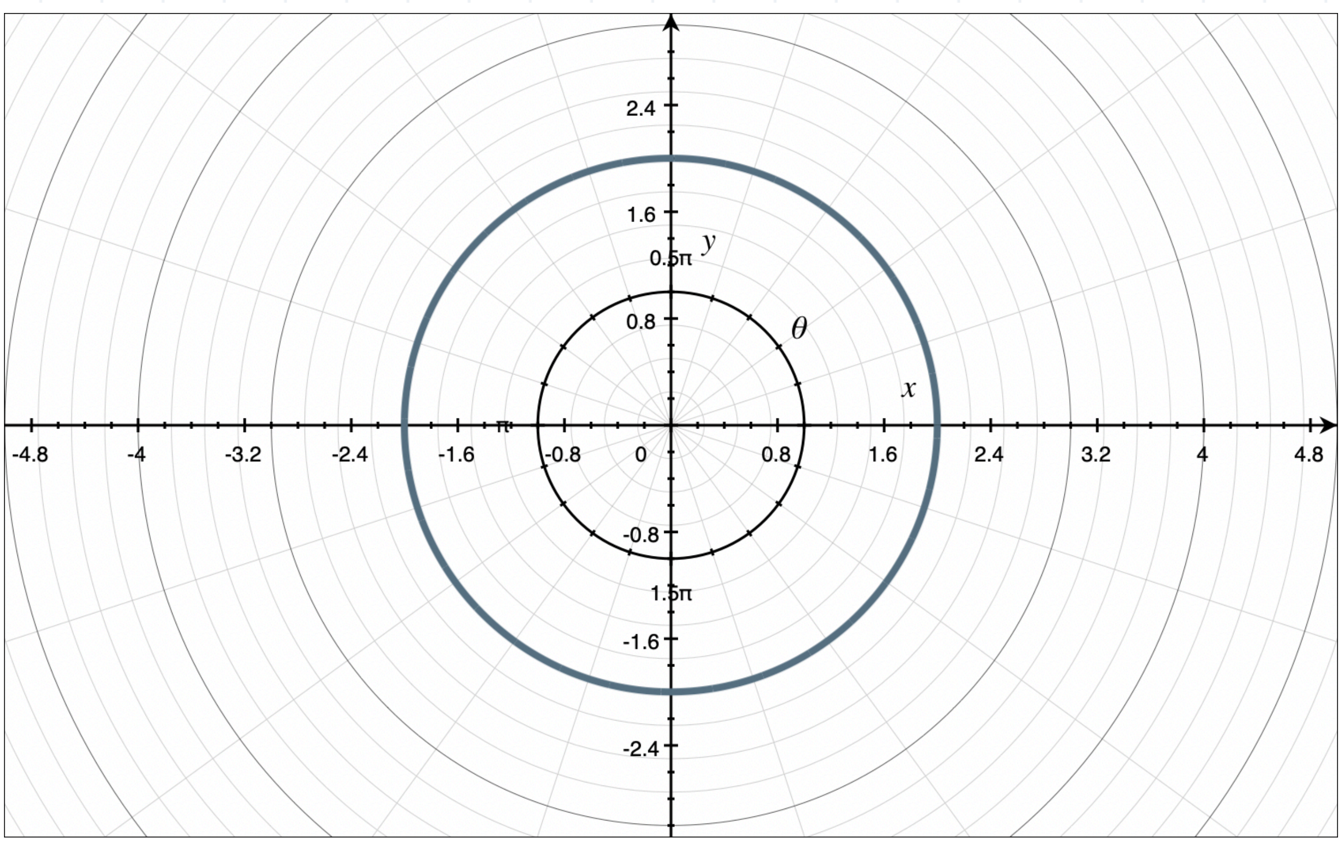
Graphing circles

In the same way that we can graph rectangular equations in the xy-plane, we can also graph polar equations, and we want to look at how to graph many of the shapes that polar curves traditionally take on.

Circles centered at the origin

We'll start by looking at the graphs of circles. In polar coordinates, the equation of a circle centered at the origin has the form $r = a$, where a is the radius of the circle. We can tell that $r = a$ represents a circle because it shows us that, regardless of the value of θ, the distance from the origin will always be the same.

For instance, $r = 2$ says that, regardless of the value of θ, the distance from the origin will always be 2, so its graph is



The smaller the value of a, the smaller the circle will be, and the larger the value of a, the larger the circle will be. The equation $r=0$ represents a "circle" with radius 0, which is just the single point at the origin, $(r,\theta)=(0,0)$.

