Geometry Quizzes

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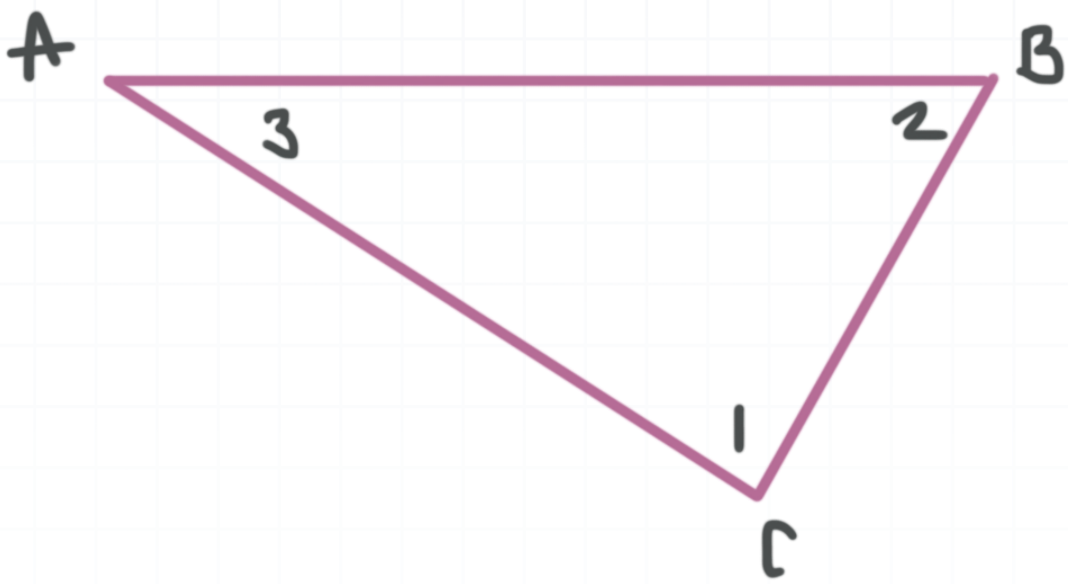
MATH

Topic: Naming simple geometric figures

Question: Which angles, in order, are the same as the angles $\angle BAC$, $\angle ACB$, $\angle CBA$?



Answer choices:

A $\angle 3$, $\angle 1$, $\angle 2$

B $\angle 2$, $\angle 3$, $\angle 1$

C $\angle 1$, $\angle 2$, $\angle 3$

D $\angle 3$, $\angle 2$, $\angle 1$

Solution: A

$\angle BAC$ is the angle in the upper-left part of the figure, and could also be named $\angle A$ or $\angle 3$.

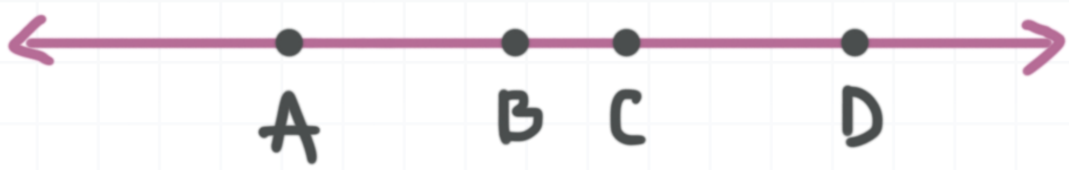
$\angle ACB$ is the angle in the lower-right part of the figure, and could also be named $\angle C$ or $\angle 1$.

$\angle CBA$ is the angle in the upper-right part of the figure, and could also be named $\angle B$ or $\angle 2$.

Therefore, the answer is A.

Topic: Naming simple geometric figures

Question: Which pairs of segments, rays, or lines overlap?



I	Lines AB and CD
II	Rays BA and CD
III	Segments AC and BD

Answer choices:

A I and II

B II and III

C I and III

D Only I

Solution: C

I. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are both lines, so they extend forever in both directions and will overlap on their entire length.

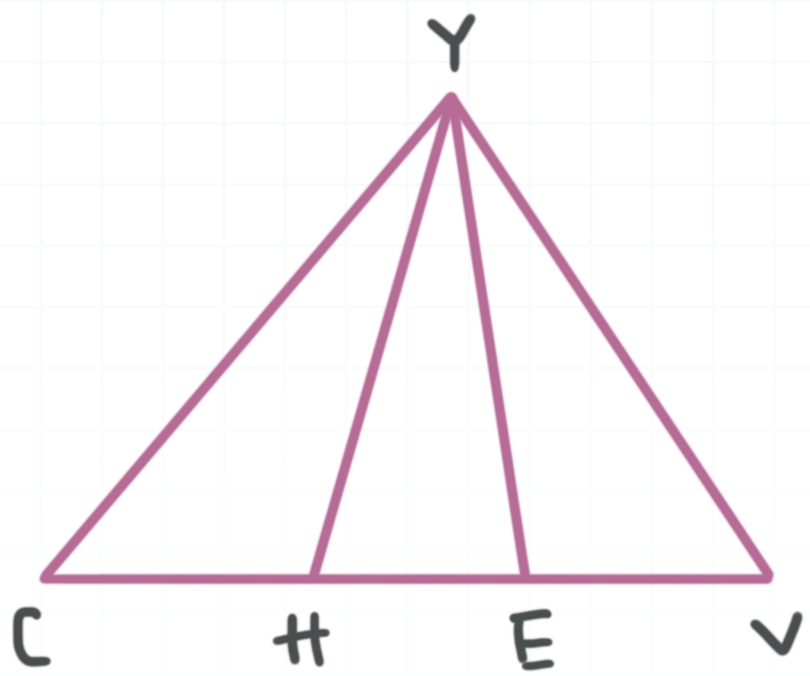
II. $\overrightarrow{BA}$ and $\overrightarrow{CD}$ are rays. $\overrightarrow{BA}$ starts at B and extends to the left through A, but $\overrightarrow{CD}$ starts at C and extends to the right through D. They don't overlap at all.

III. $\overline{AC}$ and $\overline{BD}$ are line segments. They overlap from B to C.

Therefore, both I and III have overlaps.

Topic: Naming simple geometric figures

Question: How many angles have their vertex at point Y?



Answer choices:

A 3

B 4

C 5

D 6

Solution: D

There are three small angles:

$$\angle CYH, \angle HYE, \angle EYV$$

Taking angles made of pairs of consecutive small angles gives two more angles:

$$\angle CYE, \angle HYV$$

There is only one angle made of three smaller angles:

$$\angle CYV$$

That makes a total of six angles with their vertex at Y.

Topic: Length of a line segment

Question: Points A, B, C, D and E lie, in order from left to right, on a number line. Where on the number line does E lie?

$\overline{AC} = 5$, $\overline{BC} = 3$, $\overline{BD} = 7$, and $\overline{BE} = 9$.

C is at 0.

Answer choices:

A -4

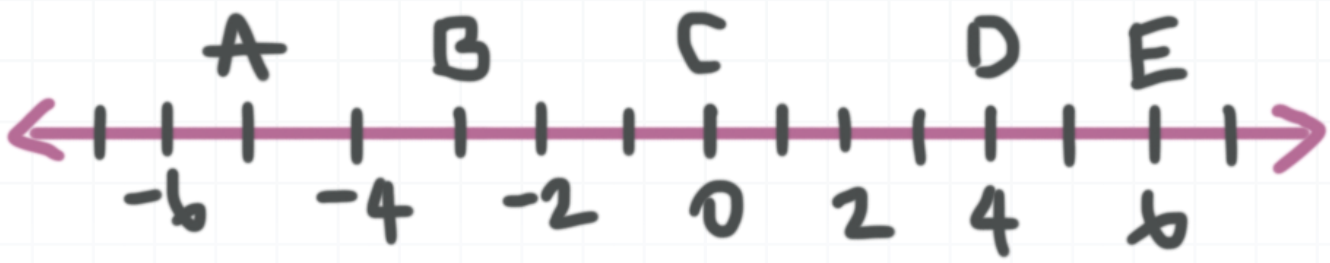
B 2

C 5

D 6

Solution: D

We know point C is at 0 and $\overline{BC} = 3$, which lets us locate point B at -3.



Since B is at -3 and $\overline{BE} = 9$, point E is at a distance of 9 to the right of -3. Which puts E at 6 on the number line.

Topic: Length of a line segment

Question: If $W = -4$, $X = -2$, $Y = 3$, and $Z = 6$ on a number line, then what is the value of $\overline{XZ} - \overline{WY}$?

Answer choices:

A 1

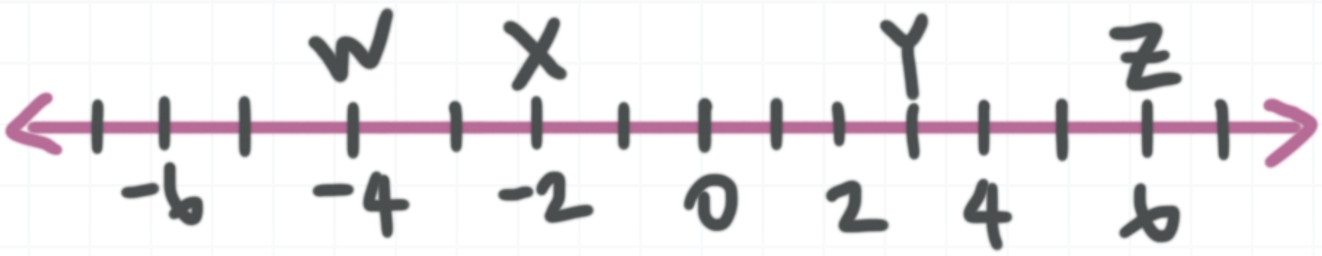
B 2

C 3

D 4

Solution: A

Let's plot the given points on a number line.



We see that

$$\overline{XZ} = |6 - (-2)| = 8$$

$$\overline{WY} = |3 - (-4)| = 7$$

Therefore,

$$\overline{XZ} - \overline{WY} = 8 - 7 = 1$$

Topic: Length of a line segment

Question: Points N, P, and R lie, in order from left to right, on a number line. $\overline{NP}$ is four less than twice $\overline{PR}$. If $\overline{NP} = 2$, what is the value of $\overline{NR}$?

Answer choices:

A 5

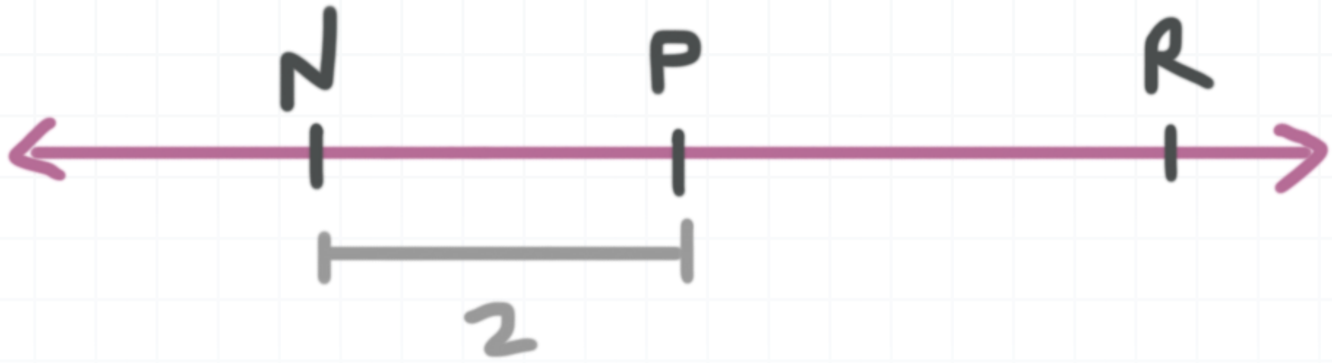
B 4

C 3

D 6

Solution: A

Let's use a number line to show what we know.



Now let $x = \overline{PR}$. We know that $\overline{NP}$ is four less than twice $\overline{PR}$, so $\overline{NP} = 2x - 4$. We also know that $\overline{NP} = 2$, so we can write

$$2x - 4 = 2$$

$$2x = 6$$

$$x = 3$$

So $\overline{PR} = 3$. Therefore,

$$\overline{NR} = \overline{NP} + \overline{PR}$$

$$\overline{NR} = 2 + 3$$

$$\overline{NR} = 5$$

Topic: Slope and midpoint of a line segment

Question: Which pair of endpoints are the endpoints of a line segment that has a slope of $m = 3/5$?

Answer choices:

A $(1,2)$ and $(7,5)$

B $(-9,0)$ and $(-4,3)$

C $(-4,-1)$ and $(-10,-5)$

D $(8,0)$ and $(2,-2)$

Solution: B

Calculate the slope for each pair of points.

For answer choice A:

$$\frac{5-2}{7-1}=\frac{3}{6}=\frac{1}{2}$$

For answer choice B:

$$\frac{3-0}{-4-(-9)}=\frac{3}{5}$$

For answer choice C:

$$\frac{-5-(-1)}{-10-(-4)}=\frac{-4}{-6}=\frac{2}{3}$$

For answer choice D:

$$\frac{-2-0}{2-8}=\frac{-2}{-6}=\frac{1}{3}$$

Only answer choice B gives the desired slope.

Topic: Slope and midpoint of a line segment

Question: Each pair of points in the table are the endpoints of some line segment. Which two segments have the same midpoint?

Segment	Point 1	Point 2
Segment AB	(1,-3)	(9,5)
Segment CD	(2,4)	(6,-3)
Segment EF	(3,0)	(6,6)
Segment GH	(1,1)	(9,1)

Answer choices:

A $\overline{AB}$ and $\overline{GH}$

B $\overline{CD}$ and $\overline{EF}$

C $\overline{EF}$ and $\overline{GH}$

D $\overline{AB}$ and $\overline{CD}$

Solution: A

Calculate the midpoint of each segment.

The midpoint of $\overline{AB}$:

$$(x, y) = \left(\frac{1+9}{2}, \frac{-3+5}{2}\right) = (5,1)$$

The midpoint of $\overline{CD}$:

$$(x, y) = \left(\frac{2+6}{2}, \frac{4+(-3)}{2}\right) = \left(4, \frac{1}{2}\right)$$

The midpoint of $\overline{EF}$:

$$(x, y) = \left(\frac{3+6}{2}, \frac{0+6}{2}\right) = \left(\frac{9}{2}, 3\right)$$

The midpoint of $\overline{GH}$:

$$(x, y) = \left(\frac{1+9}{2}, \frac{1+1}{2}\right) = (5,1)$$

The two segments with the same midpoint are $\overline{AB}$ and $\overline{GH}$.

Topic: Slope and midpoint of a line segment

Question: A line segment has one endpoint at $(5,8)$, and its midpoint at $(3,2)$. Find the position of the other endpoint.

Answer choices:

A $(7,14)$

B $(9,0)$

C $(0,-6)$

D $(1,-4)$

Solution: D

Use the midpoint formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

We know one endpoint is $(5,8)$, so let $x_1 = 5$ and $y_1 = 8$.

The midpoint is $(3,2)$, so

$$\left(\frac{5 + x_2}{2}, \frac{8 + y_2}{2}\right) = (3,2)$$

From this equation, we get equations that we can solve for x_2 and y_2.

$$\frac{5 + x_2}{2} = 3$$

$$5 + x_2 = 6$$

$$x_2 = 1$$

and

$$\frac{8 + y_2}{2} = 2$$

$$8 + y_2 = 4$$

$$y_2 = -4$$

Putting these together, we can say that the other endpoint is at $(1, -4)$.

Topic: Parallel, perpendicular, or neither

Question: Each pair of points in the table below are points that lie on the given line. Which two lines are perpendicular to each other?

Line	Point 1	Point 2
Line AB	(-2,2)	(1,8)
Line CD	(3,6)	(5,2)
Line EF	(3,0)	(7,-2)

Answer choices:

A $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

B $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$

C $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$

D None are perpendicular

Solution: C

Use the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

for each line.

$$\overleftrightarrow{AB}: m = \frac{8-2}{1-(-2)} = 2$$

$$\overleftrightarrow{CD}: m = \frac{2-6}{5-3} = -2$$

$$\overleftrightarrow{EF}: m = \frac{-2-0}{7-3} = -\frac{1}{2}$$

$\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ have slopes that are negative reciprocals of each other, so they're perpendicular.

Topic: Parallel, perpendicular, or neither

Question: Each pair of points in the table below are points that lie on the given line. Which lines are parallel to each other?

Line	Point 1	Point 2
Line AB	(0,3)	(6,7)
Line CD	(5,4)	(8,6)
Line EF	(1,-2)	(7,2)

Answer choices:

A $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

B $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$

C $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$

D All three are parallel

Solution: D

Use the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

for each line.

$$\overleftrightarrow{AB}: m = \frac{7-3}{6-0} = \frac{2}{3}$$

$$\overleftrightarrow{CD}: m = \frac{6-4}{8-5} = \frac{2}{3}$$

$$\overleftrightarrow{EF}: m = \frac{2-(-2)}{7-1} = \frac{2}{3}$$

All three lines have the same slope, so all three are parallel unless two of them (or all three) are actually one and the same line.

Topic: Parallel, perpendicular, or neither

Question: Which statement is true?

Point	Coordinates
A	(-7,2)
B	(2,5)
C	(4,-1)
D	(-5,-4)

Answer choices:

A Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, and lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular.

B Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, and lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are parallel.

C Lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$ are perpendicular, and lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{BD}$ are perpendicular.

D Lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{BD}$ are perpendicular, and lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{AB}$ are perpendicular.

Solution: A

Use the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

to get the slopes of the lines $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BD}$, and $\overleftrightarrow{AD}$.

$$\overleftrightarrow{AB}: m = \frac{5-2}{2-(-7)} = \frac{1}{3}$$

$$\overleftrightarrow{BC}: m = \frac{-1-5}{4-2} = -3$$

$$\overleftrightarrow{CD}: m = \frac{-4-(-1)}{-5-4} = \frac{1}{3}$$

$$\overleftrightarrow{AC}: m = \frac{-1-2}{4-(-7)} = \frac{-3}{11}$$

$$\overleftrightarrow{BD}: m = \frac{-4-5}{-5-2} = \frac{9}{7}$$

$$\overleftrightarrow{AD}: m = \frac{-4-2}{-5-(-7)} = -3$$

Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ have the same slope, so they're parallel, unless they're one and the same line.

Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ have slopes that are negative reciprocals of each other, so they're perpendicular. Combining these two results, we see that answer choice A is correct.

Let's check the other three answer choices.

Answer choice B can't be correct, because lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ have different slopes, so they can't be parallel.

Answer choice C can't be correct, because the slopes of the lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$ aren't negative reciprocals of each other, so they can't be perpendicular.

Answer choice D can't be correct, because the slopes of lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{BD}$ aren't negative reciprocals of each other, so they can't be perpendicular.

Topic: Distance between two points in three dimensions

Question: Calculate the distance between P_1 and P_2.

$$P_1 = (3,6,1)$$

$$P_2 = (0,1,5)$$

Answer choices:

A 5

B $5\sqrt{2}$

C $3\sqrt{5}$

D 7

Solution: B

Plug the coordinates of the given points into the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(3 - 0)^2 + (6 - 1)^2 + (1 - 5)^2}$$

$$d = \sqrt{9 + 25 + 16}$$

$$d = \sqrt{50}$$

$$d = \sqrt{25 \cdot 2}$$

$$d = \sqrt{25}\sqrt{2}$$

$$d = 5\sqrt{2}$$

Topic: Distance between two points in three dimensions

Question: Calculate the distance between P_1 and P_2.

$$P_1 = (-5, -2, 6)$$

$$P_2 = (1, 2, 4)$$

Answer choices:

A $2\sqrt{14}$

B $2\sqrt{5}$

C 6

D $2\sqrt{10}$

Solution: A

Plug the coordinates of the given points into the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(-5 - 1)^2 + (-2 - 2)^2 + (6 - 4)^2}$$

$$d = \sqrt{36 + 16 + 4}$$

$$d = \sqrt{56}$$

$$d = \sqrt{4 \cdot 14}$$

$$d = \sqrt{4}\sqrt{14}$$

$$d = 2\sqrt{14}$$

Topic: Distance between two points in three dimensions

Question: Which point is closest to the point $P = (2,4,6)$?

Answer choices:

A $Q_1 = (5, -2,3)$

B $Q_2 = (4,5,10)$

C $Q_3 = (6,1,9)$

D $Q_4 = (1,8,5)$

Solution: D

We'll use the distance formula to find the distance of each of the four points from P. To make it easier to identify the point that's closest to P, we won't simplify the radicals.

The distance between P and Q_1 is

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$d=\sqrt{(2-5)^2+(4-(-2))^2+(6-3)^2}$$

$$d=\sqrt{(-3)^2+6^2+3^2}$$

$$d=\sqrt{9+36+9}$$

$$d=\sqrt{54}$$

The distance between P and Q_2 is

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$d=\sqrt{(2-4)^2+(4-5)^2+(6-10)^2}$$

$$d=\sqrt{(-2)^2+(-1)^2+(-4)^2}$$

$$d=\sqrt{4+1+16}$$

$$d=\sqrt{21}$$

The distance between P and Q_3 is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2 - 6)^2 + (4 - 1)^2 + (6 - 9)^2}$$

$$d = \sqrt{(-4)^2 + 3^2 + (-3)^2}$$

$$d = \sqrt{16 + 9 + 9}$$

$$d = \sqrt{34}$$

The distance between P and Q_4 is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2 - 1)^2 + (4 - 8)^2 + (6 - 5)^2}$$

$$d = \sqrt{1^2 + (-4)^2 + 1^2}$$

$$d = \sqrt{1 + 16 + 1}$$

$$d = \sqrt{18}$$

The distances of Q_1, Q_2, Q_3, and Q_4 from P are $\sqrt{54}$, $\sqrt{21}$, $\sqrt{34}$, and $\sqrt{18}$, respectively, so Q_4 is the point closest to P.

Topic: Midpoint of a line segment in three dimensions

Question: Find the midpoint of the line segment with endpoints $P_1 = (2,7,5)$ and $P_2 = (4,1,1)$.

Answer choices:

A $(-1,3,2)$

B $(6,8,6)$

C $(2,7,0)$

D $(3,4,3)$

Solution: D

We'll use the formula for the midpoint M of a line segment in three dimensions,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

and plug in the coordinates of the endpoints. We'll use $(x_1, y_1, z_1) = (2,7,5)$ and $(x_2, y_2, z_2) = (4,1,1)$.

$$M = \left(\frac{2 + 4}{2}, \frac{7 + 1}{2}, \frac{5 + 1}{2}\right)$$

$$M = \left(\frac{6}{2}, \frac{8}{2}, \frac{6}{2}\right)$$

$$M = (3,4,3)$$

Topic: Midpoint of a line segment in three dimensions

Question: Find the midpoint of the line segment with endpoints $P_1 = (4, -3, -1)$ and $P_2 = (3, 5, -7)$.

Answer choices:

A $(3.5, 1, -4)$

B $(3.5, 4, -4)$

C $(1, 4, 4)$

D $(4, 0, 9)$

Solution: A

We'll use the formula for the midpoint M of a line segment in three dimensions,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

and plug in the coordinates of the endpoints. We'll use $(x_1, y_1, z_1) = (4, -3, -1)$ and $(x_2, y_2, z_2) = (3, 5, -7)$.

$$M = \left(\frac{4 + 3}{2}, \frac{-3 + 5}{2}, \frac{-1 + (-7)}{2}\right)$$

$$M = \left(\frac{7}{2}, \frac{2}{2}, \frac{-8}{2}\right)$$

$$M = (3.5, 1, -4)$$

Topic: Midpoint of a line segment in three dimensions

Question: Find the coordinates of P_2 if $P_1 = (6,5,-3)$ and $M = (4,-4,-5)$ is the midpoint of $\overline{P_1P_2}$.

Answer choices:

A $\left(5,\frac{1}{2},-4\right)$

B $(1,-4.5,-1)$

C $(2,-13,-7)$

D $(-3,6,5)$

Solution: C

We'll use the formula for the midpoint M of a line segment in three dimensions,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

and plug in what we know. We'll use $(x_1, y_1, z_1) = (6,5, -3)$ and $M = (4, -4, -5)$.

$$(4, -4, -5) = \left(\frac{6 + x_2}{2}, \frac{5 + y_2}{2}, \frac{-3 + z_2}{2}\right)$$

Then we'll equation the numbers on the left-hand side to the corresponding expressions on the right-hand side, and solve the resulting three equations separately. We'll get

$$4 = \frac{6 + x_2}{2}$$

$$8 = 6 + x_2$$

$$x_2 = 2$$

and

$$-4 = \frac{5 + y_2}{2}$$

$$-8 = 5 + y_2$$

$$y_2 = -13$$

and

$$-5 = \frac{-3 + z_2}{2}$$

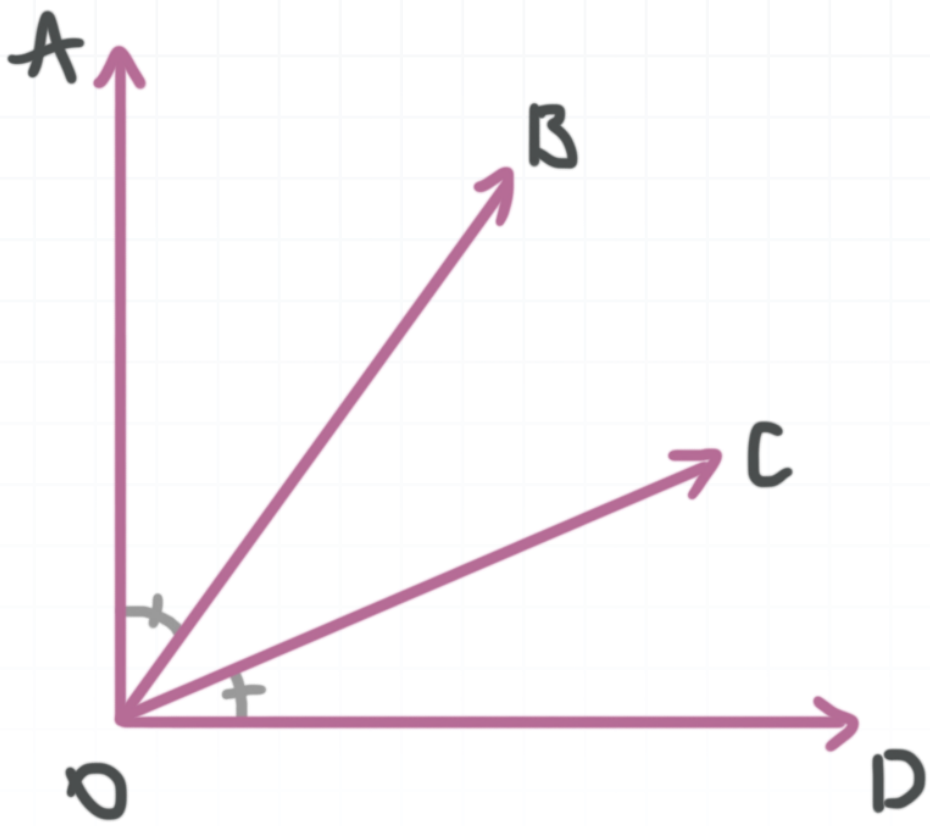
$$-10 = -3 + z_2$$

$$z_2 = -7$$

Putting these values together tells us that the coordinates of P_2 are $(2, -13, -7)$.

Topic: Measures of angles

Question: If $m\angle BOC = 37^\circ$ and $m\angle AOD = 90^\circ$, what is $m\angle AOC$?



Answer choices:

A 15°

B 26.5°

C 53°

D 63.5°

Solution: D

Let $x = m\angle AOB$. We also know that angle COD is congruent to angle AOB, so $x = m\angle COD$ as well.

We know that

$$m\angle AOB + m\angle BOC + m\angle COD = m\angle AOD$$

Substituting the expressions for the angle measures into this equation gives

$$x + 37^\circ + x = 90^\circ$$

$$2x + 37^\circ = 90^\circ$$

$$2x = 53^\circ$$

$$x = 26.5^\circ$$

Now we have enough to find $m\angle AOC$.

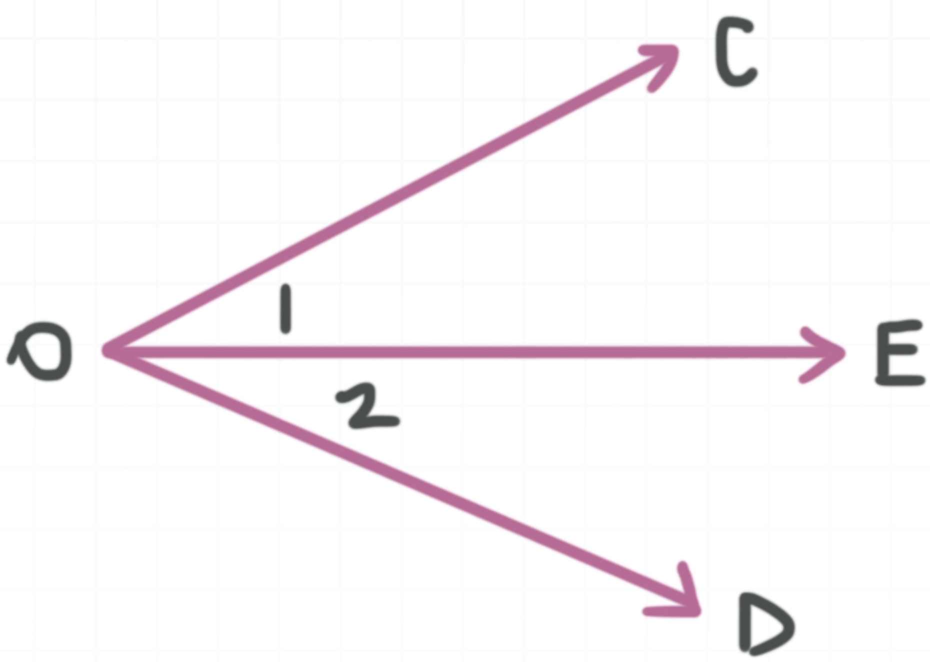
$$m\angle AOB + m\angle BOC = m\angle AOC$$

$$26.5^\circ + 37^\circ = m\angle AOC$$

$$63.5^\circ = m\angle AOC$$

Topic: Measures of angles

Question: If $m\angle 1 = 4x$, $m\angle 2 = 2x$, and $m\angle COD = 5x + 9^\circ$, where x is in degrees, what is $m\angle 1$?



Answer choices:

A 9°

B 18°

C 27°

D 36°

Solution: D

We see that

$$m\angle 1 + m\angle 2 = m\angle COD$$

Substituting the expressions for the angle measures into this equation gives

$$4x + 2x = 5x + 9^\circ$$

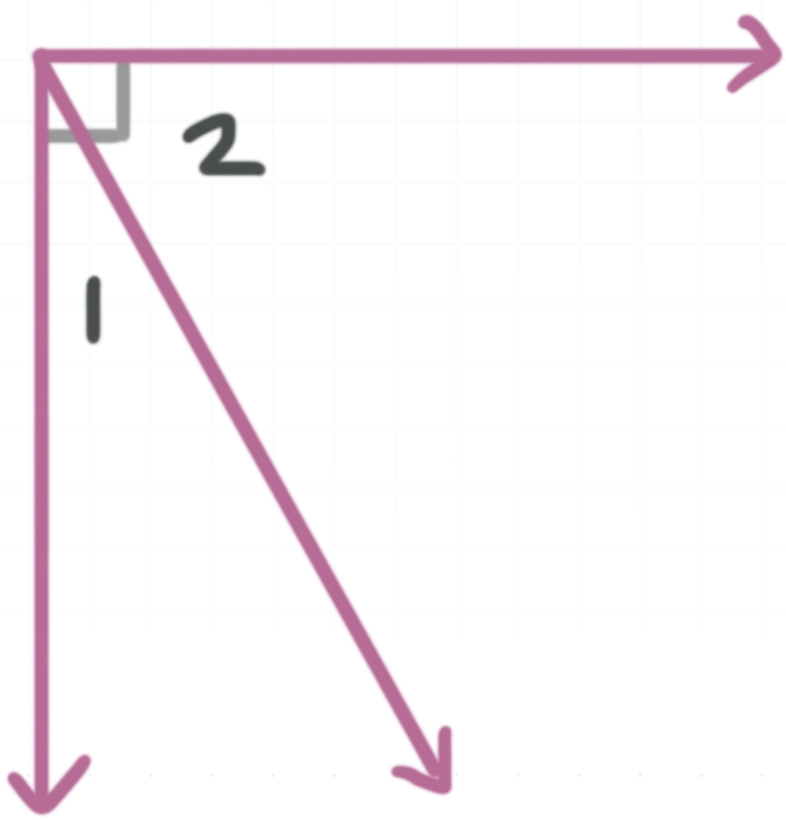
$$6x = 5x + 9^\circ$$

$$x = 9^\circ$$

$$m\angle 1 = 4x = 4(9^\circ) = 36^\circ$$

Topic: Measures of angles

Question: If $m\angle 1 = x + 2^\circ$ and $m\angle 2 = 3x$, where x is in degrees, what is $m\angle 2$? Hint: A little square at the vertex of an angle indicates that the measure of that angle is 90°. We call these “right-angles,” and the rays that form a right angle are perpendicular to each other.



Answer choices:

A 22°

B 24°

C 66°

D 88°

Solution: C

We see that

$$m\angle 1 + m\angle 2 = 90^\circ$$

Substituting the expressions for the angle measures into this equation gives

$$x + 2^\circ + 3x = 90^\circ$$

$$4x + 2^\circ = 90^\circ$$

$$4x = 88^\circ$$

$$x = 22^\circ$$

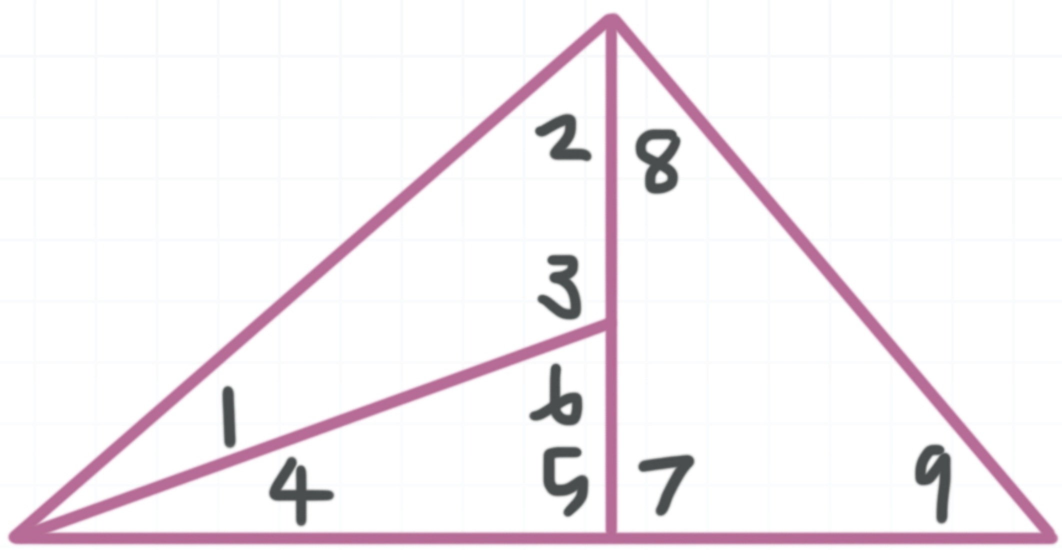
Which means that $m\angle 2$ must be

$$m\angle 2 = 3(22^\circ)$$

$$m\angle 2 = 66^\circ$$

Topic: Adjacent and nonadjacent angles

Question: Which pair of angles are not adjacent angles?



Answer choices:

A $\angle 1$, $\angle 4$

B $\angle 2$, $\angle 3$

C $\angle 3$, $\angle 6$

D $\angle 5$, $\angle 7$

Solution: B

We'll first look for a pair of angles that don't have the same vertex. If we find that each of the given pairs of angles share a vertex, we'll to determine which pair have overlapping interiors.

The figure shows that $\angle 2$ and $\angle 3$ don't share a vertex, so they aren't adjacent angles.

Topic: Adjacent and nonadjacent angles

Question: Suppose a figure with several angles is drawn and there are five points on it (all different) labeled A, B, C, D, and E. The pairs of angles below are from that figure. Which pair of angles are definitely nonadjacent?

Hint: Without the figure, you can't tell for sure which angles *are* adjacent, but the letters in the names of the angles can tell you which pair are *not* adjacent.

Answer choices:

A $\angle BAE$, $\angle DAE$

B $\angle CED$, $\angle BEC$

C $\angle ACD$, $\angle BCA$

D $\angle BCE$, $\angle CBE$

Solution: D

When an angle is named with a sequence of three letters, the letter in the middle is always the vertex.

For example, looking at the angles from answer choice D:

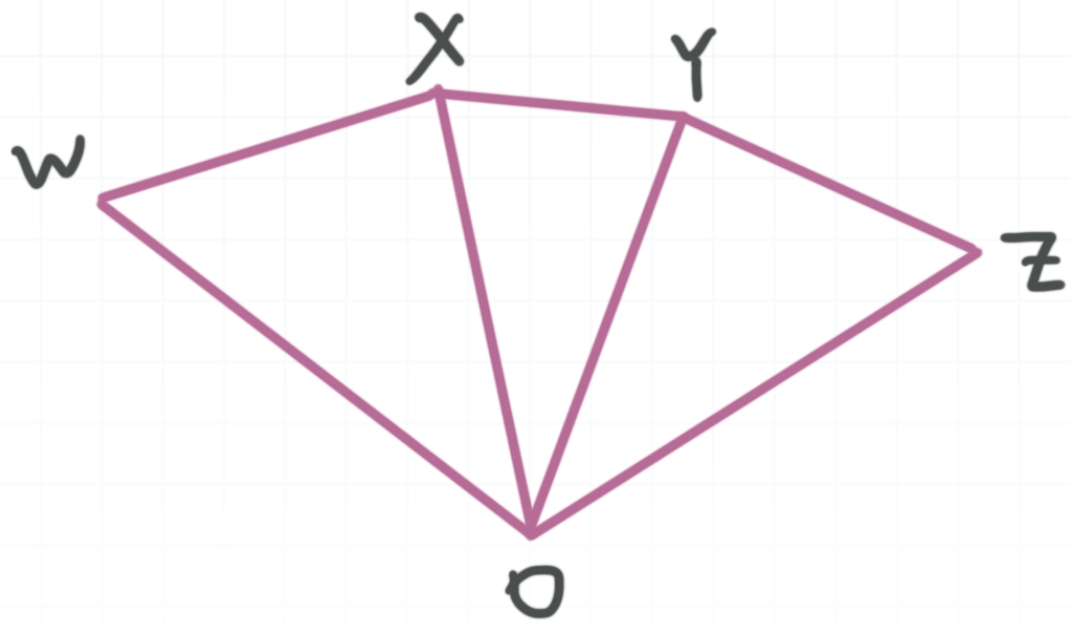
$\angle BCE$ has its vertex at C.

$\angle CBE$ has its vertex at B.

Since those angles don't share a vertex, they can't be adjacent.

Topic: Adjacent and nonadjacent angles

Question: Which pair of angles below are adjacent angles?



Answer choices:

A $\angle WOX$, $\angle WOY$

B $\angle ZYO$, $\angle ZYX$

C $\angle OYX$, $\angle ZYO$

D $\angle OWX$, $\angle OXW$

Solution: C

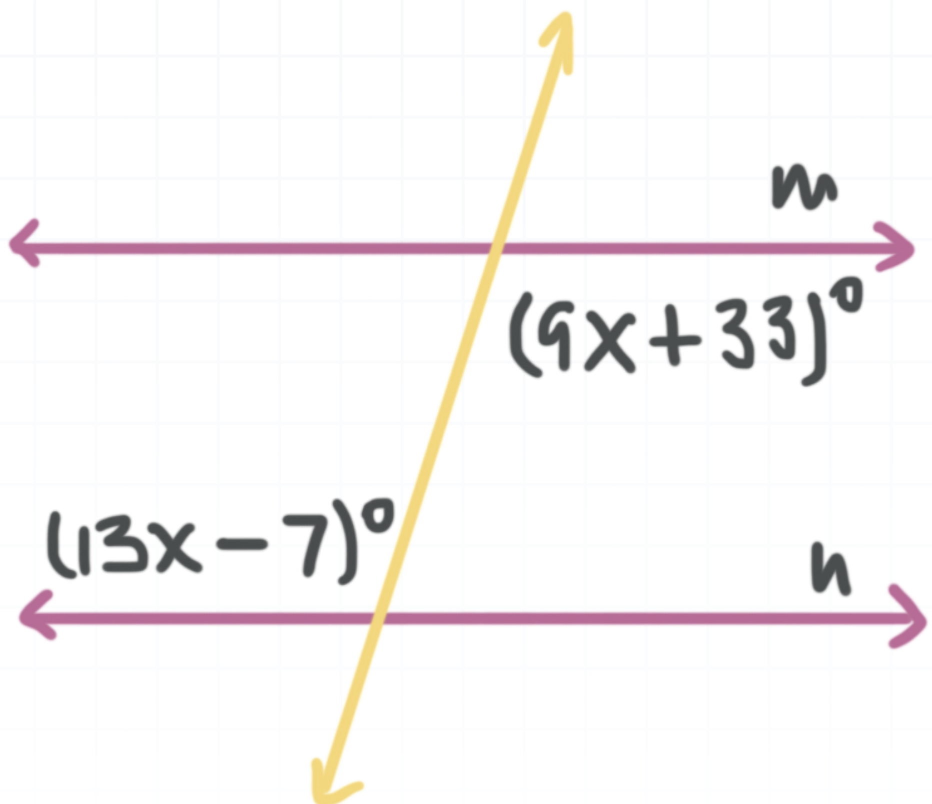
$\angle OYX$ and $\angle ZYO$ share a vertex, Y, and one side, $\overline{OY}$, and their interiors don't overlap.

That fits the definition of adjacent angles, so answer choice C is correct.

The angles in answer choice A have overlapping interiors, as do the angles in answer choice B, and the angles in answer choice D don't have the same vertex. Therefore, we can rule out answer choices A, B, and D.

Topic: Angles and transversals

Question: Find x, given that $m \parallel n$.



Answer choices:

A 5

B 6

C 8

D 10

Solution: D

When parallel lines intersect a transversal, alternate interior angles are congruent. The angles of measure $(13x - 7)^\circ$ and $(9x + 33)^\circ$ are a pair of vertical angles, so they're congruent.

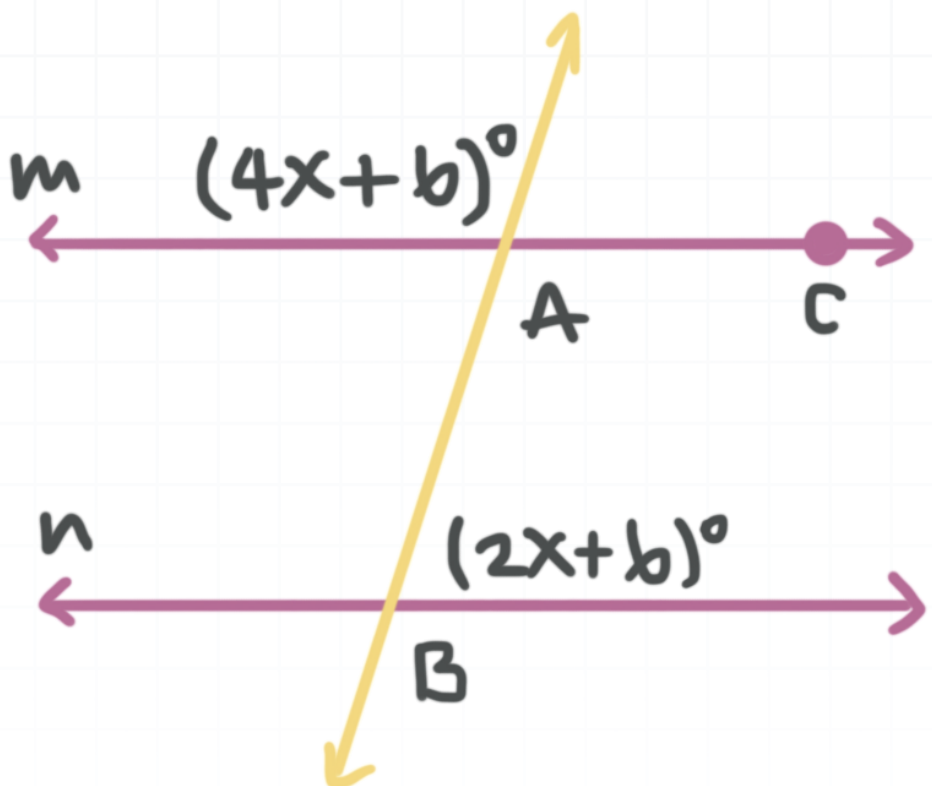
$$13x - 7 = 9x + 33$$

$$4x = 40$$

$$x = 10$$

Topic: Angles and transversals

Question: Find x, given that $m \parallel n$.



Answer choices:

A $\quad$ 28

B $\quad$ 32

C $\quad$ 36

D $\quad$ 40

Solution: A

Angle BAC and the angle of measure $(4x + 6)^\circ$ are a pair of vertical angles, so they're congruent.

$$m\angle BAC = (4x + 6)^\circ$$

When parallel lines intersect a transversal, consecutive interior angles are supplementary. Angle BAC and the angle of measure $(2x + 6)^\circ$ are a pair of consecutive interior angles. Therefore,

$$(4x + 6) + (2x + 6) = 180$$

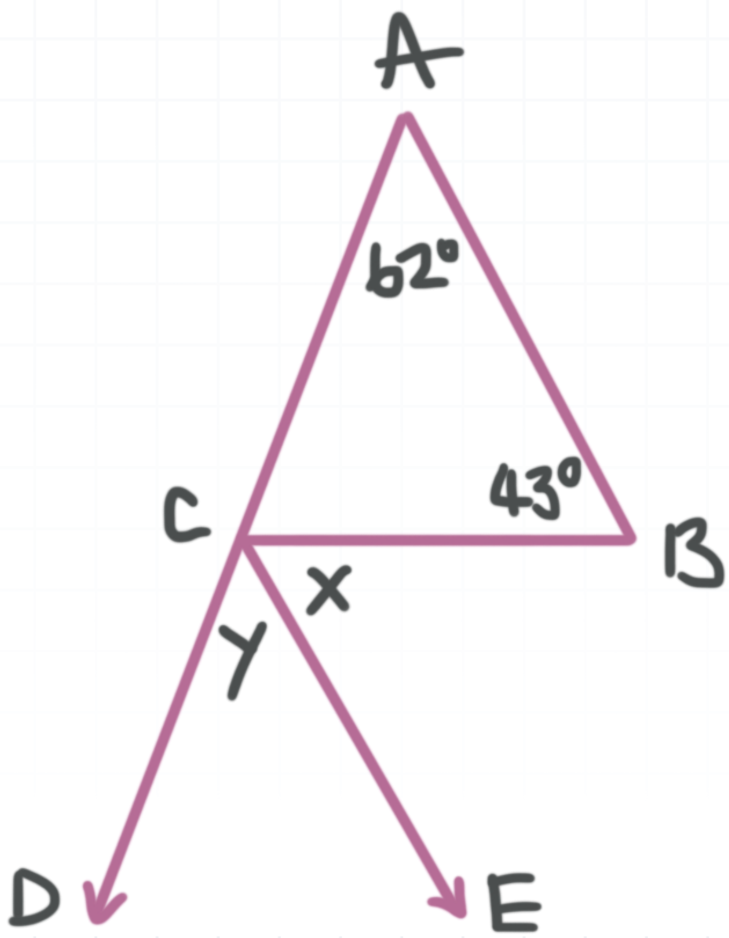
$$6x + 12 = 180$$

$$6x = 168$$

$$x = 28$$

Topic: Angles and transversals

Question: Given that $\overline{AB} \parallel \overline{CE}$, find the value of $y - x$, where both x and y are in degrees.



Answer choices:

A $\quad 11^\circ$

B $\quad 15^\circ$

C $\quad 19^\circ$

D $\quad 21^\circ$

Solution: C

When parallel lines intersect a transversal, alternate interior angles are congruent. Angles BCE and ABC are a pair of alternate interior angles. Therefore,

$$m\angle BCE = m\angle ABC$$

$$x = 43^\circ$$

When parallel lines intersect a transversal, corresponding angles are congruent. Angles DCE and CAB are a pair of corresponding angles. Therefore,

$$m\angle DCE = m\angle CAB$$

$$y = 62^\circ$$

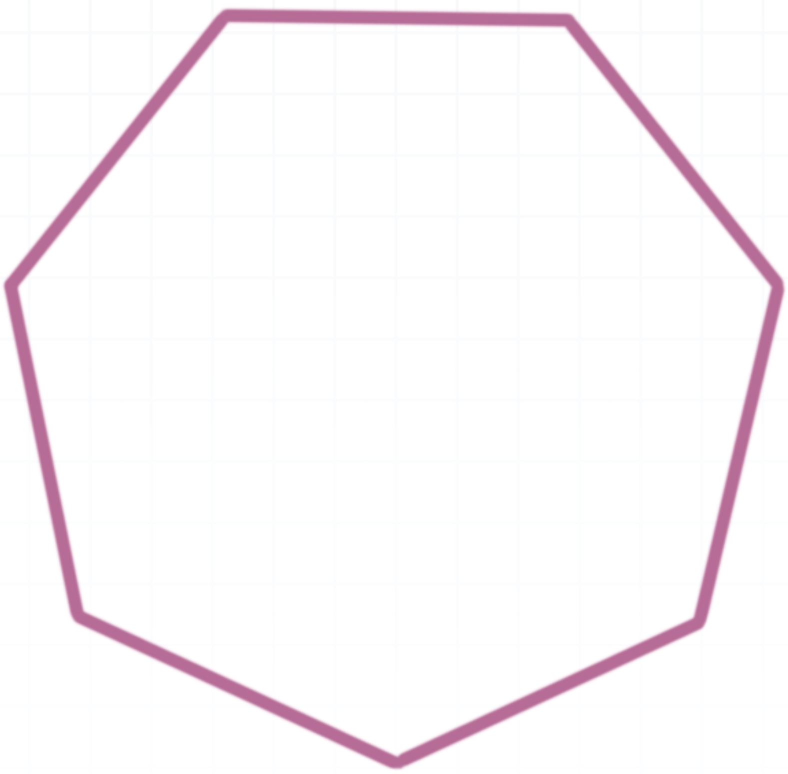
So we see that

$$y - x = 62^\circ - 43^\circ$$

$$y - x = 19^\circ$$

Topic: Interior angles of polygons

Question: A regular heptagon has all sides and all angles congruent. What is the measure of each interior angle, to the nearest tenth of a degree?



Answer choices:

A 128.6°

B 134.4°

C 139.7°

D 150.0°

Solution: A

The sum of the measures of the interior angles in a polygon is

$$(n-2)180^\circ$$

where n is the number of sides in the polygon. For a heptagon, which is a seven-sided figure, that would be

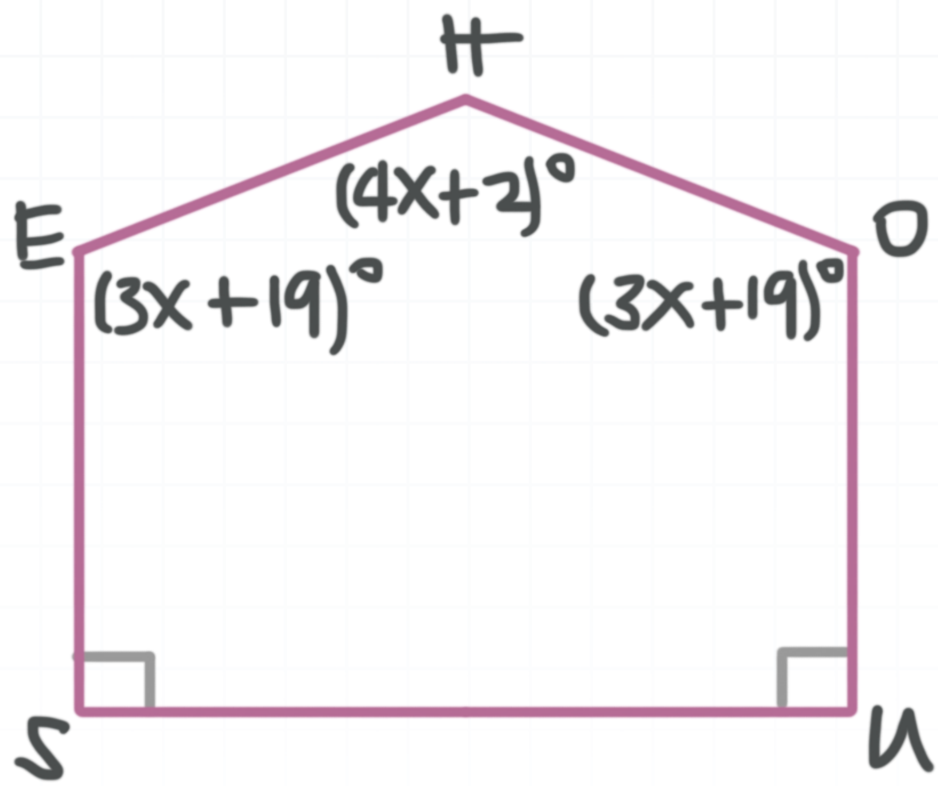
$$(7-2)180^\circ = 900^\circ$$

There are seven angles, so

$$900^\circ \div 7 = 128.6^\circ$$

Topic: Interior angles of polygons

Question: Find $m\angle H$.



Answer choices:

A 115°

B 130°

C 140°

D 145°

Solution: B

The sum of the measures of the interior angles in a polygon with n sides is

$$(n-2)180^\circ$$

For a pentagon, that would be

$$(5-2)180^\circ = 540^\circ$$

Set the sum of the five angles equal to 540° and solve.

$$(4x+2)^\circ + (3x+19)^\circ + 90^\circ + 90^\circ + (3x+19)^\circ = 540^\circ$$

$$(4x+3x+3x)^\circ + (2+19+90^\circ+90^\circ+19)^\circ = 540^\circ$$

$$10x^\circ + 220^\circ = 540^\circ$$

$$10x^\circ = 320^\circ$$

$$x = 32$$

Substitute $x = 32$ for x in $(4x+2)^\circ$ to find $m\angle H$.

$$m\angle H = (4x+2)^\circ$$

$$m\angle H = [4(32)+2]^\circ$$

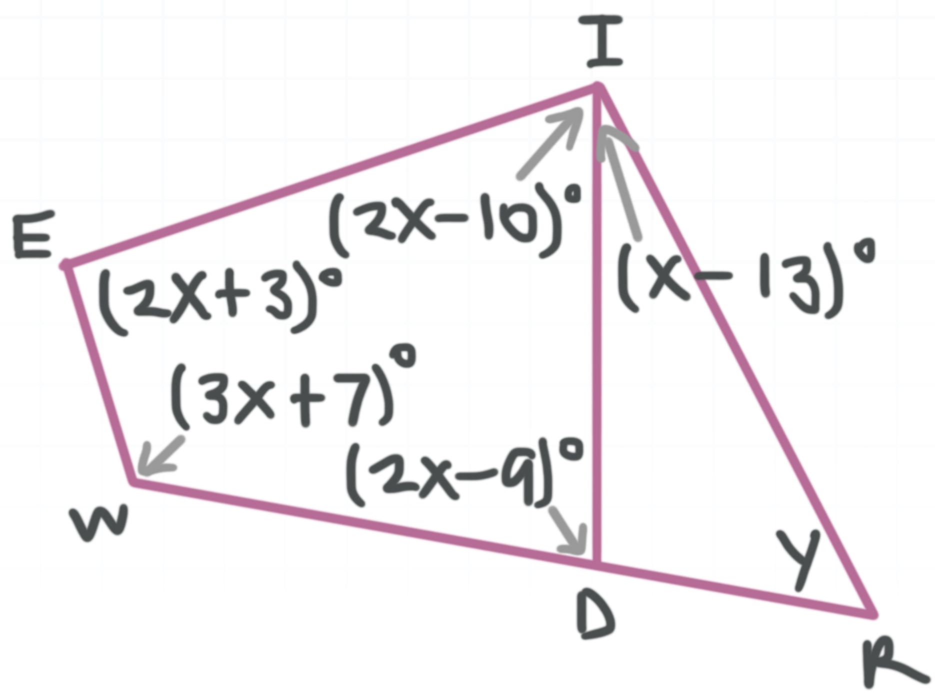
$$m\angle H = (128+2)^\circ$$

$$m\angle H = 130^\circ$$

Topic: Interior angles of polygons

Question: Find the value of y in degrees.

Hint: Any two adjacent angles that lie along the same line, like angles WDI and RDI, are supplementary, and the measures of every pair of supplementary angles always sums to 180°.



Answer choices:

A 38°

B 43°

C 45°

D 48°

Solution: C

In quadrilateral $WEID$ set the sum of the measures of the four interior angles equal to 360° and solve.

$$(3x+7)^\circ + (2x+3)^\circ + (2x-10)^\circ + (2x-9)^\circ = 360^\circ$$

$$(3x+2x+2x+2x)^\circ + (7+3-10-9)^\circ = 360^\circ$$

$$9x^\circ - 9^\circ = 360^\circ$$

$$9x^\circ = 369^\circ$$

$$x = 41$$

Find $m\angle WDI$.

$$m\angle WDI = (2x-9)^\circ$$

$$m\angle WDI = [2(41)-9]^\circ$$

$$m\angle WDI = (82-9)^\circ$$

$$m\angle WDI = 73^\circ$$

In $\triangle RDI$ (triangle RDI), find $m\angle RDI$ and $m\angle DIR$. $\angle WDI$ and $\angle RDI$ are adjacent and together make a straight line, so they're supplementary.

$$73^\circ + m\angle RDI = 180^\circ$$

$$m\angle RDI = 180^\circ - 73^\circ$$

$$m\angle RDI = 107^\circ$$

Find $m\angle DIR$.

$$m\angle DIR = (x - 13)^\circ$$

$$m\angle DIR = (41 - 13)^\circ$$

$$m\angle DIR = 28^\circ$$

Now find y, using the fact that the interior angles in triangle RDI sum to 180°.

$$m\angle RDI + m\angle DIR + m\angle IRD = 180^\circ$$

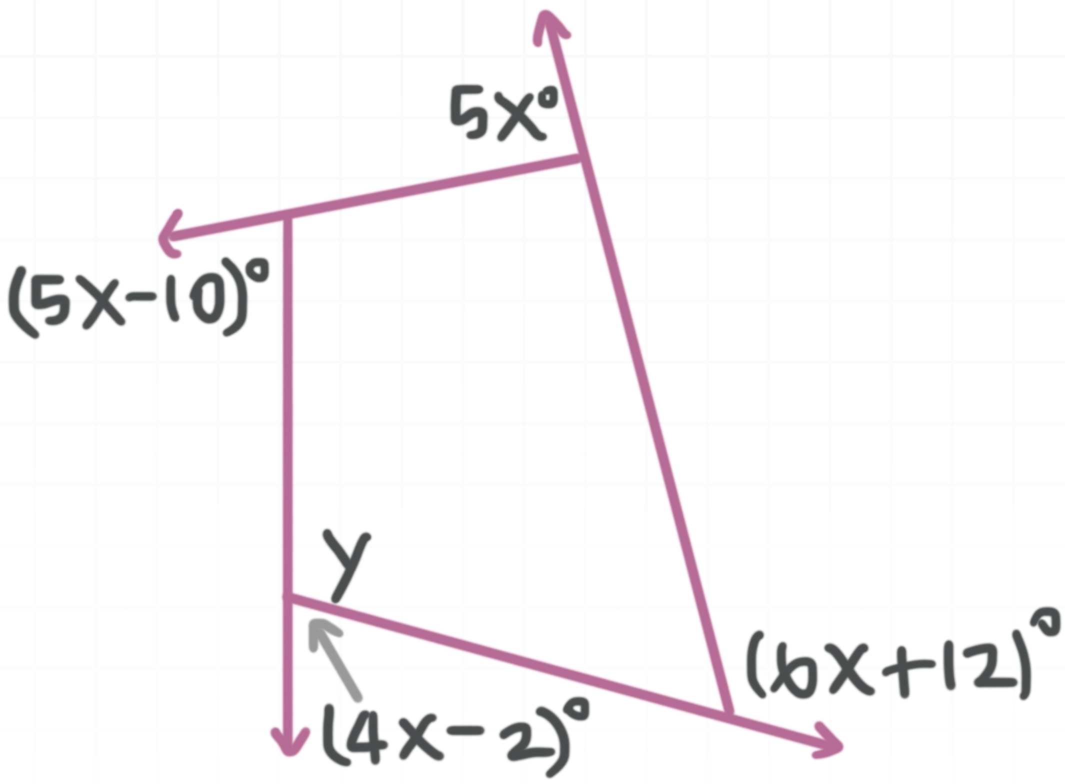
$$107^\circ + 28^\circ + y = 180^\circ$$

$$135^\circ + y = 180^\circ$$

$$y = 45^\circ$$

Topic: Exterior angles of polygons

Question: Find the value of y.



Answer choices:

A 105°

B 110°

C 115°

D 120°

Solution: B

The sum of the measures of the exterior angles is 360°. Therefore,

$$(4x-2)^\circ + (5x-10)^\circ + 5x^\circ + (6x+12)^\circ = 360^\circ$$

$$(4x^\circ + 5x^\circ + 5x^\circ + 6x^\circ) + (-2^\circ - 10^\circ + 12^\circ) = 360^\circ$$

$$20x^\circ + 0^\circ = 360^\circ$$

$$20x^\circ = 360^\circ$$

$$x = 18$$

The interior angle of measure y and the exterior angle of measure $(4x-2)^\circ$ are supplementary, so

$$y + (4x-2)^\circ = 180^\circ$$

Substitute 18 for x and solve for y.

$$y + [4(18) - 2]^\circ = 180^\circ$$

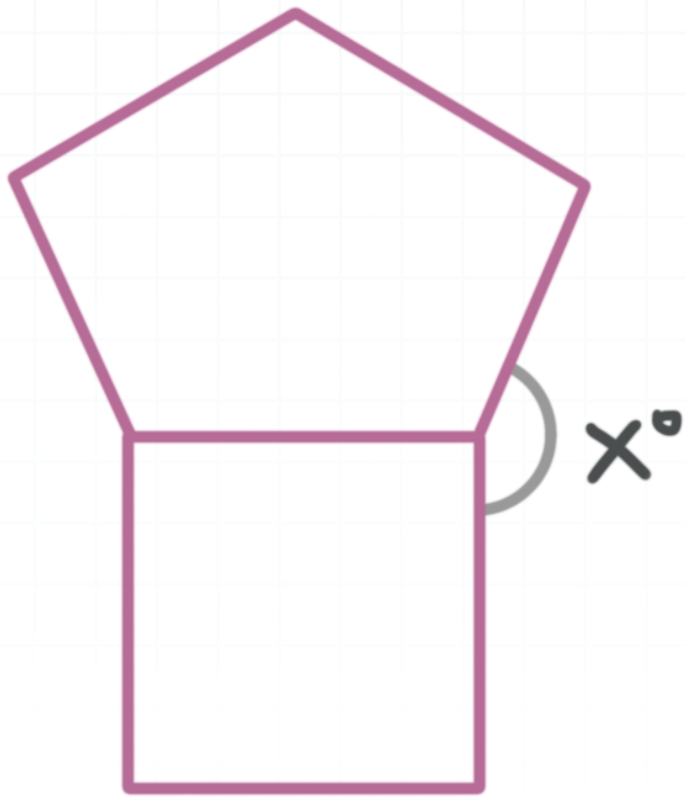
$$y + (72-2)^\circ = 180^\circ$$

$$y + 70^\circ = 180^\circ$$

$$y = 110^\circ$$

Topic: Exterior angles of polygons

Question: The figure shows a regular pentagon and a square (a regular quadrilateral). Find the value of x.



Answer choices:

A 132°

B 142°

C 152°

D 162°

Solution: D

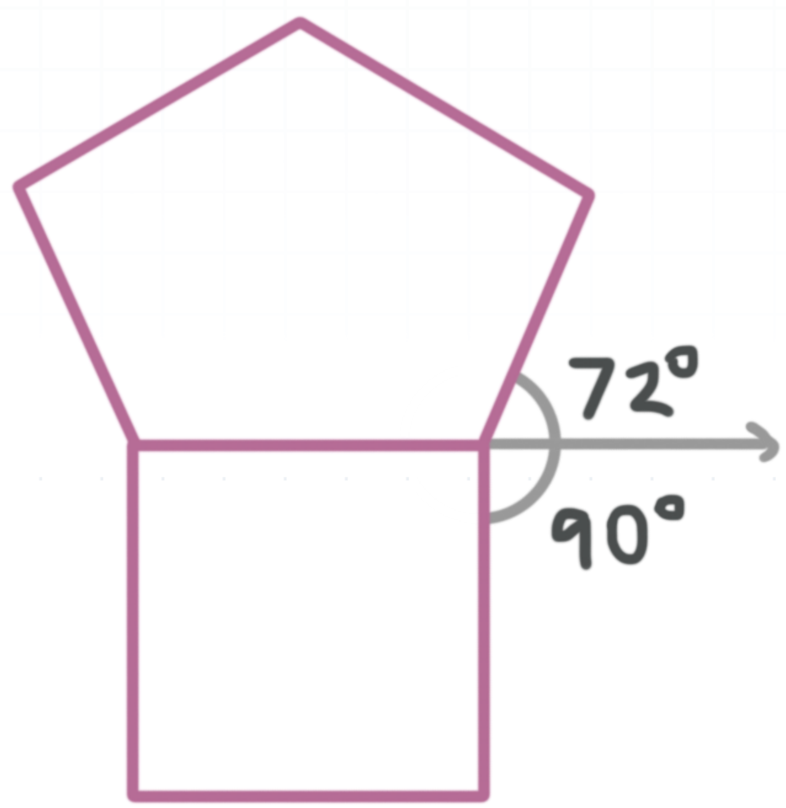
Notice that the angle of measure x is formed from an exterior angle of the pentagon and an exterior angle of the square, which are a pair of adjacent angles, and that the sum of their measures is x.

An exterior angle of a regular pentagon has a measure of

$$360^\circ \div 5 = 72^\circ$$

An exterior angle of a square has a measure of

$$360^\circ \div 4 = 90^\circ$$



Therefore,

$$x = 72^\circ + 90^\circ$$

$$x = 162^\circ$$

Topic: Exterior angles of polygons

Question: A certain regular polygon has an exterior angle of measure $2x+8^\circ$ and an exterior angle of measure $56^\circ - x$. What is the sum of the interior angles of the polygon?

Answer choices:

A $1{,}080^\circ$

B $1{,}260^\circ$

C $1{,}440^\circ$

D $1{,}620^\circ$

Solution: B

All the exterior angles of a regular polygon are congruent, so

$$2x + 8^\circ = 56^\circ - x$$

$$3x + 8^\circ = 56^\circ$$

$$3x = 48^\circ$$

$$x = 16^\circ$$

Using the exterior angle of measure $56^\circ - x$, we find that the measure of each exterior angle is

$$56^\circ - 16^\circ = 40^\circ$$

The number of sides, n, will be

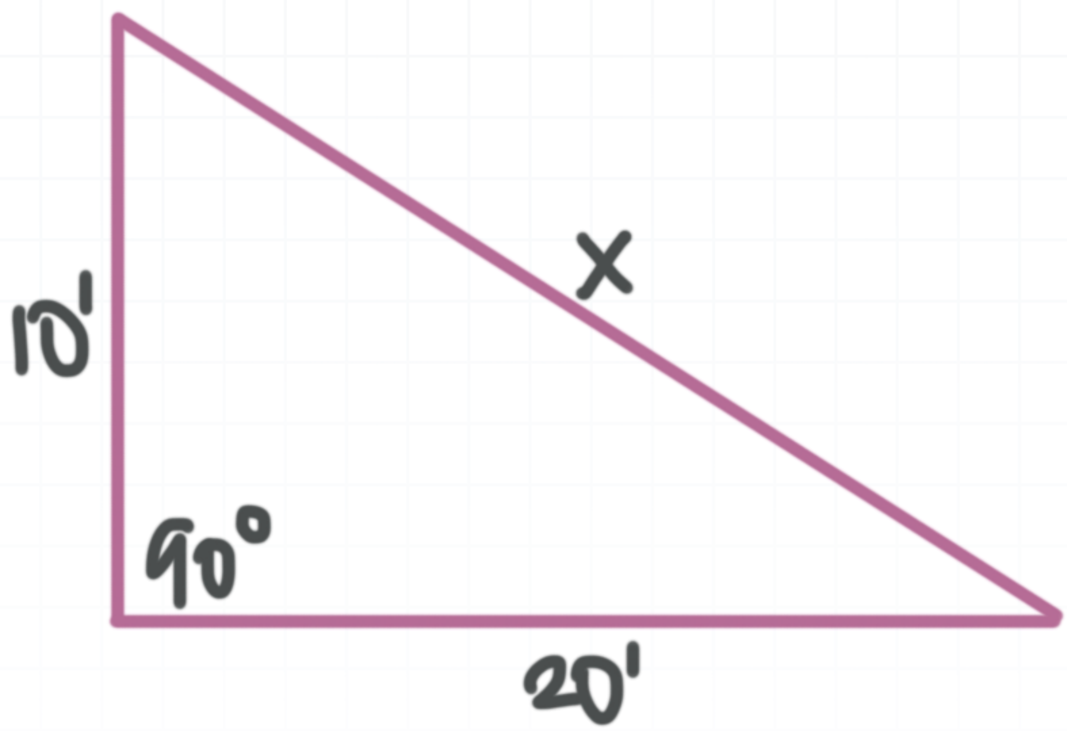
$$360^\circ \div 40^\circ = 9$$

The sum of the measures of the interior angles is therefore

$$(9 - 2)180^\circ = 1{,}260^\circ$$

Topic: Pythagorean theorem

Question: Find the length of the hypotenuse.



Answer choices:

A $5\sqrt{5}$

B $10\sqrt{5}$

C $2\sqrt{10}$

D $4\sqrt{5}$

Solution: B

Since this is a right triangle and we already know the lengths of two of its sides, we can use the Pythagorean theorem to find the length of the third side. The Pythagorean theorem is

$$a^2 + b^2 = c^2$$

where a and b are the lengths of the legs (the sides that form the 90° angle) and c is the length of the hypotenuse.

Plugging in the lengths we've been given, we get

$$(10 \text{ ft})^2 + (20 \text{ ft})^2 = c^2$$

$$100 \text{ ft}^2 + 400 \text{ ft}^2 = c^2$$

$$500 \text{ ft}^2 = c^2$$

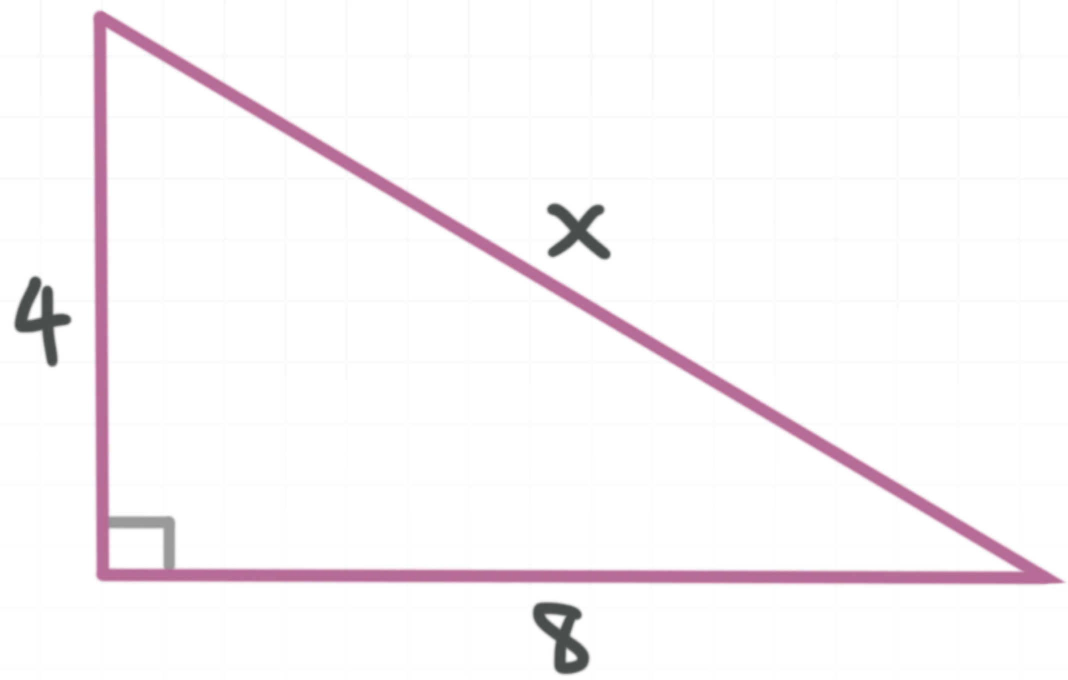
$$c = \sqrt{500} \text{ ft}$$

$$c = \sqrt{100 \cdot 5} \text{ ft}$$

$$c = 10\sqrt{5} \text{ ft}$$

Topic: Pythagorean theorem

Question: Find the length of the unknown side.



