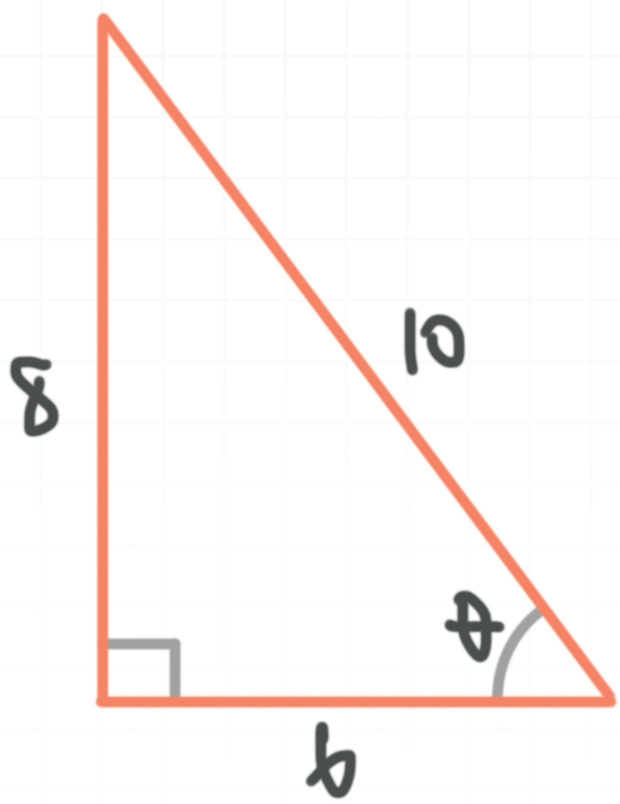
Trigonometry Workbook Solutions

The six trig functions

krista king MATH

SINE, COSINE, AND TANGENT

1. Find cosine of the angle θ.



Solution:

Given the position of the angle θ in the right triangle, the length of the opposite side is 8, the length of the adjacent side is 6, and the length of the hypotenuse is 10.

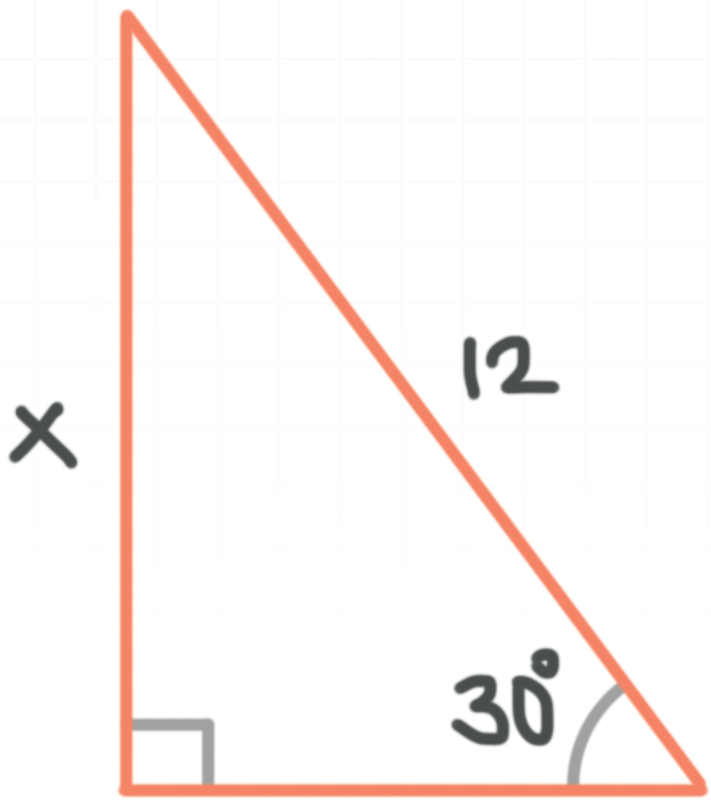
The cosine of that angle θ is equivalent to the length of the side adjacent to the angle θ, divided by the length of the hypotenuse.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute, and we get

$$\cos\theta = \frac{6}{10} = \frac{3}{5}$$

■ 2. Find the measure of the unknown angle of the triangle.



