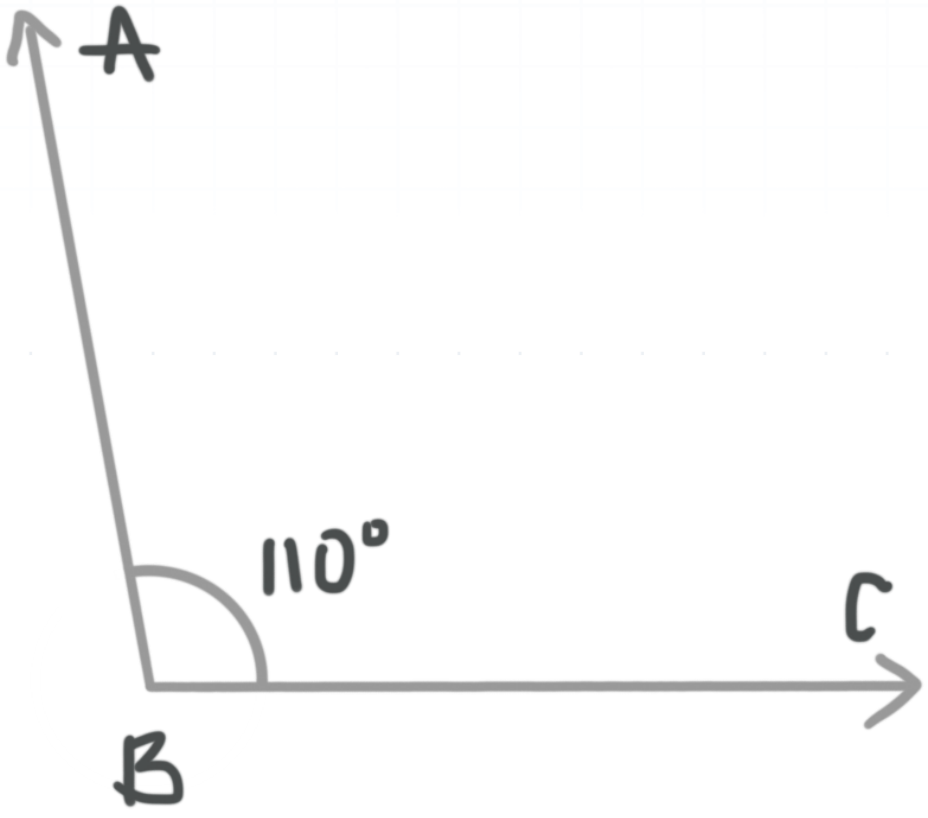
Measures of angles

In this lesson we'll look at how to find the measures of angles, in degrees, algebraically.

The measure of angles

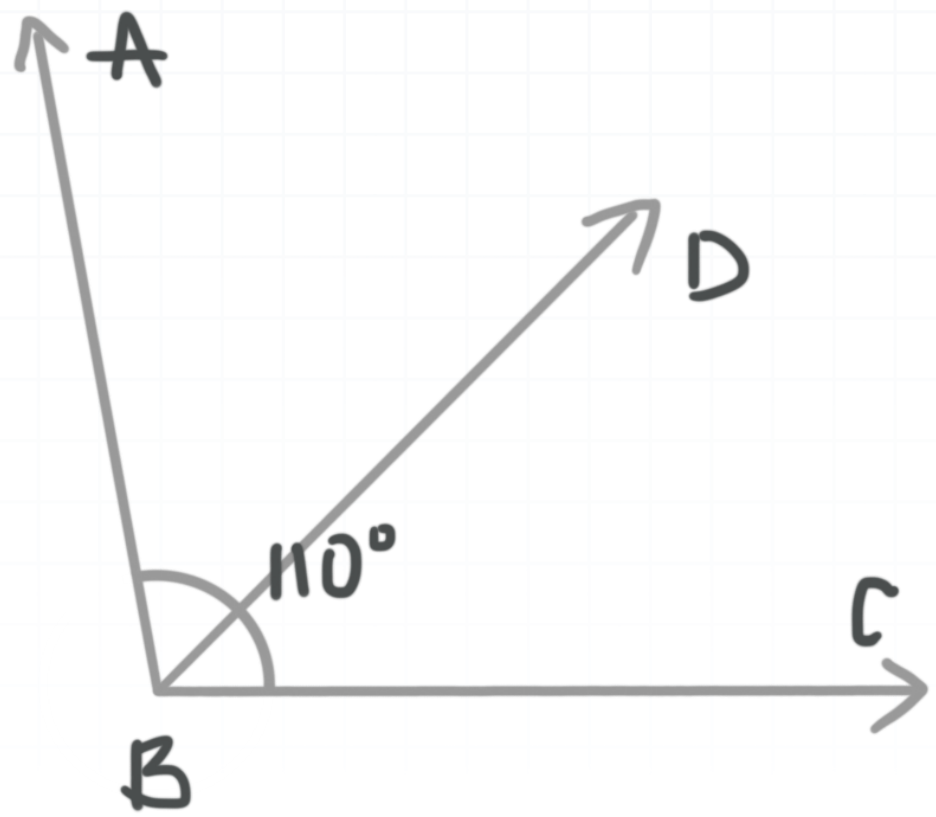
An angle is a fraction of a circle. The measure of an angle is the size of the "turn" (rotation) that's needed to get from the ray that forms one side of the angle to the ray that forms its other side. Angles are measured in degrees (or in radians, which you'll learn about if you study trigonometry).



