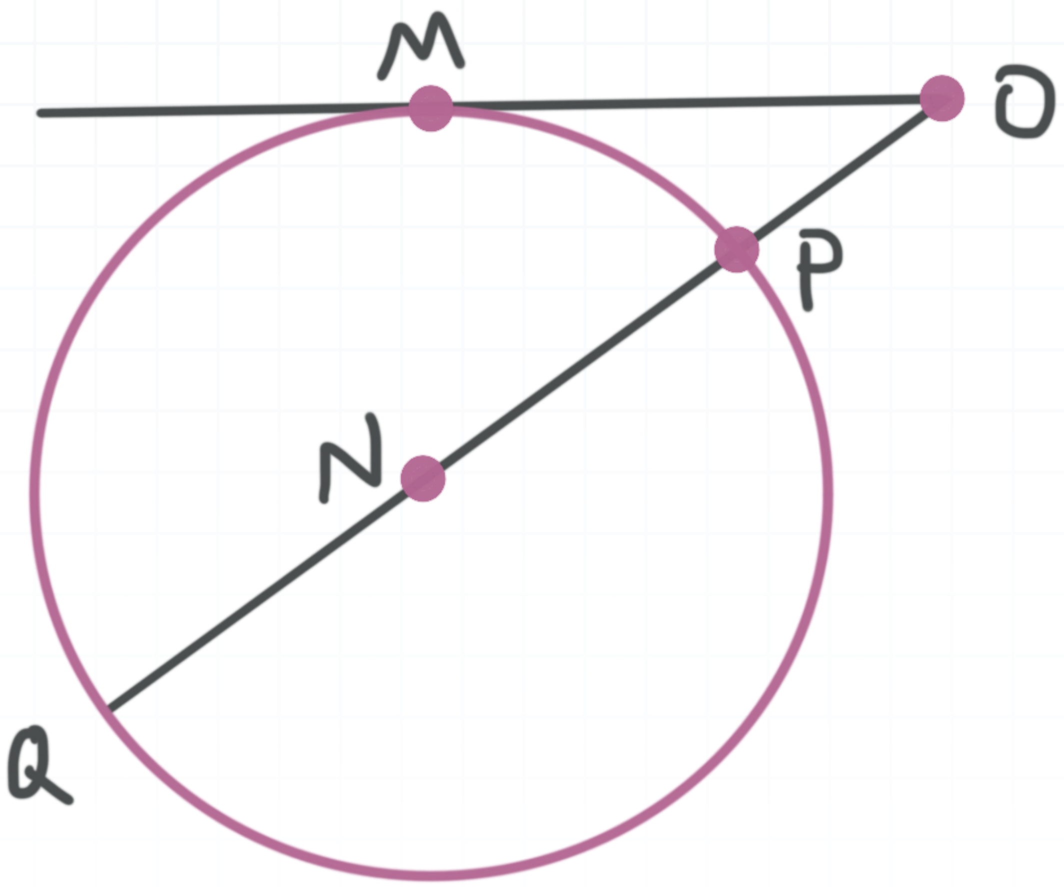
$$6^2 + 8^2 = AC^2$$

$$100 = AC^2$$

$$10 = AC$$

Then $DC = 10 - AD = 10 - 6 = 4$.

■ 2. $\overline{MO}$ is a tangent line of $\odot N$. If $MO = 12$ and $PO = 8$, find the length of the radius.



Solution:

Let the length of the radius be x. Then use the Pythagorean Theorem.

$$MN^2 + MO^2 = ON^2$$

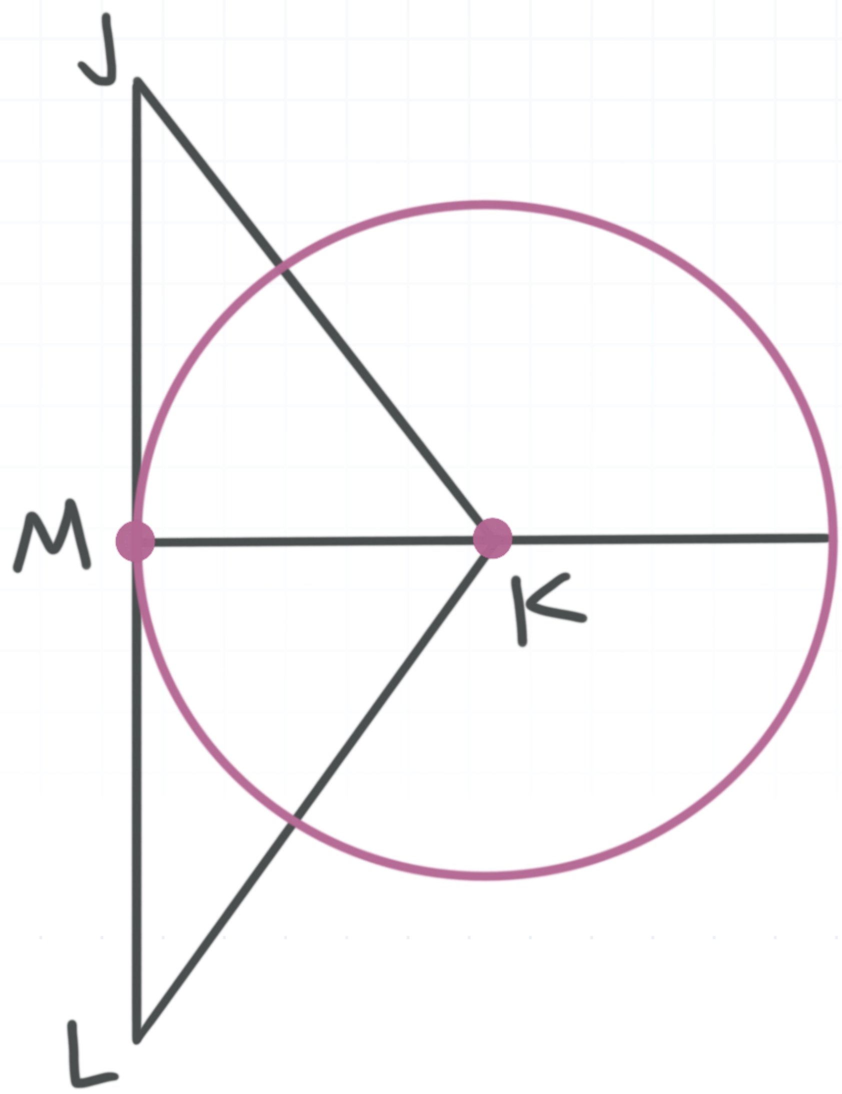
$$x^2 + 12^2 = (x + 8)^2$$

$$x^2 + 144 = x^2 + 16x + 64$$

$$16x = 80$$

$x = 5$

■ 3. $\triangle JKL$ is isosceles, $\overline{JL}$ is a tangent line, $JM = LM$ and $m\angle JKL = 120^\circ$. If $MK = 8$, find JL.



