Reflecting figures in coordinate space

In this lesson we'll look at reflection of a figure in a coordinate plane and how to determine the location and orientation of the figure after the reflection takes place.

A **reflection** is a type of transformation that flips a figure across some line. The line is called the **line of reflection**, or the mirror line. The line of reflection, which remains fixed (the points on the line aren't moved by the reflection), can be horizontal, vertical, or diagonal.

Pre-image/image

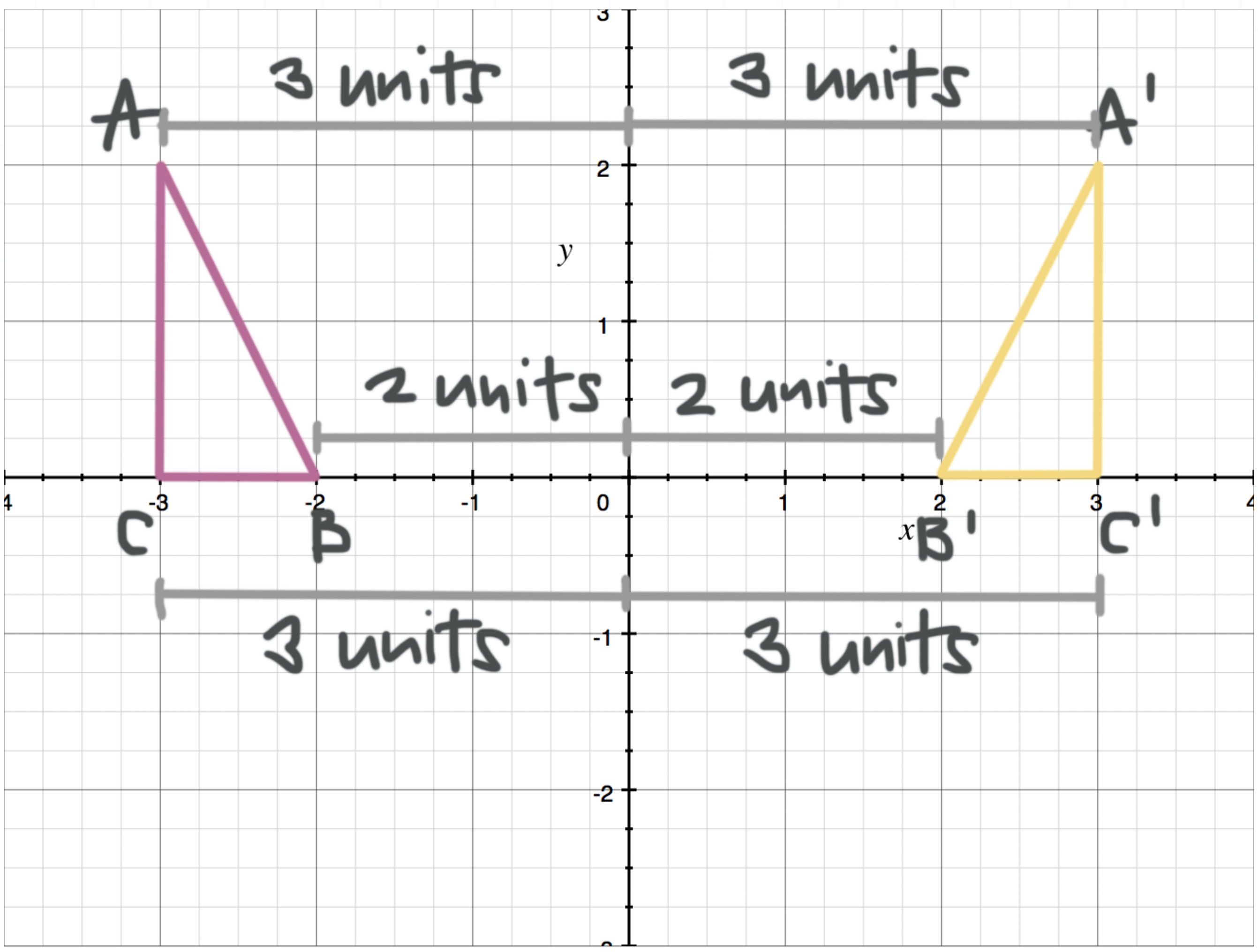
Before a reflection, we have the **pre-image** (the figure in its original location and orientation). Points in the pre-image are usually labeled with capital letters. After the reflection, we have the **image** (the figure in its final location and orientation). Points in the image are usually labeled with the same capital letters, plus the prime symbol $'$ after each letter. So if figure $ABCD$ is reflected, its image becomes figure $A'B'C'D'$.

In a reflection, the image and pre-image are always congruent, because a reflection never changes the measures of angles or the lengths of line segments and curves in the figure.