Solution:

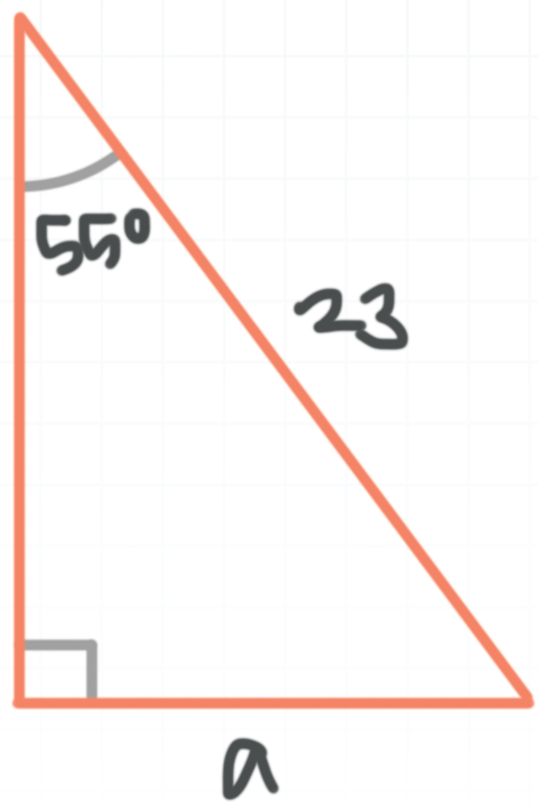
The figure shows a 90° angle and a 30° angle in the triangle. To find the measure of the unknown angle, we need to use the fact that the sum of the interior angles of a triangle is always 180°.

$$180^\circ - 90^\circ - 30^\circ$$

$$60^\circ$$

The measure of the third interior angle is 60°.

3. Find the equation that would be used to solve for a.



Solution:

Given the position of the angle $\theta = 55^\circ$ in the right triangle, the length of the opposite side is a and the length of the hypotenuse is 23.

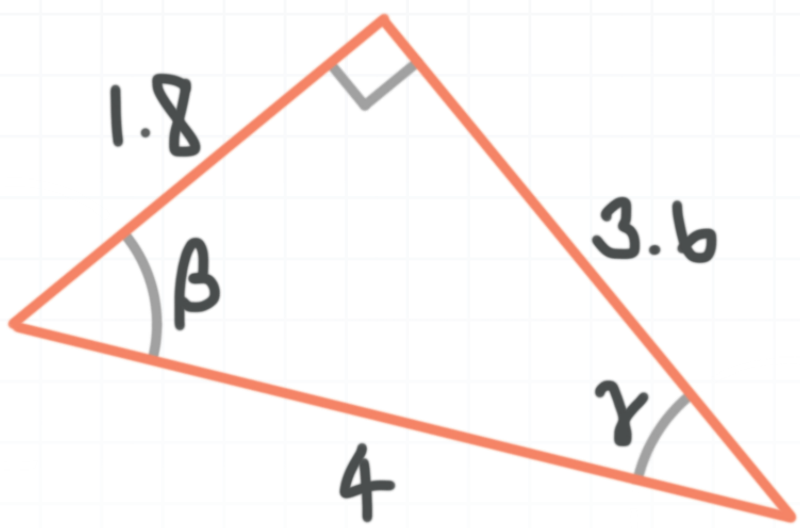
The sine of that angle θ is equivalent to the length of the side opposite the angle θ, divided by the length of the hypotenuse.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute and get

$$\sin 55^\circ = \frac{a}{23}$$

■ 4. Find the sine, cosine, and tangent for β and γ.



Solution:

Given the position of the angle β in the right triangle, the length of the opposite side is 3.6, the length of the adjacent side is 1.8, and the length of the hypotenuse is 4.

Then the values of sine, cosine, and tangent for the angle β are

$$\sin\beta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3.6}{4} = 0.9$$

$$\cos\beta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1.8}{4} = 0.45$$

$$\tan\beta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3.6}{1.8} = 2$$

Given the position of the angle γ in the right triangle, the length of the opposite side is 1.8, the length of the adjacent side is 3.6, and the length of the hypotenuse is 4.

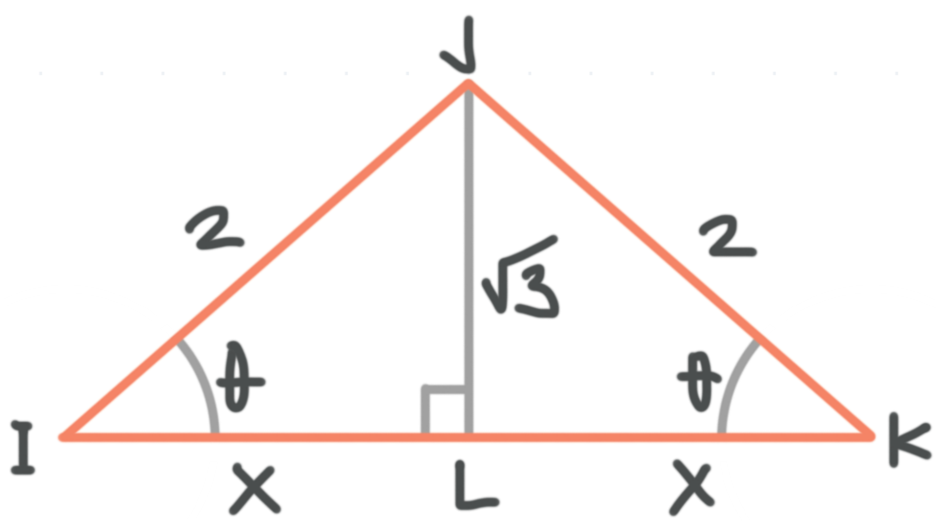
Then the values of sine, cosine, and tangent for the angle γ are

$$\sin\gamma = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1.8}{4} = 0.45$$

$$\cos\gamma = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3.6}{4} = 0.9$$

$$\tan\gamma = \frac{\text{opposite}}{\text{adjacent}} = \frac{1.8}{3.6} = \frac{1}{2}$$

5. Find the value of sine of the angle θ, given that the triangle is isosceles (two of the sides have equal length, and the base angles are equal).



