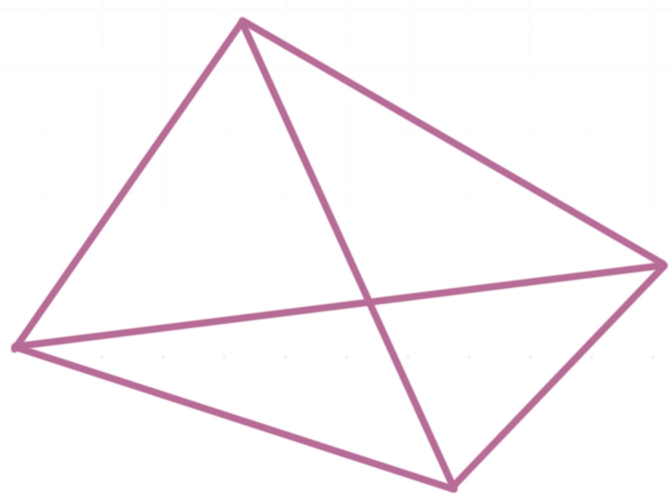
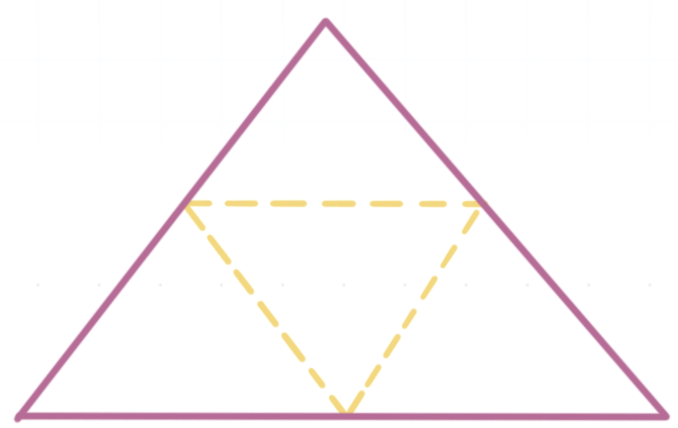
Nets/volume/surface area of pyramids

A **pyramid** has one base, in the shape of any polygon, and the rest of the faces are triangles. Each side of the base coincides with a side of exactly one of the triangular faces, and there's exactly one point of a pyramid at which all of the triangular faces intersect.

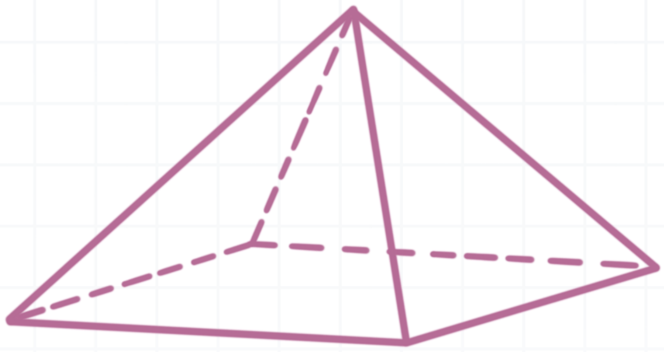
A pyramid is named by the shape of its base. So a **triangular pyramid** is a pyramid with a triangular base, a **rectangular pyramid** is a pyramid with a rectangular base, a **square pyramid** is a pyramid with a square base, etc.

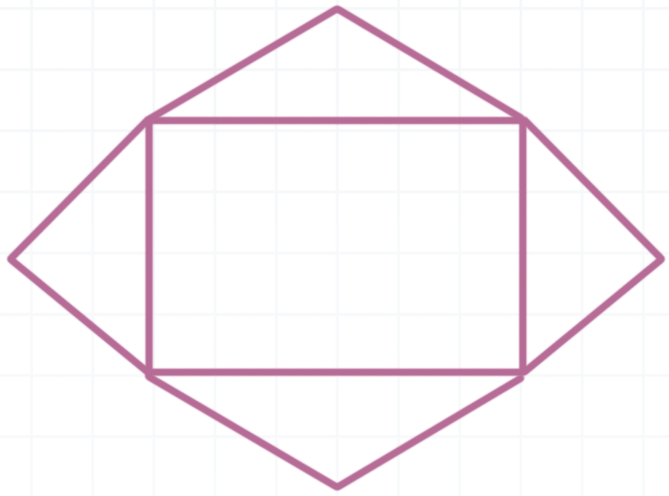
This is a triangular pyramid and its net. The net of a triangular pyramid has three triangular faces and one triangular base.





