Topic: Midpoint of a line segment in three dimensions

Question: Find the midpoint of the line segment with endpoints $P_1 = (2,7,5)$ and $P_2 = (4,1,1)$.

Answer choices:

A $(-1,3,2)$

B $(6,8,6)$

C $(2,7,0)$

D $(3,4,3)$

Solution: D

We'll use the formula for the midpoint M of a line segment in three dimensions,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

and plug in the coordinates of the endpoints. We'll use $(x_1, y_1, z_1) = (2,7,5)$ and $(x_2, y_2, z_2) = (4,1,1)$.

$$M = \left(\frac{2 + 4}{2}, \frac{7 + 1}{2}, \frac{5 + 1}{2} \right)$$

$$M = \left(\frac{6}{2}, \frac{8}{2}, \frac{6}{2} \right)$$

$$M = (3,4,3)$$

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Question: Find the midpoint of the line segment with endpoints $P_1 = (4, -3, -1)$ and $P_2 = (3{,}5, -7)$.

Answer choices:

A $(3.5{,}1, -4)$

B $(3.5{,}4, -4)$

C $(1{,}4{,}4)$

D $(4{,}0{,}9)$

Solution: A

We'll use the formula for the midpoint M of a line segment in three dimensions,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

and plug in the coordinates of the endpoints. We'll use $(x_1, y_1, z_1) = (4, -3, -1)$ and $(x_2, y_2, z_2) = (3{,}5, -7)$.

$$M = \left(\frac{4 + 3}{2}, \frac{-3 + 5}{2}, \frac{-1 + (-7)}{2} \right)$$

$$M = \left(\frac{7}{2}, \frac{2}{2}, \frac{-8}{2} \right)$$

$$M = (3.5, 1, -4)$$

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Question: Find the coordinates of P_2 if $P_1 = (6{,}5, -3)$ and $M = (4, -4, -5)$ is the midpoint of $\overline{P_1P_2}$.

Answer choices:

A $\left(5, \frac{1}{2}, -4\right)$

B $(1, -4.5, -1)$

C $(2, -13, -7)$

D $(-3{,}6{,}5)$

Solution: C

We'll use the formula for the midpoint M of a line segment in three dimensions,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

and plug in what we know. We'll use $(x_1, y_1, z_1) = (6{,}5, -3)$ and $M = (4, -4, -5)$.

$$(4, -4, -5) = \left(\frac{6 + x_2}{2}, \frac{5 + y_2}{2}, \frac{-3 + z_2}{2} \right)$$

Then we'll equation the numbers on the left-hand side to the corresponding expressions on the right-hand side, and solve the resulting three equations separately. We'll get

$$4 = \frac{6 + x_2}{2}$$

$$8 = 6 + x_2$$

$$x_2 = 2$$

and

$$-4 = \frac{5 + y_2}{2}$$

$$-8 = 5 + y_2$$

$$y_2 = -13$$

and

$$-5 = \frac{-3 + z_2}{2}$$

$$-10 = -3 + z_2$$

$$z_2 = -7$$

Putting these values together tells us that the coordinates of P_2 are $(2, -13, -7)$.