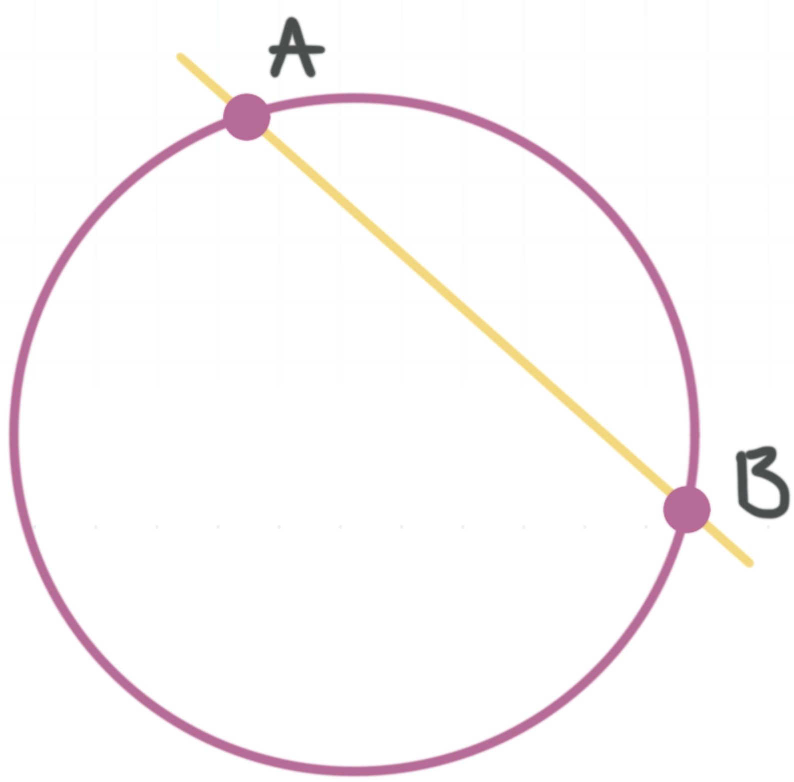
Intersecting tangents and secants

In this lesson we'll look at the relationships formed from intersecting tangents and secants in circles.

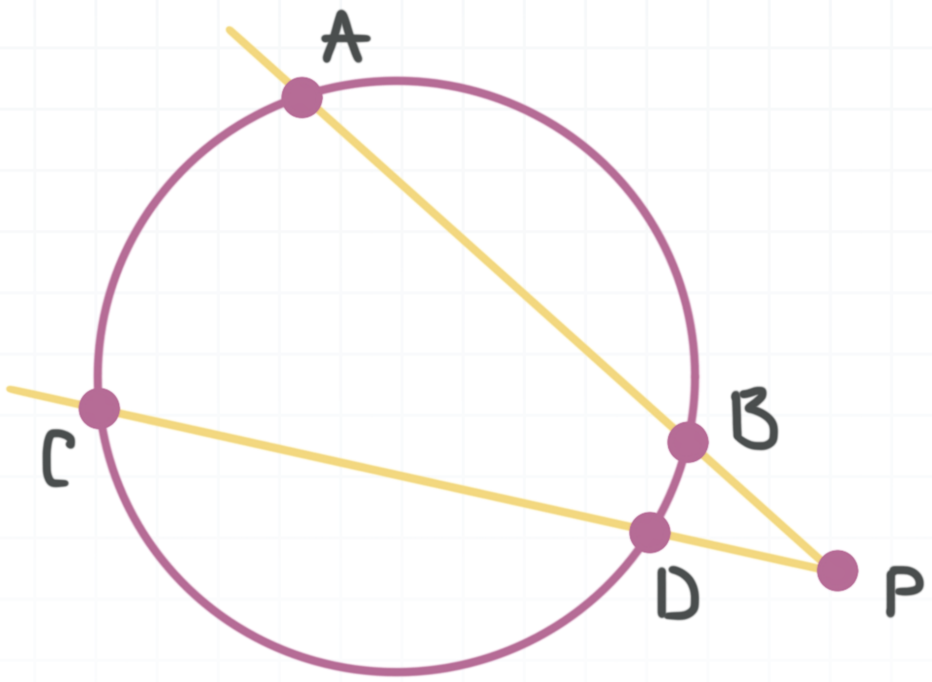
Secant

A **secant** of a circle is a line or line segment that intersects the circle at two points. $\overline{AB}$ is a secant of this circle.