Reflecting figures

When we reflect a figure across a line (which we also refer to as reflecting a figure *in* that line), the distance of any point in the pre-image from the line of reflection is equal to the distance of the corresponding point in the image from that line.

In this reflection, which is a reflection across the y-axis, each point in the pre-image is at the same distance from the y-axis as the corresponding point in the image.



Notice that when we reflect across the y-axis, the sign of the x-coordinate of any point in the image will be opposite that of the corresponding point in the pre-image, and that the y-coordinate of any point in the image will be equal to that of the corresponding point in the pre-image.

$A = (-3,2)$ $\qquad\qquad\qquad$ $A' = (3,2)$

$$B = (-2,0) \quad \rightarrow \quad B' = (2,0)$$

$$C = (-3,0) \qquad\qquad C' = (3,0)$$

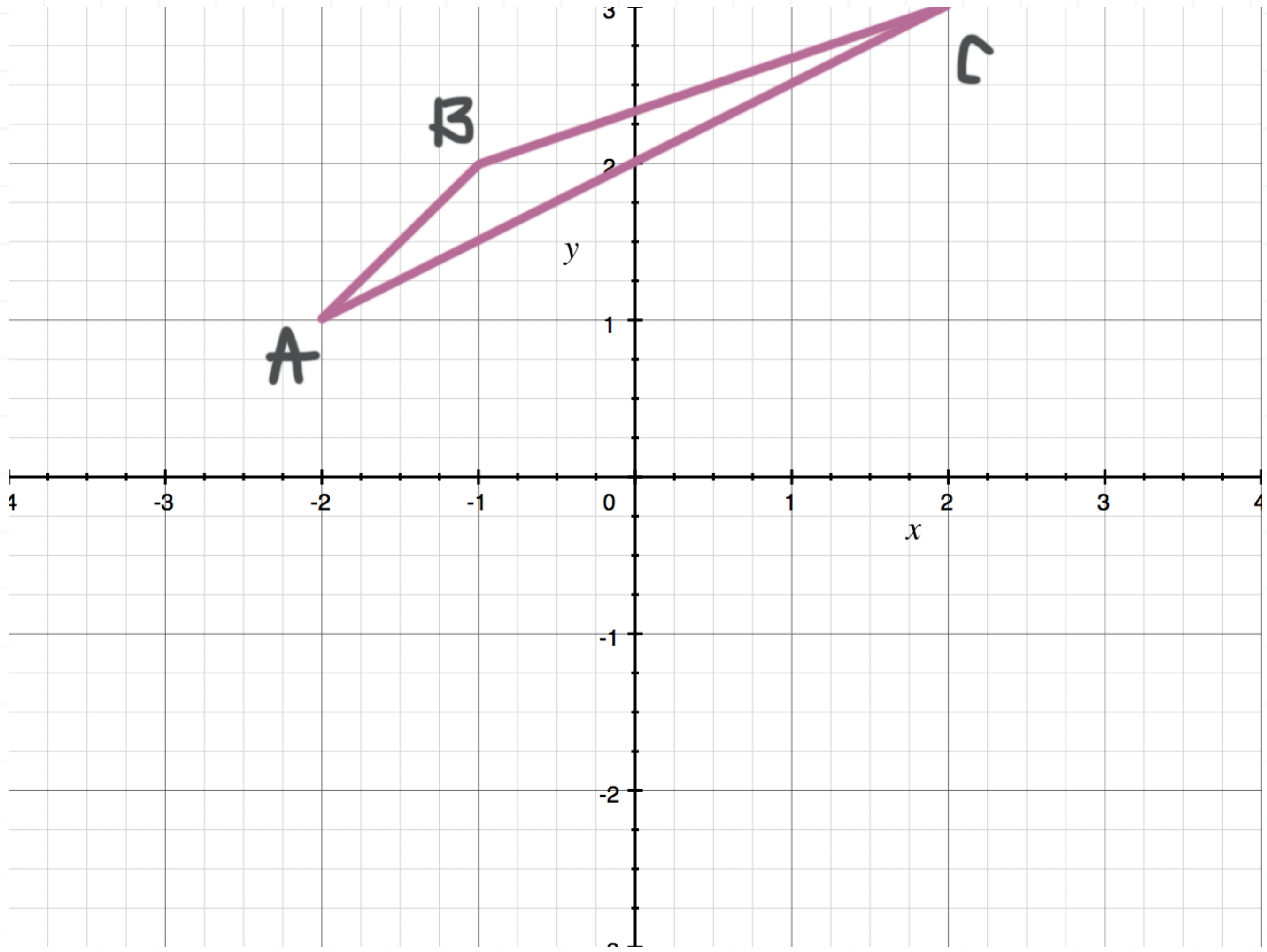
Here are some commonly used reflections and their rules.