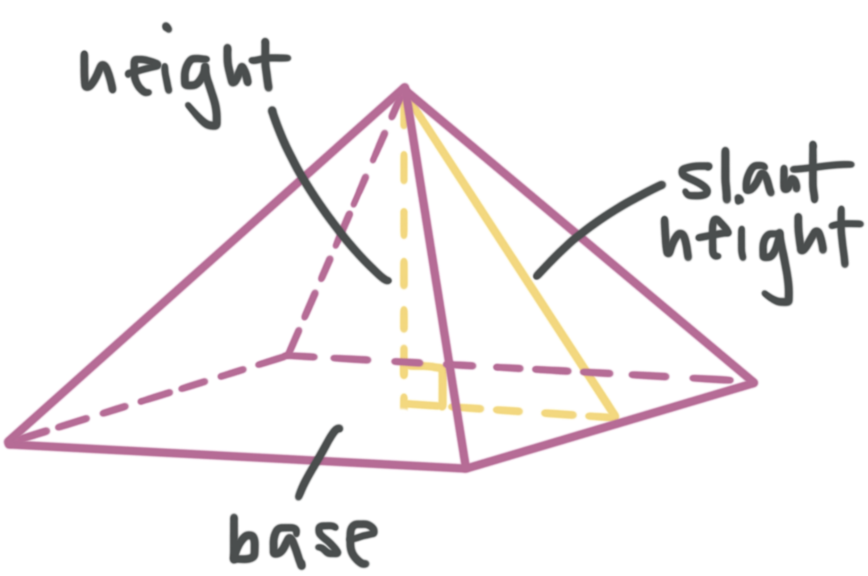
This is a rectangular pyramid and its net. The net of a rectangular pyramid has four triangular faces and one rectangular base.





You'll need to know the names for the different parts of a pyramid. The **apex** of a pyramid is the point at which all the triangular faces intersect. A **right pyramid** (the only kind of pyramid we're dealing with in this lesson) is one in which the line segment from the apex to the center of the base is perpendicular to the base.

The **height** of a right pyramid is the length of that line segment. A **right regular pyramid** is one in which all the triangular faces are congruent. The **slant height** of a right regular pyramid, which is the length of a line segment from the apex of they pyramid to the midpoint of any side of the base, is also called the **lateral height**, and is often represented in formulas with the variable l.



Volume and surface area

The volume of a pyramid is given by

$$V = \frac{1}{3}Bh$$

where V is the volume of the pyramid, B is the area of the base of the pyramid, and h is the height of the pyramid.

