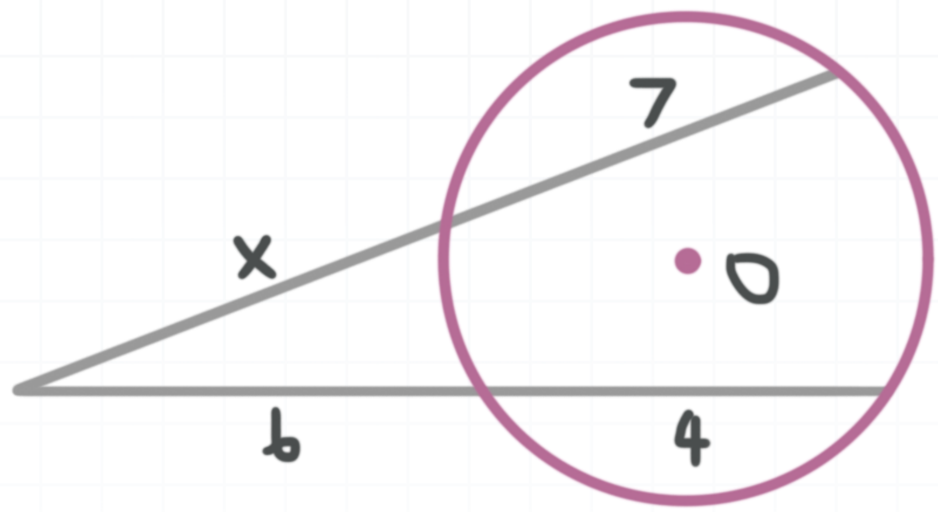
Topic: Intersecting tangents and secants

Question: Given the lengths in the figure, find x.



Answer choices:

A $x = 3$

B $x = 4$

C $x = 5$

D $x = 6$

Solution: C

Because there are two secants that intersect outside the circle, we can follow the pattern

outside · whole = outside · whole

Plugging the lengths shown in the figure into this equation, we get

$$x(x+7)=6(6+4)$$

$$x^2+7x=60$$

$$x^2+7x-60=0$$

$$(x+12)(x-5)=0$$

$x+12=0$ or $x-5=0$

$x=-12$ or $x=5$

A line segment can't have a negative length, so rule out $x=-12$ and conclude that $x=5$.

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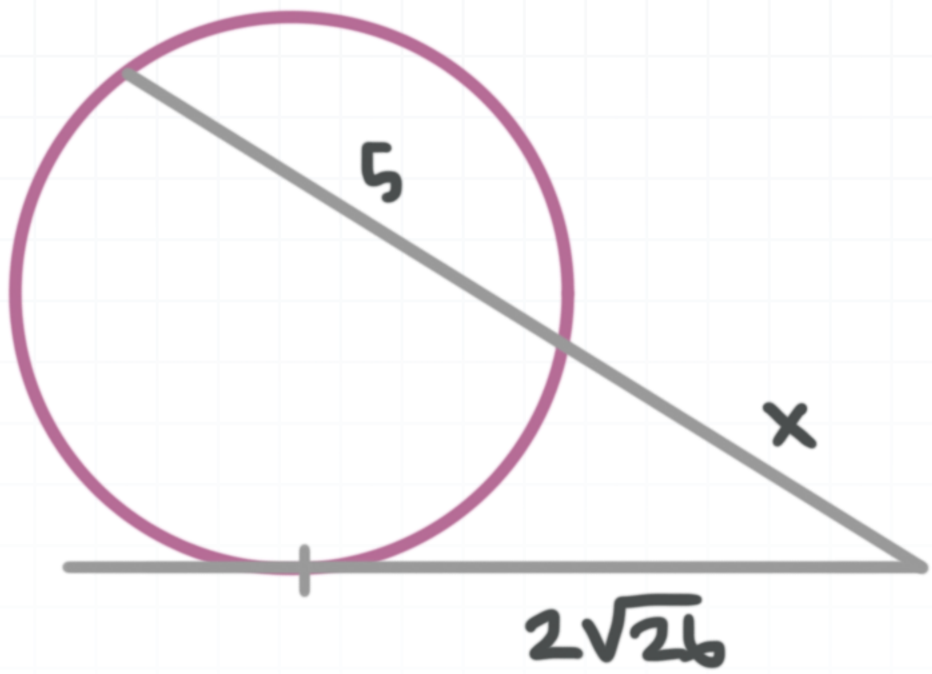


Answer choices:

A $x = 5$

B $x = 6$

C $x = 7$

D $x = 8$

Solution: D

Because there is a secant that intersects with a tangent outside the circle, we can follow the pattern

$$\text{tangent}^2 = \text{outside} \cdot \text{whole}$$

Plugging the lengths shown in the figure into this equation, we get

$$\left(2\sqrt{26}\right)^2 = x(x+5)$$

$$4(26) = x^2 + 5x$$

$$104 = x^2 + 5x$$

$$0 = x^2 + 5x - 104$$

$$(x+13)(x-8) = 0$$

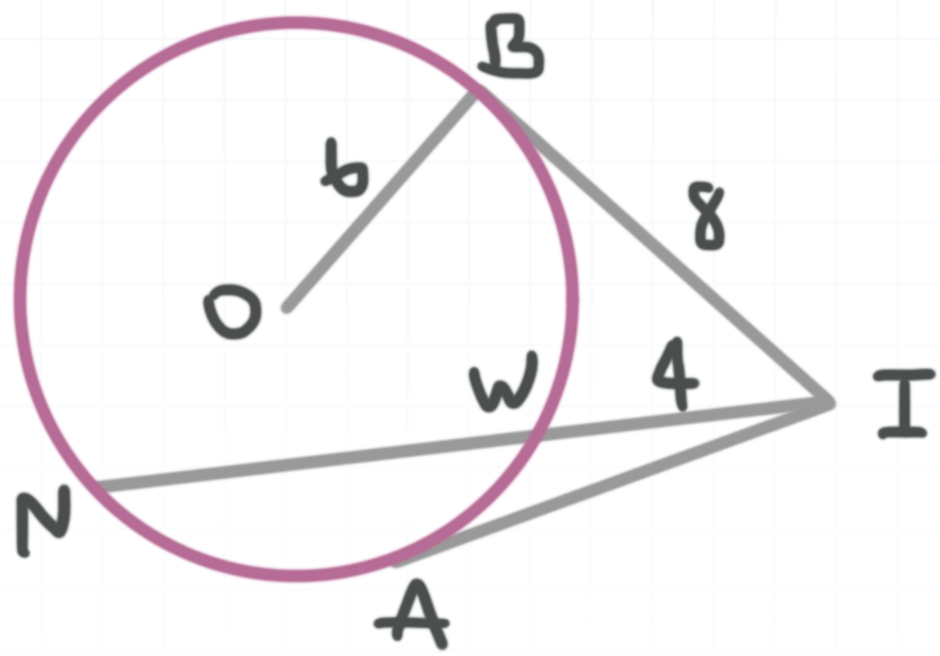
$x + 13 = 0$ or $x - 8 = 0$

$x = -13$ or $x = 8$

A line segment can't have a negative length, so rule out $x = -13$ and conclude that $x = 8$.

Topic: Intersecting tangents and secants

Question: In the figure, $\overline{IB}$ and $\overline{IA}$ are tangent to the circle (with center at O) at points B and A, respectively. Given the lengths shown, find $\overline{NW}+\overline{IA}$.



Answer choices:

A 12

B 14

C 18

D 20

Solution: D

Since $\overline{IB}$ is tangent to the circle at point B, $\overline{OB}$ is a radius. This means that the radius of the circle is 6 and that $\overline{OB}$ is perpendicular to $\overline{IB}$. Apply Pythagorean theorem to right triangle OBI.

$$(\overline{OB})^2 + (\overline{BI})^2 = (\overline{OI})^2$$

$$6^2 + 8^2 = (\overline{OI})^2$$

$$36 + 64 = (\overline{OI})^2$$

$$100 = (\overline{OI})^2$$

Similarly, $\overline{IA}$ is tangent to the circle at point A, so $\overline{OA}$ is a radius. This means that $\overline{OA} = 6$ and that $\overline{OA}$ is perpendicular to $\overline{IA}$. Apply the Pythagorean theorem to right triangle OAI.

$$(\overline{OA})^2 + (\overline{AI})^2 = (\overline{OI})^2$$

$$6^2 + (\overline{AI})^2 = 100$$

$$36 + (\overline{AI})^2 = 100$$

$$(\overline{AI})^2 = 64$$

$$\overline{AI} = 8$$

Then we can apply the pattern

$$\text{tangent}^2 = \text{outside} \cdot \text{whole}$$

to tangent $\overline{IA}$ and secant $\overline{IN}$.

$$8^2 = 4(4 + \overline{NW})$$

$$64 = 16 + 4(\overline{NW})$$

$$48 = 4(\overline{NW})$$

$$\overline{NW} = 12$$

Therefore,

$$\overline{NW} + \overline{IA} = 12 + 8$$

$$\overline{NW} + \overline{IA} = 20$$