

# Translating figures in coordinate space

In this lesson we'll look at translation of a figure in a coordinate plane and how to determine where the figure is located after the translation takes place.

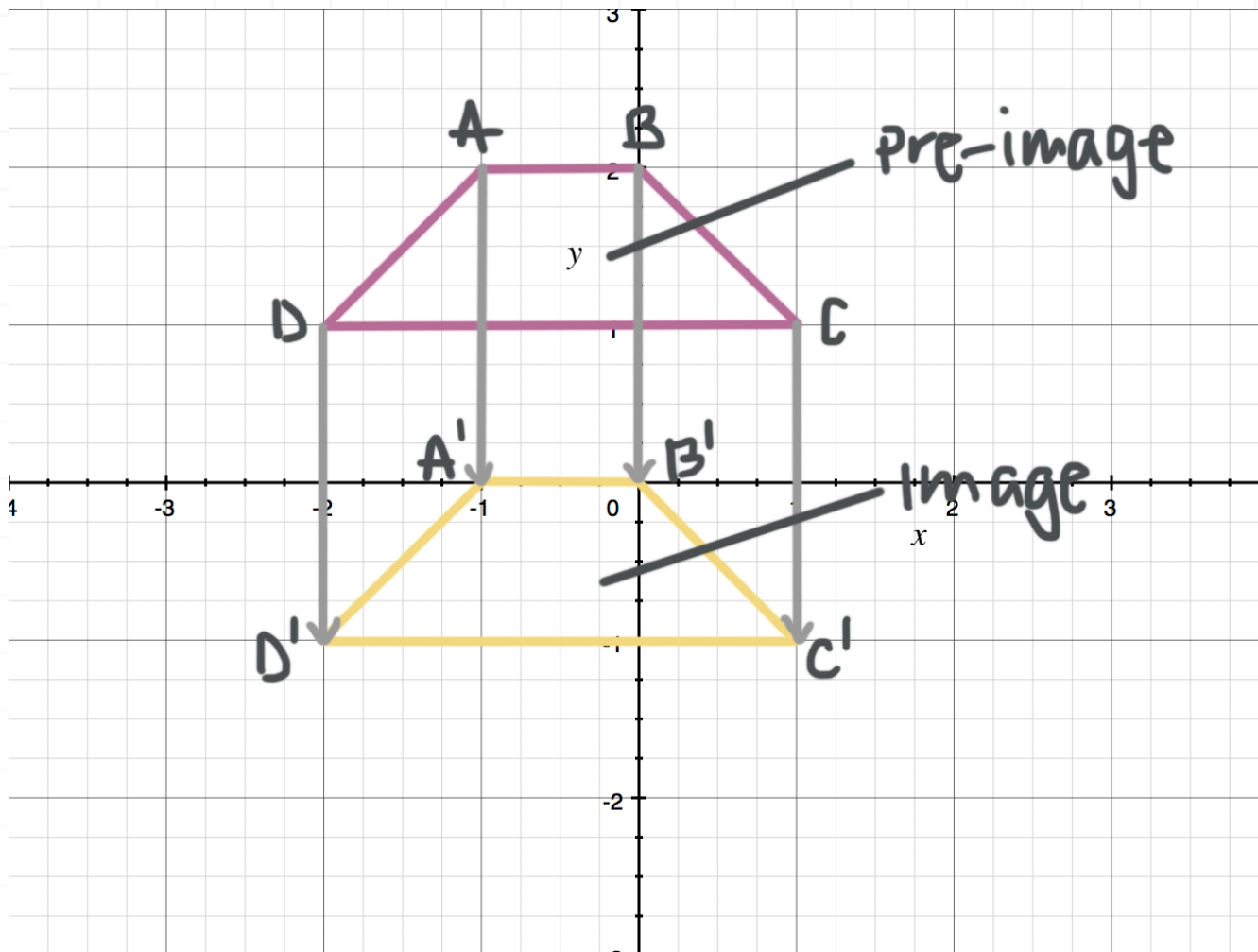
A **translation** is a transformation that moves a figure from one location to another. A translation can be thought of as a slide with no rotation. The slide won't change the shape or size of the figure, and because there's no rotation, the orientation won't change either.

## The pre-image and image

Before a translation, we have the **pre-image** (the figure in its original location and orientation). Points in the pre-image are usually labeled with capital letters. After the translation, we have the **image** (the figure in its final location and orientation). Points in the image are usually labeled with the same capital letters, plus the prime symbol ' after each letter. So if figure  $ABCD$  is translated, its image becomes figure  $A'B'C'D'$ .