The name of this angle is $\angle ABC$. When we talk about the measure of an angle, we use an m in front of the angle sign. For this angle, which has a measure of $110°$, we write $m\angle ABC = 110°$.

Angle addition

The individual parts of an angle add together to form the entire angle. For instance, in this figure,



the two smaller angles add together to equal the larger angle.

$$m\angle ABC = m\angle ABD + m\angle DBC$$

Which means that, if you know $m\angle DBC = 55^\circ$ and $m\angle ABC = 110^\circ$, you can find $m\angle ABD$ using angle addition.

$$m\angle ABC = m\angle ABD + m\angle DBC$$

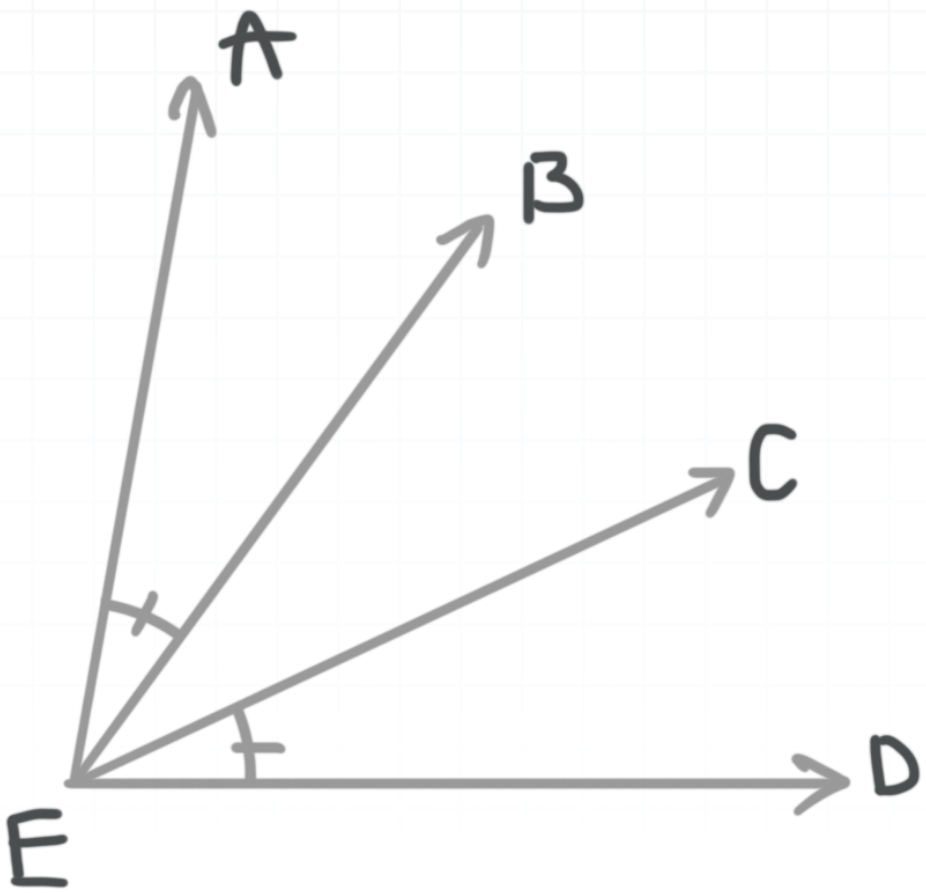
$$110^\circ = m\angle ABD + 55^\circ$$

$$55^\circ = m\angle ABD$$

Let's do a few examples.