Answer choices:

A 5

B $5\sqrt{4}$

C 6

D $4\sqrt{5}$

Solution: D

The unknown side of this triangle is the hypotenuse, so plug the lengths of the two legs for a and b into the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

$$4^2 + 8^2 = x^2$$

$$16 + 64 = x^2$$

$$80 = x^2$$

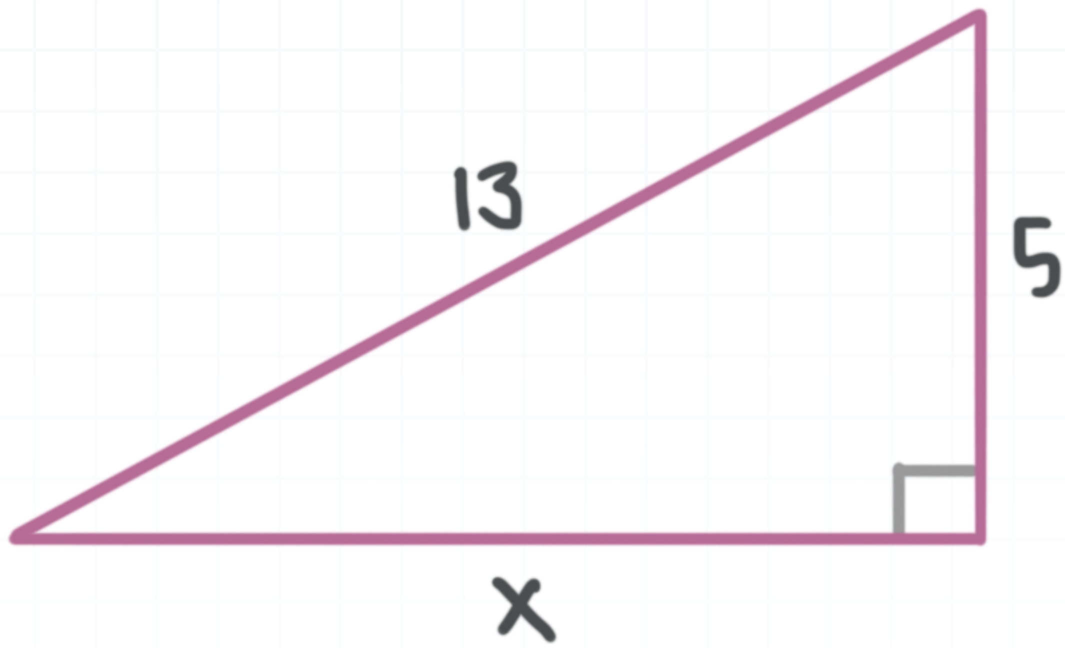
$$x = \sqrt{80}$$

$$x = \sqrt{16 \cdot 5}$$

$$x = 4\sqrt{5}$$

Topic: Pythagorean theorem

Question: Find the value of x.



Answer choices:

A $2\sqrt{5}$

B $5\sqrt{5}$

C 6

D 12

Solution: D

The figure we've been given is a rectangle, which means that the side opposite the side of length 5 must also have length 5.

In the figure, there are actually two right triangles (one in the top half of the rectangle and the other in the bottom half). We'll use the Pythagorean theorem to find x, which is the length of one of the legs of the right triangle in the bottom half of the figure.

If we plug all three side lengths, including x, into the Pythagorean theorem, we get

$$a^2 + b^2 = c^2$$

$$5^2 + x^2 = 13^2$$

$$25 + x^2 = 169$$

$$x^2 = 169 - 25$$

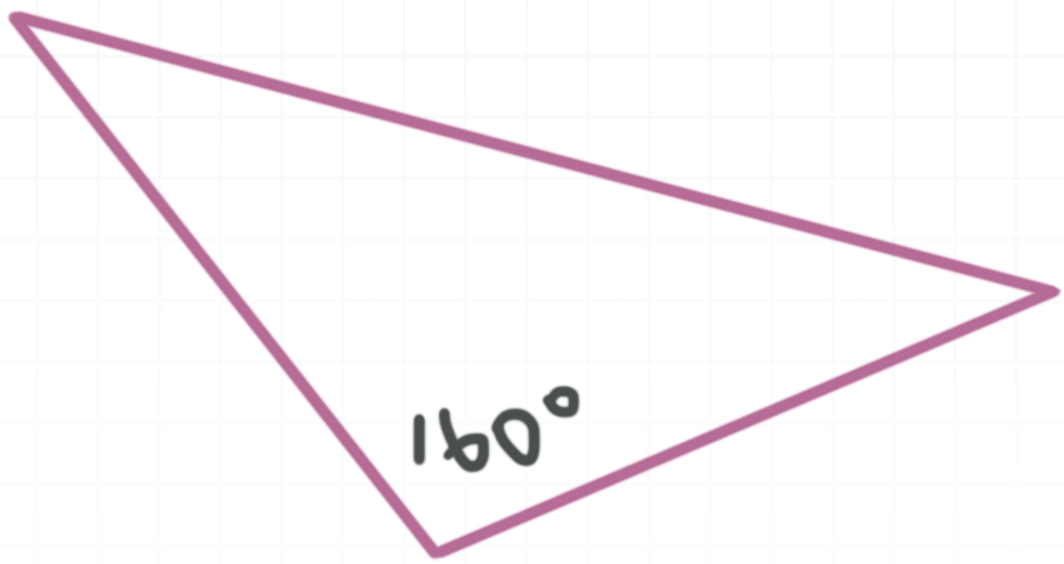
$$x^2 = 144$$

$$x = \sqrt{144}$$

$$x = 12$$

Topic: Pythagorean inequalities

Question: Classify the triangle.



Answer choices:

A Equilateral

B Acute

C Right

D Obtuse

Solution: D

The triangle is obtuse, because it has an obtuse angle, which is an angle that’s greater than 90°.

Topic: Pythagorean inequalities

Question: Each of the triangles below is obtuse and has sides of length a, b, and c, with $a < b < c$. In which triangle is the measure of the obtuse angle the largest?

Triangle	a	b	c
I	9	12	16
II	9	12	17
III	9	12	18
IV	9	12	19

Answer choices:

A I

B II

C III

D IV

Solution: D

In every triangle ABC with obtuse angle C, the inequality $c^2 > a^2 + b^2$ must be satisfied, but you don't need to do any calculations for this one.

The lengths of the two shortest sides ($a = 9$ and $b = 12$) are the same in all four triangles, so the larger the length of the third side (c) is, the larger the measure of angle C is. In the table, Triangle IV has the largest value of c, so it will have the obtuse angle C with the largest measure.

Topic: Pythagorean inequalities

Question: Each of the following triangles has sides of length a, b, and c, with $a \leq b \leq c$. Which triangles are right triangles?

Triangle	a	b	c
I	5	$5\sqrt{3}$	10
II	9	12	15
III	$\sqrt{2}$	$\sqrt{2}$	2

Answer choices:

A II and III

B I and III

C I, II, and III

D None of these

Solution: C

For each triangle in the table, we need to check to see if $a^2 + b^2 = c^2$.

For triangle I:

$$a^2 + b^2 = c^2$$

$$5^2 + (5\sqrt{3})^2 = 10^2$$

$$25 + 25(3) = 100$$

$$25 + 75 = 100$$

$$100 = 100$$

For triangle II:

$$a^2 + b^2 = c^2$$

$$9^2 + 12^2 = 15^2$$

$$81 + 144 = 225$$

$$225 = 225$$

For triangle III:

$$a^2 + b^2 = c^2$$

$$(\sqrt{2})^2 + (\sqrt{2})^2 = 2^2$$

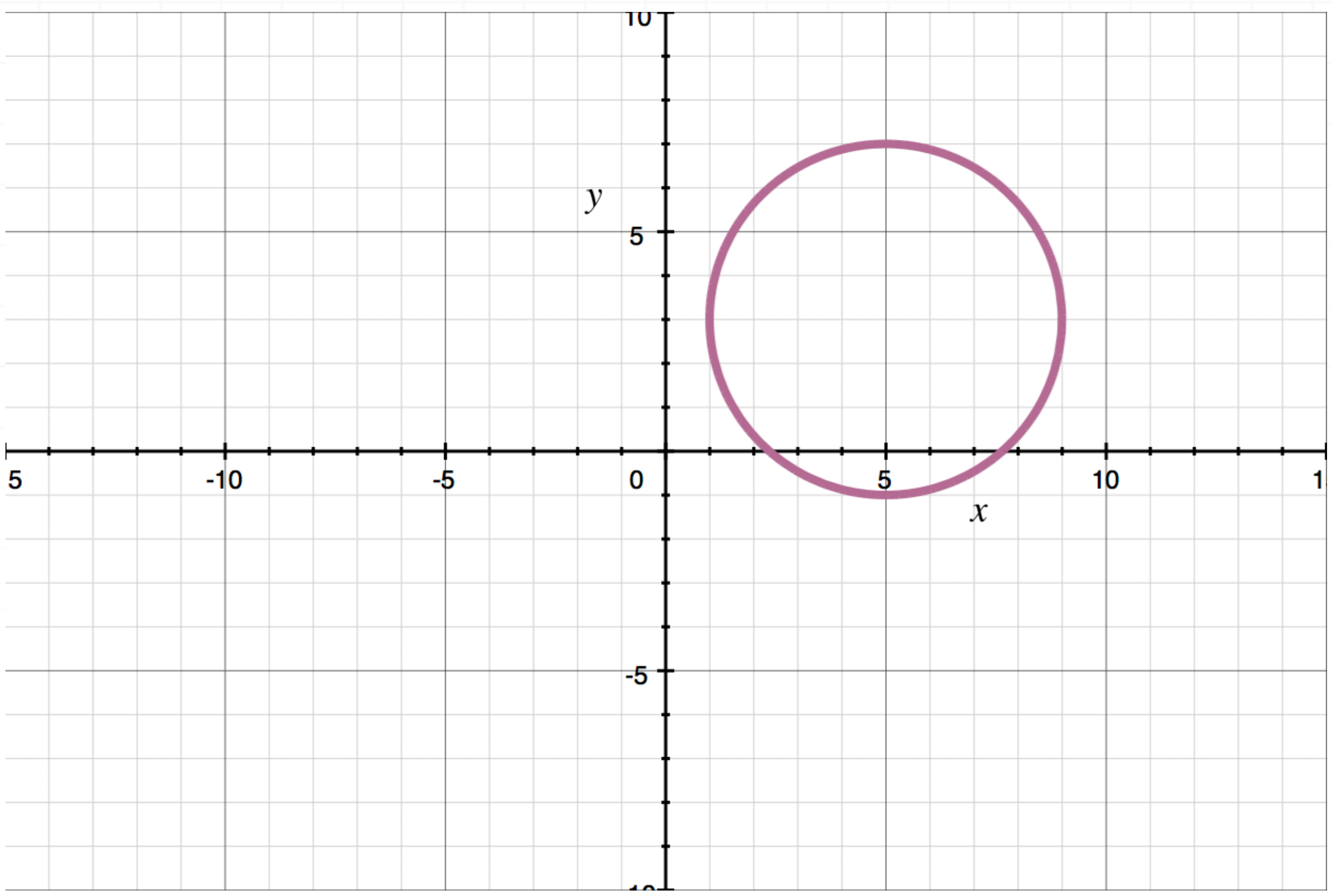
$$2 + 2 = 4$$

$$4 = 4$$

Because all three of these equations were true, all three triangles are right triangles.

Topic: Equation of a circle

Question: What is the equation of the circle?



Answer choices:

A $(x+5)^2+(y+3)^2=4$

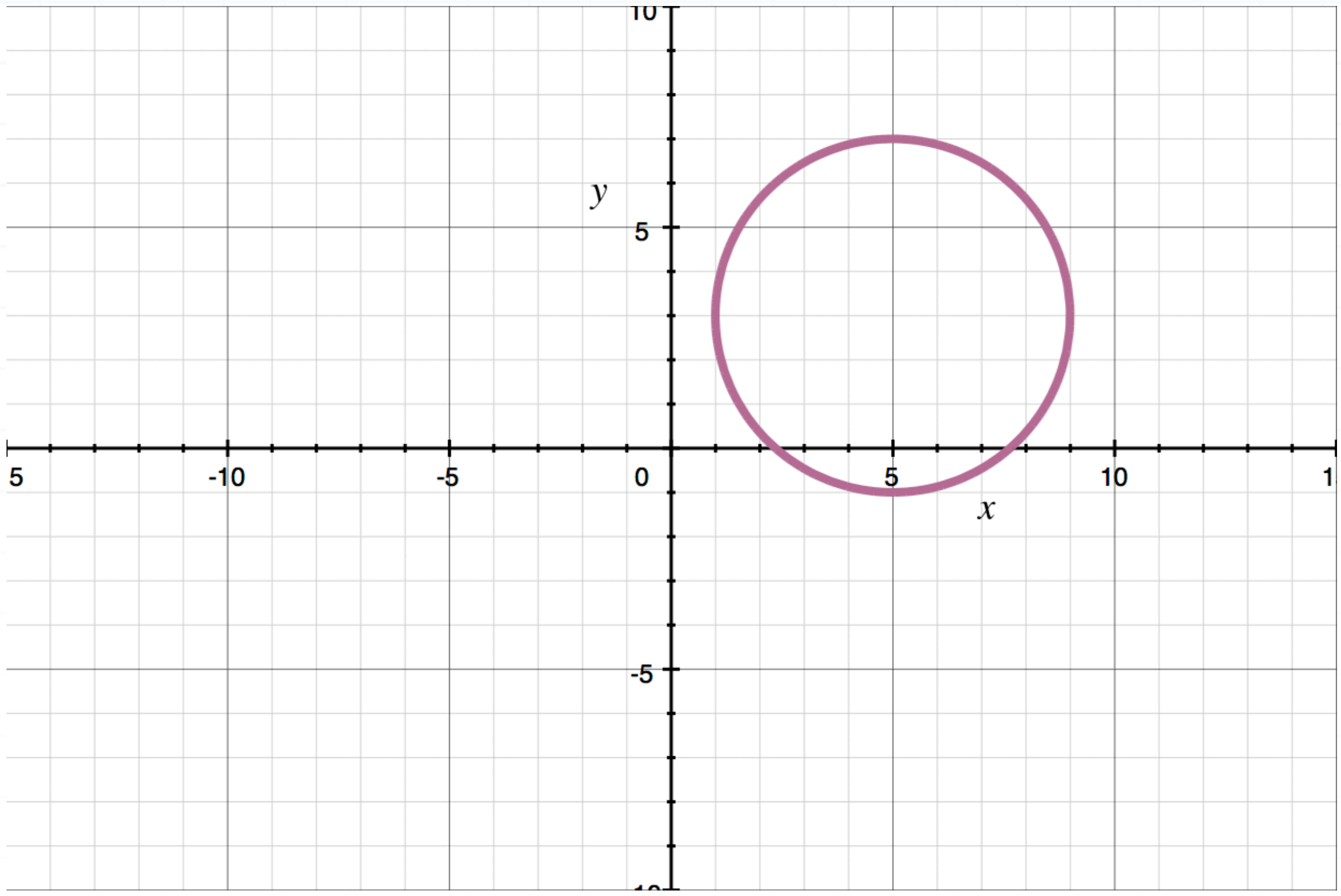
B $(x-5)^2+(y-3)^2=4$

C $(x-5)^2+(y+3)^2=4^2$

D $(x-5)^2+(y-3)^2=16$

Solution: D

The center of the circle is $(5,3)$.



That means that, in the equation of the circle, 5 will be h and 3 will be k.

The radius is the distance from the center to any point on the circle. If we use the point $(1,3)$, then its distance from the center is 4. Plugging the values of h, k, and r into the formula for the equation of a circle gives

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-5)^2+(y-3)^2=16$$

Topic: Equation of a circle

Question: What are the x-intercepts of the circle $(x+4)^2+(y-3)^2=25$?

Answer choices:

A $(1,0)$, $(-9,0)$

B $(-4,0)$, $(3,0)$

C $(0,0)$, $(-8,0)$

D $(5,0)$, $(-4,0)$

Solution: C

The x-intercepts are the points at which $y = 0$, so substitute $y = 0$ into the equation of the circle.

$$(x+4)^2 + (y-3)^2 = 25$$

$$(x+4)^2 + (0-3)^2 = 25$$

$$x^2 + 8x + 16 + 9 = 25$$

$$x^2 + 8x = 0$$

$$x(x+8) = 0$$

$x = 0$ or $x = -8$

So the x-intercepts are $(0,0)$ and $(-8,0)$.

Topic: Equation of a circle

Question: What are the center and radius of the circle $(x+5)^2 + y^2 = 11$?

Answer choices:

A $(-5,0),\ 11$

B $(-5,0),\ \sqrt{11}$

C $(5,0),\ \sqrt{11}$

D $(0,5),\ 11$

Solution: B

Following the pattern $(x-h)^2+(y-k)^2=r^2$ tells us that

$$h=-5$$

$$k=0$$

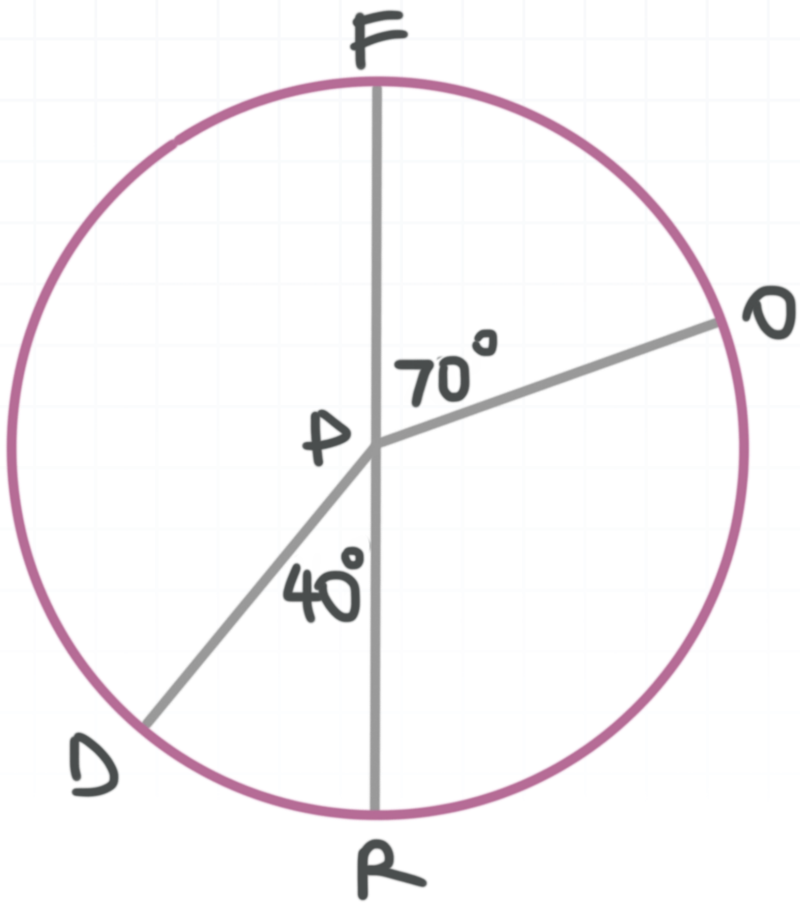
$$r^2=11$$

$$r=\sqrt{11}$$

The center of the circle is $(-5,0)$ and the radius is $\sqrt{11}$.

Topic: Degree measure of an arc

Question: $\overline{FR}$ is a diameter of the circle (with center at P). What is the sum of the measures of $\overset{\frown}{RO}$ and $\overset{\frown}{FD}$?



Answer choices:

A 110°

B 140°

C 250°

D 360°

Solution: C

$\overline{FR}$ is a diameter, so the sum of the measures of $\overarc{RO}$ and $\overarc{OF}$ is 180°.

$$m\overarc{RO} + m\overarc{OF} = 180^\circ$$

$$m\overarc{RO} + 70^\circ = 180^\circ$$

$$m\overarc{RO} = 110^\circ$$

Likewise, the sum of the measures of $\overarc{FD}$ and $\overarc{DR}$ is 180°.

$$m\overarc{FD} + m\overarc{DR} = 180^\circ$$

$$m\overarc{FD} + 40^\circ = 180^\circ$$

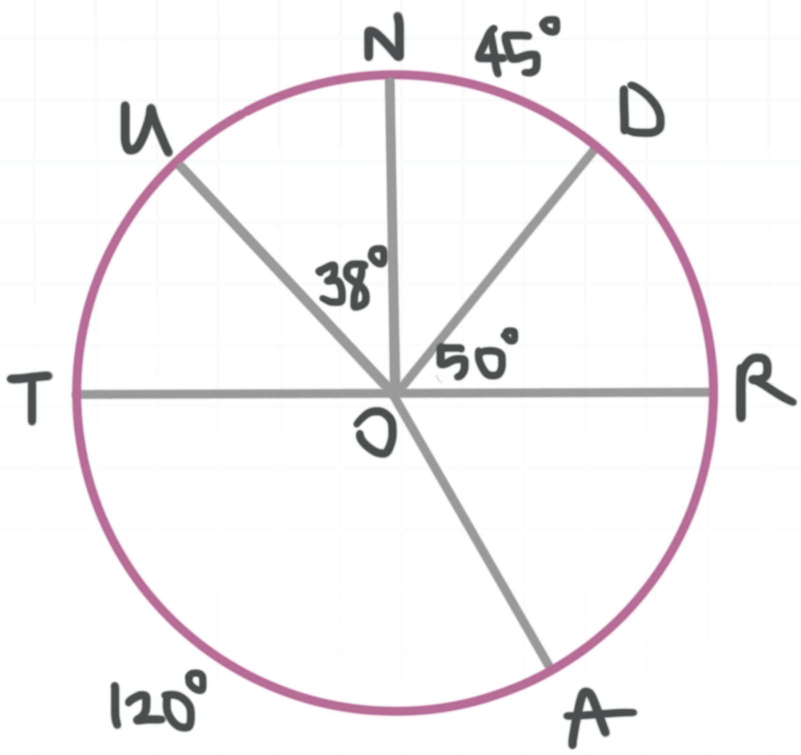
$$m\overarc{FD} = 140^\circ$$

The sum of the measures of arcs $\overarc{RO}$ and $\overarc{FD}$ is

$$110^\circ + 140^\circ = 250^\circ$$

Topic: Degree measure of an arc

Question: $\overline{TR}$ is a diameter of the circle (with center at O) in the figure. What is the difference between the measures of $\widehat{AR}$ and $\widehat{UT}$?



Answer choices:

A 82°

B 38°

C 22°

D 13°

Solution: D

$\overline{TR}$ is a diameter, so the sum of the measures of $\overset{\frown}{TA}$ and $\overset{\frown}{AR}$ is 180°.

$$m\overset{\frown}{TA} + m\overset{\frown}{AR} = 180^\circ$$

$$120^\circ + m\overset{\frown}{AR} = 180^\circ$$

$$m\overset{\frown}{AR} = 60^\circ$$

Likewise, the sum of the measures of $\overset{\frown}{RU}$ and $\overset{\frown}{UT}$ is 180°, and the measure of $\overset{\frown}{RU}$ can be written as the sum of the measures of arcs $\overset{\frown}{RD}$, $\overset{\frown}{DN}$, and $\overset{\frown}{NU}$. Therefore,

$$m\overset{\frown}{RU} + m\overset{\frown}{UT} = 180^\circ$$

$$(m\overset{\frown}{RD} + m\overset{\frown}{DN} + m\overset{\frown}{NU}) + m\overset{\frown}{UT} = 180^\circ$$

$$(50^\circ + 45^\circ + 38^\circ) + m\overset{\frown}{UT} = 180^\circ$$

$$133^\circ + m\overset{\frown}{UT} = 180^\circ$$

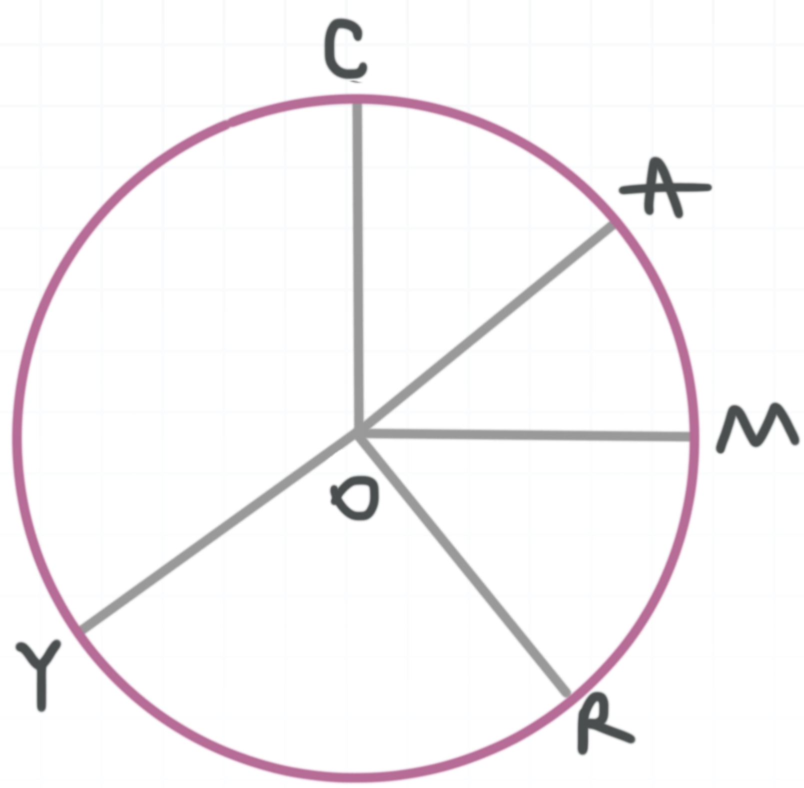
$$m\overset{\frown}{UT} = 47^\circ$$

The difference between the measures of $\overset{\frown}{AR}$ and $\overset{\frown}{UT}$ is

$$60^\circ - 47^\circ = 13^\circ$$

Topic: Degree measure of an arc

Question: Angles $\angle MOC$ and $\angle ROA$ are right angles. $m\angle MOA = 40^\circ$ and $m\angle YOR = 100^\circ$. Which arc has the largest measure?



Answer choices:

A $\quad \overset{\frown}{RC}$

B $\quad \overset{\frown}{YRC}$

C $\quad \overset{\frown}{YMA}$

D $\quad \overset{\frown}{MCY}$

Solution: B

Use the fact that $\angle MOA = 40^\circ$, and that $\angle MOC$ and $\angle ROA$ each have measure 90°, to figure out that $\angle AOC$ and $\angle ROM$ each have measure 50°.

Now you know the measures of the following four central angles: $\angle YOR$, $\angle ROM$, $\angle MOA$, and $\angle AOC$.

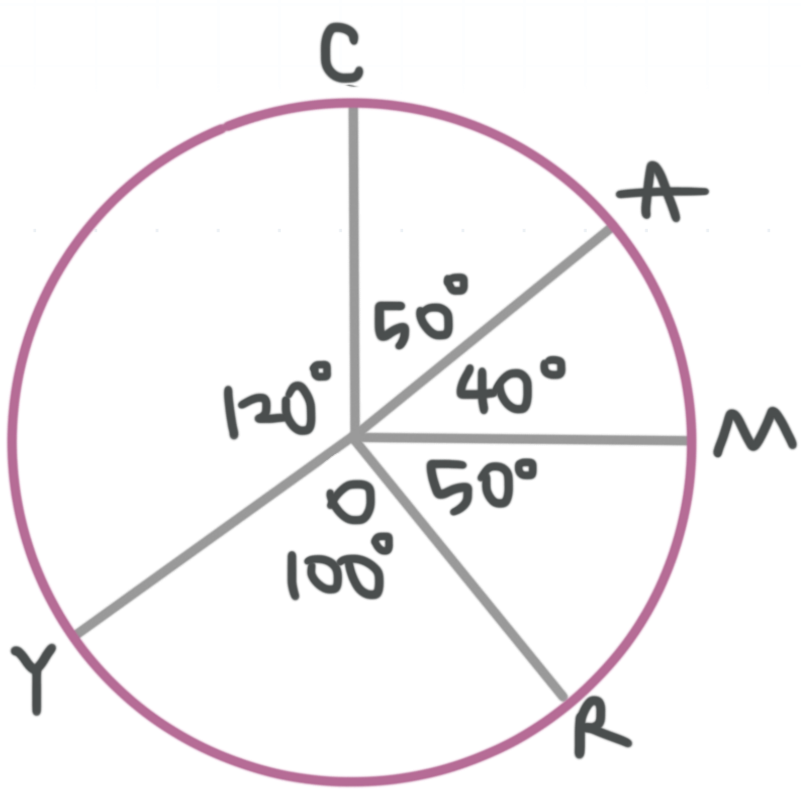
$$m\angle YOR = 100^\circ$$

$$m\angle ROM = 50^\circ$$

$$m\angle MOA = 40^\circ$$

$$m\angle AOC = 50^\circ$$

Subtract their total (240°) from 360° to get $m\angle COY = 120^\circ$.



Knowing the measures of those five central angles, you can figure out the measures of the arcs given as the answer choices.

$$m\overset{\frown}{RC} = 50^\circ + 40^\circ + 50^\circ = 140^\circ$$

$$m\overset{\frown}{YRC} = 100^\circ + 50^\circ + 40^\circ + 50^\circ = 240^\circ$$

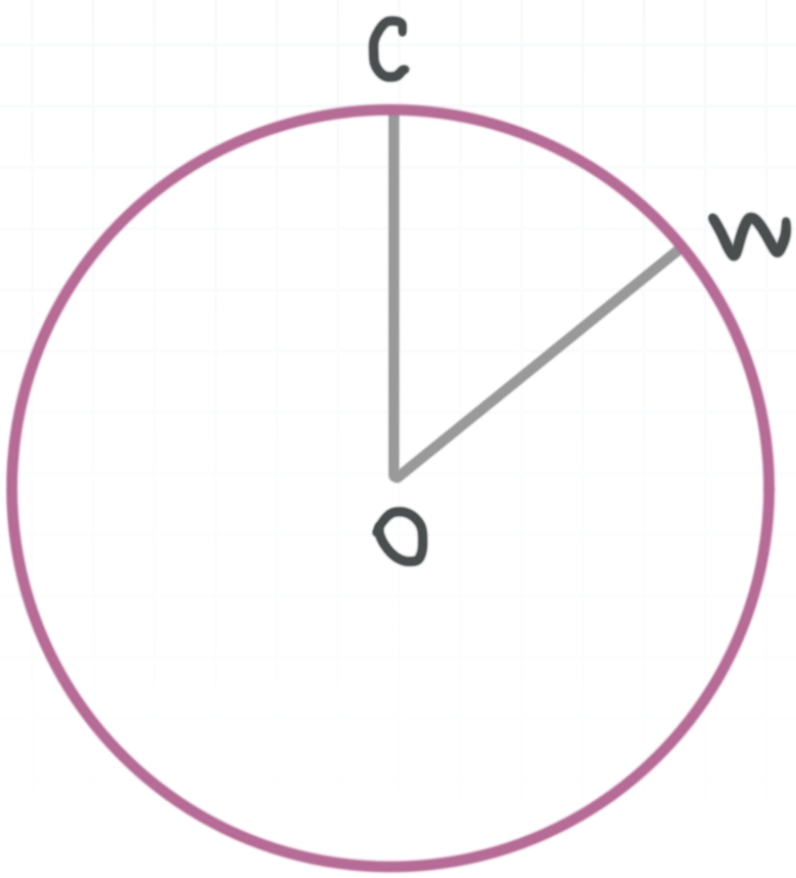
$$m\overset{\frown}{YMA} = 100^\circ + 50^\circ + 40^\circ = 190^\circ$$

$$m\overset{\frown}{MCY} = 40^\circ + 50^\circ + 120^\circ = 210^\circ$$

Of these, $\overset{\frown}{YRC}$ has the largest measure.

Topic: Arc length

Question: The center of the circle in the figure is at O, the length of $\overline{OC}$ is 18, and $m\angle WOC = 40^\circ$. What is the length of $\widehat{WC}$?



Answer choices:

A 4π

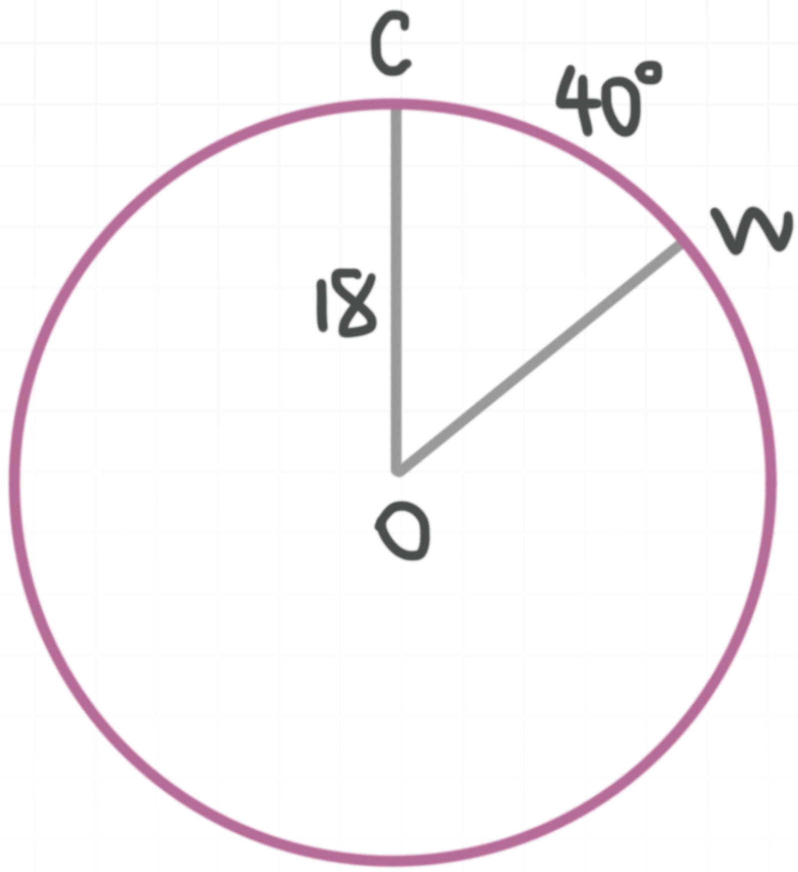
B 8π

C 10π

D 12π

Solution: A

We know that the measure of the central angle that corresponds to $\overset{\frown}{WC}$ is 40°, and that $\overline{OC}$ is a radius, so the radius of the circle is 18.



Plugging the values of m and r into the arc length formula gives

$$L = \frac{m}{360} \cdot 2\pi r$$

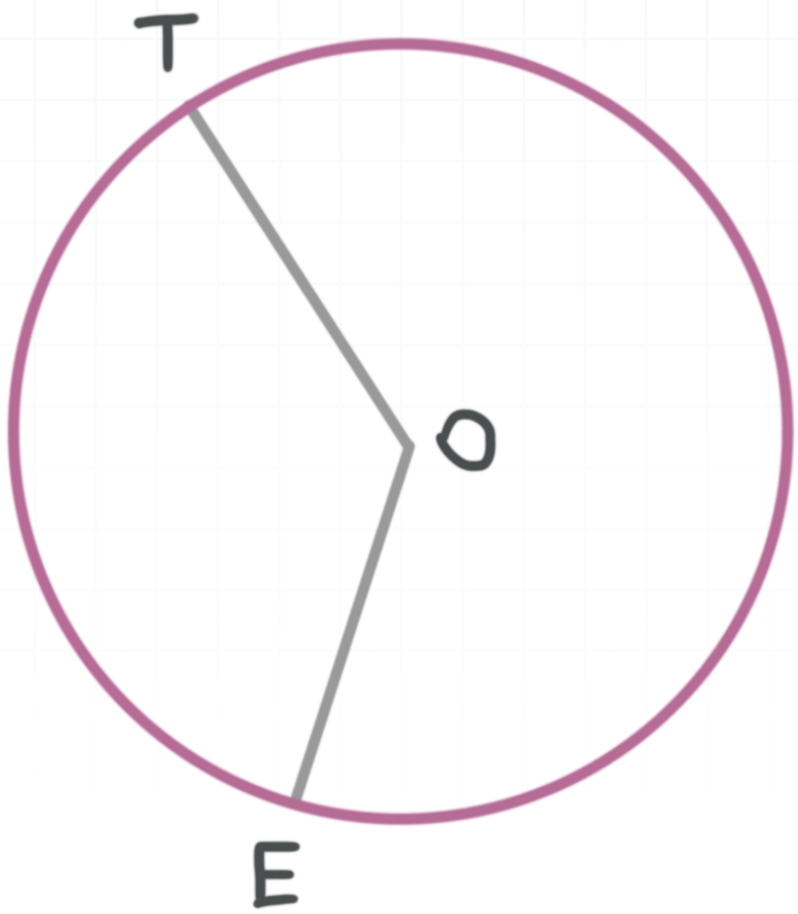
$$L = \frac{40}{360} \cdot 2\pi \cdot 18$$

$$L = \frac{1}{9} \cdot 36\pi$$

$$L = 4\pi$$

Topic: Arc length

Question: The center of the circle in the figure is at O, the length of $\overline{OT}$ is 6, and $m\angle TOE = 150^\circ$. What is the length of $\overset{\frown}{TE}$?



Answer choices:

A 4π

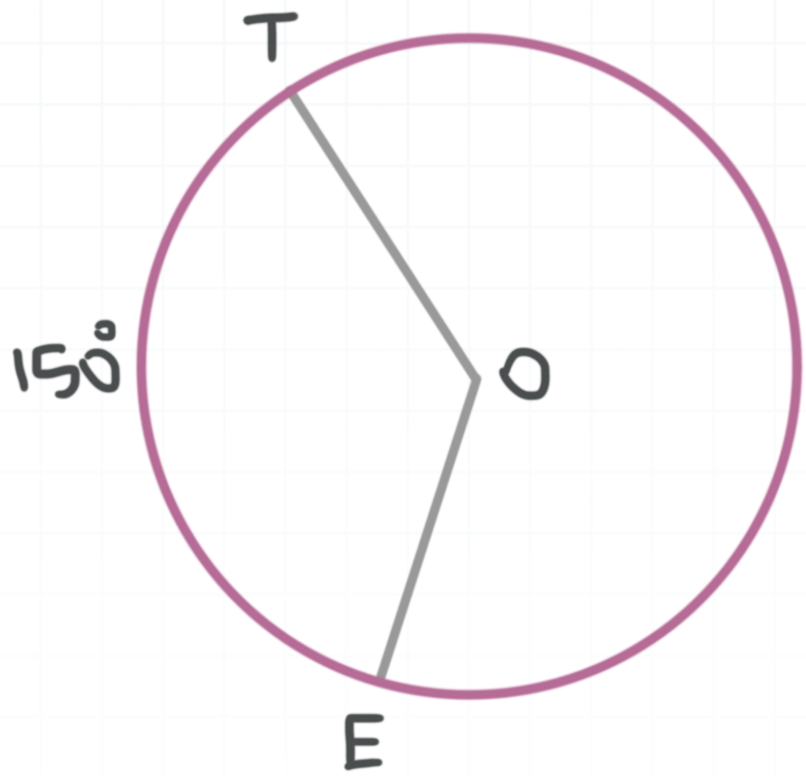
B 5π

C 6π

D 12π

Solution: B

We know that the measure of the central angle that corresponds to $\overset{\frown}{TE}$ is 150°, and that $\overline{OT}$ is a radius, so the radius of the circle is 6.



Plugging the values of m and r into the arc length formula gives

$$L = \frac{m}{360} \cdot 2\pi r$$

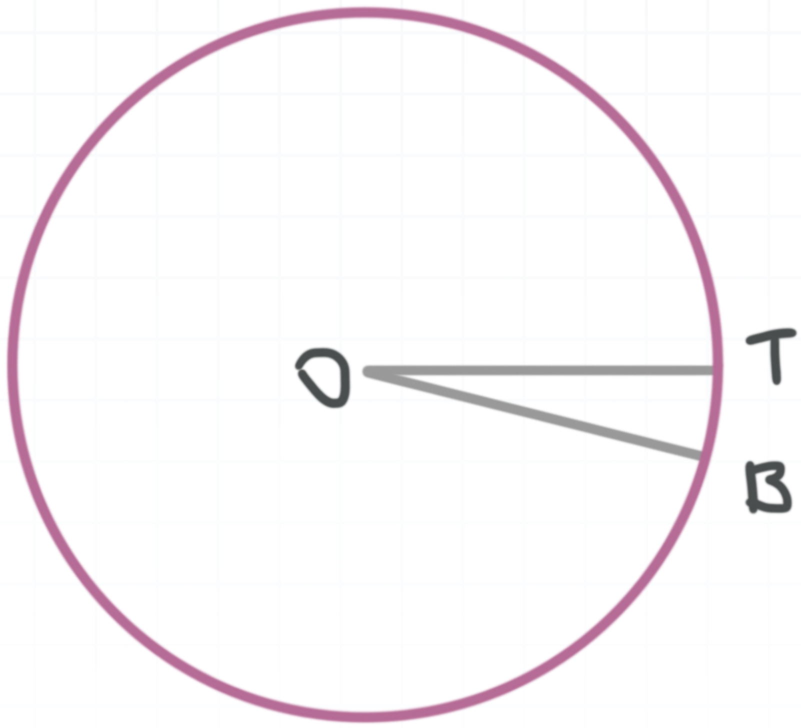
$$L = \frac{150}{360} \cdot 2\pi \cdot 6$$

$$L = \frac{5}{12} \cdot 12\pi$$

$$L = 5\pi$$

Topic: Arc length

Question: The center of the circle in the figure is at O, the length of $\overline{OT}$ is 15, and the length of $\overset{\frown}{BT}$ is $5\pi/6$. What is the measure of $\angle BOT$?



Answer choices:

A 5°

B 8°

C 10°

D 12°

Solution: C

We know that the length of $\overset{\frown}{BT}$ is $5\pi/6$ and that the radius of the circle is 15. Substituting these values into the arc length formula gives

$$L = \frac{m}{360} \cdot 2\pi r$$

$$\frac{5\pi}{6} = \frac{m}{360} \cdot 2\pi \cdot 15$$

$$\frac{5\pi}{6} = \frac{30\pi m}{360}$$

$$\frac{5\pi}{6} = \frac{\pi m}{12}$$

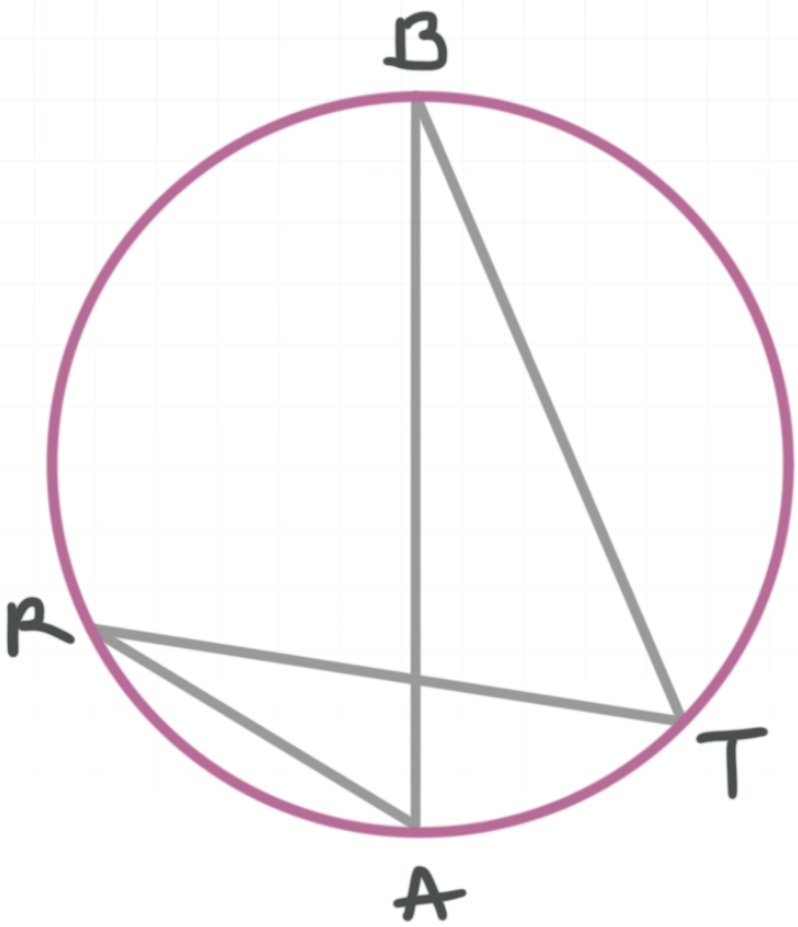
$$10\pi = \pi m$$

$$m = 10$$

So $m\angle BOT = 10^\circ$.

Topic: Inscribed angles of circles

Question: In the circle, the measure of $\widehat{AT}$ is 36°. Find the sum of the angle measures, $m\angle ABT + m\angle ART$.



Answer choices:

A 0°

B 36°

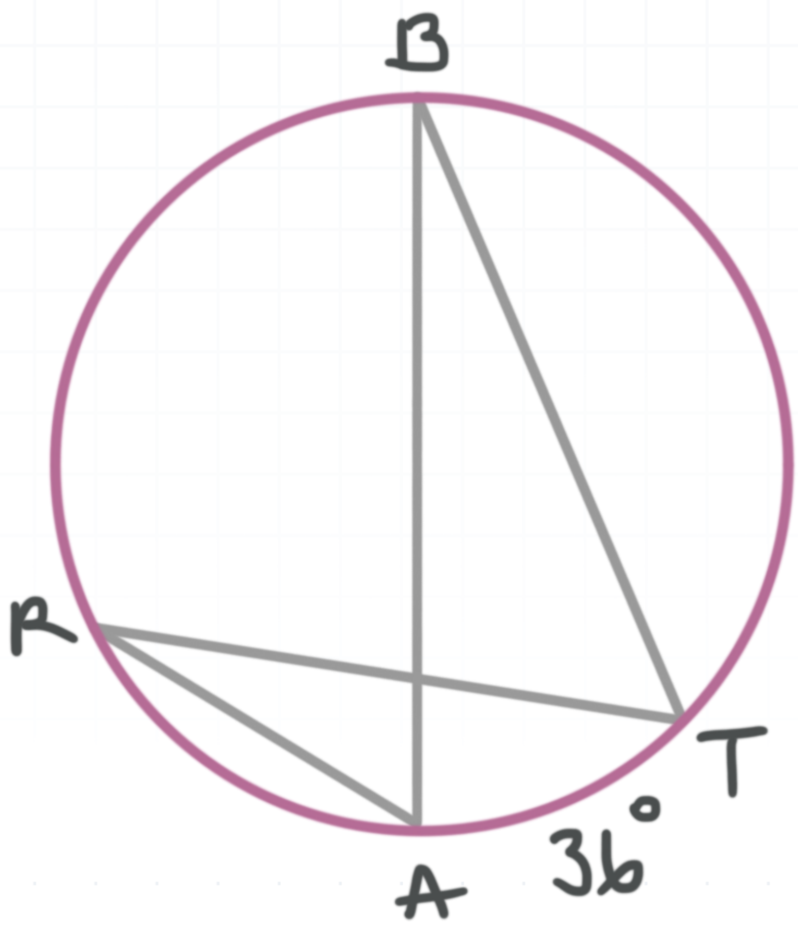
C 48°

D 72°

Solution: B

The angle $\angle ABT$ is an inscribed angle, so its measure is half that of its intercepted arc, $\overset{\frown}{AT}$.

$$m\angle ABT = \frac{1}{2}(36^\circ) = 18^\circ$$



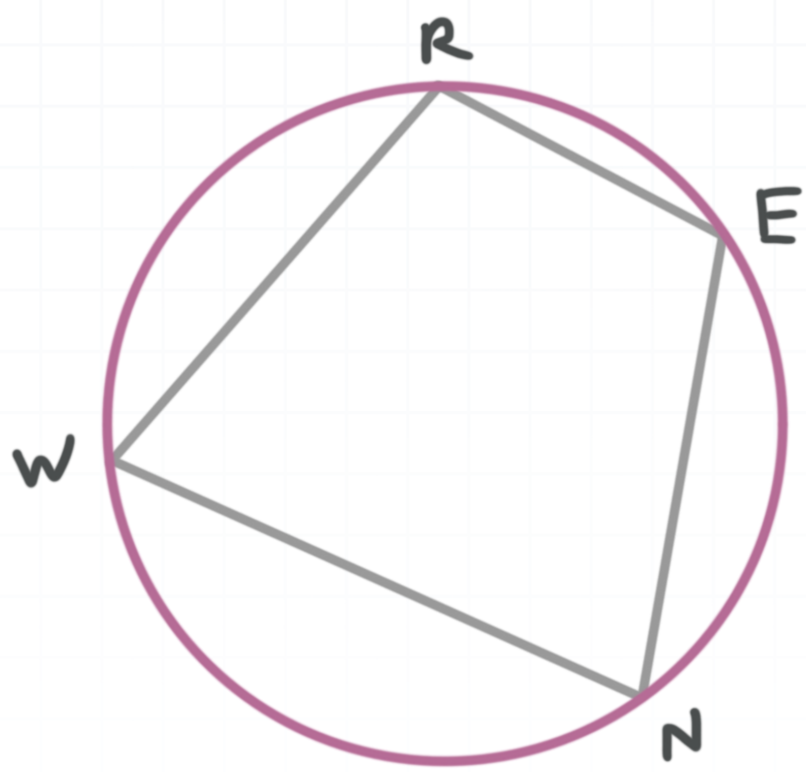
$m\angle ART$ is also an inscribed angle with intercepted arc $\overset{\frown}{AT}$, so its measure is also half that of $\overset{\frown}{AT}$, which means that $m\angle ART = 18^\circ$. Therefore,

$$m\angle ABT + m\angle ART = 18^\circ + 18^\circ$$

$$m\angle ABT + m\angle ART = 36^\circ$$

Topic: Inscribed angles of circles

Question: In the circle, $m\angle ENW = 50^\circ$, the measure of $\overset{\frown}{RW}$ is 70°, and the measure of $\overset{\frown}{EN}$ is 110°. What is $m\angle NWR$?



Answer choices:

A 55°

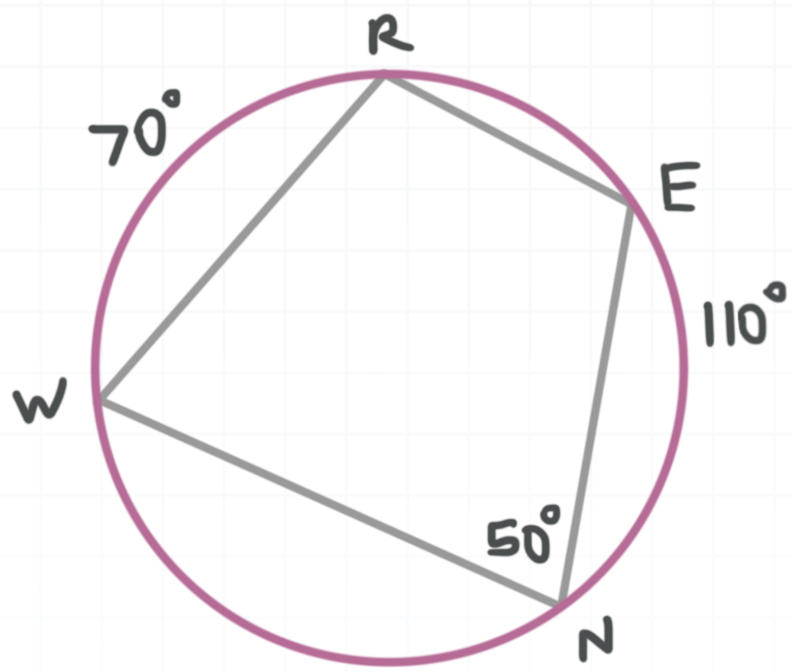
B 60°

C 65°

D 70°

Solution: D

The angle measure $m\angle ENW$ (which is 50°) is half that of $\overset{\frown}{EW}$.



So the measure of $\overset{\frown}{EW}$ is 100°. Also,

$$m\overset{\frown}{EW} = m\overset{\frown}{ER} + m\overset{\frown}{RW}$$

$$100^\circ = m\overset{\frown}{ER} + 70^\circ$$

$$m\overset{\frown}{ER} = 30^\circ$$

Now we know that

$$m\overset{\frown}{NR} = m\overset{\frown}{NE} + m\overset{\frown}{ER} = 110^\circ + 30^\circ = 140^\circ$$

Notice that $\overset{\frown}{NR}$ is the arc intercepted by inscribed angle $\angle NWR$, so

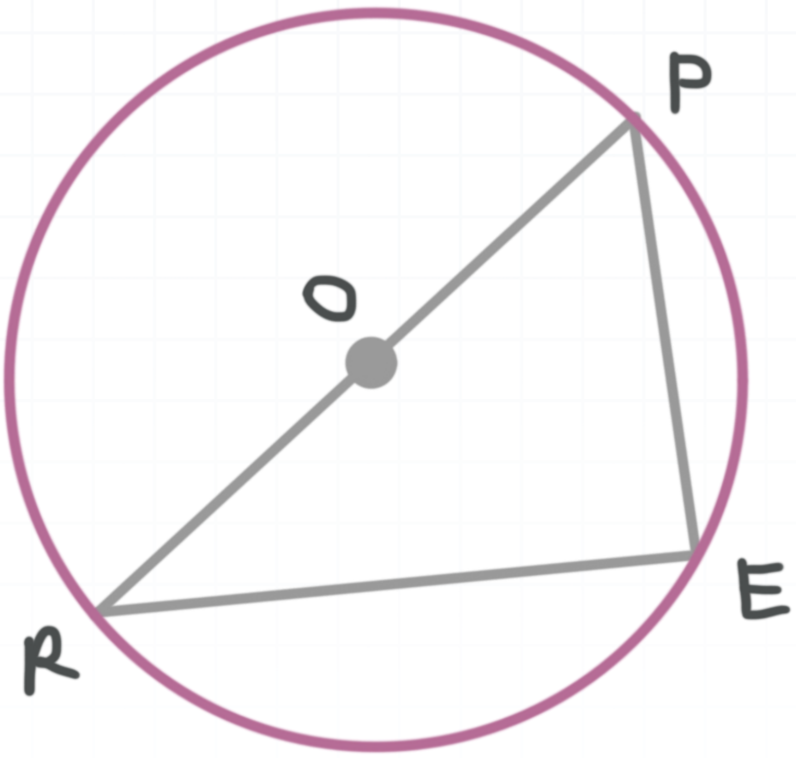
$$m\angle NWR = \frac{1}{2}m\overset{\frown}{NR}$$

$$m\angle NWR = \frac{1}{2}(140^\circ)$$

$m\angle NWR = 70^\circ$

Topic: Inscribed angles of circles

Question: $\overline{RP}$ is a diameter of the circle (with center at O), $m\angle ERP = (5x + 6)^\circ$, and $m\angle RPE = (6x + 7)^\circ$. What is $m\angle ERP$?



Answer choices:

A 41°

B 45°

C 49°

D 54°

Solution: A

The arc intercepted by inscribed angle $\angle PER$ is a semicircle, so

$$m\angle PER = \frac{1}{2}(180^\circ) = 90^\circ$$

Earlier, when we talked about interior angles of polygons, we learned that the sum of the three interior angles of a triangle is 180°. The interior angles of triangle RPE are $\angle ERP$, $\angle PER$, and $\angle RPE$. Therefore,

$$m\angle ERP + m\angle PER + m\angle RPE = 180^\circ$$

Since $m\angle PER = 90^\circ$,

$$m\angle ERP + 90^\circ + m\angle RPE = 180^\circ$$

$$m\angle ERP + m\angle RPE = 90^\circ$$

Substitute the expressions for the measures of $\angle ERP$ and $\angle RPE$, and solve for x.

$$(5x + 6)^\circ + (6x + 7)^\circ = 90^\circ$$

$$11x^\circ + 13^\circ = 90^\circ$$

$$11x^\circ = 77^\circ$$

$$x^\circ = 7^\circ$$

$$x = 7$$

So

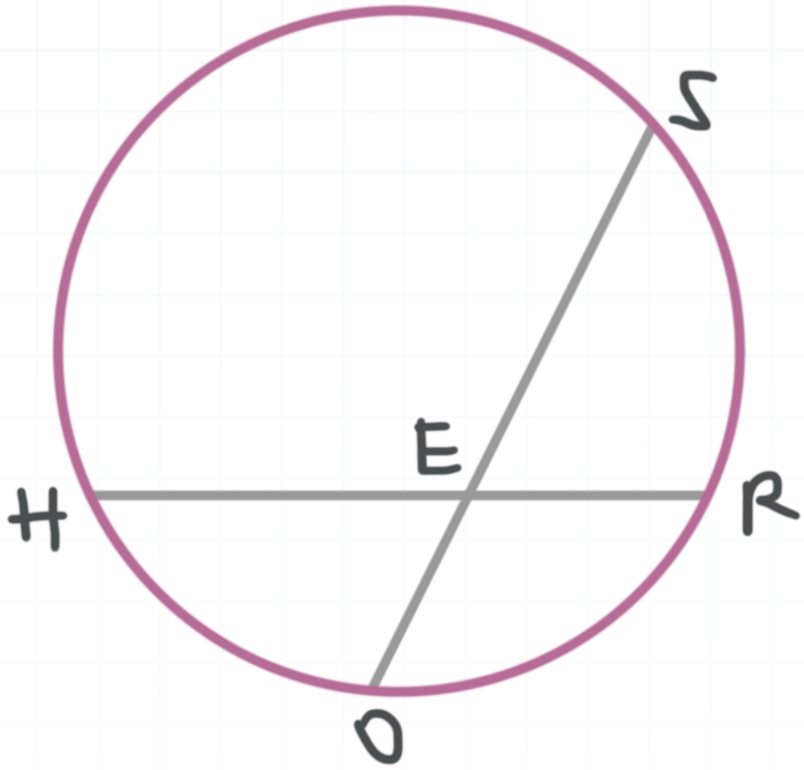
$$m\angle ERP = (5x + 6)^\circ$$

$$m\angle ERP = (5(7) + 6)^\circ$$

$$m\angle ERP = 41^\circ$$

Topic: Vertex on, inside, and outside the circle

Question: In the figure, $m\overset{\frown}{RS} = 90^\circ$ and $m\overset{\frown}{HO} = 50^\circ$. What is $m\angle OER$?



Answer choices:

A 50°

B 70°

C 90°

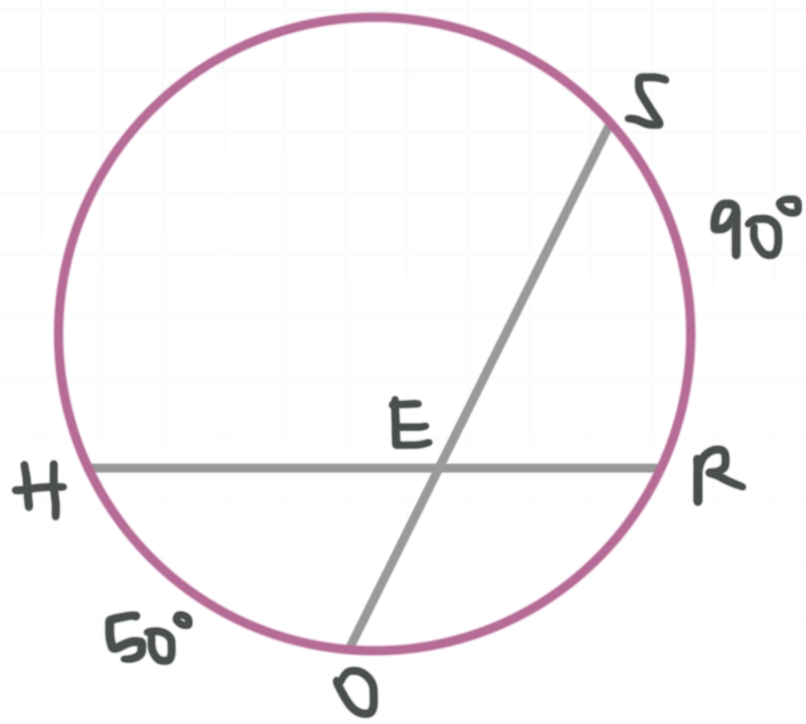
D 110°

Solution: D

We can find the measure of $\angle HEO$ given the arc lengths we already know.

$$m\angle HEO = \frac{1}{2}(m\widehat{HO} + m\widehat{RS})$$

$$m\angle HEO = \frac{1}{2}(50^\circ + 90^\circ)$$



$$m\angle HEO = 70^\circ$$

Because $\angle HEO$ and $\angle OER$ are a pair of adjacent angle that together form a straight line,

$$m\angle HEO + m\angle OER = 180^\circ$$

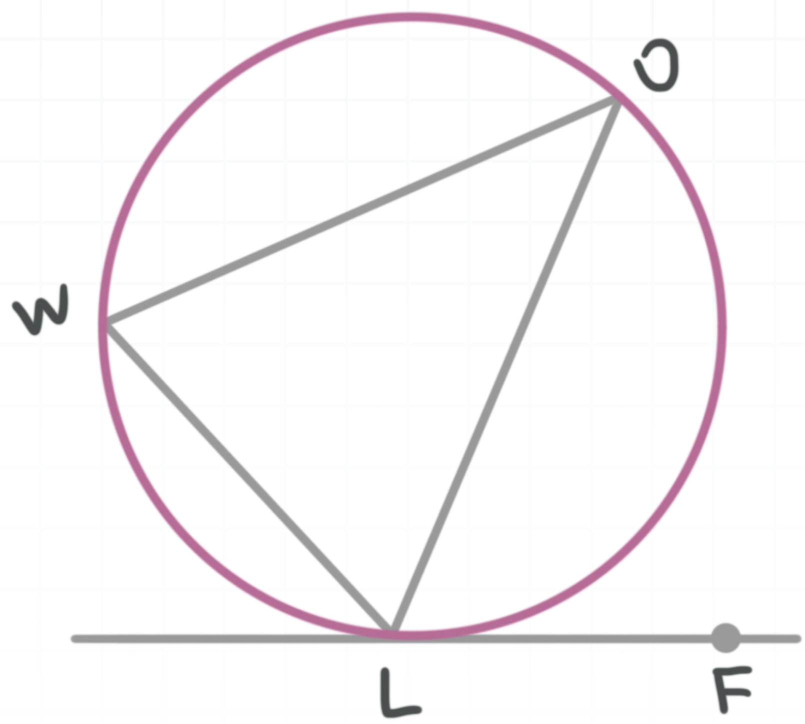
$$70^\circ + m\angle OER = 180^\circ$$

$$m\angle OER = 180^\circ - 70^\circ$$

$$m\angle OER = 110^\circ$$

Topic: Vertex on, inside, and outside the circle

Question: In the figure, $m\angle WOL = 40^\circ$ and $m\angle OLW = 80^\circ$. Also, $\overline{LF}$ is tangent to the circle at L. What is $m\angle FLO$?



Answer choices:

A 30°

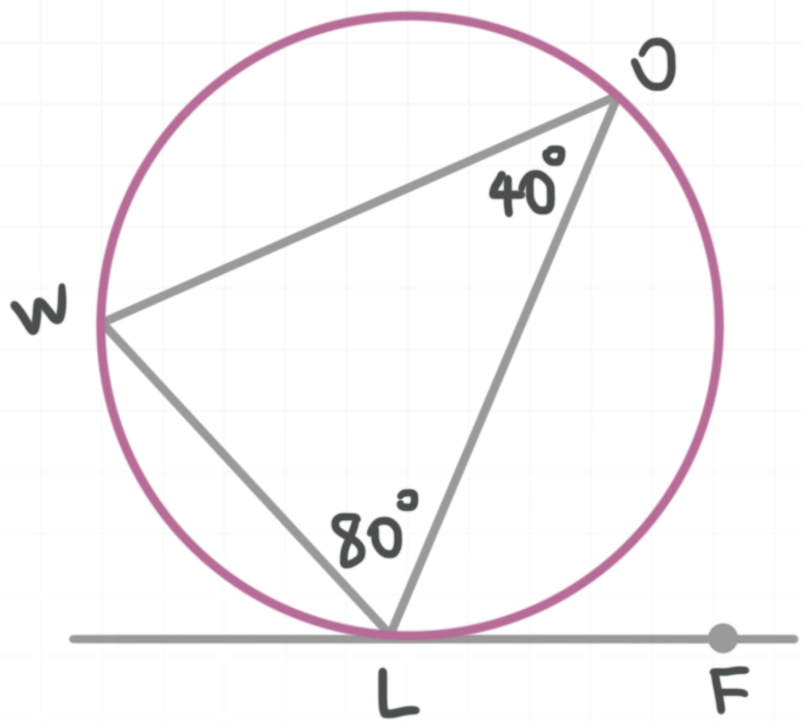
B 40°

C 50°

D 60°

Solution: D

In $\triangle OWL$, the measures of the three interior angles total 180°. We know that two of them are 40° and 80°.



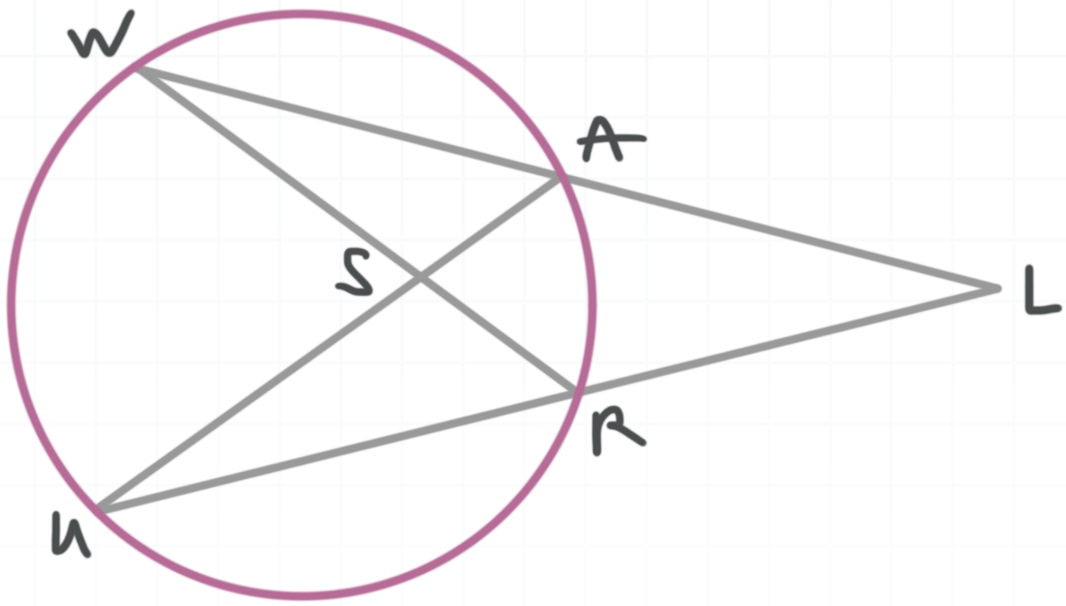
Those two total 120°, which leaves 60° for $m\angle LWO$, which is an inscribed angle, so its intercepted arc $\overset{\frown}{LO}$ has measure 120°. $\angle FLO$ has its vertex on the circle, so its measure is half that of its intercepted arc, which is $\overset{\frown}{LO}$. Therefore,

$$m\angle FLO = \frac{1}{2}m\overset{\frown}{LO} = \frac{1}{2}(120^\circ)$$

$$m\angle FLO = \frac{1}{2}m\overset{\frown}{LO} = 60^\circ$$

Topic: Vertex on, inside, and outside the circle

Question: In the figure, $m\angle RWL = 20^\circ$ and $m\angle RSA = 75^\circ$. What is $m\angle WLU$?



Answer choices:

A 30°

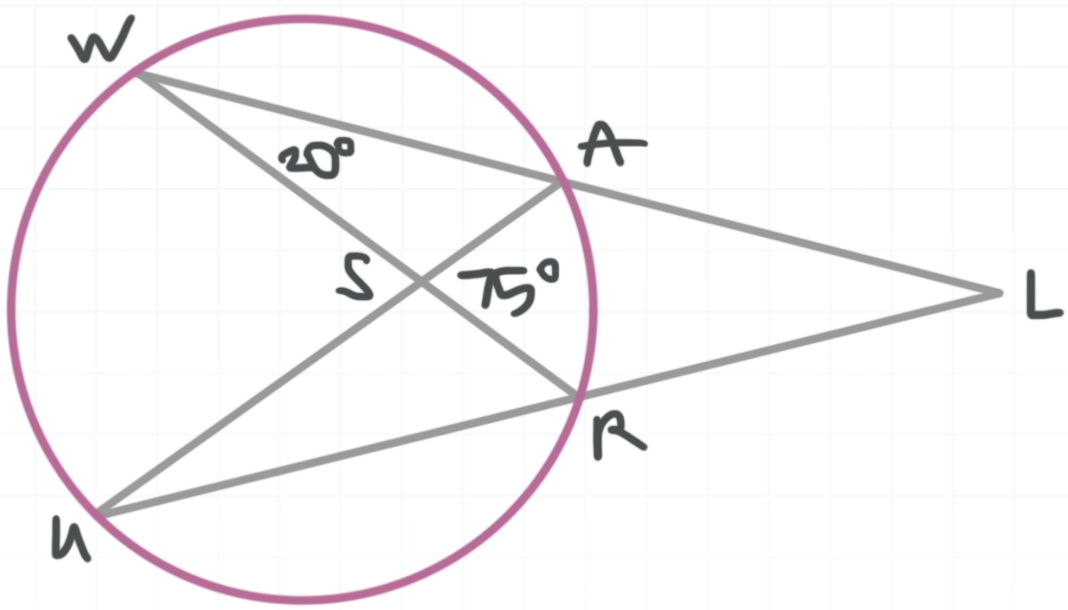
B 35°

C 40°

D 45°

Solution: B

From the information in the problem, we can fill out the figure.



First we'll find the measure of $\overset{\frown}{RA}$, which is the arc intercepted by $\angle RWL$ (an inscribed angle).

$$m\angle RWL = \frac{1}{2}m\overset{\frown}{RA}$$

$$20^\circ = \frac{1}{2}m\overset{\frown}{RA}$$

$$m\overset{\frown}{RA} = 40^\circ$$

Now we'll use this to find the measure of $\overset{\frown}{WU}$. Notice that $\angle RSA$ and $\angle WSU$ are a pair of vertical angles, and that their intercepted arcs are $\overset{\frown}{RA}$ and $\overset{\frown}{WU}$, respectively. Since their common vertex is inside the circle,

$$m\angle RSA = \frac{1}{2}(m\overset{\frown}{RA} + m\overset{\frown}{WU})$$

$$75^\circ = \frac{1}{2}(40^\circ + m\overset{\frown}{WU})$$

$$150^\circ = 40^\circ + m\widehat{WU}$$

$$110^\circ = m\widehat{WU}$$

Finally, we can find $m\angle WLU$. The arcs intercepted by $\angle WLU$ are $\widehat{WU}$ and $\widehat{RA}$. Since the vertex of $\angle WLU$ is outside the circle,

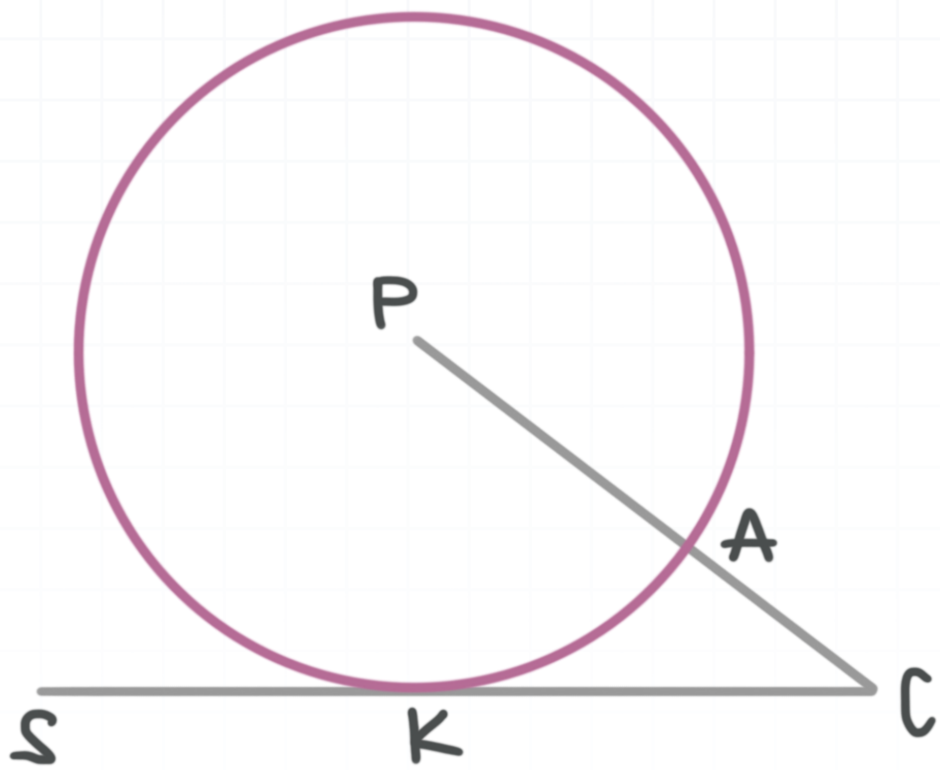
$$m\angle WLU = \frac{1}{2}(m\widehat{WU} - m\widehat{RA})$$

$$m\angle WLU = \frac{1}{2}(110^\circ - 40^\circ)$$

$$m\angle WLU = 35^\circ$$

Topic: Tangent lines of circles

Question: In the circle in the figure (with center at P), the radius is 6 and $\overline{CS}$ is tangent to the circle at K. If $\overline{AC} = 4$, how long is $\overline{CK}$?



