Geometry Workbook Solutions

Lines

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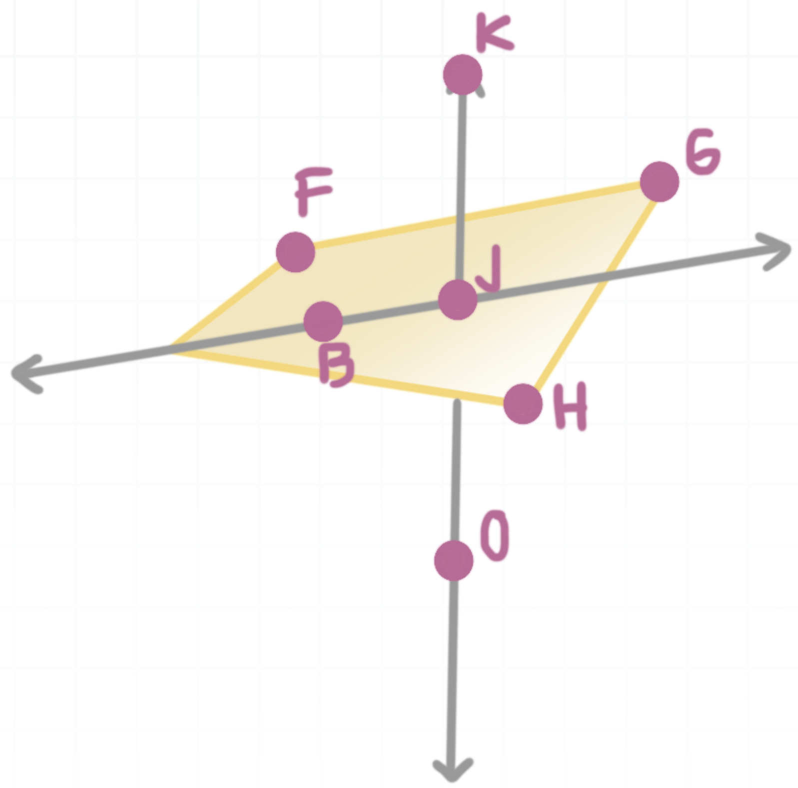
NAMING SIMPLE GEOMETRIC FIGURES

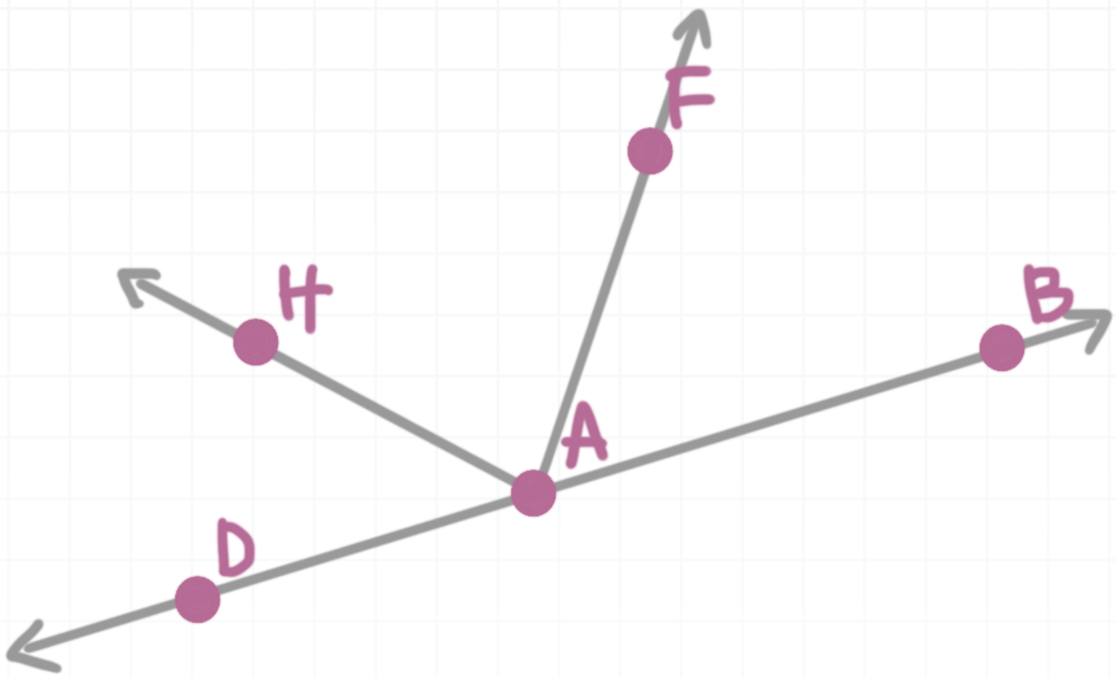
■ 1. Name the intersection of $\overline{BJ}$ and $\overline{KO}$.



Solution:

The intersection is the point J. The intersection of a two lines or line segments is always a point. A line is made up an infinite number of points. To find the intersection of two lines, we need to find the point that lies in both.