A pyramid has a surface area given by

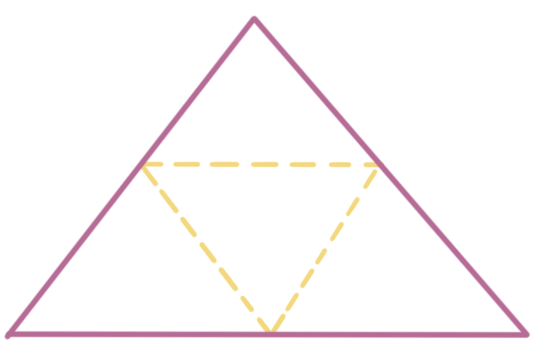
$$S = \frac{1}{2}lp + B$$

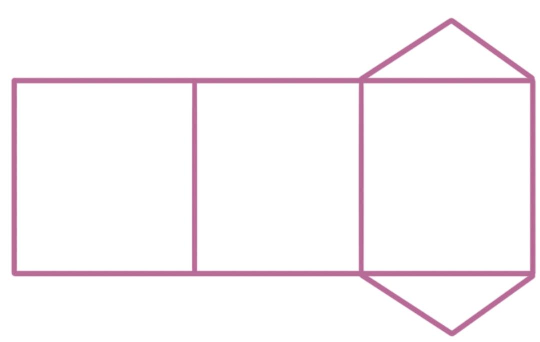
where p is the perimeter of the base, l is the slant height of the pyramid, and B is the area of the base.

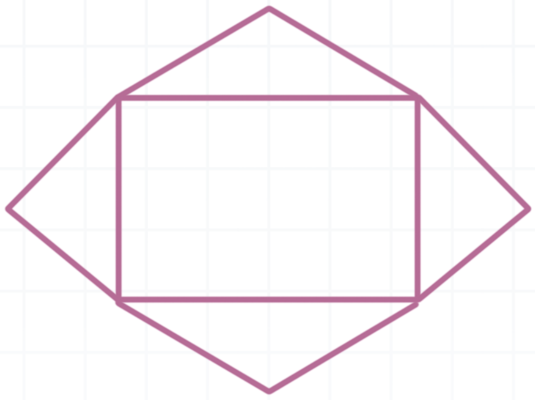
Let's work through a few examples.

Example

Which net does not belong to a pyramid?

A. 

B. 



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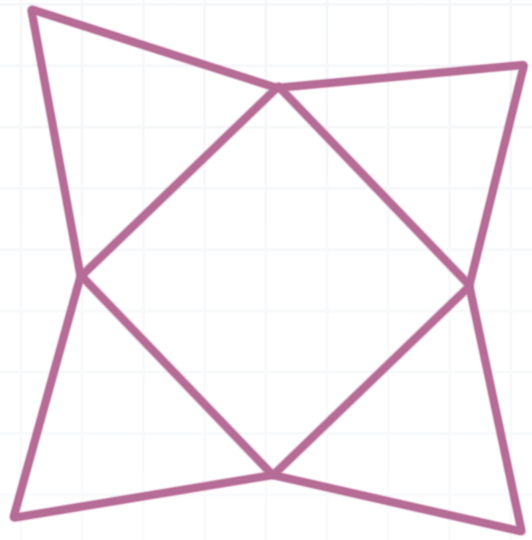
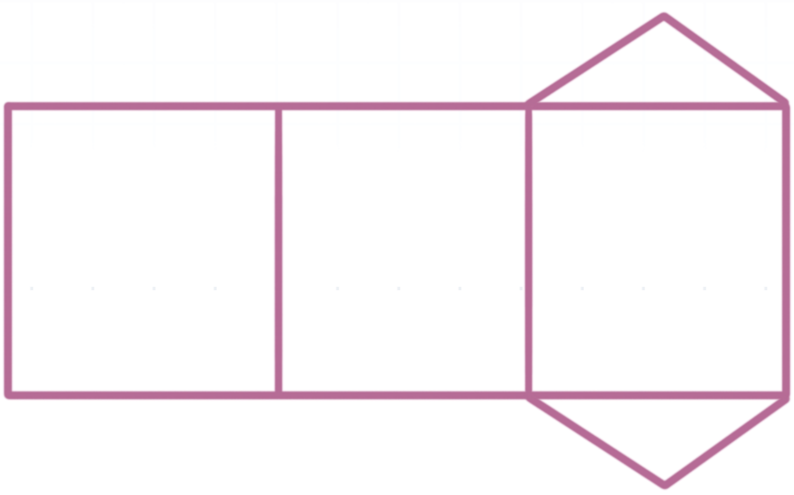


D.