Answer choices:

A 5

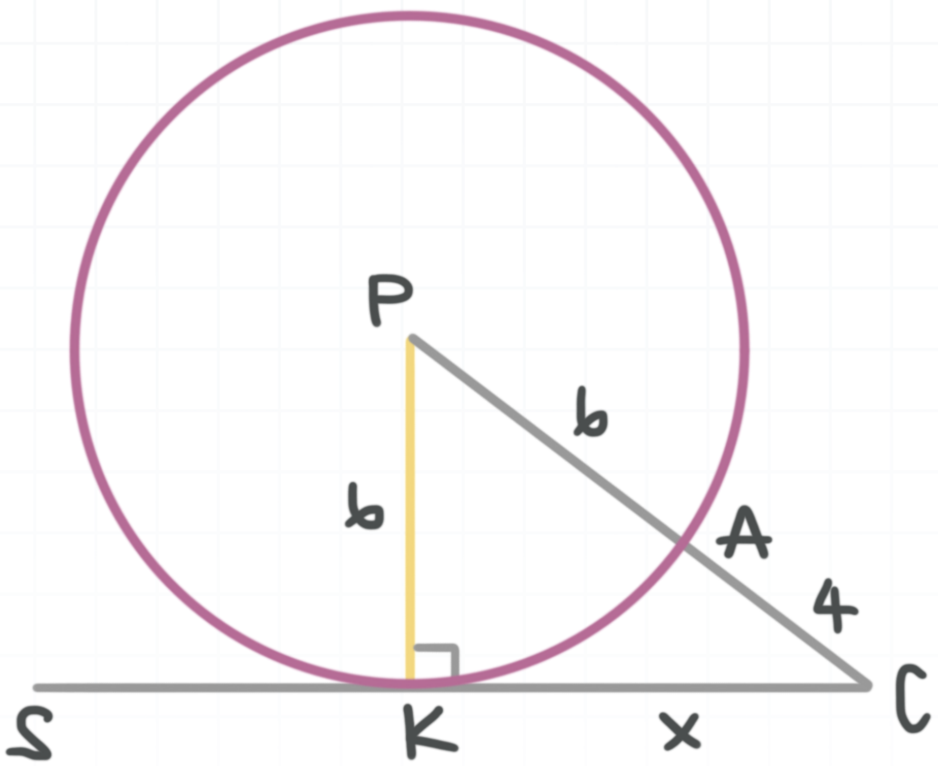
B 6

C 7

D 8

Solution: D

A radius drawn to K will be perpendicular to $\overline{CK}$, making a right triangle. Let $x = \overline{CK}$, and label the segments as shown in the figure.



Use the Pythagorean theorem to find x.

$$(\overline{CK})^2 + (\overline{PK})^2 = (\overline{PC})^2$$

$$(\overline{CK})^2 + (\overline{PK})^2 = (\overline{PA} + \overline{AC})^2$$

$$x^2 + 6^2 = 10^2$$

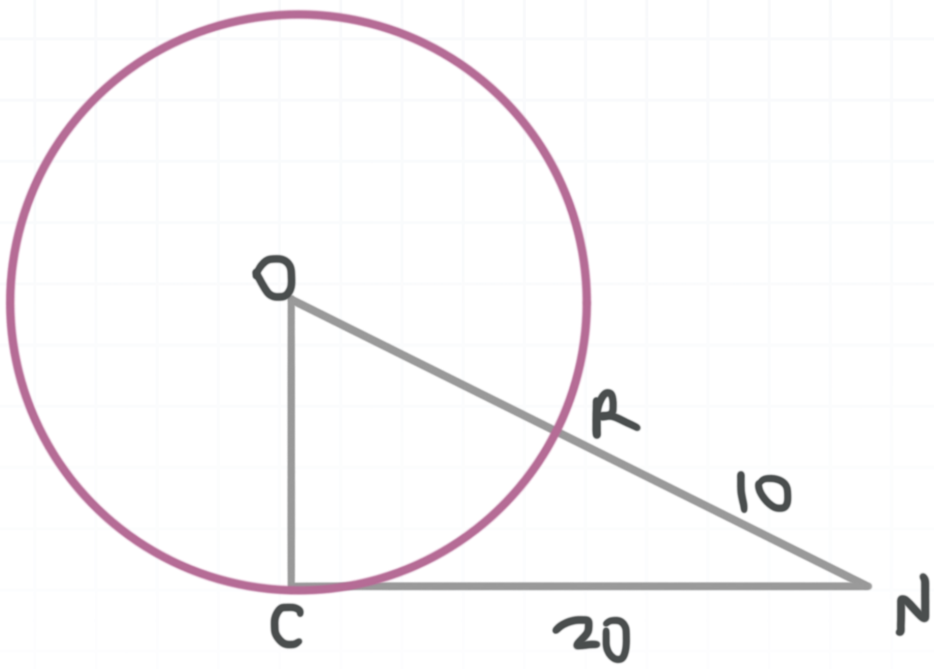
$$x^2 + 36 = 100$$

$$x^2 = 64$$

$$x = 8$$

Topic: Tangent lines of circles

Question: In the circle in the figure (with center at O), $\overline{CN}$ is tangent to the circle at C. If $\overline{CN} = 20$ and $\overline{RN} = 10$, what is the radius of the circle?



Answer choices:

A 15

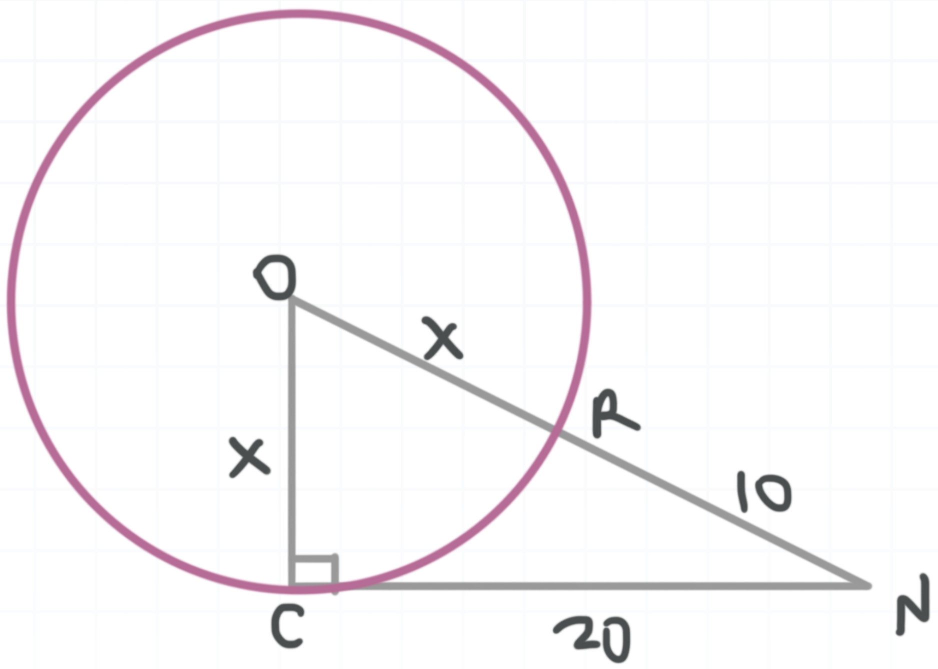
B 12

C 9

D 6

Solution: A

A radius drawn to C will be perpendicular to $\overline{CN}$, forming a right triangle.



Let $x = \overline{OC}$ (and therefore that $\overline{OR} = x$ as well) and use the Pythagorean theorem.

$$x^2 + 20^2 = (x + 10)^2$$

$$x^2 + 400 = x^2 + 20x + 100$$

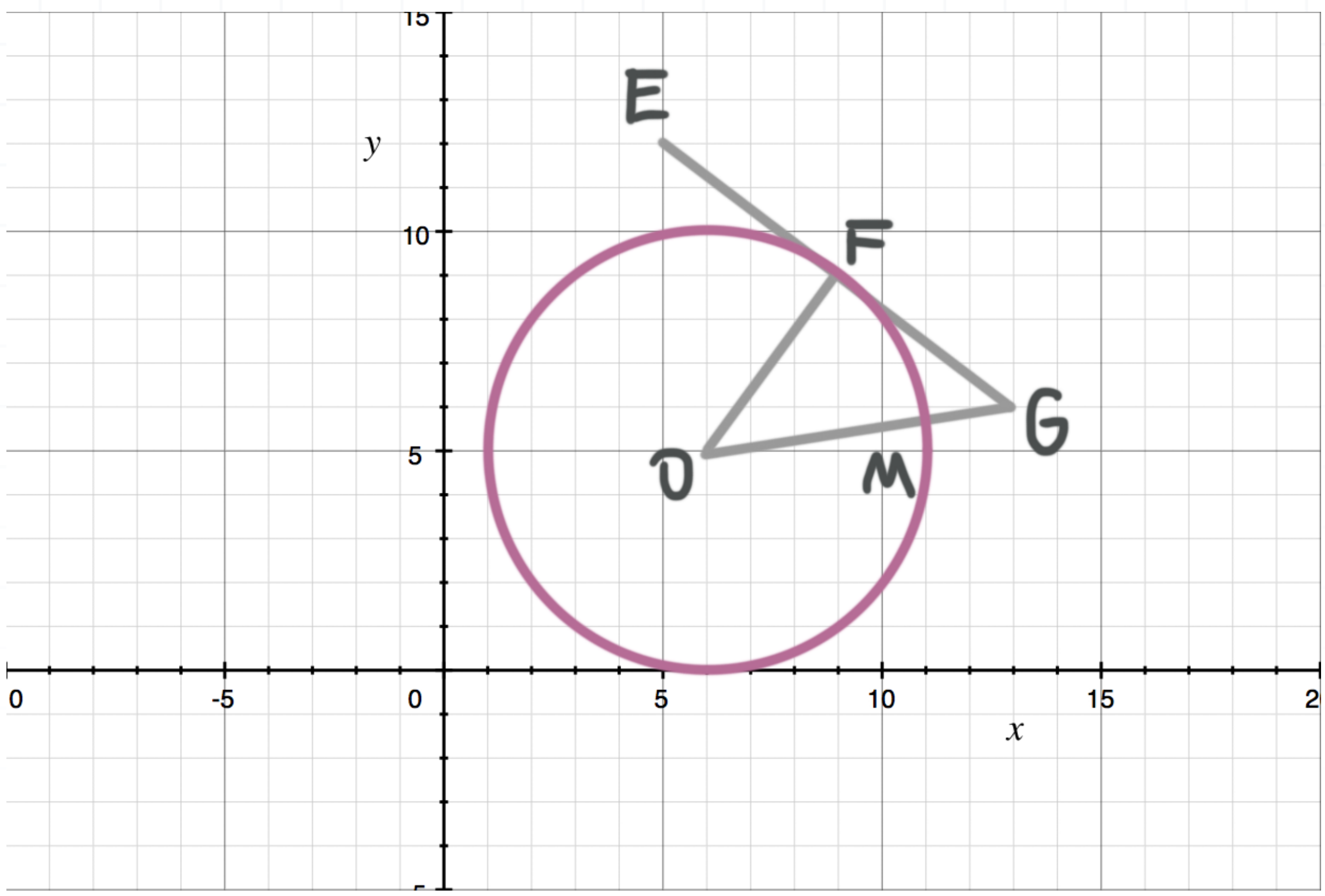
Subtract x^2 and 100 from each side.

$$300 = 20x$$

$$x = 15$$

Topic: Tangent lines of circles

Question: In the circle in the figure, the center (point O) is at $(6,5)$, F is at $(9,9)$, and G is at $(13,6)$. $\overline{EG}$ is tangent to the circle at F. How long is $\overline{MG}$?



Answer choices:

A $5\sqrt{2}$

B $5\sqrt{2}+5$

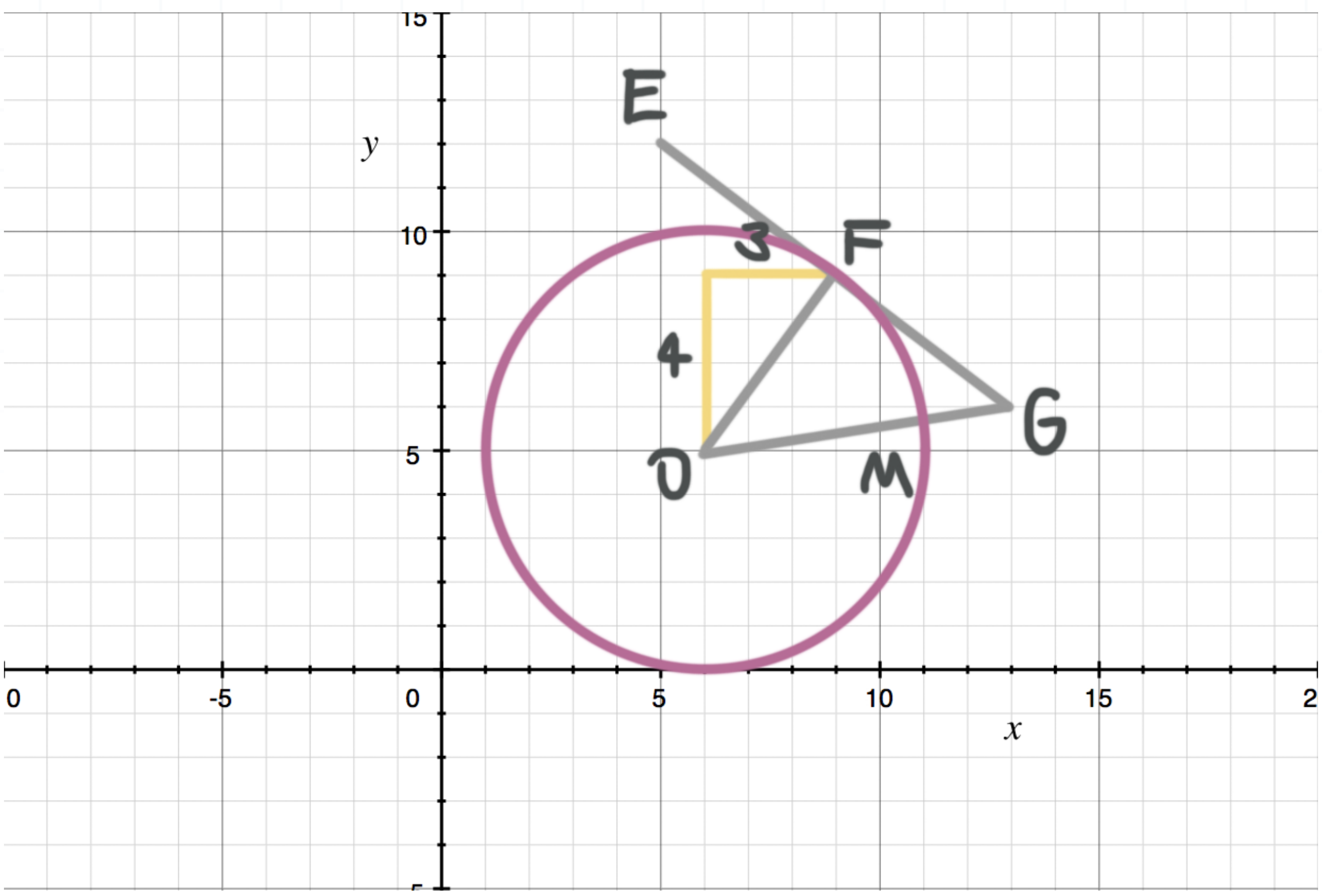
C $2\sqrt{5}$

D $5\sqrt{2}-5$

Solution: D

Notice that $\overline{OF}$ is the hypotenuse of a right triangle with legs of length 4 and 3.

The leg with length 4 is the vertical line segment from O, which is at $(6,5)$, to the point at $(6,9)$; those two points are 4 units apart. The leg with length 3 is the horizontal line segment from the point at $(6,9)$ to F, which is at $(9,9)$; those two points are 3 units apart.



We can find $\overline{OF}$ by applying the Pythagorean theorem to that right triangle.

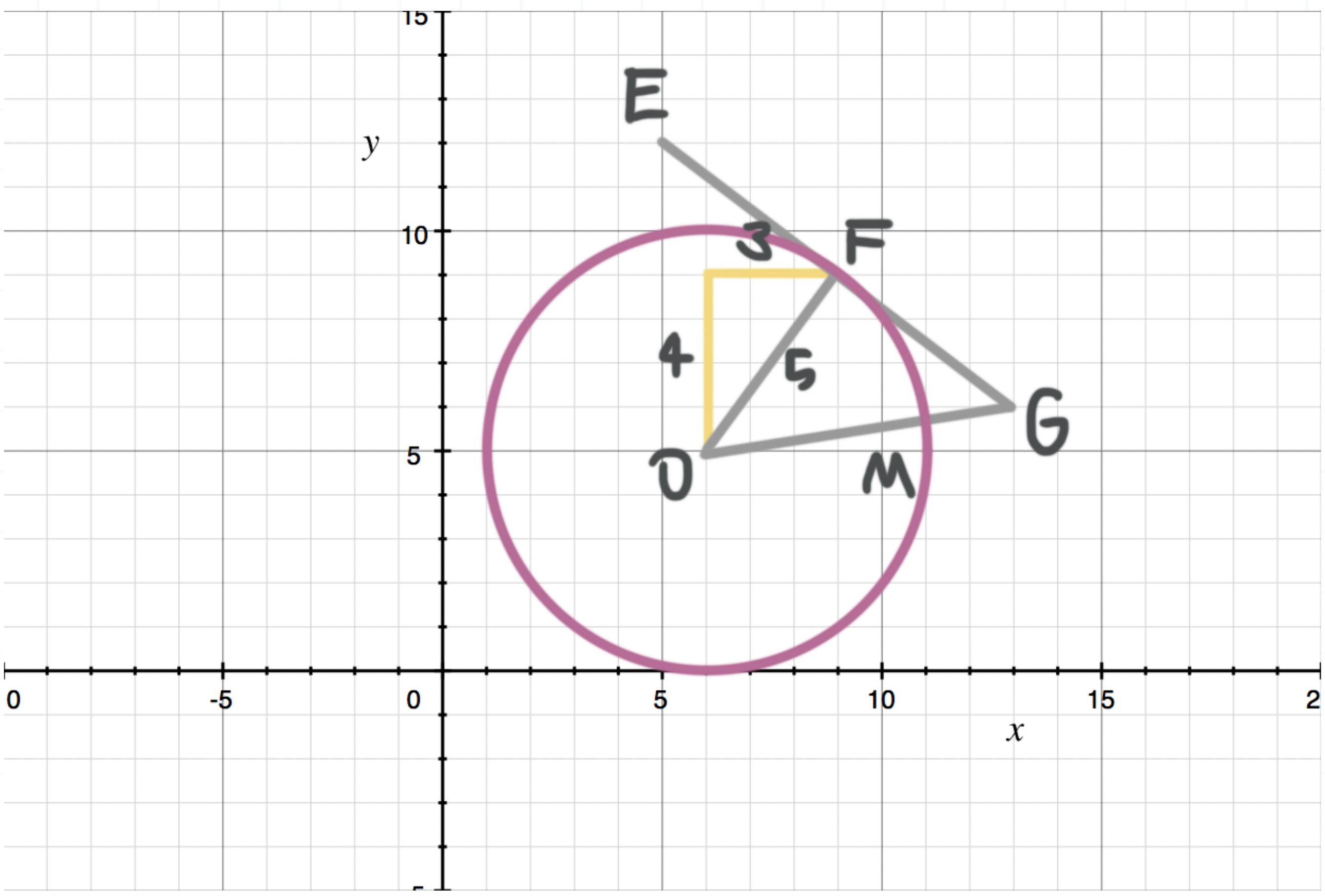
$$4^2 + 3^2 = (\overline{OF})^2$$

$$16 + 9 = (\overline{OF})^2$$

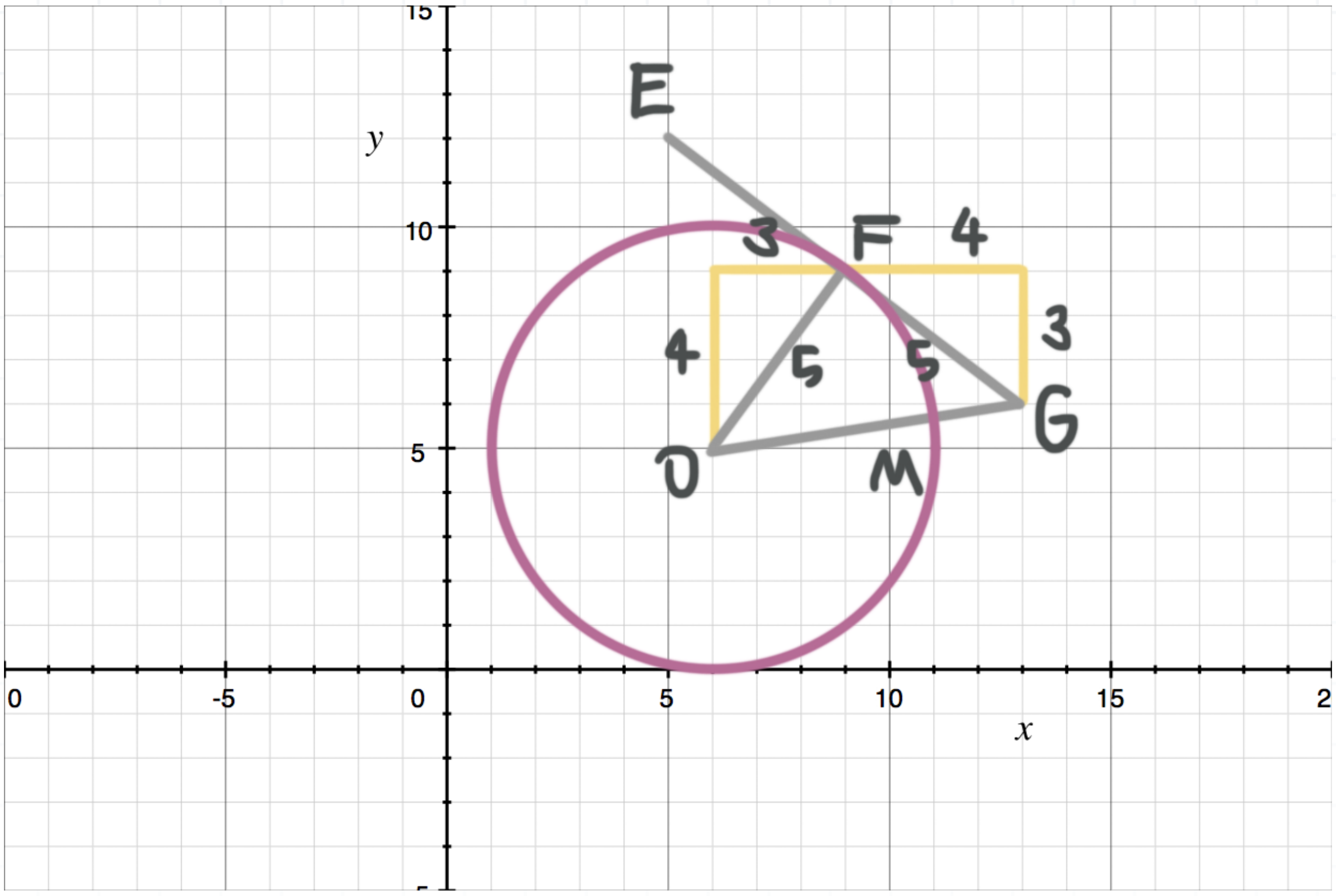
$$25 = (\overline{OF})^2$$

$$5 = \overline{OF}$$

Since $\overline{EG}$ is tangent to the circle at F, that makes $\overline{OF}$ a radius of the circle, so the radius is 5.



Also notice that $\overline{FG}$ is the hypotenuse of a different right triangle with legs of length 4 and 3. That makes $\overline{FG} = 5$ also.



Now focus on $\triangle FOG$, and notice that $\overline{OF} \perp \overline{FG}$ ($\overline{OF}$ is perpendicular to $\overline{FG}$), making $\triangle FOG$ a right triangle with legs $\overline{OF}$ and $\overline{FG}$ and hypotenuse $\overline{OG}$.

Using the Pythagorean theorem, we can find the length of $\overline{OG}$.

$$(\overline{OF})^2 + (\overline{FG})^2 = (\overline{OG})^2$$

$$5^2 + 5^2 = (\overline{OG})^2$$

$$25 + 25 = (\overline{OG})^2$$

$$50 = (\overline{OG})^2$$

$$\sqrt{50} = \overline{OG}$$

$$\sqrt{25 \cdot 2} = \overline{OG}$$

$$5\sqrt{2} = \overline{OG}$$

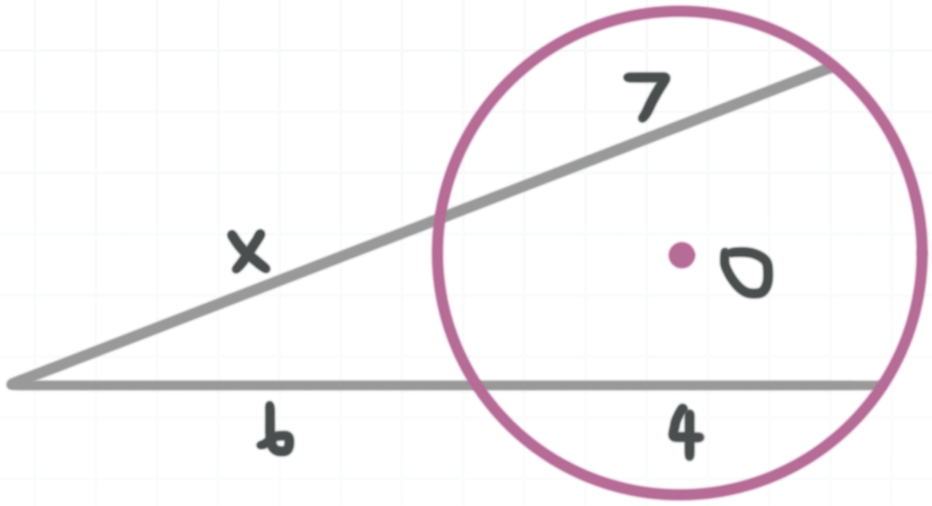
Notice that $\overline{OG} = \overline{OM} + \overline{MG}$, and that $\overline{OM}$ is a radius of the circle (so $\overline{OM} = 5$). Therefore,

$$5\sqrt{2} = 5 + \overline{MG}$$

$$\overline{MG} = 5\sqrt{2} - 5$$

Topic: Intersecting tangents and secants

Question: Given the lengths in the figure, find x.



Answer choices:

A $x = 3$

B $x = 4$

C $x = 5$

D $x = 6$

Solution: C

Because there are two secants that intersect outside the circle, we can follow the pattern

outside · whole = outside · whole

Plugging the lengths shown in the figure into this equation, we get

$$x(x+7)=6(6+4)$$

$$x^2+7x=60$$

$$x^2+7x-60=0$$

$$(x+12)(x-5)=0$$

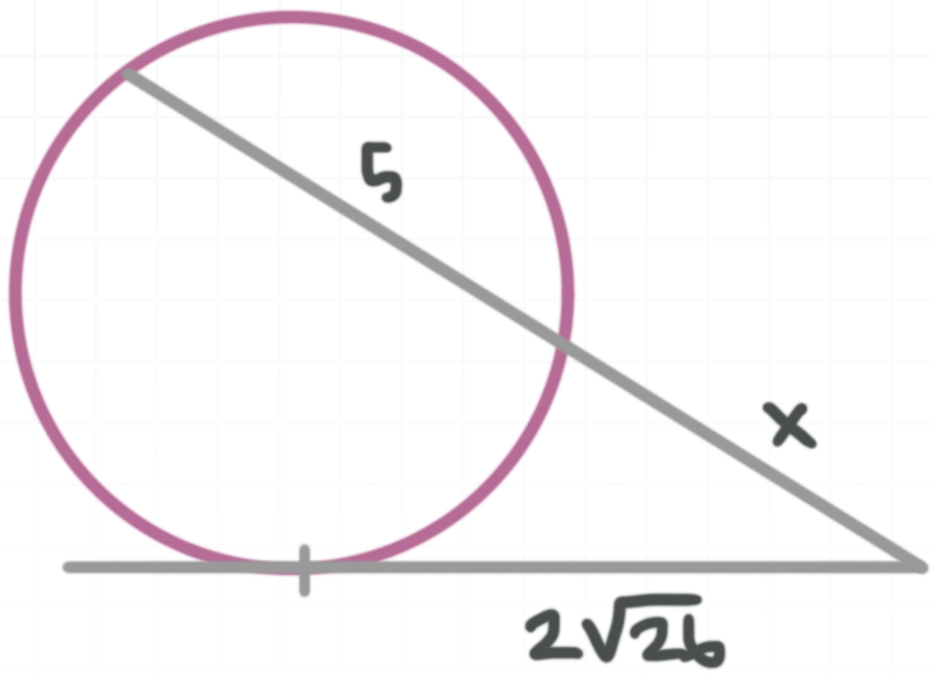
$x+12=0$ or $x-5=0$

$x=-12$ or $x=5$

A line segment can't have a negative length, so rule out $x=-12$ and conclude that $x=5$.

Topic: Intersecting tangents and secants

Question: Given the lengths in the figure, find x.



Answer choices:

A $x = 5$

B $x = 6$

C $x = 7$

D $x = 8$

Solution: D

Because there is a secant that intersects with a tangent outside the circle, we can follow the pattern

$$\text{tangent}^2 = \text{outside} \cdot \text{whole}$$

Plugging the lengths shown in the figure into this equation, we get

$$\left(2\sqrt{26}\right)^2 = x(x+5)$$

$$4(26) = x^2 + 5x$$

$$104 = x^2 + 5x$$

$$0 = x^2 + 5x - 104$$

$$(x+13)(x-8) = 0$$

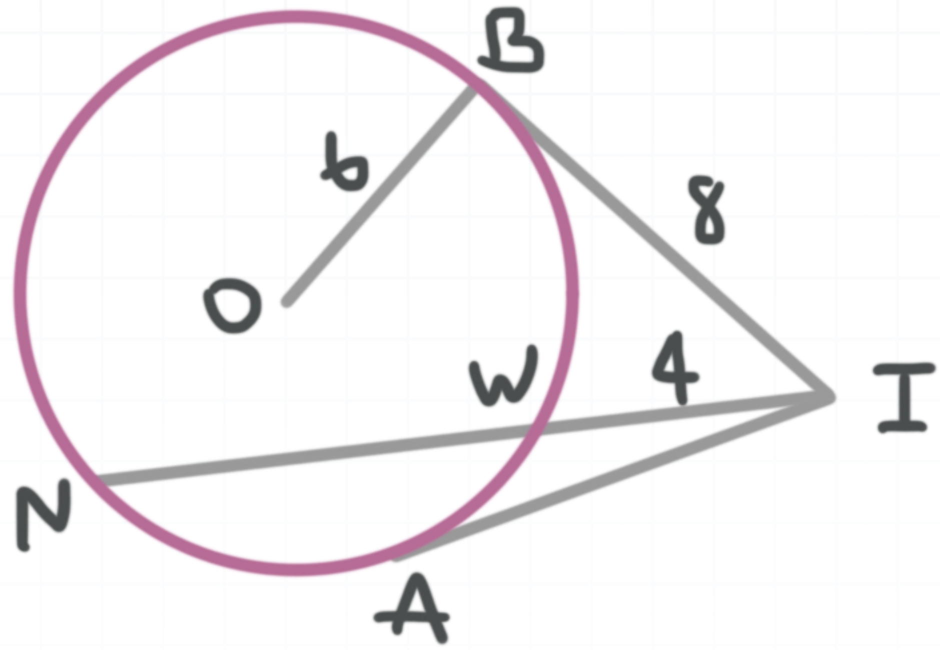
$$x + 13 = 0 \text{ or } x - 8 = 0$$

$$x = -13 \text{ or } x = 8$$

A line segment can't have a negative length, so rule out $x = -13$ and conclude that $x = 8$.

Topic: Intersecting tangents and secants

Question: In the figure, $\overline{IB}$ and $\overline{IA}$ are tangent to the circle (with center at O) at points B and A, respectively. Given the lengths shown, find $\overline{NW}+\overline{IA}$.



Answer choices:

A 12

B 14

C 18

D 20

Solution: D

Since $\overline{IB}$ is tangent to the circle at point B, $\overline{OB}$ is a radius. This means that the radius of the circle is 6 and that $\overline{OB}$ is perpendicular to $\overline{IB}$. Apply Pythagorean theorem to right triangle OBI.

$$(\overline{OB})^2 + (\overline{BI})^2 = (\overline{OI})^2$$

$$6^2 + 8^2 = (\overline{OI})^2$$

$$36 + 64 = (\overline{OI})^2$$

$$100 = (\overline{OI})^2$$

Similarly, $\overline{IA}$ is tangent to the circle at point A, so $\overline{OA}$ is a radius. This means that $\overline{OA} = 6$ and that $\overline{OA}$ is perpendicular to $\overline{IA}$. Apply the Pythagorean theorem to right triangle OAI.

$$(\overline{OA})^2 + (\overline{AI})^2 = (\overline{OI})^2$$

$$6^2 + (\overline{AI})^2 = 100$$

$$36 + (\overline{AI})^2 = 100$$

$$(\overline{AI})^2 = 64$$

$$\overline{AI} = 8$$

Then we can apply the pattern

$$\text{tangent}^2 = \text{outside} \cdot \text{whole}$$

to tangent $\overline{IA}$ and secant $\overline{IN}$.

$$8^2 = 4(4 + \overline{NW})$$

$$64 = 16 + 4(\overline{NW})$$

$$48 = 4(\overline{NW})$$

$$\overline{NW} = 12$$

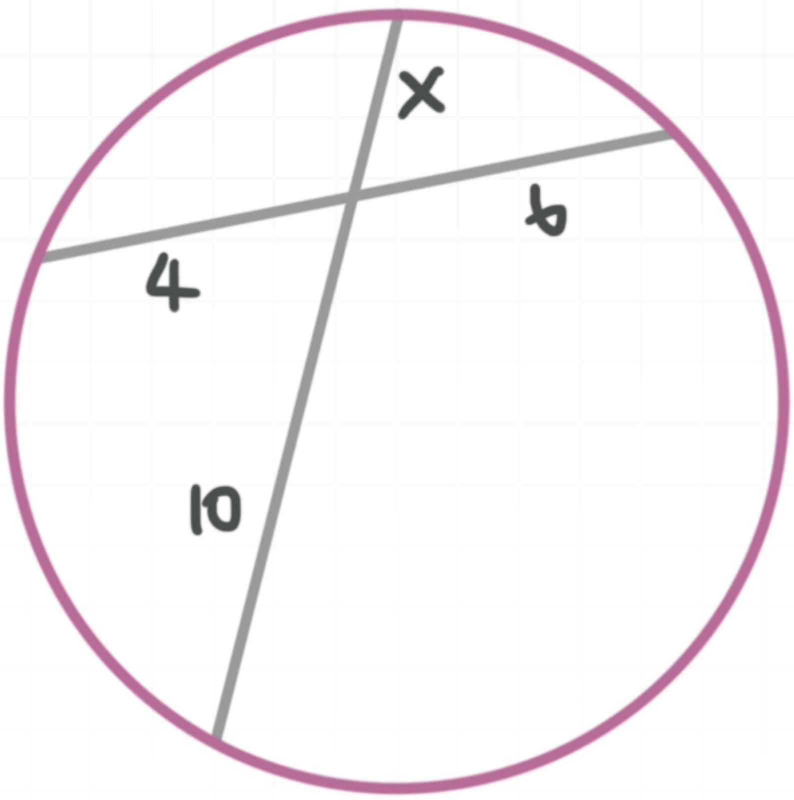
Therefore,

$$\overline{NW} + \overline{IA} = 12 + 8$$

$$\overline{NW} + \overline{IA} = 20$$

Topic: Intersecting chords

Question: Using the lengths of chord segments in the circle, find x.



Answer choices:

A 2.4

B 4.6

C 5

D 6

Solution: A

The products of the chord segments are equal.

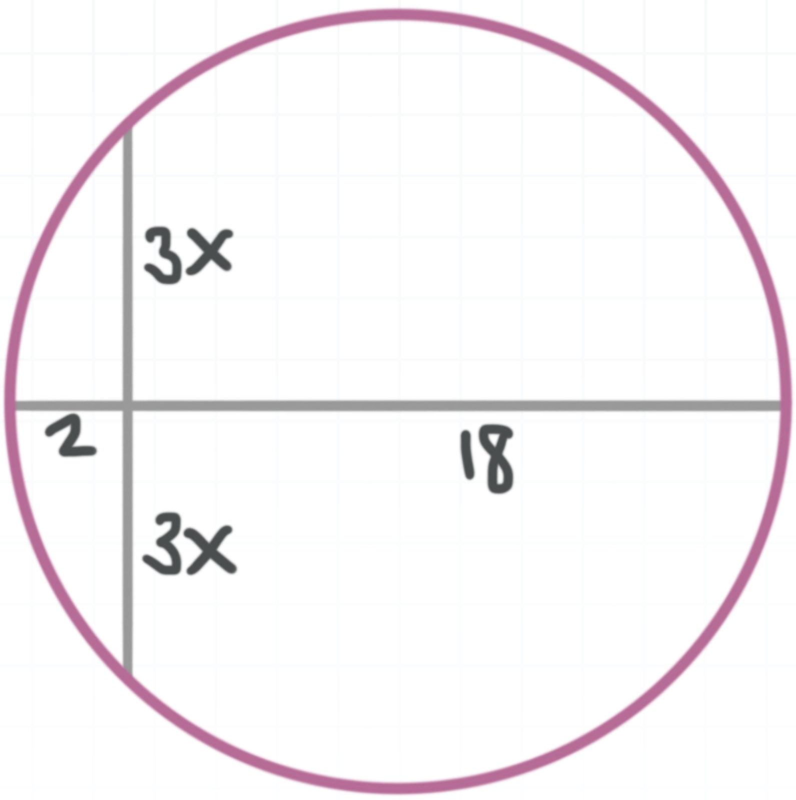
$$10 \cdot x = 4 \cdot 6$$

$$10x = 24$$

$$x = 2.4$$

Topic: Intersecting chords

Question: Using the lengths of chord segments in the circle, find x.



Answer choices:

A 2

B 3

C 4

D 5

Solution: A

The products of the chord segments are equal.

$$3x \cdot 3x = 2 \cdot 18$$

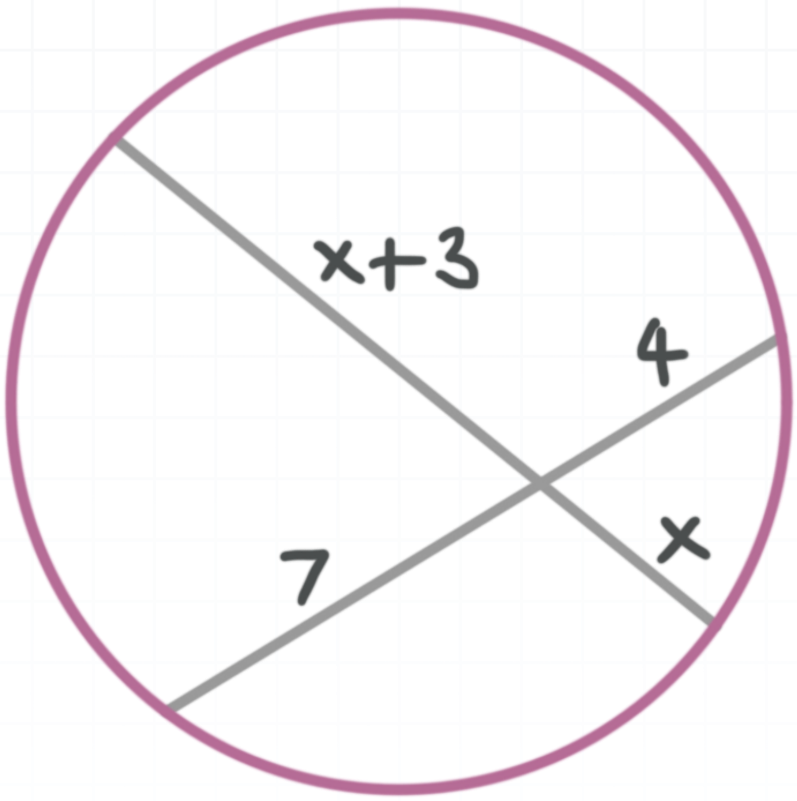
$$9x^2 = 36$$

$$x^2 = 4$$

$$x = 2$$

Topic: Intersecting chords

Question: Using the lengths of chord segments in the circle, find x.



Answer choices:

A 1

B 2

C 3

D 4

Solution: D

The products of the chord segments are equal.

$$x(x+3) = 4 \cdot 7$$

$$x^2 + 3x = 28$$

$$x^2 + 3x - 28 = 0$$

$$(x+7)(x-4) = 0$$

$x + 7 = 0$ or $x - 4 = 0$

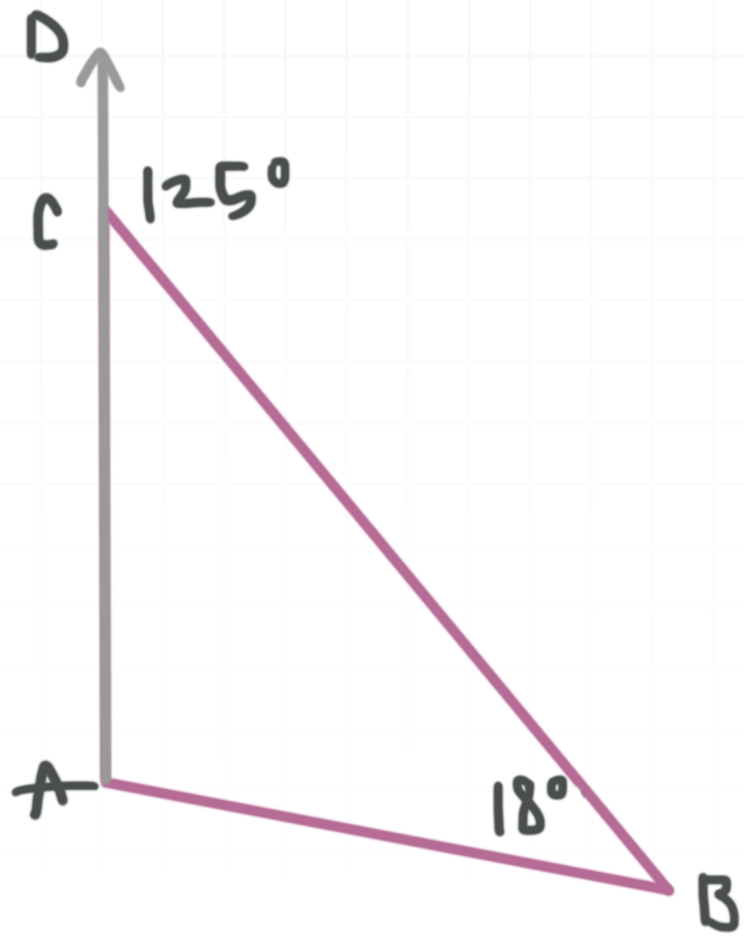
$x = -7$ or $x = 4$

Rule out the negative value.

$$x = 4$$

Topic: Interior angles of triangles

Question: What is $m\angle BAC + m\angle CBA$?



Answer choices:

A 55°

B 73°

C 107°

D 125°

Solution: D

The angles $\angle ACB$ and $\angle BCD$ form a straight line, so

$$m\angle ACB + 125^\circ = 180^\circ$$

$$m\angle ACB = 55^\circ$$

The measures of the three interior angles of a triangle add up to 180°, so

$$55^\circ + m\angle BAC + 18^\circ = 180^\circ$$

$$m\angle BAC = 107^\circ$$

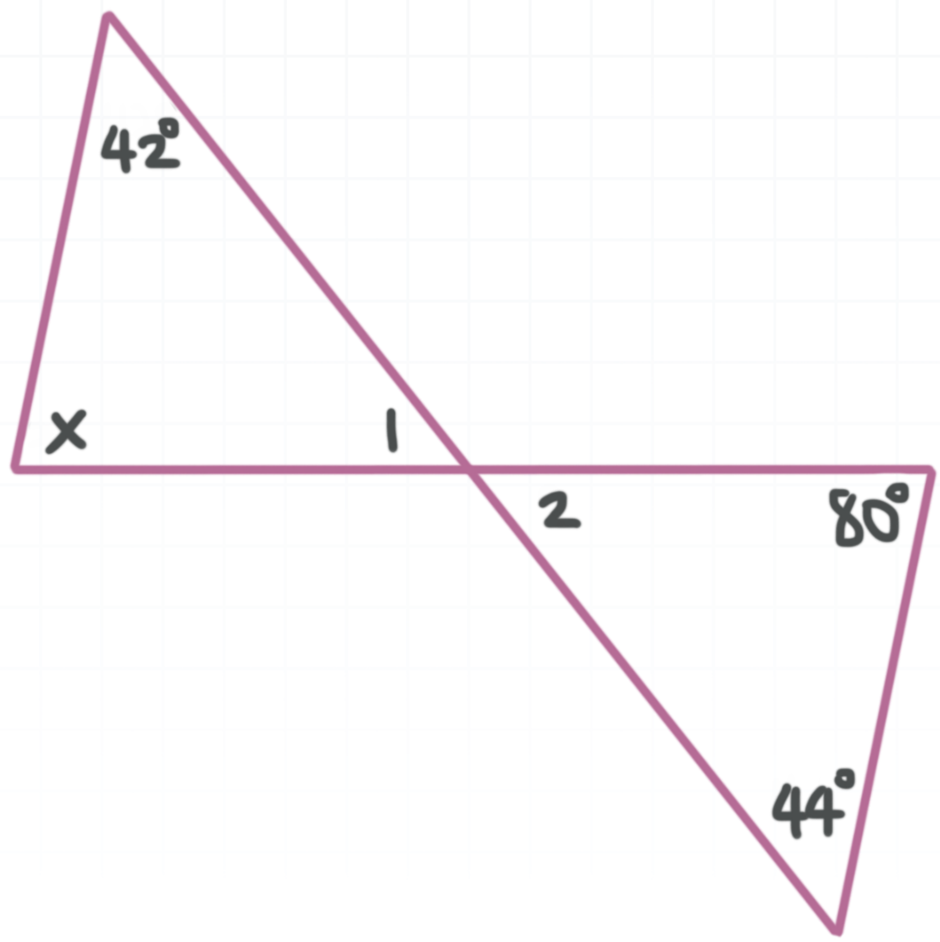
Therefore,

$$m\angle BAC + m\angle CBA = 107^\circ + 18^\circ = 125^\circ$$

If you know the theorem that states that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles (the two interior angles that aren't adjacent to that exterior angle), then you could use that to easily solve this problem.

Topic: Interior angles of triangles

Question: What is the value of x?



Answer choices:

A 56°

B 82°

C 88°

D 92°

Solution: B

The measures of the three interior angles of a triangle add up to 180°, so

$$m\angle 2 + 80^\circ + 44^\circ = 180^\circ$$

$$m\angle 2 = 56^\circ$$

Angle 1 and angle 2 are a pair of vertical angles, and vertical angles are congruent, so

$$m\angle 1 = m\angle 2 = 56^\circ$$

Again, the measures of the three interior angles of a triangle add up to 180°, so we see that

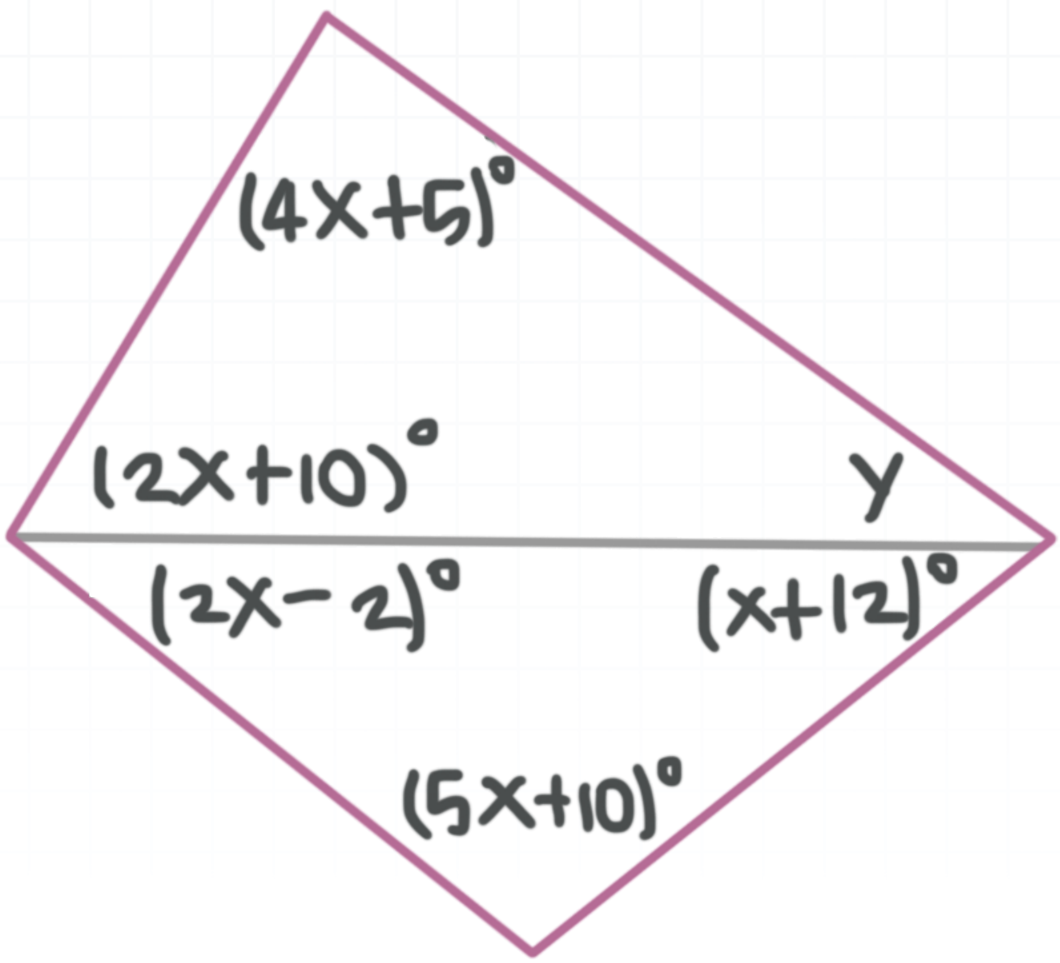
$$m\angle 1 + 42^\circ + x = 180^\circ$$

$$56^\circ + 42^\circ + x = 180^\circ$$

$$x = 82^\circ$$

Topic: Interior angles of triangles

Question: Find the value of y.



Answer choices:

A 35°

B 42°

C 45°

D 51°

Solution: C

The measures of the three interior angles of a triangle add up to 180°, so in the bottom triangle, we have

$$(2x-2)^\circ + (x+12)^\circ + (5x+10)^\circ = 180^\circ$$

$$8x^\circ + 20^\circ = 180^\circ$$

$$8x^\circ = 160^\circ$$

$$x^\circ = 20^\circ$$

$$x = 20$$

Applying this value of x to the top triangle, and using the fact that the sum of the measures of the interior angles is 180°, we get

$$(2x+10)^\circ + (4x+5)^\circ + y = 180^\circ$$

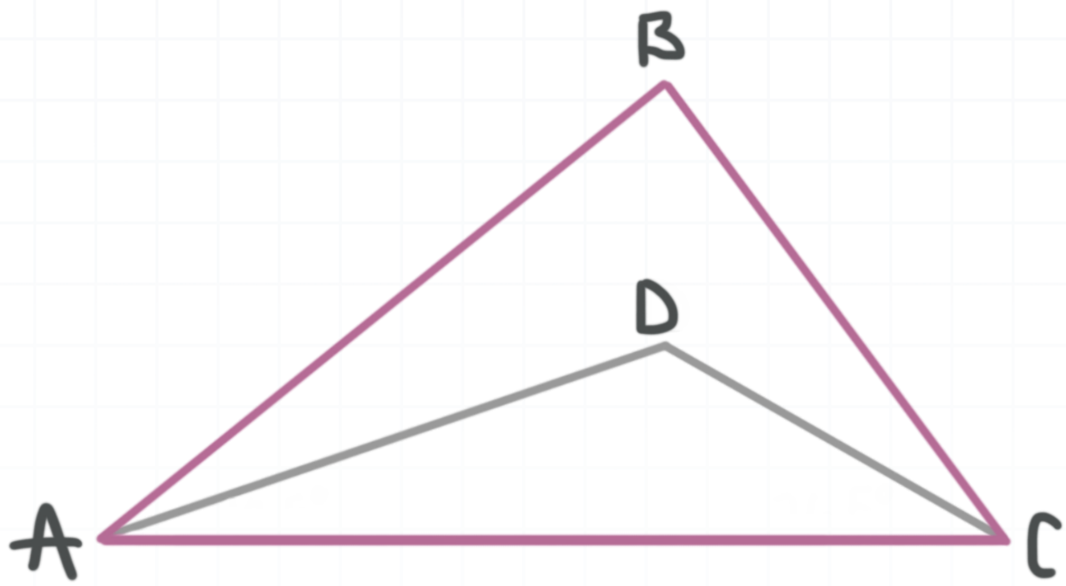
$$2(20^\circ) + 10^\circ + 4(20^\circ) + 5^\circ + y = 180^\circ$$

$$135^\circ + y = 180^\circ$$

$$y = 45^\circ$$

Topic: Perpendicular and angle bisectors

Question: The line segments $\overline{AD}$ and $\overline{CD}$ are bisectors of $\angle CAB$ and $\angle BCA$, respectively. What is $m\angle ADC$, if $m\angle CAB = 39^\circ$ and $m\angle BCA = 53^\circ$?



Answer choices:

A 88°

B 112°

C 123°

D 134°

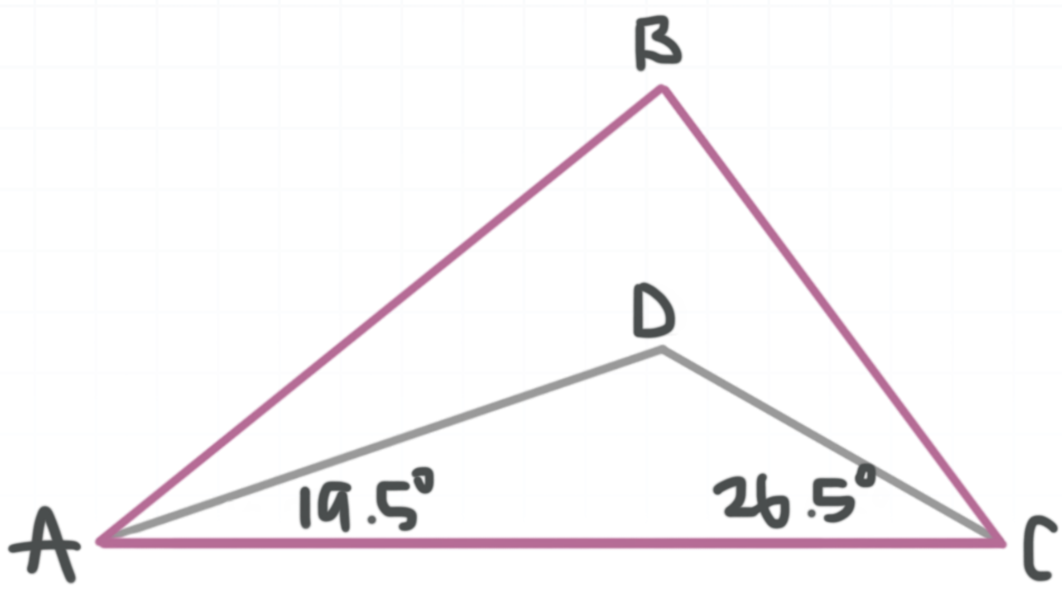
Solution: D

Using what we already know, we see that

$$m\angle CAD = \frac{1}{2}m\angle CAB = \frac{1}{2}(39^\circ) = 19.5^\circ$$

$$m\angle DCA = \frac{1}{2}m\angle BCA = \frac{1}{2}(53^\circ) = 26.5^\circ$$

Add these measures to the figure.



The measures of the three interior angles of $\triangle ADC$ (or any triangle) add up to 180°. Therefore,

$$m\angle CAD + m\angle DCA + m\angle ADC = 180^\circ$$

$$19.5^\circ + 26.5^\circ + m\angle ADC = 180^\circ$$

$$46^\circ + m\angle ADC = 180^\circ$$

$$m\angle ADC = 134^\circ$$

Topic: Perpendicular and angle bisectors

Question: The perpendicular bisector of a line segment does which of these things?

Answer choices:

A Forms at least two right angles

B Forms two line segments of equal length

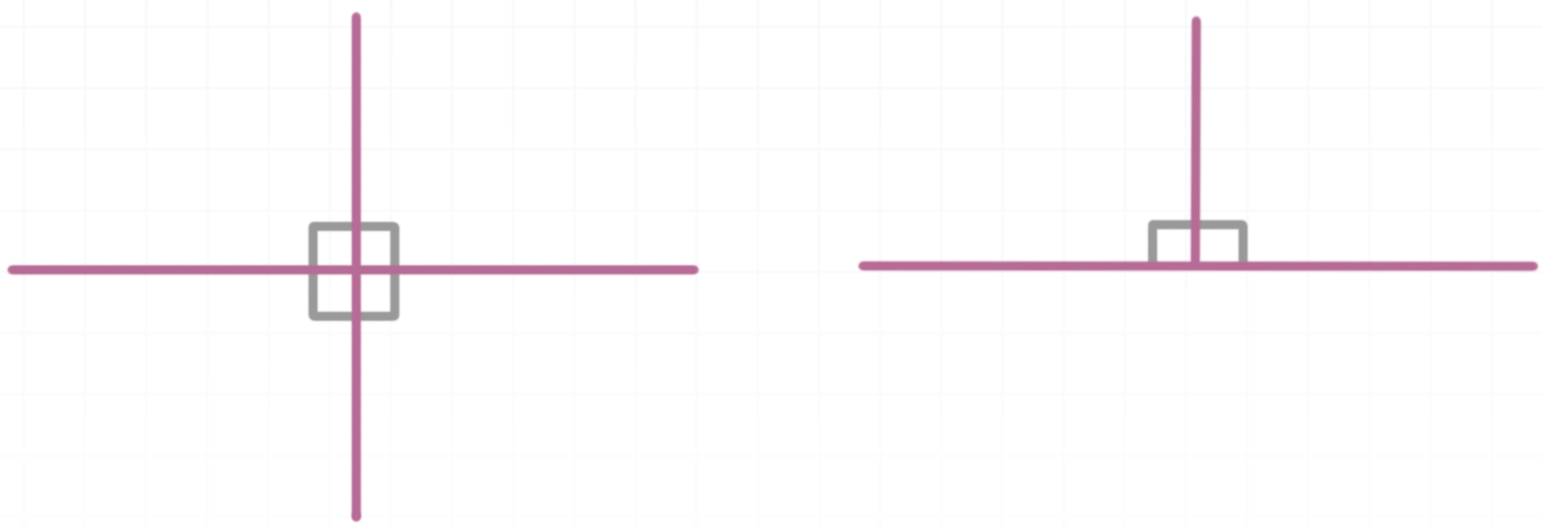
C Passes through the midpoint of the original segment

D All of these

Solution: D

All of these are true.

a) The figures below show how you could get two or four right angles.

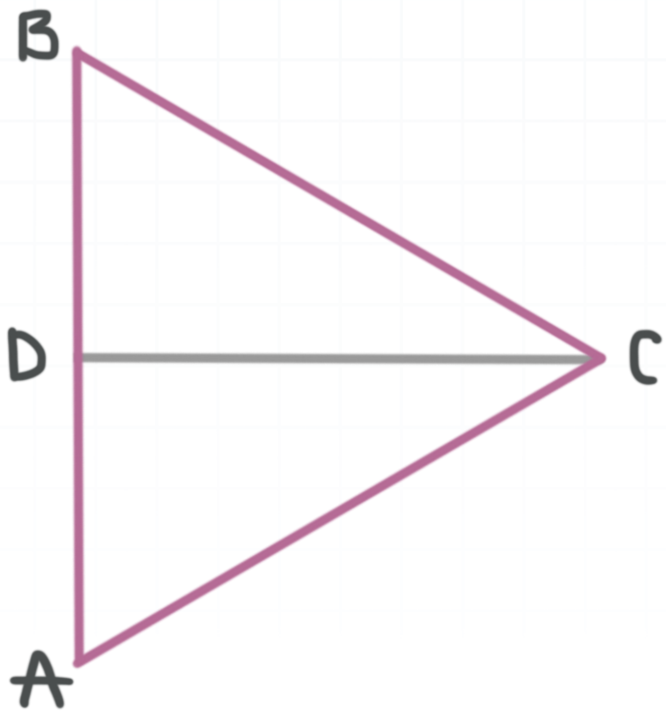


b) The word bisector tells us that the segment is split into two equal parts.

c) The point at which any line segment is divided into two equal segments is its midpoint.

Topic: Perpendicular and angle bisectors

Question: $\triangle ABC$ is an equilateral triangle (a triangle in which all three sides are of equal length). $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. $\overline{AD} = 5x - 2$ and $\overline{DB} = 3x + 6$. What is the perimeter of $\triangle ABC$ (the sum of the lengths of its sides)?



Answer choices:

A 24

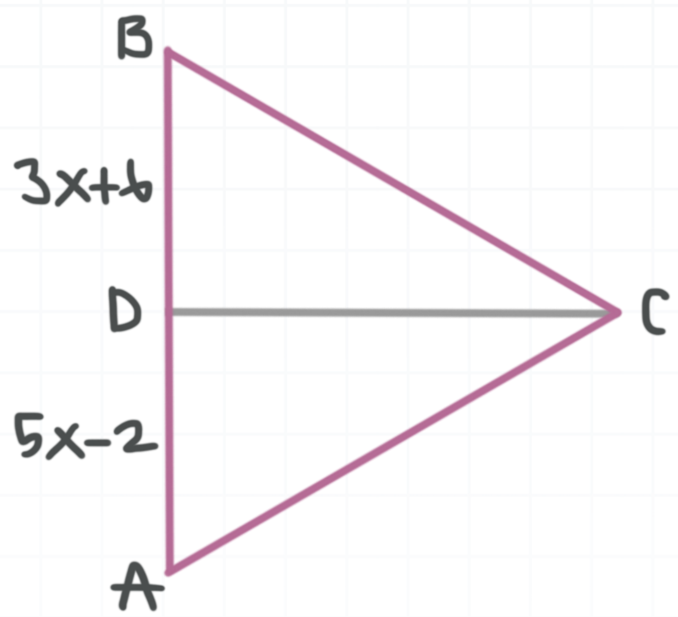
B 54

C 108

D 156

Solution: C

Let's label what we've been given.



Because $\overline{CD}$ bisects $\overline{AB}$,

$$3x + 6 = 5x - 2$$

$$8 = 2x$$

$$4 = x$$

Notice that

$$\overline{AB} = \overline{AD} + \overline{DB}$$

$$\overline{AB} = (5x - 2) + (3x + 6)$$

Substituting 4 for x, we get

$$\overline{AB} = 5(4) - 2 + 3(4) + 6$$

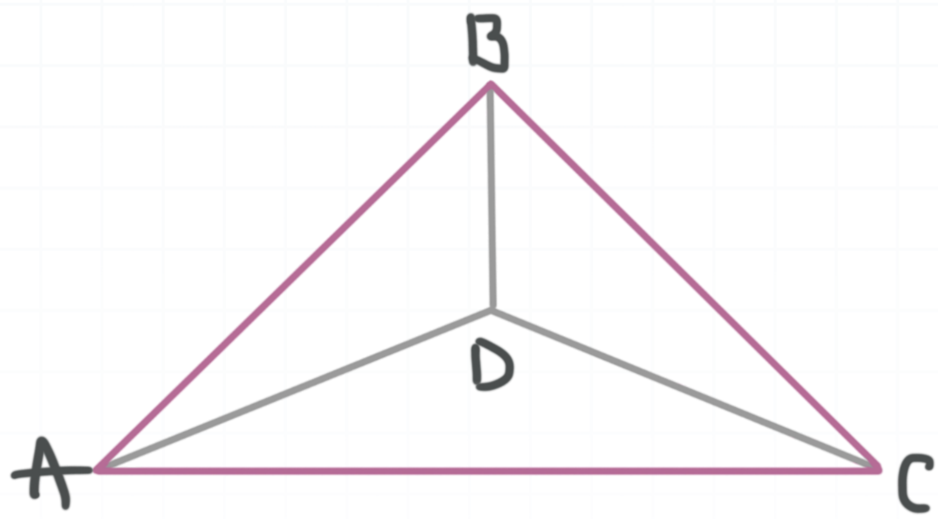
$$\overline{AB} = 36$$

Since $\triangle ABC$ is equilateral, its perimeter is

$$3 \cdot \overline{AB} = 3 \cdot 36 = 108$$

Topic: Circumscribed and inscribed circles of a triangle

Question: $\overline{AD}$, $\overline{BD}$, and $\overline{CD}$ are bisectors of $\angle CAB$, $\angle ABC$, and $\angle BCA$, respectively. Their point of intersection, D, is the incenter of $\triangle ABC$. Assuming the figure is drawn to scale, which of the following statements is true?



Answer choices:

A Point D is equidistant from A, B, and C but not equidistant from the sides of $\triangle ABC$.

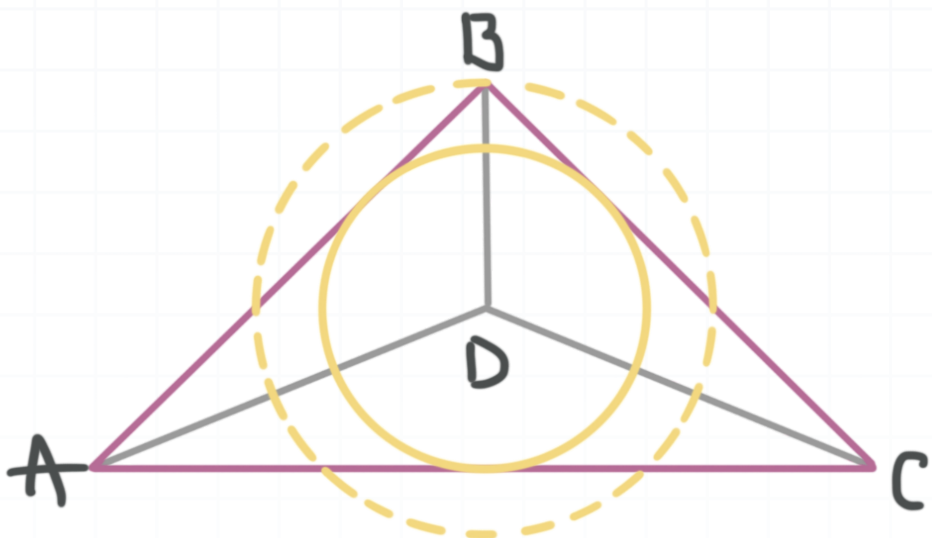
B Point D is equidistant from the sides of $\triangle ABC$ but not equidistant from A, B, and C.

C Point D is equidistant from A, B, and C and equidistant from the sides of $\triangle ABC$.

D Point D is neither equidistant from A, B, and C nor equidistant from the sides of $\triangle ABC$.

Solution: B

The incenter of any triangle, which is the point of intersection of the angle bisectors of the interior angles of the triangle, is equidistant from the sides of the triangle. So point D, which is the incenter of $\triangle ABC$, is equidistant from the sides of $\triangle ABC$. This means you can rule out answer choice D.

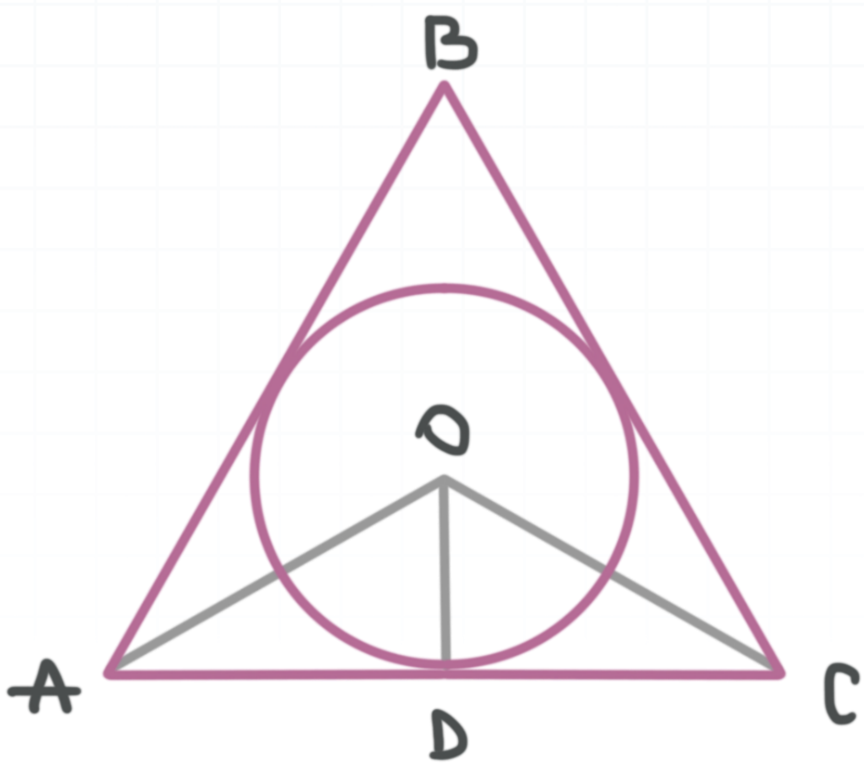


Notice that if you were to draw a circle of larger radius (with center at D), you couldn't make it hit all three vertices. That tells you that D isn't equidistant from A, B, and C, so you can also rule out answer choices A and C.

The only statement that's true is the one in answer choice B.

Topic: Circumscribed and inscribed circles of a triangle

Question: In equilateral triangle ABC, the sides are of length 8, $\overline{OA}$ and $\overline{OC}$ are bisectors of $\angle DAB$ and $\angle BCD$, respectively. The circle (with center at point O) is inscribed in $\triangle ABC$. If $\overline{DC} = \overline{OD} \cdot \sqrt{3}$, what is the radius of the circle (the length of $\overline{OD}$)?