Intersecting secants theorem

There's a special relationship between two secants that intersect at some point P outside a circle. The product of the lengths of the "outside" and "whole" segments of one of the secants is equal to the product of the lengths of the "outside" and "whole" segments of the other secant. In the following circle, secants $\overline{AP}$ and $\overline{CP}$ intersect at point P,



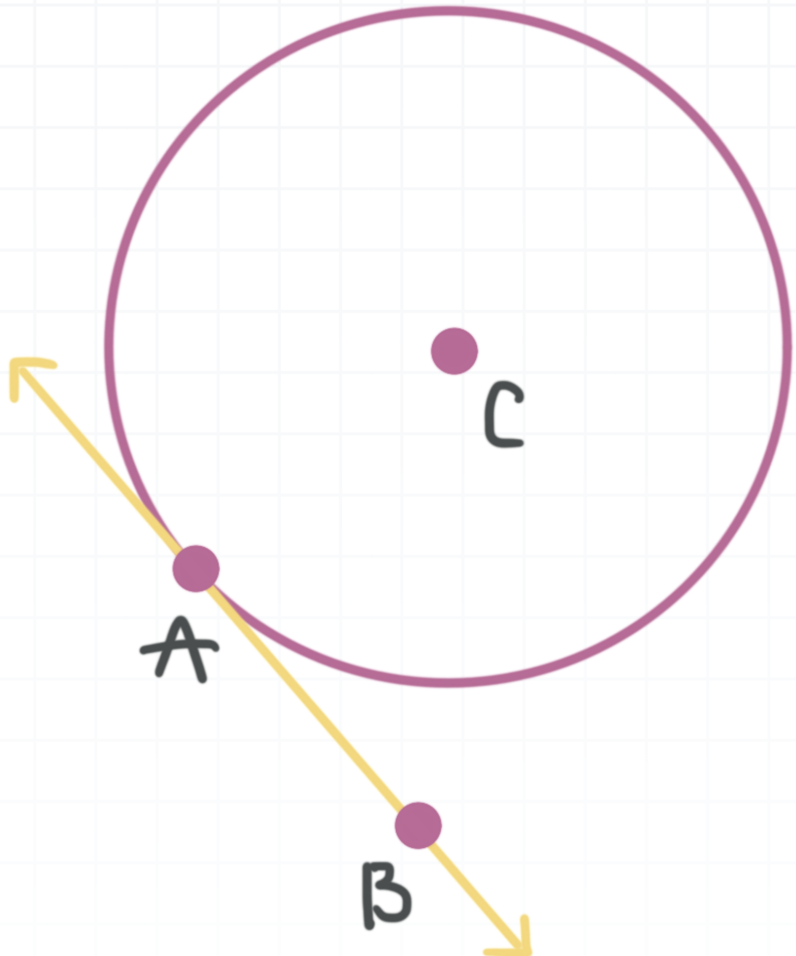
so

outside · whole = outside · whole

$$\overline{BP} \cdot \overline{AP} = \overline{DP} \cdot \overline{CP}$$

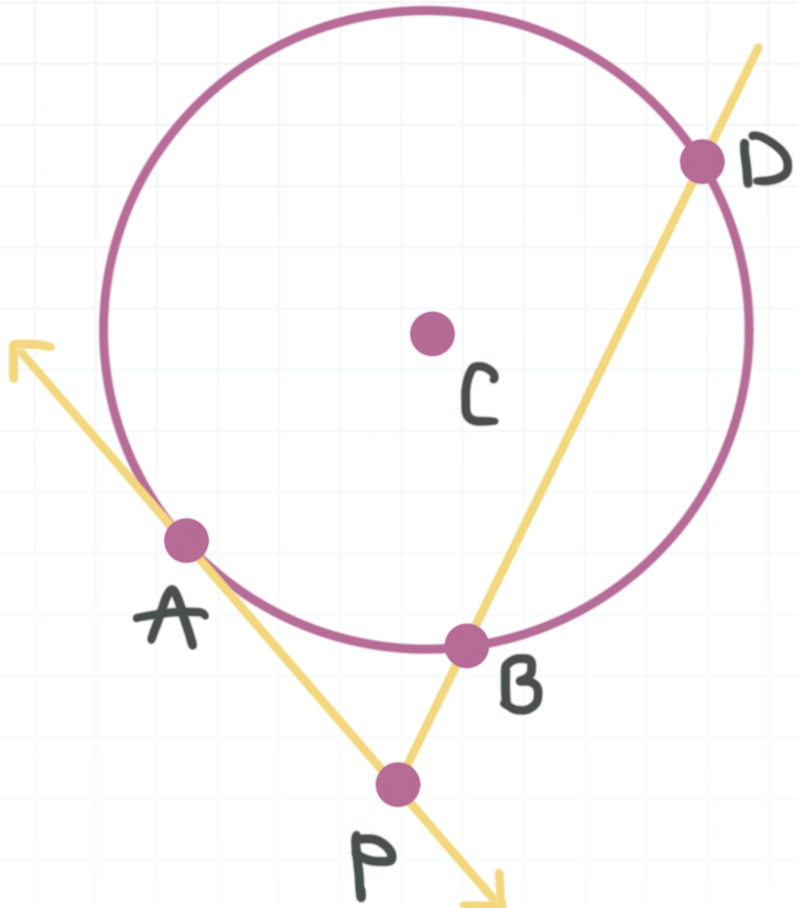
Tangents