Solution:

The triangle IJK is isosceles, so, $IJ = JK = 2$.

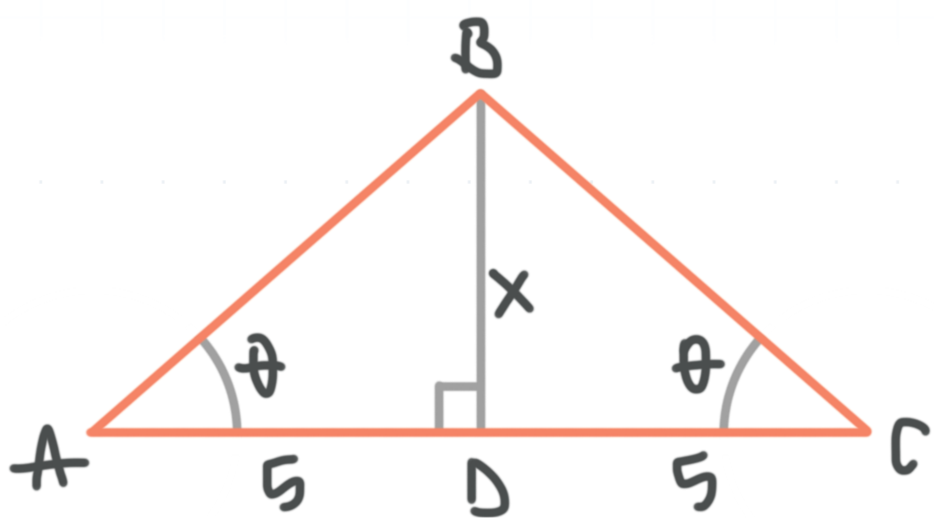
The sine of θ is equivalent to the length of the side opposite the angle θ, divided by the length of the hypotenuse.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Given the position of the angle θ in the right triangle IJL, the length of the opposite side is $\sqrt{3}$, the length of the adjacent side is x, and the length of the hypotenuse is 2. Substitute and get

$$\sin\theta = \frac{\sqrt{3}}{2}$$

■ 6. Find the equation that would be used to solve for x, given $\overline{AB} = \overline{BC}$ and $\theta = 45^\circ$.



Solution:

Given the position of the angle $\theta = 45^\circ$ in the right triangle ABD, the length of the opposite side is x and the length of the adjacent side is 5.

The tangent of the angle θ is equivalent to the length of the side opposite the angle θ, divided by the length of the side adjacent to the angle θ.

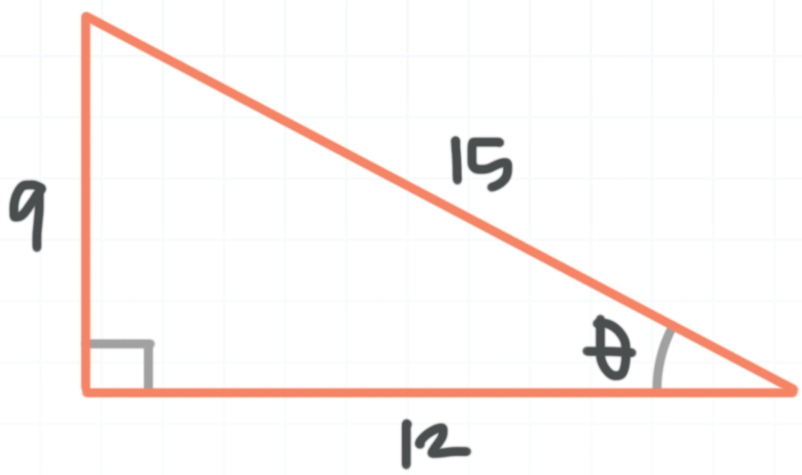
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute to get

$$\tan 45^\circ = \frac{x}{5}$$

COSECANT, SECANT, COTANGENT, AND THE RECIPROCAL IDENTITIES

1. Find the value of secant of θ.



Solution:

Given the position of the angle θ in the right triangle, the length of the opposite side is 9, the length of the adjacent side is 12, and the length of the hypotenuse is 15.