Answer choices:

A $\quad 4$

B $\quad 3$

C $\quad 4\sqrt{3}$

D $\quad \dfrac{4\sqrt{3}}{3}$

Solution: D

Let r be the radius of the inscribed circle of $\triangle ABC$ (r is the length of $\overline{OD}$). We've been given that $\overline{DC} = \overline{OD} \cdot \sqrt{3}$, which means that

$$4 = r\sqrt{3}$$

$$r = \frac{4}{\sqrt{3}}$$

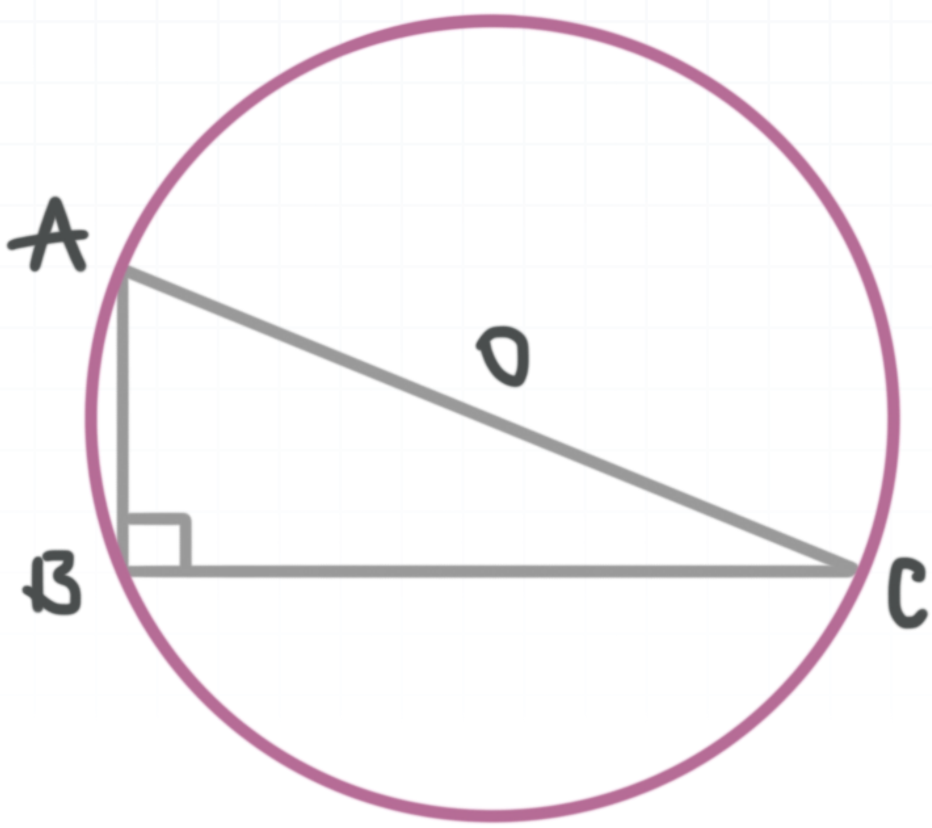
We'll rationalize the denominator by multiplying both the numerator and denominator by $\sqrt{3}$.

$$r = \frac{4}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$r = \frac{4\sqrt{3}}{3}$$

Topic: Circumscribed and inscribed circles of a triangle

Question: In the right triangle ABC, $\overline{AB} = 5$ and $\overline{BC} = 12$. The circle (with center at O) is circumscribed around $\triangle ABC$. What is the radius of the circle?



Answer choices:

A 13

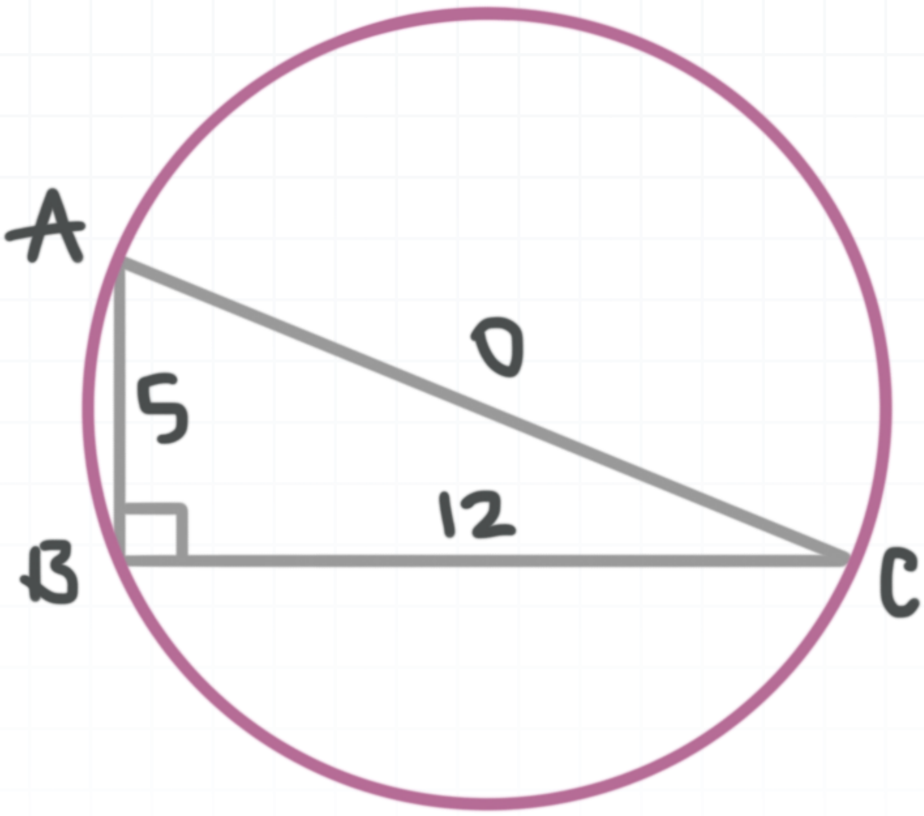
B 12.5

C 6.5

D 6

Solution: C

Fill in the figure with what we know.



Remember, the measure of an inscribed angle is half that of its intercepted arc. So because $\angle CBA$ is a 90° inscribed angle, its intercepted arc, $\overset{\frown}{CA}$, has measure 180°.

This guarantees that $\overline{AC}$ is a diameter of the circle.

Use the Pythagorean theorem to find $\overline{AC}$.

$$(\overline{AB})^2 + (\overline{BC})^2 = (\overline{AC})^2$$

$$5^2 + 12^2 = (\overline{AC})^2$$

$$169 = (\overline{AC})^2$$

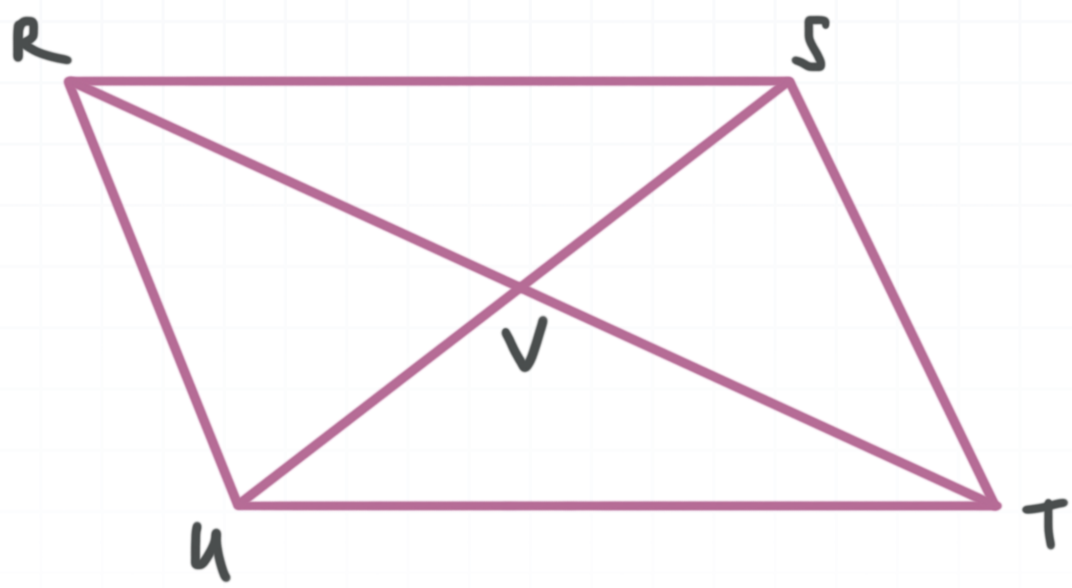
$$\overline{AC} = 13$$

The radius is half the diameter (half the length of $\overline{AC}$).

$$\frac{1}{2}(\overline{AC}) = \frac{1}{2}(13) = 6.5$$

Topic: Measures of quadrilaterals

Question: In parallelogram $RSTU$, $\overline{RT} = 11$, $\overline{US} = 8$, and $\overline{RU} = 5.5$. What is the perimeter of $\triangle STV$? Hint: The perimeter of any polygon is the sum of the lengths of its sides.



Answer choices:

A 11

B 15

C 19

D 24.5

Solution: B

The diagonals of a parallelogram bisect each other, so

$$\overline{VT} = \frac{1}{2}(\overline{RT}) = \frac{1}{2}(11) = 5.5$$

Likewise,

$$\overline{VS} = \frac{1}{2}(\overline{US}) = \frac{1}{2}(8) = 4$$

Opposite sides of a parallelogram are congruent, so

$$\overline{ST} = \overline{RU} = 5.5$$

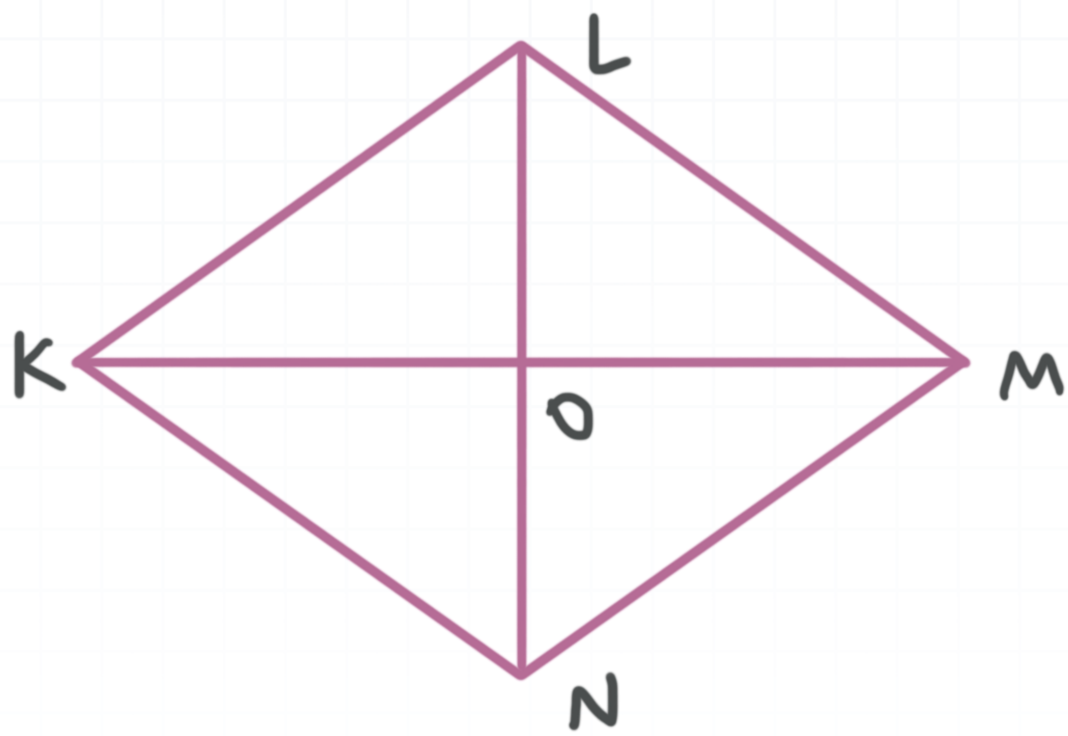
Therefore, the perimeter of $\triangle STV$ is

$$\overline{VT} + \overline{VS} + \overline{ST} = 5.5 + 4 + 5.5$$

$$\overline{VT} + \overline{VS} + \overline{ST} = 15$$

Topic: Measures of quadrilaterals

Question: In rhombus $KLMN$, $\overline{LN} = 9$ and $\overline{OM} = 6$. What is the perimeter of $KLMN$?



Answer choices:

A $\quad 24$

B $\quad 26$

C $\quad 28$

D $\quad 30$

Solution: D

The diagonals of a rhombus are perpendicular bisectors of each other, so

$$LO = \frac{1}{2}(\overline{LN}) = \frac{1}{2}(9) = 4.5$$

And $m\angle MOL = 90^\circ$. Since $\triangle LOM$ is a right triangle, we can use the Pythagorean theorem to find the length $\overline{LM}$.

$$(\overline{LO})^2 + (\overline{OM})^2 = (\overline{LM})^2$$

$$4.5^2 + 6^2 = (\overline{LM})^2$$

$$20.25 + 36 = (\overline{LM})^2$$

$$56.25 = (\overline{LM})^2$$

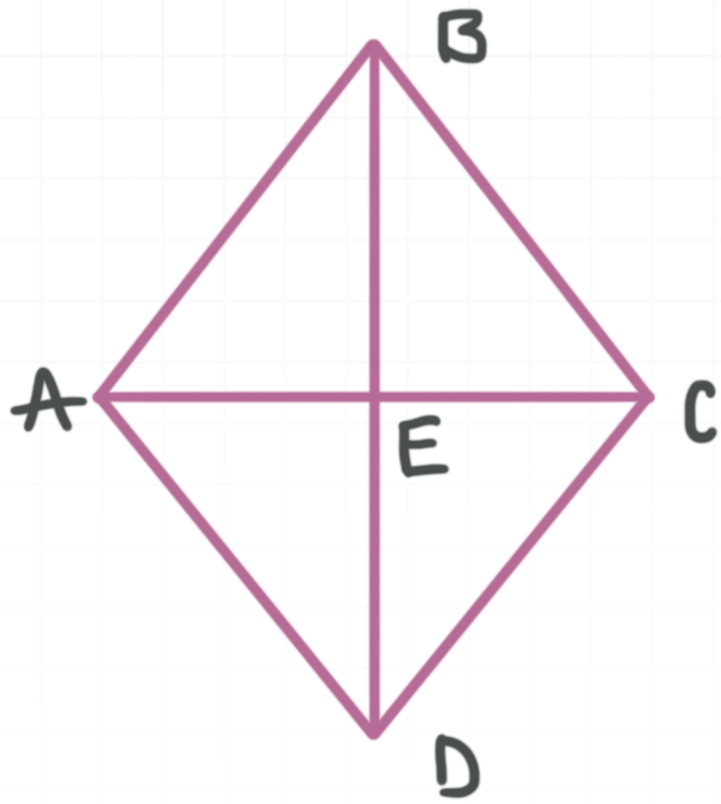
$$7.5 = \overline{LM}$$

All sides of a rhombus are congruent, so the perimeter of $KLMN$ is

$$4(\overline{LM}) = 4(7.5) = 30$$

Topic: Measures of quadrilaterals

Question: In rhombus $ABCD$, $\overline{BC} = 6$ and $m\angle BCE = 60^\circ$. What is $m\angle EDA$?



Answer choices:

A 15°

B 30°

C 45°

D 60°

Solution: B

The diagonals of a rhombus are perpendicular to each other, so

$$m\angle CEB = 90^\circ$$

The sum of the measures of the interior angles of a triangle is 180°, which means that in triangle BCE,

$$m\angle EBC + m\angle CEB + m\angle BCE = 180^\circ$$

$$m\angle EBC + 90^\circ + 60^\circ = 180^\circ$$

$$m\angle EBC + 150^\circ = 180^\circ$$

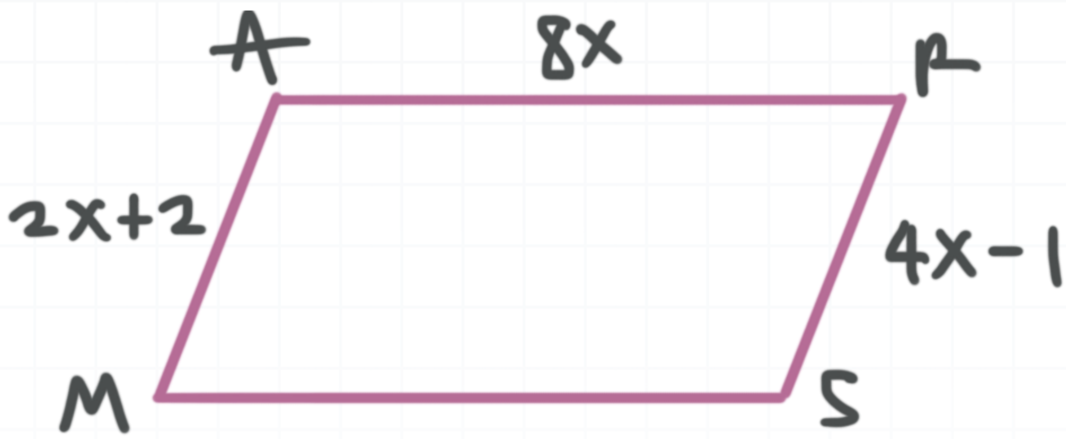
$$m\angle EBC = 30^\circ$$

Since $ABCD$ is a rhombus, $\overline{BC}$ is parallel to $\overline{AD}$. Therefore, the diagonal $\overline{BD}$ is a transversal that crosses a pair of parallel lines (the extensions of $\overline{BC}$ and $\overline{AD}$ to infinity in both directions). Notice that $\angle EDA$ and $\angle EBC$ are a pair of alternate interior angles, which means they're congruent, so

$$m\angle EDA = m\angle EBC = 30^\circ$$

Topic: Measures of parallelograms

Question: In parallelogram $MARS$, find the length of $\overline{MS}$.



Answer choices:

A 1.5

B 5

C 8.5

D 12

Solution: D

Opposite sides of a parallelogram are congruent, so

$$4x - 1 = 2x + 2$$

$$2x = 3$$

$$x = 1.5$$

Therefore,

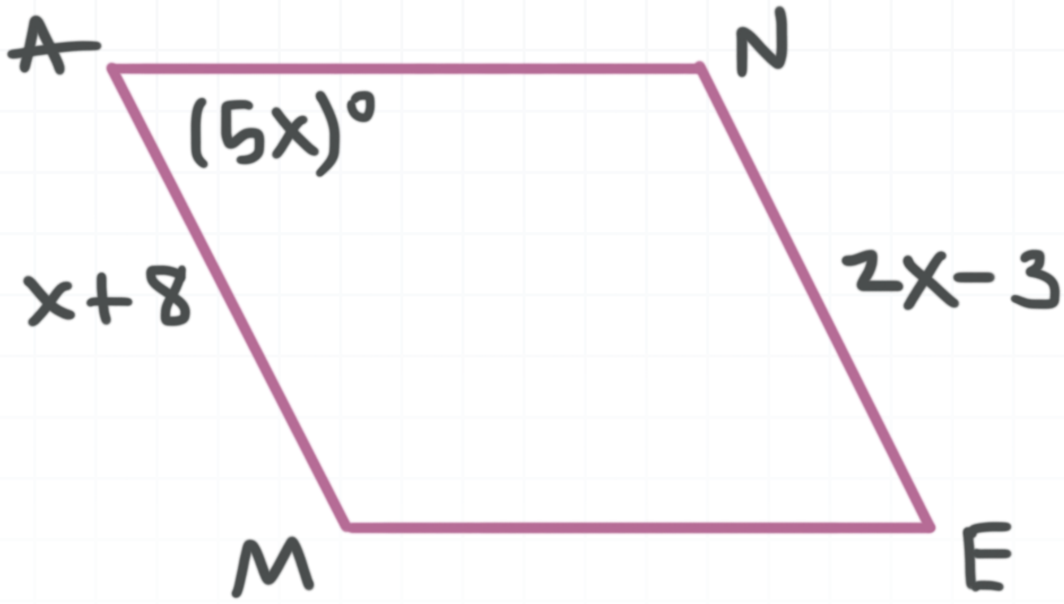
$$\overline{MS} = \overline{AR} = 8x$$

$$\overline{MS} = \overline{AR} = 8(1.5)$$

$$\overline{MS} = \overline{AR} = 12$$

Topic: Measures of parallelograms

Question: In parallelogram $MANE$, find $m\angle NEM$.



Answer choices:

A 55°

B 65°

C 85°

D 125°

Solution: A

Opposite sides of a parallelogram are congruent, so

$$2x - 3 = x + 8$$

$$x = 11$$

Therefore,

$$m\angle MAN = (5x)^\circ$$

$$m\angle MAN = (5 \cdot 11)^\circ$$

$$m\angle MAN = 55^\circ$$

Opposite angles of a parallelogram are congruent, so

$$m\angle NEM = m\angle MAN = 55^\circ$$

Topic: Measures of parallelograms

Question: In parallelogram $JAYZ$, find the length of $\overline{AJ}$.

Answer choices:

A $\quad$ 5

B $\quad$ 6

C $\quad$ 8

D $\quad$ 10

Solution: C

Consecutive angles of a parallelogram are supplementary (angles next to each other add up to 180°), so

$$(3x - 3)^\circ + (8x + 7)^\circ = 180$$

$$11x^\circ + 4^\circ = 180^\circ$$

$$11x^\circ = 176^\circ$$

$$x^\circ = 16^\circ$$

Therefore,

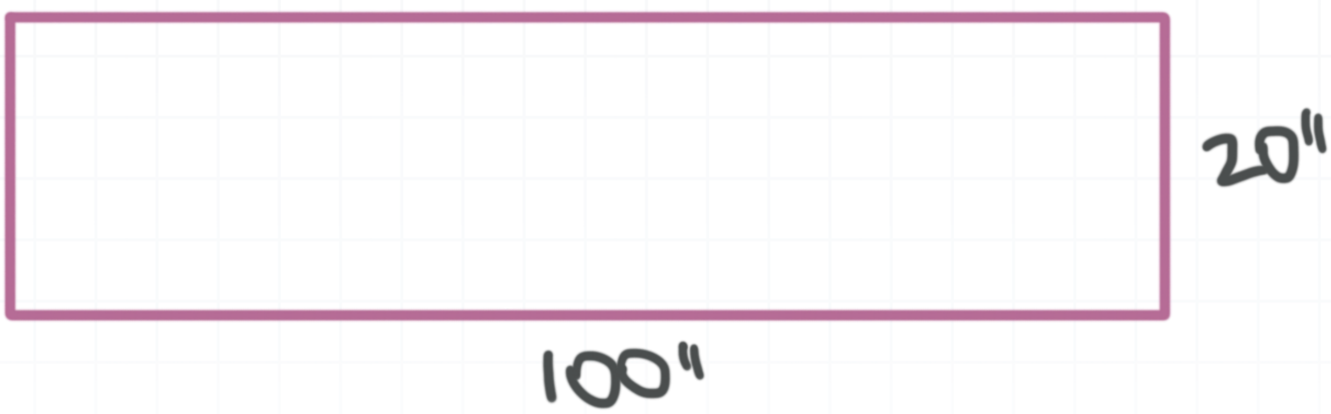
$$\overline{AJ} = 0.5x$$

$$\overline{AJ} = 0.5(16)$$

$$\overline{AJ} = 8$$

Topic: Area of a rectangle

Question: Find the area of the rectangle.



Answer choices:

A $2{,}000\ \text{in}^2$

B $2{,}000\ \text{in}^3$

C $200\ \text{in}^2$

D $200\ \text{in}^3$

Solution: A

Plugging the dimensions of the rectangle into the formula for the area of a rectangle, we get

$$A = bh$$

$$A = (100 \text{ in})(20 \text{ in})$$

$$A = 2{,}000 \text{ in}^2$$

Topic: Area of a rectangle

Question: A large house has three garage doors, each of which is made of four rectangular panels. Each panel is 24 inches high and 108 inches long. The home owner wants to repaint them and needs to know the total area of the three doors so she can buy enough paint. Find the total area, in square feet, of the three doors.

Answer choices:

A 18 ft^2

B 72 ft^2

C 216 ft^2

D 432 ft^2

Solution: C

The dimensions of each panel in feet are

$$\text{height} = 24 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = 2 \text{ feet}$$

$$\text{base} = 108 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = 9 \text{ feet}$$

The area of each panel is given by

$$bh = 9 \text{ ft} \cdot 2 \text{ ft} = 18 \text{ ft}^2$$

Three doors with four panels each gives us a total of

$$3 \cdot 4 = 12 \text{ panels}$$

The total area of all 12 panels is

$$12 \cdot 18 \text{ ft}^2 = 216 \text{ ft}^2$$

Topic: Area of a rectangle

Question: A rectangular wall has a height that's $1/3$ of its base. If the area of the wall is 24 m^2, how long is the base of the wall?

Answer choices:

A 2.83 m

B 8.00 m

C 8.48 m

D 10.17 m

Solution: C

Let x be the height of the wall, which will make the base $3x$. Their product is the area.

$$A = bh = 3x \cdot x = 3x^2$$

Since the area is 24 m^2, we have

$$3x^2 = 24$$

$$x^2 = 8$$

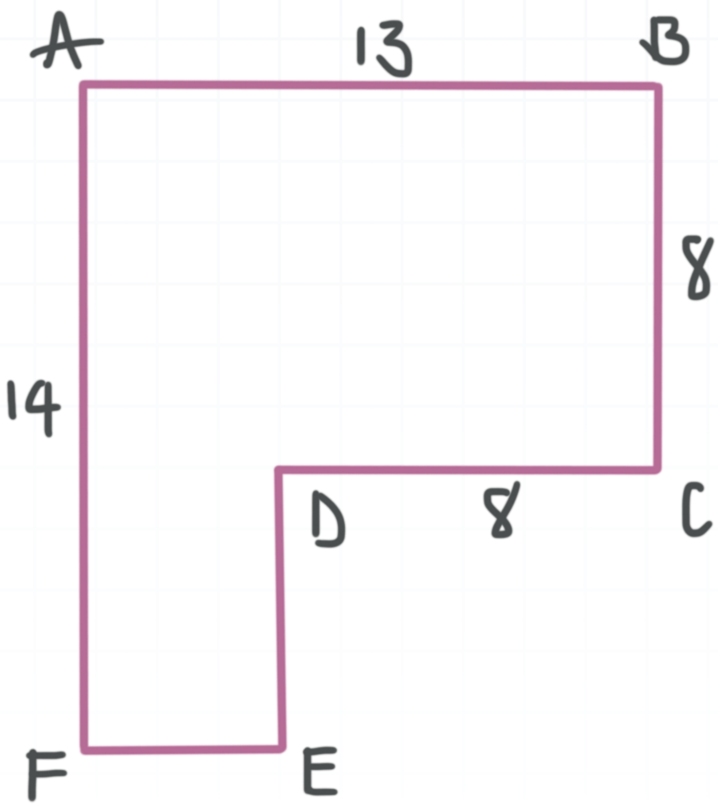
$$x = \sqrt{8}$$

According to the way we set up the problem, this is the height of the wall, which we can now use to find the base.

$$b = 3x = 3\sqrt{8} \approx 3 \cdot 2.828 = 8.484 \approx 8.48 \text{ m}$$

Topic: Area of a rectangle using sums and differences

Question: The figure is made by combining rectangles. What is the area of the figure?



Answer choices:

A 96

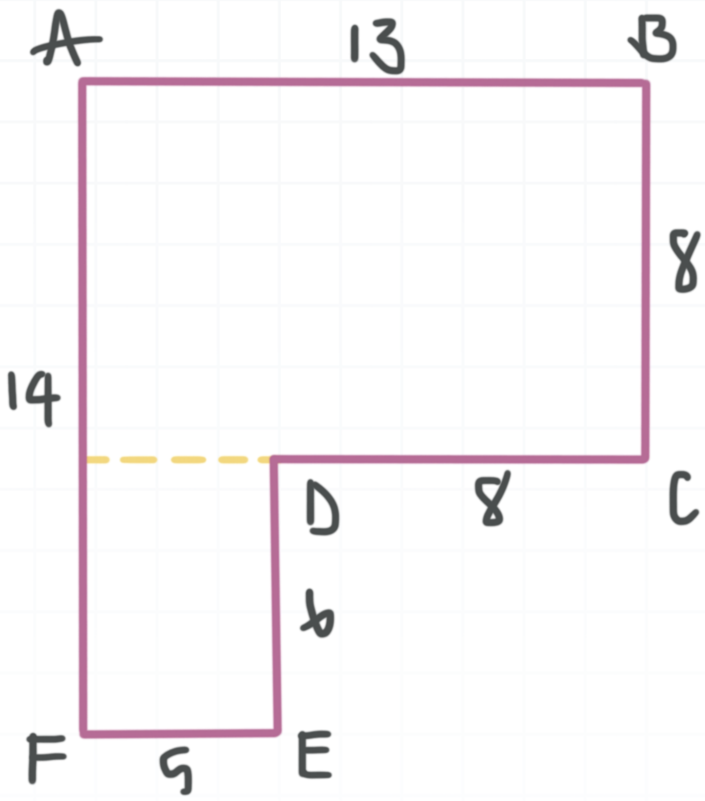
B 104

C 134

D 182

Solution: C

Draw a dashed segment to divide the figure into two rectangles.



The height of the figure is 14, so $\overline{DE} = \overline{AF} - \overline{BC} = 14 - 8 = 6$.

The width of the figure is 13, so $\overline{FE} = \overline{AB} - \overline{DC} = 13 - 8 = 5$.

The area of the upper rectangle is

$$\text{area} = bh = 13 \cdot 8 = 104$$

The area of the lower rectangle is

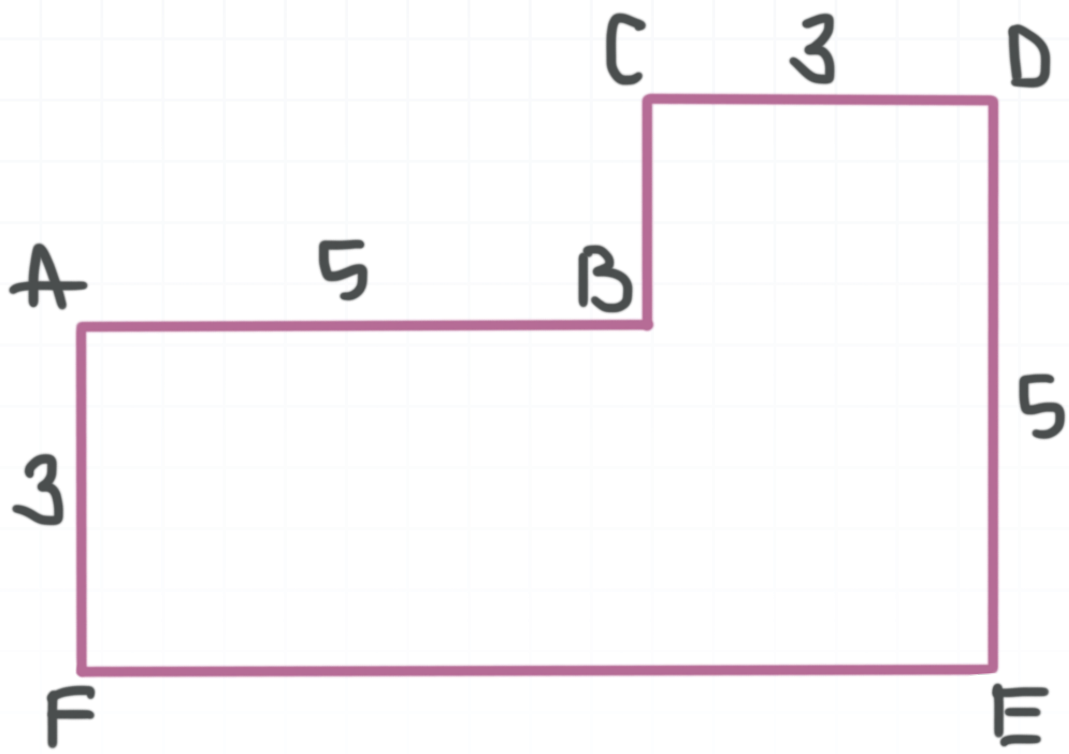
$$\text{area} = bh = 5 \cdot 6 = 30$$

So the total area of the figure is

$$\text{area} = 104 + 30 = 134$$

Topic: Area of a rectangle using sums and differences

Question: The figure is made by combining rectangles. What is the area of the figure?



Answer choices:

A 30

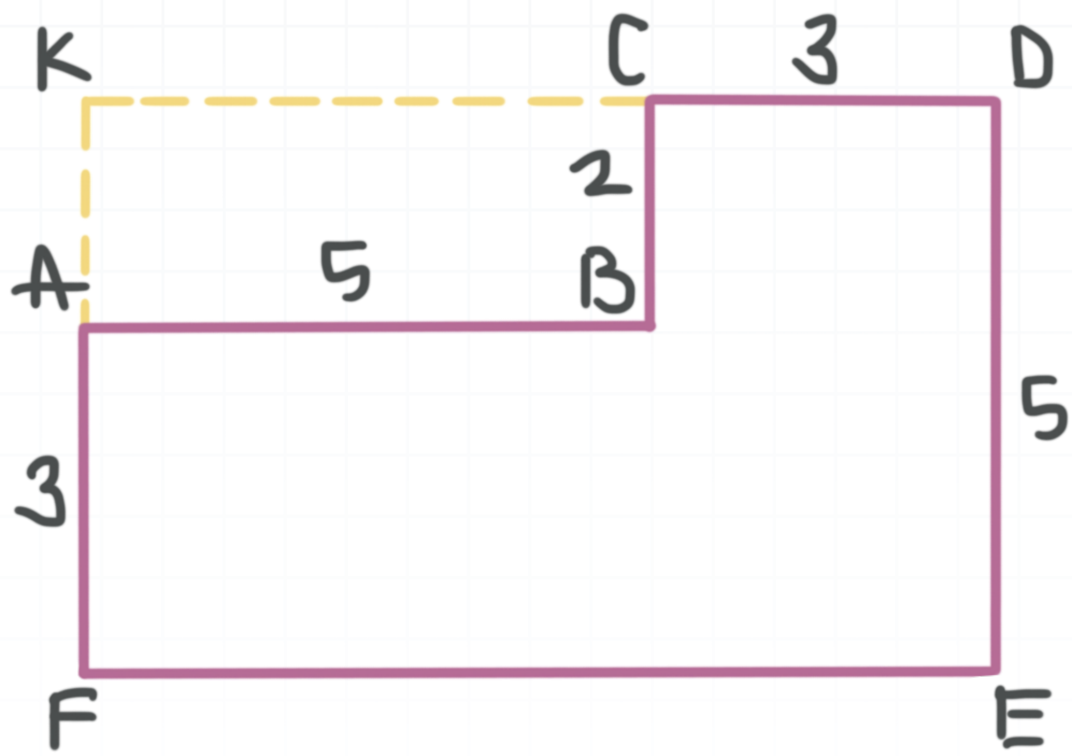
B 40

C 42

D 52

Solution: A

Form a new, large rectangle by drawing a rectangle that fills in the empty space, and draw dashed line segments from A and C to K, where K is the vertex in the upper-left corner of the new, large rectangle we formed.



The height of the new, large rectangle we formed is 5, so $\overline{CB} = \overline{DE} - \overline{AF} = 5 - 3 = 2$.

By adding $\overline{AB}$ and $\overline{CD}$, we find that the base of the new, large rectangle we formed is 8.

The area of the new, large rectangle we formed, $KDEF$, is

$$\text{area} = bh = 8 \cdot 5 = 40$$

The area of $ABCK$ is

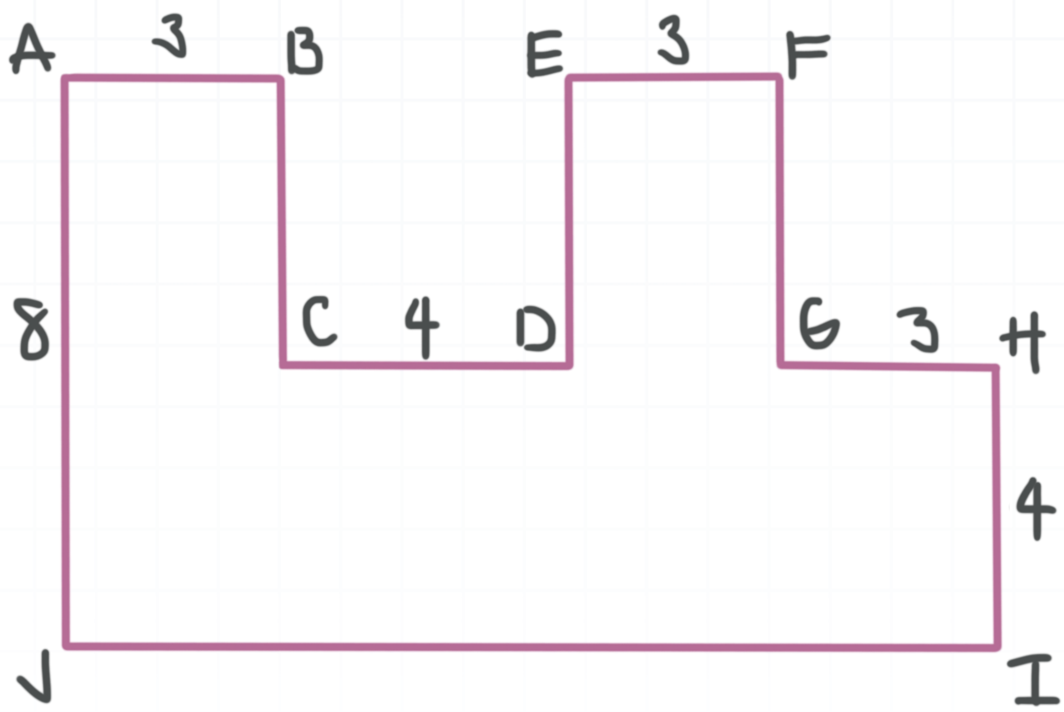
$$\text{area} = bh = 5 \cdot 2 = 10$$

To get the area of the original figure, subtract the area of the rectangle $ABCK$ from the area of the rectangle $KDEF$.

$$\text{area} = 40 - 10 = 30$$

Topic: Area of a rectangle using sums and differences

Question: The figure is made by combining rectangles. What is the area of the figure?



Answer choices:

A 76

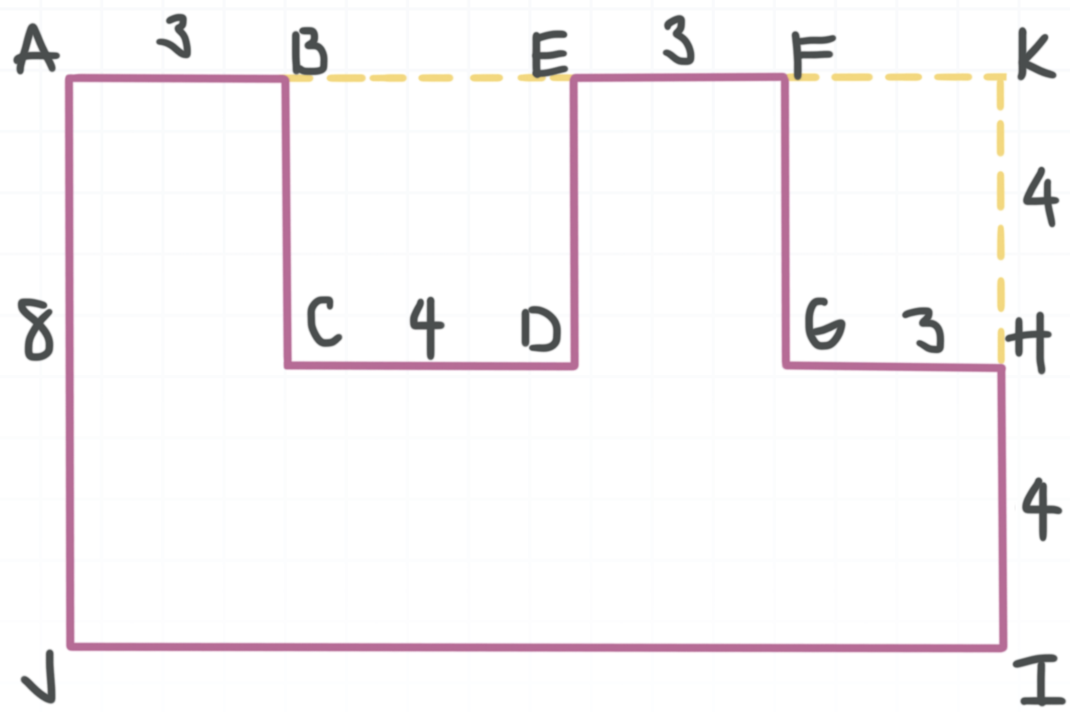
B 82

C 94

D 108

Solution: A

Form a new, large rectangle by drawing two rectangles that fill in the empty spaces. To do this, draw a dashed line segment from B to E (to get rectangle $BEDC$), and draw dashed line segments from F and H to a new point K (to get rectangle $FKHG$).



The height of the new, large rectangle we formed is 8, so $\overline{HK} = \overline{IK} - \overline{IH} = 8 - 4 = 4$. Likewise, $\overline{DE}$ must also be 4.

By adding the lengths of $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, and $\overline{GH}$, we get a total width of 13.

The area of the new, large rectangle we formed is therefore

$$\text{area} = bh = 13 \cdot 8 = 104$$

The area of $BEDC$ is

$$\text{area} = bh = 4 \cdot 4 = 16$$

The area of $FKHG$ is

area $= bh = 3 \cdot 4 = 12$

To get the area of the original figure, subtract the sum of the areas of the rectangles $BEDC$ and $FKHG$ from the area of the new, large rectangle we formed, $AKIJ$.

area $= 104 - (16 + 12) = 104 - 28 = 76$

Topic: Perimeter of a rectangle

Question: What is the perimeter of a rectangle with one vertex at $(0,0)$ and the opposite vertex at $(9,6)$, if its sides are parallel to the coordinate axes?

Answer choices:

A 15

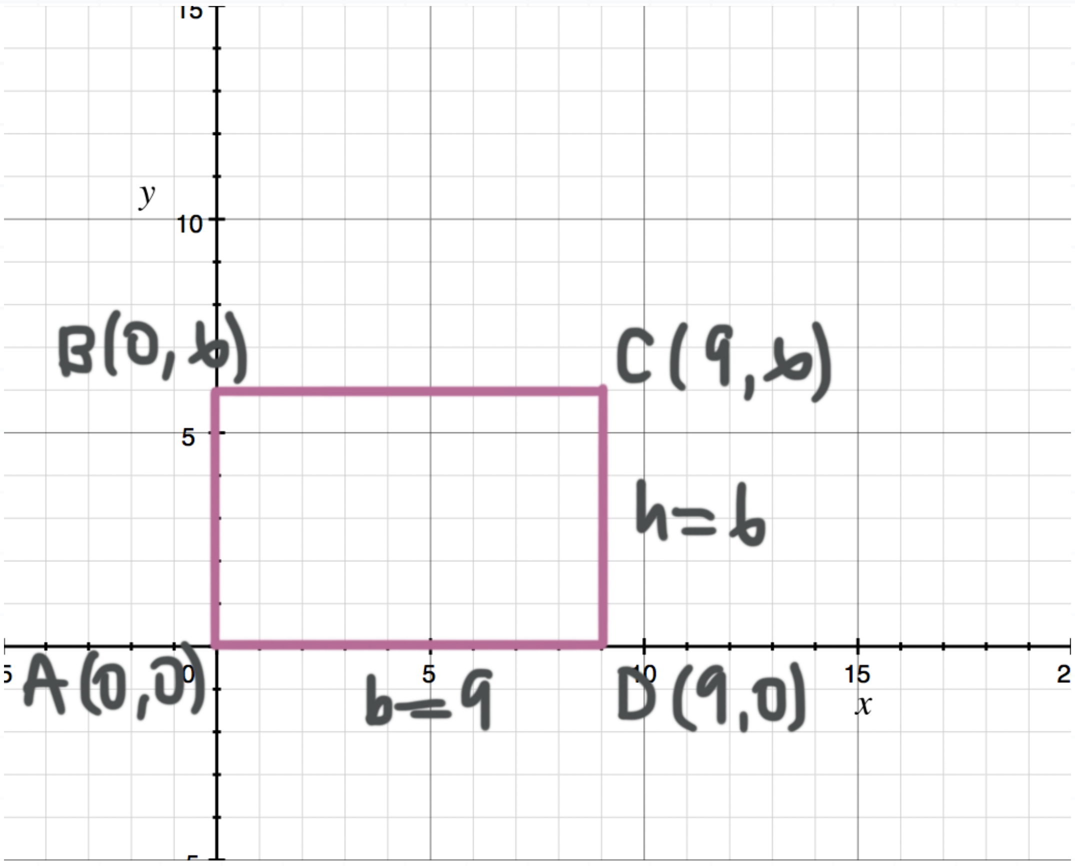
B 30

C 45

D 54

Solution: B

The perimeter of rectangle $ABCD$,



is

$$\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA} = 6 + 9 + 6 + 9$$

$$\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA} = 30$$

Or we could write

perimeter $= 2b + 2h$

perimeter $= 2(9) + 2(6)$

perimeter $= 18 + 12$

perimeter $= 30$

Topic: Perimeter of a rectangle

Question: Find the perimeter of a rectangle with a base length of 6 and an area of 54.

Answer choices:

A 12

B 15

C 22

D 30

Solution: D

The formula for area of a rectangle is

$$A = bh$$

We know that $A = 54$ and $b = 6$, so

$$54 = 6 \cdot h$$

Solving for h, we get

$$h = \frac{54}{6} = 9$$

The rectangle will have two sides of length 6 and two sides of length 9.

$$\text{perimeter} = 2(6) + 2(9)$$

$$\text{perimeter} = 12 + 18$$

$$\text{perimeter} = 30$$

Topic: Perimeter of a rectangle

Question: What is the height of a rectangle with a height that's twice the base, and a perimeter of 36?

Answer choices:

A 6

B 8

C 9

D 12

Solution: D

Let x be the base, which would make the height $2x$. Then the perimeter is the sum of twice the base and twice the height.

twice the base $= 2(x) = 2x$

twice the height $= 2(2x) = 4x$

So we see that

perimeter $= 2x + 4x = 6x$

and we know that the perimeter is 36. Therefore,

$$6x = 36$$

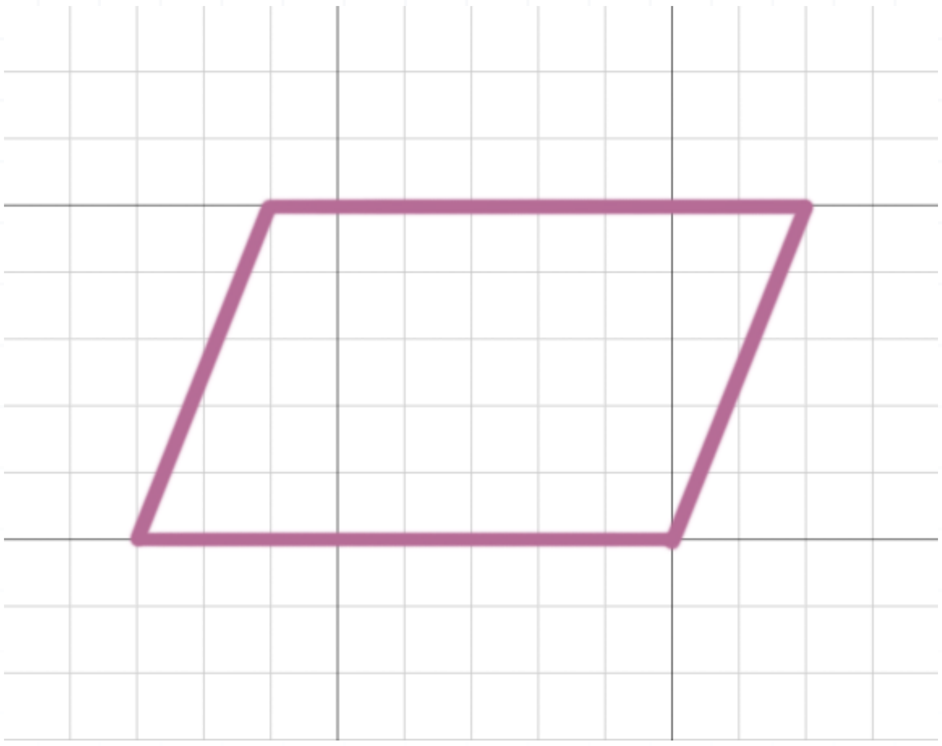
$$x = 6$$

Which means that

height $= 2x = 2(6) = 12$

Topic: Area of a parallelogram

Question: What is the area of the parallelogram, assuming that the lines in the grid are each 1 cm apart?



Answer choices:

A $\quad 26$ cm^2

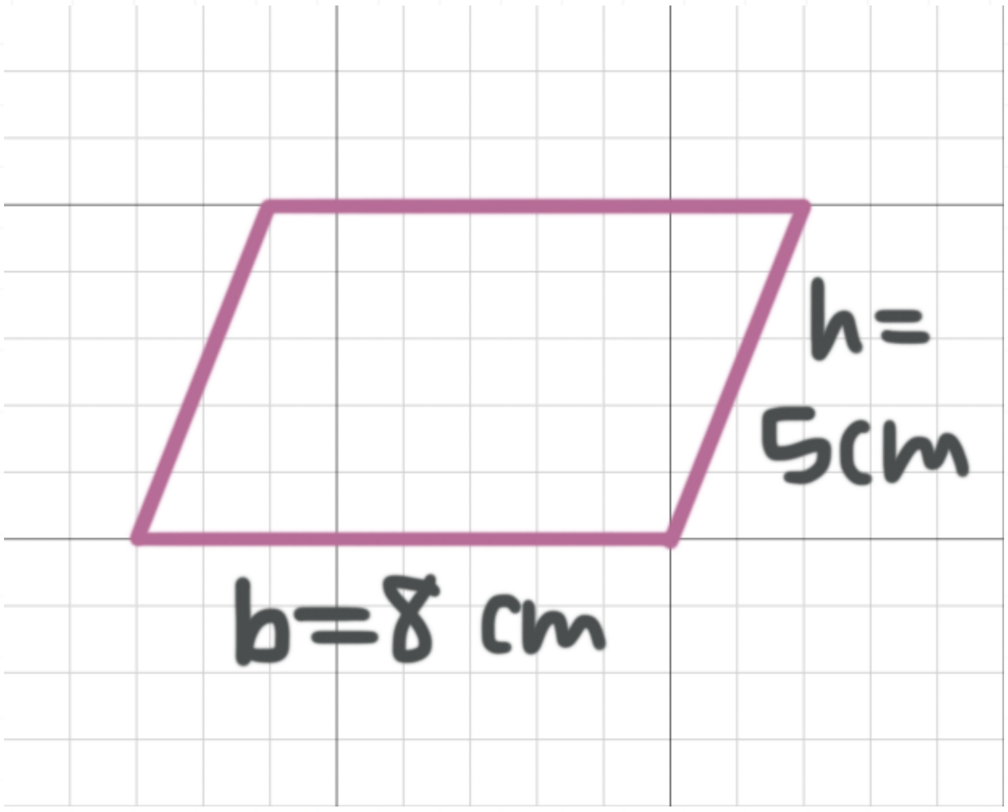
B $\quad 32$ cm^2

C $\quad 38$ cm^2

D $\quad 40$ cm^2

Solution: D

In the figure, we see that the base of the parallelogram is 8 cm and the height is 5 cm.



Plugging these dimensions into the area formula, we get

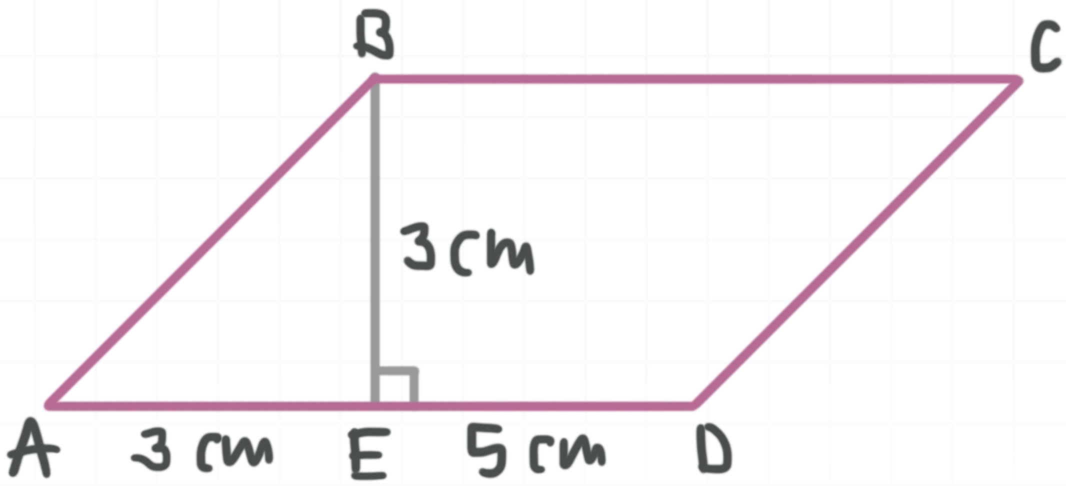
$$A = bh$$

$$A = (8 \text{ cm})(5 \text{ cm})$$

$$A = 40 \text{ cm}^2$$

Topic: Area of a parallelogram

Question: What is the area of the parallelogram?



Answer choices:

A $\quad 9\text{ cm}^2$

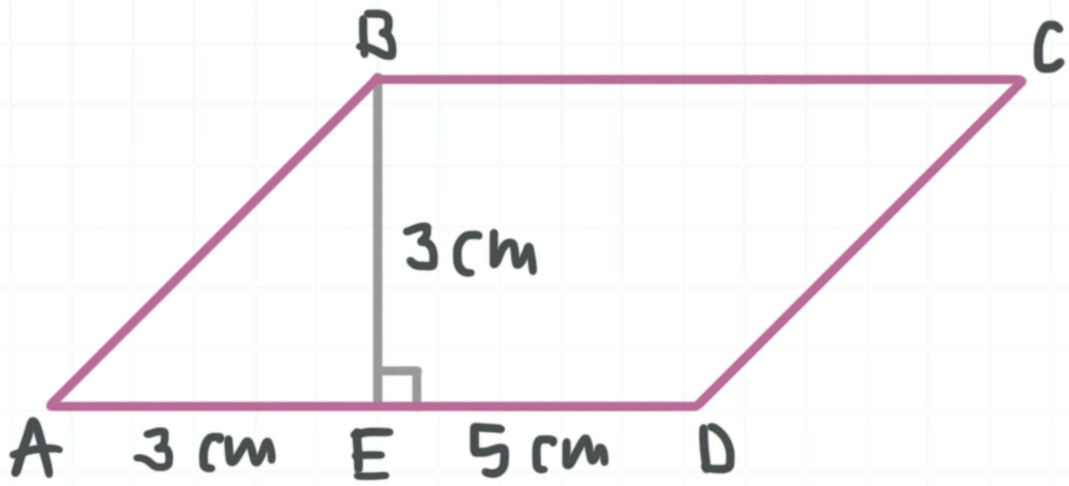
B $\quad 12\text{ cm}^2$

C $\quad 24\text{ cm}^2$

D $\quad 35\text{ cm}^2$

Solution: C

In the figure, we see that the base of the parallelogram is $3 + 5 = 8$ cm and the height is 3 cm.



Plugging these dimensions into the area formula, we get

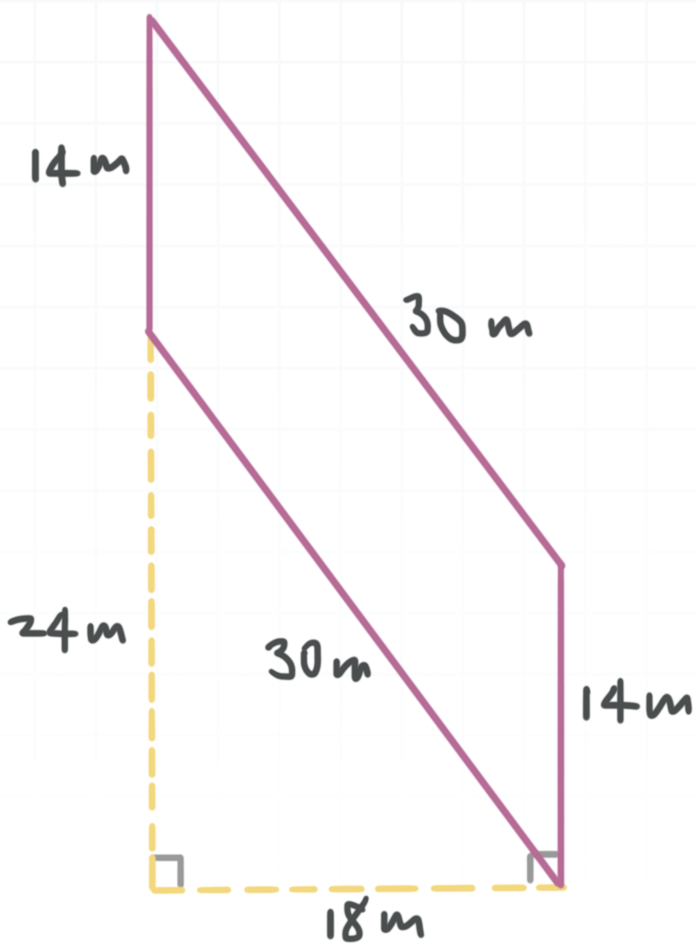
$$A = bh$$

$$A = (8 \text{ cm})(3 \text{ cm})$$

$$A = 24 \text{ cm}^2$$

Topic: Area of a parallelogram

Question: What is the area of the parallelogram?



Answer choices:

A 252 m^2

B 420 m^2

C 432 m^2

D 720 m^2

Solution: A

Imagine rotating the parallelogram until the 14 m side that started out on the left is horizontal (and becomes the base). From that base, the height to the opposite 14 m side is 18 m.

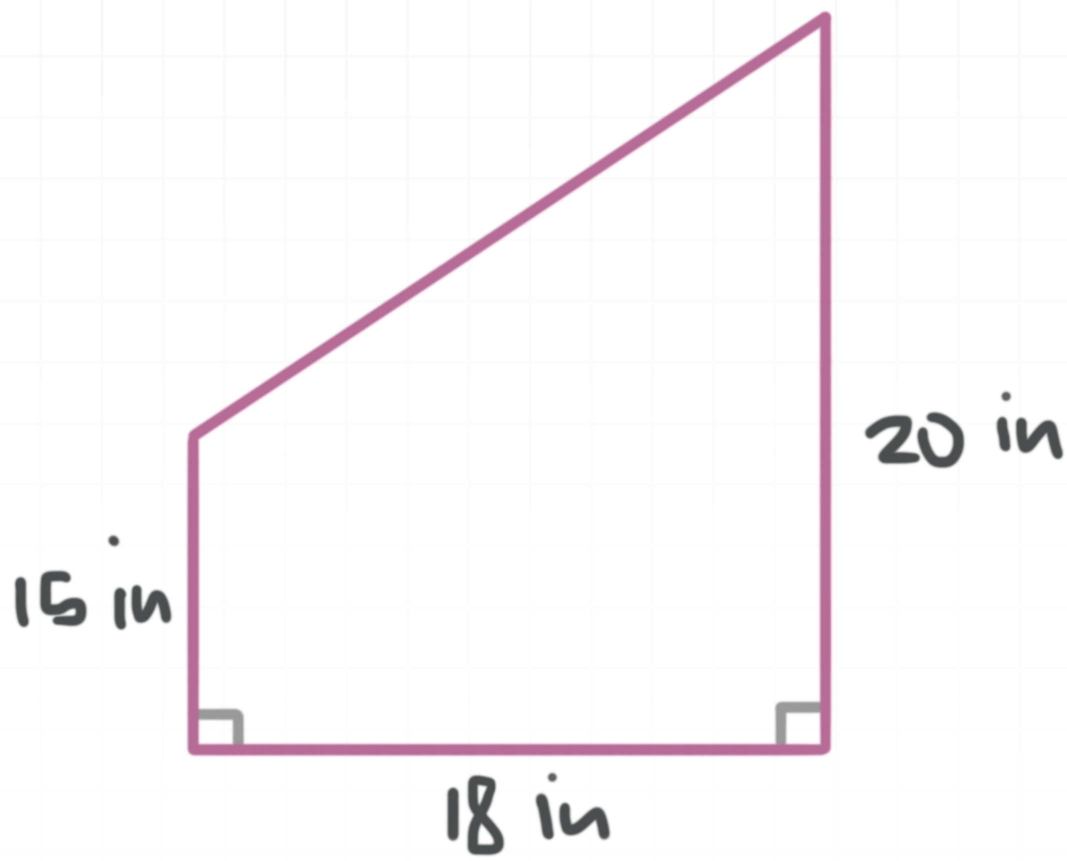
$$A = bh$$

$$A = (14 \text{ m})(18 \text{ m})$$

$$A = 252 \text{ m}^2$$

Topic: Area of a trapezoid

Question: What is the area of the trapezoid?



Answer choices:

A 315 in^2

B 360 in^2

C 424 in^2

D 630 in^2

Solution: A

Even though the two parallel sides are vertical in the figure, they're still the bases (because they are parallel). We can mentally rotate the figure by 90° if that helps.

So the bases are the sides of length 15 in and 20 in.

Likewise, even though the 18 in side is on the bottom, it's still considered the height. So the height is 18 in.

$$\text{area} = \frac{1}{2}(15\text{ in} + 20\text{ in})(18\text{ in})$$

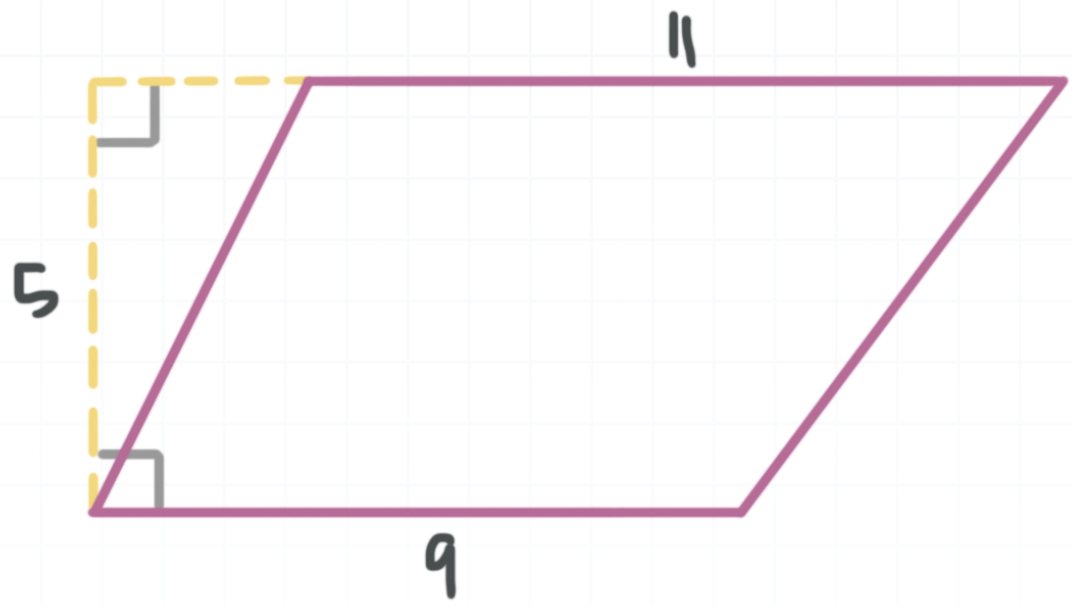
$$\text{area} = \frac{1}{2}(35\text{ in})(18\text{ in})$$

$$\text{area} = \frac{1}{2}(630\text{ in}^2)$$

$$\text{area} = 315\text{ in}^2$$

Topic: Area of a trapezoid

Question: What is the area of the trapezoid?



Answer choices:

A 30

B 45

C 50

D 55

Solution: C

The bases are the sides of lengths 9 and 11. The segment of length 5 is the height because it's perpendicular to the bottom base (the base of length 9) and to the (extension of the) top base (the base of length 11).

$$\text{area} = \frac{1}{2}(9+11)(5)$$

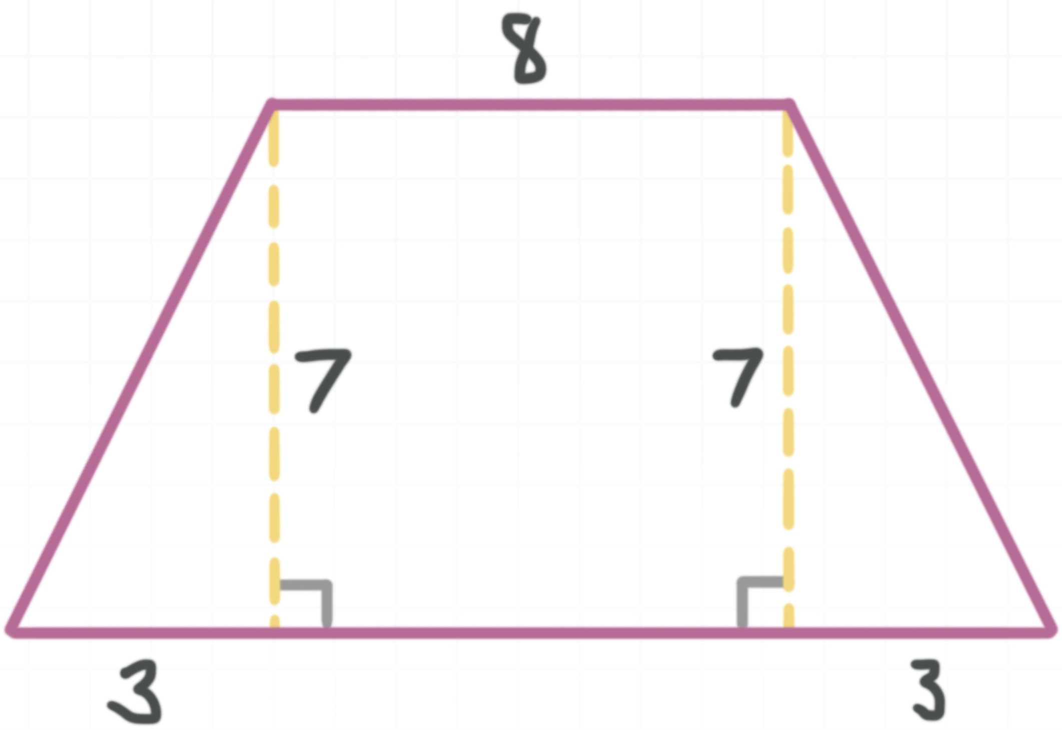
$$\text{area} = \frac{1}{2}(20)(5)$$

$$\text{area} = \frac{1}{2}(100)$$

$$\text{area} = 50$$

Topic: Area of a trapezoid

Question: What is the area of the trapezoid?



Answer choices:

A 49

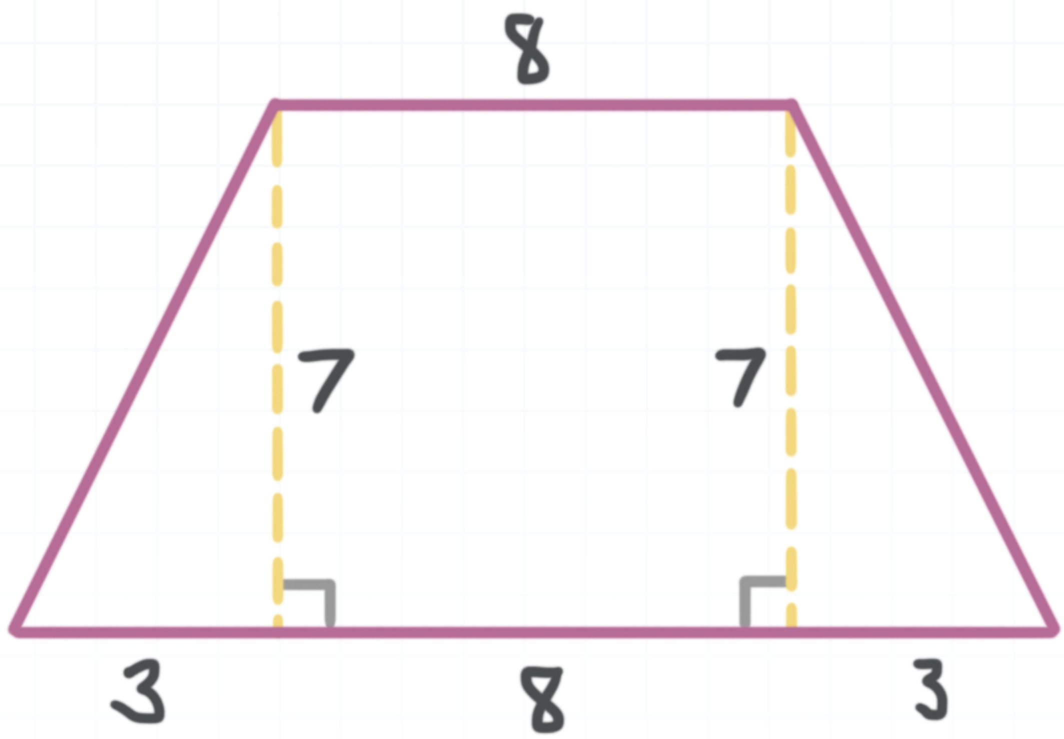
B 56

C 62

D 77

Solution: D

By drawing in the two segments of length 7 (which are both perpendicular to the bottom base of the trapezoid), we get a rectangle.



The base of the rectangle is 8.

This gives us the length of the bottom base of the parallelogram as

$$3 + 8 + 3 = 14$$

Which means the area is given by

$$\text{area} = \frac{1}{2}(8 + 14)(7)$$

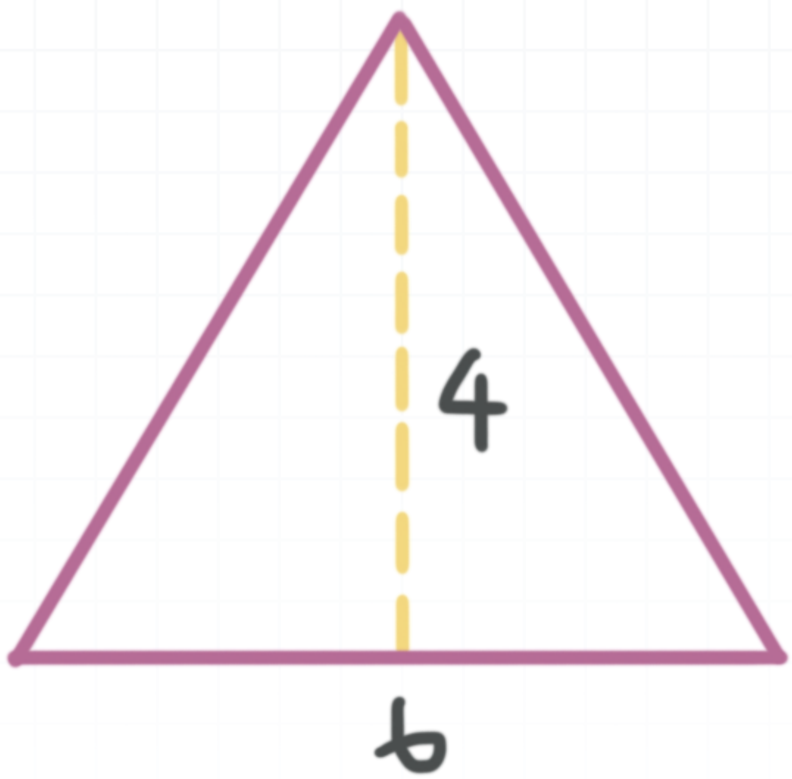
$$\text{area} = \frac{1}{2}(22)(7)$$

$$\text{area} = \frac{1}{2}(154)$$

area $= 77$

Topic: Area of a triangle

Question: Find the area of the triangle.



Answer choices:

A 24 ft^2

B 12 ft^3

C 12 ft

D 12 ft^2

Solution: D

Plugging in the dimensions of the triangle we've been given into the formula for the area of a triangle, we get

$$A = \frac{1}{2}bh$$

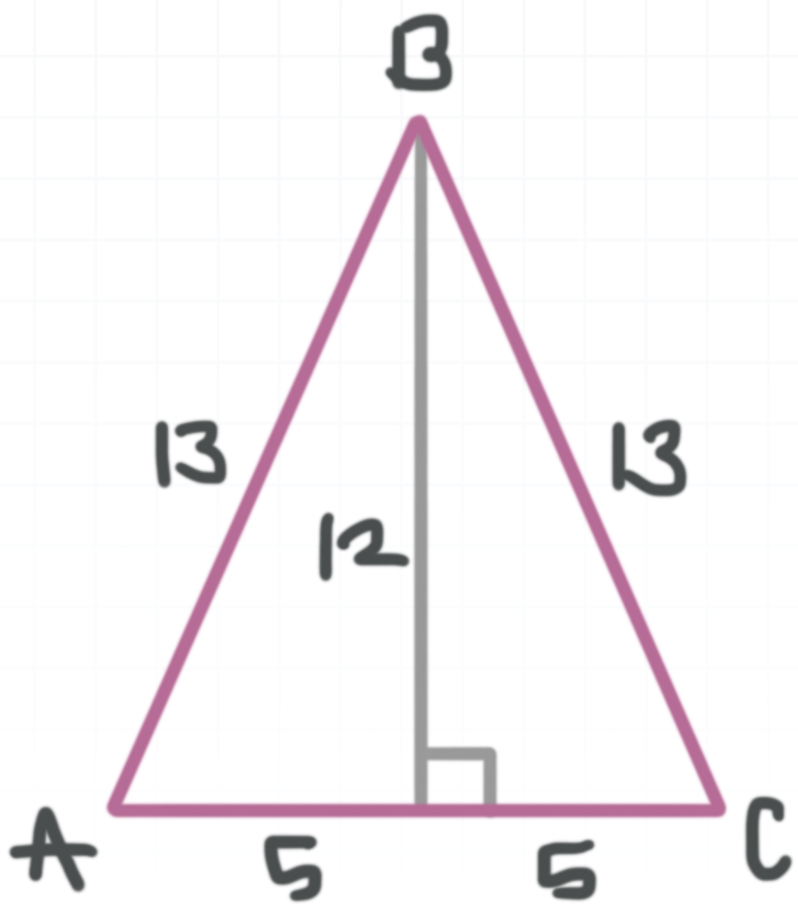
$$A = \frac{1}{2}(6 \text{ ft})(4 \text{ ft})$$

$$A = \frac{1}{2}(24 \text{ ft}^2)$$

$$A = 12 \text{ ft}^2$$

Topic: Area of a triangle

Question: What is the area of triangle ABC?