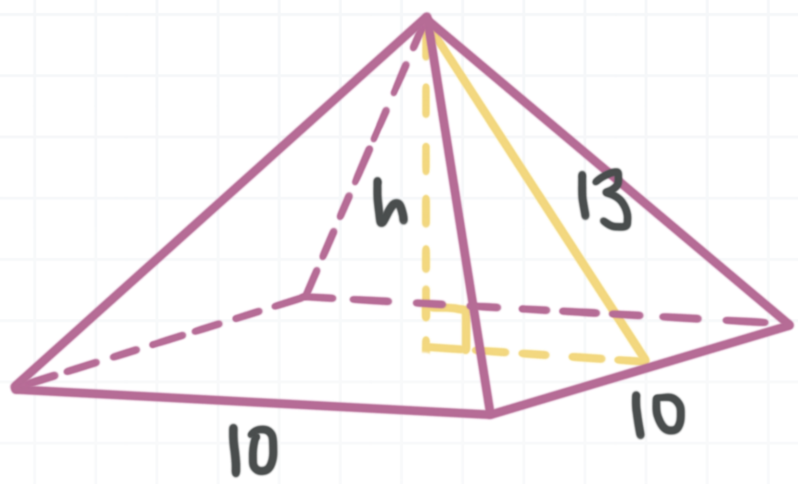
Net B can't be a net of a pyramid, because it has only two triangular faces. The base of a pyramid is a polygon, so the base has at least three sides. Each side of the base of a pyramid coincides with a side of exactly one of the triangular faces, so a pyramid has at least three triangular faces.



Let's look at another example.

Example

What is the volume of the square pyramid, which has a 10×10 base and a lateral height of 13?



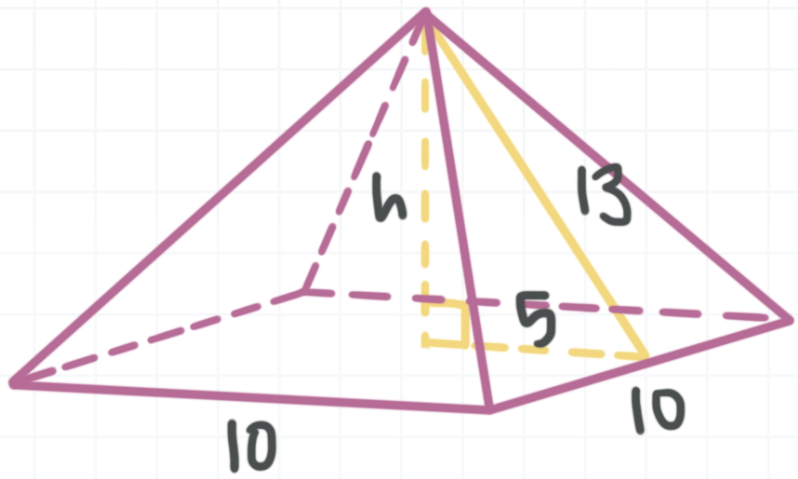
The volume of a pyramid is

$$V = \frac{1}{3}Bh$$

The area of the square base is $B = (10)(10) = 100$, and we need to use the Pythagorean Theorem to find the height of the pyramid.

To find the height h (the length of the line segment from the apex of the pyramid to the center of the base), we can use a right triangle that has that line segment as one of its legs, and where the second leg is a line segment from the center of the base of the pyramid to the midpoint of a side of its base.

The length of a side of the base of this pyramid is 10, so the length of the second leg of the right triangle is 5. Then the length of the hypotenuse of the right triangle is equal to the lateral height l of the pyramid, which is 13.



$$5^2 + h^2 = 13^2$$

$$25 + h^2 = 169$$

$$h^2 = 144$$

$$h = 12$$

Now we'll plug the values of B and h into the volume formula.