Realize that the value of r can also be negative, but changing r's value from positive to negative doesn't change the graph. If we're graphing $r=-2$, then for every angle θ, we plot a point on the opposite side of the origin from θ, which will ultimately still just give us the same circle with radius 2.

Circles centered off the origin

Circles that are centered off the origin, but on the horizontal or vertical axis have an equation in one of these forms:

$$r=c\sin\theta$$

$$r=c\cos\theta$$

where c is a non-zero constant. For equations like these, we'll graph the circle by first plotting points in the xy-plane, and then transferring those points onto our polar graph. This procedure is one that will use for graphing every type of polar curve, and we'll use the following steps.

1. Set the argument of the trigonometric function equal to $\pi/2$, and then solve the equation for θ.

2. Evaluate the polar curve at multiples of the θ-value we solved for in Step 1, starting with $\theta = 0$, and plot the resulting points on the polar graph.

3. Connect the points on the polar graph with a smooth curve.

In Step 1 above, the reason we set the argument equal to $\pi/2$ is because this will give us values that are easy to evaluate. If we think back to what we learned about the unit circle in Trigonometry, we should remember that the values of the basic sine and cosine functions are 0, 1, or -1 at multiples of $\pi/2$. For instance, if $\theta = \pi/2$, then $\sin\theta = \sin(\pi/2) = 1$ and $\cos\theta = \cos(\pi/2) = 0$.

We could use other angles, but these 0, 1, and -1 values are really convenient to work with, so we'll make it easy on ourselves by using these $\pi/2$ multiples.

Let's make these steps come to life by working through an example, so that we can see how to create the graph of the polar curve.

Example

Sketch the graph of $r = 6\cos\theta$.

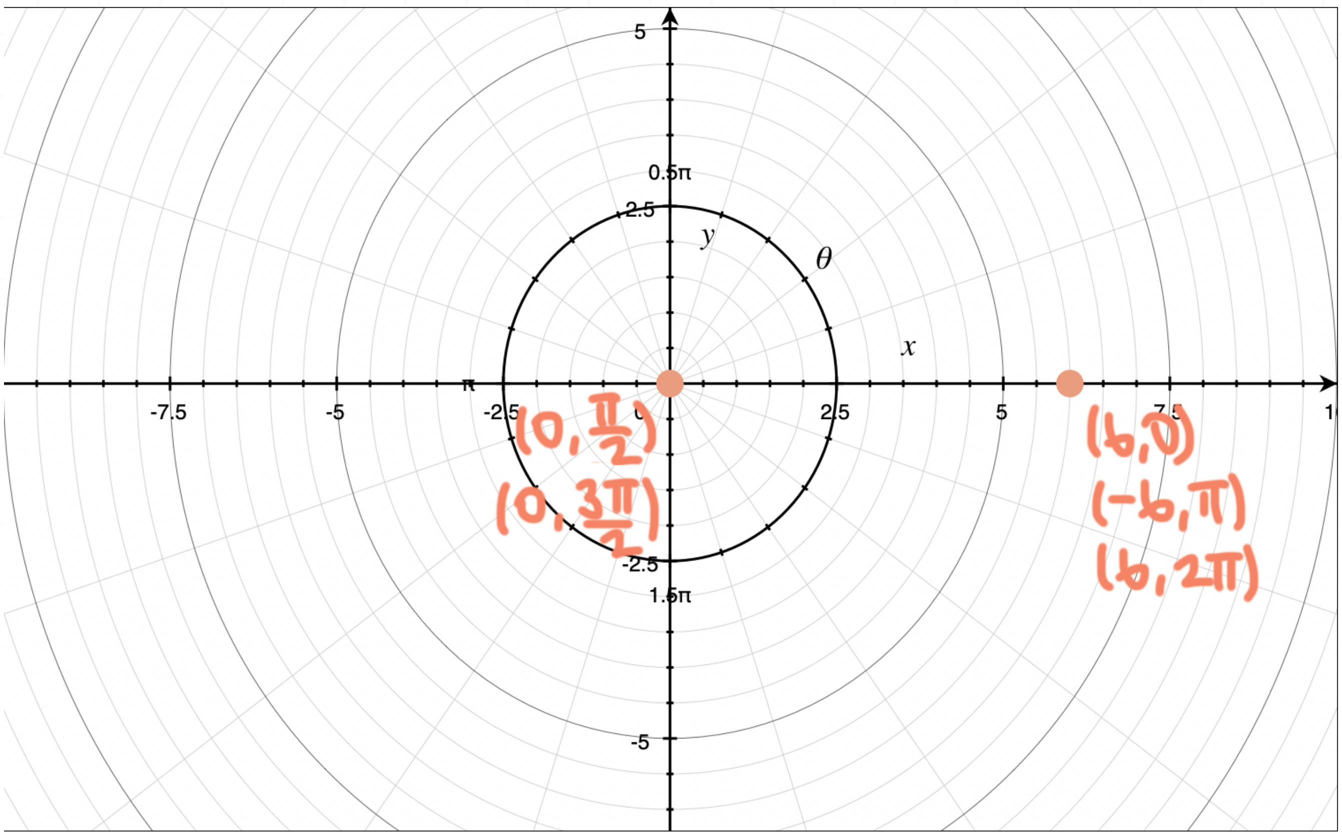
The trigonometric function in this polar equation is $\cos\theta$, and its argument (the angle at which cosine is evaluated) is θ. So we'll set