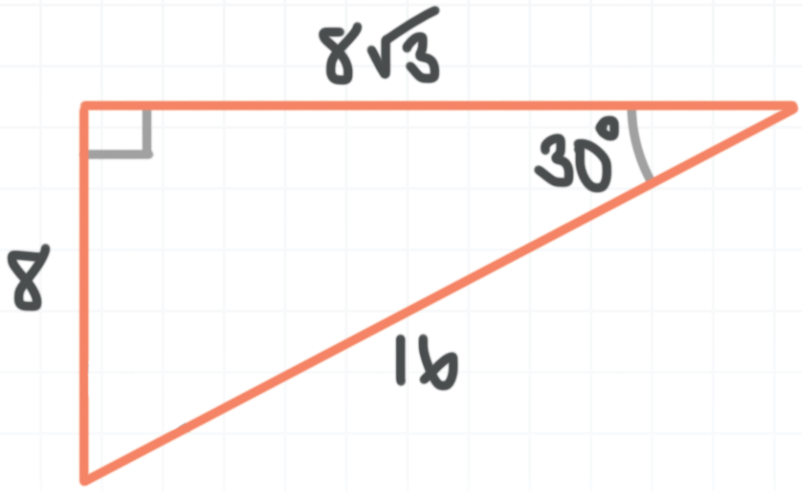
Then the value of secant for the angle is

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

Substitute, and we get

$$\sec\theta = \frac{15}{12} = \frac{5}{4}$$

2. Find the exact value of the six trigonometric functions for $\theta = 30^\circ$.



Solution:

Given the position of the angle θ in the right triangle, the length of the opposite side is 8, the length of the adjacent side is $8\sqrt{3}$, and the length of the hypotenuse is 16.

Then the values of the six trigonometric functions for $\theta = 30^\circ$ are

$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{16} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$$

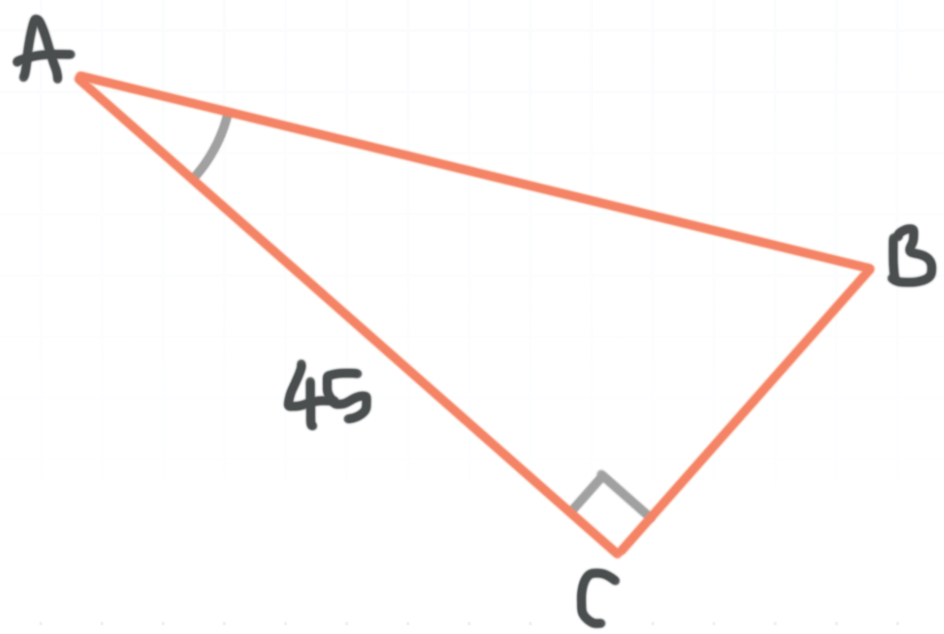
$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{8\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\csc 30^\circ = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{16}{8} = 2$$

$$\sec 30^\circ = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{16}{8\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot 30^\circ = \frac{\text{adjacent}}{\text{opposite}} = \frac{8\sqrt{3}}{8} = \sqrt{3}$$

■ 3. Given right triangle ABC, $\sin A = 28/53$. Find the exact value of secant, cosecant, and cotangent for the angle A.



Solution:

Given the position of the angle A in the right triangle, the length of the adjacent side is 45. Since $\sin A = 28/53$, we know the length of the opposite side is 28 and the length of the hypotenuse is 53.

Then the values of cosecant, secant, and cotangent of the angle A are

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{53}{28}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{53}{45}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{45}{28}$$

■ 4. Find $\csc\theta$, if $\sin\theta = 8/17$.

Solution:

The sine and cosecant functions are related to each other by the reciprocal identities, so we can substitute the value of $\sin\theta$ into the reciprocal identity for $\csc\theta$.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$$

■ 5. If $\sec\theta = 61/60$ and $\tan\theta = 11/60$, determine the values of the other four trigonometric functions.