Answer choices:

A 60

B 65

C 120

D 156

Solution: A

The base b is $\overline{AC}$, which has a length of 10, and the height h is 12. So the area is given by

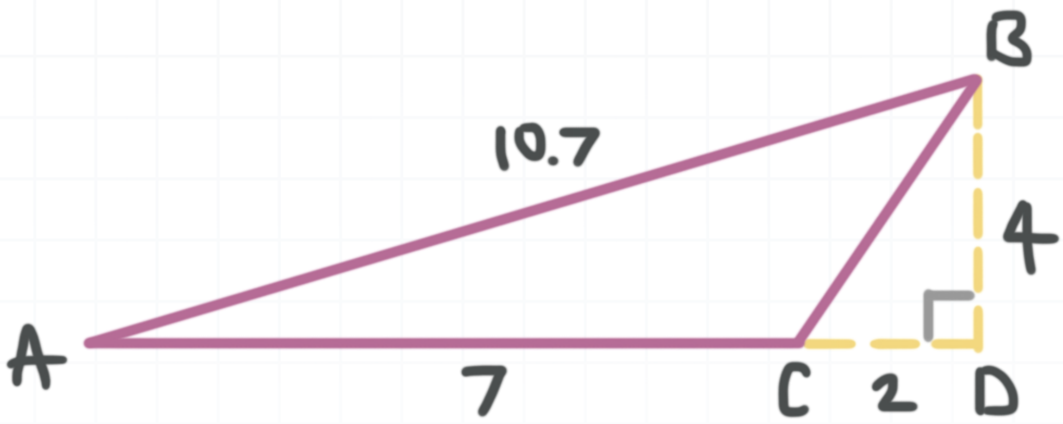
$$\text{area} = \frac{1}{2}(10)(12)$$

$$\text{area} = \frac{1}{2}(120)$$

$$\text{area} = 60$$

Topic: Area of a triangle

Question: What is the area of triangle ABC?



Answer choices:

A 4

B 8

C 14

D 18

Solution: C

The base b is $\overline{AC}$, which has a length of 7. The height h is $\overline{BD}$, which has a length of 4. So the area is given by

$$\text{area} = \frac{1}{2}(7)(4)$$

$$\text{area} = \frac{1}{2}(28)$$

$$\text{area} = 14$$

Topic: Perimeter of a triangle

Question: Find the perimeter of a right triangle with leg lengths 8 and 15.

Answer choices:

A $P = 38$

B $P = 40$

C $P = 42$

D $P = 44$

Solution: B

We can substitute the leg lengths into the Pythagorean theorem to find the length of the triangle's hypotenuse. Given $a = 8$ and $b = 15$, we get

$$a^2 + b^2 = c^2$$

$$8^2 + 15^2 = c^2$$

$$64 + 225 = c^2$$

$$289 = c^2$$

$$c = \sqrt{289}$$

$$c = 17$$

Now that we know all three side lengths, we can add them together to find the triangle's perimeter.

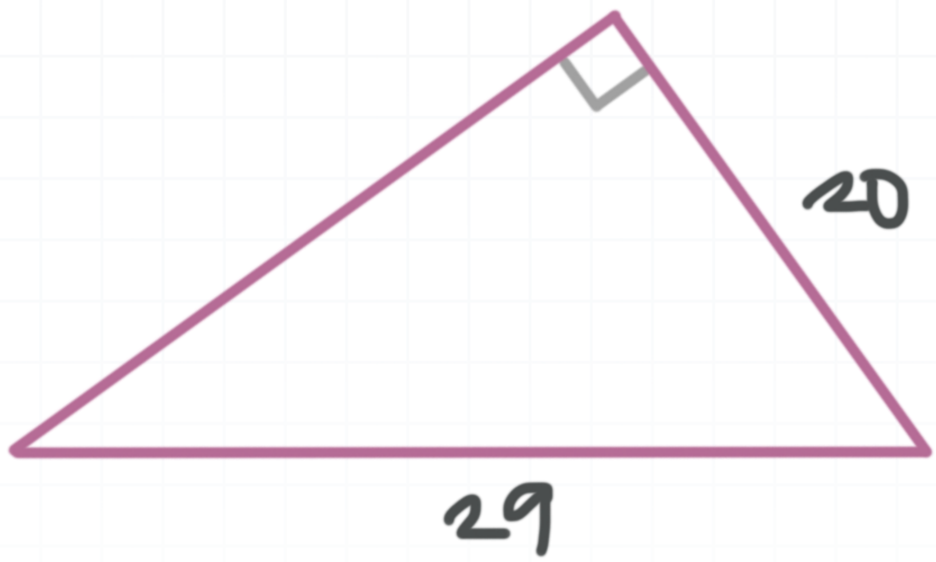
$$P = a + b + c$$

$$P = 8 + 15 + 17$$

$$P = 40$$

Topic: Perimeter of a triangle

Question: What is the sum of the side lengths of the triangle?



Answer choices:

A $P = 77$

B $P = 75$

C $P = 73$

D $P = 70$

Solution: D

The triangle is right, which means we can use the Pythagorean theorem to find the length of the unknown leg. The hypotenuse is always opposite of the right angle, so $a = 20$, $c = 29$, and we get

$$a^2 + b^2 = c^2$$

$$20^2 + b^2 = 29^2$$

$$400 + b^2 = 841$$

$$b^2 = 441$$

$$b = \sqrt{441}$$

$$b = 21$$

Then the perimeter of the triangle is

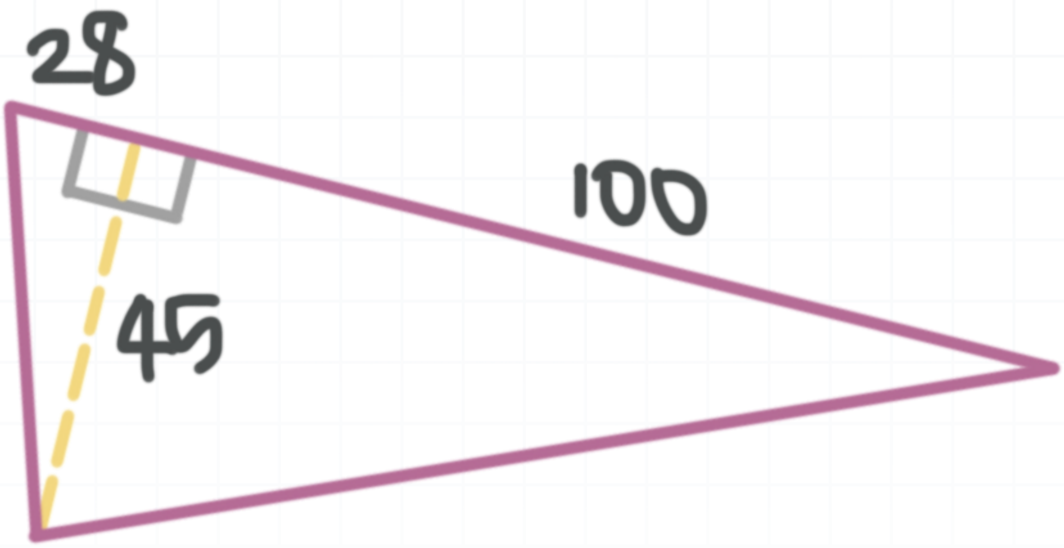
$$P = a + b + c$$

$$P = 20 + 21 + 29$$

$$P = 70$$

Topic: Perimeter of a triangle

Question: Find the perimeter of the oblique triangle.



Answer choices:

A $P = 181 + 5\sqrt{481}$

B $P = 191 + 5\sqrt{481}$

C $P = 201 + 5\sqrt{481}$

D $P = 211 + 5\sqrt{481}$

Solution: A

The oblique triangle can be split into two right triangles. The right triangle on the left has legs 28 and 45, so its hypotenuse is

$$28^2 + 45^2 = c^2$$

$$784 + 2{,}025 = c^2$$

$$2{,}809 = c^2$$

$$c = \sqrt{2{,}809}$$

$$c = 53$$

and the right triangle on the right has legs 45 and 100, so its hypotenuse is

$$45^2 + 100^2 = c^2$$

$$2{,}025 + 10{,}000 = c^2$$

$$12{,}025 = c^2$$

$$c = \sqrt{12{,}025}$$

$$c = 5\sqrt{481}$$

Therefore, the perimeter of the oblique triangle is

$$P = 53 + 5\sqrt{481} + 28 + 100$$

$$P = 181 + 5\sqrt{481}$$

Topic: Area of a circle

Question: What is the diameter of a circle with an area of 8π?

Answer choices:

A $\sqrt{2}$

B $2\sqrt{2}$

C $3\sqrt{2}$

D $4\sqrt{2}$

Solution: D

The formula for the area of a circle is $A = \pi r^2$, and the area is 8π. Therefore,

$$\pi r^2 = 8\pi$$

Dividing both sides by π gives

$$r^2 = 8$$

$$r = \sqrt{8}$$

$$r = \sqrt{4} \cdot \sqrt{2}$$

$$r = 2\sqrt{2}$$

Since $r = 2\sqrt{2}$, and the diameter d is double the radius, we get

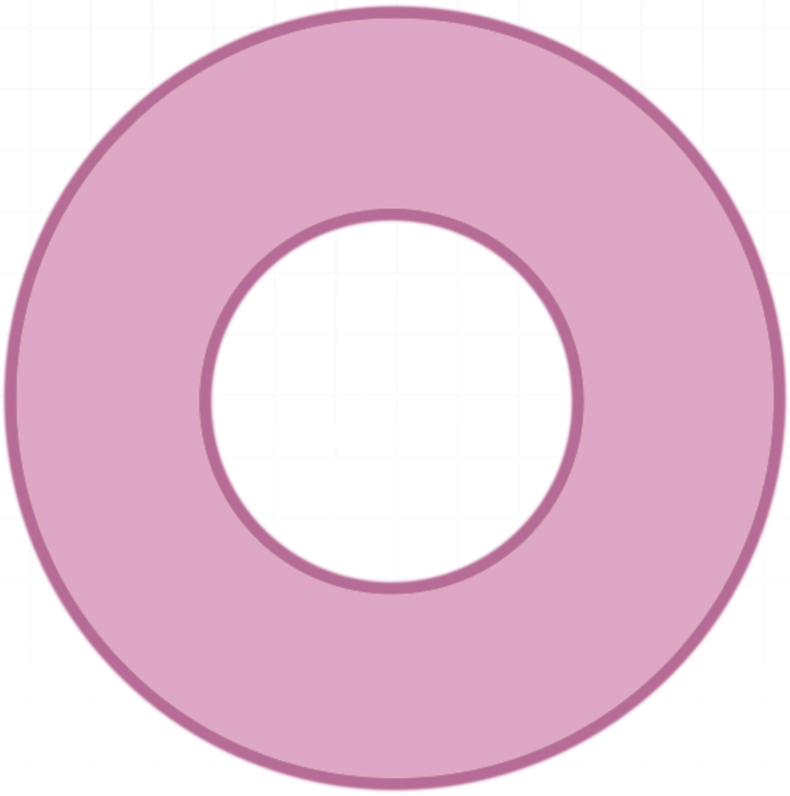
$$d = 2r$$

$$d = 2 \cdot 2\sqrt{2}$$

$$d = 4\sqrt{2}$$

Topic: Area of a circle

Question: The radius of the outer circle is 4 ft and the radius of the inner circle is 2 ft. What is the area of the shaded region?



Answer choices:

A 2π ft^2

B 6π ft^2

C 12π ft^2

D 16π ft^2

Solution: C

To get the area of the shaded region, you need to find the area of the outer circle and subtract from it the area of the inner circle.

For the outer circle:

$$A = \pi \cdot (4 \text{ ft})^2 = 16\pi \text{ ft}^2$$

For the inner circle:

$$A = \pi \cdot (2 \text{ ft})^2 = 4\pi \text{ ft}^2$$

Therefore, the area of the shaded region is

$$16\pi \text{ ft}^2 - 4\pi \text{ ft}^2 = 12\pi \text{ ft}^2$$

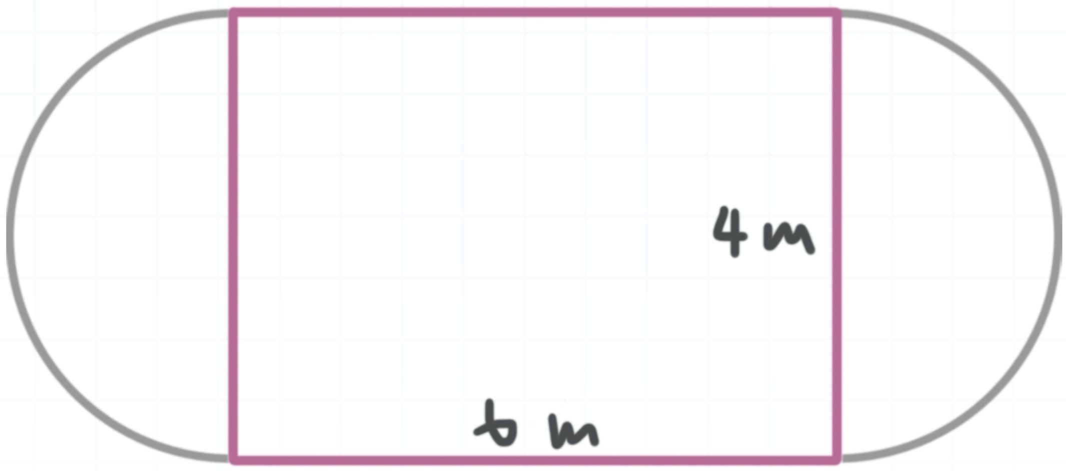
As a side note, 12π is the exact value of the answer, but if you wanted only an approximate value, you could use 3.14 as an approximation of π:

$$\text{area} \approx (12 \cdot 3.14) \text{ ft}^2$$

$$\text{area} \approx 37.68 \text{ ft}^2$$

Topic: Area of a circle

Question: The figure shows a 4 m by 6 m rectangle with a semicircle on each end. What is the area of the figure?



Answer choices:

A $(4\pi + 24)$ m^2

B 24π m^2

C $(16\pi + 24)$ m^2

D 28π m^2

Solution: A

You can see that the diameter of each semicircle is 4 m, so the radius of each is 2 m.

Two semicircles add up to one complete circle, so we need to add the area of one circle of radius 2 m and a rectangle with dimensions 4 m by 6 m.

For the circle:

$$A = \pi r^2 = \pi \cdot (2 \text{ m})^2 = 4\pi \text{ m}^2$$

For the rectangle:

$$A = lw = (4 \text{ m})(6 \text{ m}) = 24 \text{ m}^2$$

Therefore, the area of the given figure is $(4\pi + 24)$ m^2.

You could also find the approximate value using $\pi \approx 3.14$.

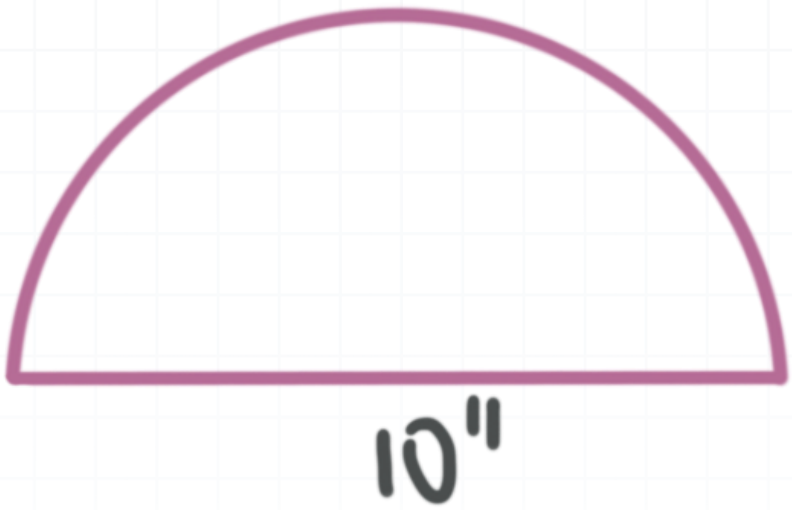
$$\text{area} \approx [(4 \cdot 3.14) + 24] \text{ m}^2$$

$$\text{area} \approx (12.56 + 24) \text{ m}^2$$

$$\text{area} \approx 36.56 \text{ m}^2$$

Topic: Circumference of a circle

Question: Find the length of the semicircle, assuming $\pi \approx 3.14$.



Answer choices:

A 15.7 in

B 16.7 in

C 14.7 in

D 31.4 in

Solution: A

Imagine drawing a full circle by combining two semicircles that are each of the same size as the one in the figure. Notice that the length of the line segment in the figure (10 in) is equal to the diameter of that circle. So the circumference of the circle is

$$C = \pi d$$

$$C \approx (3.14)(10 \text{ in})$$

$$C = 31.4 \text{ in}$$

Since we're looking for the length of just the semicircle, which is half the circumference of the full circle, we need to divide the circumference of the full circle by 2.

$$\text{length of semicircle} = \frac{C}{2} \approx \frac{31.4 \text{ in}}{2} \approx 15.7 \text{ in}$$

Topic: Circumference of a circle

Question: What is the circumference of a circle whose area is 12π?

Answer choices:

A $2\pi\sqrt{3}$

B 6π

C $4\pi\sqrt{3}$

D 8π

Solution: C

Since we've been given the area of the circle, we'll plug it into the formula for the area of a circle (which gives the area as a function of the radius) and then solve for the radius.

$$A = \pi r^2$$

$$12\pi = \pi r^2$$

Divide both sides by π and rearrange.

$$r^2 = 12$$

$$r = \sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$$

Using $r = 2\sqrt{3}$, calculate the circumference.

$$C = 2\pi r$$

$$C = (2\pi)(2\sqrt{3}) = 4\pi\sqrt{3}$$

Topic: Circumference of a circle

Question: A mountain bike wheel has a diameter of 26 inches. Someone rode the bike, and stopped once the wheel had made exactly 100 revolutions. Using $\pi \approx 3.14$, estimate how far the bike moved.

Answer choices:

A 680 ft

B $1{,}110$ ft

C $2{,}607$ ft

D $8{,}164$ ft

Solution: A

The formula for the circumference of a circle is $C = \pi d$. Plugging in the approximation $\pi \approx 3.14$ that we're told to use, and the value we're given for the diameter, we get

$$C = \pi(26 \text{ in})$$

$$C \approx 3.14(26 \text{ in})$$

$$C \approx 81.64$$

This tells us that one revolution of the wheel moved the bike 81.64 inches, which means that 100 revolutions moved the bike

$$100 \cdot 81.64 = 8{,}164 \text{ inches}$$

We'll convert that to feet to get

$$8{,}164 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 680.3 \text{ ft} \approx 680 \text{ ft}$$

Topic: Nets/volume/surface area of prisms

Question: Find the surface area of the rectangular box with length $10'$, width $4'$, and height $5'$.

Answer choices:

A 200 ft^2

B 220 ft

C 220 ft^2

D 180 ft^2

Solution: C

The surface area of a rectangular box will be the sum of the areas of its six sides:

Top and bottom	length $\times$ width
Left and right	width $\times$ height
Front and back	length $\times$ height

We'll use the formula

$$A = 2lw + 2wh + 2lh$$

Plugging in the dimensions of the box we've been given, we get

$$A = 2[(10 \text{ ft})(4 \text{ ft})] + 2[(4 \text{ ft})(5 \text{ ft})] + 2[(10 \text{ ft})(5 \text{ ft})]$$

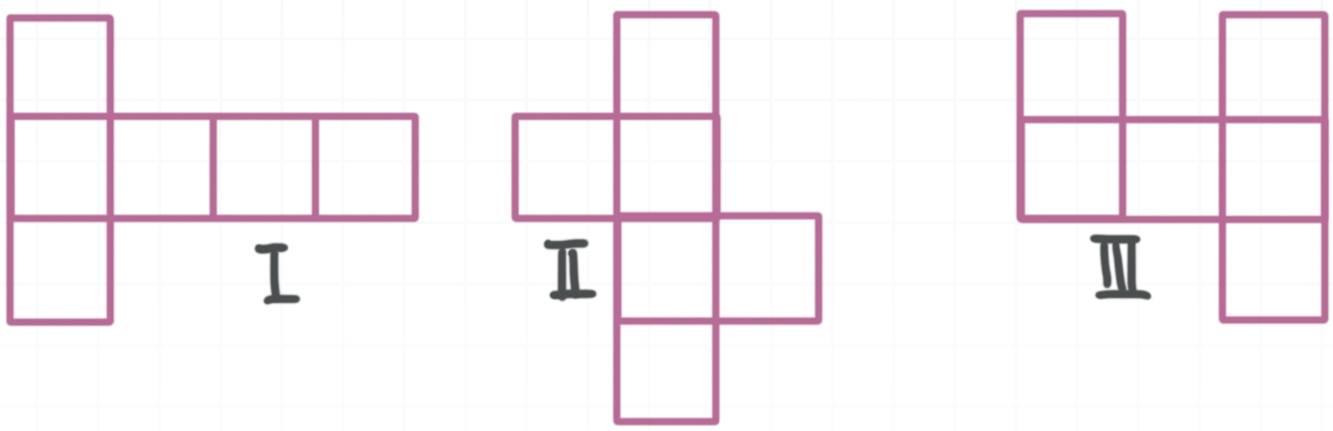
$$A = 2[(40 \text{ ft}^2)] + 2[(20 \text{ ft}^2)] + 2[(50 \text{ ft}^2)]$$

$$A = 80 \text{ ft}^2 + 40 \text{ ft}^2 + 100 \text{ ft}^2$$

$$A = 220 \text{ ft}^2$$

Topic: Nets/volume/surface area of prisms

Question: Which net would not form a cube (a rectangular prism where all the faces are squares)?



Answer choices:

A I

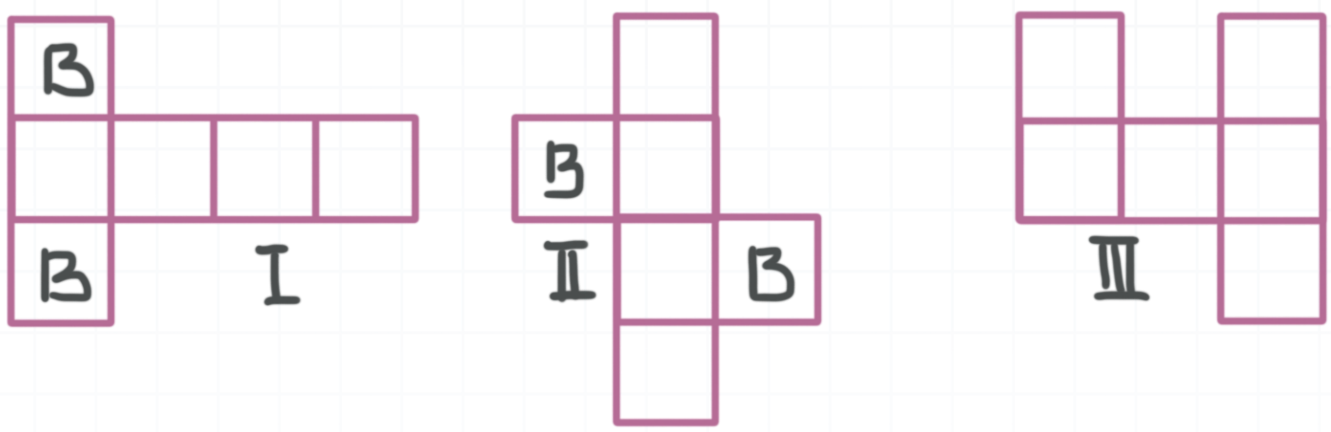
B II

C III

D Each of the nets would form a cube

Solution: C

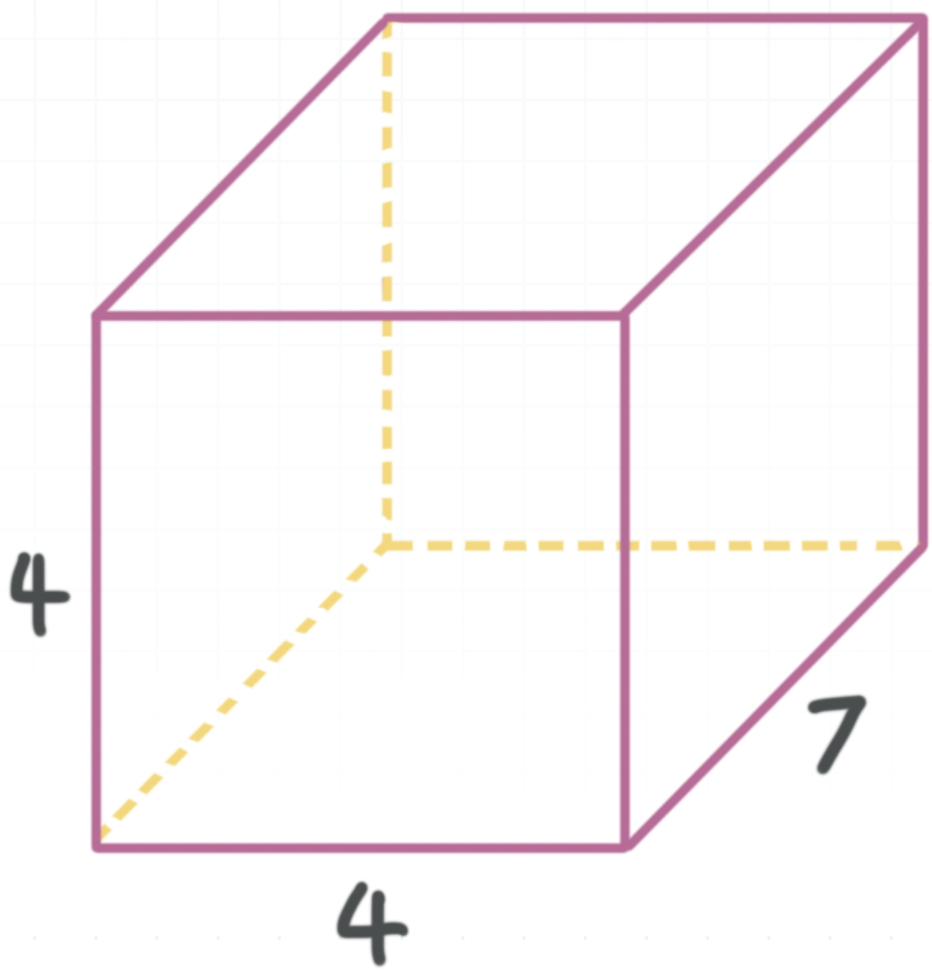
A cube has six faces. In I and II, the row of four squares can be folded to make four faces of the cube. Then the other two squares can be folded to make the two bases.



In III, there aren't four squares in a row, which makes it impossible to fold that net into a cube.

Topic: Nets/volume/surface area of prisms

Question: What is the surface area of the given right rectangular prism (a rectangular prism in which all the faces are rectangles)?



Answer choices:

A 88

B 112

C 144

D 160

Solution: C

The surface area formula is

$$A = 2lw + 2wh + 2lh$$

And we've been given the dimensions

$$h = 4,\ w = 4,\ l = 7$$

So plugging these into the surface area formula, we get

$$A = (2 \cdot 7 \cdot 4) + (2 \cdot 4 \cdot 4) + (2 \cdot 7 \cdot 4)$$

$$A = 56 + 32 + 56$$

$$A = 144$$

Topic: Surface area to volume ratio of prisms

Question: Find the surface area to volume ratio of a right rectangular prism that's 2 cm high, 3 cm wide, and 5 cm long.

Answer choices:

A 2.1 cm^{-1}

B 2.6 cm^{-1}

C 3.5 cm^{-1}

D 3.8 cm^{-1}

Solution: A

Plug the measurements we've been given into the surface area formula.

$$S = 2lw + 2wh + 2lh$$

$$S = 2(5\text{ cm} \cdot 3\text{ cm}) + 2(3\text{ cm} \cdot 2\text{ cm}) + 2(5\text{ cm} \cdot 2\text{ cm})$$

$$S = 2(15\text{ cm}^2) + 2(6\text{ cm}^2) + 2(10\text{ cm}^2)$$

$$S = 30\text{ cm}^2 + 12\text{ cm}^2 + 20\text{ cm}^2$$

$$S = 62\text{ cm}^2$$

Use $V = lwh$ to find the volume.

$$V = (5\text{ cm}) \cdot (3\text{ cm}) \cdot (2\text{ cm})$$

$$V = 30\text{ cm}^3$$

Now it's easy to find the ratio.

$$\frac{S}{V} = \frac{62\text{ cm}^2}{30\text{ cm}^2} = \frac{31}{15}\text{ cm}^{-1} \approx 2.1\text{ cm}^{-1}$$

Topic: Surface area to volume ratio of prisms

Question: Find the surface area to volume ratio of a cube with dimensions $1 \times 1 \times 1$. Then calculate the same ratio for a $2 \times 2 \times 2$ cube. When we double the length of each side, the surface area to volume ratio...

Answer choices:

A stays the same.

B doubles.

C quadruples.

D gets cut in half.

Solution: D

The surface area and volume of the $1 \times 1 \times 1$ cube are

$$S = 2lw + 2wh + 2lh$$

$$S = 2(1 \text{ cm} \cdot 1 \text{ cm}) + 2(1 \text{ cm} \cdot 1 \text{ cm}) + 2(1 \text{ cm} \cdot 1 \text{ cm})$$

$$S = 2(1 \text{ cm}^2) + 2(1 \text{ cm}^2) + 2(1 \text{ cm}^2)$$

$$S = 2 \text{ cm}^2 + 2 \text{ cm}^2 + 2 \text{ cm}^2$$

$$S = 6 \text{ cm}^2$$

and

$$V = lwh$$

$$V = 1 \text{ cm} \cdot 1 \text{ cm} \cdot 1 \text{ cm}$$

$$V = 1 \text{ cm}^3$$

So the surface area to volume ratio of the $1 \times 1 \times 1$ is

$$\frac{S}{V} = \frac{6 \text{ cm}^2}{1 \text{ cm}^3} = 6 \text{ cm}^{-1}$$

The surface area and volume of the $2 \times 2 \times 2$ cube are

$$S = 2lw + 2wh + 2lh$$

$$S = 2(2 \text{ cm} \cdot 2 \text{ cm}) + 2(2 \text{ cm} \cdot 2 \text{ cm}) + 2(2 \text{ cm} \cdot 2 \text{ cm})$$

$$S = 2(4 \text{ cm}^2) + 2(4 \text{ cm}^2) + 2(4 \text{ cm}^2)$$

$$S = 8\text{ cm}^2 + 8\text{ cm}^2 + 8\text{ cm}^2$$

$$S = 24\text{ cm}^2$$

and

$$V = lwh$$

$$V = 2\text{ cm} \cdot 2\text{ cm} \cdot 2\text{ cm}$$

$$V = 8\text{ cm}^3$$

So the surface area to volume ratio of the $2 \times 2 \times 2$ is

$$\frac{S}{V} = \frac{24\text{ cm}^2}{8\text{ cm}^3} = 3\text{ cm}^{-1}$$

Comparing these, we can see that the ratio has been cut in half, from 6 to 3.

Topic: Surface area to volume ratio of prisms

Question: Calculate the surface area to volume ratio of a right rectangular prism with dimensions $x \times x \times 2x$.

Answer choices:

A $\frac{5}{x}$

B $\frac{5}{1}$

C $\frac{4x}{1}$

D $\frac{1}{10x}$

Solution: A

Find the surface area.

$$S = 2lw + 2wh + 2lh$$

$$S = 2(2x \cdot x) + 2(x \cdot x) + 2(2x \cdot x)$$

$$S = 2(2x^2) + 2(x^2) + 2(2x^2)$$

$$S = 4x^2 + 2x^2 + 4x^2$$

$$S = 10x^2$$

Find the volume.

$$V = lwh$$

$$V = 2x \cdot x \cdot x$$

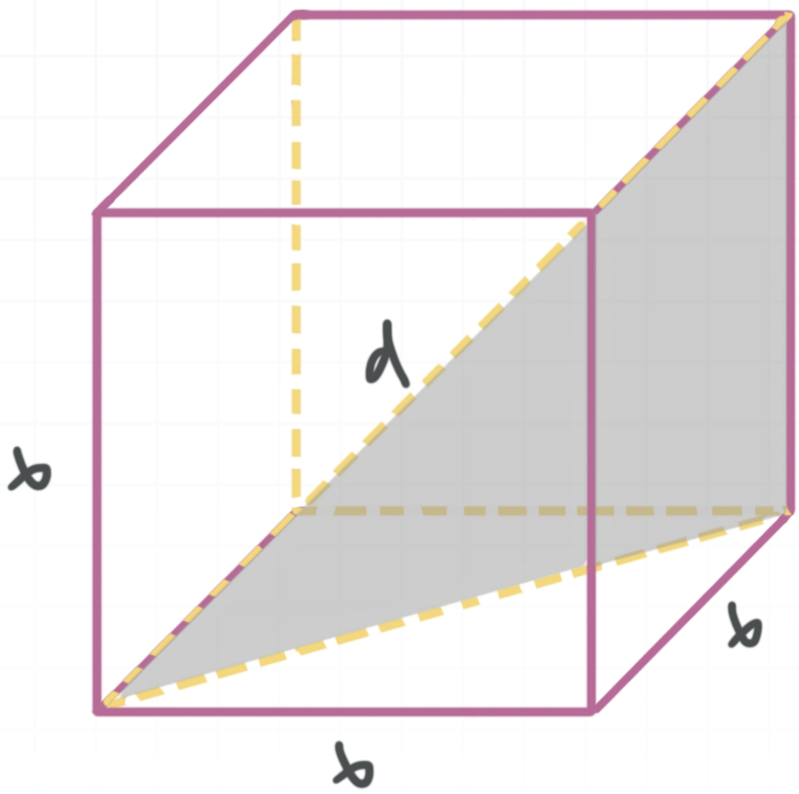
$$V = 2x^3$$

The ratio is of the surface area to volume is

$$\frac{S}{V} = \frac{10x^2}{2x^3} = \frac{5}{x}$$

Topic: Diagonal of a right rectangular prism

Question: What is the length of the diagonal of the cube?



Answer choices:

A 36

B $6\sqrt{2}$

C 108

D $6\sqrt{3}$

Solution: D

Plugging the dimensions we've been given into the formula for the diagonal, we get

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{6^2 + 6^2 + 6^2}$$

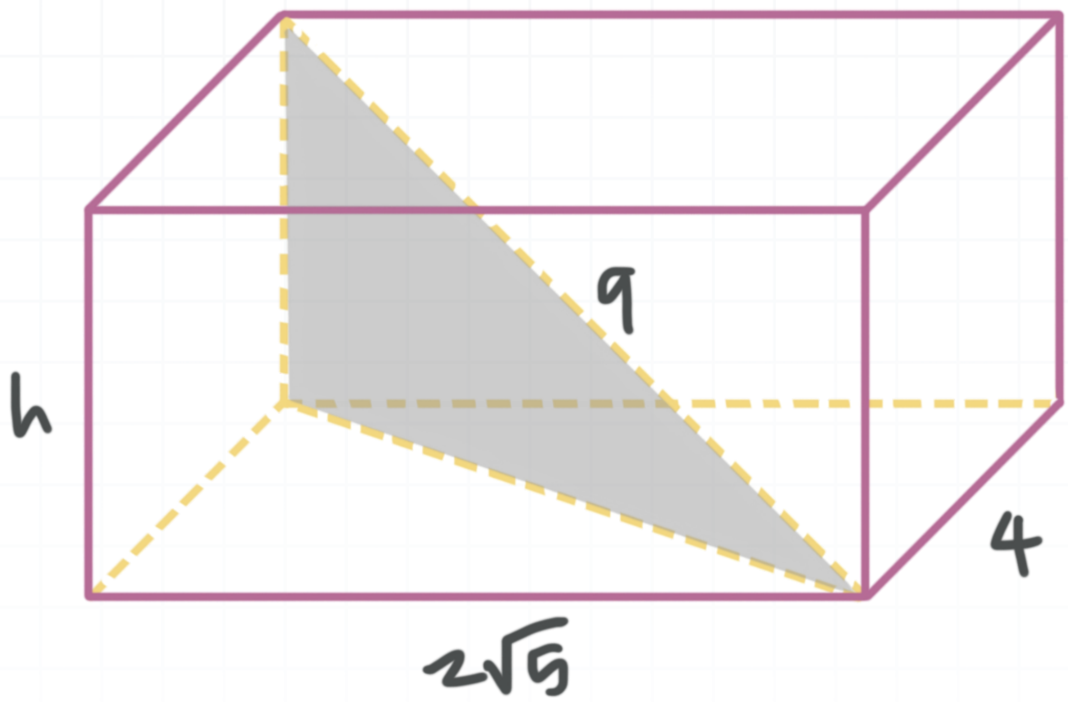
$$d = \sqrt{36 + 36 + 36}$$

$$d = \sqrt{36 \cdot 3}$$

$$d = 6\sqrt{3}$$

Topic: Diagonal of a right rectangular prism

Question: What is the height of the right rectangular prism?



Answer choices:

A $5\sqrt{2}$

B $3\sqrt{5}$

C $2\sqrt{6}$

D $6\sqrt{2}$

Solution: B

Plugging the dimensions we've been given into the formula for the diagonal, we get

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$9 = \sqrt{(2\sqrt{5})^2 + 4^2 + h^2}$$

$$9 = \sqrt{4(5) + 4^2 + h^2}$$

$$9 = \sqrt{20 + 16 + h^2}$$

$$9 = \sqrt{36 + h^2}$$

Square both sides.

$$81 = 36 + h^2$$

$$h^2 = 45$$

$$h^2 = 9 \cdot 5$$

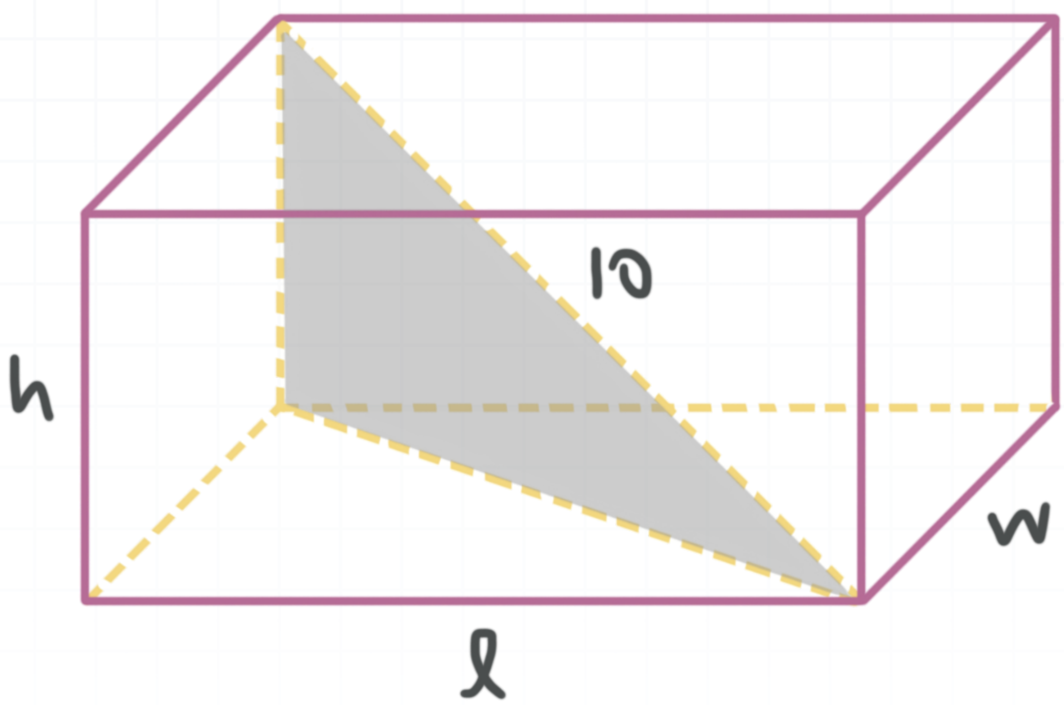
$$h = \pm 3\sqrt{5}$$

Ruling out the negative value for h leaves us with

$$h = 3\sqrt{5}$$

Topic: Diagonal of a right rectangular prism

Question: In this right rectangular prism with a diagonal of 10, which dimensions of $l \times w \times h$ could not be correct?



Answer choices:

A $l = 2\sqrt{6},\ w = 2\sqrt{10},\ h = 6$

B $l = 5\sqrt{2},\ w = 4\sqrt{2},\ h = 3\sqrt{2}$

C $l = 3\sqrt{5},\ w = 2\sqrt{6},\ h = 4\sqrt{2}$

D $l = 8,\ w = 4,\ h = 2\sqrt{5}$

Solution: C

We'll try each of the answer choices in the formula for the diagonal of a right rectangular prism,

$$d = \sqrt{l^2 + w^2 + h^2}$$

For answer choice A:

$$d = \sqrt{(2\sqrt{6})^2 + (2\sqrt{10})^2 + 6^2}$$

$$d = \sqrt{(4 \cdot 6) + (4 \cdot 10) + 36}$$

$$d = \sqrt{24 + 40 + 36}$$

$$d = \sqrt{100}$$

$$d = 10$$

For answer choice B:

$$d = \sqrt{(5\sqrt{2})^2 + (4\sqrt{2})^2 + (3\sqrt{2})^2}$$

$$d = \sqrt{(25 \cdot 2) + (16 \cdot 2) + (9 \cdot 2)}$$

$$d = \sqrt{50 + 32 + 18}$$

$$d = \sqrt{100}$$

$$d = 10$$

For answer choice C:

$$d = \sqrt{(3\sqrt{5})^2 + (2\sqrt{6})^2 + (4\sqrt{2})^2}$$

$$d = \sqrt{(9 \cdot 5) + (4 \cdot 6) + (16 \cdot 2)}$$

$$d = \sqrt{45 + 24 + 32}$$

$$d = \sqrt{101}$$

For answer choice D:

$$d = \sqrt{8^2 + 4^2 + (2\sqrt{5})^2}$$

$$d = \sqrt{64 + 16 + (4 \cdot 5)}$$

$$d = \sqrt{64 + 16 + 20}$$

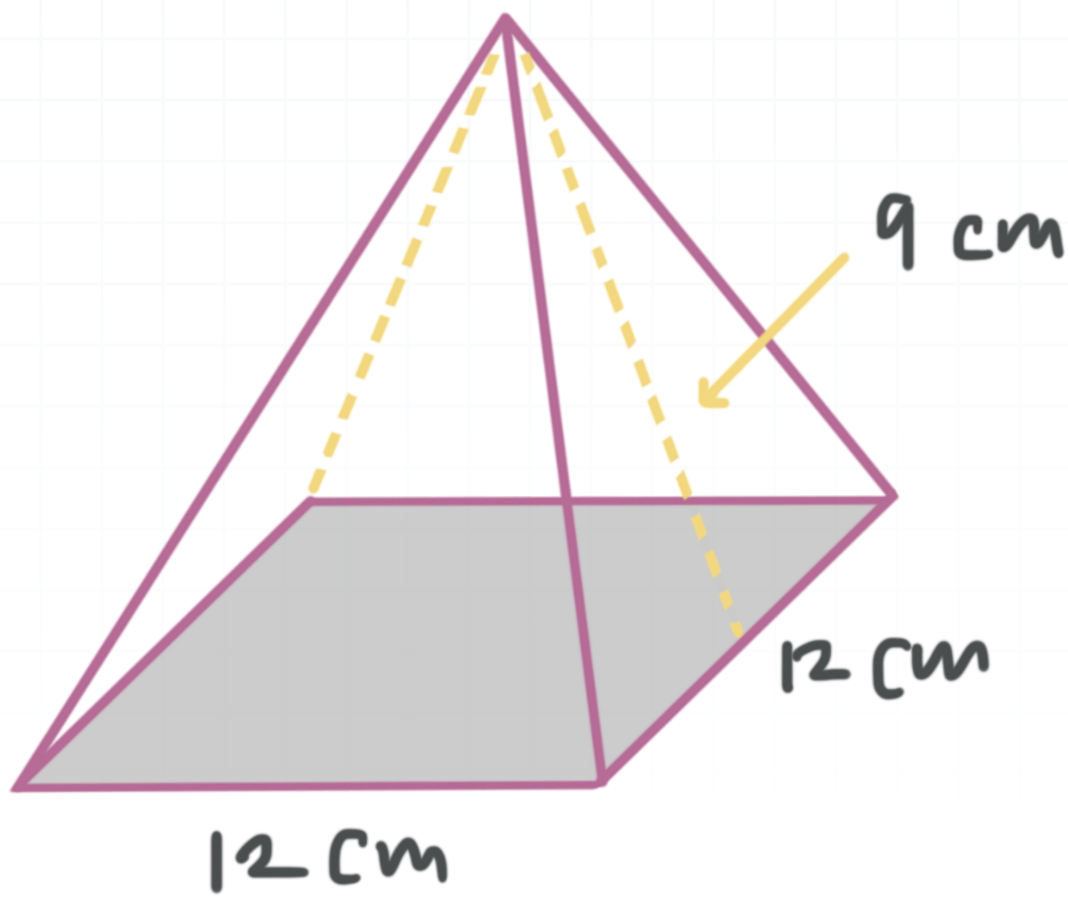
$$d = \sqrt{100}$$

$$d = 10$$

Answer choice C is the only answer choice that doesn't come out to 10 as the value for the diagonal.

Topic: Nets/volume/surface area of pyramids

Question: What is the surface area of a square pyramid with a base of 12 cm by 12 cm and a lateral height of 9 cm?



Answer choices:

A 216 cm^2

B 360 cm^2

C 442 cm^2

D 576 cm^2

Solution: B

Plugging the dimensions we've been given into the formula for the surface area of a pyramid, we get

$$S = \frac{1}{2}lp + B$$

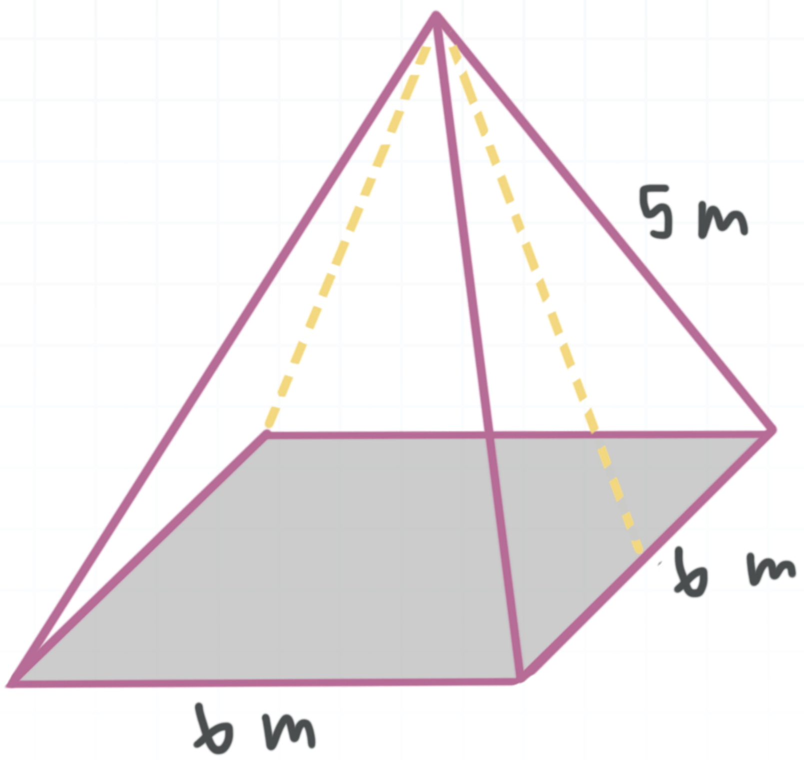
$$S = \frac{1}{2}(9 \text{ cm})(4 \cdot 12 \text{ cm}) + (12 \text{ cm})^2$$

$$S = 216 \text{ cm}^2 + 144 \text{ cm}^2$$

$$S = 360 \text{ cm}^2$$

Topic: Nets/volume/surface area of pyramids

Question: What is the surface area of a square pyramid with a base of 6 m by 6 m if the length of an edge of the pyramid is 5 m?



Answer choices:

A 84 m^2

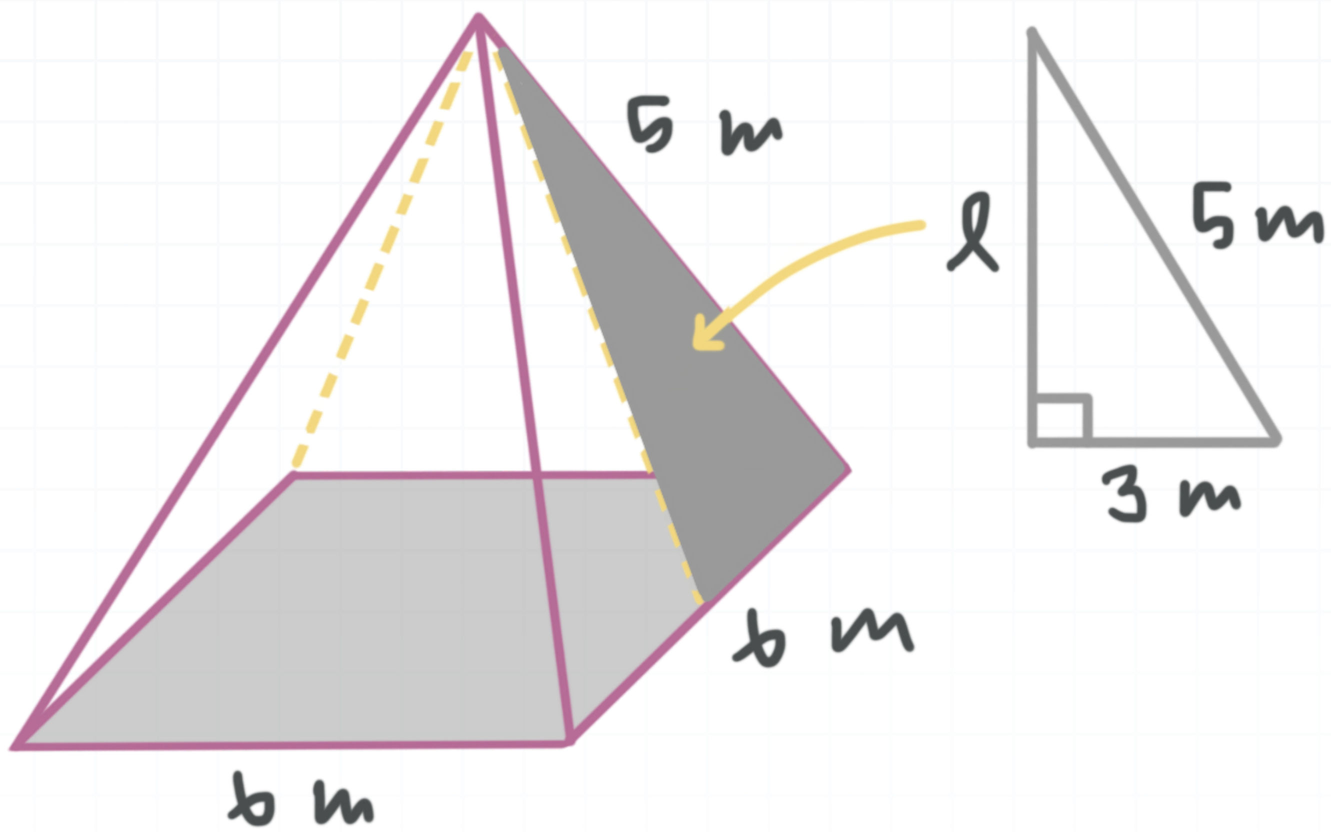
B 96 m^2

C 114 m^2

D 132 m^2

Solution: A

First, find l by using the shaded triangle,



with the Pythagorean theorem.

$$(3 \text{ m})^2 + l^2 = (5 \text{ m})^2$$

$$9 \text{ m}^2 + l^2 = 25 \text{ m}^2$$

$$l^2 = 16 \text{ m}^2$$

$$l = 4 \text{ m}$$

Now use the formula for surface area of a pyramid.

$$S = \frac{1}{2}lp + B$$

$$S = \frac{1}{2}(4 \text{ m})(4 \cdot 6 \text{ m}) + (6 \text{ m})^2$$

$$S = \frac{1}{2}(4 \text{ m})(24 \text{ m}) + (6 \text{ m})^2$$

$$S = 48 \text{ m}^2 + 36 \text{ m}^2$$

$$S = 84 \text{ m}^2$$

Topic: Nets/volume/surface area of pyramids

Question: A rectangular pyramid has a height of 6, and a base with length 8 and unknown width. If the pyramid's volume is 32, what is the width of the base?

Answer choices:

A 2

B 3

C 4

D 5

Solution: A

Because $B = lw$, where l and w are the length and width of the base, we can rewrite the volume formula

$$V = \frac{1}{3}Bh$$

as

$$V = \frac{1}{3}lwh$$

Substituting for V, l, and h, we get

$$32 = \frac{1}{3} \cdot 8 \cdot w \cdot 6$$

$$32 = 16w$$

$$w = 2$$

Topic: Nets/volume/surface area of cylinders

Question: Find the volume of a right circular cylinder with base radius $10'$ and height $10'$, assuming $\pi \approx 3.14$.

Answer choices:

A $3{,}140$ ft^3

B 314 ft^3

C 3.14 ft^2

D $1{,}413$ ft^3

Solution: A

Plugging the dimensions of the cylinder we've been given into the formula for the volume of a cylinder, we get

$$V = \pi r^2 h$$

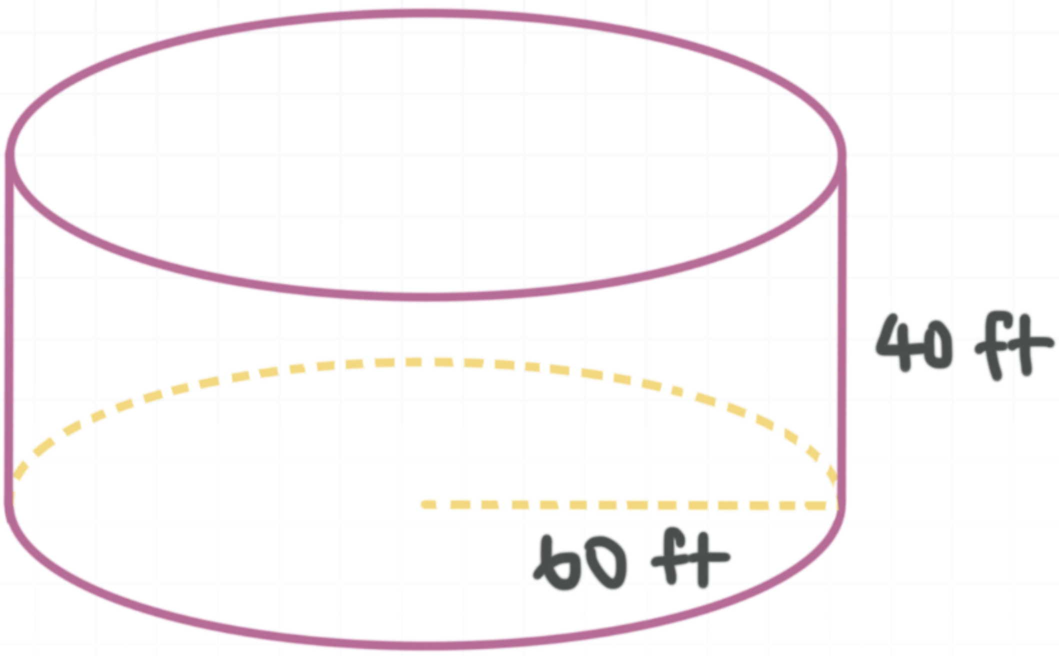
$$V \approx 3.14(10 \text{ ft})^2(10 \text{ ft})$$

$$V \approx 3.14(100 \text{ ft}^2)(10 \text{ ft})$$

$$V = 3{,}140 \text{ ft}^3$$

Topic: Nets/volume/surface area of cylinders

Question: How many square feet of steel will be needed to build a cylindrical steel tank with a height of 40 ft and a radius of 60 ft.



Answer choices:

A $\quad 8{,}000\pi$ ft^2

B $\quad 12{,}000\pi$ ft^2

C $\quad 18{,}000\pi$ ft^2

D $\quad 24{,}000\pi$ ft^2

Solution: B

Substituting the values we've been given into the formula for the surface area of a cylinder, we get

$$S = 2\pi rh + 2\pi r^2$$

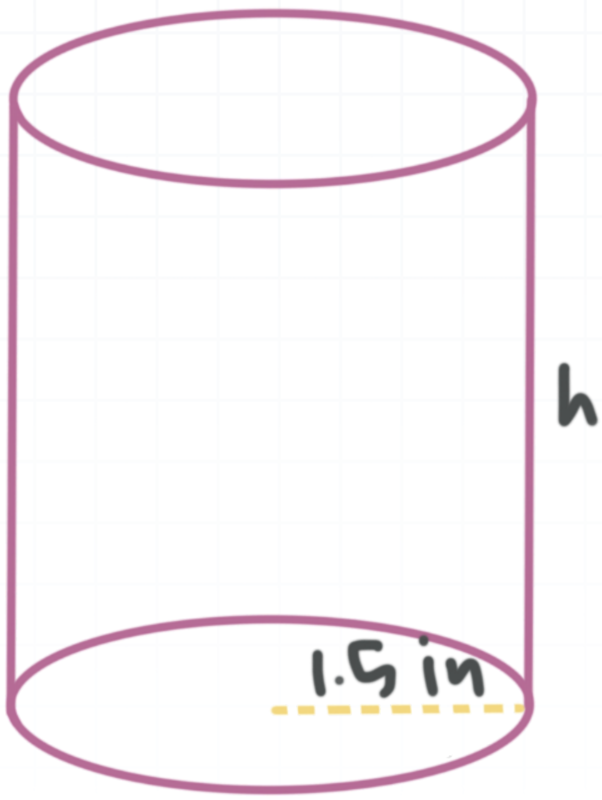
$$S = 2\pi(60 \text{ ft})(40 \text{ ft}) + 2\pi(60 \text{ ft})^2$$

$$S = 4{,}800\pi \text{ ft}^2 + 7{,}200\pi \text{ ft}^2$$

$$S = 12{,}000\pi \text{ ft}^2$$

Topic: Nets/volume/surface area of cylinders

Question: A cylindrical soup can has a surface area of 16.5π in^2 and a radius of 1.5 in. What is the height of the can?



Answer choices:

A 3 in

B 4 in

C 5 in

D 6 in

Solution: B

Plugging what we know into the formula for the surface area of a cylinder, we get

$$S = 2\pi rh + 2\pi r^2$$

$$16.5\pi \text{ in}^2 = 2\pi(1.5 \text{ in})(h) + 2\pi(1.5 \text{ in})^2$$

$$16.5\pi \text{ in}^2 = (3\pi \text{ in})(h) + 4.5\pi \text{ in}^2$$

$$12\pi \text{ in}^2 = (3\pi \text{ in})(h)$$

$$4 \text{ in} = h$$

Topic: Nets/volume/surface area of cones

Question: A round guard tower has a cone-shaped roof with a radius of 12 ft and a height (not a slant height) of 16 ft. Approximately how many square feet of roof cover the tower?

Answer choices:

A 240 ft^2

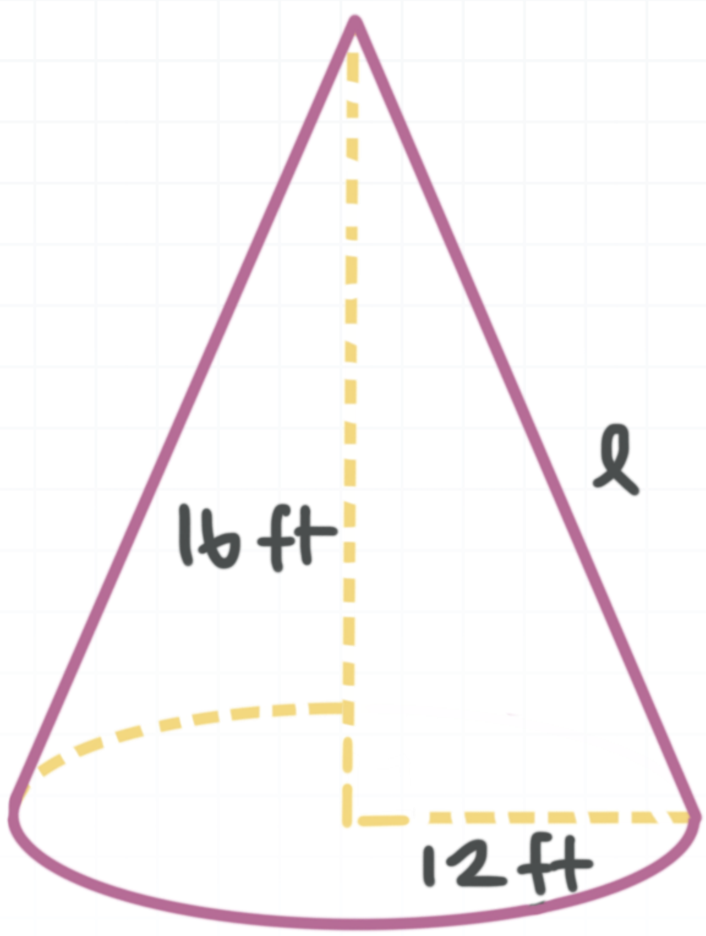
B 406 ft^2

C 639 ft^2

D 754 ft^2

Solution: D

The area of the roof is the lateral area of a cone.



To find it, we need to first get the slant height using $r^2 + h^2 = l^2$.

$$(12\text{ ft})^2 + (16\text{ ft})^2 = l^2$$

$$144\text{ ft}^2 + 256\text{ ft}^2 = l^2$$

$$400\text{ ft}^2 = l^2$$

$$l = 20\text{ ft}$$

Then the lateral surface area of the cone, and therefore the surface area of the roof, is given by

$$L = \pi r l$$

$$L \approx 3.14(12 \text{ ft})(20 \text{ ft})$$

$$L \approx 753.60 \text{ ft}^2$$

$$L \approx 754 \text{ ft}^2$$

Topic: Nets/volume/surface area of cones

Question: An ice cream cone has a diameter of 3 in and a height of 6 in. When it's filled level with the top, what volume of ice cream will it hold?

Answer choices:

A 4.5π in^3

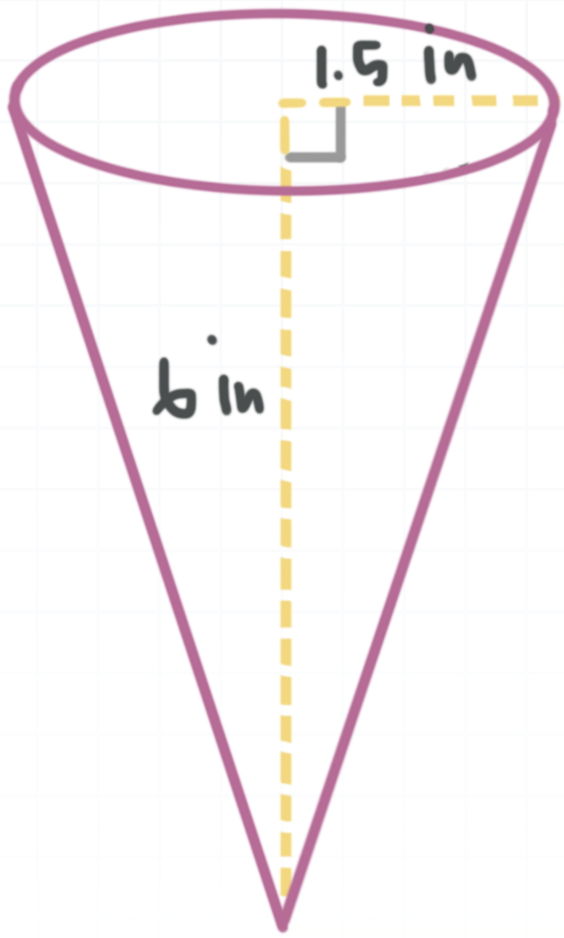
B 9.25π in^3

C 13.5π in^3

D 37.5π in^3

Solution: A

A sketch of the figure is



Plugging what we know into the formula for volume of a cone gives

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(1.5 \text{ in})^2(6 \text{ in})$$

$$V = 4.5\pi \text{ in}^3$$

Topic: Nets/volume/surface area of cones

Question: Two identical cones are glued together base-to-base to make a single double-cone solid. The diameter of the base of one cone is 10, and the length of the solid (the distance from the apex of one cone to the apex of the other cone) is 24. What is the surface area to volume ratio of the solid?

Answer choices:

A $\frac{7}{10}$

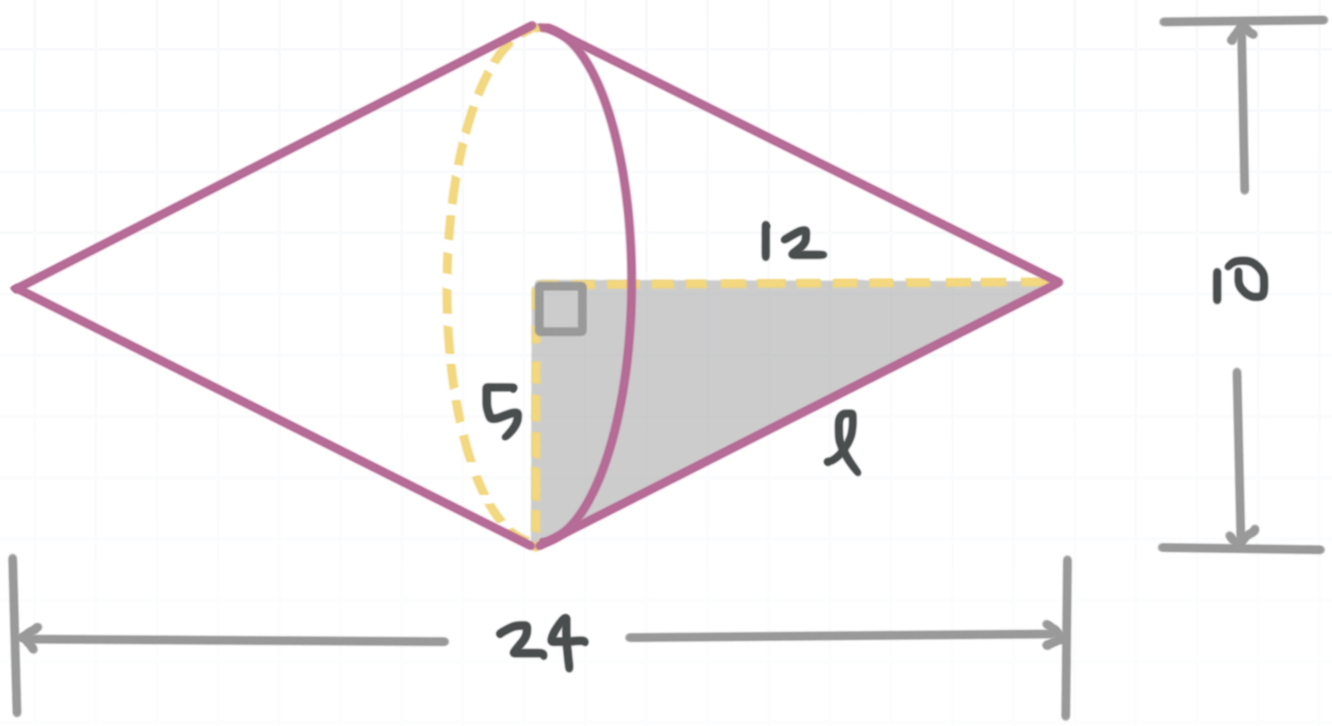
B $\frac{5}{6}$

C $\frac{5}{12}$

D $\frac{13}{20}$

Solution: D

The surface area of the solid is twice the lateral area of one cone, and the volume of the solid is twice the volume of one cone.



First, use $r^2 + h^2 = l^2$ to find l. Notice that h is half the length of the solid, so $h = 24/2 = 12$. Also, r is half the diameter of one cone, so $r = 10/2 = 5$.

$$5^2 + 12^2 = l^2$$

$$169 = l^2$$

$$13 = l$$

Second, find the lateral area of one cone.

$$L = \pi rl = \pi(5)(13) = 65\pi$$

Twice the lateral area of one cone is $2 \cdot 65\pi = 130\pi$.

Third, find the volume of one cone.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(5^2)(12)$$

$$V = 100\pi$$

Twice the volume of one cone is $2 \cdot 100\pi = 200\pi$.

Therefore, the surface area to volume ratio is

$$\frac{S}{V} = \frac{130\pi}{200\pi} = \frac{13}{20}$$

Topic: Volume/surface area of spheres

Question: What is the surface area of a sphere with a diameter of π cm?

Answer choices:

A $\frac{\pi^2}{2}$ cm^2

B π^3 cm^2

C $2\pi^3$ cm^2

D $\frac{\pi^3}{4}$ cm^2

Solution: B

If the diameter is π cm, then the radius is

$$r = \frac{d}{2} = \frac{\pi \text{ cm}}{2} = \frac{\pi}{2} \text{ cm}$$

Use the formula for surface area of a sphere, and plug in the radius.

$$S = 4\pi r^2$$

$$S = 4\pi \left(\frac{\pi}{2} \text{ cm} \right)^2$$

$$S = 4\pi \left(\frac{\pi^2}{4} \text{ cm}^2 \right)$$

$$S = \pi^3 \text{ cm}^2$$

Topic: Volume/surface area of spheres

Question: Find the surface area to volume ratio of a sphere with radius 6.

Answer choices:

A $\frac{1}{1}$

B $\frac{2}{1}$

C $\frac{1}{2}$

D $\frac{1}{4}$

Solution: C

The surface area of a sphere is given by

$$S = 4\pi r^2$$

Plugging in the radius gives

$$S = 4\pi(6)^2$$

$$S = 144\pi$$

The volume of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$

Plugging in the radius gives

$$V = \frac{4}{3}\pi(6)^3$$

$$V = \frac{4}{3}\pi(216)$$

$$V = 288\pi$$

Therefore, the surface area to volume ratio is

$$\frac{144\pi}{288\pi} = \frac{1}{2}$$

Topic: Volume/surface area of spheres

Question: A water tower in the shape of a sphere has a diameter of 30 ft. Assuming π ft$^3 \approx 24$ gallons, how many gallons of water does it hold?

Answer choices:

A $54{,}000$ gallons

B $108{,}000$ gallons

C $205{,}000$ gallons

D $234{,}000$ gallons

Solution: B

Because the diameter is 30 ft, the radius of the sphere is 15 ft. Plug the radius into the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi(15 \text{ ft})^3$$

$$V = \frac{4}{3}\pi(3{,}375 \text{ ft}^3)$$

$$V = 4{,}500\pi \text{ ft}^3$$

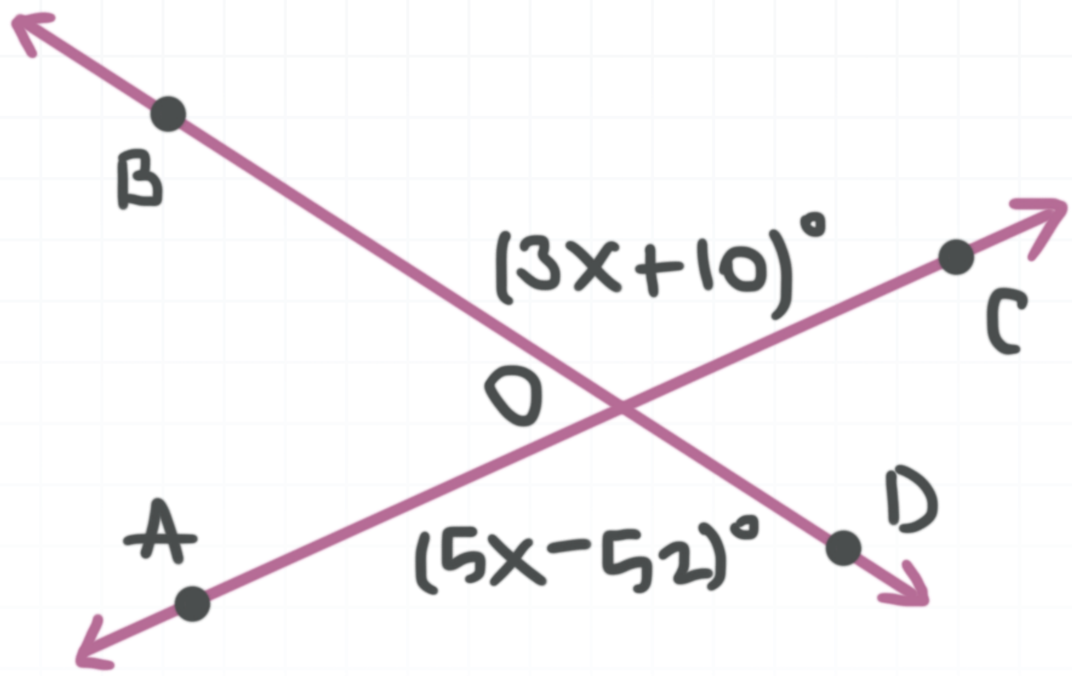
Using $\pi \text{ ft}^3 \approx 24$ gallons, we get

$$V \approx 4{,}500(24 \text{ gallons})$$

$$V = 108{,}000 \text{ gallons}$$

Topic: Congruent angles

Question: What is the measure of $\angle BOA$?



