Triangle congruence with SSS, ASA, SAS

In this lesson we'll look at how to use triangle congruence theorems to prove that triangles, or parts of triangles, are congruent.

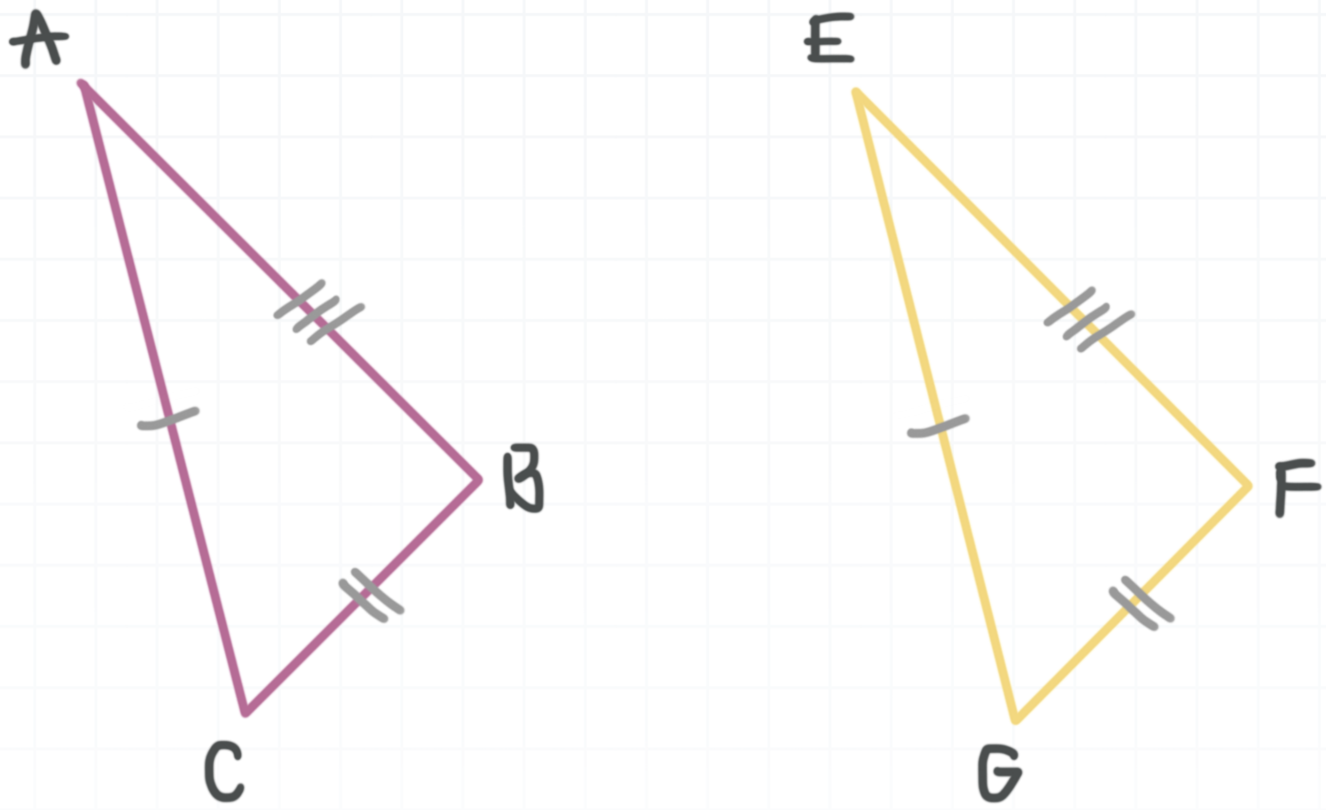
Congruent triangles

A pair of congruent triangles have exactly the same size and shape. That means that we could place one triangle on top of the other in such a way that they're identical, this is, corresponding sides have the same length and corresponding angles have the same measure.

The good news is that to prove that two triangles are congruent, we don't have to show that all three pairs of sides and all three pairs of angles match up. There are some triangle theorems that you can use as a short cut to prove that two triangles are congruent.