Solution:

We know that the secant of an angle is the ratio of the hypotenuse to the adjacent side, and that the tangent of an angle is the ratio of the opposite side to the adjacent side.

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Given the definitions of secant and tangent we've been given, the length of the opposite side is 11, the length of the adjacent side is 60, and the length of the hypotenuse is 61.

Then the values of sine, cosine, cosecant, and cotangent are

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{11}{61}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{60}{61}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{61}{11}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{60}{11}$$

■ 6. Given the value of $\cot\theta$, find the value of $\tan\theta$.

$$\cot\theta = \frac{63}{16}$$

Solution:

The cotangent and tangent functions are reciprocals of one another, so we can substitute the value of $\cot\theta$ into the reciprocal identity for $\tan\theta$.

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{63}{16}} = \frac{16}{63}$$

THE QUOTIENT IDENTITIES

■ 1. If $\sin\theta = 16/65$ and $\cos\theta = 63/65$, find $\cot\theta$.

Solution:

The cotangent of an angle is defined in terms of the sine and cosine by the quotient identity for cotangent.

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{63}{65}}{\frac{16}{65}} = \frac{63}{65}\left(\frac{65}{16}\right) = \frac{63}{16}$$

■ 2. If $\tan\theta = 4/3$ and $\cos\theta = 3/5$, find $\sin\theta$.

Solution:

We can find sine of θ just by plugging these tangent and cosine values into the quotient identity for tangent.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Substitute, and we get

$$\frac{4}{3} = \frac{\sin\theta}{\frac{3}{5}}$$

$$\sin\theta = \frac{4}{3}\left(\frac{3}{5}\right) = \frac{4}{5}$$

■ 3. If $\cot\theta = \sqrt{13}/6$ and $\csc\theta = -7/6$, find $\cos\theta$.

Solution:

Use the reciprocal identity for sine, substituting the value of cosecant.

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{7}{6}} = -\frac{6}{7}$$

Now we can find cosine of θ just by plugging the cotangent and sine values into the quotient identity for cotangent.

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Substitute, and we get

$$\frac{\sqrt{13}}{6} = \frac{\cos\theta}{-\frac{6}{7}}$$

$$\cos\theta = \frac{\sqrt{13}}{6}\left(-\frac{6}{7}\right) = -\frac{\sqrt{13}}{7}$$

4. If $\cot\theta = -12/5$ and $\cos\theta = 12/13$, find $\sin\theta$.

Solution:

We can find sine of θ just by plugging these cotangent and cosine values into the quotient identity for cotangent.

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Substitute, and we get

$$-\frac{12}{5} = \frac{\frac{12}{13}}{\sin\theta}$$

$$\sin\theta = \frac{\frac{12}{13}}{-\frac{12}{5}} = \frac{12}{13}\left(-\frac{5}{12}\right) = -\frac{5}{13}$$

5. If $\sin\theta = 39/89$ and $\tan\theta = -39/80$, find $\cos\theta$.

Solution:

We can find cosine of θ just by plugging these tangent and sine values into the quotient identity for tangent.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Substitute, and we get

$$-\frac{39}{80} = \frac{\frac{39}{89}}{\cos\theta}$$

$$\cos\theta = \frac{\frac{39}{89}}{-\frac{39}{80}} = \frac{39}{89}\left(-\frac{80}{39}\right) = -\frac{80}{89}$$

■ 6. If $\tan\theta = 8/15$ and $\sec\theta = 17/15$, find $\sin\theta$.

Solution:

Use the reciprocal identity for cosine, substituting the value of secant.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{17}{15}} = \frac{15}{17}$$

We can find sine of θ just by plugging the tangent and cosine values into the quotient identity for tangent.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Substitute and get

$$\frac{8}{15} = \frac{\sin\theta}{\frac{15}{17}}$$

$$\sin\theta = \frac{8}{15}\left(\frac{15}{17}\right) = \frac{8}{17}$$

THE PYTHAGOREAN IDENTITIES

■ 1. Find the positive value of $\cos(49.3^\circ)$ if $\sin(49.3^\circ)=0.758$.

Solution:

We'll use a rewritten form of the Pythagorean identity with sine and cosine.

$$\cos^2\theta = 1-\sin^2\theta$$

$$\cos^2(49.3^\circ)=1-\sin^2(49.3^\circ)$$

$$\cos^2(49.3^\circ)=1-(0.758)^2$$

$$\cos^2(49.3^\circ)\approx 1-0.575$$

$$\cos^2(49.3^\circ)\approx 0.425$$

$$\cos(49.3^\circ)\approx \pm\sqrt{0.425}$$

Since we need to find the positive value of cosine, we ignore the negative value and say

$$\cos(49.3^\circ)\approx \sqrt{0.425}$$

$$\cos(49.3^\circ)\approx 0.652$$

2. In a right triangle, sine of the acute angle is $1/5$. What are the positive values of the cosine and cotangent of this angle?

