Geometry Formulas

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Two- and three-dimensional shapes

Two-dimensional shapes

	Area	Perimeter
Square	$A = s^2$	$P = 4s$
Rectangle	$A = lw$	$P = 2l + 2w$
Parallelogram	$A = bh$	
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$	
Triangle	$A = \frac{1}{2}bh$	
Circle	$A = \pi r^2$	$C = 2\pi r$ (circumference)

Three-dimensional shapes

	Surface area	Volume
Cube	$SA = 6s^2$	$V = s^3$
Right rectangular prism	$SA = 2lw + 2lh + 2wh$	$V = lwh$
Right triangle prism	$SA = lw + 2ls + wh$	$V = \frac{1}{2}whl$

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Right circular cylinder	$SA = 2\pi r^2 + 2\pi rh$	$V = \pi r^2 h$
Regular square pyramid	$SA = s^2 + 2sl$	$V = \frac{1}{3}s^2 h$
Right circular cone	$SA = \pi r^2 + \pi rl$	$V = \frac{1}{3}\pi r^2 h$
Sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Angles

Individual angles

Acute angle	less than 90°
Obtuse angle	greater than 90°
Right angle	exactly 90°
Straight angle	exactly 180°

Pairs of angles

Adjacent angles	angles that share a common side
Supplementary angles	angles whose sum is 180°

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If the endpoint of a ray falls on a line so that two angles are formed, then the angles are known as a linear pair. By the linear pair property, if two angles form a linear pair, then they are supplementary.

Vertical angles	non-adjacent angles formed by intersecting lines
Complementary angles	angles whose sum is 90°

Angle addition

If point S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.

Overlapping angles

Given $\angle AOD$ with points B and C in its interior, the following statements are true:

1. If $m\angle AOB = m\angle COD$, then $m\angle AOC = m\angle BOD$

2. If $m\angle AOC = m\angle BOD$, then $m\angle AOB = m\angle COD$

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Transversals

Transversal

A transversal is a line, ray, or segment that intersects two or more coplanar lines, rays, or segments, each at a different point.

Corresponding angles postulate

If two lines cut by a transversal are parallel, then corresponding angles are congruent.

Converse of corresponding angles

If two lines are cut by a transversal in such a way that corresponding angles are congruent, then the two lines are parallel.

Alternate interior angles

If two lines cut by a transversal are parallel, then alternate interior angles are congruent.

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Alternate exterior angles

If two lines cut by a transversal are parallel, then alternate exterior angles are congruent.

Same-side interior angles

If two lines cut by a transversal are parallel, then same-side interior angles are supplementary.

Converse of same-side interior angles

If two lines are cut by a transversal in such a way that same-side interior angles are supplementary, then the two lines are parallel.

Converse of alternate interior angles

If two lines are cut by the transversal in such a way that alternate interior angles are congruent, then the two lines are parallel.

Converse of alternate exterior angles

If two lines are cut by a transversal in such a way that alternate exterior angles are congruent, then the two lines are parallel.

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Two transversal proportionality

Three or more parallel lines divide two intersecting transversals proportionally.

Interior and exterior angles

Sum of the interior angles of a triangle

The sum of the measures of the angles of a triangle is 180°.

Exterior angle of a triangle

The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

Measure of the interior angle of a regular polygon

The measure, m, of an interior angle of a regular polygon with n sides is

$$m = 180^\circ - \frac{360^\circ}{n}$$

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Sum of the interior angles of a polygon

The sum, s, of the measures of the interior angles of a polygon with n sides is given by

$$s = (n - 2)180^\circ$$

Sum of the exterior angles of a polygon

The sum of the measures of the exterior angles of a polygon is 360°.

Points, lines and planes

Points

Points are often shown as dots, but unlike physical dots, geometric points have no size. They are named by capital letters.

Lines

Geometric lines have no thickness, are perfectly straight, and extend forever. They are name by two points on the line with a double-headed arrow over the letters, or by a single lowercase letter.

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Planes

A geometric plane extends infinitely in all directions along a flat surface. A plane can be named by three points that lie in the plane, as long as the three points are not on the same line. A plane can also be named by a script capital letter.

Slope

The slope of a non-vertical line that contains the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Perpendicular and parallel lines

Two lines are perpendicular if their intersection forms a right angle. If the lines are non-vertical, the product of their slopes is -1.

Two lines are parallel if they are coplanar but don't intersect. If the lines are non-vertical, they have the same slope.

A line parallel to a plane

A line that is not contained in a given plane is parallel to the plane if and only if it is parallel to a line contained in the plane.

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A line perpendicular to a plane

A line is perpendicular to a plane to a point P if and only if it is perpendicular to every line in the plane that passes through P.

Distance formulas

On a coordinate plane, the distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance, d, between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in space is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Midpoint formula

The midpoint of a segment with endpoints at (x_1, y_1, z_1) and (x_2, y_2, z_2) in space is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

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Collinear

Points are collinear if a single line can contain them all. Any two points are collinear.