Side, side, side (SSS)

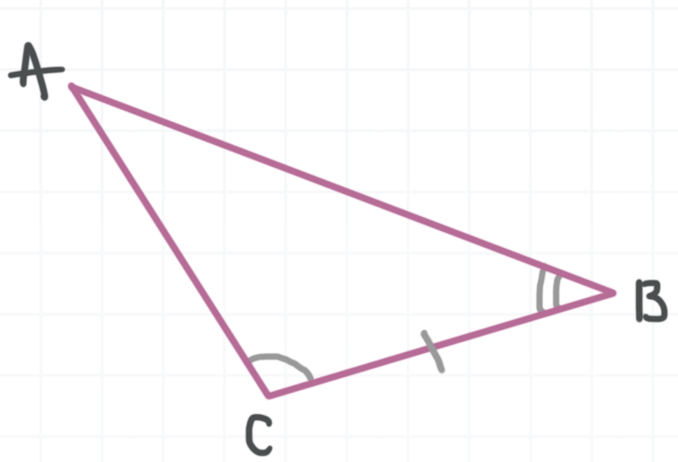
If you can show that all three pairs of sides of two triangles are congruent, then you'll have proven that the triangles are congruent, without needing to check any pair of angles. In the figure below, $\triangle ABC \cong \triangle EFG$ by side, side, side (often called SSS).



Angle, side, angle (ASA)

For “angle, side, angle,” you need to have two pairs of congruent angles and the corresponding pair of “included sides” must be congruent. The **included side** of two angles of a triangle is the side that connects those two angles.

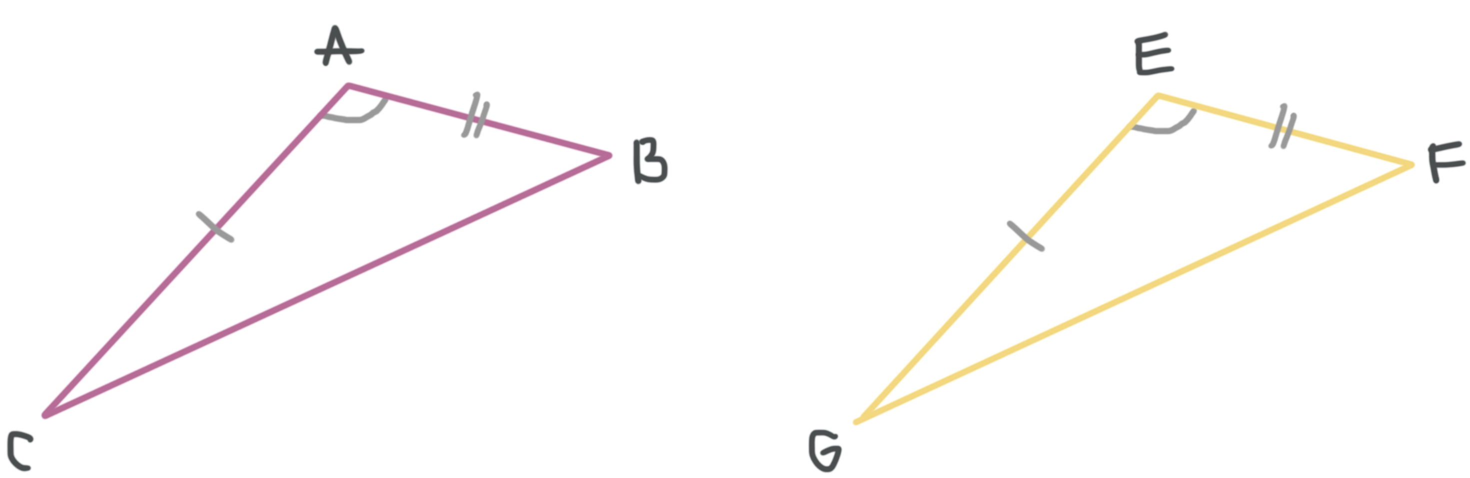
If we can prove that two triangles have these three congruences, then we’ve proven that the triangles are congruent, without checking the third pair of angles or the other two pairs of sides. In the figure below, $\triangle ABC \cong \triangle EFG$ by angle, side, angle (often called ASA): The included side of angles C and B in $\triangle ABC$ is $\overline{BC}$, and the included side of angles G and F in $\triangle EFG$ is $\overline{FG}$.