Line of reflection	Rule
y-axis	(x, y) to (-x, y) The x-coordinates will change sign.
x-axis	(x, y) to (x,-y) The y-coordinates will change sign.
y=x	(x, y) to (y, x) The x- and y-coordinates will change places.

Let's look at some examples.

Example

Draw $\triangle A'B'C'$, the reflection of $\triangle ABC$ across the x-axis.



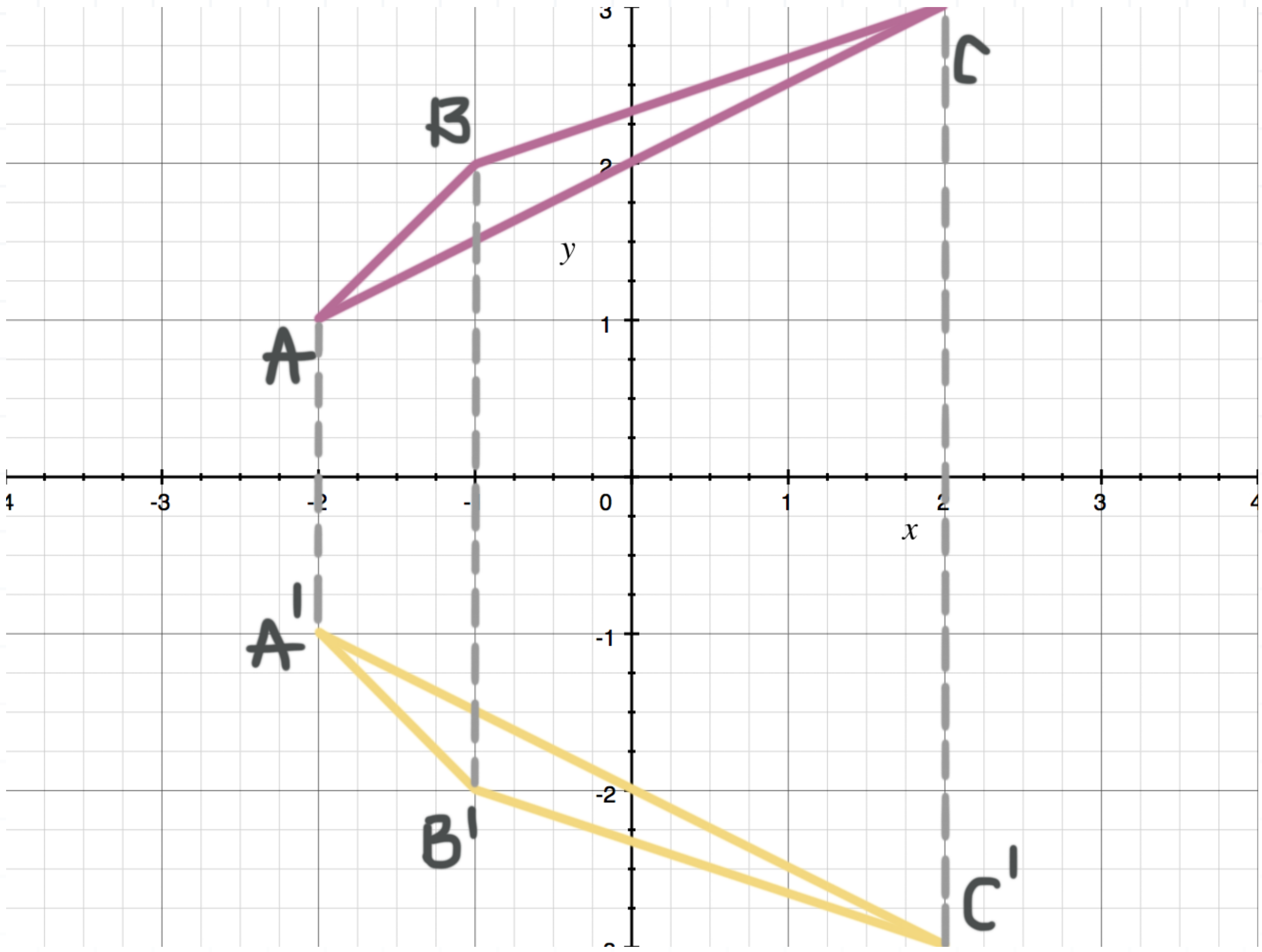
Let's look at one vertex of the triangle at a time.

Point A is 1 unit above the x-axis, so A' will be 1 unit below the x-axis.

Point B is 2 units above the x-axis, so B' will be 2 units below the x-axis.

Point C is 3 units above the x-axis, so C' will be 3 units below the x-axis.