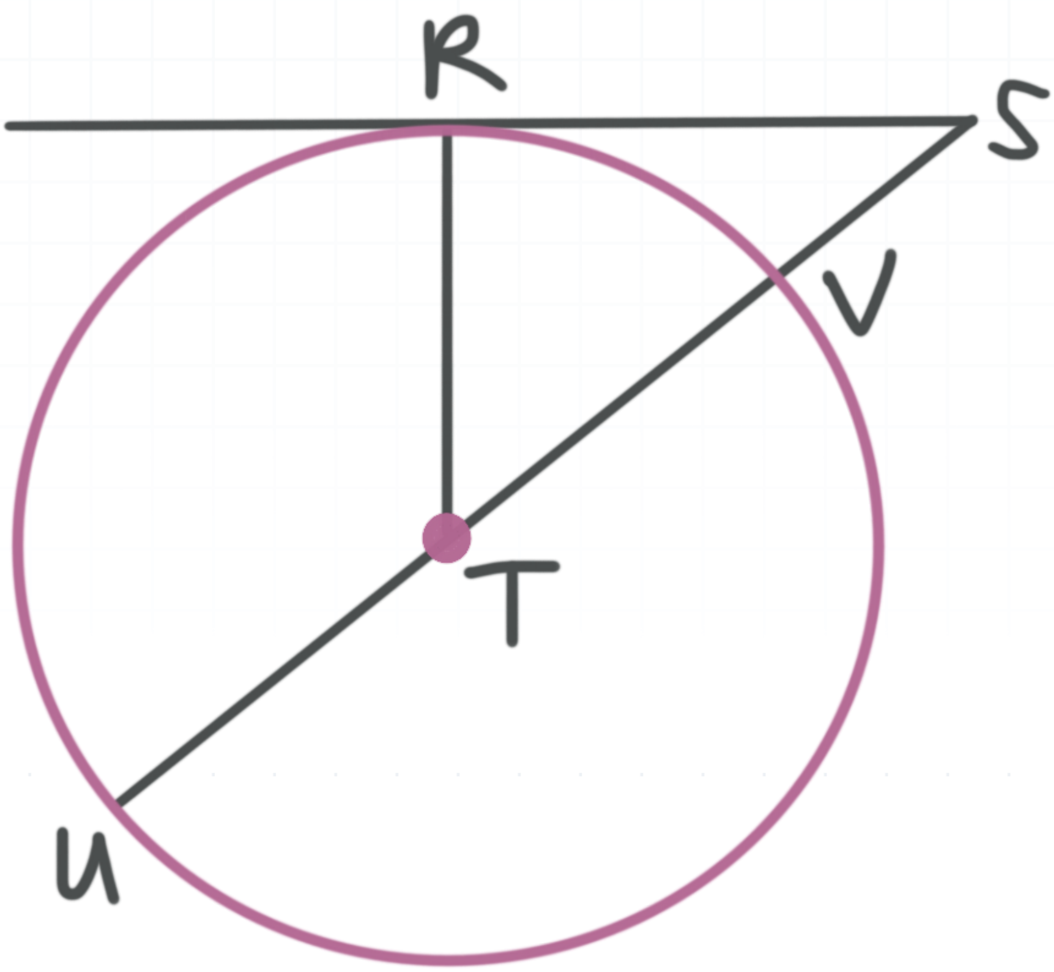
Solution:

The triangles $\triangle JMK$ and $\triangle LMK$ are congruent and $\overline{MK}$ bisects $\angle JKL$. We know $m\angle JKL = 120^\circ$, so $m\angle JKM = m\angle LKM = 60^\circ$. $m\angle KMJ$ must be 90° because $\overline{MK}$ is a radius and $\overline{JL}$ is a tangent line. Use the rules for $30-60-90$ triangles to find JM.

$$JM = MK\sqrt{3} = 8\sqrt{3}$$

$$JL = 2JM = 2\left(8\sqrt{3}\right) = 16\sqrt{3}$$

■ 4. In $\odot T$, $\overline{RS}$ is a tangent line and the diameter $\overline{UV}$ has length of 6. Find VS if $RS = 4$.



Solution:

We know that $TV = RT = 3$ because they are both radii and are half the length of the diameter. From the Pythagorean Theorem,