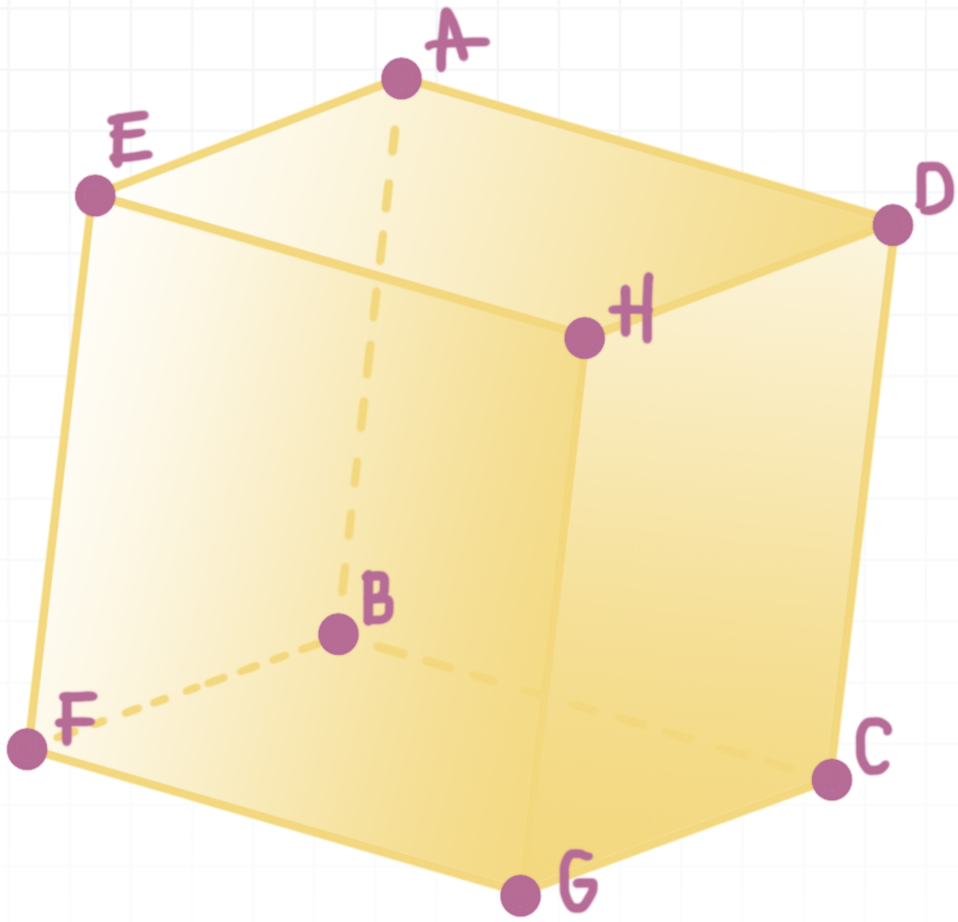
■ 2. Name the angle that forms a linear pair with $\angle DAF$.



Solution:

$\angle FAB$ (can also be named $\angle BAF$). Angles form a linear pair when they're adjacent, which means they share a common side and are supplementary, which means the degree measures of the pair have a sum of 180°.

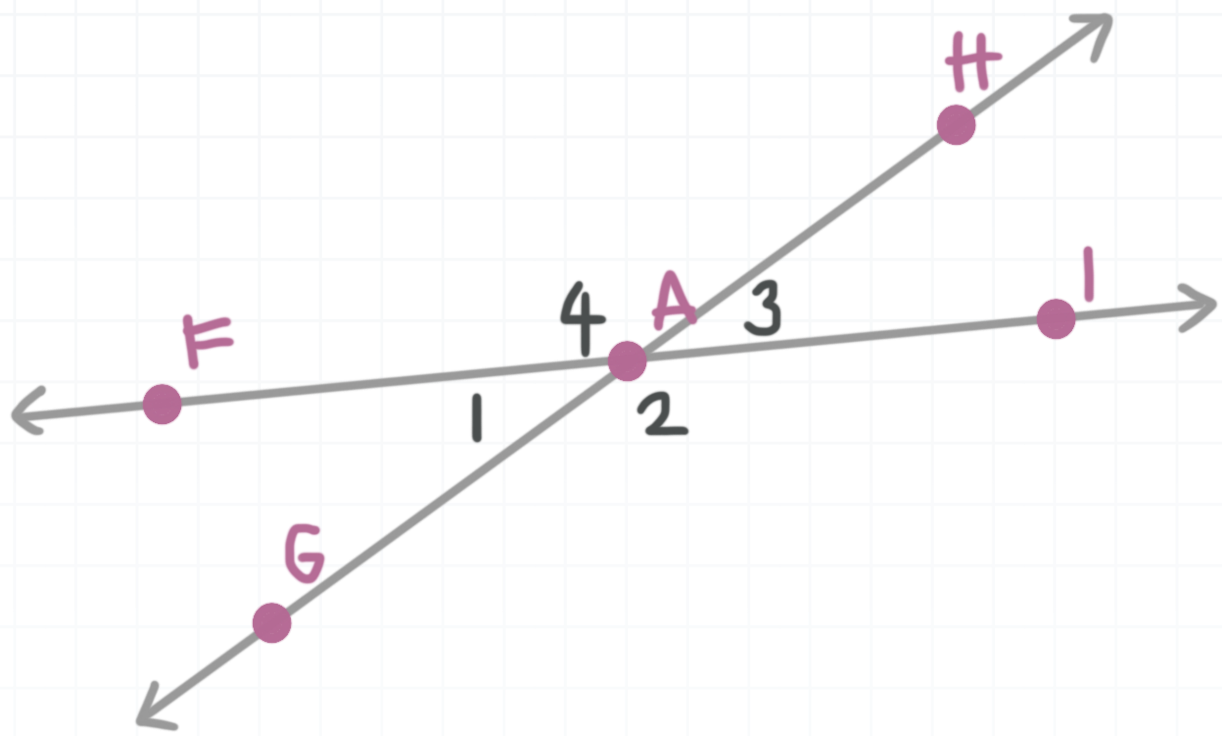
■ 3. Name three non-collinear points.