$$\theta = \frac{\pi}{2}$$

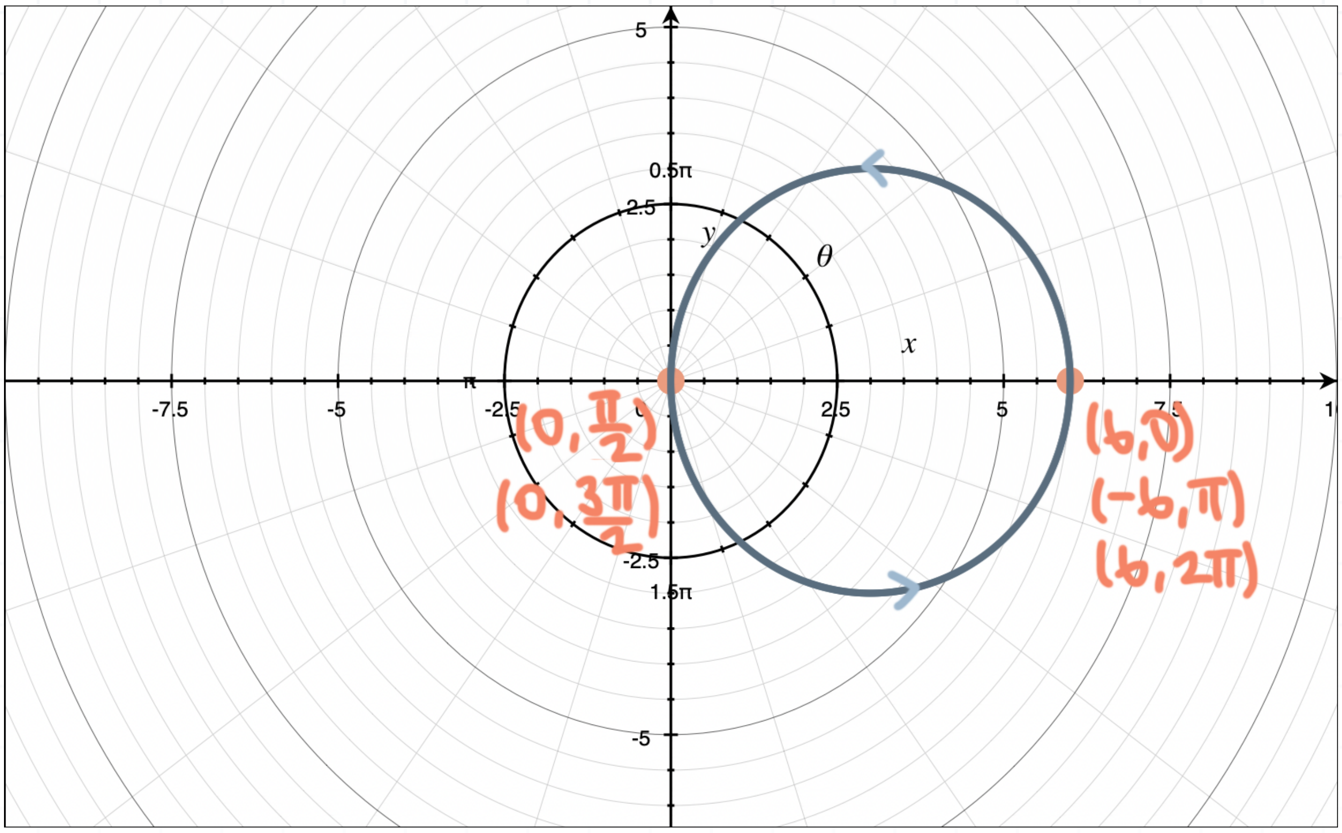
Now we'll make a table with multiples of $\pi/2$, like $\theta = 0$, $\pi/2$, π, $3\pi/2$, 2π, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	6	0	-6	0	6

Plotting these points on the polar graph gives



And if we connect these points with a smooth curve, in order, we see the graph of the circle. We start at $(6,0)$, loop up around to the origin at $(0,\pi/2)$, then loop back down around to $(-6,\pi)$, which is actually the same point as $(6,0)$. From there on, we're retracing the same pieces of the circle over and over.