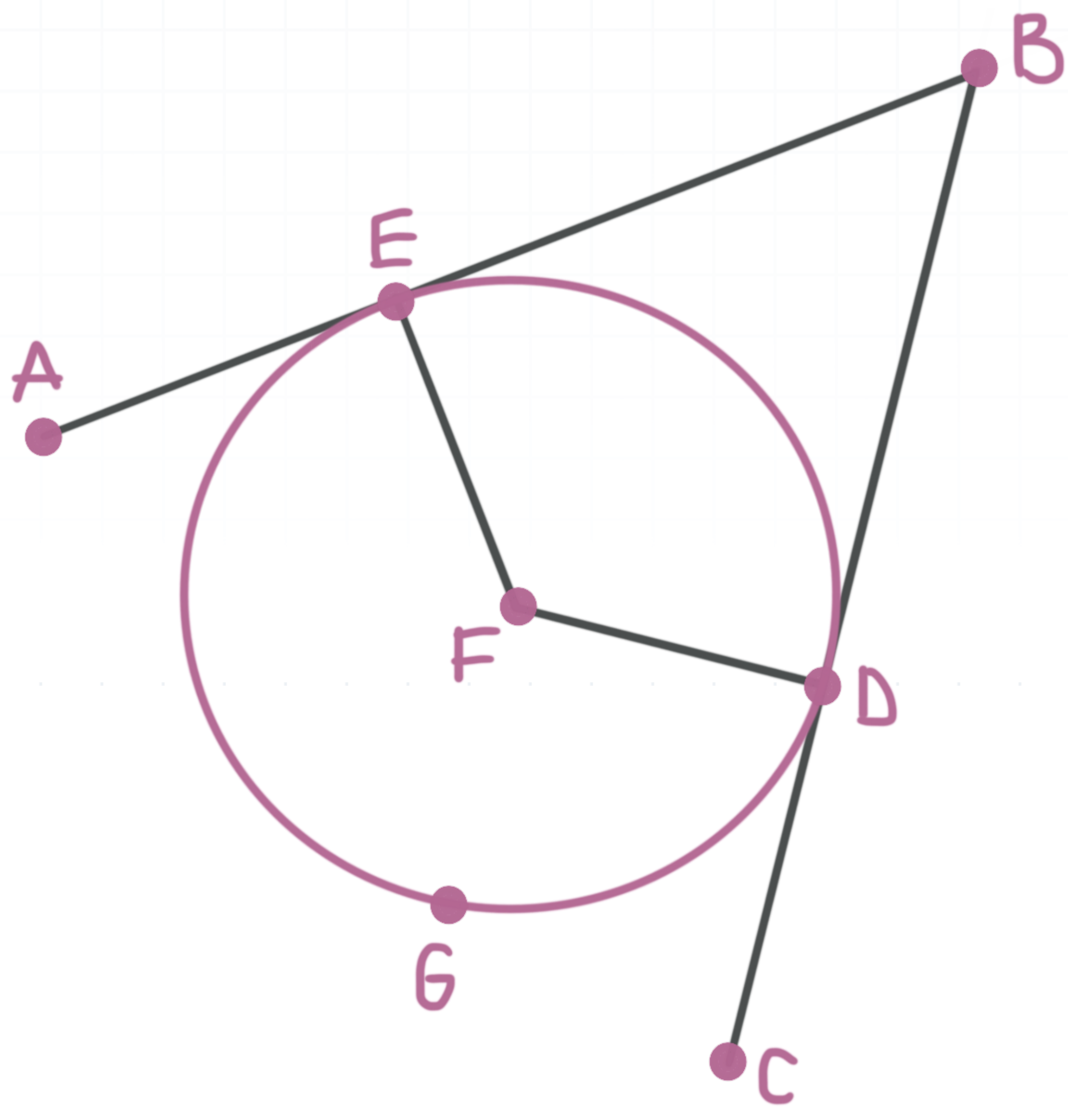
$$RT^2 + RS^2 = TS^2$$

$$3^2 + 4^2 = TS^2$$

$$TS = 5$$

Which means the length of VS is $VS = TS - TV = 5 - 3 = 2$.

■ 5. arc $EGD = 240^\circ$ and $\overline{BF}$ bisects $\angle EFD$. Find the length of the radius of $\odot F$ if $FB = 14$.

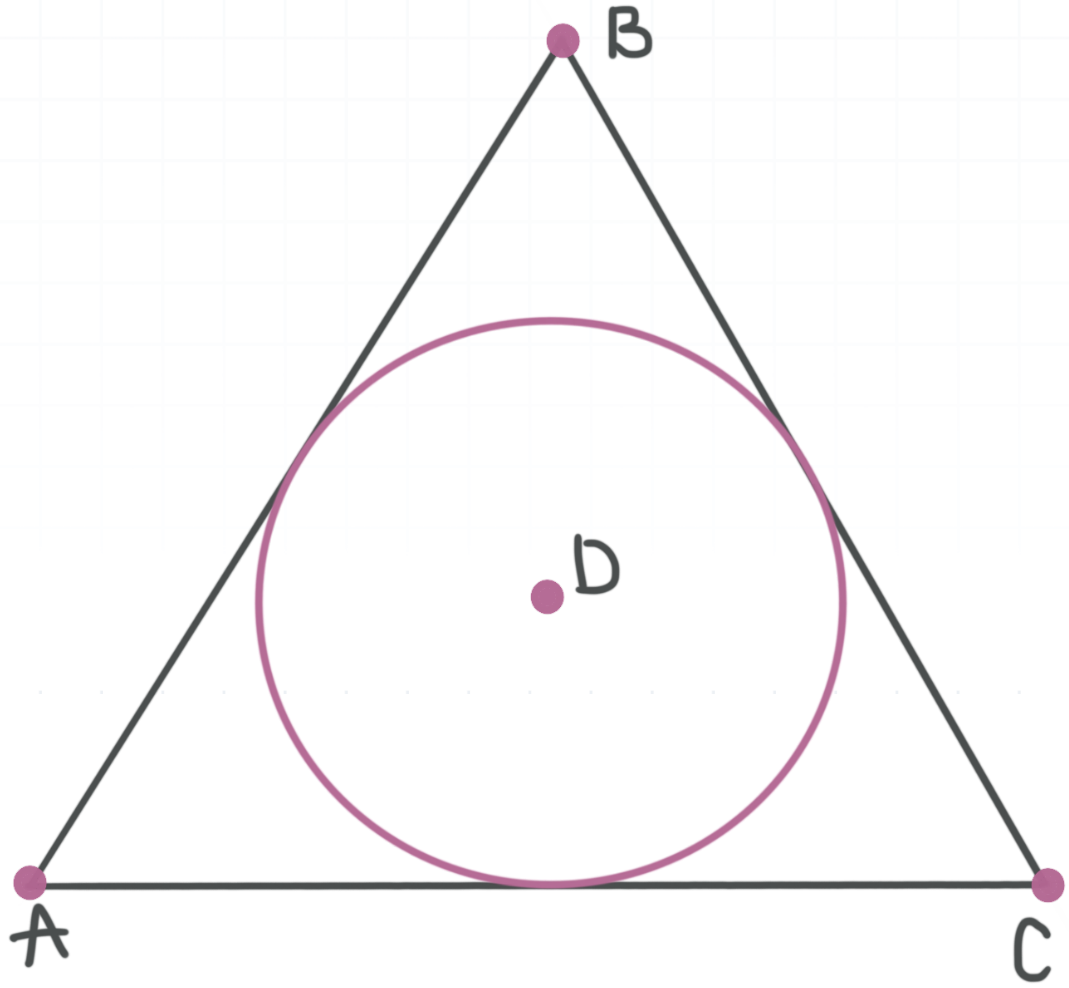


Solution:

We know that $m\angle EFD = 360^\circ - 240^\circ = 120^\circ$.

Draw in $\overline{FB}$ and note $\triangle FEB$ and $\triangle DFB$ are congruent $30-60-90$ triangles. $\overline{FB}$ is a hypotenuse of these triangles and has a measure of 14. $\overline{EF}$ and $\overline{FD}$ are the shortest legs of these triangles and have a measure of $14/2=7$.

■ 6. Find the perimeter of $\triangle ABC$ if the radius of $\odot D$ is 10 feet and $\triangle ABC$ is equilateral.



Solution:

We know that $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are congruent since $\triangle ABC$ is equilateral. Which means $\angle A=\angle B=\angle C=60^\circ$.