We'll use a rewritten form of the Pythagorean identity with sine and cosine.

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2(\theta) = 1 - \left(\frac{1}{5}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{25}$$

$$\cos^2\theta = \frac{24}{25}$$

$$\cos\theta = \pm\sqrt{\frac{24}{25}}$$

Since we need to find the positive value of cosine, we ignore the negative value and say

$$\cos\theta = \sqrt{\frac{24}{25}}$$

$$\cos\theta = \frac{2\sqrt{6}}{5}$$

Now that we know the value of both sine and cosine of the angle, we'll use the quotient identity to find the value of cotangent.

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = \frac{2\sqrt{6}}{5}\left(\frac{5}{1}\right) = 2\sqrt{6}$$

■ 3. If $\sin\theta = 12/13$, what is the negative value of $\cot\theta$?

Solution:

We'll use a rewritten form of the Pythagorean identity with cotangent and cosecant.

$$\cot^2\theta = \csc^2\theta - 1$$

Use the reciprocal identity for cosecant to rewrite the Pythagorean identity as

$$\cot^2\theta = \left(\frac{1}{\sin\theta}\right)^2 - 1$$

Substitute, and we get

$$\cot^2\theta = \left(\frac{1}{\frac{12}{13}}\right)^2 - 1$$

$$\cot^2\theta = \left(\frac{13}{12}\right)^2 - 1$$

$$\cot^2\theta = \frac{169}{144} - 1$$

$$\cot^2\theta = \frac{25}{144}$$

$$\cot\theta = \pm\frac{5}{12}$$

Since we need to find the negative value of cotangent, we ignore the positive value and say

$$\cot\theta = -\frac{5}{12}$$

■ 4. If $\theta = 6\pi/5$ and $\sin\theta = -0.588$, what is the negative value of $\cos\theta$?

Solution:

We'll use a rewritten form of the Pythagorean identity with sine and cosine.

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos^2\left(\frac{6\pi}{5}\right) = 1 - \sin^2\left(\frac{6\pi}{5}\right)$$

$$\cos^2\left(\frac{6\pi}{5}\right) = 1 - (-0.588)^2$$

$$\cos^2\left(\frac{6\pi}{5}\right) \approx 1 - 0.346$$

$$\cos^2\left(\frac{6\pi}{5}\right) \approx 0.654$$

$$\cos\left(\frac{6\pi}{5}\right) \approx \pm\sqrt{0.654}$$

Since we need to find the negative value of cosine, we ignore the positive value and say

$$\cos\left(\frac{6\pi}{5}\right) \approx -\sqrt{0.654}$$

$$\cos\left(\frac{6\pi}{5}\right) \approx -0.809$$

■ 5. If θ is an angle in the second quadrant such that $\cos\theta = -0.412$, what is the negative value of $\tan\theta$?

Solution:

We'll use a rewritten form of the Pythagorean identity with tangent and secant.

$$\tan^2\theta = \sec^2\theta - 1$$

Use the reciprocal identity for secant to rewrite the Pythagorean identity as

$$\tan^2\theta = \left(\frac{1}{\cos\theta}\right)^2 - 1$$

Substitute, and we get

$$\tan^2\theta = \left(\frac{1}{-0.412}\right)^2 - 1$$

$$\tan^2\theta = 5.891 - 1$$

$$\tan^2\theta = 4.891$$

$$\tan\theta = \pm 2.212$$

Since we need to find the negative value of tangent, we ignore the positive value and say

$$\tan\theta = -2.212$$

■ 6. Evaluate the expression if $\cos\theta = 1/\sqrt{3}$.

$$\tan^2\theta + \sin^2\theta + \sec^2\theta$$

Solution:

Use the Pythagorean identity with sine and cosine to find the value of sine.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\sin^2\theta = 1 - \frac{1}{3}$$

$$\sin^2\theta = \frac{2}{3}$$

If we use the quotient identity for tangent, then we get

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta = \frac{\frac{2}{3}}{\frac{1}{3}}$$

$$\tan^2\theta = \frac{2}{3}\left(\frac{3}{1}\right)$$

$$\tan^2\theta = 2$$

If we use the reciprocal identity for secant, we get

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec^2\theta = \frac{1}{\cos^2\theta}$$

$$\sec^2\theta = \frac{1}{\frac{1}{3}}$$

$$\sec^2\theta = 3$$

Then the value of the expression is

$$\tan^2\theta + \sin^2\theta + \sec^2\theta$$

$$2 + \frac{2}{3} + 3$$

$$\frac{6}{3} + \frac{2}{3} + \frac{9}{3}$$

$$\frac{17}{3}$$

SIGNS BY QUADRANT

1. Find $\sin\theta$ if the angle θ lies in the interval $[0^\circ,180^\circ)$ and $\cos^2\theta - 0.36 = 0$.

Solution:

Rewrite the cosine equation.

$$\cos^2\theta - 0.36 = 0$$

$$\cos^2\theta = 0.36$$

Now that we have a value for $\cos^2\theta$, we can substitute it into a rewritten form of the Pythagorean identity for sine and cosine.