Solution:

There are many correct answers, one of which could be points A, B, and G. Collinear points all lie on the same line. Non-collinear points are points that do not all lie on the same line. So choose any three points that are not on the same line.

■ 4. Name a pair of vertical angles.



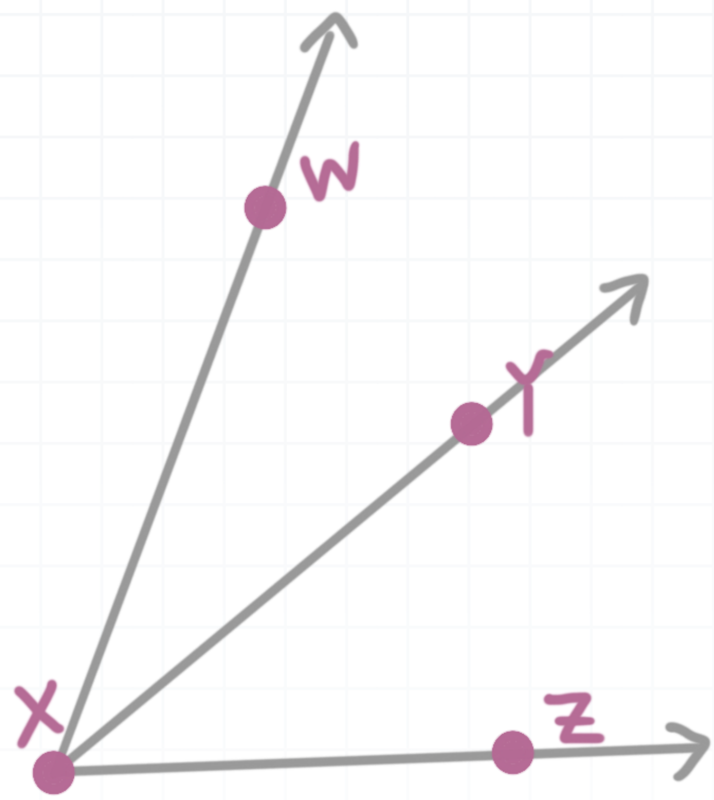
Solution:

$\angle 1$ and $\angle 3$, or $\angle 2$ and $\angle 4$. Vertical angles are formed when two lines intersect. They are the angles across from one another and are always congruent.

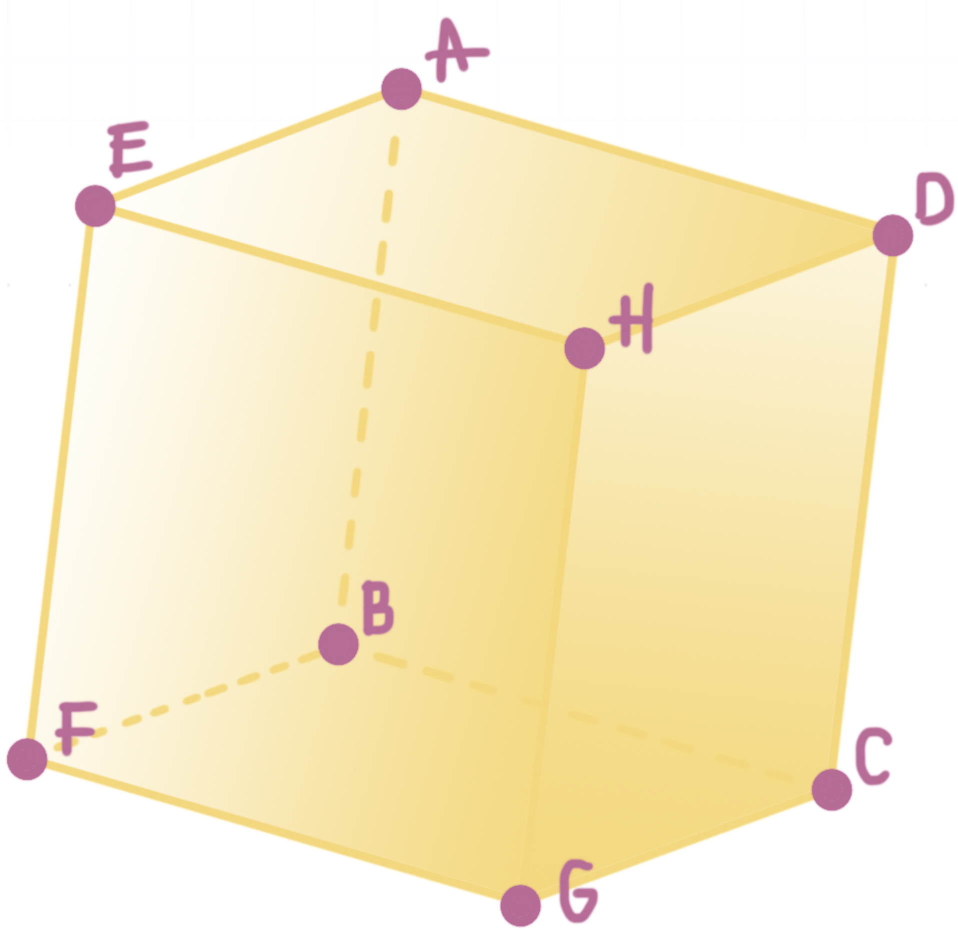
■ 5. $\overline{XY}$ is an angle bisector of $\angle WXZ$. Write the congruence statement that follows.

Solution:

$\angle WXY \cong \angle ZXY$



■ 6. Name the intersection of plane AEH and plane GCD.