Since the diameter is 10, the length of half of each side must be $10\sqrt{3}$ if you draw segments at $\overline{AD}$, $\overline{DB}$, and $\overline{DC}$. Each side must have a length of

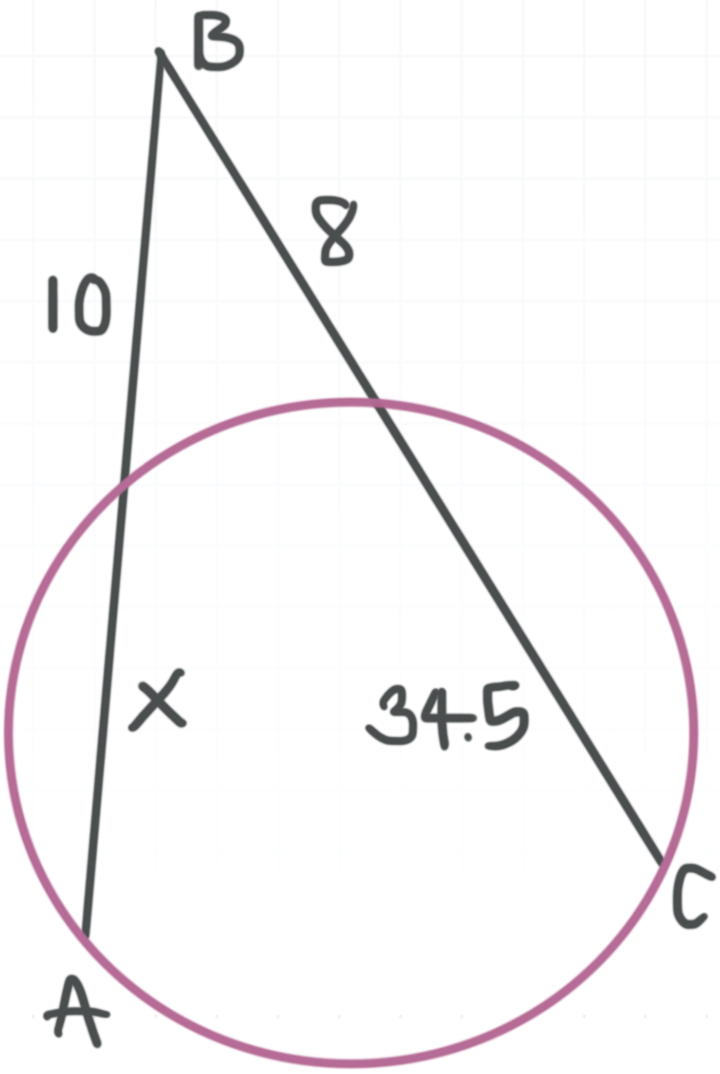
$$2\left(10\sqrt{3}\right)=20\sqrt{3}$$

Find the perimeter by adding up the three sides to get

$$P=20\sqrt{3}+20\sqrt{3}+20\sqrt{3}=60\sqrt{3}$$

INTERSECTING TANGENTS AND SECANTS

■ 1. $\overline{AB}$ and $\overline{CB}$ are secants and intersect at B. Find the value of x.



Solution:

From the figure, we know that

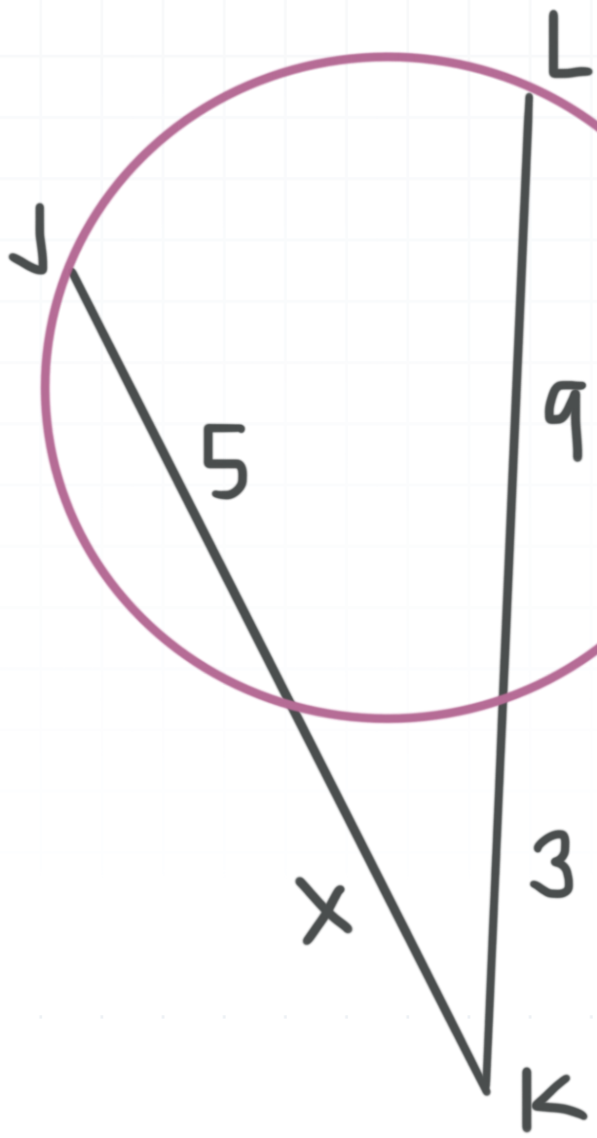
$$10(10 + x) = 8(8 + 34.5)$$

$$100 + 10x = 340$$

$$10x = 240$$

$$x = 24$$

■ 2. $\overline{JK}$ and $\overline{LK}$ are secants and intersect at K. Find the value of x.



Solution:

From the figure, we know that

$$x(x + 5) = 3(3 + 9)$$

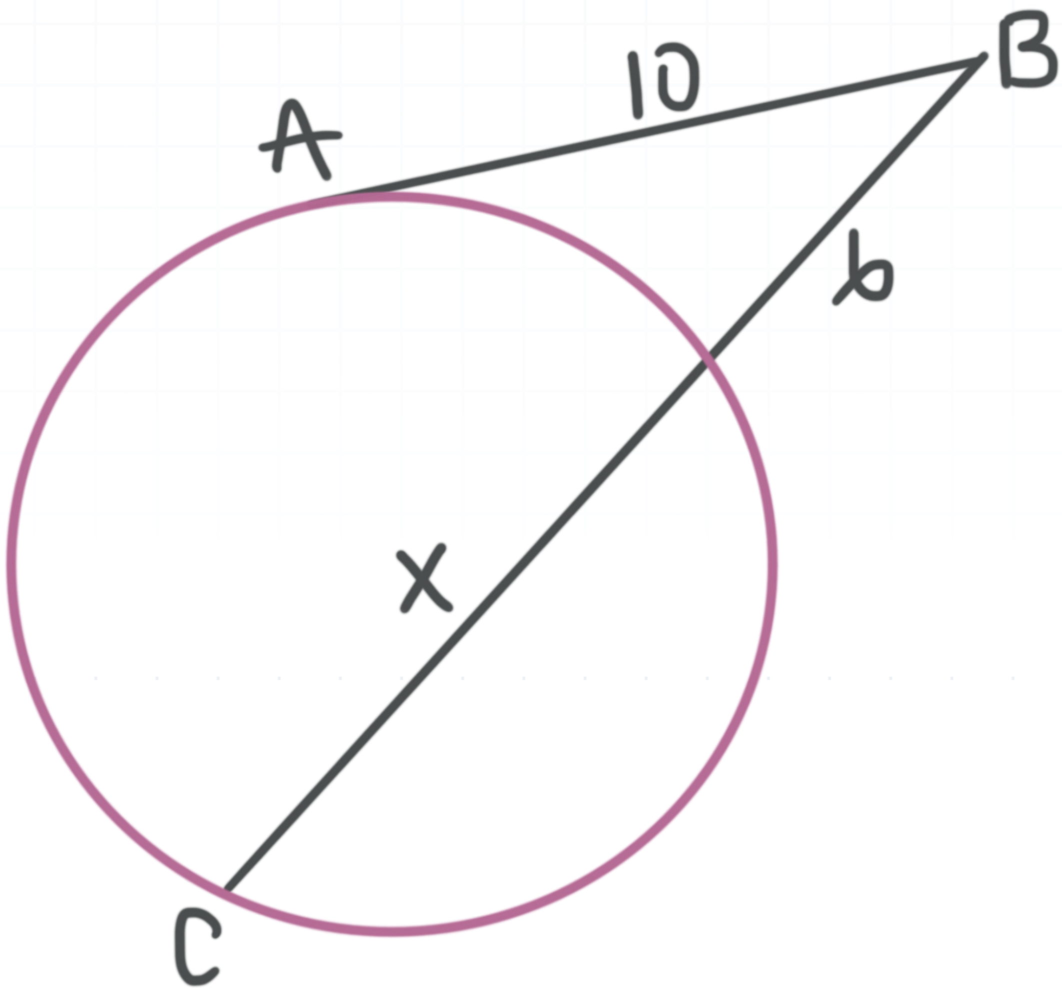
$$x^2 + 5x = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x + 9)(x - 4) = 0$$