Coplanar

Points are coplanar if a single plane can contain them all. Any three points are coplanar.

Line segment

A segment is a part of a line that begins at one point and ends at another. The points are called the endpoints of the segment.

Length of a segment

Let A and B be points on a number line, with coordinates a and b. Then the measure of $\overline{AB}$, which is called its length, is $|a-b|$ or $|b-a|$.

Segment bisector

A segment bisector is a line that divides a segment into two congruent parts. The point where a bisector intersects a segment is the midpoint of

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the segment. A bisector that is perpendicular to a segment is called a perpendicular bisector.

Angle bisector

An angle bisector is a line or ray that divides an angle into two congruent angles.

Segment addition

If point R is between points P and Q on a line, then $PR + RQ = PQ$.

Linear pair property

If two angles form a linear pair, then they are supplementary.

Overlapping segments

Given a segment with points A, B, C, and D (in order) the following statements are true:

1. If $AB = CD$, then $AC = BD$.

2. If $AC = BD$, then $AB = CD$.

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Polygons

Polygon

A polygon is a plane figure formed from three or more segments such that each segment intersects exactly two other segments, one at each endpoint, and no two segments with a common endpoint are collinear. The segments are called the sides of the polygon, and the common endpoints are called the vertices of the polygon.

Classifying polygons

Number of sides	Name	Number of sides	Name
3	Triangle	9	Nonagon
4	Quadrilateral	10	Decagon
5	Pentagon	11	Undecagon
6	Hexagon	12	Dodecagon
7	Heptagon	n	n-gon
8	Octagon		

Area of a regular polygon

The area of a regular polygon with apothem a and perimeter p is

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$$A = \frac{1}{2}ap$$

Polygon similarity

- Two polygons are similar if and only if there is a way of setting up a correspondence between their sides and angles such that:
- Each pair of corresponding angles is congruent.
- Each pair of corresponding sides is proportional.

Polygon congruence

Two polygons are congruent if and only if there is a way of settling up a correspondence between their sides and angles, in order, such that

1. all pairs of corresponding angles are congruent, and
2. all pairs of corresponding sides are congruent.

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Triangles

Pythagorean theorem

If a and b are the lengths of the legs of a right-triangle, and c is the length of its hypotenuse, then

$$a^2 + b^2 = c^2$$

Converse of the pythagorean theorem

If the square of the length of one side of a triangle equals the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

Pythagorean inequalities

For any triangle, ABC, with c as the length of the longest side:

If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a **right** triangle.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is an **obtuse** triangle.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is an **acute** triangle.

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45-45-90 triangles

If any 45-45-90 triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

30-60-90 triangles

If any 30-60-90 triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Trigonometry of a right triangle

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	SOH	$\dfrac{\text{oscar}}{\text{had}}$
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	CAH	$\dfrac{\text{a}}{\text{hold}}$
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	TOA	$\dfrac{\text{on}}{\text{arthur}}$

Triangles classified by congruent sides

Three congruent sides	equilateral
Two or more congruent sides	isosceles

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No congruent sides	scalene

Midsegments

A midsegment of a triangle is a segment whose endpoints are the midpoints of two sides.

A midsegment of a triangle is parallel to a side of the triangle, and its length is equal to half the length of that side.

Triangle congruence

SSS (side-side-side)

If the sides of one triangle are congruent to the sides of another triangle, then the two triangles are congruent.

SAS (side-angle-side)

If two sides and their included angle in one triangle are congruent to two sides and their included angle in another triangle, then the two triangles are congruent.

ASA (angle-side-angle)

If two angles and their included side in one triangle are congruent to two angles and their included side in another triangle, then the two triangles are congruent.

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AAS (angle-angle-side)

If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

HL (hypotenuse-leg)

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the two triangles are congruent.

Isosceles triangle theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Converse of the isosceles triangle theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Triangle similarity

AA (angle-angle)

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

SSS (side-side-side)

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If the three sides of one triangle are proportional to the three sides of another triangle, then the triangles are similar.

SAS (side-angle-side)

If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

Proportional triangles

Proportional altitudes

If two triangles are similar, then their corresponding altitudes have the same ratio as their corresponding sides.

Proportional medians

If two triangles are similar, then their corresponding medians have the same ratio as their corresponding sides.

Proportional angle bisectors

If two triangles are similar, then their corresponding angle bisectors have the same ratio as their corresponding sides.

Proportional segments

An angle bisector of a triangle divides the opposite side into two segments that have the same ratio as the other two sides.

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Side-splitting

A line parallel to one side of a triangle divides the other two sides proportionally.

Triangle inequality

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Inscribed and circumscribed circles

An inscribed circle is a circle drawn inside a triangle that just touches its three sides.