Answer choices:

A 31°

B 54°

C 77°

D 82°

Solution: C

Vertical angles are congruent, so

$$5x - 52 = 3x + 10$$

$$2x = 62$$

$$x = 31$$

Therefore,

$$m\angle COB = (3x + 10)^\circ = (3(31) + 10)^\circ = 103^\circ$$

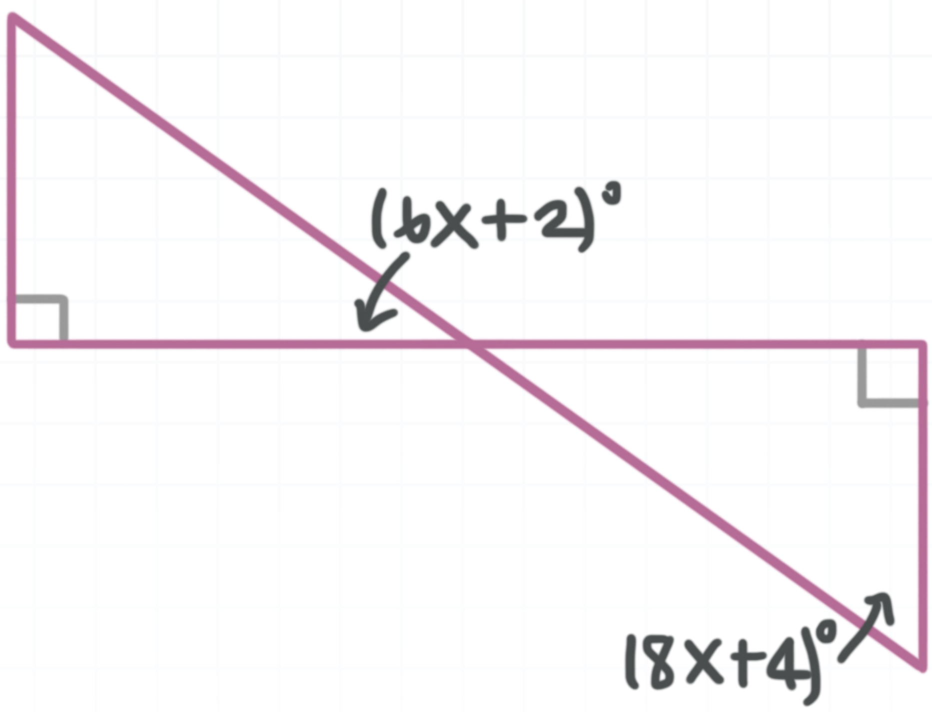
The measures of $\angle BOA$ and $\angle COB$ add up to 180°, so

$$m\angle BOA + 103^\circ = 180^\circ$$

$$m\angle BOA = 77^\circ$$

Topic: Congruent angles

Question: Solve for x.



Answer choices:

A -1

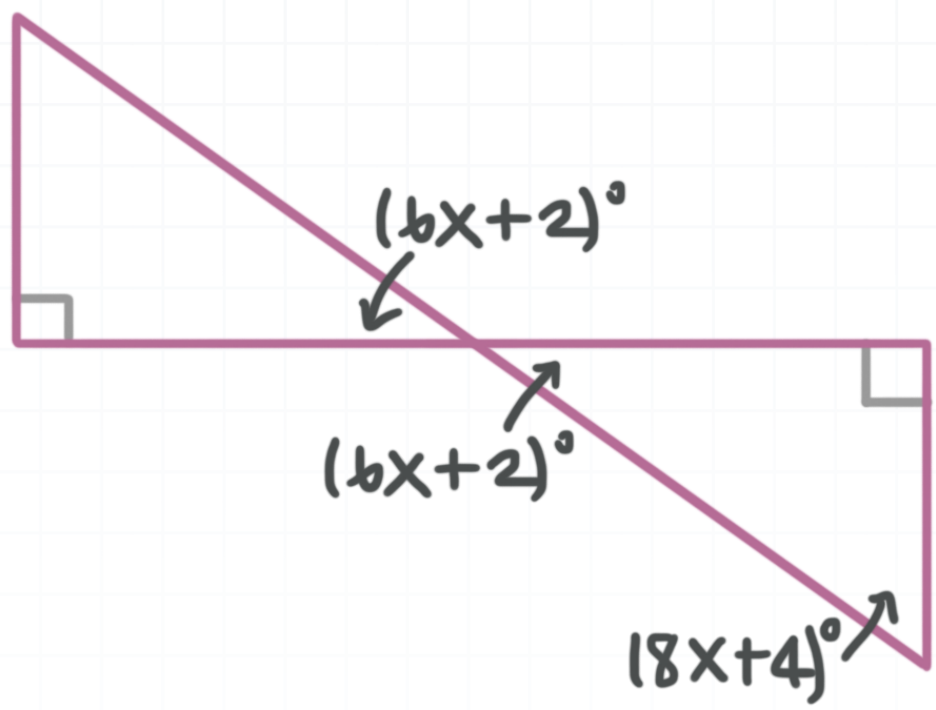
B 6

C 11

D 14

Solution: B

Vertical angles are congruent, so the angle opposite the one of measure $(6x+2)^\circ$ also has measure $(6x+2)^\circ$.



The sum of the measures of the interior angles of the triangle on the right must be 180°. Therefore,

$$(6x+2)+(8x+4)+90=180$$

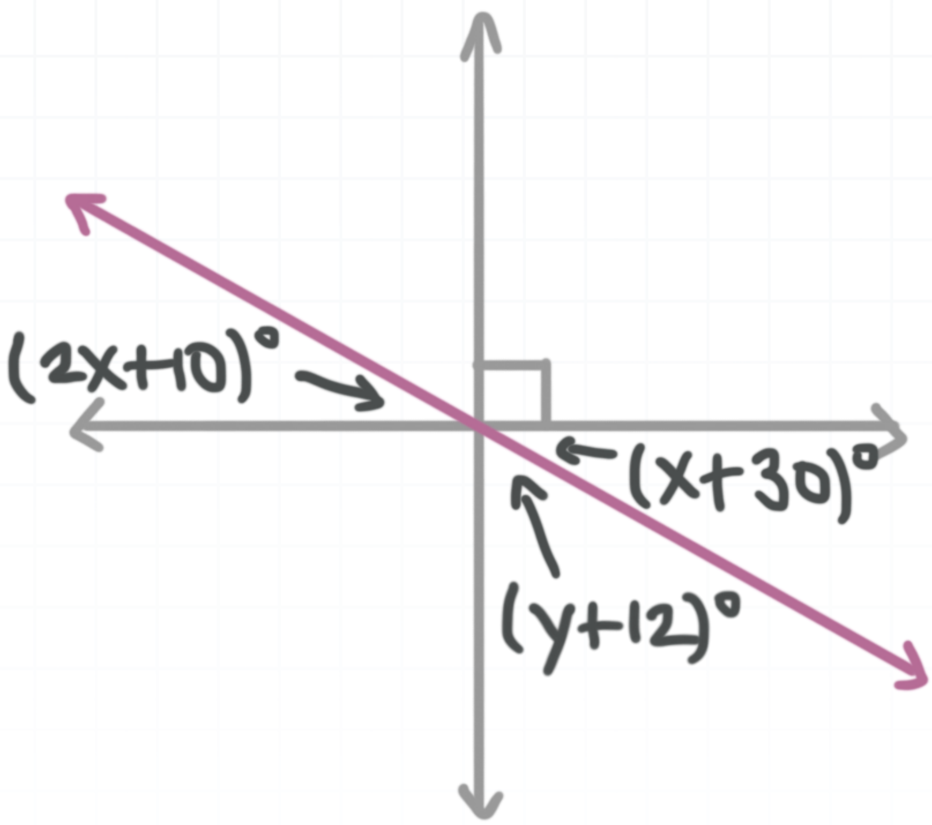
$$14x+96=180$$

$$14x=84$$

$$x=6$$

Topic: Congruent angles

Question: Solve for y.



Answer choices:

A 20

B 28

C 38

D 40

Solution: B

Vertical angles are congruent, so

$$2x + 10 = x + 30$$

$$x = 20$$

Taken together, the angles of measure $(x + 30)^\circ$ and $(y + 12)^\circ$ form a right angle, so

$$(x + 30) + (y + 12) = 90$$

We can now substitute 20 for x and solve for y.

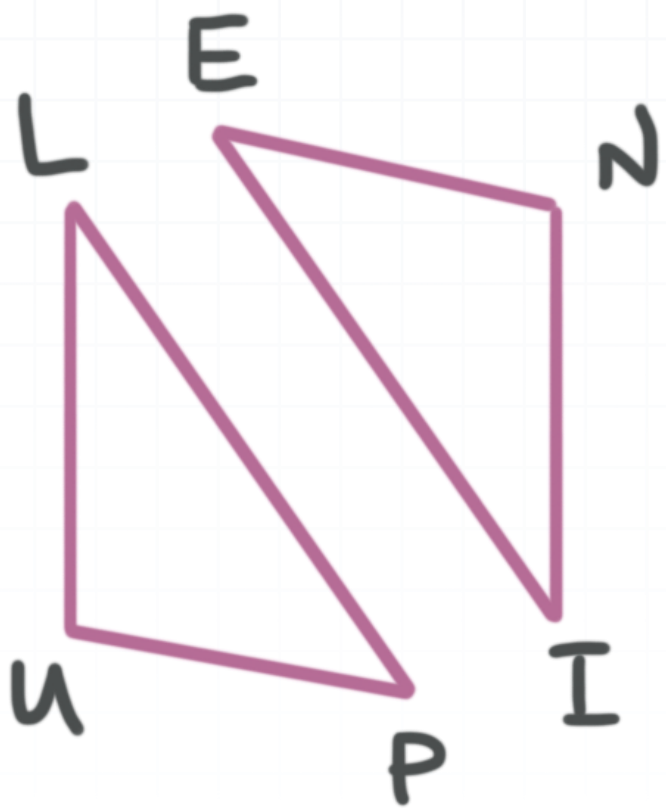
$$(20 + 30) + (y + 12) = 90$$

$$62 + y = 90$$

$$y = 28$$

Topic: Triangle congruence with SSS, ASA, SAS

Question: Given triangles $\triangle LUP$ and $\triangle INE$, and $\overline{LU} \cong \overline{IN}$, $\angle P \cong \angle E$, and $\overline{LP} \cong \overline{IE}$, which theorem could you use to prove the triangles congruent?



Answer choices:

A SSS

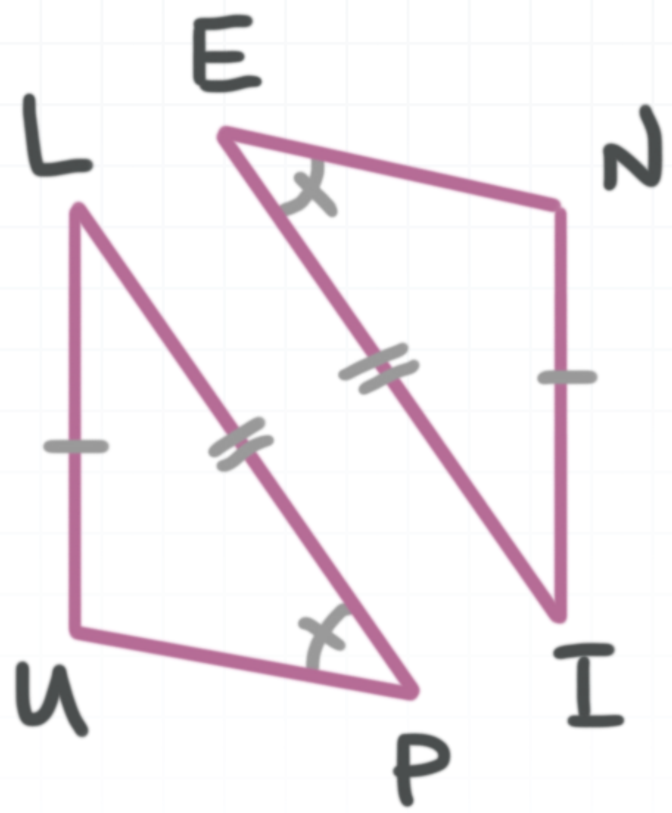
B SAS

C ASA

D None of these

Solution: D

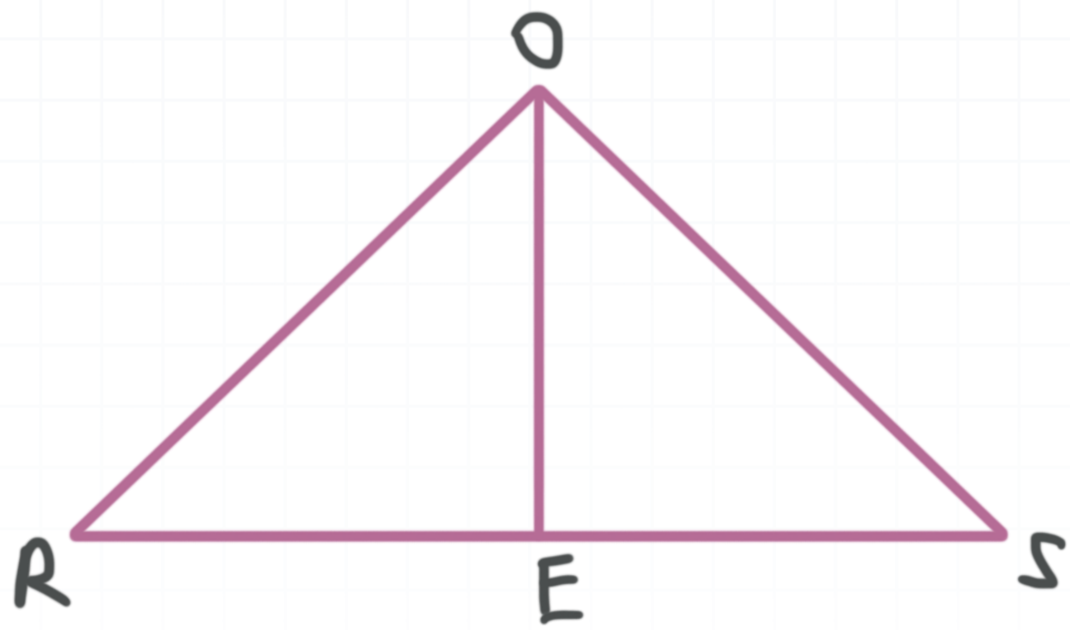
Because only two pairs of sides and one pair of angles are given as congruent, you can rule out SSS and ASA.



Because $\angle P$ isn't the included angle of sides $\overline{LU}$ and $\overline{LP}$ in $\triangle LUP$, you can rule out SAS. If you look at this from the standpoint of $\triangle INE$, you would arrive at the same conclusion, because $\angle E$ isn't the included angle of sides $\overline{IN}$ and $\overline{IE}$.

Topic: Triangle congruence with SSS, ASA, SAS

Question: Given $\triangle ROE$ and $\triangle SOE$, and $\overline{OE} \perp \overline{RS}$ and $\angle ROE \cong \angle EOS$, which theorem could you use to prove the triangles congruent?



Answer choices:

A SSS

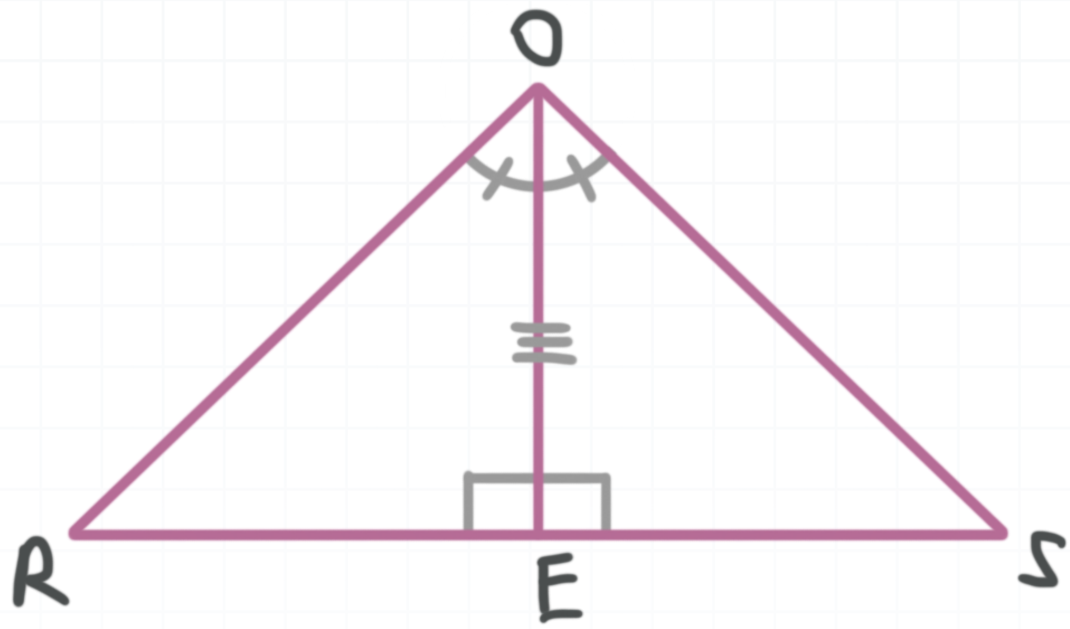
B SAS

C ASA

D None of these

Solution: C

Fill in the diagram with the given information.



Then from the figure, we can say

A: $\angle ROE \cong \angle EOS$

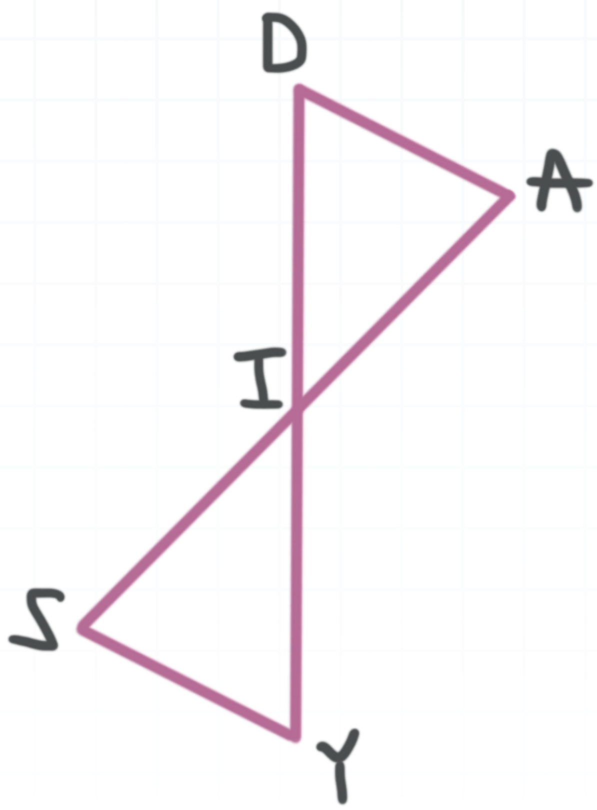
S: $\overline{OE} \cong \overline{OE}$ by the reflexive property

A: $\overline{OE} \perp \overline{RS}$ so $\angle OER \cong \angle SEO$ because right angles are congruent.

This makes the triangles congruent by ASA: $\overline{OE}$ is the included side of $\angle ROE$ and $\angle OER$ in $\triangle ROE$, and $\overline{OE}$ is also the included side of $\angle EOS$ and $\angle SEO$ in $\triangle SOE$.

Topic: Triangle congruence with SSS, ASA, SAS

Question: Given $\triangle DAI$ and $\triangle YSI$, and $\overline{AI} \cong \overline{SI}$ and $\overline{DI} \cong \overline{YI}$, which theorem could you use to prove the triangles congruent?



Answer choices:

A SSS

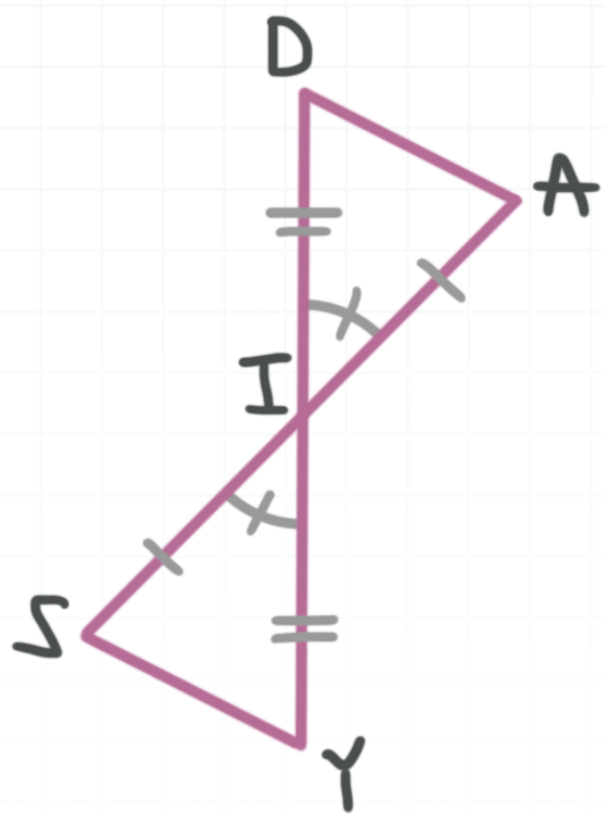
B SAS

C ASA

D None of these

Solution: B

Fill in the diagram with the given information.



Then from the figure, we can say

S: $\overline{AI} \cong \overline{SI}$

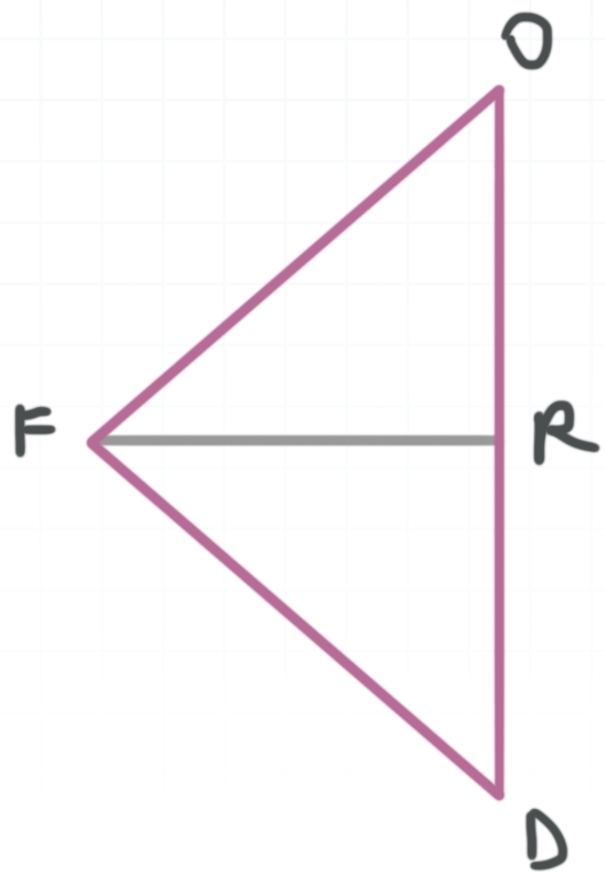
A: $\angle AID \cong \angle SIY$ because vertical angles are congruent.

S: $\overline{DI} \cong \overline{YI}$

This makes the triangles congruent by SAS: The included angle of sides $\overline{AI}$ and $\overline{DI}$ in $\triangle DAI$ is $\angle AID$, and the included angle of sides $\overline{SI}$ and $\overline{YI}$ in $\triangle YSI$ is $\angle SIY$.

Topic: Triangle congruence with AAS, HL

Question: Given $\triangle FOR$ and $\triangle FDR$, and $\overline{FO} \cong \overline{FD}$ and $\overline{FR} \perp \overline{OD}$, which theorem would be used to prove the triangles congruent?



Answer choices:

A SAS

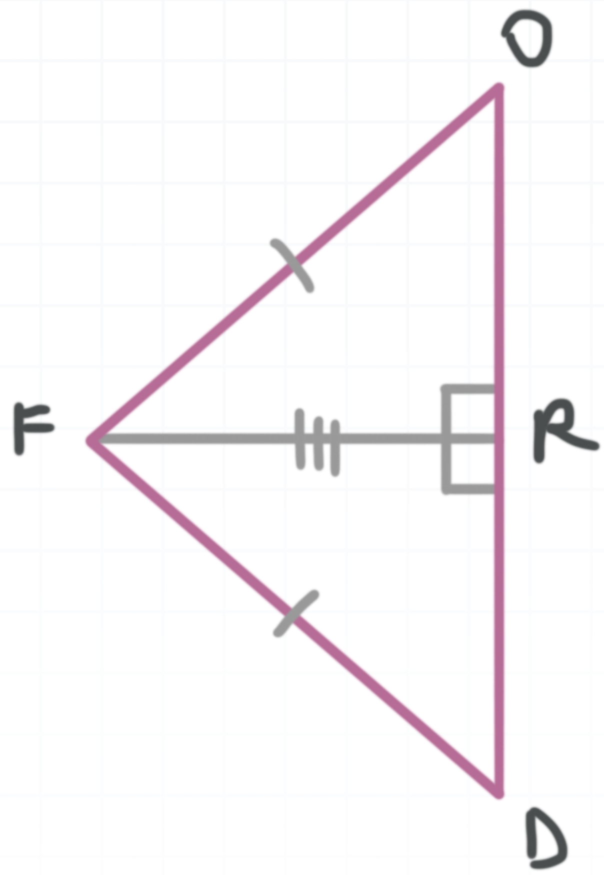
B AAS

C HL

D None of these

Solution: C

Add the information we've been given into the figure.



$\overline{FR} \perp \overline{OD}$, so $\angle ORF$ and $\angle FRD$ are right angles. It follows that $\triangle FOR$ and $\triangle FDR$ are right triangles.

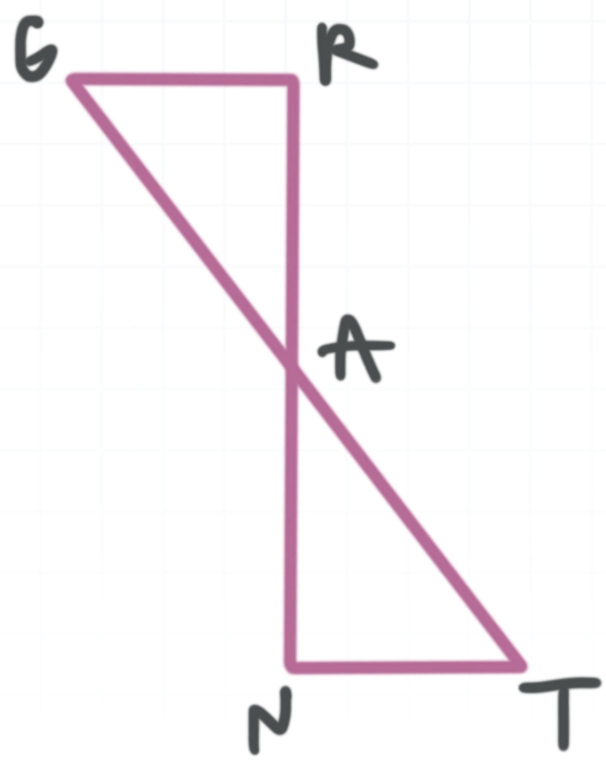
Hypotenuse: $\overline{FO} \cong \overline{FD}$ because this is given information.

Leg: $\overline{FR} \cong \overline{FR}$ by the reflexive property.

This makes the triangles congruent by HL.

Topic: Triangle congruence with AAS, HL

Question: Given $\triangle GRA$ and $\triangle TNA$, $\overline{GR} \cong \overline{TN}$, and the fact that $\angle GRA$ and $\angle TNA$ are right angles, which theorem would be used to prove the triangles congruent?



Answer choices:

A SAS

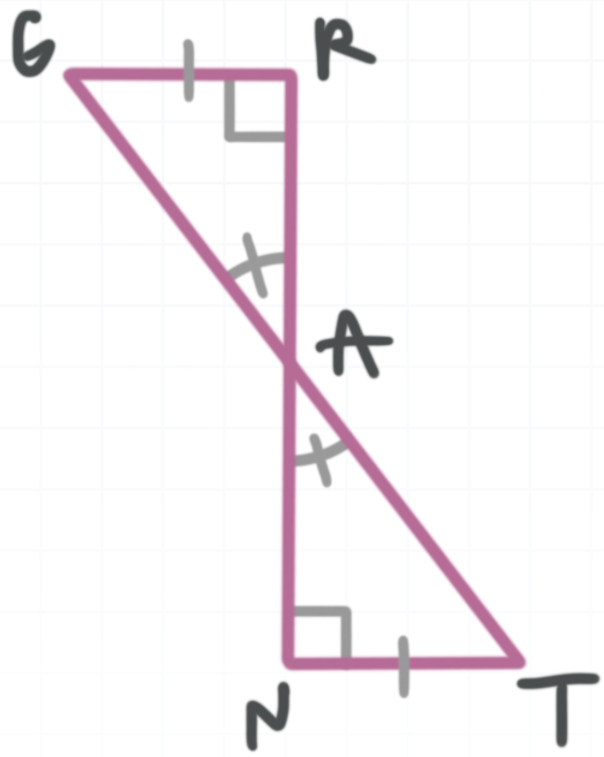
B AAS

C HL

D None of these

Solution: B

Fill out the figure with the given information.



A: $\angle GRA$ and $\angle TNA$ are right angles, so $\angle GRA \cong \angle TNA$.

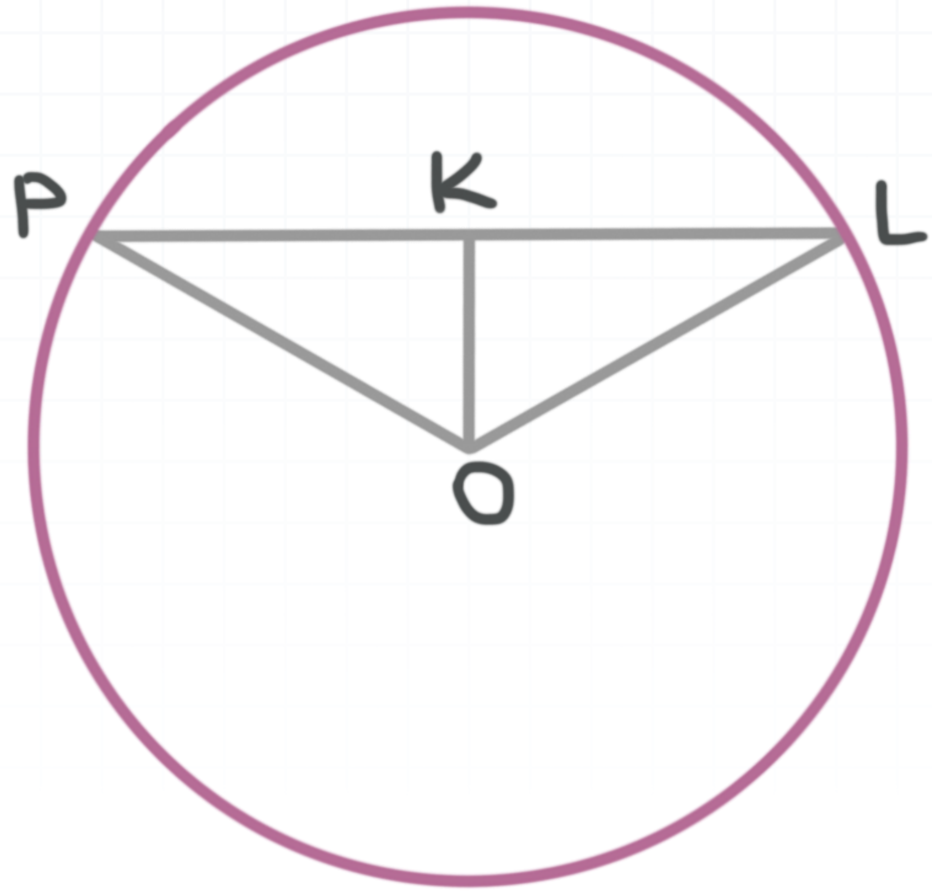
A: $\angle RAG \cong \angle NAT$ because vertical angles are congruent.

S: $\overline{GR} \cong \overline{TN}$ because this is given information.

This makes the triangles congruent by AAS: In $\triangle GRA$, $\overline{GR}$ is the side opposite $\angle RAG$ (an angle in one of the pairs of congruent angles, namely the pair $\angle RAG$ and $\angle NAT$); and in $\triangle TNA$, $\overline{TN}$ is the side opposite $\angle NAT$ (the other angle in that congruent pair).

Topic: Triangle congruence with AAS, HL

Question: In the circle with center at O, $\overline{KO}$ bisects $\angle LOP$. Which theorem would be used to prove the triangles congruent?



Answer choices:

A ASA

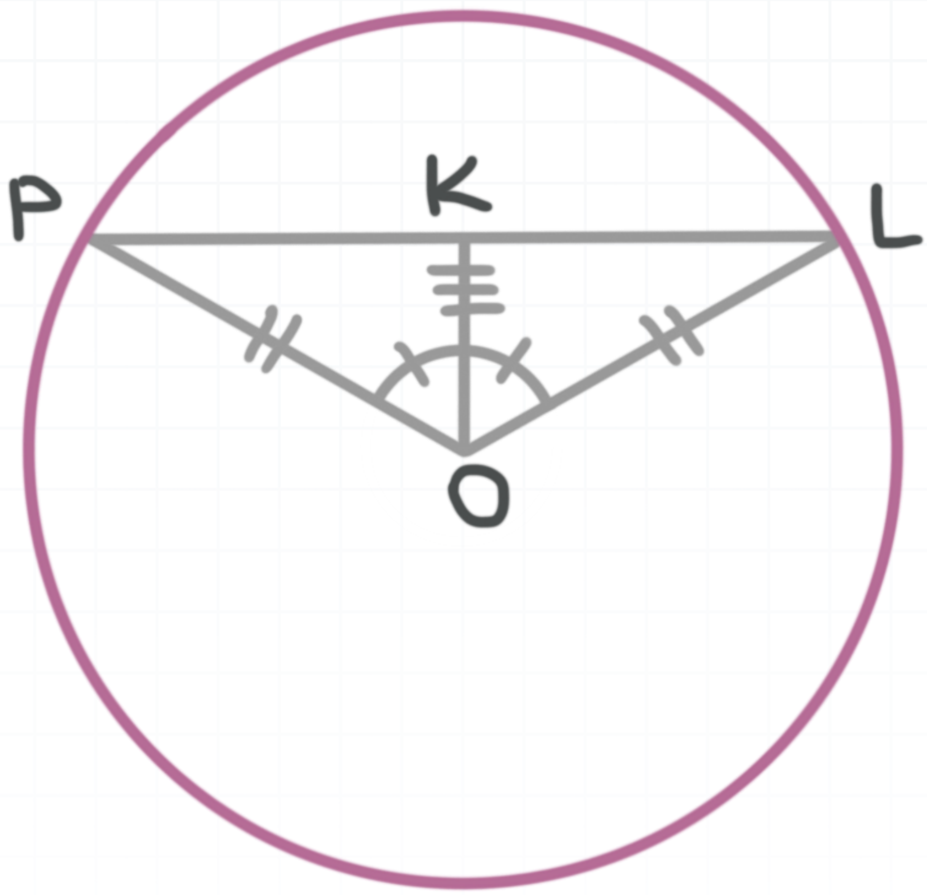
B AAS

C HL

D None of these

Solution: D

Fill in the figure with the given information.



S: $\overline{OP} \cong \overline{OL}$ because radii of a circle are congruent.

A: $\angle KOP \cong \angle LOK$ because an angle bisector forms a pair of congruent angles.

S: $\overline{KO} \cong \overline{KO}$ by the reflexive property.

This makes the triangles congruent by SAS, because $\angle KOP$ is the included angle of sides $\overline{KO}$ and $\overline{OP}$ in $\triangle KOP$, and $\angle LOK$ is the included angle of sides $\overline{KO}$ and $\overline{OL}$ in $\triangle KOL$. However, SAS isn't one of the answer choices.

It looks as though $\overline{KO} \perp \overline{PL}$. If that were the case, then answer choices A and C are both correct:

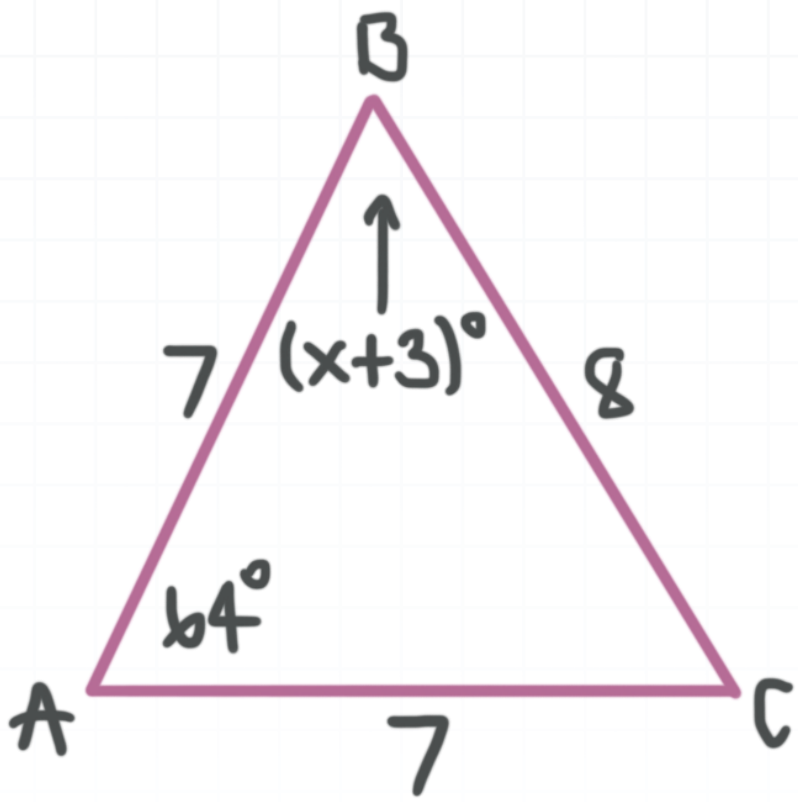
- answer choice A (ASA) because $\overline{KO}$ is the included side for $\angle KOP$ and $\angle PKO$ in $\triangle KOP$, and $\overline{KO}$ is also the included side for $\angle LOK$ and $\angle OKL$ in $\triangle KOL$

- answer choice C (HL) because $\overline{OP}$ and $\overline{KO}$ are the hypotenuse and one leg, respectively, of $\triangle KOP$; $\overline{OL}$ and $\overline{KO}$ are the hypotenuse and the corresponding leg, respectively, of $\triangle KOL$; and $\overline{OP} \cong \overline{OL}$

However, we haven't been told that $\overline{KO} \perp \overline{PL}$, so we can't be sure that that's true. Therefore, "None of these" is the only answer choice that follows from the given information (and nothing more).

Topic: Isosceles triangle theorem

Question: Solve for x.



Answer choices:

A 49

B 52

C 55

D 58

Solution: C

We know that $\overline{AC} \cong \overline{AB}$, so $\angle B \cong \angle C$ and $m\angle C = (x+3)^\circ$. The measures of the interior angles of a triangle always sum to 180°, so

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$64^\circ + (x+3)^\circ + (x+3)^\circ = 180^\circ$$

$$70^\circ + 2x^\circ = 180^\circ$$

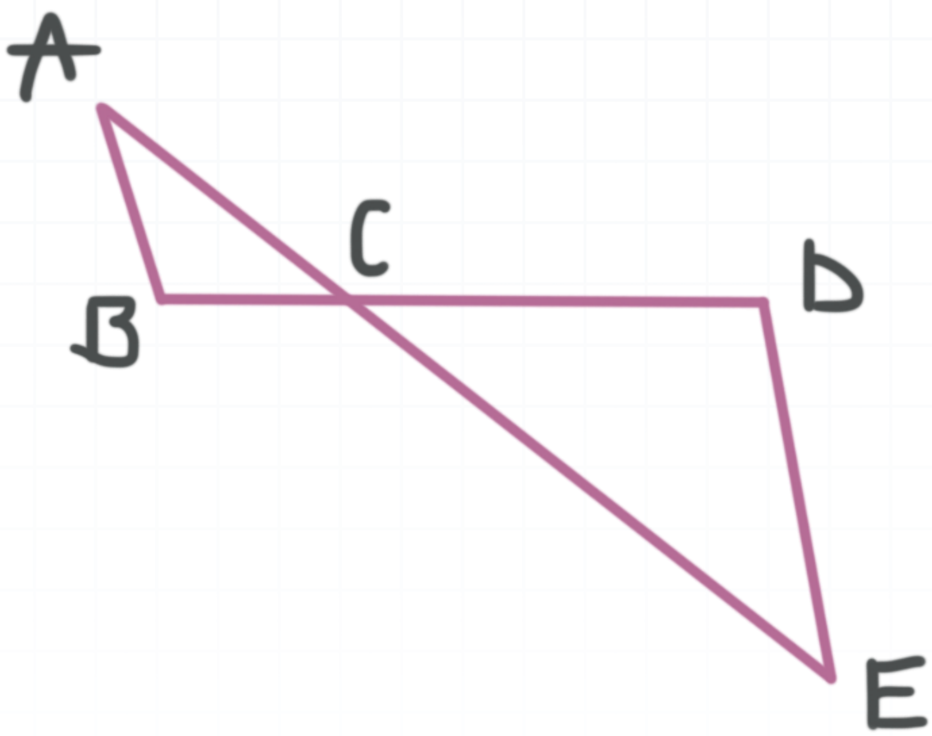
$$2x^\circ = 110^\circ$$

$$x^\circ = 55^\circ$$

$$x = 55$$

Topic: Isosceles triangle theorem

Question: Find the measure of $\angle CDE$, given that $\overline{AB} \cong \overline{BC}$, $\overline{CD} \cong \overline{DE}$, and $m\angle BAC = 35^\circ$.



Answer choices:

A 95°

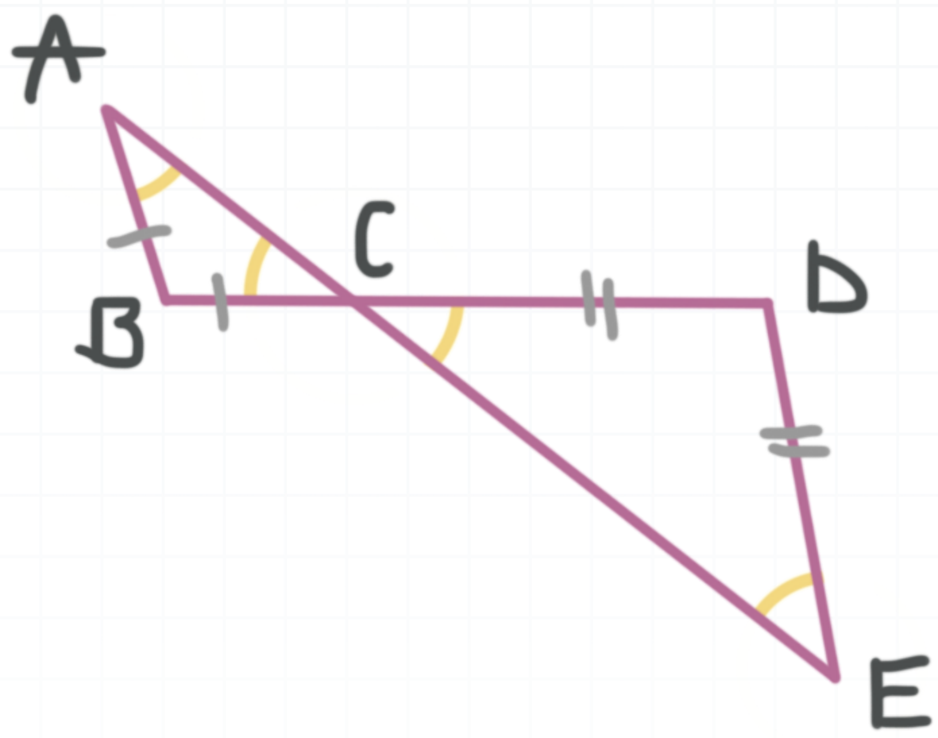
B 110°

C 118°

D 122°

Solution: B

Fill in the figure with the given information.



Because we know $\overline{AB} \cong \overline{BC}$, we can say $\angle BAC \cong \angle ACB$ and $m\angle ACB = 35^\circ$.

The angles $\angle ACB$ and $\angle ECD$ are a pair of vertical angles, so $m\angle ECD = 35^\circ$, because vertical angles are congruent.

Then we can say $\overline{CD} \cong \overline{DE}$, so $\angle DEC \cong \angle ECD$ and $m\angle DEC = 35^\circ$.

The measures of the interior angles of a triangle always sum to 180°, so

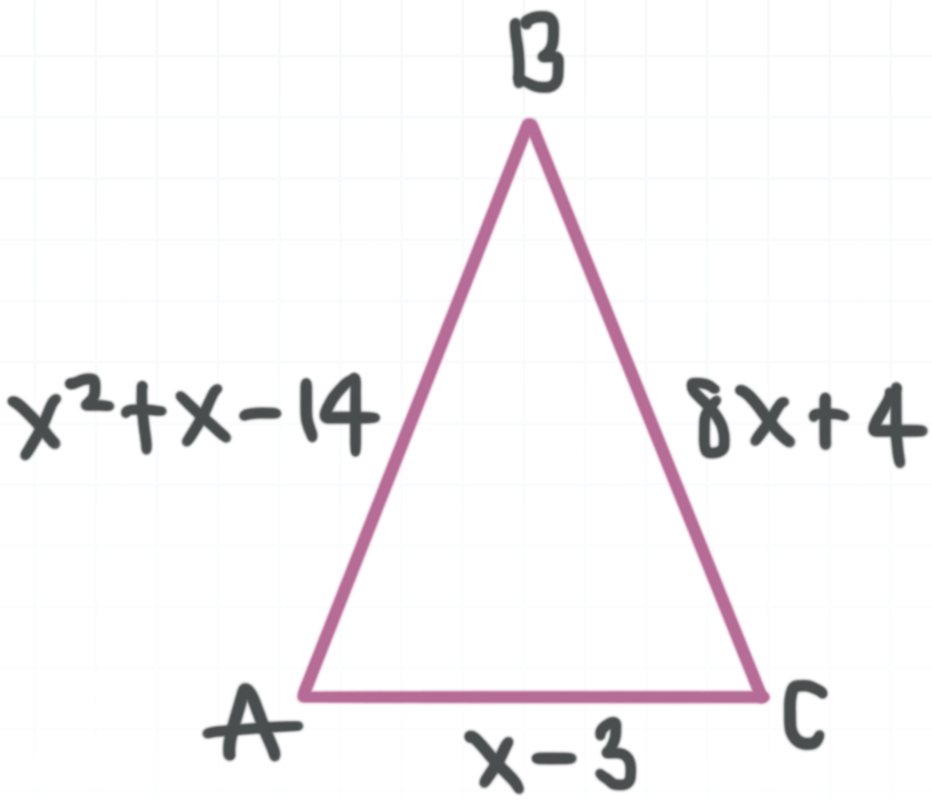
$$m\angle ECD + m\angle DEC + m\angle CDE = 180^\circ$$

$$35^\circ + 35^\circ + m\angle CDE = 180^\circ$$

$$m\angle CDE = 110^\circ$$

Topic: Isosceles triangle theorem

Question: Find the length of $\overline{AC}$, given that $\angle C \cong \angle A$.



Answer choices:

A 1

B 2

C 5

D 6

Solution: D

With the given information, $\angle C \cong \angle A$, we can say that $\overline{BA} = \overline{BC}$.

Equate the given expressions for $\overline{BA}$ and $\overline{BC}$, then solve for x.

$$x^2 + x - 14 = 8x + 4$$

$$x^2 - 7x - 18 = 0$$

$$(x - 9)(x + 2) = 0$$

$$x = 9 \text{ or } x = -2$$

Using $x = -2$ will lead to a negative value for $\overline{BA}$ (because $(-2)^2 + (-2) - 14 = 4 - 2 - 14 = -12$) and $\overline{BC}$ (because $8(-2) + 4 = -16 + 4 = -12$), so rule out $x = -2$. That leaves $x = 9$.

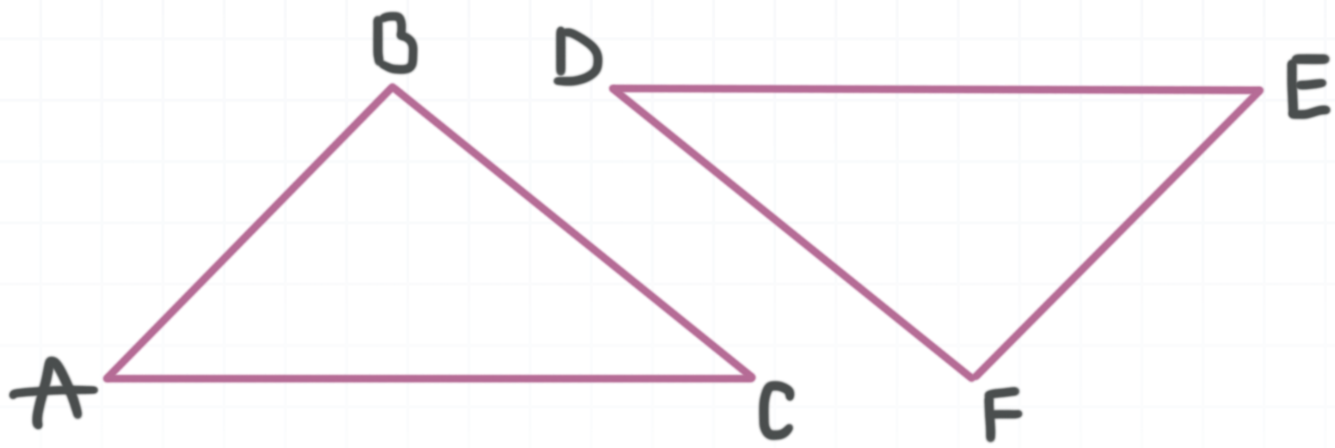
$$\overline{AC} = x - 3$$

$$\overline{AC} = 9 - 3$$

$$\overline{AC} = 6$$

Topic: CPCTC

Question: Given that $\triangle ABC \cong \triangle EFD$, which of these can be proven by CPCTC?



Answer choices:

A $\angle A \cong \angle D$

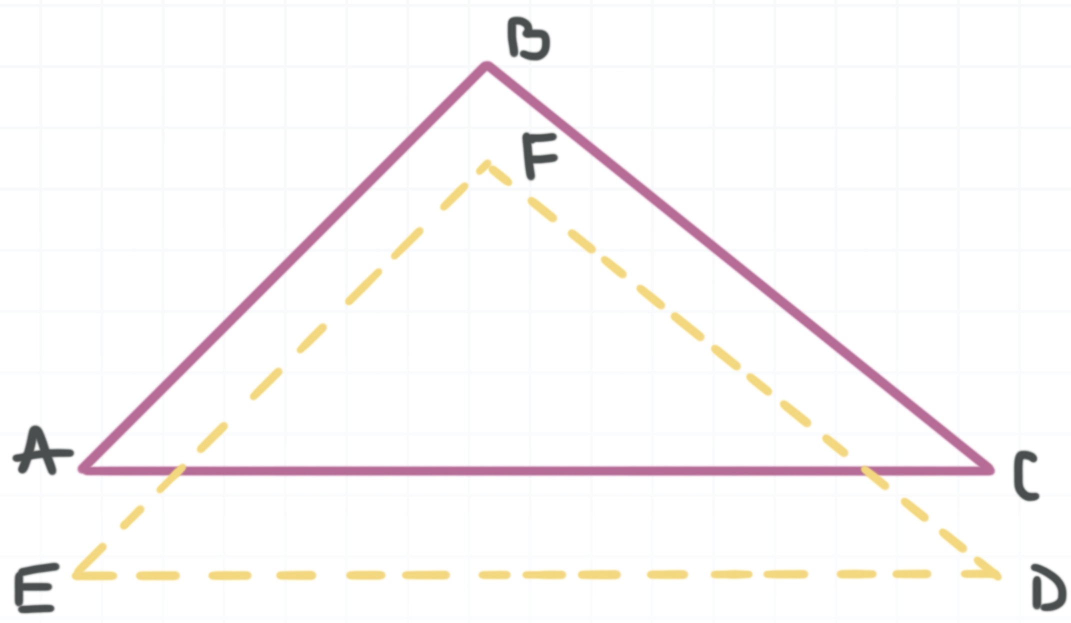
B $\overline{AC} \cong \overline{ED}$

C Both A and B

D Neither A nor B

Solution: B

We know that $\triangle ABC \cong \triangle EFD$.



The order of the letters for the vertices tells you that the triangles match up as shown above, and the congruences of angles are as follows:

$$\angle A \cong \angle E$$

$$\angle B \cong \angle F$$

$$\angle C \cong \angle D$$

That rules out answer choice A, which also rules out answer choice C.

Likewise, the congruences of sides are as follows:

$$\overline{AB} \cong \overline{EF}$$

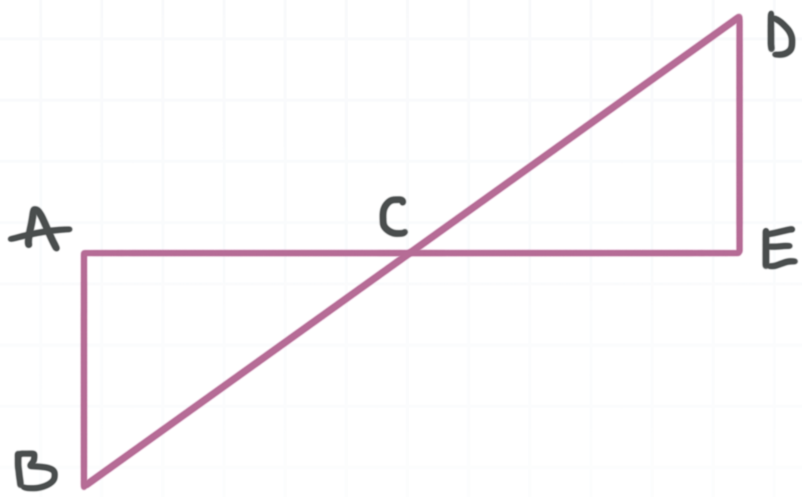
$$\overline{BC} \cong \overline{FD}$$

$$\overline{AC} \cong \overline{ED}$$

This third side congruency statement matches answer choice B.

Topic: CPCTC

Question: We've written a proof showing that $\overline{AB} \cong \overline{ED}$ in the figure. Which column in the table shows the last three steps of the proof?



A	B	C	D
CPCTC	Vertical angles are congruent	CPCTC	Vertical angles are congruent
Vertical angles are congruent	CPCTC	ASA	ASA
ASA	ASA	Vertical angles are congruent	CPCTC

Answer choices:

A Column A

B Column B

C Column C

D Column D

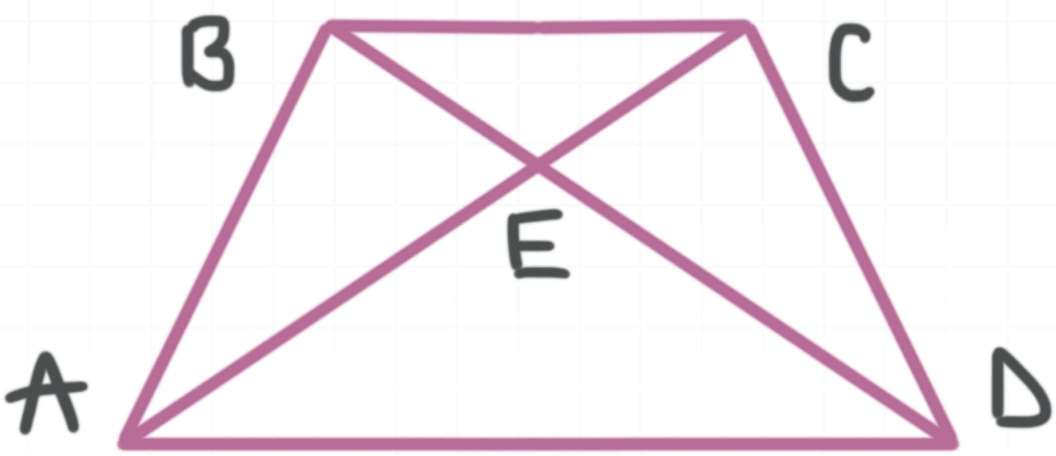
Solution: D

The proof is using ASA to prove triangles congruent. CPCTC is used only after triangles have been proven congruent, which means it must come after ASA.

Only answer choice D has ASA and CPCTC in the correct order.

Topic: CPCTC

Question: Given trapezoid $ABCD$, and $\overline{AC} \cong \overline{BD}$ and $\angle DAC \cong \angle BDA$, assuming we're trying to prove that $ABCD$ is isosceles, which is the missing step in the proof?



1. Segment *AC* congruent to segment *BD*	Given
2. Segment *AD* congruent to segment *AD*	Reflexive
3. Angle *DAC* congruent to angle *BDA*	Given
4	
5. Segment *AB* congruent to segment *CD*	CPCTC
6. Trapezoid *ABCD* is isosceles	Definition of isosceles trapezoid

Answer choices:

A $\triangle BAD \cong \triangle CAD$ by SAS

B $\triangle ABE \cong \triangle DCE$ by ASA

C $\triangle BCA \cong \triangle CBD$ by AAS

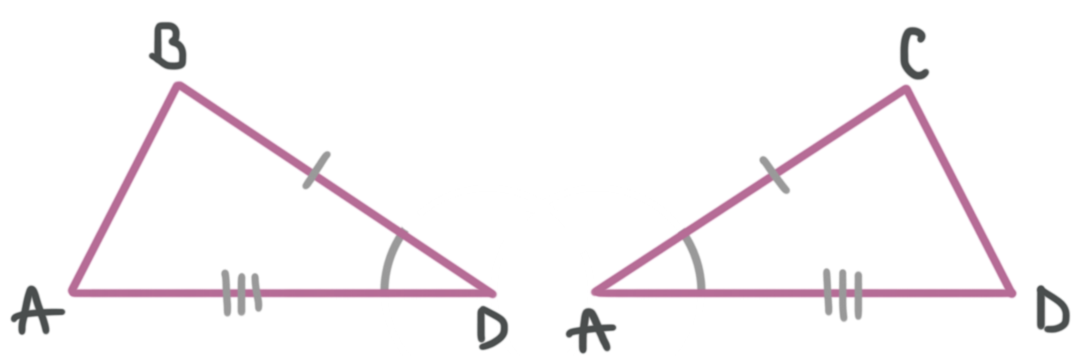
D $\triangle ABD \cong \triangle DCA$ by SAS

Solution: D

Answer choice A is ruled out, because it indicates that sides $\overline{BA}$ and $\overline{CA}$ are congruent. It's clear from the figure that that's false.

Answer choice B is ruled out because a proof of triangle congruence by ASA would include two statements of congruence of angles, whereas the given proof includes just one statement of congruence of angles. Similarly, answer choice C is ruled out because a proof of triangle congruence by AAS would include two statements of congruence of angles.

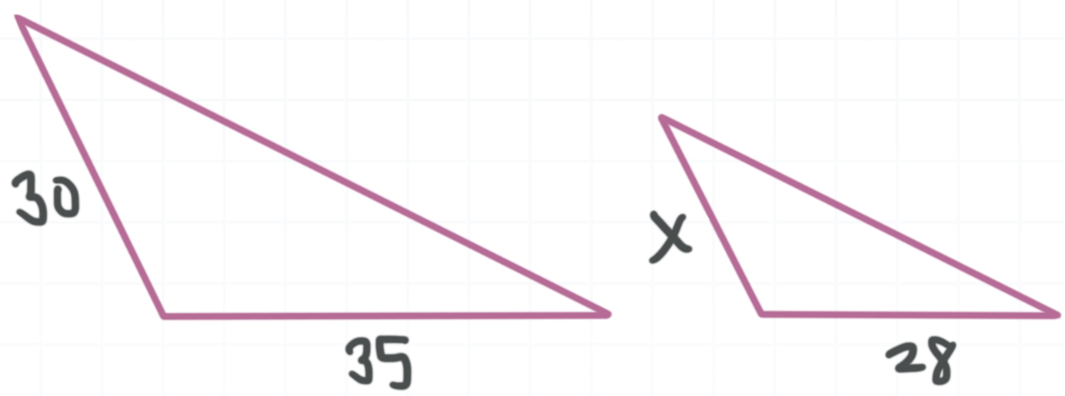
Now we'll investigate answer choice D. Separating the overlapping triangles $\triangle ABD$ and $\triangle ACD$ and marking the congruent parts indicated in the first three steps of the proof gives us



It's clear from this figure that the triangles are congruent by SAS. In $\triangle ABD$ the included angle of sides $\overline{BD}$ and $\overline{AD}$ is $\angle BDA$, and in $\triangle ACD$ the included angle of sides $\overline{AC}$ and $\overline{AD}$ is $\angle DAC$. This means that answer choice D is indeed correct.

Topic: Similar triangles

Question: The triangles in the figure are similar. What is the value of the variable?



Answer choices:

A 24

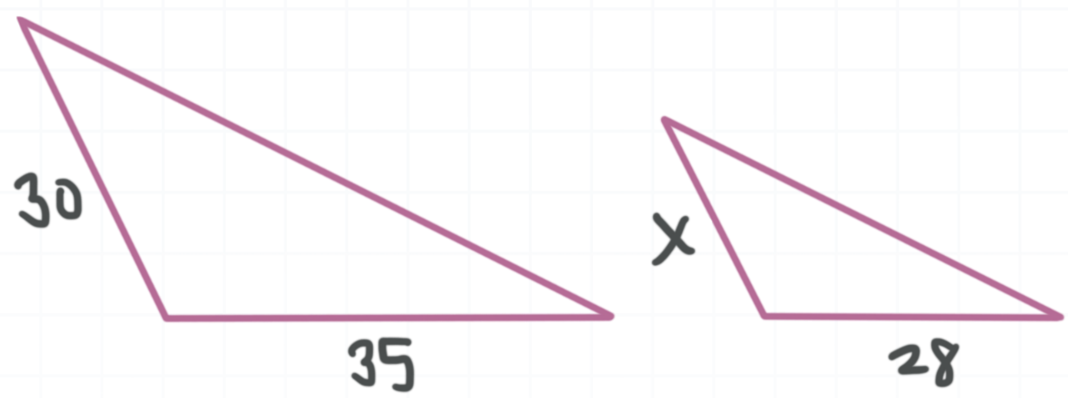
B 25.5

C 26

D 37.5

Solution: A

In a pair of similar triangles, lengths of corresponding sides are proportional. In the figure, the sides of length x and 28 in the triangle on the right correspond to the sides of length 30 and 35, respectively, in the triangle on the left.



So we have the following proportion:

$$\frac{x}{30} = \frac{28}{35}$$

$$\frac{x}{30} = \frac{4}{5}$$

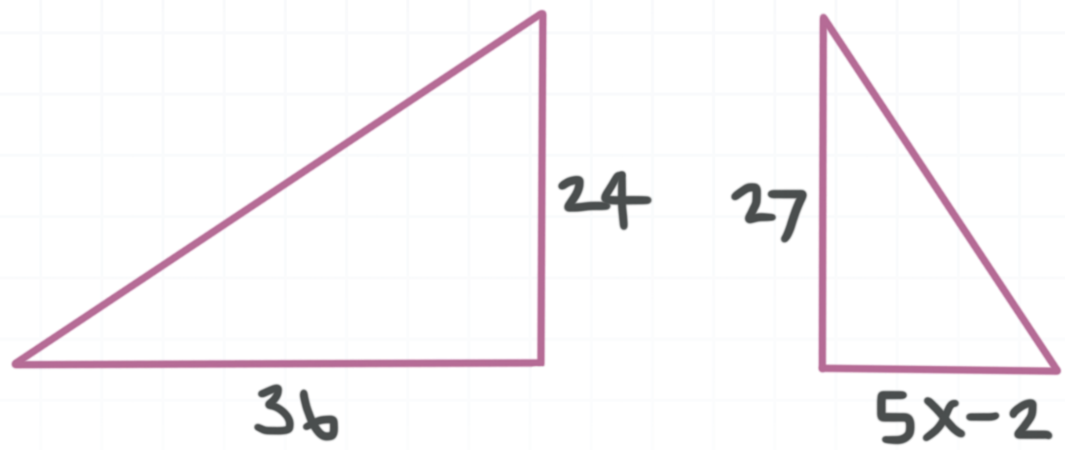
Cross multiply.

$$5x = 120$$

$$x = \frac{120}{5} = 24$$

Topic: Similar triangles

Question: The triangles in the figure are similar. What is the value of the variable?



Answer choices:

A 16

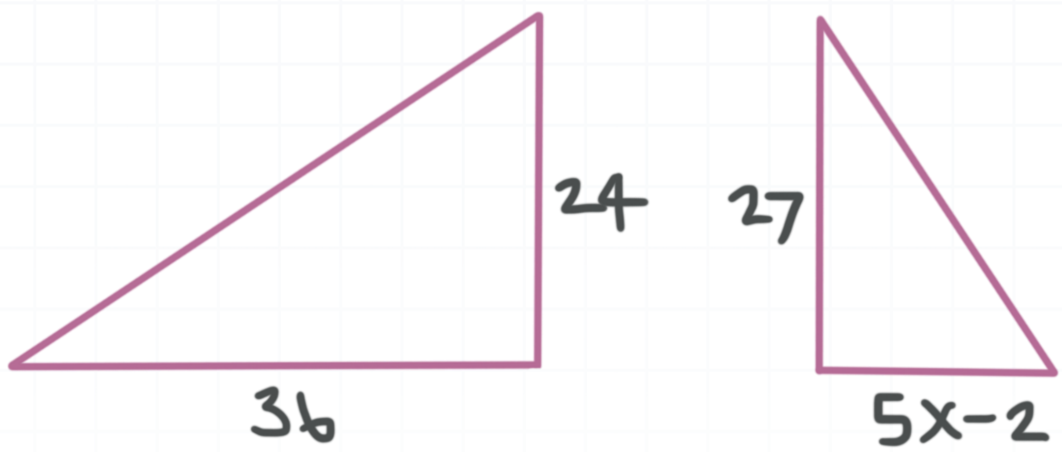
B 11

C 7

D 4

Solution: D

In a pair of similar triangles, lengths of corresponding sides are proportional. In the figure, the sides of length $5x - 2$ and 27 in the triangle on the right correspond to the sides of length 24 and 36, respectively, in the triangle on the left.



So we have the following proportion:

$$\frac{5x-2}{24} = \frac{27}{36}$$

$$\frac{5x-2}{24} = \frac{3}{4}$$

Cross multiply.

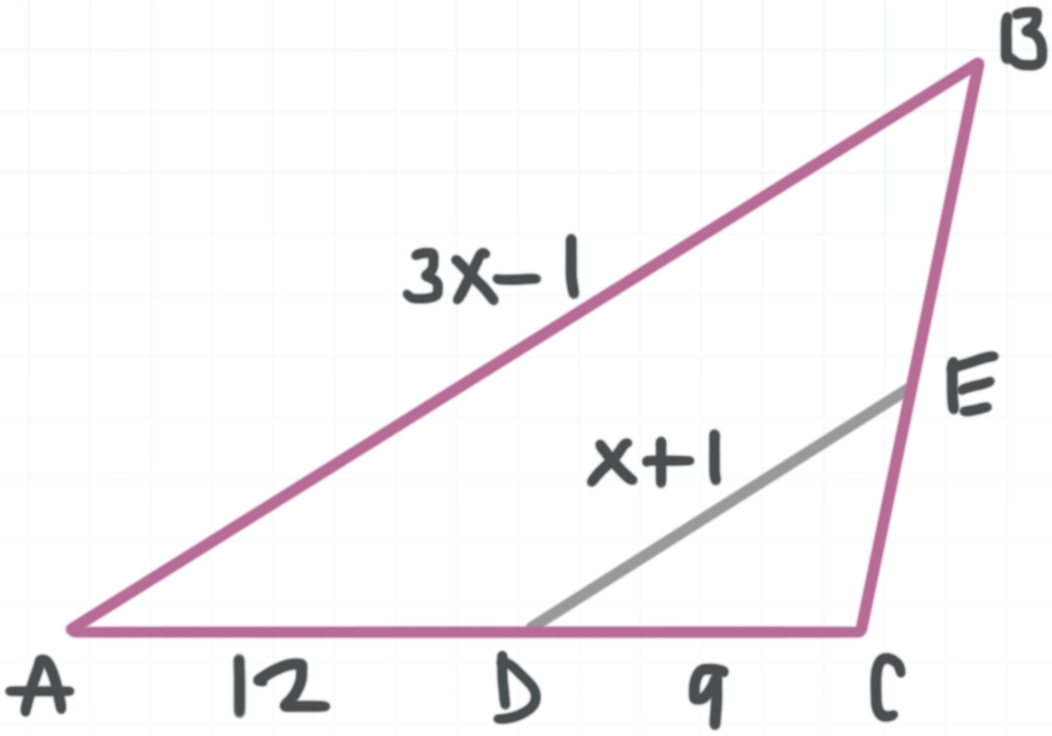
$$20x - 8 = 72$$

$$20x = 80$$

$$x = \frac{80}{20} = 4$$

Topic: Similar triangles

Question: Given that $\triangle ABC \sim \triangle DEC$, what is the value of the variable?



Answer choices:

A 4

B 5

C 6

D 9

Solution: B

In a pair of similar triangles, lengths of corresponding sides are proportional.

side $\overline{DC}$ in $\triangle DEC$ corresponds to side $\overline{AC}$ in $\triangle ABC$

side $\overline{DE}$ in $\triangle DEC$ corresponds to side $\overline{AB}$ in $\triangle ABC$

So we have the following proportion:

$$\frac{9}{12+9} = \frac{x+1}{3x-1}$$

$$\frac{9}{21} = \frac{x+1}{3x-1}$$

Cross multiply.

$$9(3x-1) = 21(x+1)$$

$$27x - 9 = 21x + 21$$

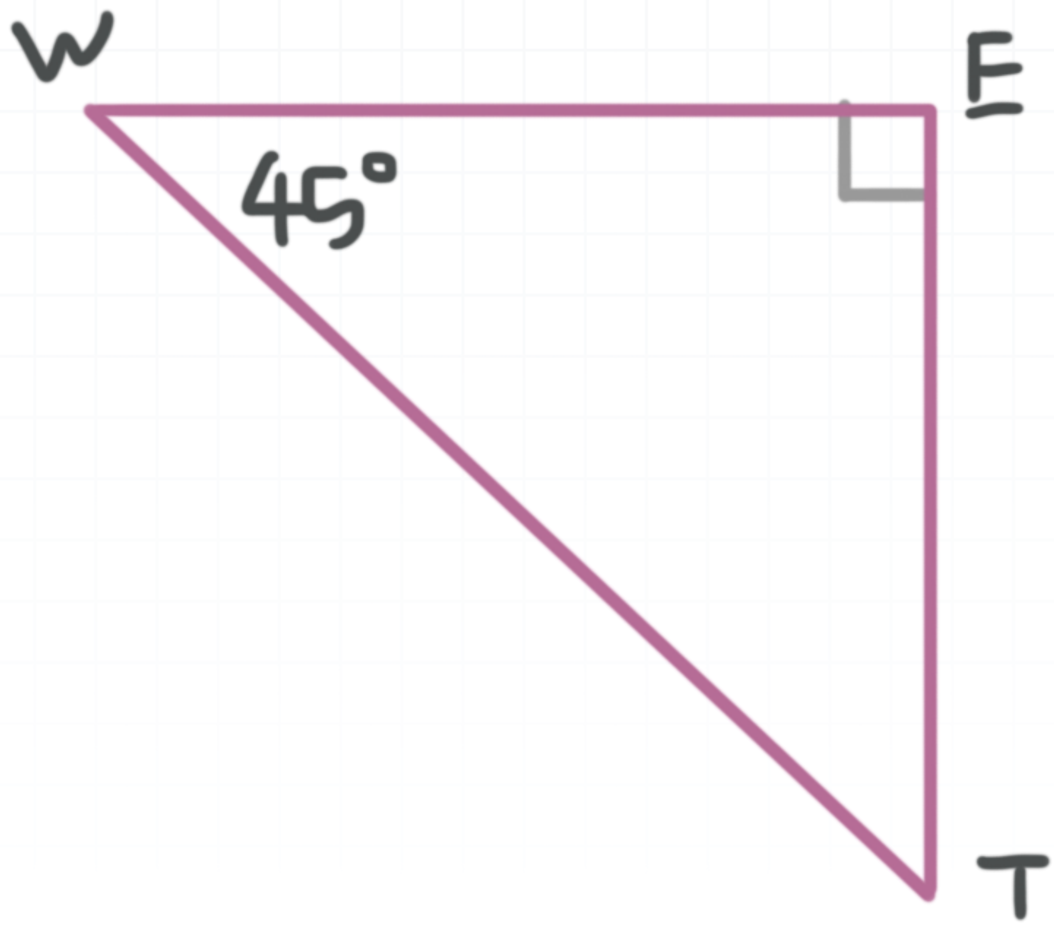
$$6x - 9 = 21$$

$$6x = 30$$

$$x = 5$$

Topic: 45-45-90 triangles

Question: If $\overline{ET} = 3$, what is $\overline{WT}$?