Either $x = -9$ or $x = 4$, but x represents a physical distance, which means it can't have a negative value, so $x = 4$.

3. $\overline{AB}$ is a tangent line and $\overline{BC}$ is a secant of the circle. Find the value of x.



Solution:

From the figure, we know that

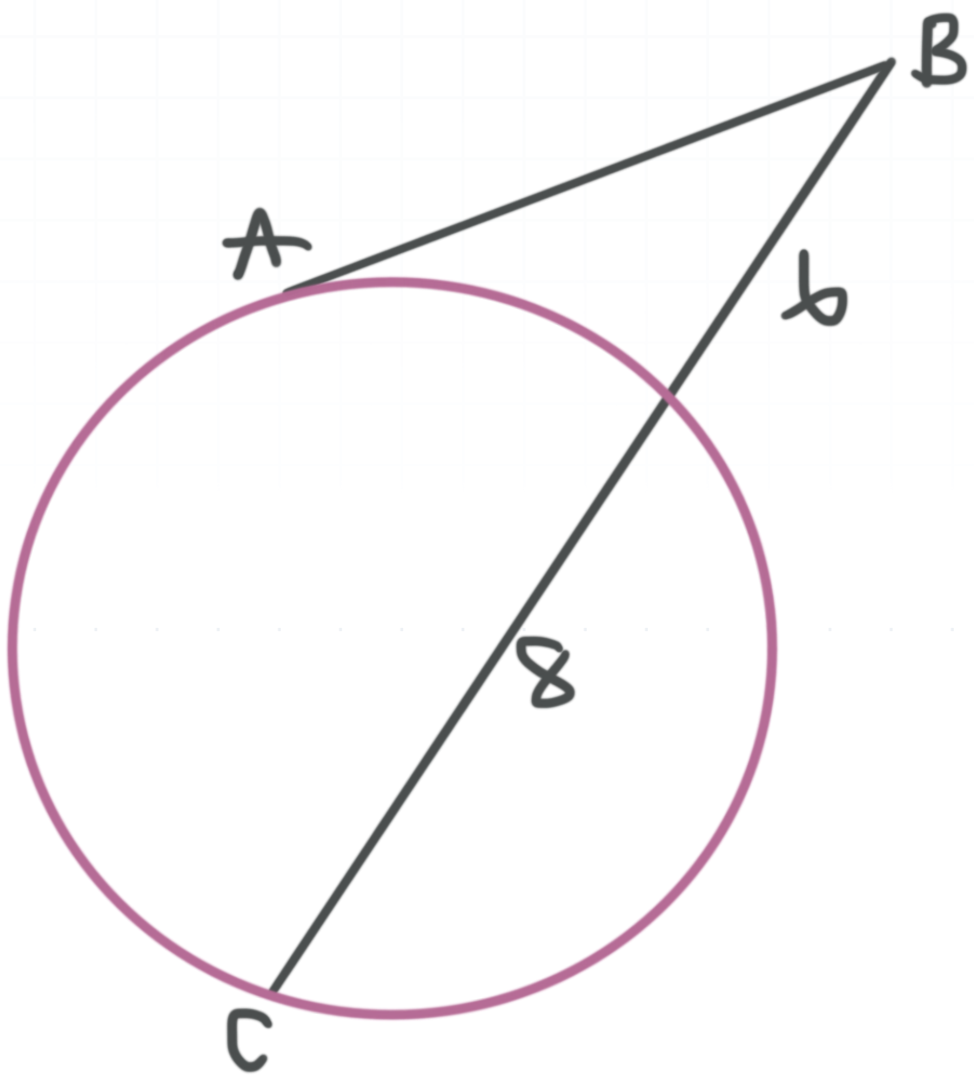
$$10^2 = 6(6 + x)$$

$$100 = 36 + 6x$$

$$6x = 64$$

$$x = \frac{32}{3}$$

■ 4. $\overline{AB}$ is a tangent line and $\overline{CB}$ is a secant of the circle. Find the length of AB.



Solution:

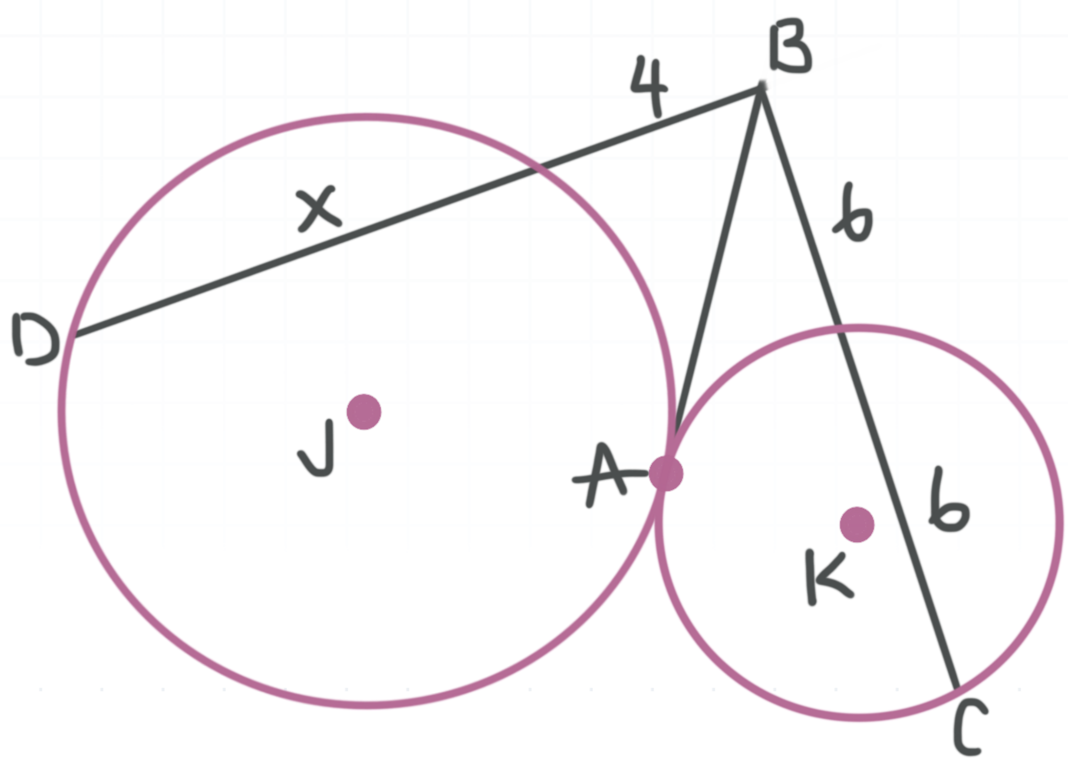
From the figure, the length of AB is

$$x^2 = 6(6 + 8)$$

$$x^2 = 84$$

$$x = \sqrt{84} = 2\sqrt{21}$$

■ 5. $\overline{DB}$ is a secant of $\odot J$ and $\overline{CB}$ is a secant of $\odot K$. $\overline{AB}$ is a tangent for both circles. Find x.



Solution:

From the figure, we know that

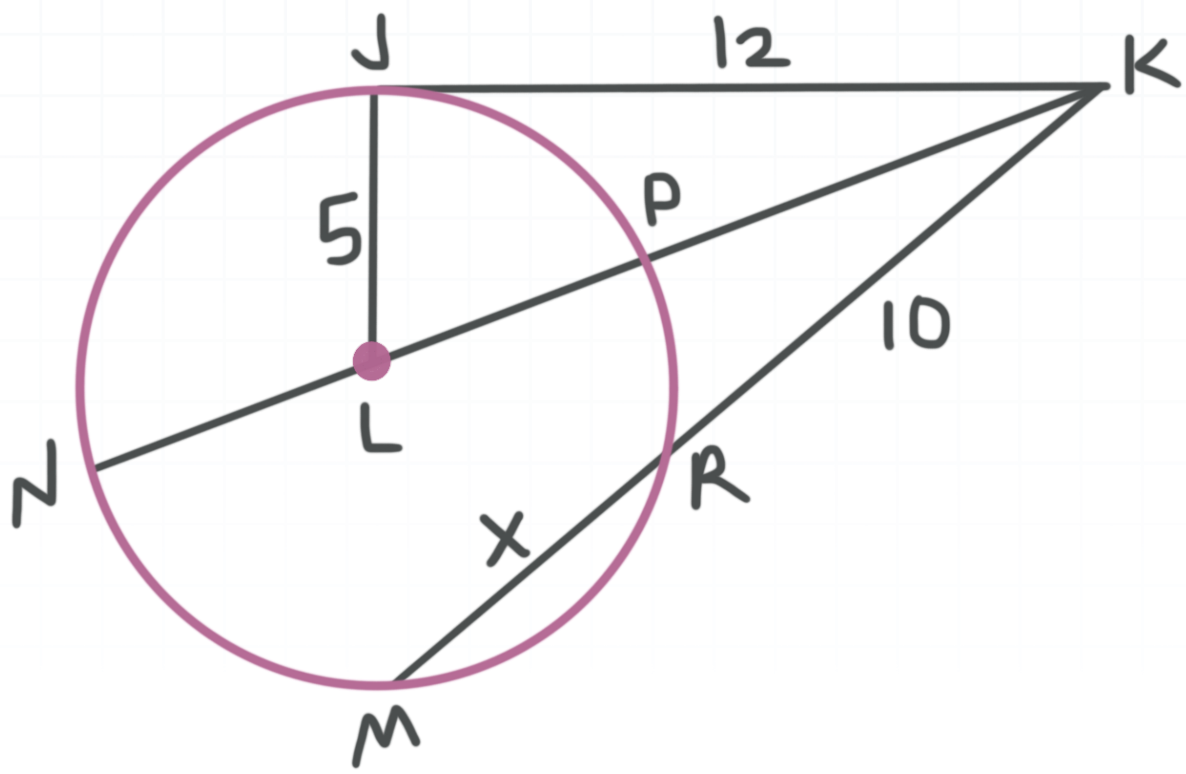
$$AB^2 = 6(6 + 6) = 72$$

$$AB^2 = 4(4 + x)$$

$$72 = 4(4 + x)$$

$$x = 14$$

■ 6. $\overline{JK}$ is a tangent line, $\overline{KN}$ and $\overline{KM}$ are secants, and $\overline{LJ}$ and $\overline{LP}$ are radii of $\odot L$. Find x.



Solution:

We can use the Pythagorean Theorem.

$$LK^2 = JK^2 + LJ^2$$

$$LK^2 = 12^2 + 5^2 = 169$$

$$LK = 13$$

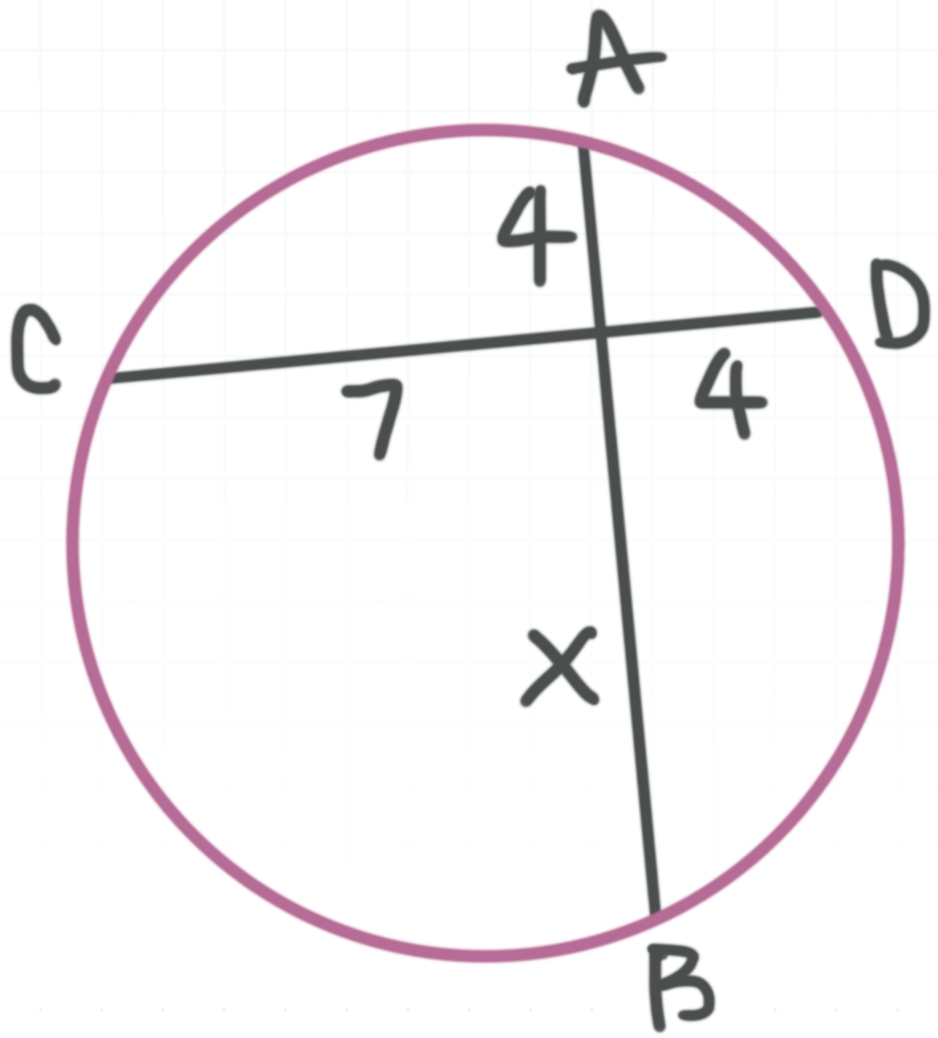
Therefore, $PK = 8$ and $NP = 10$. Which means that we can set up an equation to solve for x.