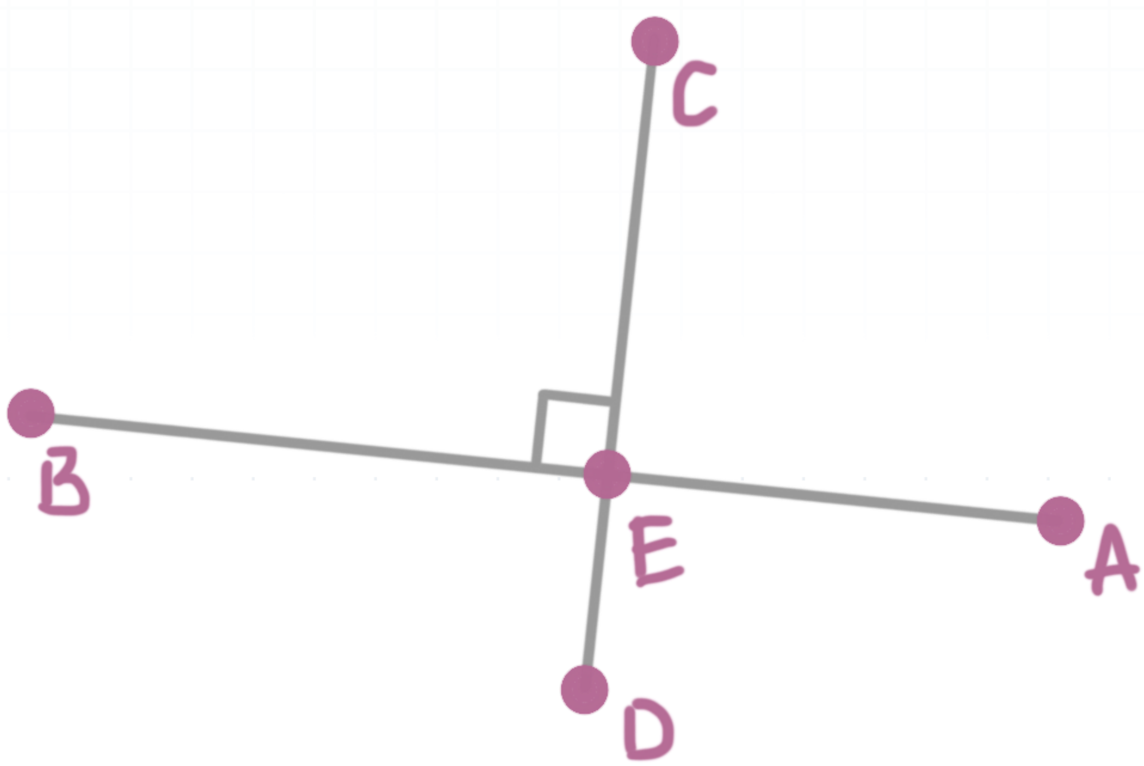


Solution:

$\overline{HD}$. The intersection of these two planes is a line. They share the segment $\overline{HD}$.

■ 7. $\overline{AB} \perp \overline{CD}$ and they intersect at E. Draw a sketch of this and include all necessary labels on your diagram.

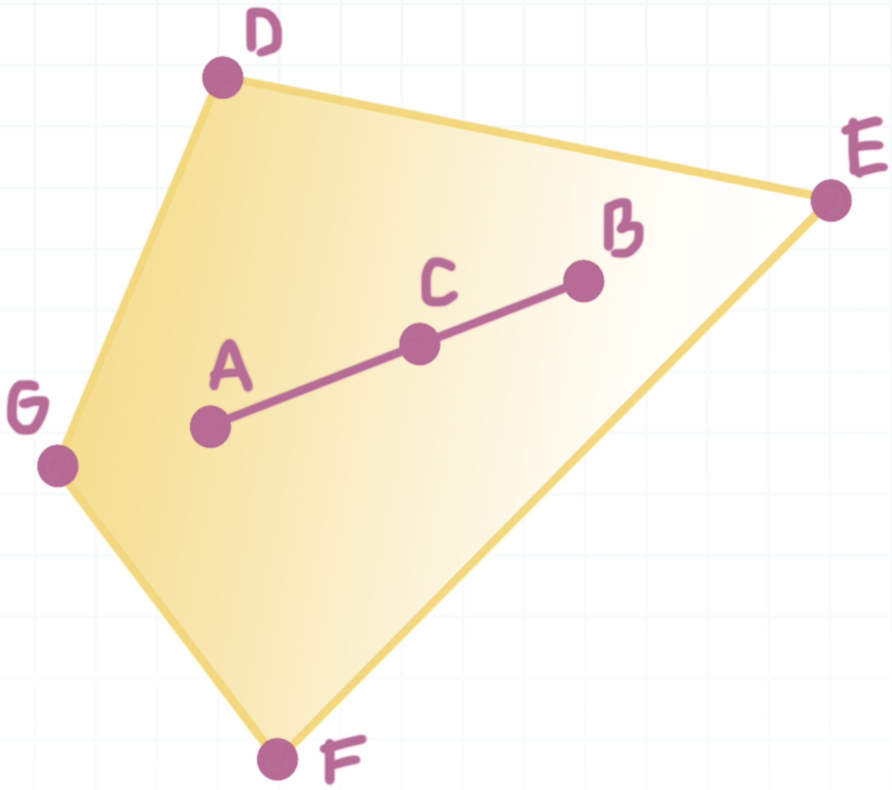
Solution:



■ 8. Sketch the following: $\overline{AB}$ lies on plane DEF and C is contained in $\overline{AB}$.