A **tangent** of a circle is a line that intersects the circle at only one point. $\overleftrightarrow{AB}$ is a tangent of this circle.



Intersecting tangent-secant theorem

There is also a special relationship between a tangent and a secant that intersect at some point P outside a circle. The square of the length of (the segment of) the tangent from point P to the point of tangency is equal to the product of the lengths of the "outside" and "whole" segments of the secant. In the following circle, tangent $\overleftrightarrow{AP}$ and secant $\overline{DP}$ intersect at point P,



so

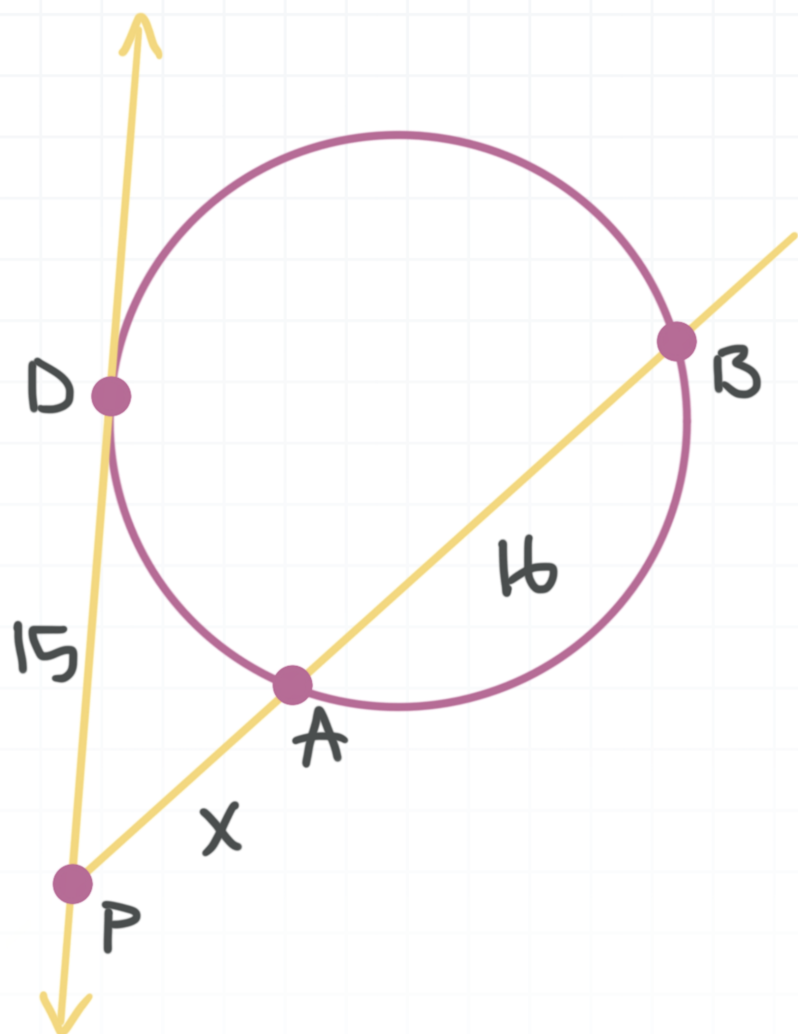
$$\text{tangent}^2 = \text{outside} \cdot \text{whole}$$