A circumscribed circle is a circle drawn outside the outside of a triangle that contains the triangles three vertices.

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Circles

Basic definitions

Chord	a line segment with both endpoints on the circle
Diameter	the longest chord in the circle; it passes through the center
Radius	distance from the center to the edge; it's half the diameter
Semicircle	a half circle outlined by the diameter and the circumference
Sector	portion of the circle outlined by the circumference and two radii

Circle

A circle is the set of all points in a plane that are equidistant from a given point in the plane known as the center of the circle. A radius (plural, radii) is a segment from the center of the circle to a point on the circle. A chord is a segment whose endpoints line on a circle. A diameter is a chord that contains the center of a circle.

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Central angle and intercepted arc

A central angle of a circle is an angle in the plane of a circle whose vertex is the center of the circle. An arc whose endpoints lie on the sides of the angle and whose other points lie in the interior of the angle is the intercepted arc of the central angle.

Degree measure of an arc

The degree measure of a minor arc is the measure of its central angle. The degree measure of a major arc is 360° minus the degree measure of its minor arc. The degree measure of a semicircle is 180°.

Chords and arcs

In a circle, or in congruent circles, the arcs of congruent chords are congruent.

Converse of the chords and arcs

In a circle, or in congruent circles, the chords of congruent arcs are congruent.

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Secants and tangents

A secant to a circle is a line that intersects the circle at two points. A tangent is a line in the plane of the circle that intersects the circle at exactly one point, which is known as the point of tangency.

Tangents

If a line is tangent to a circle, then the line is perpendicular to a radius of the circle drawn to the point of tangency.

Converse of the tangent theorem

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

Radius and chords

A radius that is perpendicular to a chord of a circle bisects the chord.

The perpendicular bisector of a chord passes through the center of the circle.

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Inscribed angles

The measure of an angle inscribed in a circle is equal to one-half the measure of the intercepted arc.

Right angle corollary

If an inscribed angle intercepts a semicircle, then the angle is a right angle.

Arc-intercept corollary

If two inscribed angles intercept the same arc, then they have the same measure.

Intersecting tangents and secants

If a tangent and a secant (or a chord) intersect on a circle at the point of tangency, then the measure of the angle formed is one-half the measure of its intercepted arc.

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Intersecting secants in the interior

The measure of an angle formed by two secants or chords that intersect in the interior of a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

Intersecting secants on the exterior

The measure of an angle formed by two secants that intersect in the exterior of a circle is one-half the difference of the measures of the intercepted arcs.

Secant-tangent vertex outside the circle

The measure of a secant-tangent angle with its vertex outside the circle is one-half the difference of the measures of the intercepted arcs.

Tangent-tangent vertex outside the circle

The measure of a tangent-tangent angle with its vertex outside the circle is one-half the difference of the measures of the intercepted arcs, or the measure of the major arc minus 180°.

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Tangent segments

If two segments are tangent to a circle from the same external point, then the segments are of equal length.

Intersecting secants

If two secants intersect outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. (Whole x Outside = Whole x Outside)

Secant tangent intersect outside the circle

If a secant and a tangent intersect outside a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. (Whole x Outside = Tangent Squared)

Chords intersect inside the circle

If two chords intersect inside a circle, then the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord.

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Transformations

Transformations

A rigid transformation is a transformation that does not change the size or shape of a figure.

Translations

In a translation, every point of a figure moves in a straight line, and all points move the same distance and in the same direction. The paths of the points are parallel.

Rotations

In a rotation, very point of a figure moves around a given point known as the center of rotation. All points move the same angle measure.

Reflections

In a mathematical reflection, a line plays the role of the mirror, and every point in a geometric figure is flipped across the line.

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Across parallel lines: Reflection across two parallel lines is equivalent to a translation of twice the distance between the lines and in a direction perpendicular to the lines.

Across intersecting lines: Reflection across two intersecting lines is equivalent to a rotation about the point of intersection through twice the measure of the angle between the lines.

Reflectional symmetry

A figure has reflectional symmetry if and only if its reflected image across a line coincides exactly with the preimage. The line is called an axis of symmetry.

Rotational symmetry

A figure has rotational symmetry if and only if it has at least one rotation image, not counting rotation images of 0° or multiples of 360°, that coincides with the original image.

Logic and reasoning

Deductive reasoning

The process of drawing logically certain conclusions using an argument.

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Inductive reasoning

The process of forming conjectures that are based on observations.

Conditional

An “if-then” statement. The statement following the “if” is the *hypothesis*, and the statement following the “then” is the *conclusion*.

Logical chain

When multiple conditionals are linked together.

Converse

The converse of a conditional is the conditional statement with its hypothesis and conclusion interchanged.

Biconditional

An “if-and-only-if” statement.

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Counterexample

An example which proves that a statement is false.

Proof

A proof is a convincing argument that something is true.

Theorem

A theorem is a statement that has been proved deductively.

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