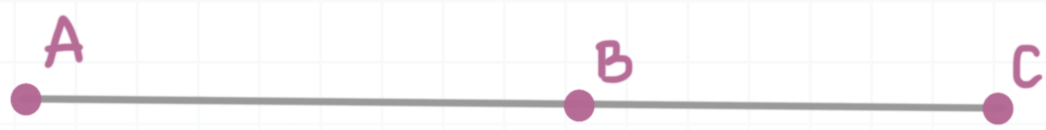
Solution:

There are many possible solutions, but one example might be



LENGTH OF A LINE SEGMENT

■ 1. In the line segment, $AB=14$ and $BC=10$. Find AC.



Solution:

$AC=24$ because

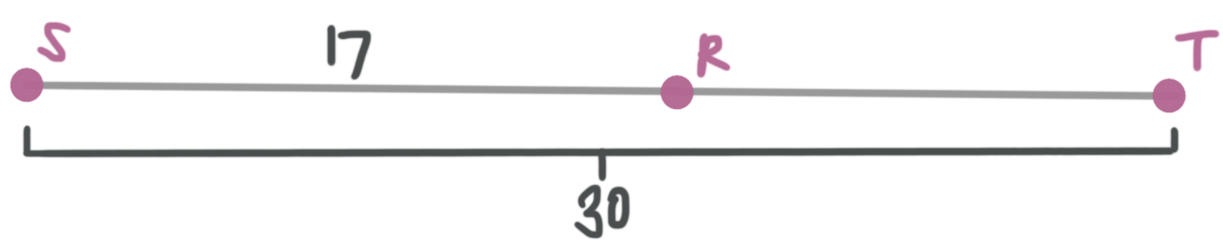
$$AB+BC=AC$$

$$14+10=24$$

■ 2. R lies between S and T. $ST=30$ and $SR=17$. Find RT.

Solution:

$RT=13$. Make a diagram of $\overline{ST}$ and place point R on the line segment. Label the lengths of ST and SR on your diagram.



Then we can say

$$SR + RT = ST$$

$$17 + RT = 30$$

$$RT = 13$$

■ 3. $JM = 2MP$ and $JP = 30$. Find JM and MP.

Solution:

$JM = 20$ and $MP = 10$. We know that $JM + MP = JP$ and we can substitute $2MP$ in for JM to get $2MP + MP = JP$. We can further substitute 30 to get $3MP = 30$. Therefore $MP = 10$ and $JM = 20$.

■ 4. B lies between L and N. $LB = x$, $BN = 2x + 5$, and $LN = 17$. Write an equation that can be used to find the value of x. Then find x.

Solution:

The equation $x + 2x + 5 = 17$ can be used to find the value of x, and $x = 4$. Make a diagram of $\overline{LN}$ and place point B between L and N. Label the given expressions on your diagram and note that $LB + BN = LN$. Substitute your expressions to get $x + 2x + 5 = 17$.

$$x + 2x + 5 = 17$$

$$3x = 12$$

$$x = 4$$

5. $\overline{AB}$ bisects $\overline{DC}$ at E. $DC = 8$ cm, $AB = 10$ cm, and $AE = 4$ cm. Find DE and EB.

Solution:

$DE = 4$ cm and $EB = 6$ cm. Make a diagram of AB bisecting DC at E. Since $\overline{AB}$ is a bisector of $\overline{DC}$, we know that $DE = EC$. Since the whole length of DC is 8 cm, we know that $DE = 4$ because it's half of 8. Then we know that

$$AE + EB = AB$$

$$4 + EB = 10$$

$$EB = 6$$

6. P lies between M and O. $MP = 3x - 4$, $PO = 2x + 2$, and $MO = 3x + 12$. Find x and MO.

Solution:

$x = 7$ and $MO = 33$. Make a diagram of $\overline{MO}$ and place point P between M and O. Label the given expressions on your diagram and note that

$$MP + PO = MO$$

$$3x - 4 + 2x + 2 = 3x + 12$$

$$5x - 2 = 3x + 12$$

$$2x = 14$$

$$x = 7$$

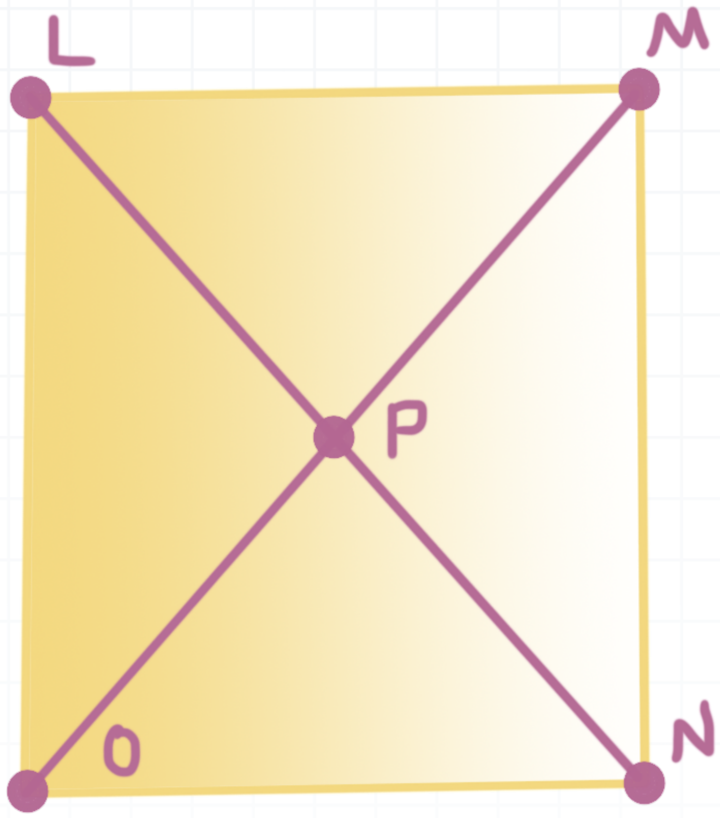
Then plug 7 into the following equation.