Example

If $m\angle AED = 80°$ and $m\angle AEB = 30°$, what is $m\angle BEC$?



Let's organize what we know. In the diagram, the short line segments that cross the circular arcs inside angles AEB and CED mean that $\angle AEB$ is congruent to $\angle CED$, which is another way of saying that the measures of angles AEB and CED are equal. So we know that $m\angle CED = m\angle AEB = 30°$.

We also know the measure of the entire angle, $m\angle AED = 80°$, and that $m\angle AED = m\angle AEB + m\angle BEC + m\angle CED$. Let's let $x = m\angle BEC$. Then we get

$$m\angle AED = m\angle AEB + x + m\angle CED$$

Now we'll substitute the measures of the other three angles.

$$80^\circ = 30^\circ + x + 30^\circ$$

$$80^\circ = 60^\circ + x$$

$$x = 20^\circ$$

So $m\angle BEC = 20^\circ$.

Here's another type of problem you might see.

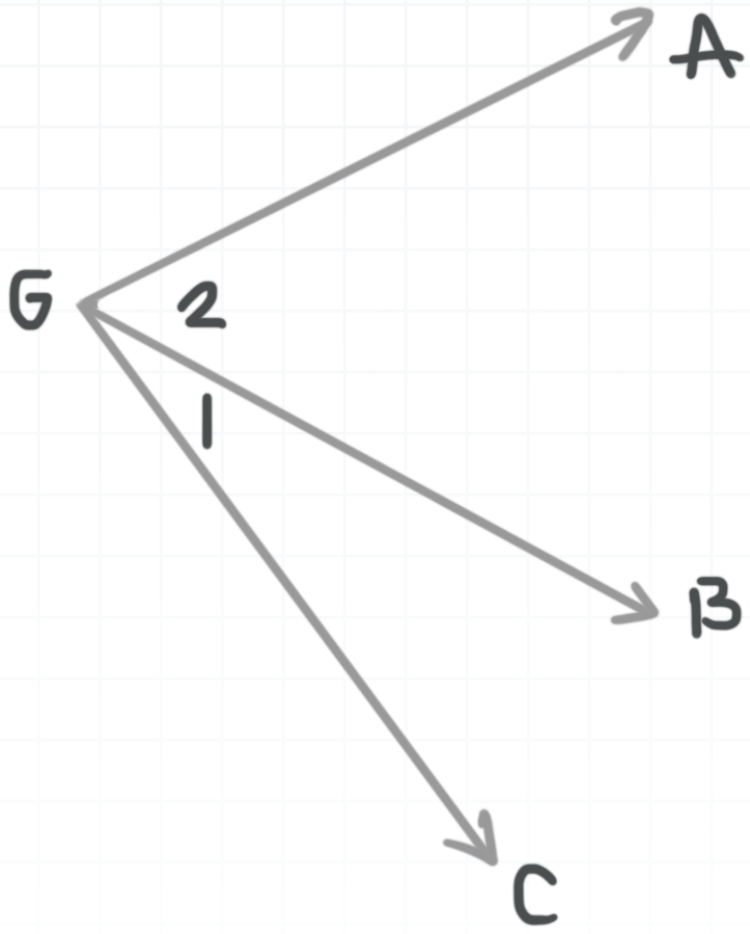
Example

Find the measure of angle 2 if x is in degrees.

$$m\angle 1 = 2x$$

$$m\angle 2 = 5x + 5^\circ$$

$$m\angle AGC = 105^\circ - 3x$$



We can set up an equation, solve for x, then substitute back in to find $m\angle 2$.

$$m\angle 1 + m\angle 2 = m\angle AGC$$

$$2x + 5x + 5^\circ = 105^\circ - 3x$$

$$7x + 5^\circ = 105^\circ - 3x$$

$$7x + 3x + 5^\circ = 105^\circ$$

$$10x + 5^\circ = 105^\circ$$

$$10x = 100^\circ$$

$$x = 10^\circ$$

Substituting $10°$ for x in the equation $m\angle 2 = 5x + 5°$, we get $m\angle 2 = 5(10°) + 5° = 55°$.