In a translation, the image and pre-image are always congruent, because a translation never changes the measures of angles or the lengths of line segments and curves in the figure.





## Translation notation

A translation can be vertical, horizontal, or both. Regardless of the direction, a translation can be written in mathematical notation. We'll use a "rule"  $T(x, y)$  that express the coordinates of a point in the image in terms of the coordinates  $(x, y)$  of the corresponding point in the pre-image.

A translation 3 units to the **left**:

$$T(x, y) = (x - 3, y)$$

A translation 2 units to the **right**:

$$T(x, y) = (x + 2, y)$$

A translation 4 units **down**:

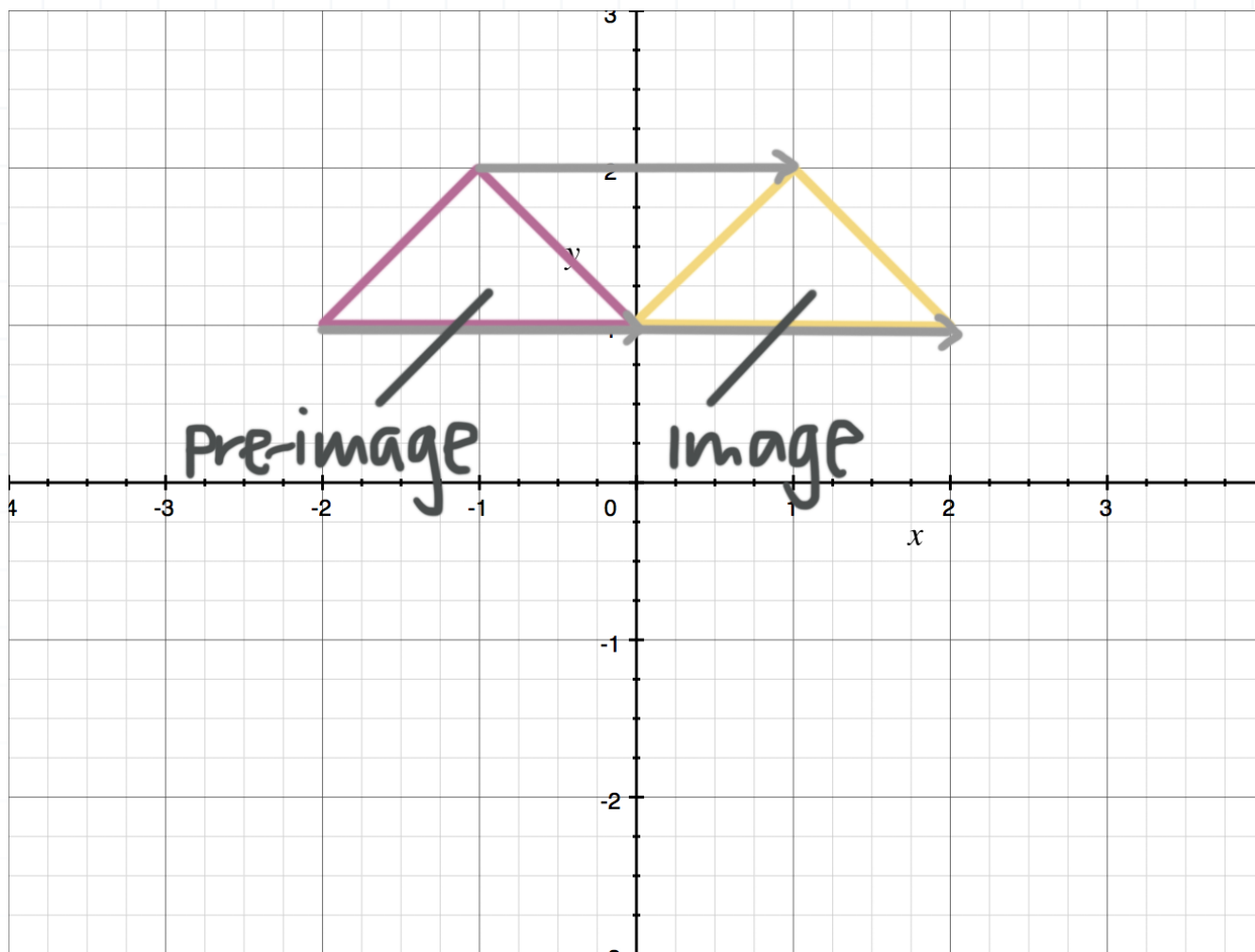
$$T(x, y) = (x, y - 4)$$

A translation 1 unit **up**:

$$T(x, y) = (x, y + 1)$$

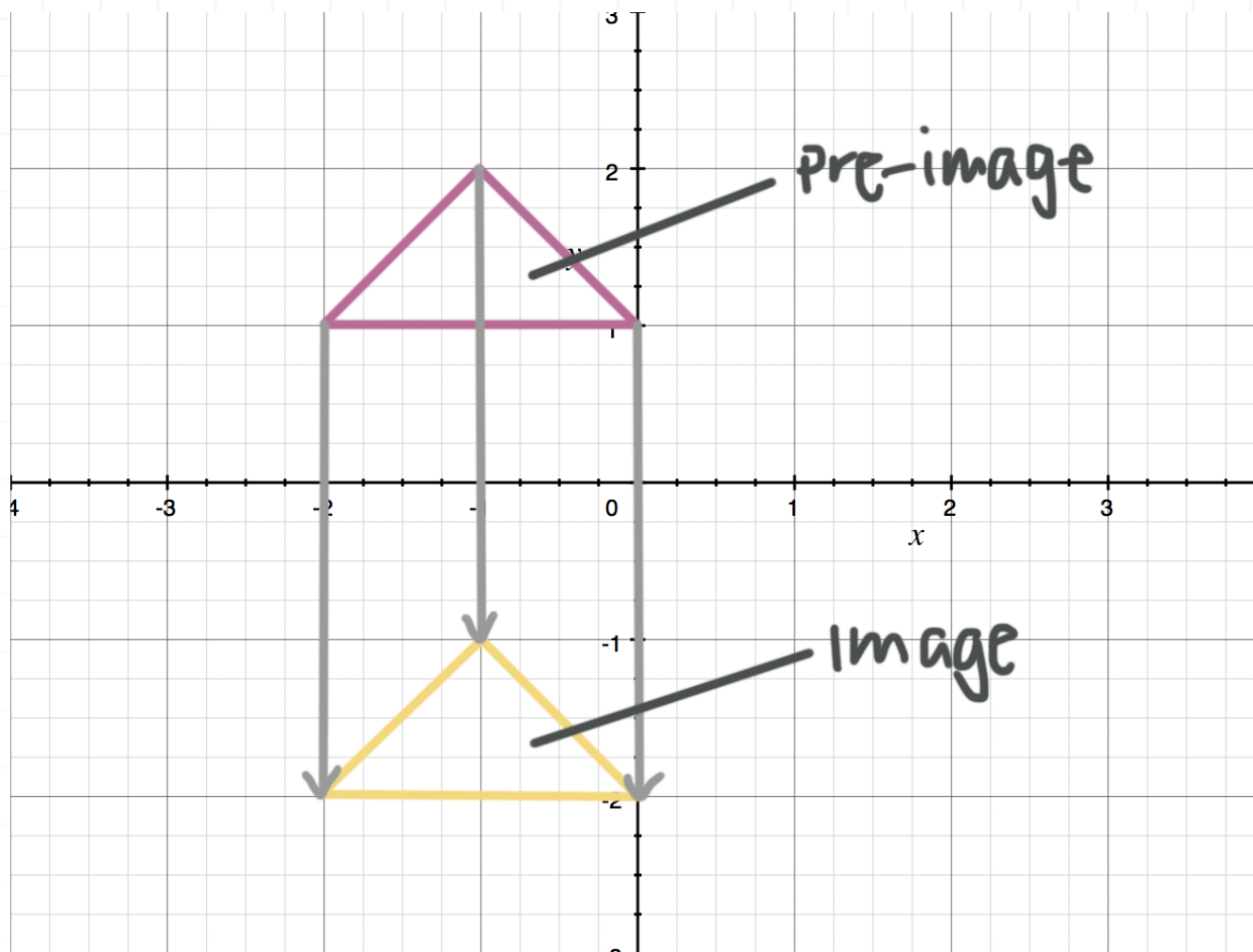


In the translation of the triangle in this figure, the pre-image is translated 2 units to the right to get the image, so  $T(x, y) = (x + 2, y)$ .



In the translation of the triangle in the next figure, the pre-image is translated 4 units down to get the image, so  $T(x, y) = (x, y - 4)$ .