Answer choices:

A $\sqrt{6}$

B $2\sqrt{3}$

C $3\sqrt{2}$

D 9

Solution: C

The triangle $\triangle WET$ is a 45-45-90 triangle, and the pattern for the lengths of the sides of a 45-45-90 triangle is x, x, and $x\sqrt{2}$, where x is the length of each leg.

In this case, $x = 3$, so the lengths of sides $\overline{ET}$, $\overline{WE}$, and $\overline{WT}$ of this 45-45-90 triangle (in which $\overline{WT}$ is the hypotenuse) are 3, 3 and $3\sqrt{2}$, respectively. This means that $\overline{WT} = 3\sqrt{2}$.

Alternatively, we could have used the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

$$3^2 + 3^2 = c^2$$

$$9 + 9 = c^2$$

$$18 = c^2$$

Take the square root of both sides to solve for c.

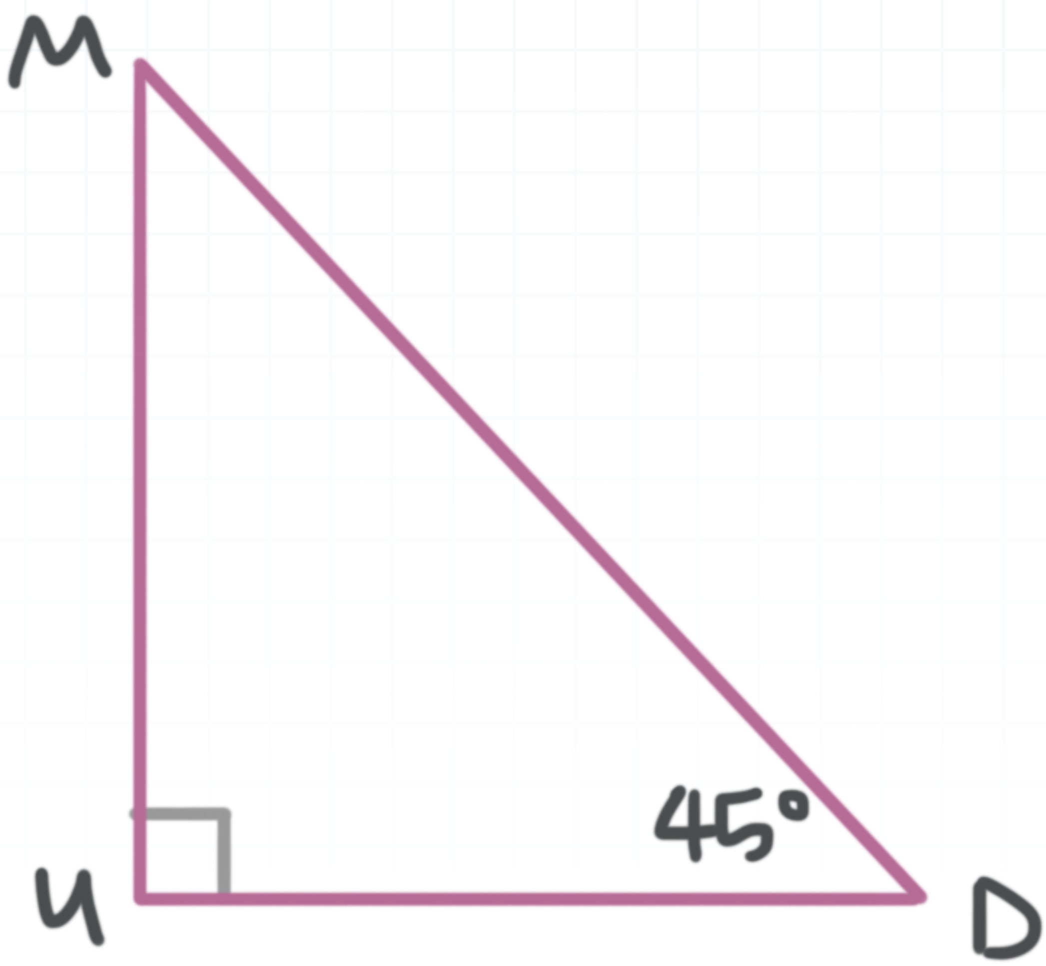
$$c = \sqrt{18}$$

$$c = \sqrt{9 \cdot 2}$$

$$c = 3\sqrt{2}$$

Topic: 45-45-90 triangles

Question: If $\overline{MU} = 2\sqrt{2}$, what is $\overline{MD}$?



Answer choices:

A $\sqrt{2}$

B $4\sqrt{2}$

C 2

D 4

Solution: D

The triangle $\triangle MUD$ is a 45-45-90 triangle, and the pattern for the sides of a 45-45-90 triangle is x, x, and $x\sqrt{2}$, where x is the length of each leg.

In this case, $x = 2\sqrt{2}$, so the lengths of sides $\overline{MU}$, $\overline{UD}$, and $\overline{MD}$ of this 45-45-90 triangle (in which $\overline{MD}$ is the hypotenuse) are $2\sqrt{2}$, $2\sqrt{2}$, and $2\sqrt{2} \cdot \sqrt{2} = 2(2) = 4$, respectively.

Alternatively, we could have used the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

$$(2\sqrt{2})^2 + (2\sqrt{2})^2 = c^2$$

$$4(2) + 4(2) = c^2$$

$$8 + 8 = c^2$$

$$16 = c^2$$

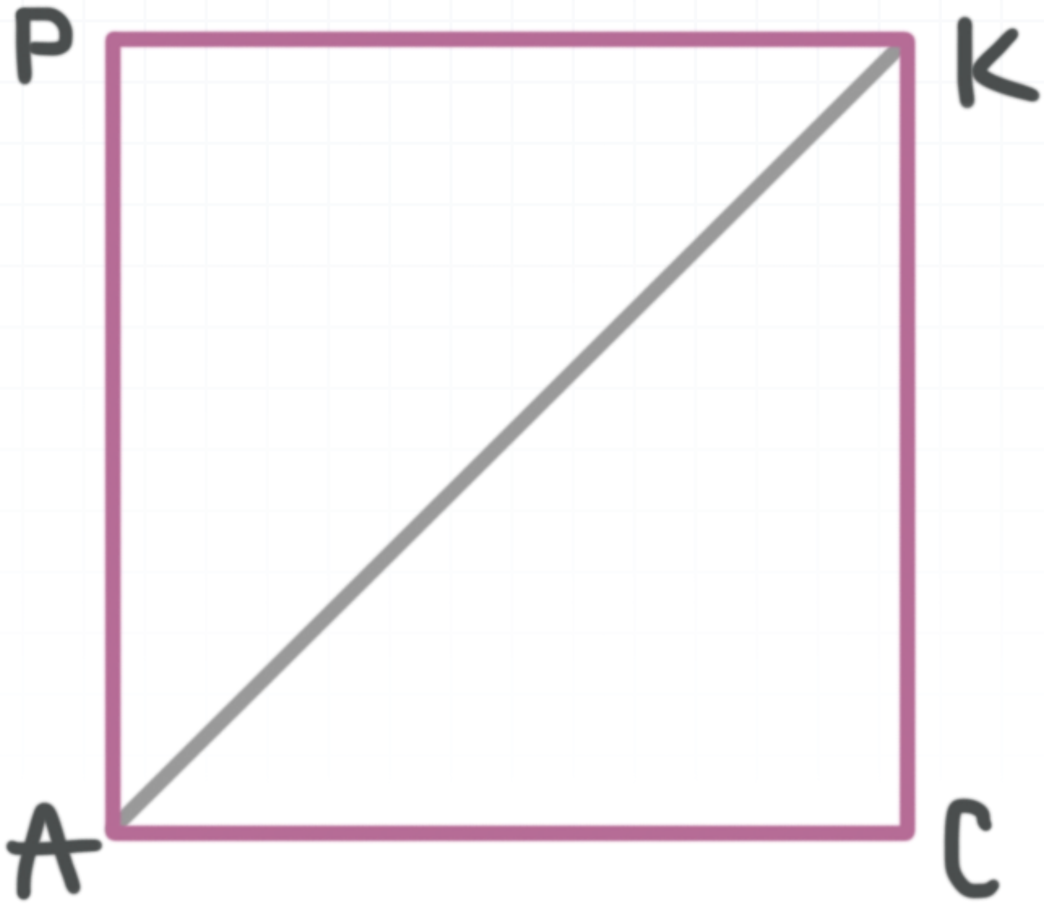
Take the square root of both sides to solve for c.

$$c = \sqrt{16}$$

$$c = 4$$

Topic: 45-45-90 triangles

Question: Quadrilateral $PACK$ is a square that has a diagonal of length 4. What is the length of $\overline{PA}$?



Answer choices:

A $2\sqrt{2}$

B $4\sqrt{2}$

C $8\sqrt{2}$

D 4

Solution: A

All four sides of a square are congruent, so $\overline{PA} = \overline{PK}$, which means that $\triangle PAK$ is isosceles. Also, the measure of each of the four interior angles of a square is 90°, so $\angle APK$ is a right angle. Combining these two results, we see that $\triangle PAK$ is a 45-45-90 triangle.

The pattern for the lengths of the sides of a 45-45-90 triangle is x, x, and $x\sqrt{2}$, where x is the length of each leg. In this case, we see that the hypotenuse of $\triangle PAK$ is side $\overline{AK}$, which is also a diagonal of square $PACK$, so its length is 4 (and is represented by $x\sqrt{2}$).

Write $x\sqrt{2} = 4$ and solve for x.

$$x\sqrt{2} = 4$$

$$x = \frac{4}{\sqrt{2}}$$

$$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{4\sqrt{2}}{2}$$

$$x = 2\sqrt{2}$$

Alternatively, we could have used the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

$$x^2 + x^2 = 4^2$$

$$2x^2 = 16$$

$$x^2 = 8$$

Take the square root of both sides to solve for c.

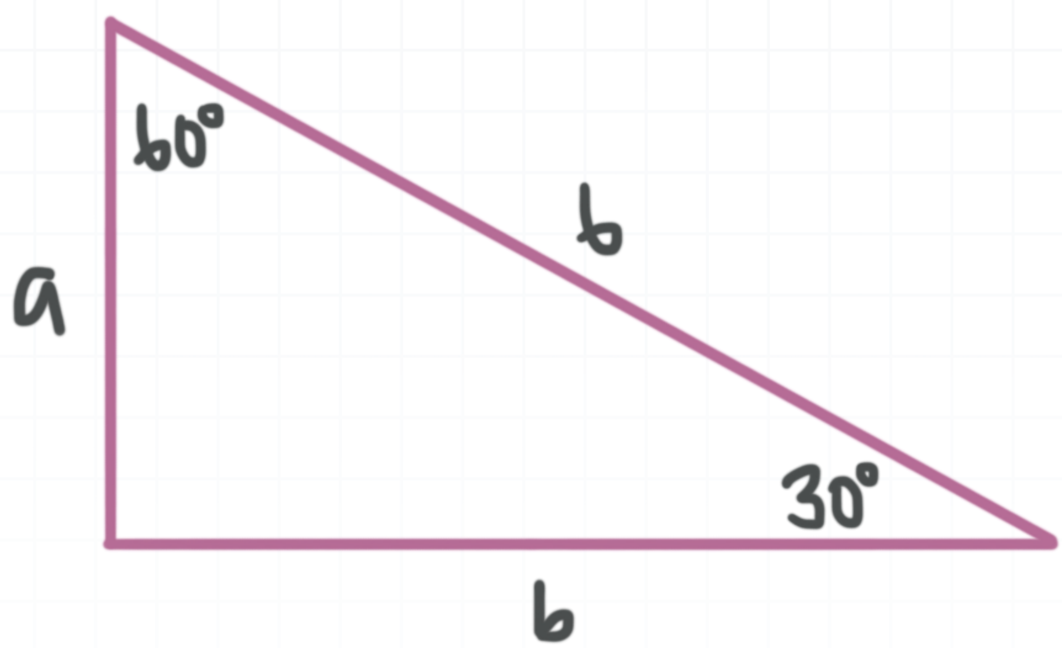
$$x = \sqrt{8}$$

$$x = \sqrt{4} \cdot \sqrt{2}$$

$$x = 2\sqrt{2}$$

Topic: 30-60-90 triangles

Question: What are the values of a and b?



Answer choices:

A $a = 3\sqrt{3}$ and $b = 3$

B $a = 3$ and $b = 3\sqrt{3}$

C $a = 3$ and $b = 4$

D $a = 3$ and $b = 3$

Solution: B

The pattern for the lengths of the sides of a 30-60-90 triangle is x for the short leg, $x\sqrt{3}$ for the long leg, and $2x$ for the hypotenuse. In this case, we know that 6 is the length of the hypotenuse, which is represented by $2x$.

Write $2x = 6$ and solve for x.

$$2x = 6$$

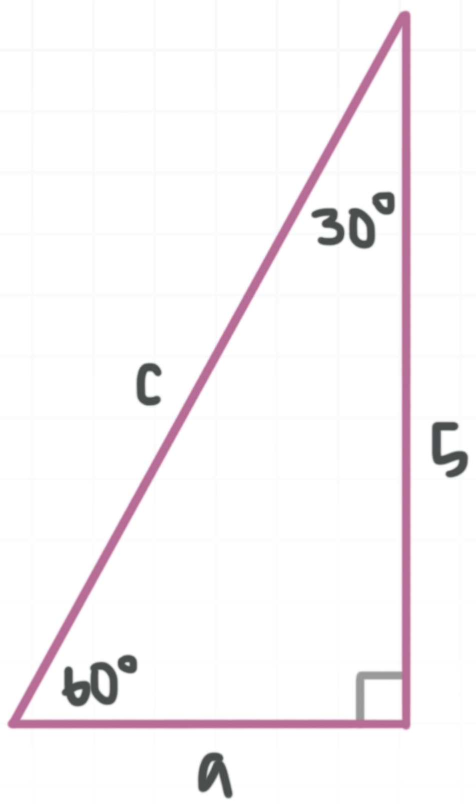
$$x = 3$$

a is the length of the short leg, so $a = x = 3$.

b is the length of the long leg, so $b = x\sqrt{3} = 3\sqrt{3}$.

Topic: 30-60-90 triangles

Question: What are the values of a and c?



Answer choices:

A $\quad a = \frac{10\sqrt{3}}{3}$ and $c = \frac{5\sqrt{3}}{3}$

B $\quad a = \frac{5\sqrt{3}}{3}$ and $c = \frac{10\sqrt{3}}{3}$

C $\quad a = 5\sqrt{3}$ and $c = 10$

D $\quad a = 5\sqrt{3}$ and $c = 5$

Solution: B

The pattern for the lengths of the sides of a 30-60-90 triangle is x for the short leg, $x\sqrt{3}$ for the long leg, and $2x$ for the hypotenuse. In this case, we know that 5 is the length of the long leg, which is represented by $x\sqrt{3}$.

Write $x\sqrt{3}=5$ and solve for x.

$$x\sqrt{3}=5$$

$$x=\frac{5}{\sqrt{3}}$$

Rationalizing the denominator, we get

$$x=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{3}$$

a is the length of the short leg, so

$$c=2x=2\left(\frac{5\sqrt{3}}{3}\right)=\frac{10\sqrt{3}}{3}$$

c is the length of the hypotenuse, so

$$c=2x=2\left(\frac{5\sqrt{3}}{3}\right)=\frac{10\sqrt{3}}{3}$$

Topic: 30-60-90 triangles

Question: $\triangle SAM$ is a 30-60-90 triangle, with S at $(0,0)$, A at $(4\sqrt{3},4)$, and M at $(4\sqrt{3},0)$ in the Cartesian coordinate system. Which angle is the 60° angle?

Answer choices:

A S

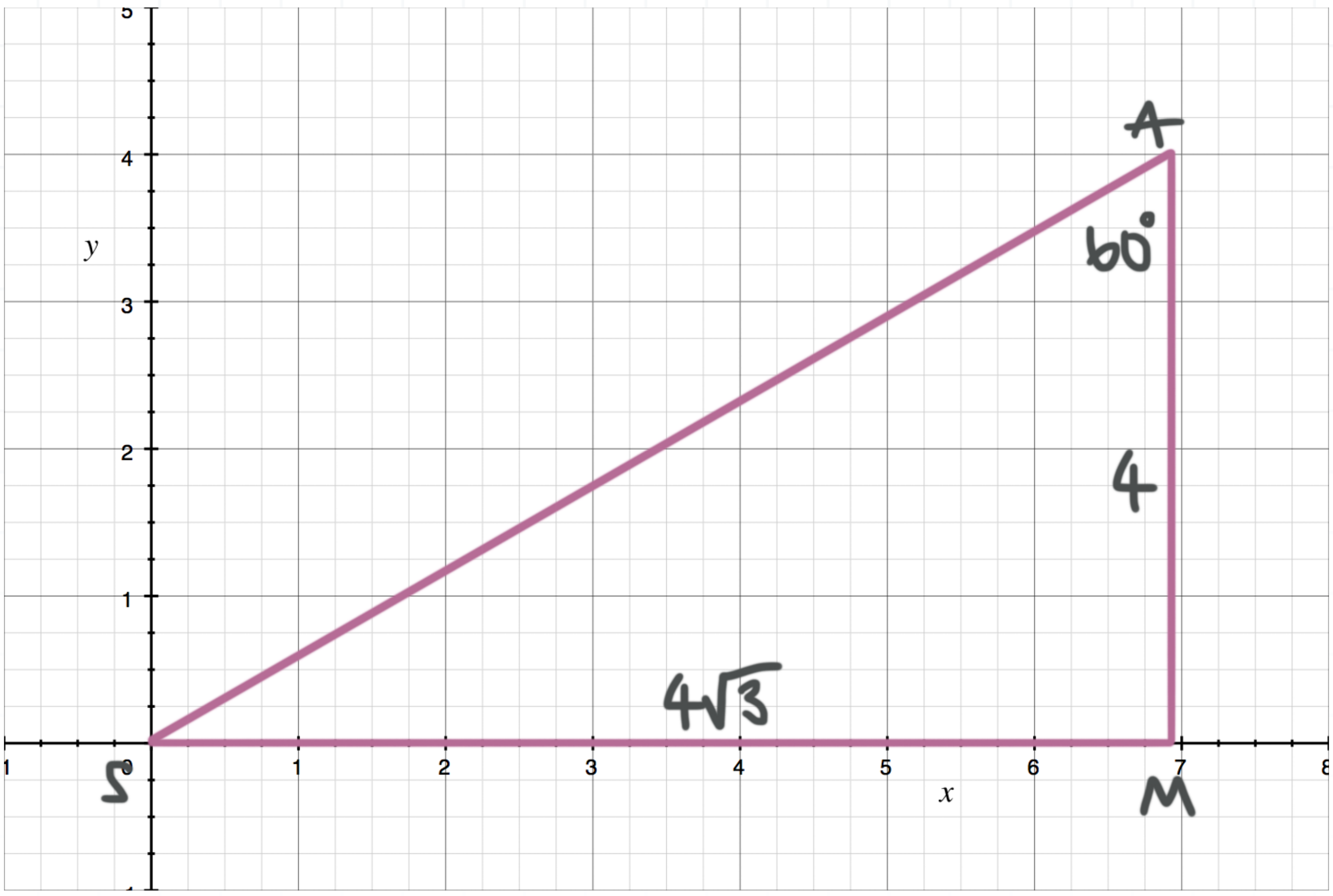
B A

C M

D None of these

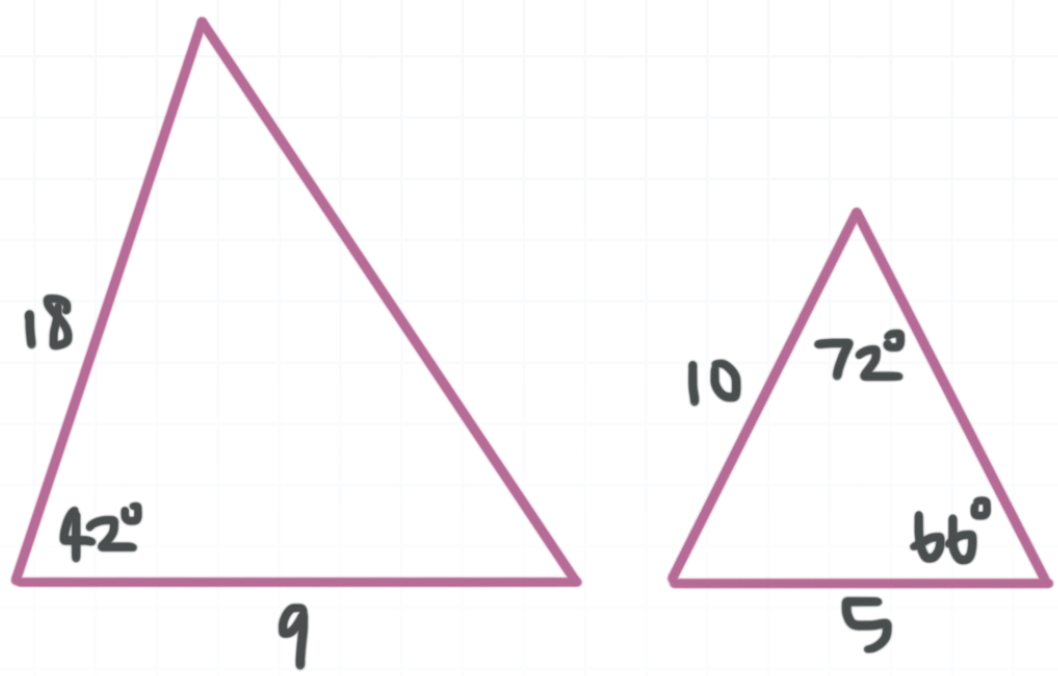
Solution: B

In a 30-60-90 triangle the angle opposite the long leg is the 60° angle, which in this case would be $\angle A$.



Topic: Triangle similarity theorems

Question: Can the two triangles in the figure be proven similar?



Answer choices:

A Yes, by AA

B Yes, by SAS

C Yes, by SSS

D No, they can't be proven similar.

Solution: B

The ratio of the lengths of one pair of sides of these two triangles is

$$\frac{10}{18} = \frac{5}{9}$$

The ratio of the lengths of a second pair of sides is also $5/9$. Therefore, the lengths of two pairs of sides are proportional.

The measure of the included angle of the sides of length 10 and 5 in the small triangle can be calculated as

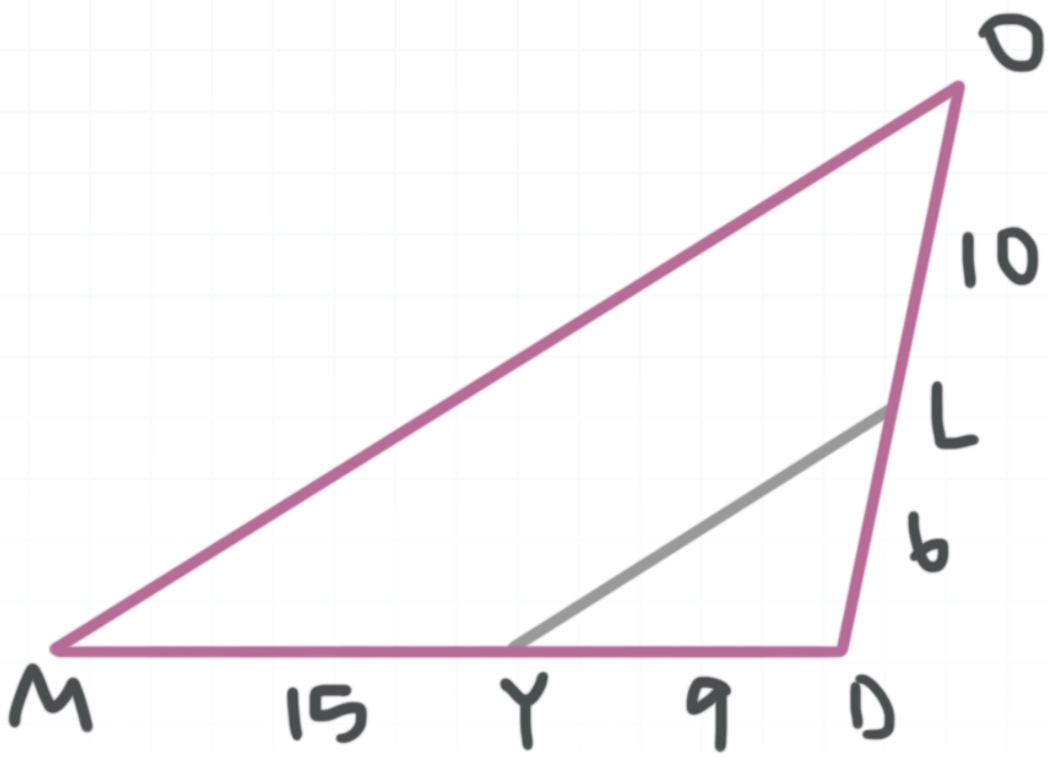
$$180^\circ - 66^\circ - 72^\circ = 42^\circ$$

That matches the measure of the included angle of the sides of length 18 and 9 in the large triangle, so we have two pairs of sides whose lengths are proportional, and the corresponding pair of included angles are congruent.

The two triangles are similar by SAS.

Topic: Triangle similarity theorems

Question: Can the two triangles in the figure be proven similar?



Answer choices:

A Yes, $\triangle ODM \sim \triangle LDY$ by SAS.

B Yes, $\triangle OMD \sim \triangle DLY$ by SAS.

C Yes, $\triangle DOM \sim \triangle DLY$ by SSS.

D The triangles can't be proven similar.

Solution: A

We see that

$$\frac{\overline{YD}}{\overline{MD}} = \frac{9}{24} = \frac{3}{8}$$

and

$$\frac{\overline{LD}}{\overline{OD}} = \frac{6}{16} = \frac{3}{8}$$

Therefore, the lengths of two pairs of sides are proportional. Also, $\angle D$ is the included angle of both of those pairs of sides, and by the reflexive property $\angle D \cong \angle D$.

The two triangles are similar by SAS.

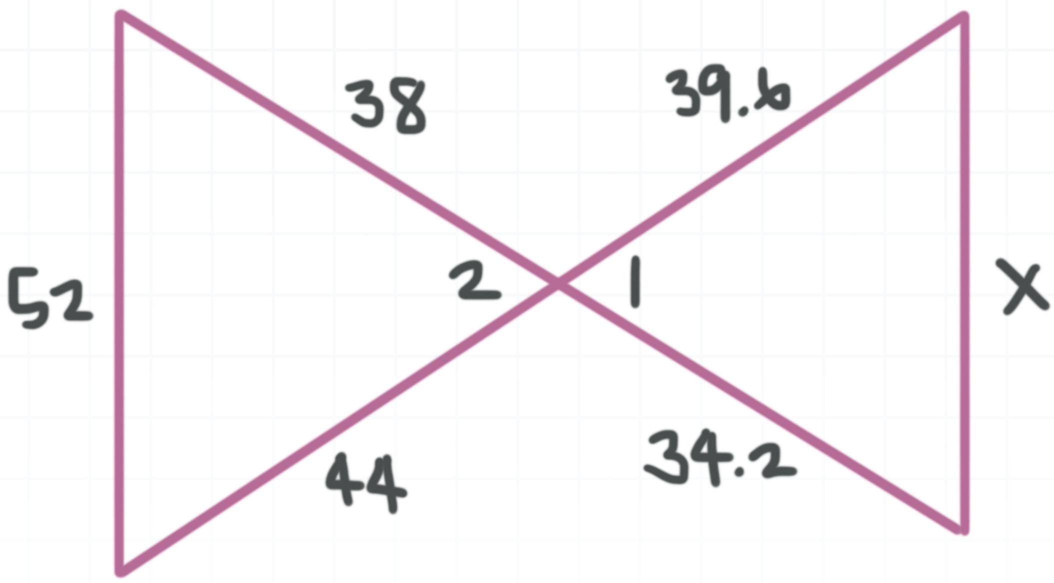
Answer choice A shows SAS and a correct ordering of vertices in the similarity statement.

Answer choice B shows SAS, but the ordering of vertices in the similarity statement is incorrect. If you write $\triangle OMD$ for the large triangle, you need to write $\triangle LYD$ for the small triangle.

We can't prove similarity of these triangles by SSS, because we've been given the lengths of only two sides of each triangle - and no information that would enable us to determine the length of the third side of each of them.

Topic: Triangle similarity theorems

Question: Solve for x.



Answer choices:

A 44.2

B 48.4

C 46.8

D Impossible to determine

Solution: C

We see that the ratio of the length of one side of the triangle on the right to the length of one side of the triangle on the left is

$$\frac{39.6}{44} = 0.9$$

and that the ratio of the length of one side of the triangle on the right to the length of one side of the triangle on the left is

$$\frac{34.2}{38} = 0.9$$

Therefore, the lengths of two pairs of sides are proportional. Also, $\angle 1$ is the included angle of the pair of sides of length 39.6 and 34.2 in the triangle on the right, and $\angle 2$ is the included angle of the pair of sides of length 44 and 38 in the triangle on the left, and $\angle 1 \cong \angle 2$ because vertical angles are congruent.

The two triangles are similar by SAS.

Now that we know the two triangles are similar, we know that the ratio of the length of the third side of the triangle on the right to the length of the third side of the triangle on the left also has to be 0.9, so

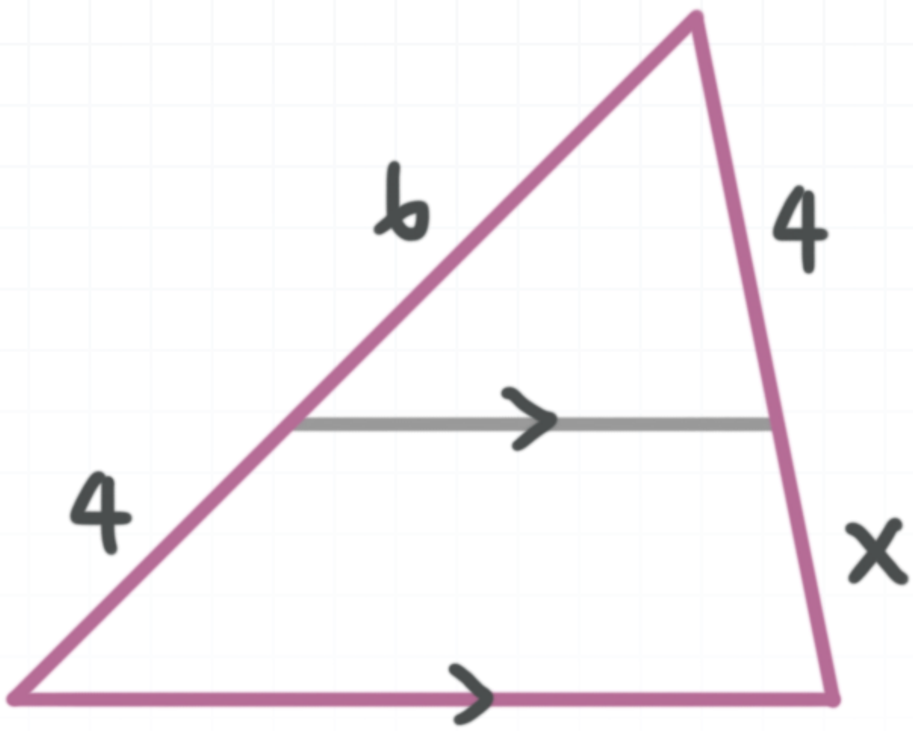
$$\frac{x}{52} = 0.9$$

$$x = 52 \cdot 0.9$$

$$x = 46.8$$

Topic: Triangle side-splitting theorem

Question: Solve for the value of x.



Answer choices:

A 2

B $\frac{8}{3}$

C 3

D $\frac{7}{2}$

Solution: B

The ratio $6/4$ has to be equal to $4/x$.

$$\frac{6}{4} = \frac{4}{x}$$

Cross multiply.

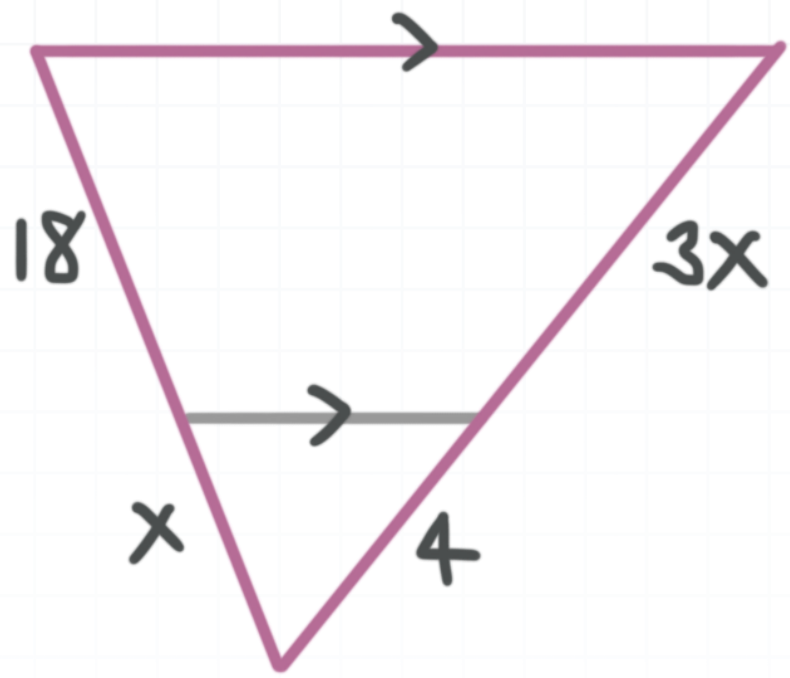
$$6x = 16$$

$$x = \frac{16}{6}$$

$$x = \frac{8}{3}$$

Topic: Triangle side-splitting theorem

Question: Solve for the value of x.



Answer choices:

A $6\sqrt{2}$

B $\frac{9}{2}$

C 6

D $2\sqrt{6}$

Solution: D

The ratio $x/18$ has to be equal to $4/3x$.

$$\frac{x}{18}=\frac{4}{3x}$$

Cross multiply.

$$3x^2=72$$

$$x^2=24$$

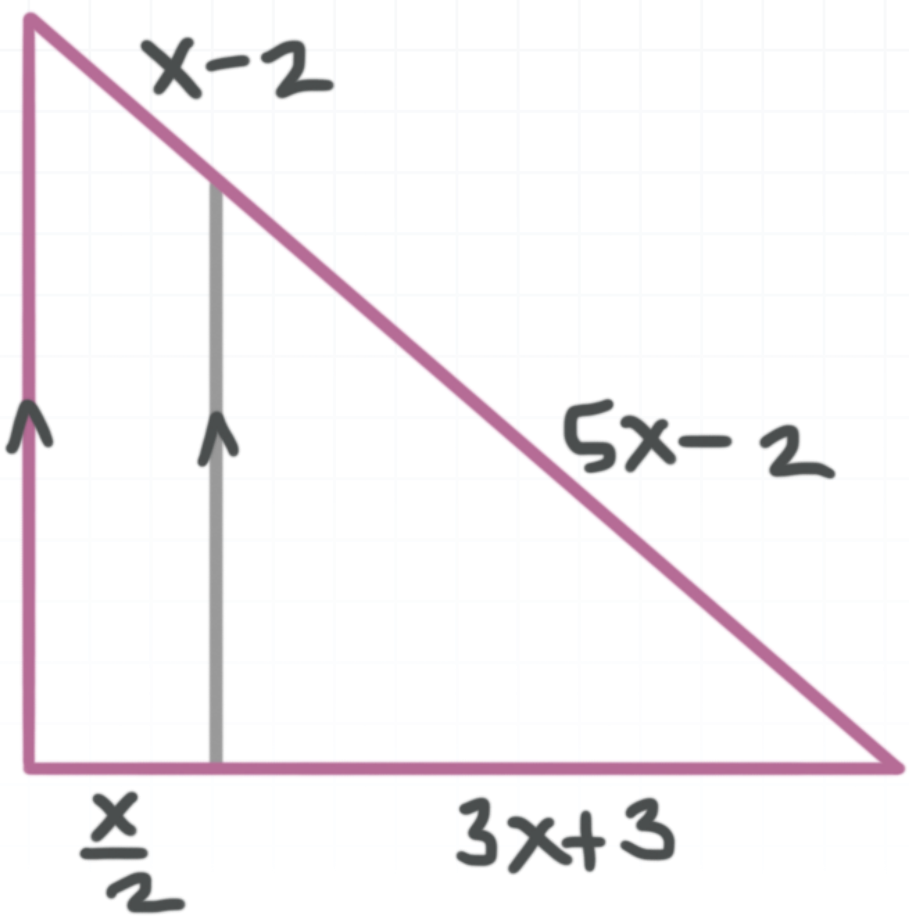
$$x=\sqrt{24}$$

$$x=\sqrt{4}\cdot\sqrt{6}$$

$$x=2\sqrt{6}$$

Topic: Triangle side-splitting theorem

Question: Solve for the value of x.



Answer choices:

A $\quad$ 6

B $\quad$ 7

C $\quad$ 4

D $\quad$ 5

Solution: A

The ratio $(x-2)/(5x-2)$ has to be equal to $(x/2)/(3x+3)$.

$$\frac{x-2}{5x-2}=\frac{\frac{x}{2}}{3x+3}$$

Cross multiply.

$$(x-2)(3x+3)=(5x-2)\left(\frac{x}{2}\right)$$

$$3x^2+3x-6x-6=\frac{5x^2}{2}-x$$

$$3x^2-3x-6=\frac{5x^2}{2}-x$$

To clear the fraction, multiply both sides of this equation by 2.

$$6x^2-6x-12=5x^2-2x$$

Combine like terms and factor.

$$x^2-4x-12=0$$

$$(x-6)(x+2)=0$$

$x=6$ or $x=-2$

Rule out $x=-2$ because that would give negative values for the lengths of both parts of each of the two sides of the triangle that are split:

$$x-2=-2-2=-4$$

$$5x - 2 = 5(-2) - 2 = -10 - 2 = -12$$

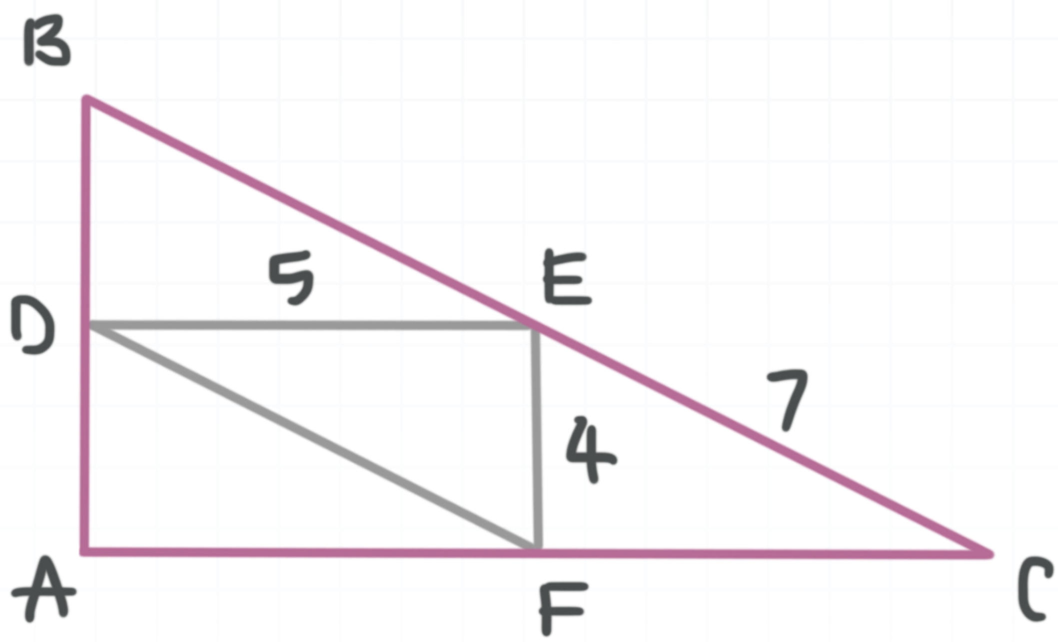
$$\frac{x}{2} = \frac{-2}{2} = -1$$

$$3x + 3 = 3(-2) + 3 = -6 + 3 = -3$$

Therefore, $x = 6$.

Topic: Midsegments of triangles

Question: In $\triangle ABC$, the midpoints of sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are D, E, and F, respectively. What is the perimeter of $\triangle ABC$?



Answer choices:

A $\quad 16$

B $\quad 21$

C $\quad 24$

D $\quad 32$

Solution: D

The points D, E, and F are the midpoints of the sides of $\triangle ABC$. Therefore,

$$\overline{AB} = 2(\overline{EF}) = 2(4) = 8$$

$$\overline{BC} = 2(\overline{EC}) = 2(7) = 14$$

$$\overline{AC} = 2(\overline{DE}) = 2(5) = 10$$

Thus the perimeter is

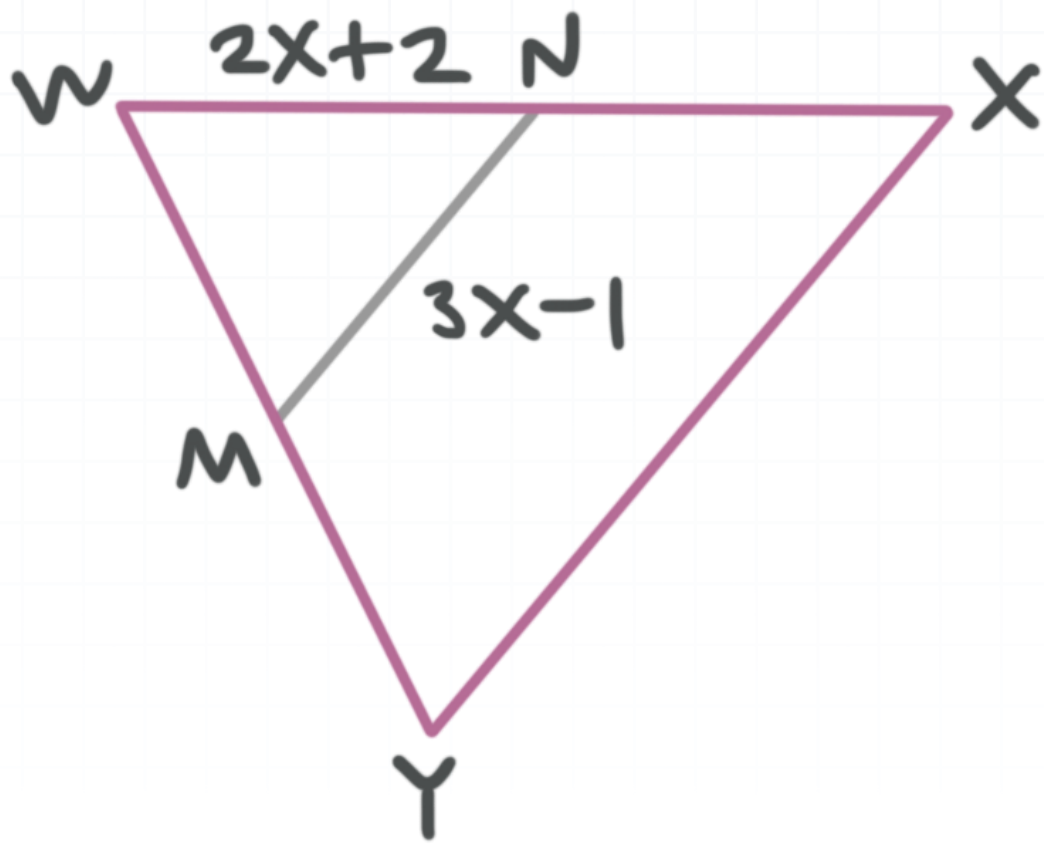
$$P = \overline{AC} + \overline{BC} + \overline{AC}$$

$$P = 8 + 14 + 10$$

$$P = 32$$

Topic: Midsegments of triangles

Question: In $\triangle WXY$, the midpoints of sides $\overline{WY}$ and $\overline{WX}$ are M and N, respectively, and $\overline{WX} = 24$. Find $\overline{XY}$.



Answer choices:

A 18

B 22

C 24

D 28

Solution: D

Because $\overline{WX} = 24$ and N is the midpoint of $\overline{WX}$, we have

$$2(2x + 2) = 24$$

$$4x + 4 = 24$$

$$4x = 20$$

$$x = 5$$

Because M and N are the midpoints of $\overline{WY}$ and $\overline{WX}$, respectively, we know that

$$\overline{MN} = \frac{1}{2}\overline{XY}$$

$$3x - 1 = \frac{1}{2}\overline{XY}$$

Substitute 5 for x.

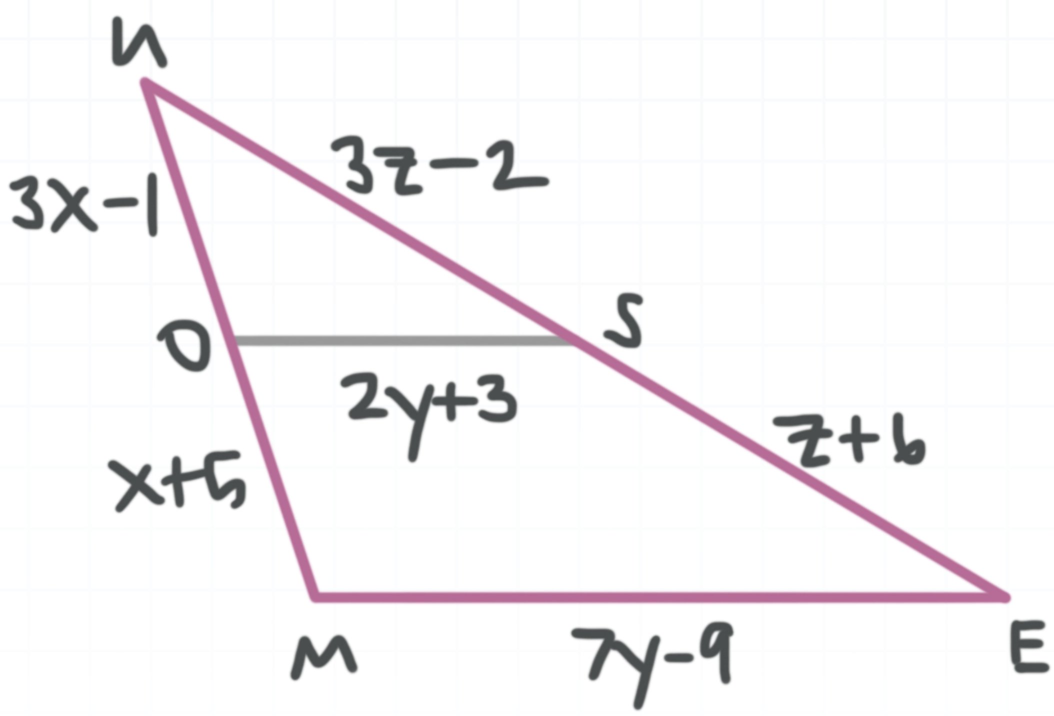
$$3(5) - 1 = \frac{1}{2}\overline{XY}$$

$$14 = \frac{1}{2}\overline{XY}$$

$$28 = \overline{XY}$$

Topic: Midsegments of triangles

Question: In $\triangle MUE$, the midpoints of sides $\overline{UM}$ and $\overline{UE}$ are O and S, respectively. Find the value of $x+y+z$.



Answer choices:

A $\quad 12$

B $\quad 15$

C $\quad 24$

D $\quad 25$

Solution: A

Because O is the midpoint of $\overline{UM}$,

$$3x - 1 = x + 5$$

$$2x = 6$$

$$x = 3$$

Because O and S are the midpoints of $\overline{UM}$ and $\overline{UE}$, respectively,

$$\overline{OS} = \frac{1}{2}\overline{ME}$$

$$2y + 3 = \frac{1}{2}(7y - 9)$$

Multiplying both sides of this equation by 2 (to clear the fraction), we get

$$4y + 6 = 7y - 9$$

$$15 = 3y$$

$$y = 5$$

Because S is the midpoint of $\overline{UE}$,

$$3z - 2 = z + 6$$

$$2z = 8$$

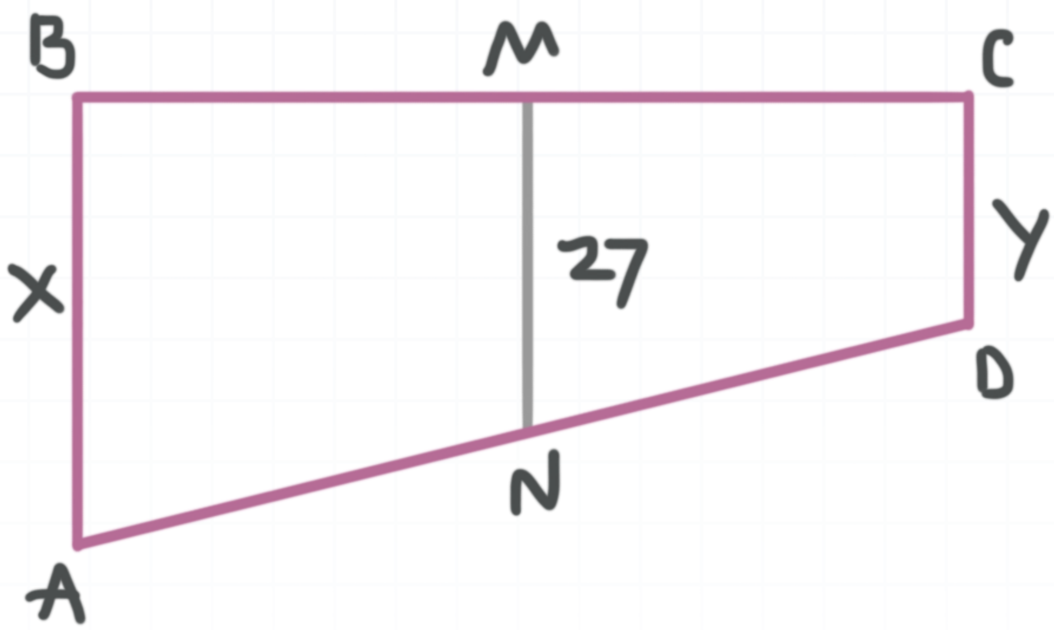
$$z = 4$$

Therefore,

$$x + y + z = 3 + 5 + 4 = 12$$

Topic: Midsegments of trapezoids

Question: In the figure, $ABCD$ is a trapezoid and M and N are midpoints of the opposite non-parallel sides. Find the value of $x+y$.



Answer choices:

A 27

B 34

C 45

D 54

Solution: D

You don't actually have to know the value of x or y to do this problem.

Because M and N are the midpoints of the opposite non-parallel sides of the trapezoid, $\overline{MN}$ is the midsegment, so

$$\overline{MN} = \frac{1}{2}(\overline{AB} + \overline{CD})$$

$$27 = \frac{1}{2}(x + y)$$

$$54 = x + y$$

Topic: Midsegments of trapezoids

Question: In the coordinate plane, a trapezoid $ABCD$ has vertices at $A=(0,8)$, $B=(12,8)$, $C=(9,1)$, and $D=(7,1)$. M and N are the midpoints of the opposite non-parallel sides. What is the length of $\overline{MN}$?

Answer choices:

A 2

B 5

C 7

D 12

Solution: C

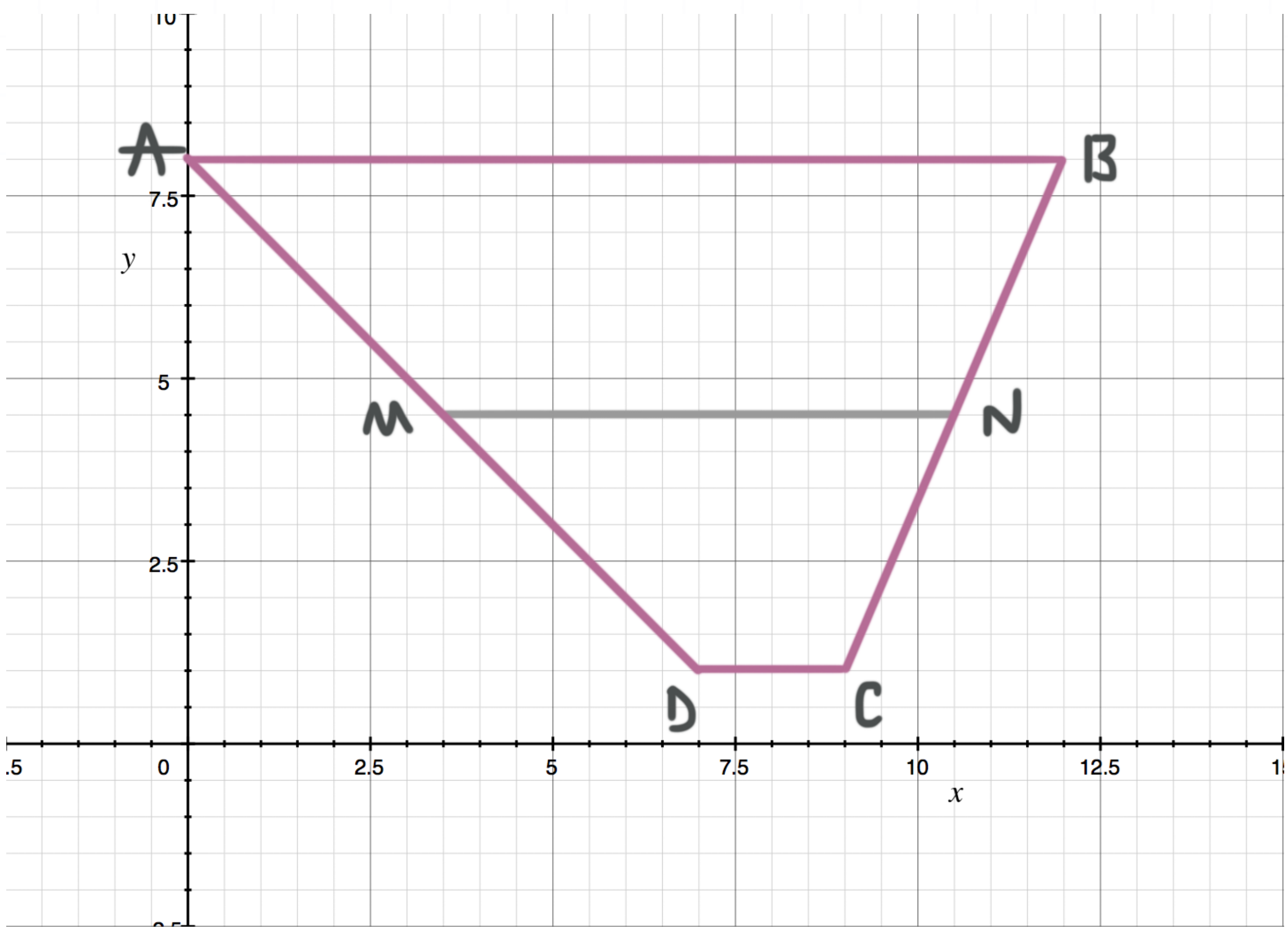
Notice that vertices A and B have the same y-coordinate of 8, and that vertices D and C have the same y-coordinate of 1. That means that $\overline{AB} \parallel \overline{CD}$ ($\overline{AB}$ and $\overline{DC}$ are the bases of the trapezoid).

Also, since the y-coordinates of vertices A and B are equal, the length of $\overline{AB}$ is the difference in their x-coordinates (which are 12 and 0), so

$$\overline{AB} = 12 - 0 = 12$$

Similarly, since the y-coordinate of vertices D and C are equal, the length of $\overline{DC}$ is the difference in their x-coordinates (which are 9 and 7), so

$$\overline{DC} = 9 - 7 = 2$$



Since M and N are the midpoints of the opposite non-parallel sides of this trapezoid, $\overline{MN}$ is the midsegment, so

$$\overline{MN} = \frac{1}{2}(\overline{AB} + \overline{DC})$$

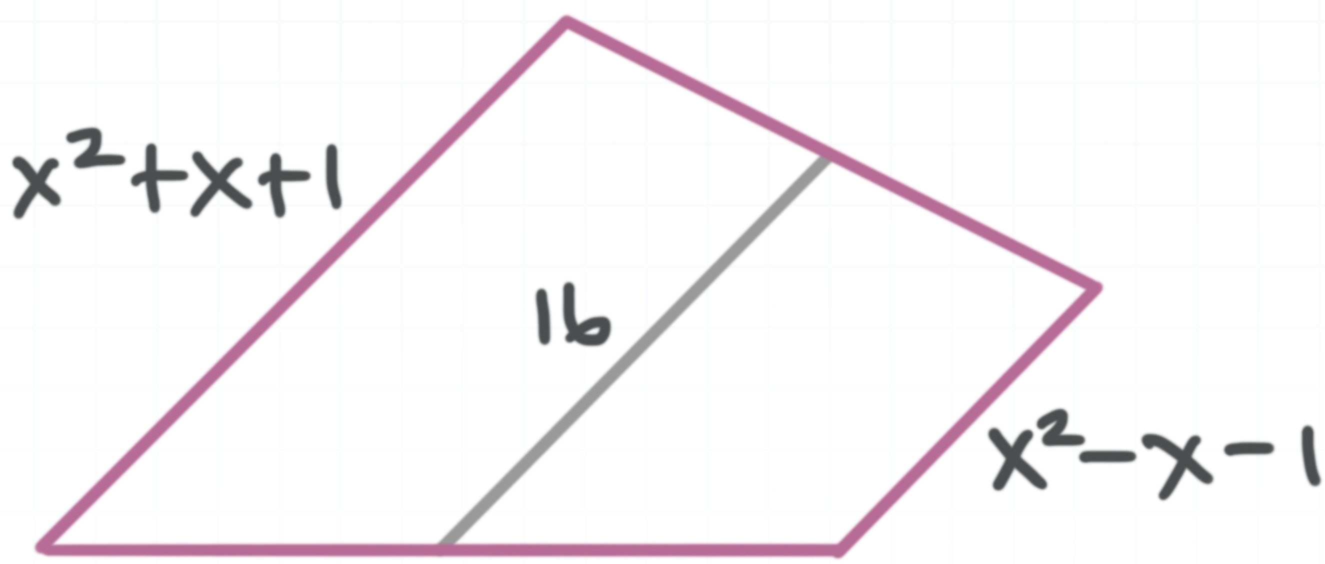
$$\overline{MN} = \frac{1}{2}(12 + 2)$$

$$\overline{MN} = \frac{1}{2}(14)$$

$$\overline{MN} = 7$$

Topic: Midsegments of trapezoids

Question: In the trapezoid the length of the midsegment is 16, the length of the longer base is x^2+x+1, and the length of the shorter base is x^2-x-1. Calculate the length of the shorter base.



Answer choices:

A $\quad 11$

B $\quad 12$

C $\quad 13$

D $\quad 14$

Solution: A

The length of the midsegment is half the sum of the lengths of the bases. Therefore,

$$16 = \frac{1}{2}\left[(x^2 + x + 1) + (x^2 - x - 1)\right]$$

$$16 = \frac{1}{2}(2x^2)$$

$$16 = x^2$$

$$\pm 4 = x$$

First, use $x = -4$ to calculate the lengths of the bases.

Length of the longer base: $(-4)^2 + (-4) + 1 = 13$

Length of the shorter base: $(-4)^2 - (-4) - 1 = 19$

These results are contradictory (they indicate that the length of the longer base is less than the length of the shorter base), so rule out $x = -4$. That leaves only $x = 4$. So we get

Length of the longer base: $4^2 + 4 + 1 = 21$

Length of the shorter base: $4^2 - 4 - 1 = 11$

Topic: Translating figures in coordinate space

Question: When a figure is translated in a coordinate plane, the image...

Answer choices:

A has the same shape as the pre-image.

B has the same size as the pre-image.

C has the same orientation as the pre-image.

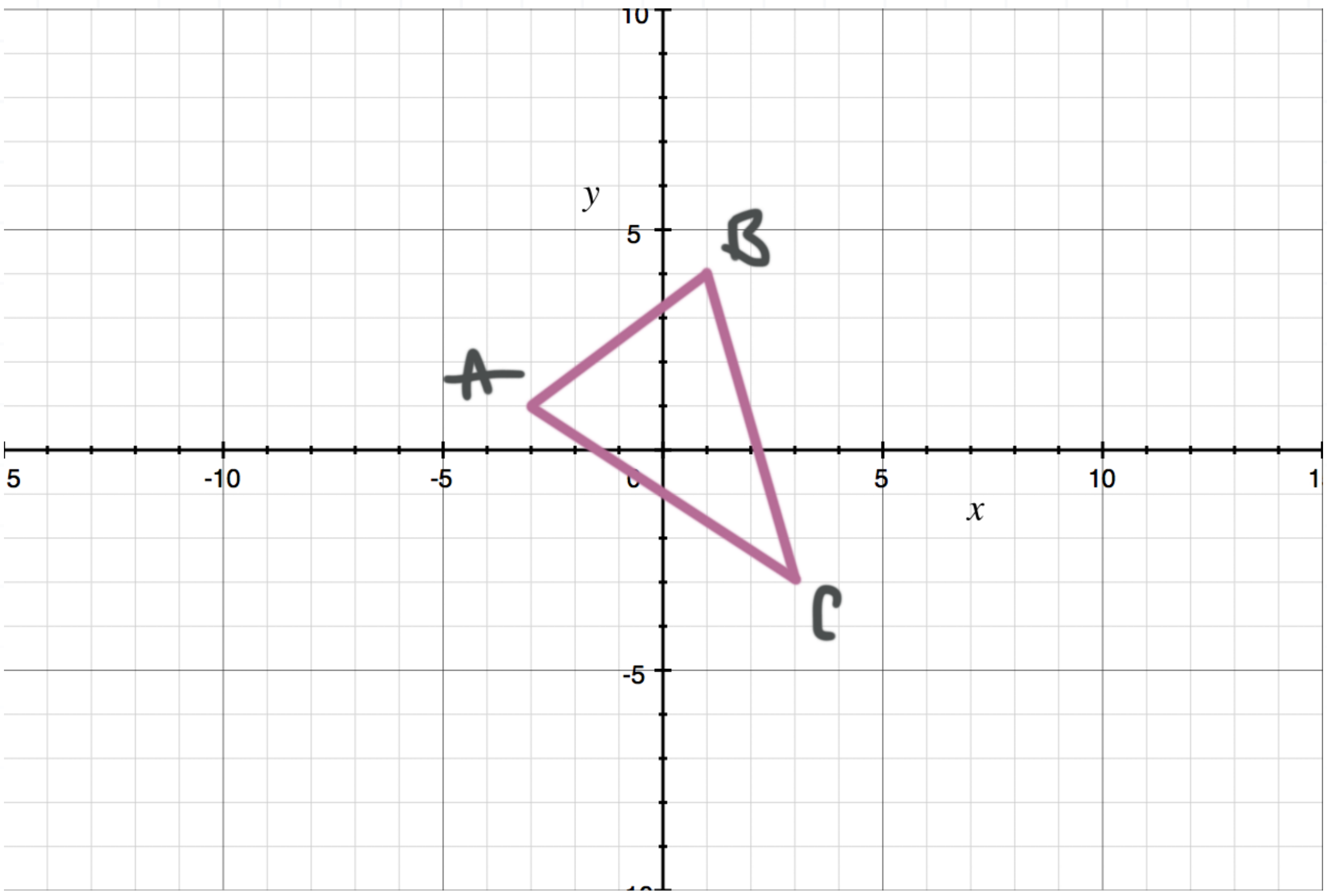
D has the same shape, size, and orientation as the pre-image.

Solution: D

A translation can be thought of as a slide with *no rotation*. The slide won't change the shape or size of the figure, and because there's no rotation, the orientation won't change either.

Topic: Translating figures in coordinate space

Question: If $\triangle ABC$ undergoes the translation described by $T(x, y) = (x + 5, y)$, to what point B' will point B be moved?



Answer choices:

A $(6,4)$

B $(1,9)$

C $(5,4)$

D $(-4,4)$

Solution: A

The translation is

$$T(x, y) = (x + 5, y)$$

The $x + 5$ tells you that the x-coordinate of any point in the image will be 5 more than the x-coordinate of the corresponding point in the pre-image, and the y tells you that the y-coordinate of any point in the image will be equal to the y-coordinate of the corresponding point in the pre-image.

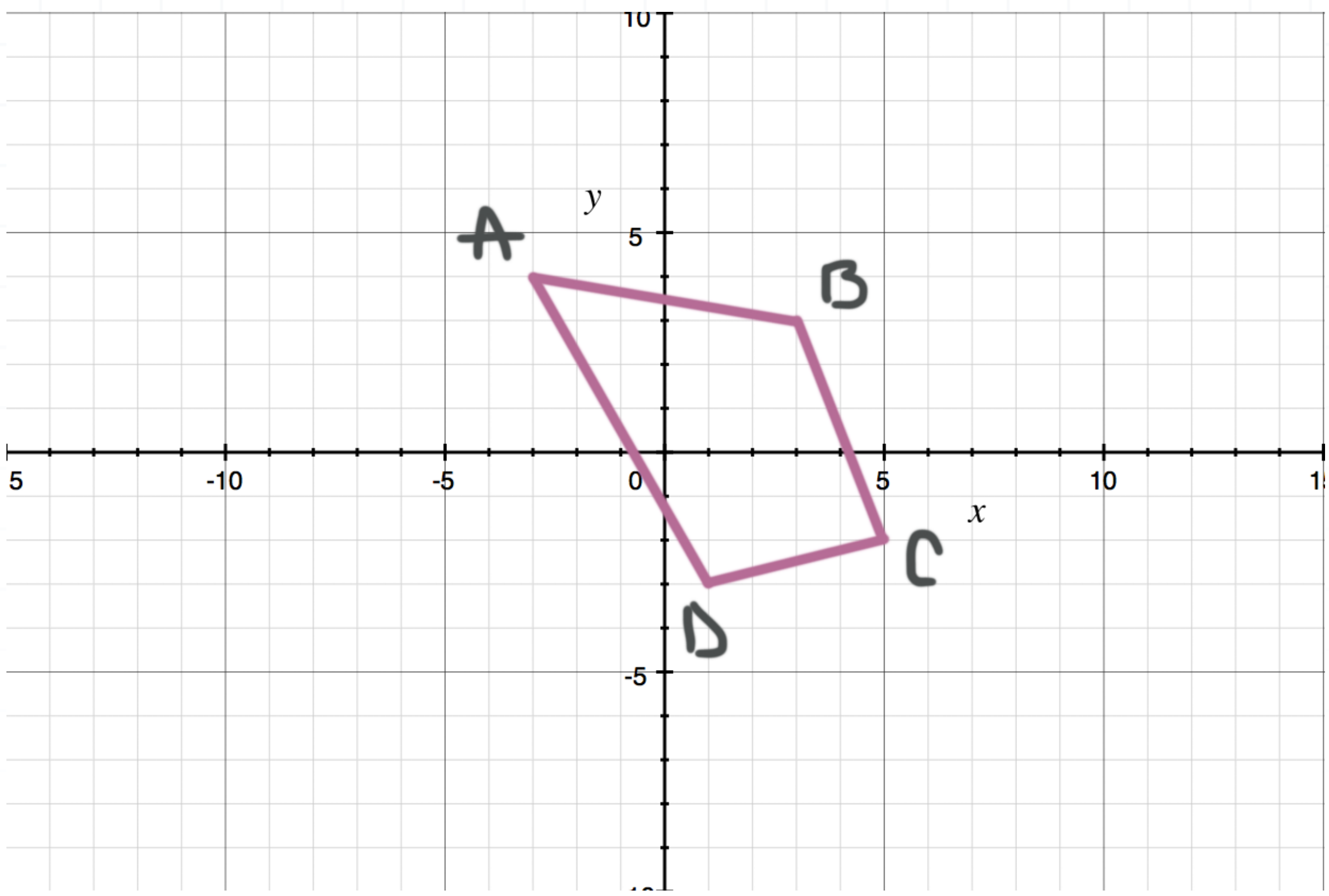
In other words, after the translation the figure will be located 5 units to the right of its original location.

The coordinates of point B are $(1,4)$, so the coordinates of point B' are given by

$$T(1,4) = (1 + 5,4) = (6,4)$$

Topic: Translating figures in coordinate space

Question: If quadrilateral $ABCD$ undergoes the translation described by $T(x, y) = (x, y - 3)$, to what point C' will point C be moved?



Answer choices:

A $(8, -2)$

B $(2, -2)$

C $(5,1)$

D $(5, -5)$

Solution: D

The translation is

$$T(x, y) = (x, y - 3)$$

The $y - 3$ tells you that the y-coordinate of any point in the image will be 3 less than the y-coordinate of the corresponding point in the pre-image, and the x tells you that the x-coordinate of any point in the image will be equal to the x-coordinate of the corresponding point in the pre-image.

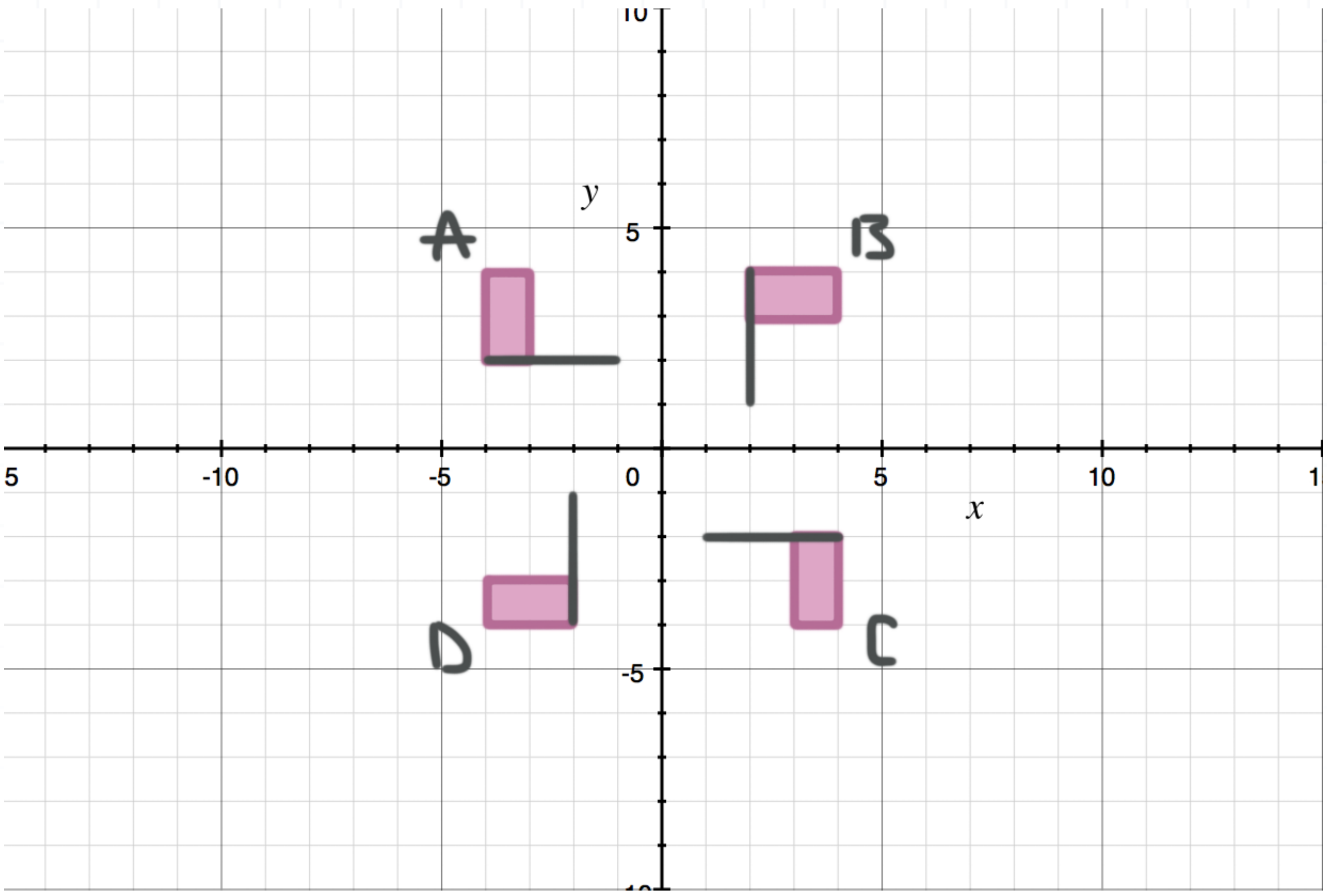
In other words, after the translation the figure will be located 3 units below its original location.