$$V = \frac{1}{3}Bh$$

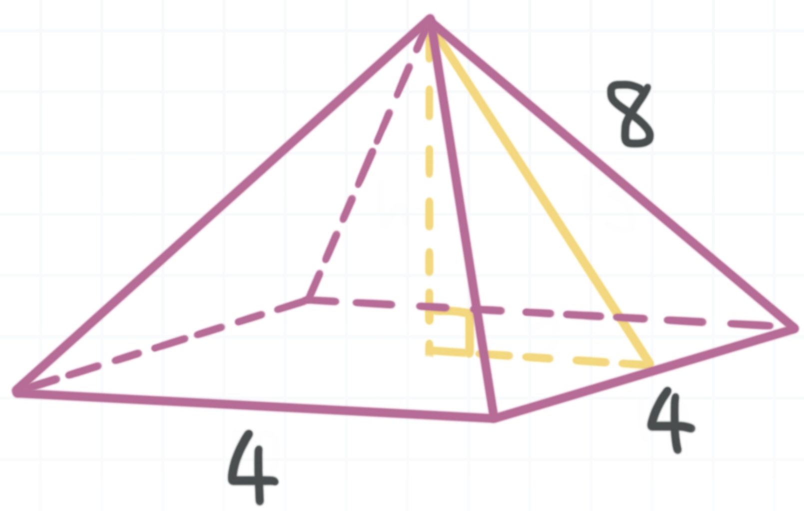
$$V = \frac{1}{3}(100)(12)$$

$$V = 400$$

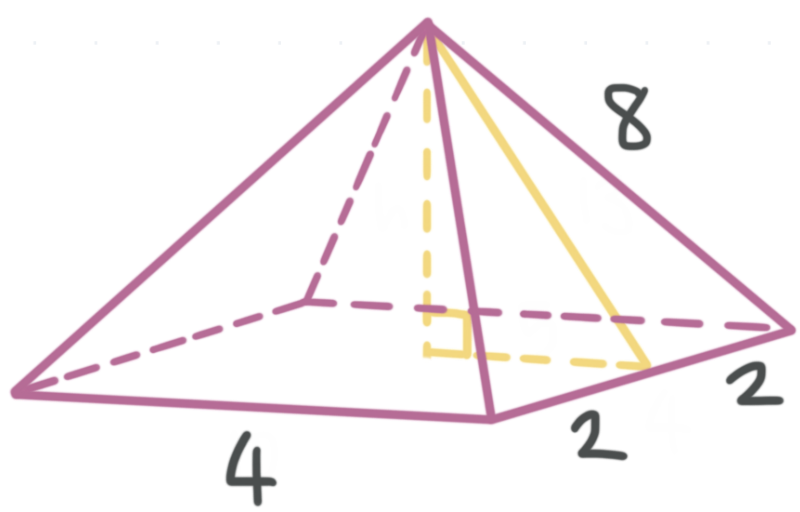
Let's do a problem with surface area.

Example

What is the surface area of a square pyramid with a base of 4 cm by 4 cm if the length of a line segment from the apex of the pyramid to a vertex of its base (the length of an edge of the pyramid) is 8 cm?



First we need to find the lateral height by using the Pythagorean theorem, applied to the right triangle that's laying on the face of the pyramid, formed by the slant height down the center of the face, the slant down the edge of the pyramid, and half of the edge of the base.



$$(2 \text{ cm})^2 + l^2 = (8 \text{ cm})^2$$

$$4 \text{ cm}^2 + l^2 = 64 \text{ cm}^2$$

$$l^2 = 60 \text{ cm}^2$$

$$l = \sqrt{60 \text{ cm}^2}$$

$$l = \sqrt{4 \cdot 15 \text{ cm}^2}$$

$$l = 2\sqrt{15} \text{ cm}$$

Since this is a square pyramid, the perimeter of the base is four times the length of a side of the base:

$$p = 4(4 \text{ cm}) = 16 \text{ cm}$$

And the area of the base is

$$B = (4 \text{ cm})^2 = 16 \text{ cm}^2$$

Now we'll plug everything into the surface area formula.

$$S = \frac{1}{2}lp + B$$

$$S = \frac{1}{2}(2\sqrt{15} \text{ cm})(16 \text{ cm}) + 16 \text{ cm}^2$$

$$S = (16\sqrt{15} + 16) \text{ cm}^2$$