$$MO = 3x + 12$$

$$MO = 3(7) + 12$$

$$MO = 33$$

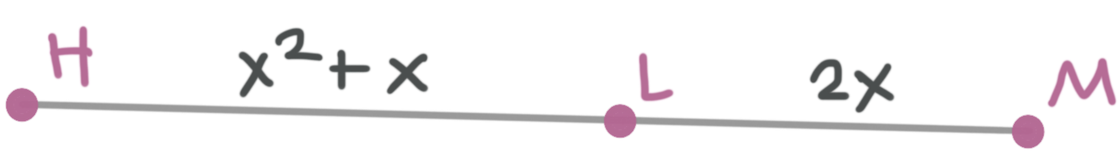
■ 7. The diagonals of a square bisect each other and are also congruent. The diagram below show diagonals $\overline{LN}$ and $\overline{MO}$ intersecting at P. Because they are bisectors, P is the midpoint of each segment. If $LP = 4.5$ inches, find MO.



Solution:

$MO=9$ inches. Since $LP=4.5$ inches, PN also equals 4.5 inches. This makes $LN=9$ inches. And because the diagonals are congruent, $LN=MO$. This make $MO=9$ inches as well.

■ 8. $HM=10$. Use the diagram to find x and HL.



Solution:

$x=2$ and $HL=6$. We know that

$$HL + LM = HM$$

$$x^2 + x + 2x = 10$$

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

$$x = -5, 2$$

Because $x = -5$ results in negative segment lengths, $x = 2$ is the only possible solution. We then substitute $x = 2$ into the expression for HL and get

$$HL = x^2 + x$$

$$HL = (2)^2 + (2)$$

$$HL = 6$$

SLOPE AND MIDPOINT OF A LINE SEGMENT

■ 1. Find the length of $\overline{AB}$ given $A(-2,3)$ and $B(4,3)$.

Solution:

$AB = 6$. The distance between points can be found using the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4 - (-2))^2 + (3 - 3)^2}$$

$$d = \sqrt{6^2 + 0^2}$$

$$d = \sqrt{36}$$

$$d = 6$$

■ 2. Find the length of $\overline{EF}$ given $E(-3, -2)$ and $F(1,1)$.

Solution:

$EF = 5$. The distance between points can be found using the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(1 - (-3))^2 + (1 - (-2))^2}$$

$$d = \sqrt{4^2 + 3^2}$$

$$d = \sqrt{25}$$

$$d = 5$$

■ 3. Find the length of $\overline{JK}$ given $J(0,6)$ and $K(2,-4)$.

Solution:

$JK = 2\sqrt{26}$. The distance between points can be found using the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

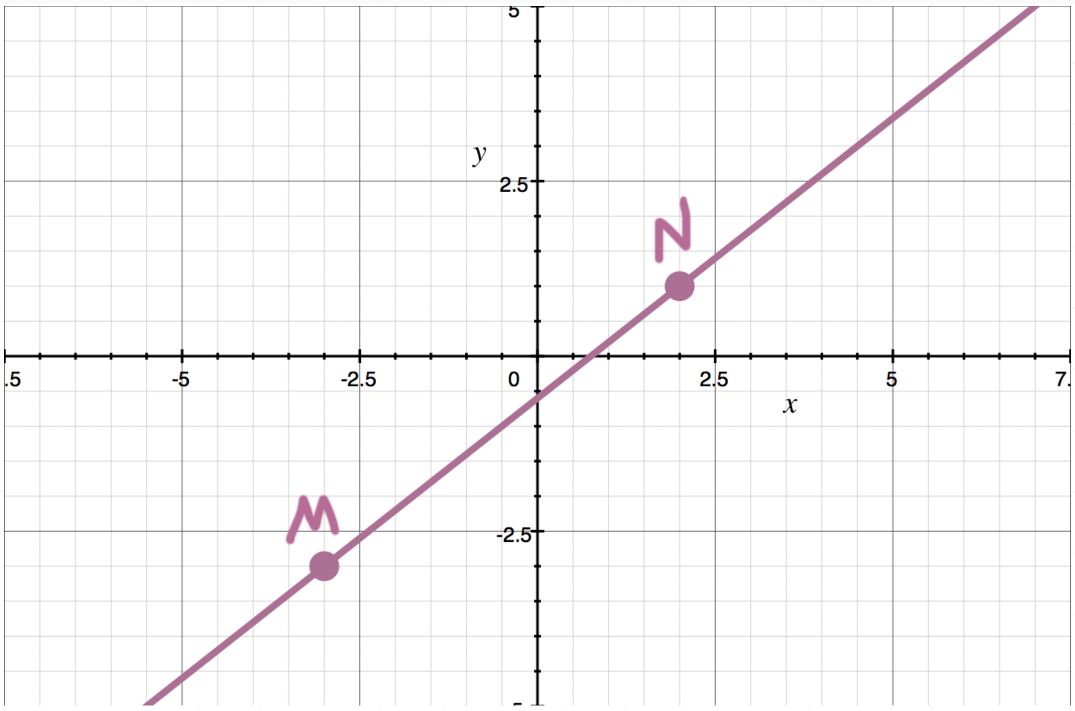
$$d = \sqrt{(2 - 0)^2 + (-4 - 6)^2}$$

$$d = \sqrt{2^2 + (-10)^2}$$

$$d = \sqrt{104}$$

$$d = 2\sqrt{26}$$

■ 4. Find the slope of line MN.