Side, angle, side (SAS)

For “side, angle, side,” we need to have two pairs of congruent sides, and the corresponding pair of included angles must be congruent. The **included angle** of two sides of a triangle is the angle whose vertex is the point of intersection of those two sides.

If we can prove that two triangles have these three congruences, then we’ve proven that the triangles are congruent, without needing to check the third pair of sides or the other two pairs of angles. In the figure below, $\triangle ABC \cong \triangle EFG$ by side, angle, side (often called SAS): The included angle of sides $\overline{CA}$ and $\overline{AB}$ in $\triangle ABC$ is $\angle A$, and the included angle of sides $\overline{GE}$ and $\overline{EF}$ in $\triangle EFG$ is $\angle E$.



Matching congruent parts

Whenever you state that two triangles are congruent, you must match the letters for corresponding vertices when you name the triangles.

Even if the letters for the vertices of one of the triangles are in alphabetical order, the letters for the corresponding vertices of the other triangle will not necessarily be in alphabetical order. Write the names so that the letters for the vertices are in the same places. Then the letters for the endpoints of pairs of congruent sides will also be in the same places.

If we have a pair of congruent triangles, $\triangle ABC$ and $\triangle DEF$, then the triangle congruency statement $\triangle ABC \cong \triangle DEF$ means that all of the following are true:

The letters A, B, and C for the vertices of $\triangle ABC$ correspond to the letters D, E, and F, respectively, for the vertices of $\triangle DEF$.

Side $\overline{AB}$ in $\triangle ABC$ is congruent to side $\overline{DE}$ in $\triangle DEF$.

Side $\overline{BC}$ in $\triangle ABC$ is congruent to side $\overline{EF}$ in $\triangle DEF$.

Side $\overline{AC}$ in $\triangle ABC$ is congruent to side $\overline{DF}$ in $\triangle DEF$.

For instance, in the figure above, it would be correct to say that $\triangle ABC \cong \triangle EFG$ because angles A and E are congruent, angles B and F are congruent, and angles C and G are congruent. But it would be incorrect to say $\triangle ABC \cong \triangle GFE$, since this statement doesn't list the letters for the vertices of the triangle on the right (in the figure) in the same order as the letters for the corresponding vertices of the triangle on the left. In other words, we can actually state the congruence of the triangles in any of these ways:

$$\triangle ABC \cong \triangle EFG$$

$$\triangle BCA \cong \triangle FGE$$

$$\triangle CAB \cong \triangle GEF$$

$$\triangle ACB \cong \triangle EGF$$

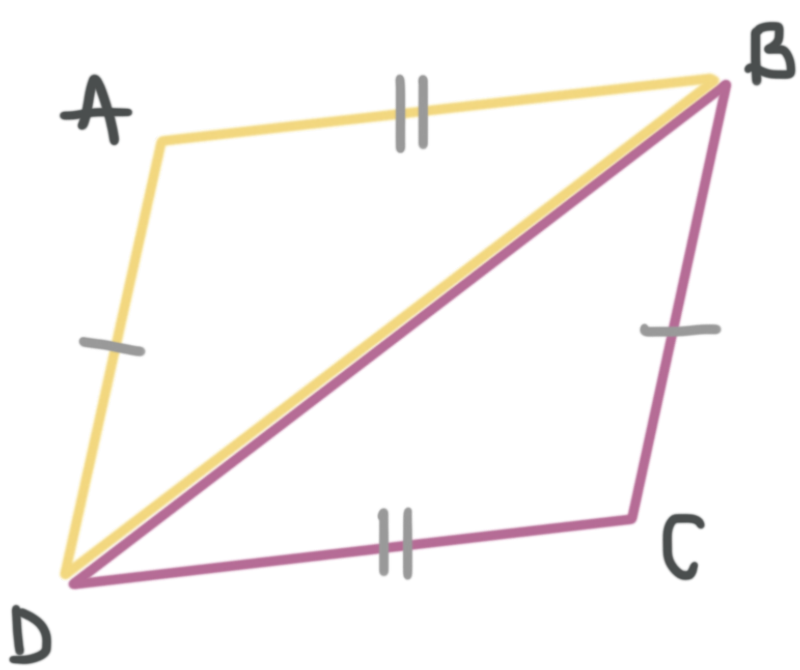
$$\triangle BAC \cong \triangle FEG$$

$$\triangle CBA \cong \triangle GFE$$

Let's start by working through an example.

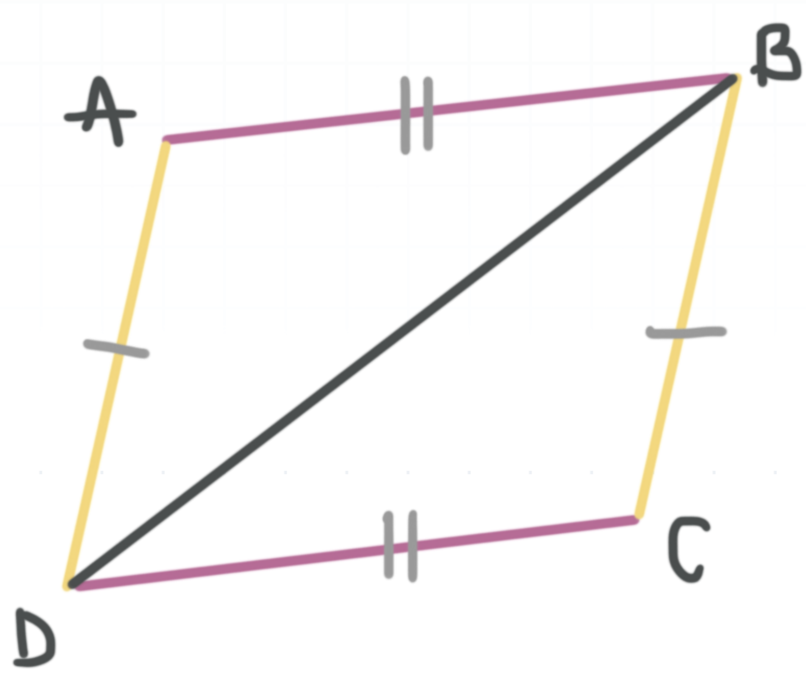
Example

Name the pair of congruent triangles in a triangle congruency statement, and state how we know that the triangles are congruent.



When we look at the two triangles, we see that $\overline{DA} \cong \overline{BC}$ and $\overline{AB} \cong \overline{CD}$. We also know that any line segment is congruent to itself, so $\overline{DB} \cong \overline{DB}$ (this is the **reflexive property**, which we'll use when we do proofs). This means we have three congruent pairs of sides, so we can prove triangle congruence by side, side, side.

In our triangle congruency statement, we need to name the triangles by matching the letters for the corresponding vertices. Sometimes color coding the given congruent parts of the triangles can help you match up the names.