$$8(8+10) = 10(10+x)$$

$$x = 4.4$$

INTERSECTING CHORDS

1. $\overline{AB}$ and $\overline{CD}$ are intersecting chords of the circle. Find x.



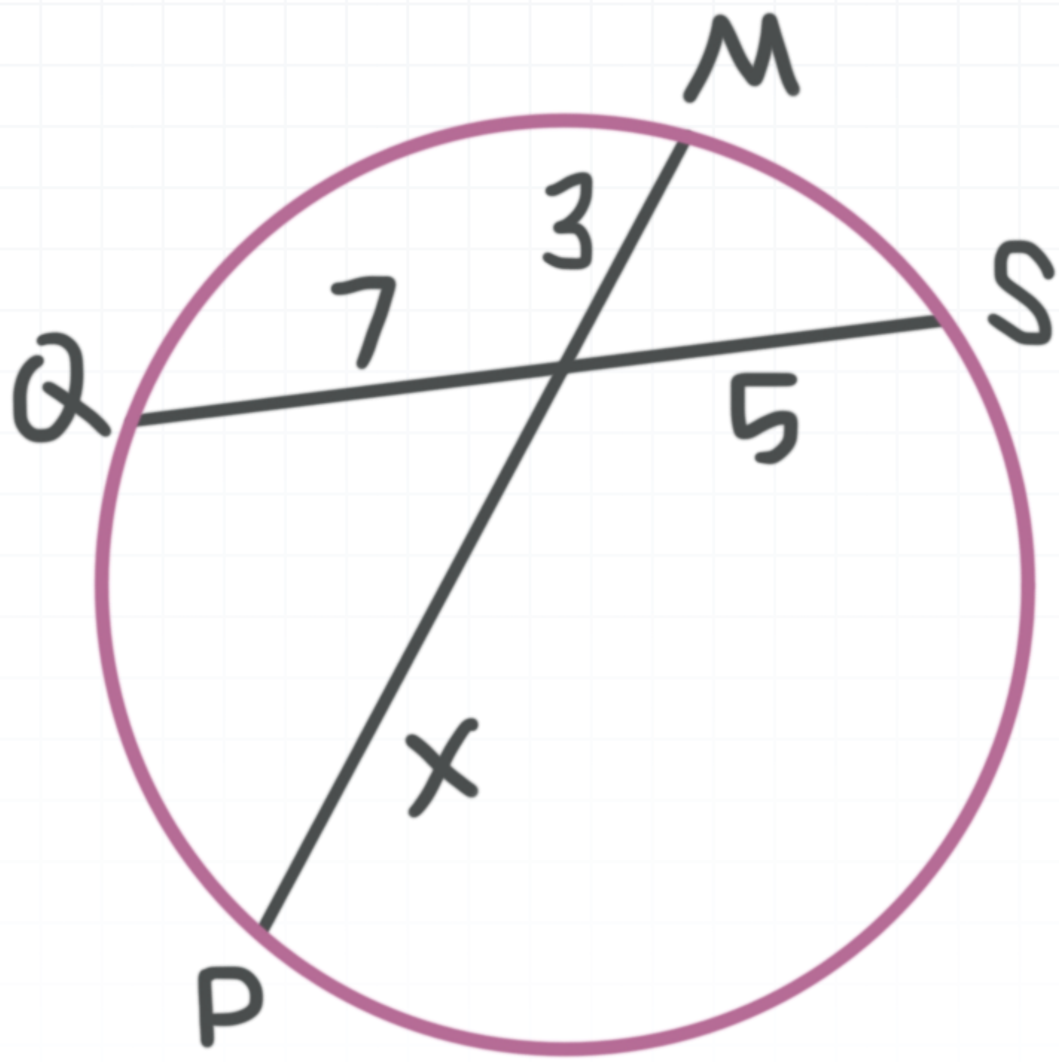
Solution:

From the figure, we know that

$$4x = 7(4)$$

$$x = 7$$

■ 2. $\overline{MP}$ and $\overline{QS}$ are intersecting chords of the circle. Find x.