Solution:

$m=4/5$. Plug both points from the graph into the formula for the slope of a line.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-(-3)}{2-(-3)}=\frac{4}{5}$$

■ 5. Find the slope of the line passing through $S(-6,6)$ and $T(2,-4)$.

Solution:

$m = -5/4$. Plug both points into the formula for the slope of a line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 6}{2 - (-6)} = -\frac{10}{8} = -\frac{5}{4}$$

■ 6. J is the midpoint of $\overline{RF}$. Find the coordinates of J if $R(-4,6)$ and $F(0,-2)$.

Solution:

$J(-2,2)$. Use the midpoint formula and plug in the given points.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{-4 + 0}{2}, \frac{6 + (-2)}{2}\right)$$

$$(-2,2)$$

■ 7. P is the midpoint of $\overline{XY}$. Find the coordinates of X if $P(-3,6)$ and $Y(0,2)$.

Solution:

$X(-6,10)$. Using the midpoint formula, we can plug in what we know.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{x_1 + 0}{2} = -3, \frac{y_1 + 2}{2} = 6\right)$$

$$(x_1 = -6, y_1 + 2 = 12)$$

$$(x_1 = -6, y_1 = 10)$$

Therefore, the coordinates of X are $X(-6,10)$.

■ 8. E is a midpoint of $\overline{LM}$. $LE = 2x + 3$ and $LM = 6x - 4$. Find x and LM.

Solution:

$x = 5$ and $LM = 26$. Draw a diagram with E as the midpoint of $\overline{LM}$. Label your diagram with the given expressions. Because E is the midpoint,

$$LE = EM = 2x + 3$$

$$2(2x + 3) = 6x - 4$$

$$4x + 6 = 6x - 4$$

$$x = 5$$

Use substitution to find $LM = 6(5) - 4 = 26$.

PARALLEL, PERPENDICULAR, OR NEITHER

■ 1. $\overline{AB} \perp \overline{CD}$. The slope of $\overline{AB}$ is $2/3$. Find the slope of $\overline{CD}$.

Solution:

The slope of $\overline{CD}$ is $-3/2$. The symbol $\perp$ means the two line segments are perpendicular. Perpendicular lines intersect and form a right angle. The slopes of perpendicular lines are always opposite reciprocals of one another. Since the slope of $\overline{AB}$ is $2/3$, the slope of $\overline{CD}$ must be $-3/2$.

■ 2. $\overline{MN} \parallel \overline{ST}$, and the slope of $\overline{MN}$ is -2. Find the slope of $\overline{ST}$.

Solution:

$\overline{MN}$ has slope of -2. Parallel lines will never intersect and will always have the same slope.

■ 3. Are $\overline{XY}$ and $\overline{AB}$ parallel, perpendicular, or neither? $X(4,-3)$, $Y(-2,1)$, $A(1,3)$, and $B(3,6)$. Use the slopes of the lines to justify your answer.

Solution:

Perpendicular. Calculate both slopes.

$$m_{\overline{XY}} = \frac{1-(-3)}{-2-4} = -\frac{4}{6} = -\frac{2}{3}$$

$$m_{\overline{AB}} = \frac{6-3}{3-1} = \frac{3}{2}$$

Since these slopes are opposite reciprocals, the two lines are perpendicular.

■ 4. Are $\overline{EF}$ and $\overline{GH}$ parallel, perpendicular, or neither? $E(-1,4)$, $F(0,2)$, $G(-1,0)$, and $H(1,4)$. Use the slope of the lines to justify your answer.

Solution:

Neither. Calculate both slopes.

$$m_{\overline{EF}} = \frac{2-4}{0-(-1)} = \frac{-2}{1} = -2$$

$$m_{\overline{GH}} = \frac{4-0}{1-(-1)} = \frac{4}{2} = 2$$

-2 and 2 are not equal, nor are they opposite reciprocals. Therefore these lines are neither parallel nor perpendicular.

■ 5. Write the equation of a line in slope-intercept form that's perpendicular to the given line and passes through $(2,3)$.

$$y = \frac{1}{2}x + 2$$

Solution:

$y = -2x + 7$. The slope of the given line is $m = 1/2$. The slope of any line perpendicular to this one is $m = -2$. Use point slope with $m = -2$ and given point $(2,3)$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - 2)$$

$$y - 3 = -2x + 4$$

$$y = -2x + 7$$

■ 6. Write the equation of a line parallel to $y = 3x - 2$ that passes through $(0,3)$.

Solution:

$y = 3x + 3$. The slope of the given line is $m = 3$. The slope of any line parallel to this one is $m = 3$. We are also given the y-intercept of $(0,3)$. Expressed in slope-intercept form, we get $y = 3x + 3$.

■ 7. A square has opposite sides parallel and consecutive sides perpendicular and all sides are congruent. Quadrilateral $SQRE$ has coordinates $S(0,3)$, $Q(4,0)$, $R(1, -4)$, and $E(-3, -1)$. Determine whether or not $SQRE$ is a square by showing that the opposite sides are parallel and consecutive sides are perpendicular and that all sides are congruent.

Solution:

$SQRE$ is a square because the slopes of $\overline{SQ}$, $\overline{QR}$, $\overline{RE}$, and $\overline{ES}$ are $4/3$, $-3/4$, $4/3$, and $-3/4$ respectively, and the length of each side is 5.