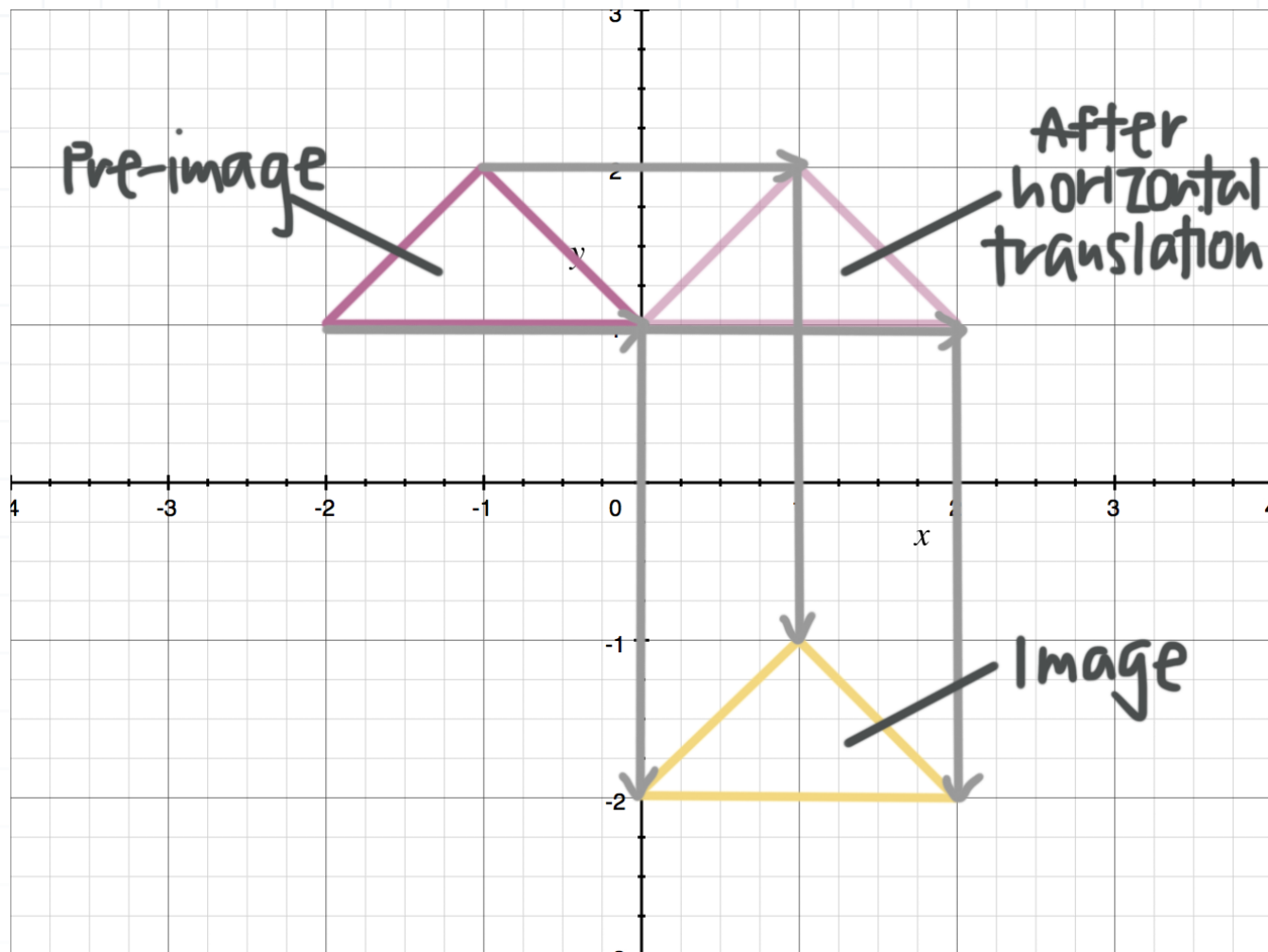


## Two translations together

We said that we can do a translation that's both horizontal and vertical. If we want to translate a figure 2 units to the right and 4 units down, then the rule for the translation is