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - 0.36$$

$$\sin^2\theta = 0.64$$

$$\sin\theta = \pm\sqrt{0.64}$$

The interval $[0^\circ,180^\circ)$ defines the first and second quadrants, which means θ must lie in one of those quadrants. All points in the first and second quadrants have a positive y-value, which means that the sine of any angle in those quadrants will be positive. So we ignore the negative value and say

$$\sin\theta = \sqrt{0.64}$$

$$\sin\theta = 0.8$$

■ 2. Find $\cot\theta$ if $\cos\theta = 0.6$ and the angle θ is in the interval $[5\pi, 6\pi)$.

Solution:

We're told that θ is between 5π and 6π, so it lies in the third of fourth quadrants. We also know that cosine is positive, and since cosine is only positive in the first and fourth quadrants, θ must lie in the fourth quadrant.

In the fourth quadrant, $\sin\theta$ is negative, which means cotangent is also negative. So we'll find the value of sine using the Pythagorean identity with sine and cosine.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - 0.6^2$$

$$\sin^2\theta = 0.64$$

Because we know sine is negative,

$$\sin\theta = -\sqrt{0.64}$$

$$\sin\theta = -0.8$$

Using the quotient identity for cotangent, $\cot\theta$ must be

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{0.6}{-0.8}$$

$$\cot\theta = -\frac{3}{4}$$

3. Find $\sin\theta$ if $\sec\theta = 3$ and $\cot\theta < 0$.

Solution:

Use the reciprocal identity to find $\cos\theta$.

$$\sec\theta = \frac{1}{\cos\theta}$$

$$3 = \frac{1}{\cos\theta}$$

$$\cos\theta = \frac{1}{3}$$

Since we were told that cotangent is negative, and cotangent is negative for angles that fall in the second and fourth quadrants, the angle must fall in the second or fourth quadrant.

We also know that cosine is positive, and since cosine is only positive for angles in the first and fourth quadrants, θ must be in first or fourth quadrant.

Comparing these results for cotangent and cosine tells us that the angle can only fall in the fourth quadrant.

In the fourth quadrant, $\sin\theta$ is negative, so we can find the sine of the angle using the Pythagorean identity with sine and cosine.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(\frac{1}{3}\right)^2$$

$$\sin^2\theta = 1 - \frac{1}{9}$$

$$\sin^2\theta = \frac{8}{9}$$

$$\sin\theta = -\sqrt{\frac{8}{9}}$$

$$\sin\theta = -\frac{2\sqrt{2}}{3}$$

■ 4. At the angle -340°, what are the signs of sine and cosine.

Solution:

The angle -340° lies in the first quadrant. Since the x- and y-values of any point in the first quadrant are positive, we know that $\sin(-340^\circ)$ and $\cos(-340^\circ)$ must both be positive.

■ 5. In which quadrant does the angle θ lie, if $\tan\theta$ is positive and $\sec\theta$ is negative?

Solution:

If $\sec\theta$ is negative, then $\cos\theta$ is also negative, which means θ must lie in the second or third quadrants.

Since $\tan\theta$ is positive and cosine is negative, $\sin\theta$ must be negative, which means θ must lie in the third of fourth quadrants.

But since the sign of $\cos\theta$ restricts the angle to the second and third quadrants, the angle θ can only lie within the third quadrant.

■ 6. Find the largest among the values of the six trig functions of θ if $\cos\theta = -0.1$ and θ lies in the third quadrant.

Solution:

Since we're looking for the largest value among the six trig functions, we only need to consider the trig functions that will have a positive value. In the third quadrant, the only positive trig functions are tangent and cotangent.

To find the value of tangent, we'll use the Pythagorean identity with sine and cosine.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - (-0.1)^2$$

$$\sin^2\theta = 0.99$$

In the third quadrant, $\sin\theta$ is negative.

$$\sin\theta = -\sqrt{0.99}$$

With the values of sine and cosine, we can find the values of tangent and cotangent.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{0.99}}{-0.1} = \sqrt{99} \approx 10$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-0.1}{-\sqrt{0.99}} = \frac{1}{\sqrt{99}} \approx 0.1$$

Because $10 > 0.1$, of the six trig functions, tangent has the largest value at this particular angle θ.

WHEN THE TRIG FUNCTIONS ARE UNDEFINED

■ 1. For what angle is $\cot\theta$ undefined in the interval $(0,2\pi]$?

Solution:

We know by the quotient identity

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

that cotangent is undefined when sine is 0, and we know that sine is 0 when $y = 0$. We know we'll have $y = 0$ along the horizontal axis, which means that, in the interval

$$(0,2\pi]$$

cotangent will be undefined at $\theta = \pi$ and $\theta = 2\pi$.