■ 8. A square has opposite sides parallel and consecutive sides perpendicular and all sides are congruent. Quadrilateral $SQRE$ has coordinates $S(0,3)$, $Q(4,0)$, $R(1, -4)$, and $E(-3, -1)$. Determine if the diagonals of the square are perpendicular. Determine if the diagonals are congruent.

Solution:

The diagonals are both perpendicular and congruent. $\overline{SR}$ has slope -7 and $\overline{EQ}$ has slope $1/7$. The diagonals are also congruent with $SR = 5\sqrt{2} = EQ$.

DISTANCE BETWEEN TWO POINTS IN TWO DIMENSIONS

■ 1. Find the length of $\overline{GH}$ given $G(-2,1)$ and $H(4,1)$.

Solution:

$GH = 6$. Use the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-2 - 4)^2 + (1 - 1)^2}$$

$$d = \sqrt{(-6)^2 + 0^2}$$

$$d = \sqrt{36}$$

$$d = 6$$

■ 2. Find the length of $\overline{XY}$ given $X(-4,1)$ and $Y(0,2)$.

Solution:

$XY = \sqrt{17}$. Use the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-4 - 0)^2 + (1 - 2)^2}$$

$$d = \sqrt{16 + 1}$$

$$d = \sqrt{17}$$

■ 3. Find the perimeter of $\triangle EFG$ if $E(1,1)$, $F(1,6)$, and $G(5,4)$.

Solution:

The perimeter of $\triangle EFG = 10 + 2\sqrt{5}$. Using the distance formula, we can find the length of each side.

$$EF = \sqrt{(1 - 1)^2 + (1 - 6)^2} = \sqrt{25} = 5$$

$$FG = \sqrt{(6 - 4)^2 + (1 - 5)^2} = \sqrt{20} = 2\sqrt{5}$$

$$GE = \sqrt{(4 - 1)^2 + (5 - 1)^2} = \sqrt{25} = 5$$

Then the perimeter is

$$EF + FG + GE = 10 + 2\sqrt{5}$$

■ 4. Find the area of square $ABCD$ given $A(-8,0)$, $B(0,6)$, $C(6,-2)$, and $D(-2,-8)$.

Solution:

The area of square $ABCD$ is 100 units^2. Calculate the length of two adjacent sides.

$$AB = \sqrt{(6-0)^2 + (0-(-8))^2} = \sqrt{100} = 10$$

$$BC = \sqrt{((-2)-6)^2 + (6-0)^2} = \sqrt{100} = 10$$

$$CD = \sqrt{((-2)-(-8))^2 + (6-((-2))^2} = \sqrt{100} = 10$$

$$DA = \sqrt{(0-(-8))^2 + ((-8)-(-2))^2} = \sqrt{100} = 10$$

Then the area of the square is

$$A = lw = (10)(10) = 100$$

DISTANCE BETWEEN TWO POINTS IN THREE DIMENSIONS

■ 1. Find the distance between points with coordinates $(3,8,0)$ and $(3,8,6)$.

Solution:

$d=6$. Use the distance formula for three dimensions.

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$d=\sqrt{(3-3)^2+(8-8)^2+(0-6)^2}$$

$$d=\sqrt{36}$$

$$d=6$$

■ 2. Find the distance between points with coordinates $(2,5,-3)$ and $(2,8,1)$.

Solution:

$d=5$. Use the distance formula for three dimensions.

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$d = \sqrt{(2-2)^2 + (5-8)^2 + (-3-1)^2}$$

$$d = \sqrt{25}$$

$$d = 5$$

■ 3. Find the distance between points with coordinates $(1,1,1)$ and $(5,5,5)$.

Solution:

$d = \sqrt{48} = 4\sqrt{3} \approx 6.93$. Use the distance formula for three dimensions.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(1-5)^2 + (1-5)^2 + (1-5)^2}$$

$$d = \sqrt{48}$$

$$d = 4\sqrt{3}$$

$$d \approx 6.93$$

■ 4. Find the distance between points with coordinates $(9,6,3)$ and $(-9,-6,-3)$.

Solution:

$d = \sqrt{504} = 6\sqrt{14} \approx 22.45$. Use the distance formula for three dimensions.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{((-9) - 9)^2 + ((-6) - 6)^2 + ((-3) - 3)^2}$$

$$d = \sqrt{324 + 144 + 36}$$

$$d = \sqrt{504}$$

$$d = 6\sqrt{14}$$

$$d \approx 22.45$$

MIDPOINT OF A LINE SEGMENT IN THREE DIMENSIONS

■ 1. Find the midpoint between points with coordinates $(3,8,0)$ and $(3,8,6)$.

Solution:

$(3,8,3)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a, b, c) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$(a, b, c) = \left(\frac{3 + 3}{2}, \frac{8 + 8}{2}, \frac{0 + 6}{2}\right)$$

$$(a, b, c) = (3,8,3)$$

■ 2. Find the midpoint between points with coordinates $(2,5,-3)$ and $(2,8,1)$.

Solution:

$(2,6.5,-1)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{2+2}{2},\frac{5+8}{2},\frac{-3+1}{2}\right)$$

$$(a,b,c)=(2,6.5,-1)$$

■ 3. Find the midpoint between points with coordinates $(1,1,1)$ and $(5,5,5)$.

Solution:

$(3,3,3)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{1+5}{2},\frac{1+5}{2},\frac{1+5}{2}\right)$$

$$(a,b,c)=(3,3,3)$$

4. Find the midpoint between points with coordinates $(9,6,3)$ and $(-9,-6,-3)$.

Solution:

$(0,0,0)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{9+(-9)}{2},\frac{6+(-6)}{2},\frac{3+(-3)}{2}\right)$$

$$(a,b,c)=(0,0,0)$$

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