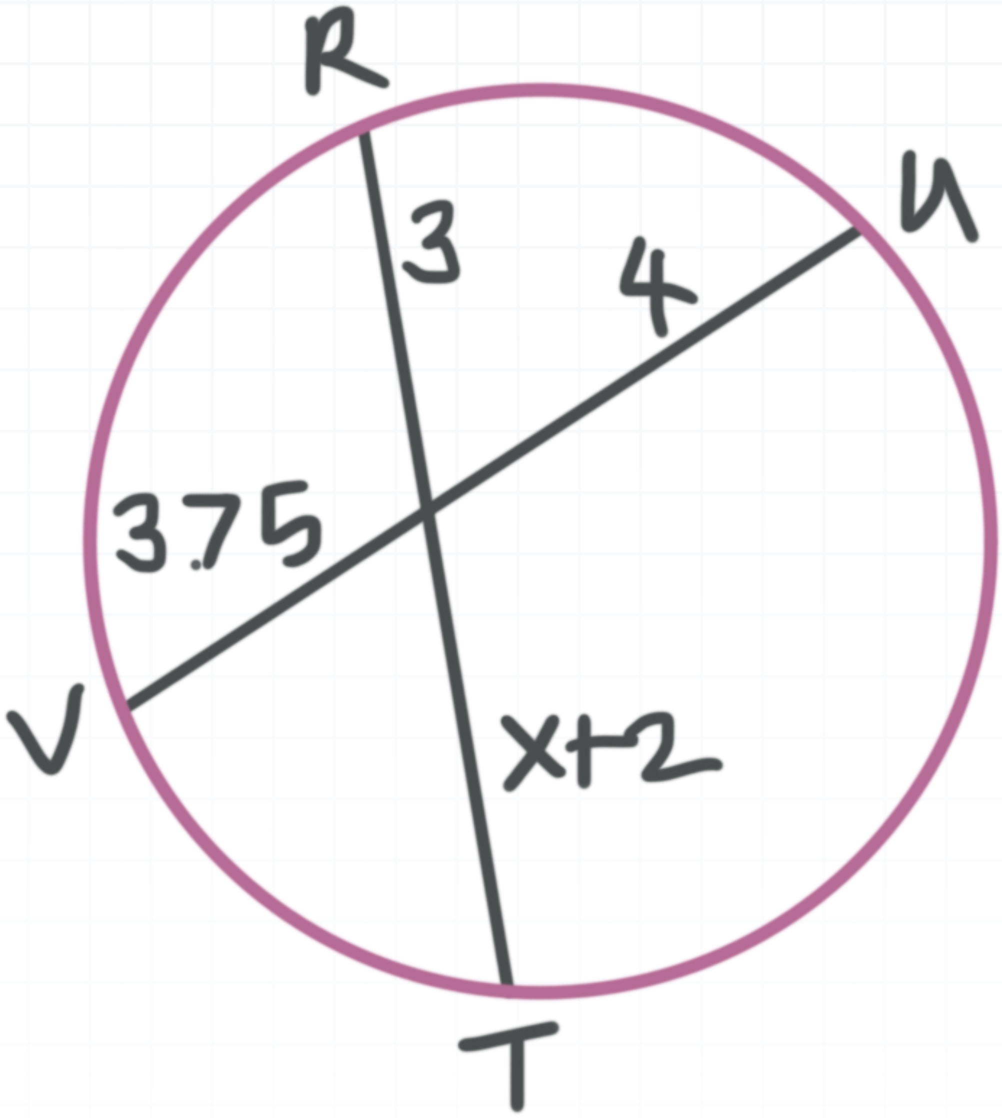
Solution:

From the figure, we know that

$$3x = (5)(7)$$

$$x = \frac{35}{3}$$

■ 3. $\overline{RT}$ and $\overline{UV}$ are intersecting chords of the circle. Find x.



Solution:

From the figure, we can say

$$x(x+2) = (3.75)(4)$$

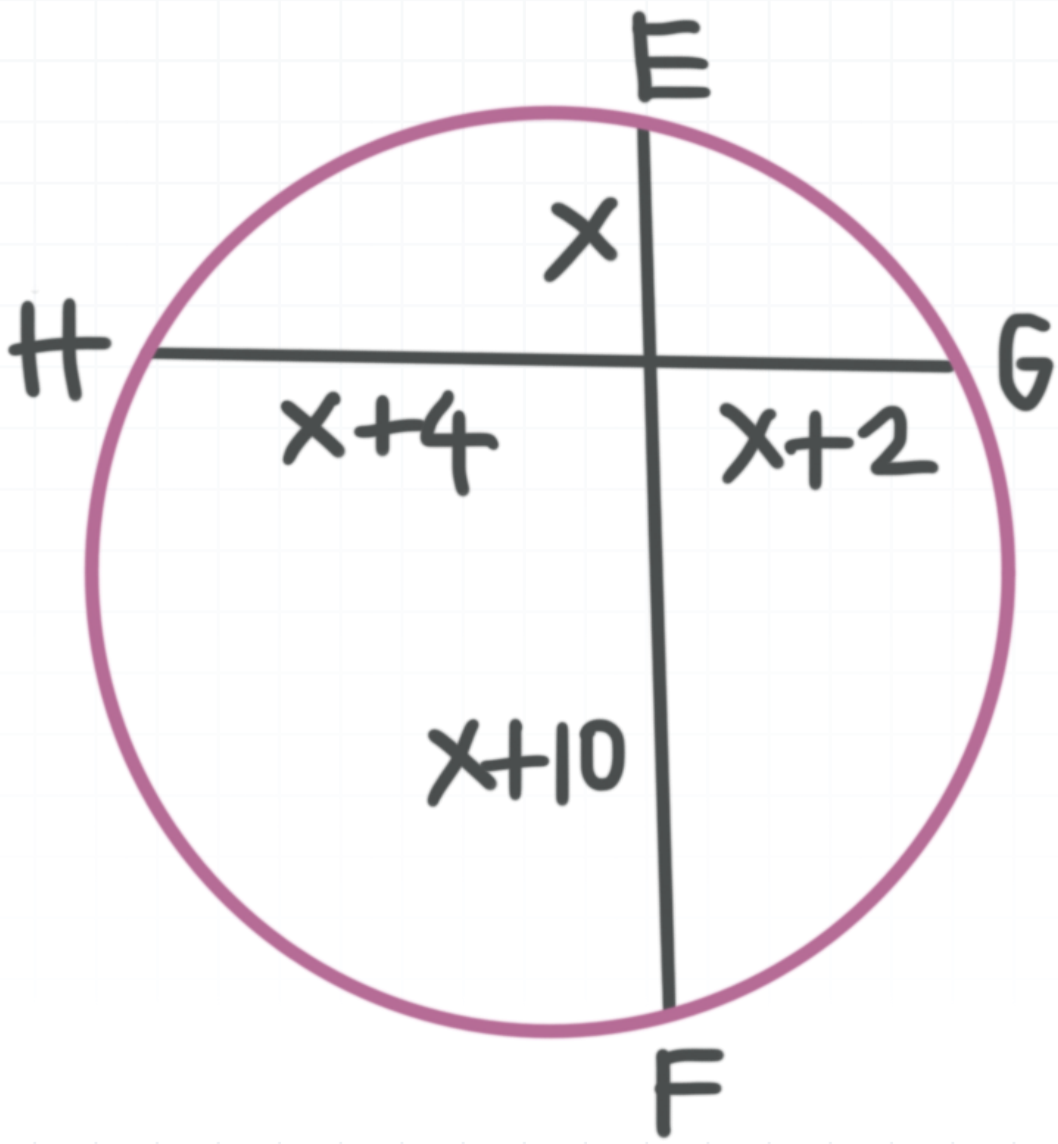
$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$x = 3$$

■ 4. $\overline{EF}$ and $\overline{HG}$ are intersecting chords of the circle. Find x.



Solution:

From the figure, we know that

$$x(x+10) = (x+2)(x+4)$$

$$x^2 + 10x = x^2 + 6x + 8$$

$$x = 2$$

W W W . K R I S T A K I N G M A T H . C O M