$$(\overline{AP})^2 = \overline{BP} \cdot \overline{DP}$$

Let's start by working through an example.

Example

Find the value of x in the figure, assuming that $\overleftrightarrow{DP}$ is a tangent.



Because there's a secant intersecting with a tangent, we can follow the formula and plug in the lengths shown in the figure.

$$\text{tangent}^2 = \text{outside} \cdot \text{whole}$$

$$15^2 = x(x + 16)$$

$$225 = x^2 + 16x$$

$$0 = x^2 + 16x - 225$$

$$0 = (x + 25)(x - 9)$$

$$0 = x + 25 \text{ or } 0 = x - 9$$

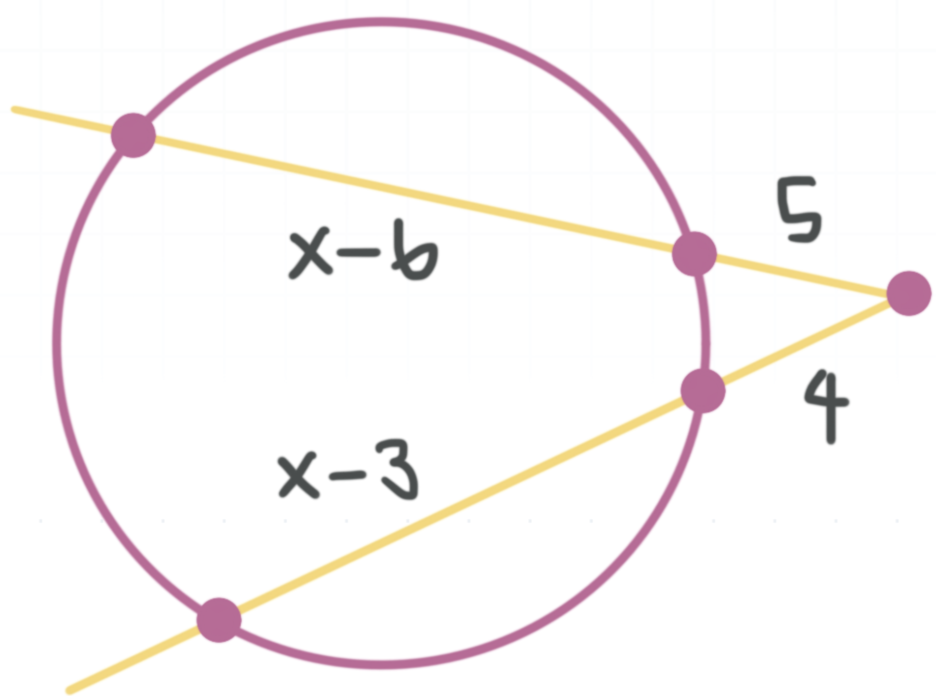
$x = -25$ or $x = 9$

A line segment can't have a negative length, so rule out $x = -25$ and conclude that $x = 9$.

Let's do one more problem.

Example

Given the lengths in the figure, find the value of x.



Because there are two secants that intersect outside the circle, we can follow the formula and plug in the lengths shown in the figure.

outside · whole = outside · whole

$$5[(x-6)+5] = 4[(x-3)+4]$$

$$5(x - 1) = 4(x + 1)$$

$$5x - 5 = 4x + 4$$

$$x - 5 = 4$$

$$x = 9$$