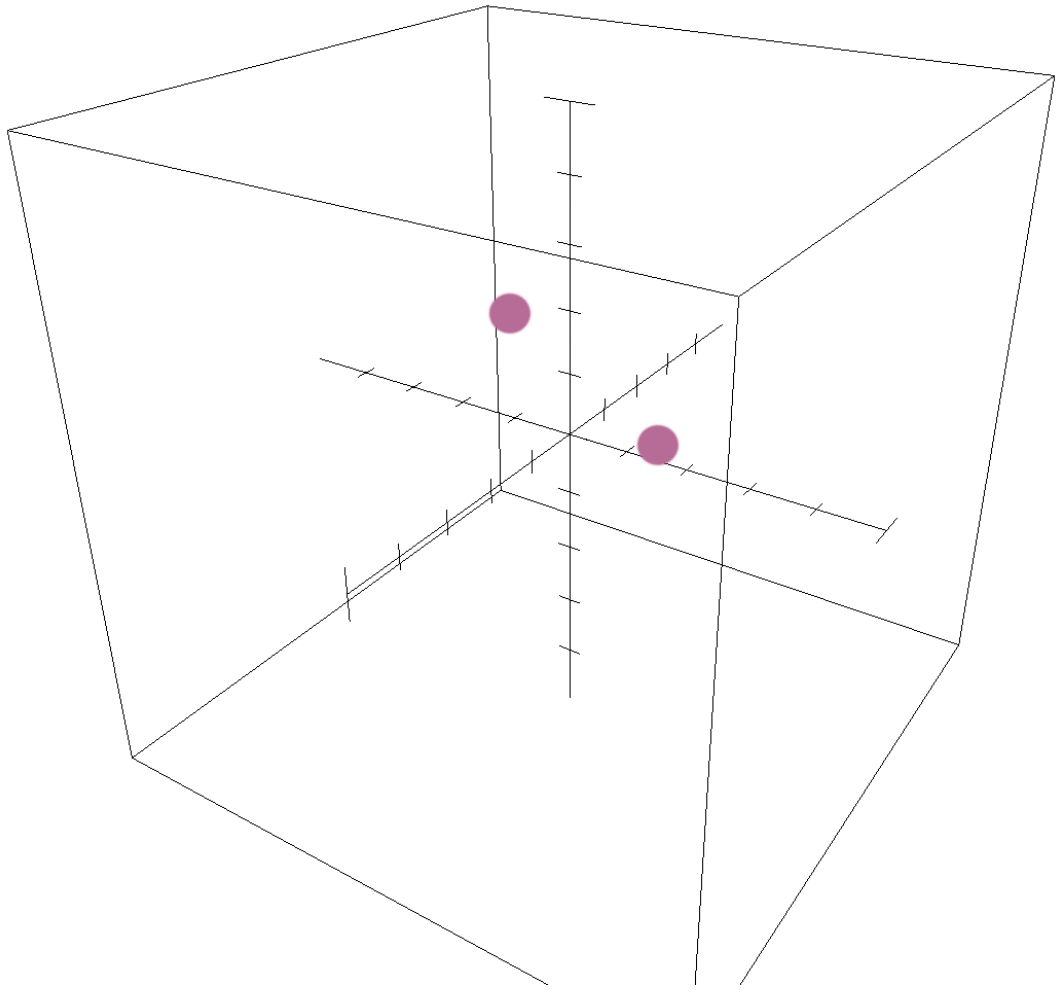
Distance between two points in three dimensions

In this lesson we'll look at points that are plotted in three dimensions, and learn how to find the distance between them.

Points

When we plot points in three dimensions, we use a coordinate system with three axes: the x-axis, the y-axis, and the z-axis. So a point is represented by three coordinates, as (x, y, z). Here are two points plotted in three-dimensional space:



Distance formula

We can use the distance formula for three dimensions to find the length of the line segment that connects two points in three-dimensional space.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

where $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$.

Simplifying radicals

Since use of the distance formula involves taking a square root, we'll need to know how to simplify radicals.

Remember that, when the value under the radical isn't a perfect square, we can simplify it by factoring out the perfect squares, separating each factor into its own root, and then taking the square root of any perfect squares. For example,

$$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

Now let's do an example of finding the distance between two points.

Example

Calculate the distance between Points B and C.

$$B = (4, -5, 8)$$

$$C = (1, -3, 2)$$

We need to use the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(4 - 1)^2 + (-5 - (-3))^2 + (8 - 2)^2}$$

$$d = \sqrt{(3)^2 + (-2)^2 + (6)^2}$$

$$d = \sqrt{9 + 4 + 36}$$

$$d = \sqrt{49}$$

$$d = 7$$

Let's try another one.

Example

Calculate the distance between Points A and D.

$$A = (3, 9, -2)$$

$$D = (0, -5, 4)$$

We need to use the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(3 - 0)^2 + (9 - (-5))^2 + (-2 - 4)^2}$$

$$d = \sqrt{(3)^2 + (14)^2 + (-6)^2}$$

$$d = \sqrt{9 + 196 + 36}$$

$$d = \sqrt{241} \approx 15.5$$