The coordinates of point C are $(5, -2)$, so the coordinates of point C' are given by

$$T(5, -2) = (5, -2 - 3) = (5, -5)$$

Topic: Rotating figures in coordinate space

Question: Suppose one of the flags A, B, or C is rotated around $(0,0)$ to position D. Which statement could not describe this rotation?



Answer choices:

A $\quad$ A is rotated 90° counterclockwise.

B $\quad$ B is rotated 90° clockwise.

C $\quad$ B is rotated 180° clockwise.

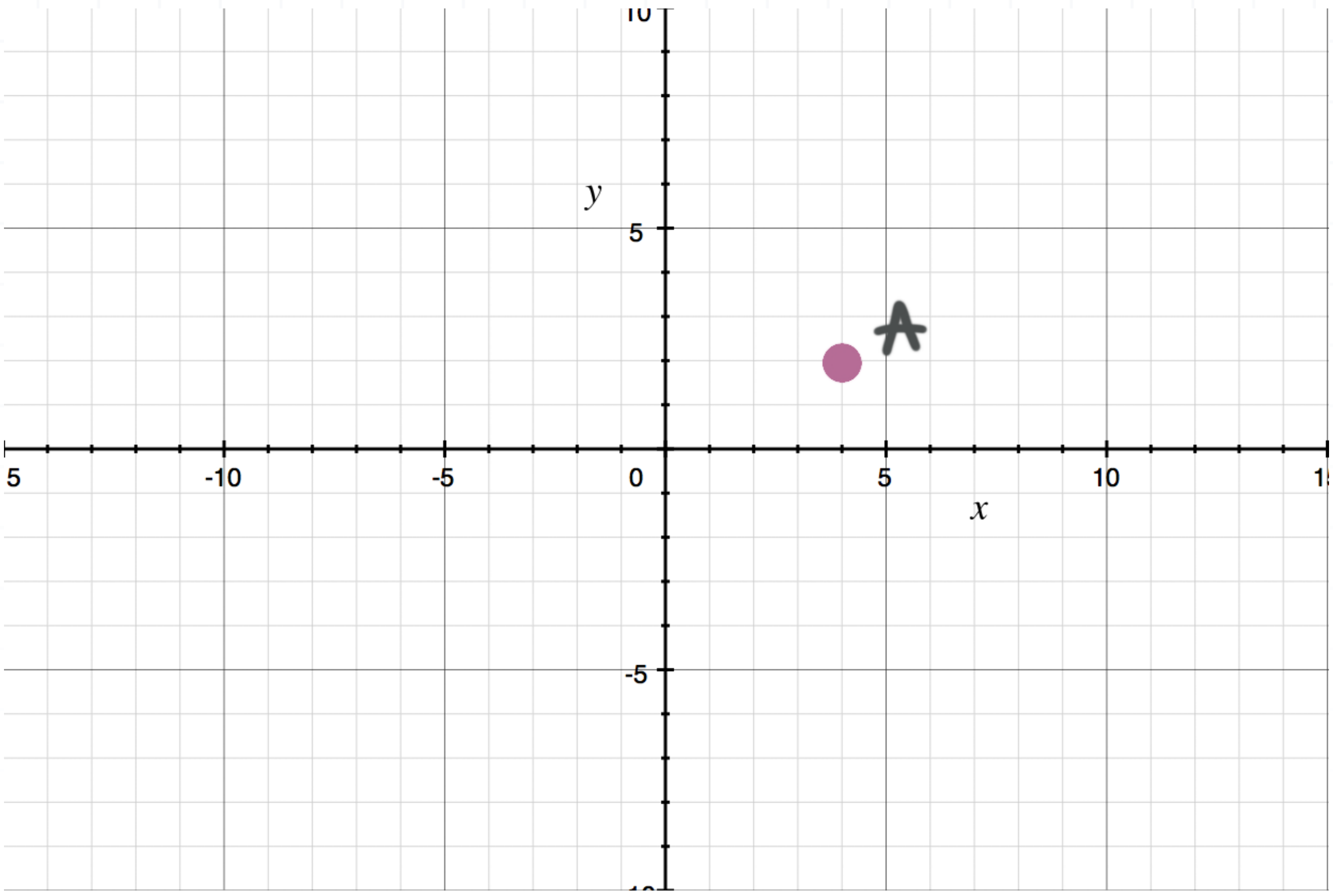
D $\quad$ C is rotated 270° counterclockwise.

Solution: B

Answer choice B, “B is rotated 90° clockwise,", would move flag B to position C, not to position D, so the statement given in answer choice B would not work.

Topic: Rotating figures in coordinate space

Question: In a 90° clockwise rotation around $(0,0)$, to what point A' will point A be moved?



Answer choices:

A $(-2,4)$

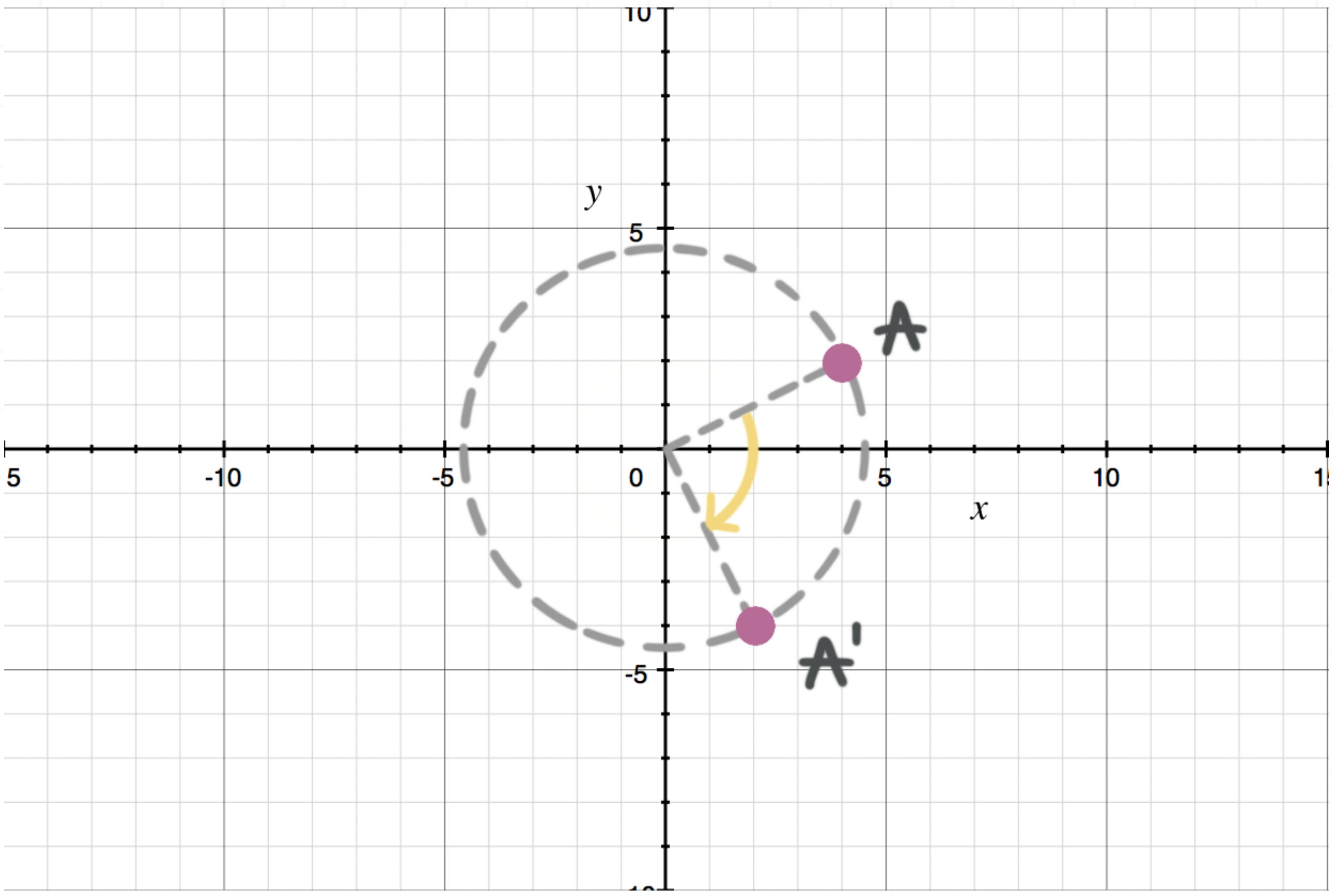
B $(-4,2)$

C $(-2,-4)$

D $(2,-4)$

Solution: D

The figure shows what happens to point A in a 90° clockwise rotation about the origin. It's moved to point $A' = (2, -4)$.



Topic: Rotating figures in coordinate space

Question: Which rotation describes a point (h, k) that's moved to the point $(-h, -k)$ in a rotation around $(0,0)$?

Answer choices:

A Clockwise 180°

B Counterclockwise 90°

C Counterclockwise 270°

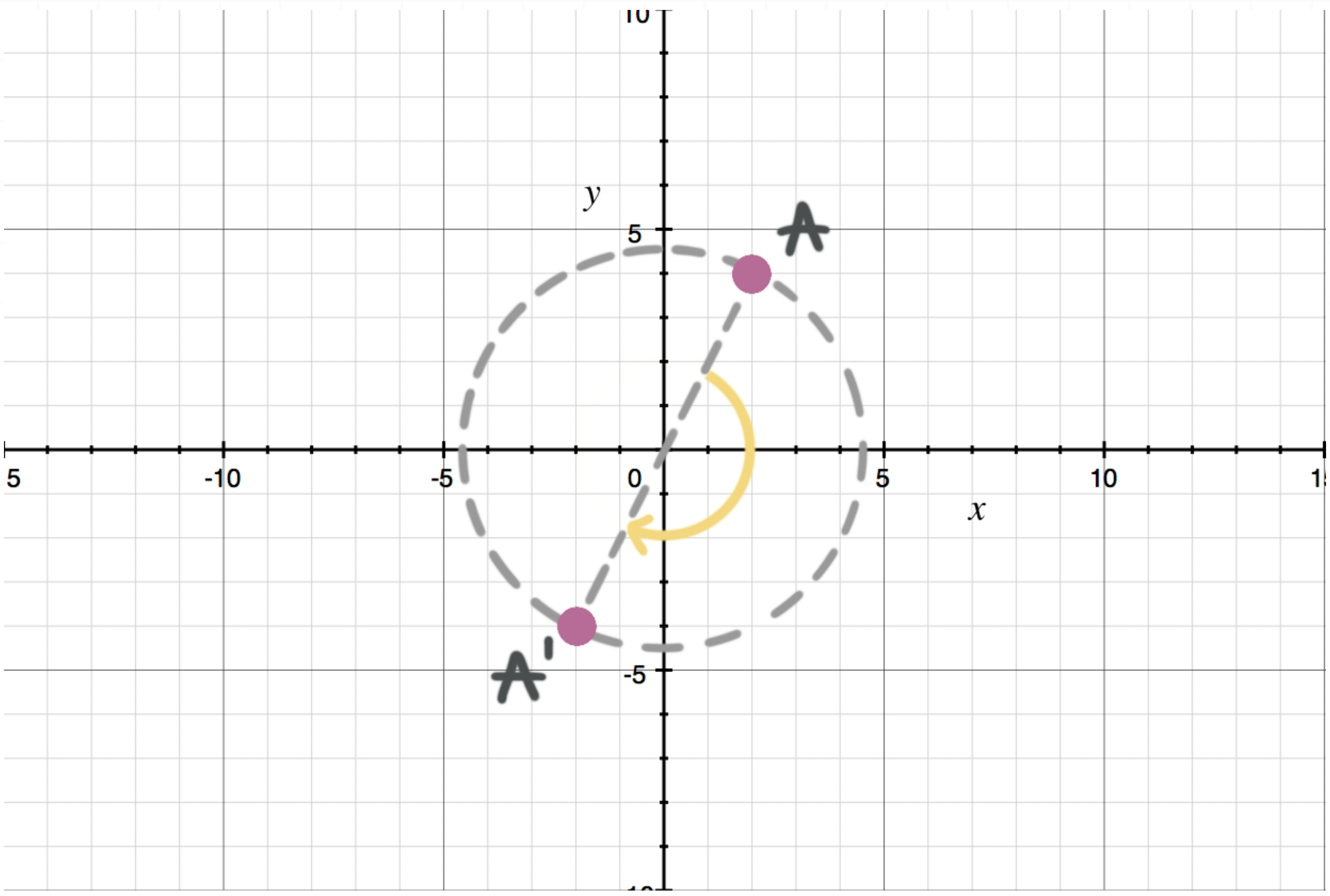
D Clockwise 270°

Solution: A

The easiest way to solve this problem is to use a numerical example.

Let $(h, k) = (2,4)$, plot this point, and label it A. Now $(-h, -k) = (-2, -4)$. Plot and label it A'.

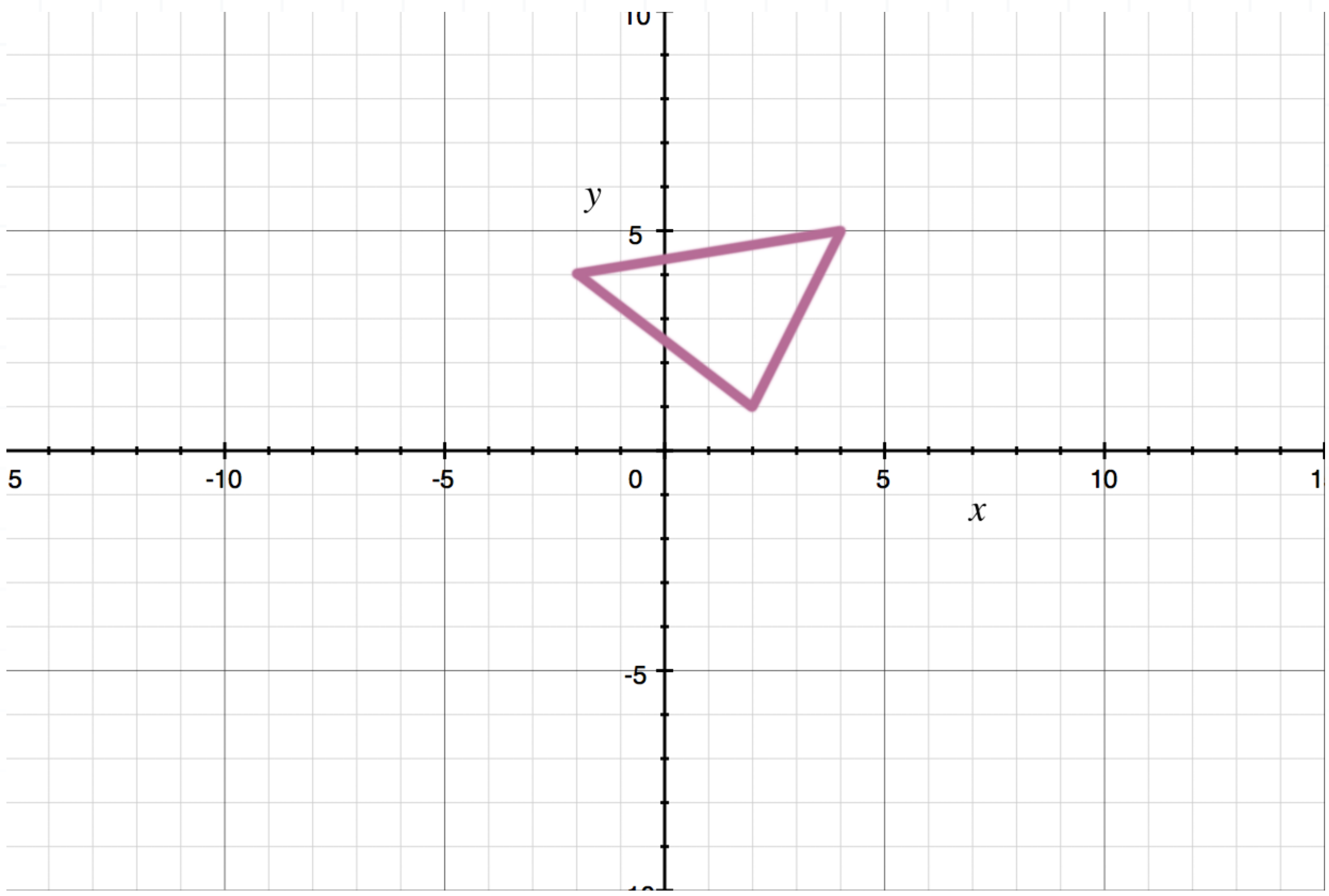
It's pretty clear that A' is on the line that passes through A and $(0,0)$, so A' is exactly "halfway around the circle from A." In other words, we see a clockwise rotation of 180°.



Answer choice A works. Note: It would have been just as valid to say a counterclockwise rotation of 180°.

Topic: Reflecting figures in coordinate space

Question: If the triangle is reflected in the x-axis, what is the lowest point in the image?



Answer choices:

A $(4, -5)$

B $(2,1)$

C $(2, -1)$

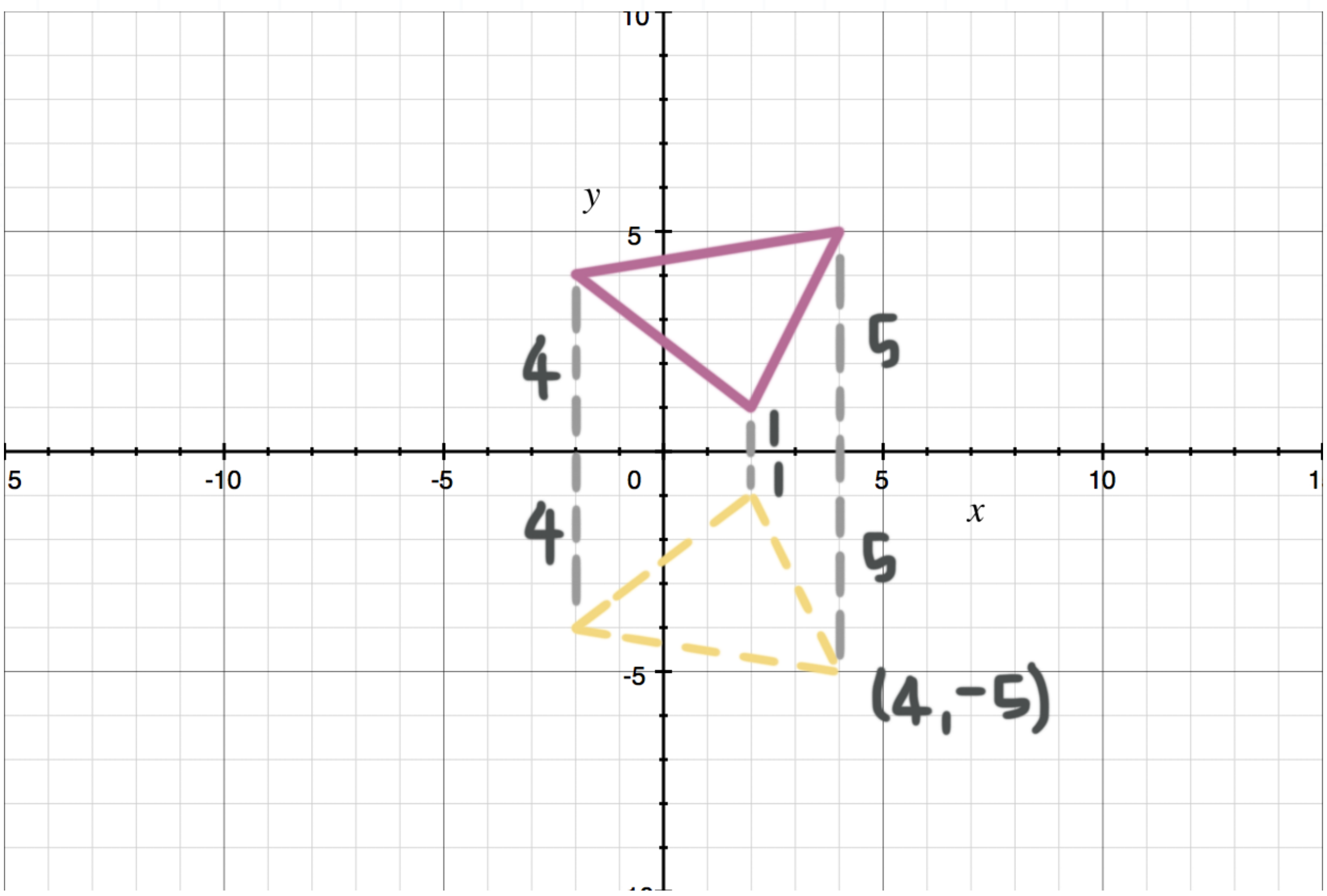
D $(-2, -4)$

Solution: A

The coordinates of the vertices of the triangle are $(-2,4)$, $(2,1)$, and $(4,5)$.

Since the triangle is being reflected across the x-axis, the lowest point in the image (the point with the least y-coordinate) will be the one that corresponds to the highest point in the pre-image (the point with the greatest y-coordinate).

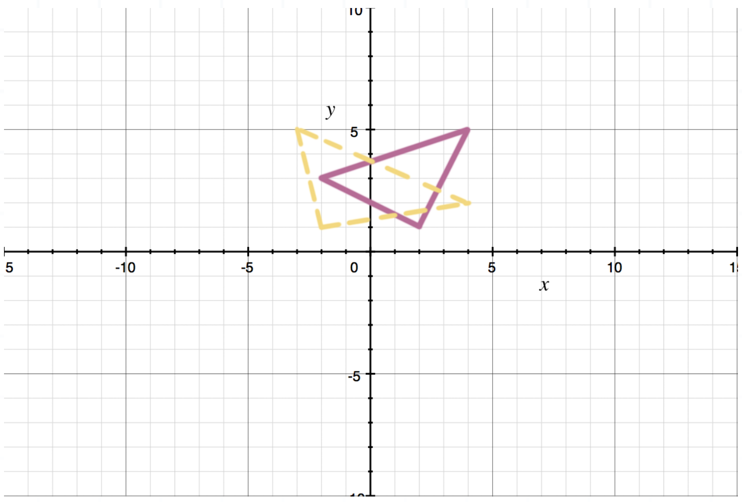
The highest point in the pre-image is $(4,5)$, so the lowest point in the image is $(4,-5)$.

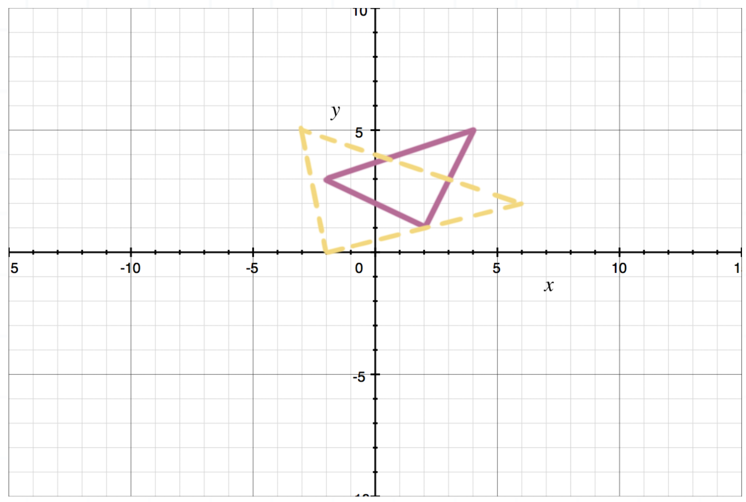


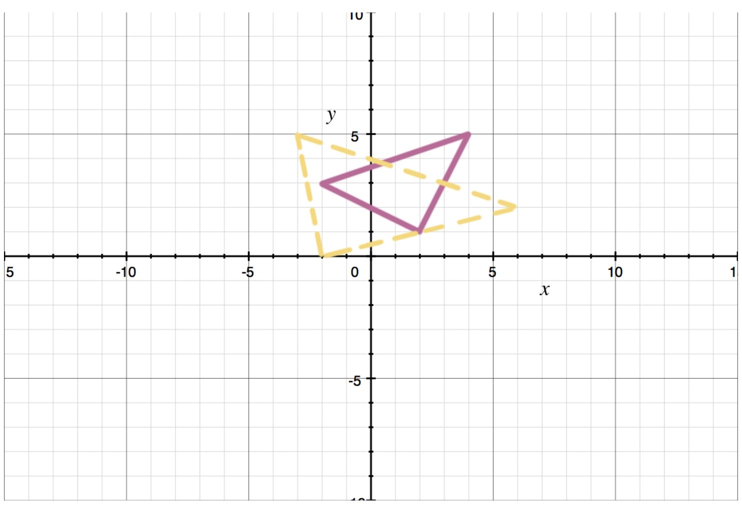
Topic: Reflecting figures in coordinate space

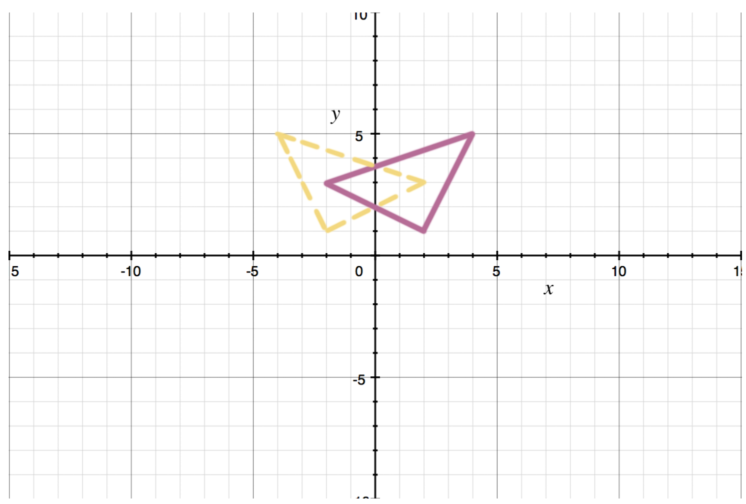
Question: The triangle is reflected in the y-axis. Which sketch correctly shows the reflection?

Answer choices:

A


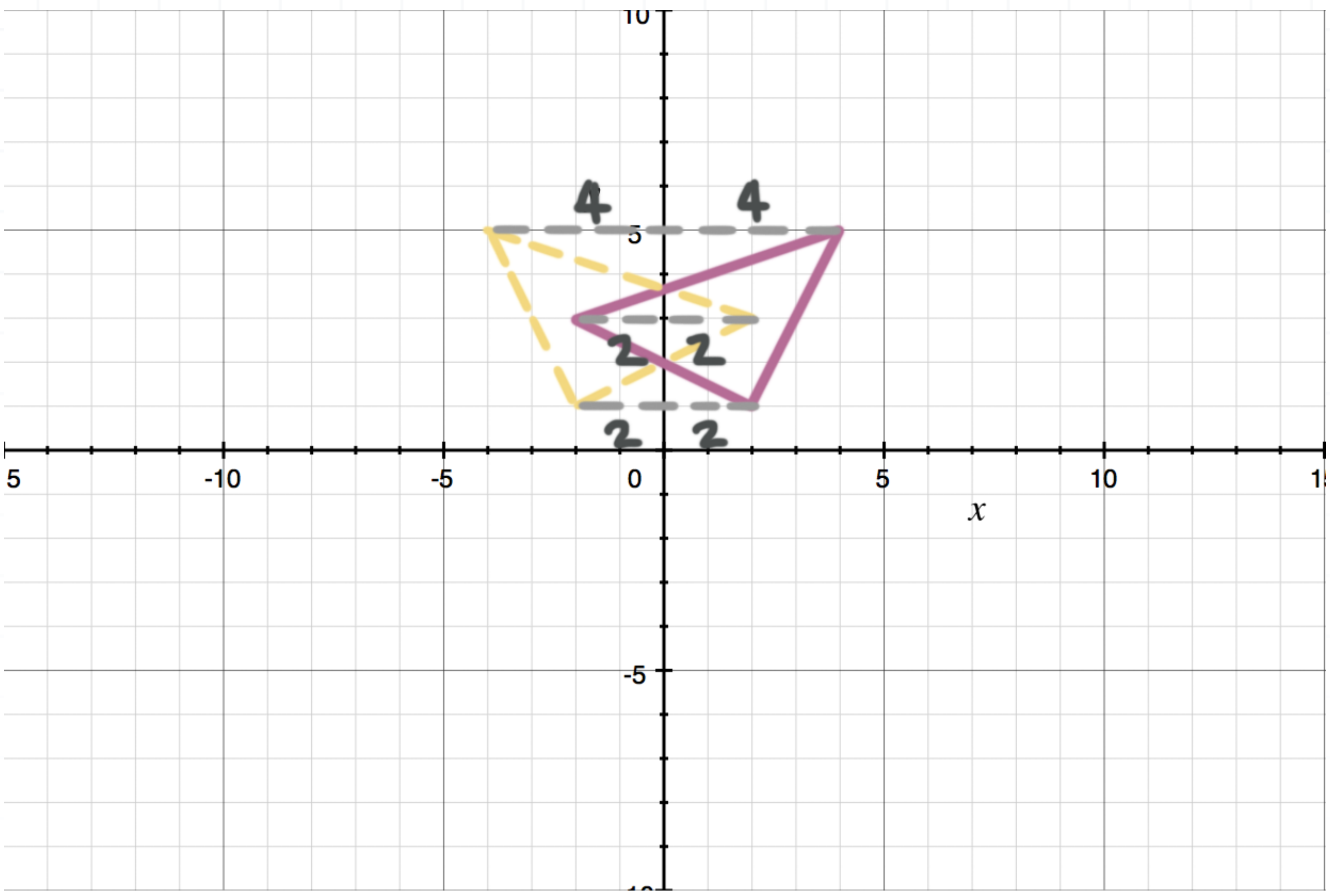
B


C


D


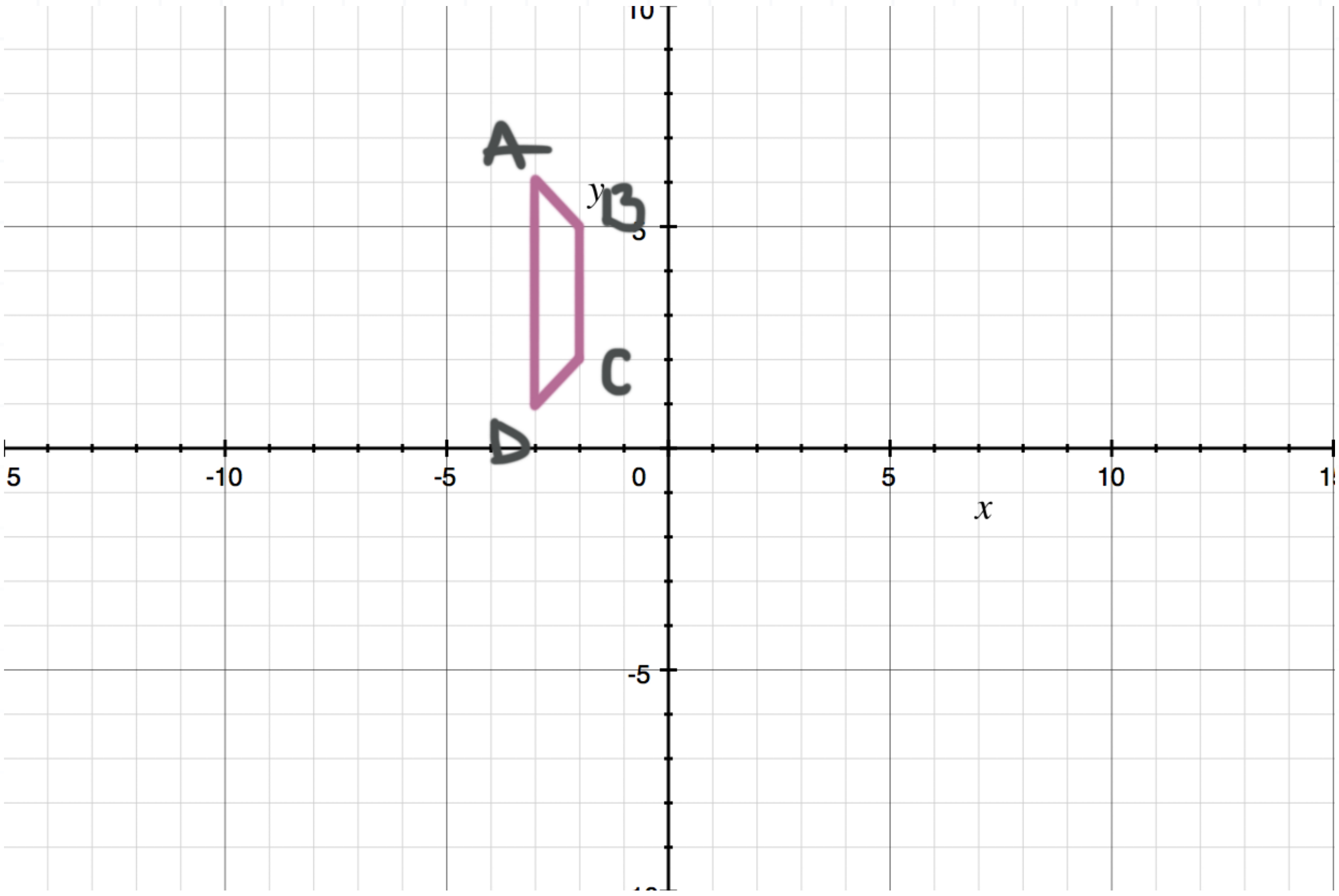
Solution: D

In figure D, each vertex of the triangle in the image and the corresponding vertex in the pre-image have the same y-coordinate and are equidistant from the y-axis.



Topic: Reflecting figures in coordinate space

Question: If the trapezoid $ABCD$ is reflected in the line $x = 2$, what is the point D' in the image that corresponds to point D in the pre-image?



Answer choices:

A $(3,1)$

B $(2,1)$

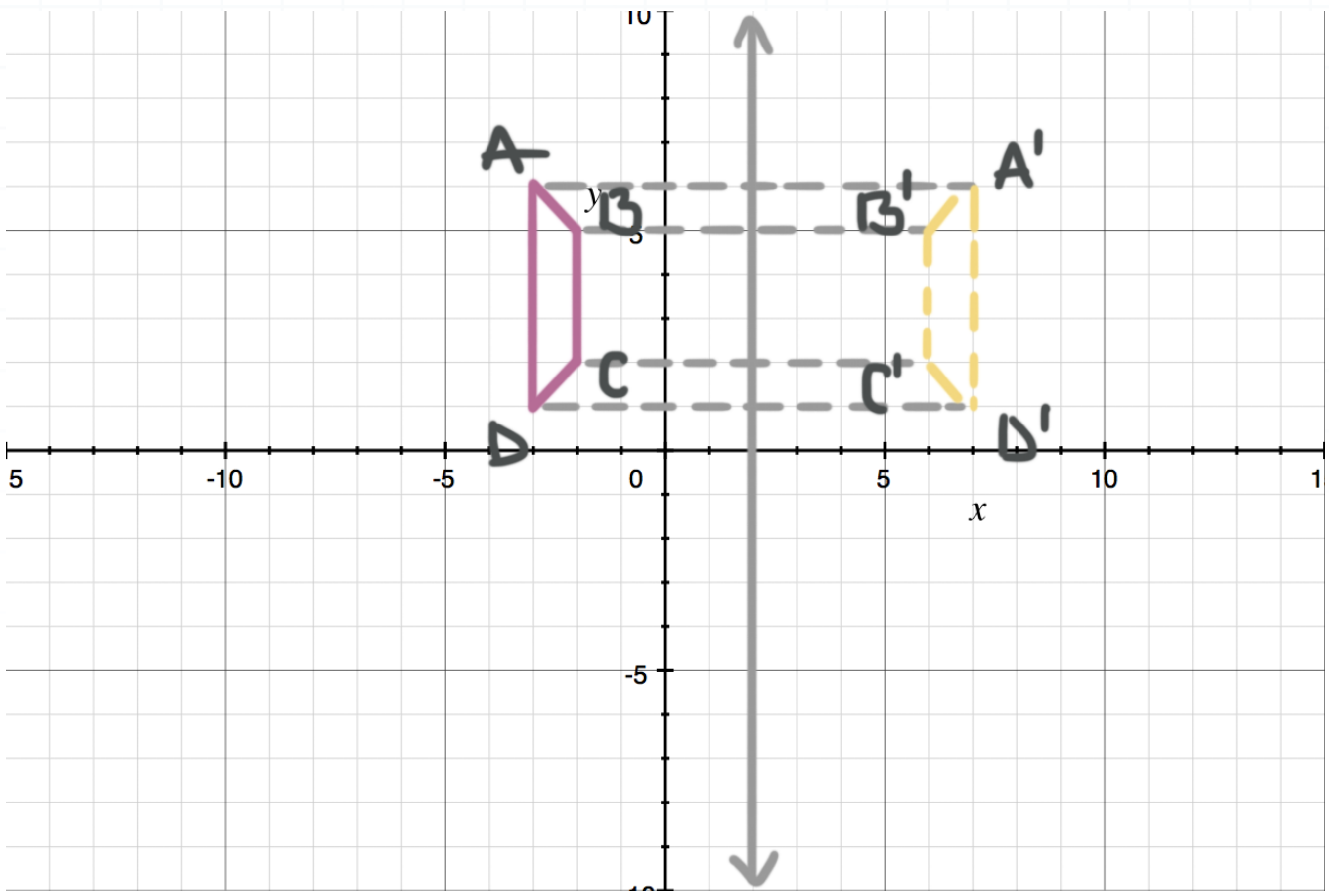
C $(7,1)$

D $(6,1)$

Solution: C

The coordinates of point D are $(-3,1)$, so the distance from D to the line $x = 2$ is $2 - (-3) = 5$.

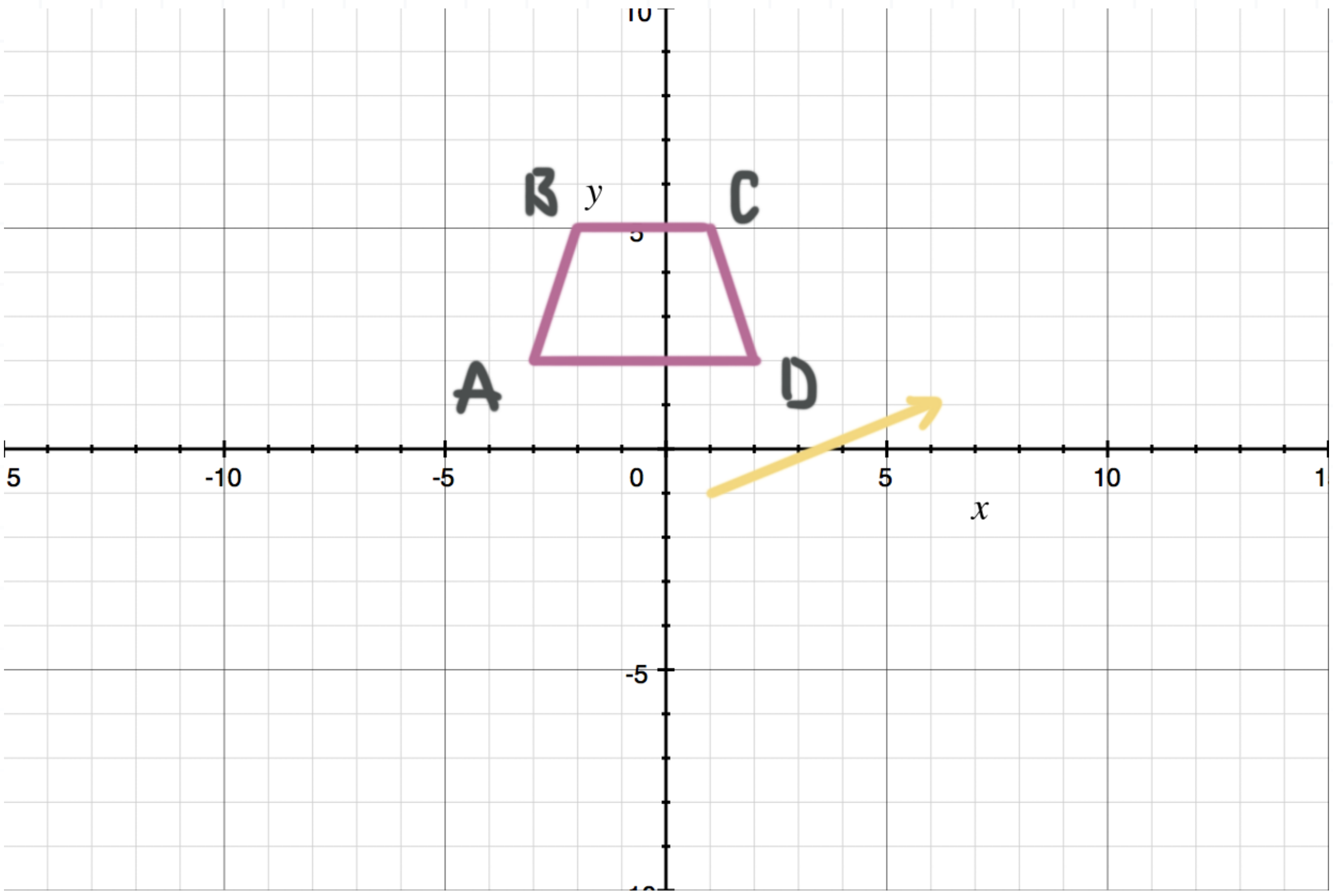
Since D is 5 units to the left of the line $x = 2$, D' is 5 units to the right of that line. Therefore, the x-coordinate of D' is $2 + 5 = 7$, and the y-coordinate of D' is equal to that of D.



That gives $(7,1)$ for the location of D'.

Topic: Translation vectors

Question: If trapezoid $ABCD$ undergoes a translation to $A'B'C'D$ as indicated by the vector shown, what are the coordinates of point C'?



Answer choices:

A $(6,7)$

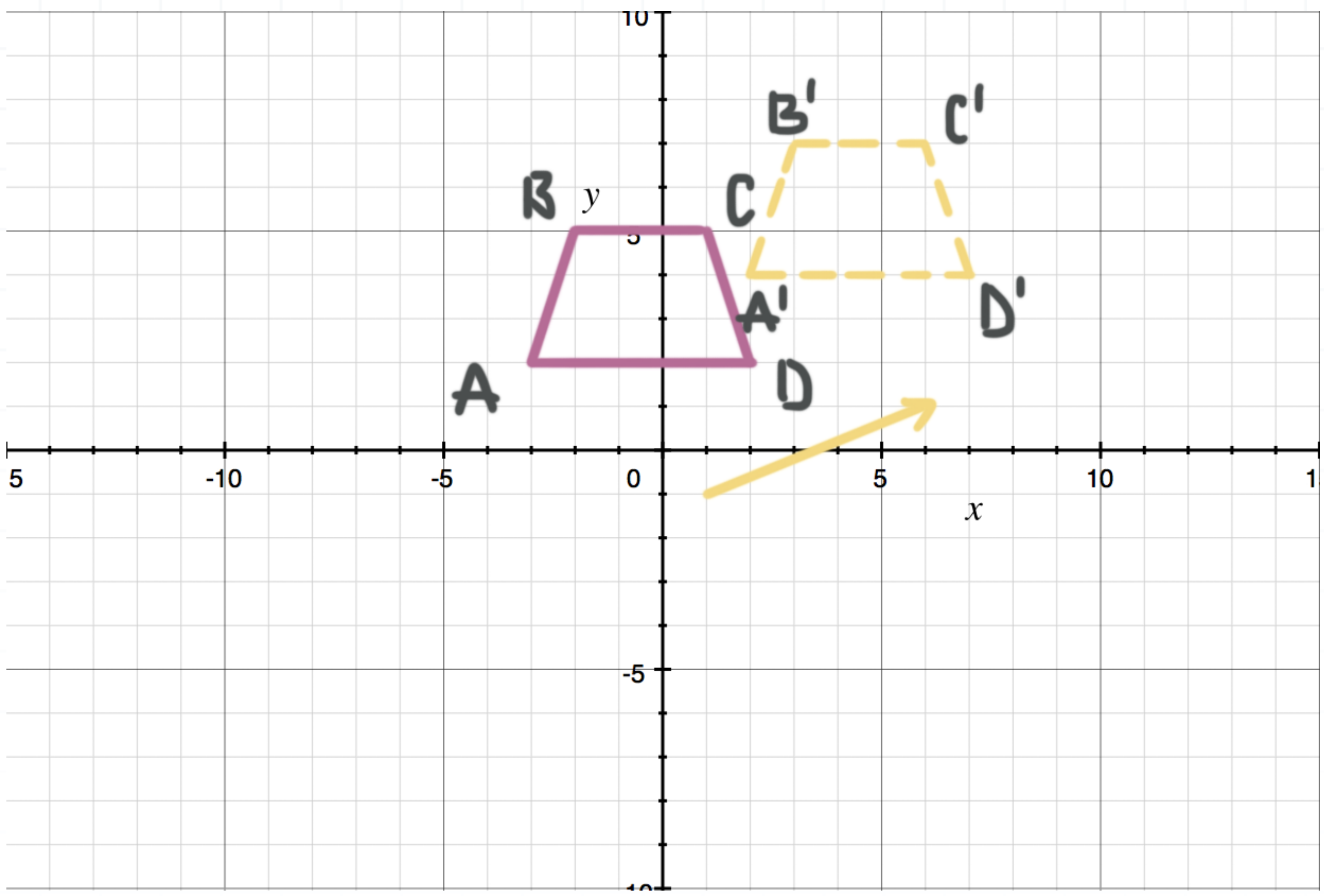
B $(5,2)$

C $(3,10)$

D $(2,4)$

Solution: A

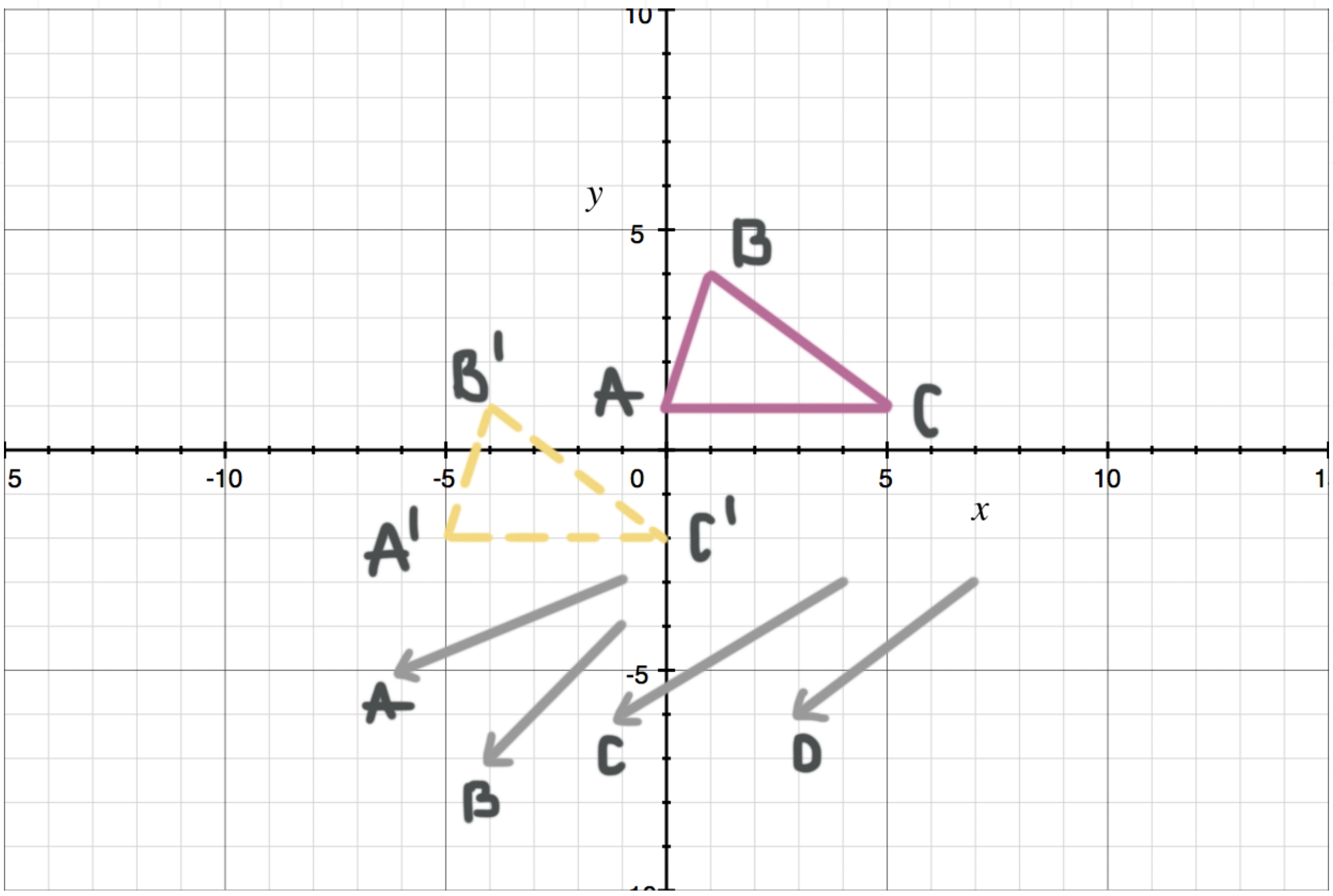
Though it's not totally necessary for solving the problem, the figure below shows the entire trapezoid being translated.



The tail and head of the translation vector are at $(1, -1)$ and $(6,1)$, respectively, which indicates a translation of $6 - 1 = 5$ units to the right and $1 - (-1) = 2$ units up. Point C is at $(1,5)$, so we need to add 5 to its x-coordinate and 2 to its y-coordinate. The result, $(6,7)$, is the location of C'.

Topic: Translation vectors

Question: $\triangle ABC$ undergoes a translation to $\triangle A'B'C'$. Which of the vectors shown would produce that translation?



Answer choices:

A A

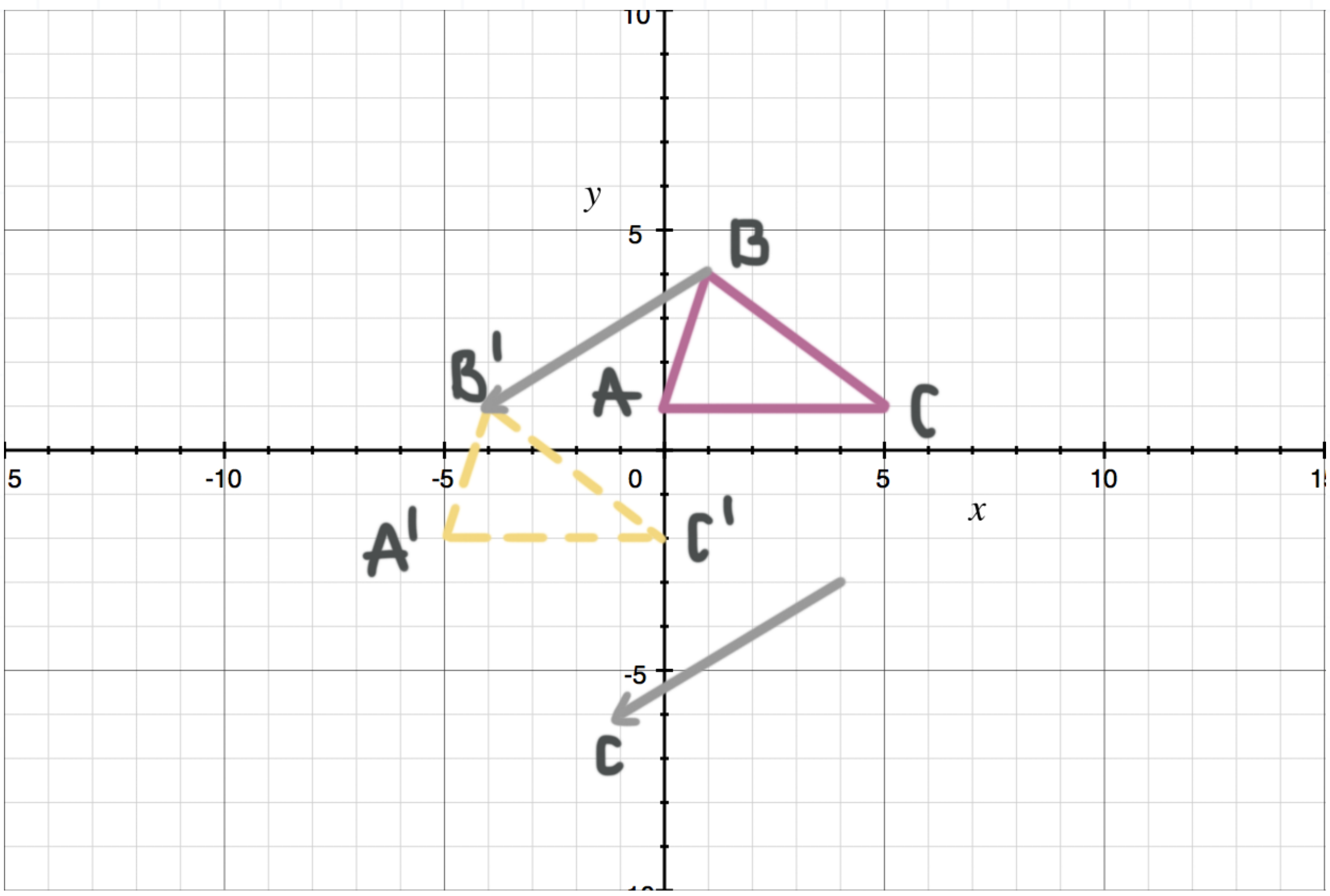
B B

C C

D D

Solution: C

The coordinates of points B and B' are $(1,4)$ and $(-4,1)$, respectively. The x-coordinate of B' is 5 less than that of B, and the y-coordinate of B' is 3 less than that of B, so we have to move 5 units to the left and 3 units down to get from B to B'.



Vector C is the vector that matches that translation, since its tail and head are at $(4,-3)$ and $(-1,-6)$, respectively, which means that C indicates a horizontal translation of $-1-4=-5$ units (5 units to the left) and a vertical translation of $-6-(-3)=-3$ units (3 units down).

Topic: Translation vectors

Question: The tail and head of translation vector A are at $(3,1)$ and $(0,7)$, respectively. The tail and head of a second translation vector B are at $(0,7)$ and $(-1,4)$, respectively. Determine a single translation vector that would accomplish the same translation as the given two vectors in succession (vector A followed by vector B). What is its length?

Answer choices:

A 5

B 4

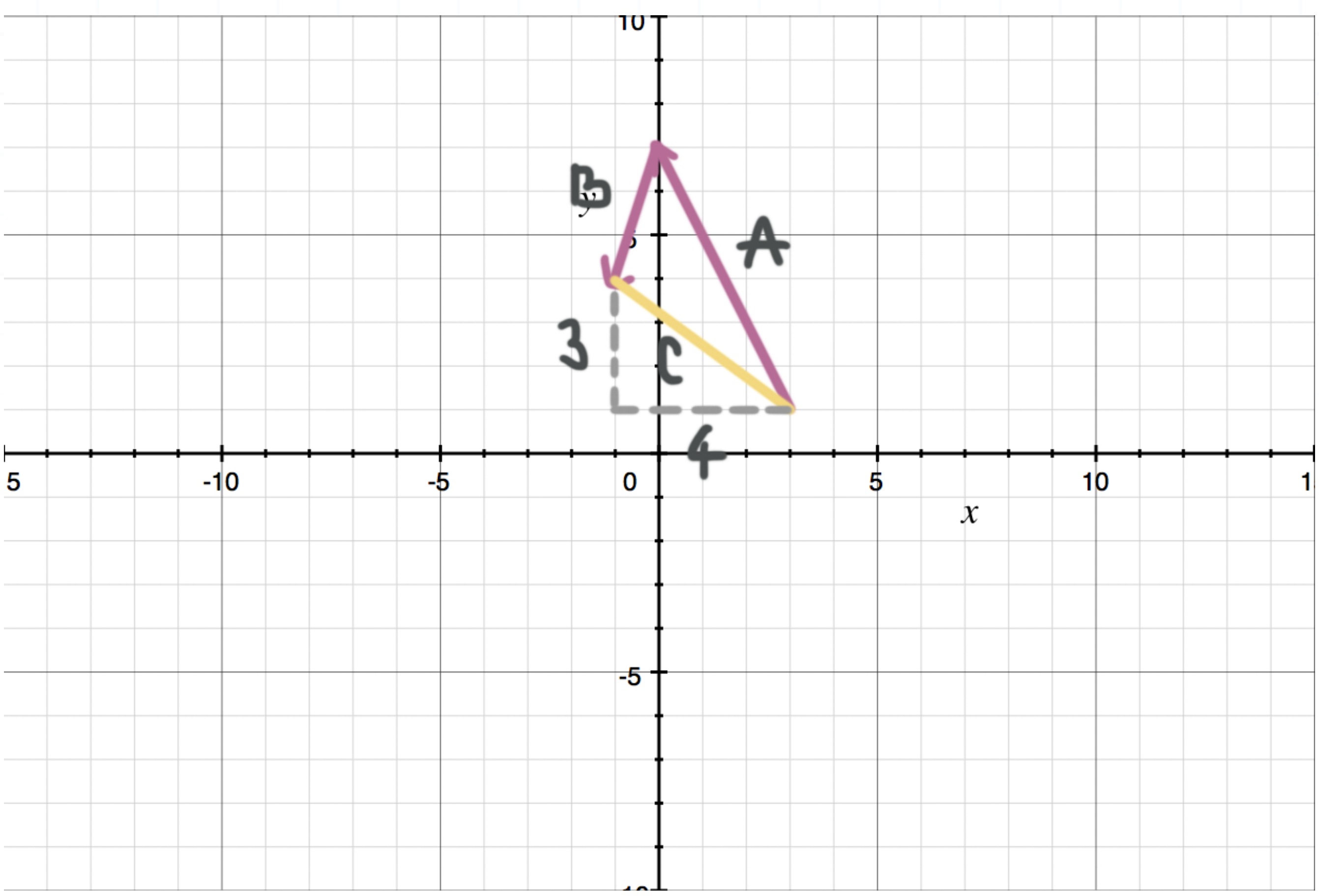
C 3

D 2

Solution: A

Since translation vector C must produce the same translation in one step as the translation vectors A and B in succession, the tail and head of vector C are at the tail of vector A and the head of vector B.

Therefore, the tail and head of vector C are at $(3,1)$ and $(-1,4)$, respectively, so translation vector C indicates a horizontal translation of $-1-3=-4$ units (4 units to the left) and a vertical translation of $4-1=3$ units (3 units up).

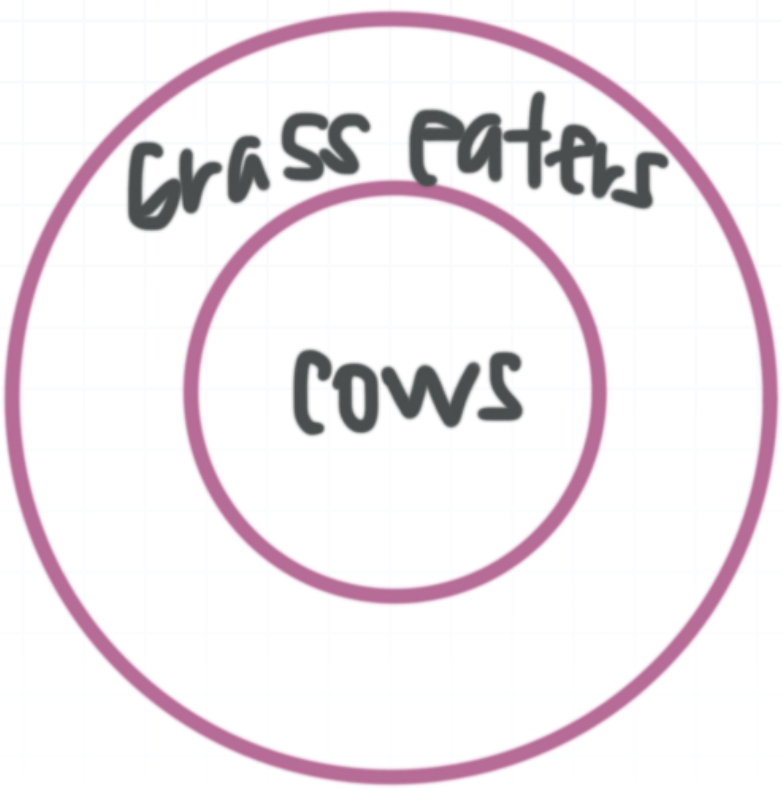


Notice that the line segment that represents vector C is the hypotenuse of a right triangle in which the lengths of the legs are 4 and 3 (the absolute values of the horizontal and vertical translations, respectively, indicated by vector C), so the length of vector C must be the length of the hypotenuse of that right triangle, which is

$$\sqrt{(4^2) + (3^2)} = \sqrt{16 + 9} = \sqrt{25} = 5$$

Topic: Conditionals and Euler diagrams

Question: Which if/then statement corresponds to the Euler diagram below?



Answer choices:

A If it's a cow, it eats grass.

B If it eats grass, it's a cow.

C If it eats oats, it's not a cow.

D If it's a cow, it doesn't eat Jell-O.

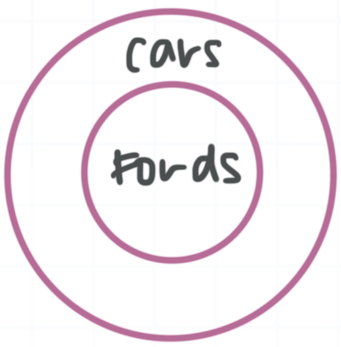
Solution: A

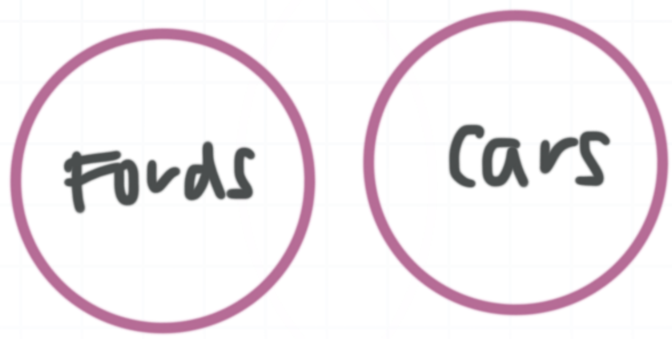
Answer choice A is correct. The set of cows is entirely inside the set of grass eaters, meaning that every cow is a grass eater.

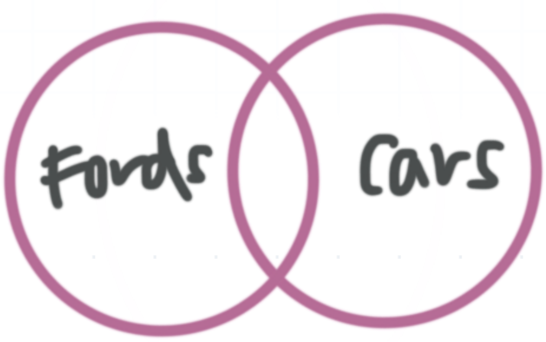
Topic: Conditionals and Euler diagrams

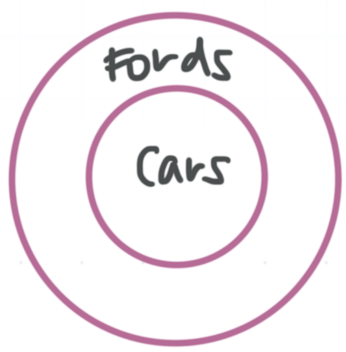
Question: Choose the Euler diagram below that corresponds to the statement “All cars are Fords.”

Answer choices:

A


B


C


D


Solution: D

Answer choice D is the right one. The statement “All cars are Fords” is equivalent to the conditional statement

“If it is a car, then it is a Ford.”

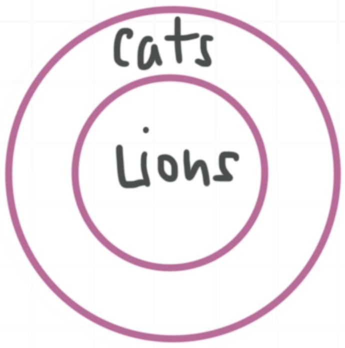
Diagram-wise, that means that the set of all cars must be totally inside the set of all Fords, as in D.

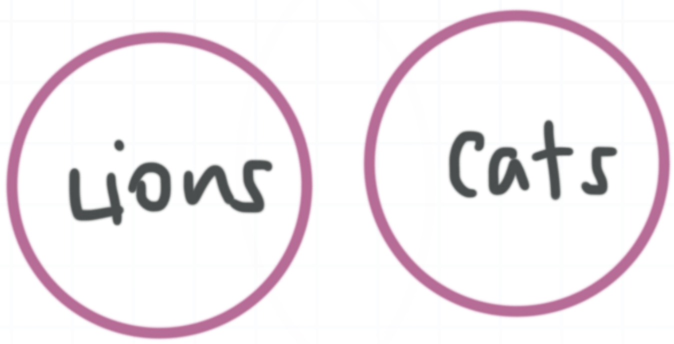
Of course, the statement “All cars are Fords” happens to be false, but D is still the correct diagram for that statement.

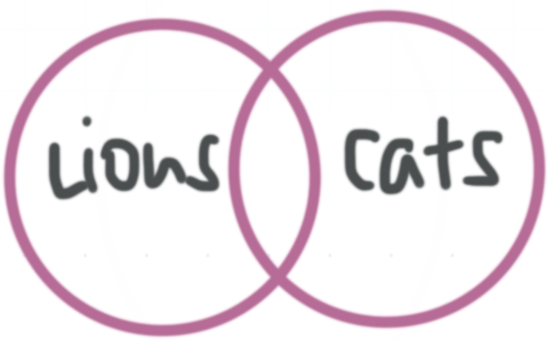
Topic: Conditionals and Euler diagrams

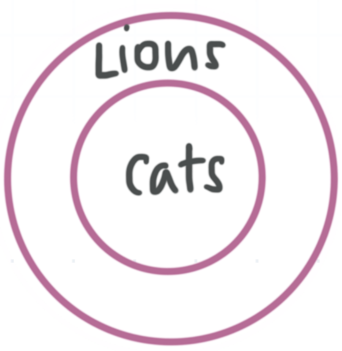
Question: Choose the Euler diagram below that corresponds to the statement “Every lion is a cat, but not every cat is a lion.”

Answer choices:

A 

B 

C 

D 

Solution: A

The statement “Every lion is a cat” tells us that the group “lions” is contained within the group “cat.” And the statement “not every cat is a lion” tells us that there are some cats which are not lions (like domesticated cats, tigers, leopards, etc.).

Notice how diagram A shows that within the set of all cats, there is a set of lions, but there's an area around lions to represent the set of cats that aren’t lions.

In other words, it shows that only some of the cats are lions, but that every single lion must also be called a cat, just what we wanted it to show.

Topic: Converses of conditionals

Question: Choose the converse statement of “If I'm lying, then my lips are moving.”

Answer choices:

A If I'm not lying, then my lips are not moving.

B If my lips are moving, then I'm lying.

C If my lips are not moving, then I'm not lying.

D If I'm telling the truth, then my lips are moving.

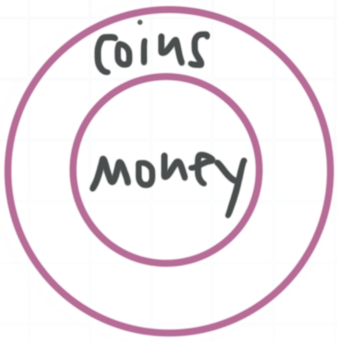
Solution: B

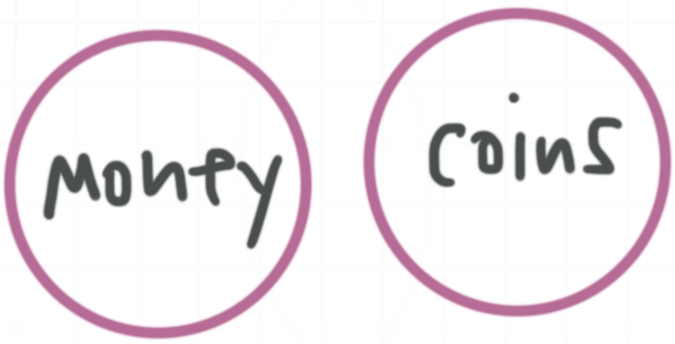
The two phrases “my lips are moving” and “I'm lying” have been switched, which is how the converse of a conditional statement is formed.

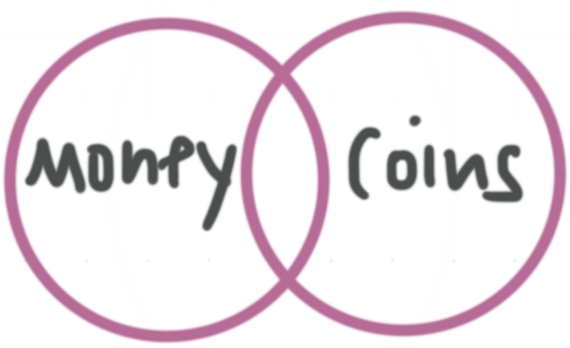
Topic: Converses of conditionals

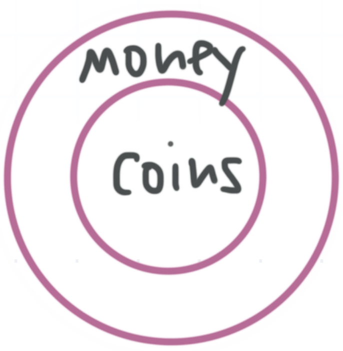
Question: Choose the Euler diagram that corresponds to the converse of the statement “All coins are money.”

Answer choices:

A 

B 

C 

D 

Solution: A

The statement “All coins are money” can be phrased as an if/then statement:

“If it's a coin, then it's money.”

The converse would be

“If it's money, then it's a coin.”

Diagram A shows that if a thing is money (in the Money circle), it's always a coin (in the Coin circle). In other words, all money is made of coins.

Topic: Converses of conditionals

Question: Suppose a certain if/then statement is true. Will the converse of that statement be true?

Answer choices:

A Sometimes

B Always

C Never

D Only on Tuesdays

Solution: A

Whether the converse is true depends totally on the original statement. Sometimes the converse is true, and sometimes it's not.

Note: If the original statement is a *definition*, its converse will be true.

Example: If it's a triangle, it has three sides.

Converse: If it has three sides, it's a triangle.

Both statements are true, because the definition of triangle is “a figure with three sides.”

Topic: Arranging conditionals in a logical chain

Question: What is the logical conclusion of the logic chain?

If the cat's away, then the mice will play chess.

If the mice play chess, then they will get smart.

If the mice get smart, then the cat will be humiliated.

Answer choices:

A If the mice play chess, then the cat's away.

B If the cat is humiliated, then it is away.

C If the mice are smart, then they will play chess.

D If the cat's away, then it will be humiliated.

Solution: D

To find the logical conclusion, we use the hypothesis of the first statement in the chain as its hypothesis, and we use the conclusion of the last statement in the chain as its conclusion.

If **the cat's away**, then the mice will play chess.

If the mice play chess, then they will get smart.

If the mice get smart, then **the cat will be humiliated**.

Therefore, the logical conclusion is

If **the cat's away**, then **it will be humiliated**.

Topic: Arranging conditionals in a logical chain

Question: Which statement would work for the missing conditional in the logic chain?

1) If the stars are out, then the sun has gone down.

2)

3) If it's time to party, then there will be refreshments.

Answer choices:

A If the sun has gone down, then it's time to party.

B If it's time to party, then the stars are out.

C If there will be refreshments, then the stars are out.

D If the sun has gone down, then the stars are out.

Solution: A

The conclusion of the first conditional gives us the hypothesis of the second conditional.

1) If the stars are out, then **the sun has gone down**.

2) If **the sun has gone down**, ..

The hypothesis of the third conditional gives us the conclusion of the second conditional.

2) .... then **it's time to party**

3) If *it's time to party*, then there will be refreshments

Putting the two halves of 2) together, we see that the missing conditional is

If **the sun has gone down**, then **it's time to party**.

Topic: Arranging conditionals in a logical chain

Question: The four statements below can be rearranged to form a logic chain. What is the logical conclusion of the logic chain?

If your dog wrecks the lawn, then you will have to buy sod.

If you have a dog, then you will give him a bone.

If your dog digs a hole, then he will wreck the lawn.

If you give your dog a bone, then he will dig a hole.

Answer choices:

A If your dog digs a hole, then he will make you mad.

B If your lawn is wrecked, then you gave your dog a bone.

C If you have a dog, then you will have to buy sod.

D If your dog has a bone, then you gave it to him.

Solution: C

Arrange the conditionals in the following order, so that the hypothesis of each conditional is the same as the conclusion of the previous conditional.

If you have a dog, then you will give him a **bone**.

If you give your dog a **bone**, then he will dig a **hole**.

If your dog digs a **hole**, then he will **wreck the lawn**.

If your dog **wrecks the lawn**, then you will have to **buy sod**.

The logical conclusion of this logic chain is

If you have a dog, then you will have to buy sod.

W W W . K R I S T A K I N G M A T H . C O M