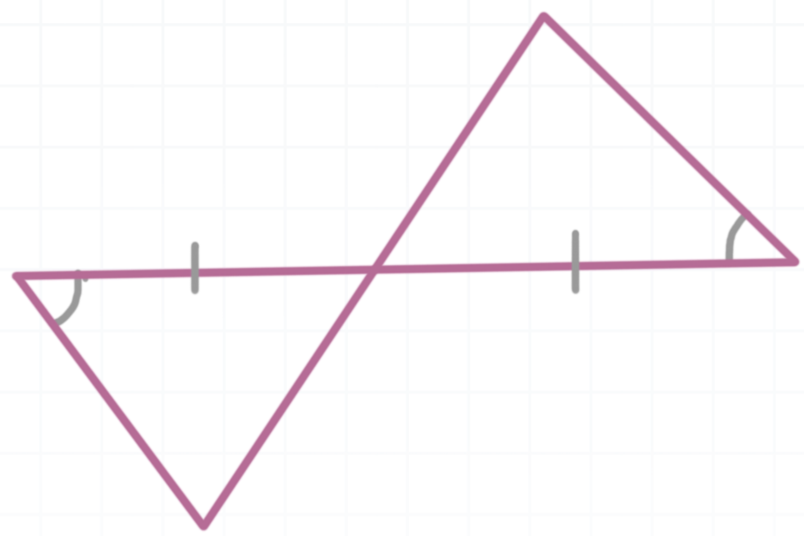
We can then write a triangle congruency statement for these triangles as follows:

$$\triangle DAB \cong \triangle BCD$$

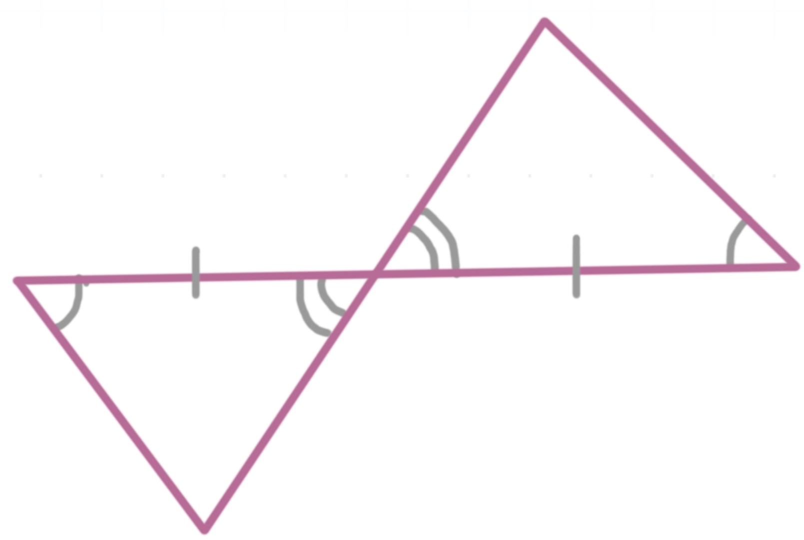
Let's try two more.

Example

State how we know that the triangles are congruent.



In these two triangles, we have a congruent pair of angles and a congruent pair of sides. We also have a pair of vertical angles here:

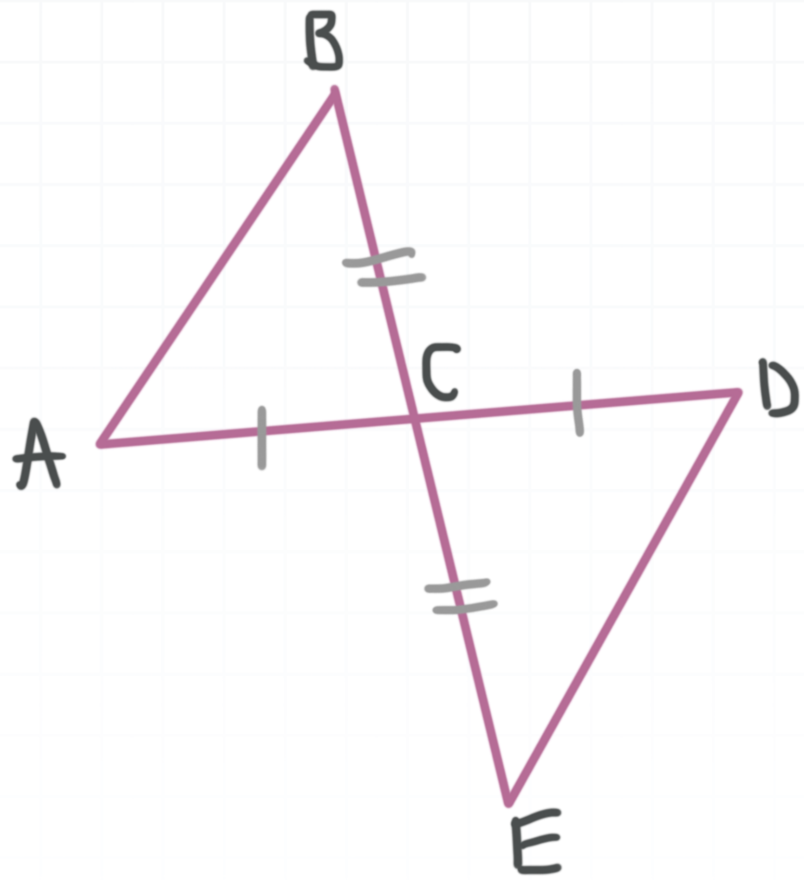


Remember that vertical angles are congruent, so these two triangles are congruent by ASA.

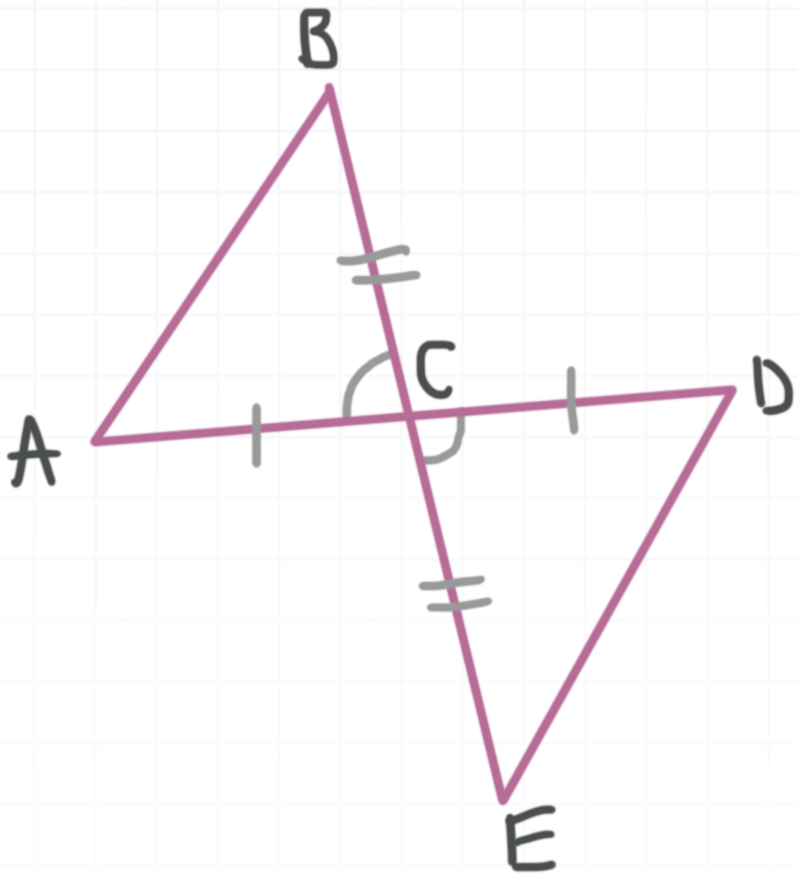
Let's do the last example.

Example

State how we know that the triangles are congruent, and write a triangle congruency statement for them.



The figure tells us that $\overline{AC} \cong \overline{DC}$ and $\overline{BC} \cong \overline{EC}$. We also know that $\angle BCA \cong \angle ECD$ because they are a pair of vertical angles.



This means $\triangle ABC \cong \triangle DEC$ by SAS: The included angle of sides $\overline{AC}$ and $\overline{BC}$ in $\triangle ABC$ is $\angle BCA$, and the included angle of sides $\overline{DC}$ and $\overline{EC}$ in $\triangle DEC$ is $\angle ECD$.