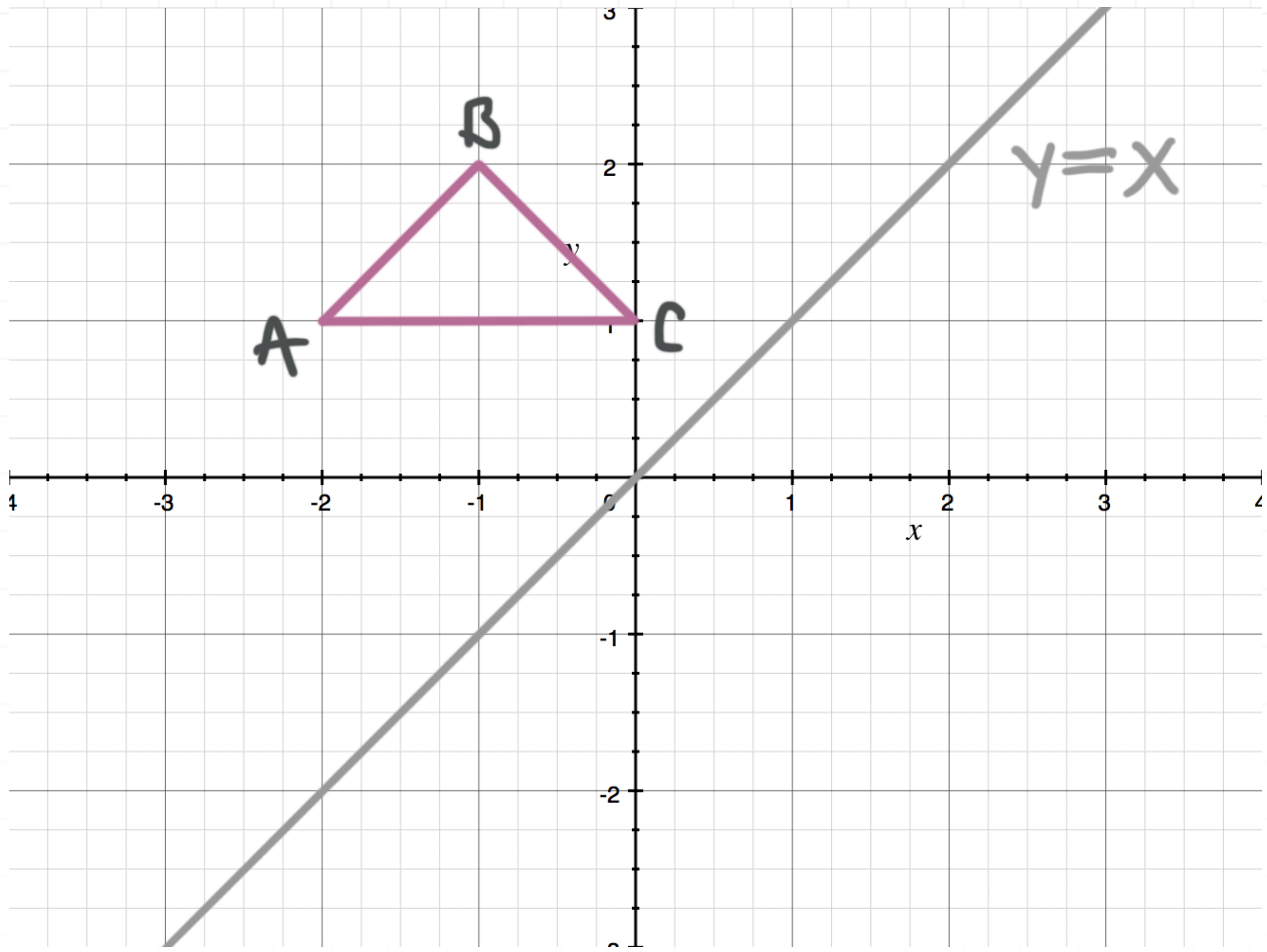
This means that when you reflect $\triangle ABC$ across the x-axis, the y-coordinates will change sign.



Let's look at another example.

Example

Reflect triangle ABC across the line $y = x$.



When you reflect a figure across the line $y = x$, the x- and y-coordinates switch places. Let's write down the coordinates of the vertices of the triangle.

$$A = (-2,1)$$

$$B = (-1,2)$$

$$C = (0,1)$$

Now to reflect the triangle across the line $y = x$, you switch the x- and y-coordinates of each vertex.

$$A = (-2,1) \qquad A' = (1,-2)$$

$$B = (-1,2) \qquad \rightarrow \qquad B' = (2,-1)$$

$$C = (0,1) \qquad\qquad\qquad C' = (1,0)$$

Then you can draw the image.