■ 2. Determine whether or not $\cot(-43\pi/4)$ is defined.

Solution:

We know by the quotient identity

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

that cotangent is undefined when sine is 0, and we know that sine is 0 when $y=0$. We know we'll have $y=0$ along the horizontal axis, so we need to determine whether or not $-43\pi/4$ lies along the horizontal axis.

If we know one full rotation is $2\pi=8\pi/4$, then we can say $-43\pi/4$ is five full negative rotations ($-40\pi/4$) and then an additional $-3\pi/4$ rotations. The angle $-3\pi/4$ doesn't fall on the horizontal axis, which means $\cot(-43\pi/4)$ will be defined.

3. Which trigonometric functions are undefined for $\theta=\pi/2$?

Solution:

The angle $\theta=\pi/2$ falls on the positive y-axis. So in a circle centered at the origin with radius 1,

$$y=\sin\left(\frac{\pi}{2}\right)=1$$

$$x=\cos\left(\frac{\pi}{2}\right)=0$$

Use the reciprocal identities to find cosecant and secant of the angle.

$$\csc\left(\frac{\pi}{2}\right)=\frac{1}{\sin\left(\frac{\pi}{2}\right)}=\frac{1}{1}=1$$

$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{\cos\left(\frac{\pi}{2}\right)} = \frac{1}{0}$$

Use the quotient identities to find tangent and cotangent of the angle.

$$\tan\left(\frac{\pi}{2}\right) = \frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)} = \frac{1}{0}$$

$$\cot\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0$$

So to summarize, because we got a 0 value in the denominator of the secant and tangent functions, we know these two trig functions are undefined at $\theta = \pi/2$.

■ 4. Which of the six trigonometric functions are undefined along the y-axis (when $x = 0$)?

Solution:

We know we'll have $x = 0$ along the vertical axis. And we know that cosine is 0 when $x = 0$. Therefore, we need to find the functions which are undefined when cosine is 0. These are the tangent and secant functions, because $\cos x$ is in the denominator in these functions.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

■ 5. Find the angle where $\tan\theta$ is undefined in the given interval.

$$\left(\frac{7\pi}{3}, \frac{25\pi}{6}\right)$$

Solution:

We need to start by first identifying an interval that's coterminal with the given interval.

We know that one full rotation is given by 2π radians.

$$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3}$$

So we know that an angle of $7\pi/3$ is one full rotation, and then another $\pi/3$ rotation. So $7\pi/3$ is coterminal with $\pi/3$. To get from $7\pi/3$ to $\pi/3$, we're subtracting one full 2π rotation. So we need to also subtract a 2π rotation from $25\pi/6$.

$$\frac{25\pi}{6} - 2\pi = \frac{25\pi}{6} - \frac{12\pi}{6} = \frac{13\pi}{6}$$

Therefore, we would rewrite the given interval as

$$\left(\frac{\pi}{3}, \frac{13\pi}{6}\right)$$

This is now an interval that spans from the first quadrant, through the second, third, and fourth quadrants, and back into the first quadrant, stopping $\pi/6$ rotations short of where we started.

We know by the quotient identity

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

that $\tan\theta$ is undefined when $\cos\theta = 0$, and we know that $\cos\theta = 0$ along the vertical axis, which occurs in the interval $(\pi/3, 13\pi/6)$ at

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

To find the angles that correspond to these in the original interval, we need to add 2π back to each angle, since we previously took 2π away from the original interval.

$$\theta = \frac{\pi}{2} + 2\pi, \frac{3\pi}{2} + 2\pi$$

$$\theta = \frac{\pi}{2} + \frac{4\pi}{2}, \frac{3\pi}{2} + \frac{4\pi}{2}$$

$$\theta = \frac{5\pi}{2}, \frac{7\pi}{2}$$

6. Find the values of all six trig functions at $\theta = \pi$, and say whether or not any of them are undefined at this angle.

Solution:

The angle $\theta = \pi$ falls on the negative x-axis. So in a circle centered at the origin with radius 1,

$$y = \sin \pi = 0$$

$$x = \cos \pi = -1$$

Use the reciprocal identities to find cosecant and secant of the angle.

$$\csc \pi = \frac{1}{\sin \pi} = \frac{1}{0}$$

$$\sec \pi = \frac{1}{\cos \pi} = \frac{1}{-1} = -1$$

Use the quotient identities to find tangent and cotangent of the angle.

$$\tan \pi = \frac{\sin \pi}{\cos \pi} = \frac{0}{-1} = 0$$

$$\cot \pi = \frac{\cos \pi}{\sin \pi} = \frac{-1}{0}$$

So to summarize, because we got a 0 value in the denominator of the cosecant and cotangent values, we know these two trig functions are undefined at $\theta = \pi$.

W W W . K R I S T A K I N G M A T H . C O M