$$T(x, y) = (x + 2, y - 4)$$



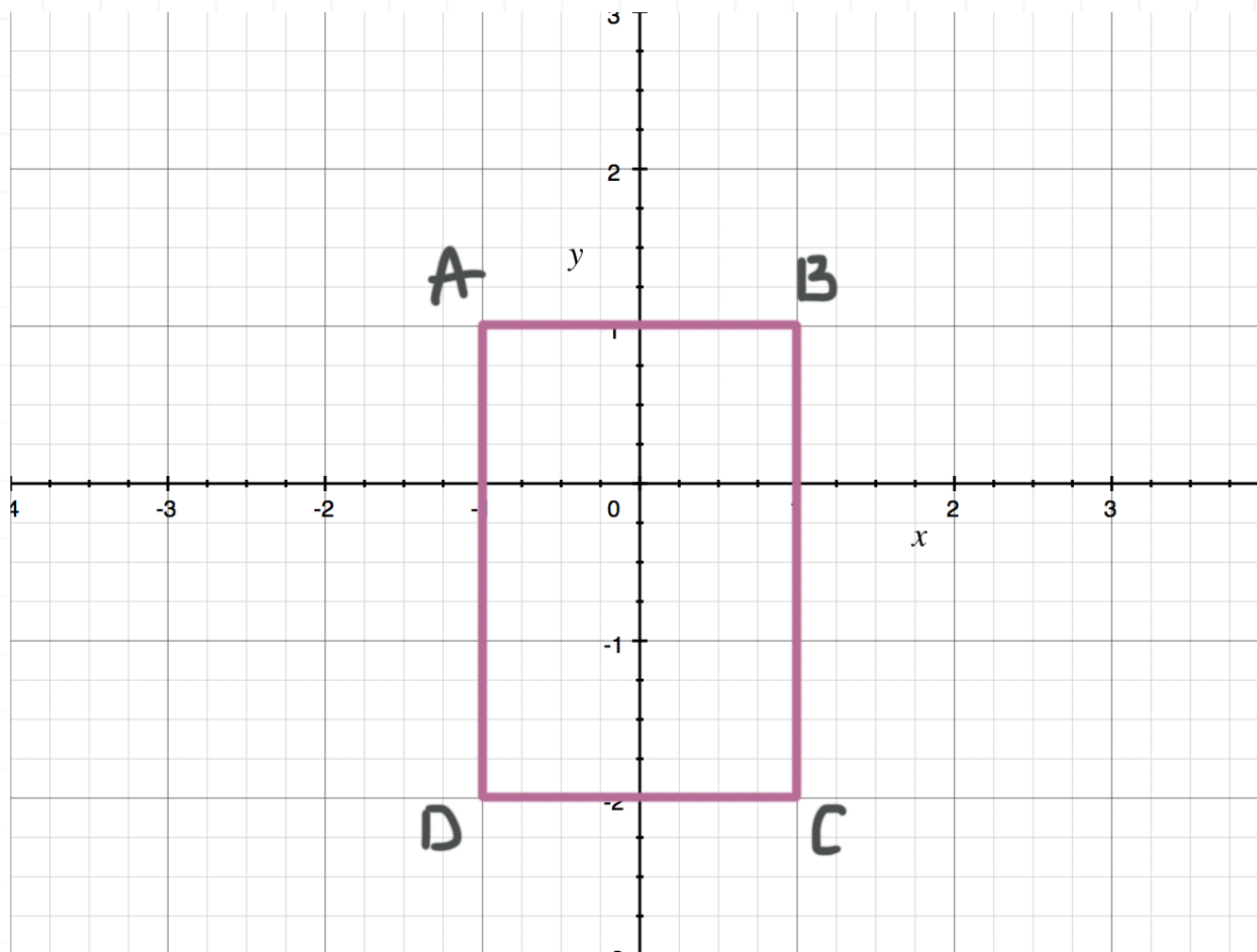


Let's start by working through an example.

### Example

If rectangle  $ABCD$  undergoes the translation described by  $T(x, y) = (x - 2, y + 3)$ , where will point  $D'$  be located?





The translation is  $T(x, y) = (x - 2, y + 3)$ . The  $x - 2$  tells you that the  $x$ -coordinate of any point in the image will be 2 less than the  $x$ -coordinate of the corresponding point in the pre-image, and that the  $y$ -coordinate of any point in the image will be 3 more than the  $y$ -coordinate of the corresponding point in the pre-image.

In other words, after the translation the figure will be located 2 units to the left of and 3 units above, its original location. The coordinates of point  $D$  are  $(-1, -2)$ , so the coordinates of point  $D'$  are given by

$$T(-1, -2) = (-1 - 2, -2 + 3) = (-3, 1)$$

So  $D' = (-3, 1)$ .



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Let's try another translation problem.

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### Example

If a translation moves a point  $A$  to a point  $A'$ , write the rule for the translation if  $A = (-3, 7)$  and  $A' = (5, -2)$ .

Let's look at what happens to each coordinate.

The  $x$ -coordinate:  $-3 \rightarrow 5$

This means we added 8 because  $-3 + 8 = 5$ .

The  $y$ -coordinate:  $7 \rightarrow -2$

This means we subtracted 9 because  $7 - 9 = -2$ .

Now we can put this all together to write the rule for the translation.

$$T(x, y) = (x + 8, y - 9)$$

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