Let's do another example with a cosine function.

Example

Sketch the graph of $r = -7\cos\theta$.

The trigonometric function in this polar equation is $\cos\theta$, and its argument (the angle at which cosine is evaluated) is θ. So we'll set

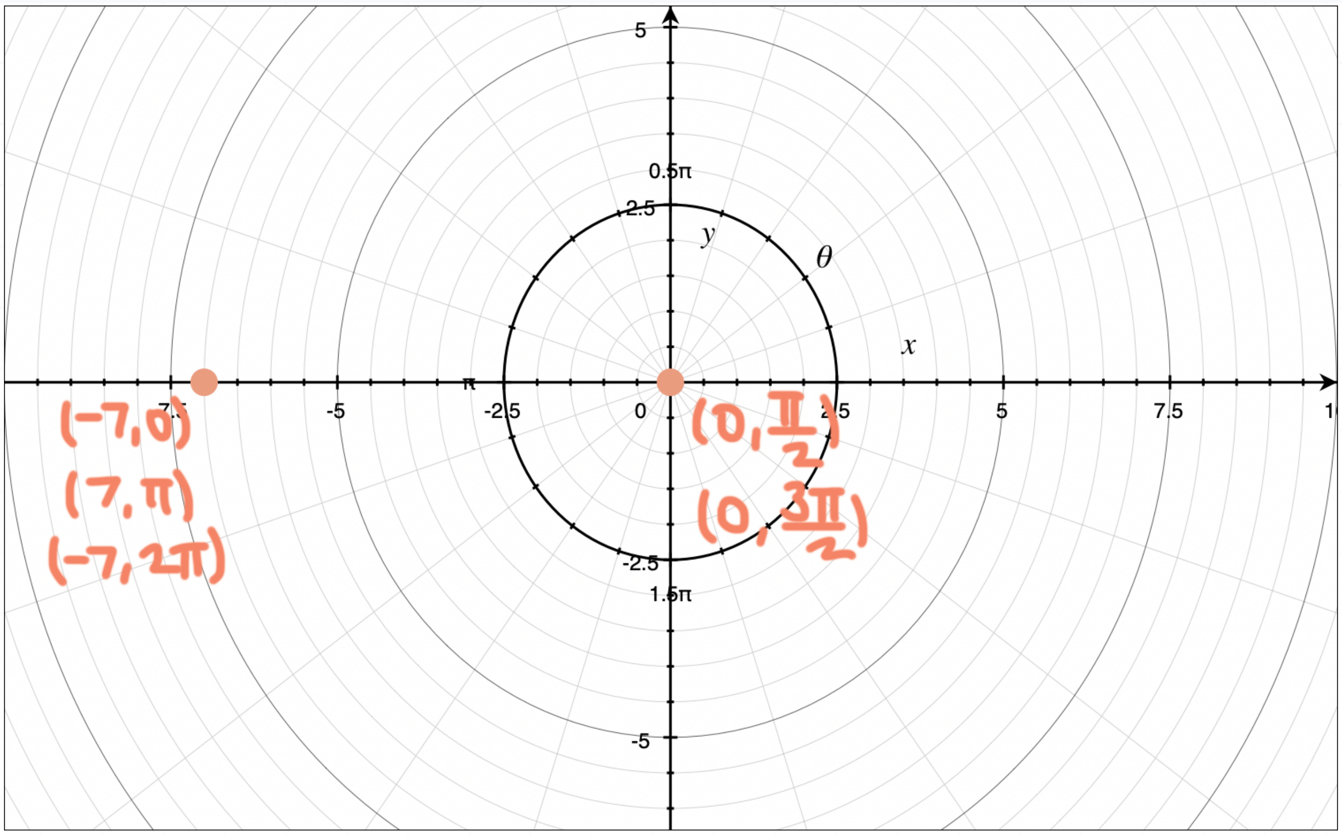
$$\theta = \frac{\pi}{2}$$

Now we'll make a table with multiples of $\pi/2$, like $\theta = 0$, $\pi/2$, π, $3\pi/2$, 2π, etc., and include the values of r that correspond to each of these θ-values.

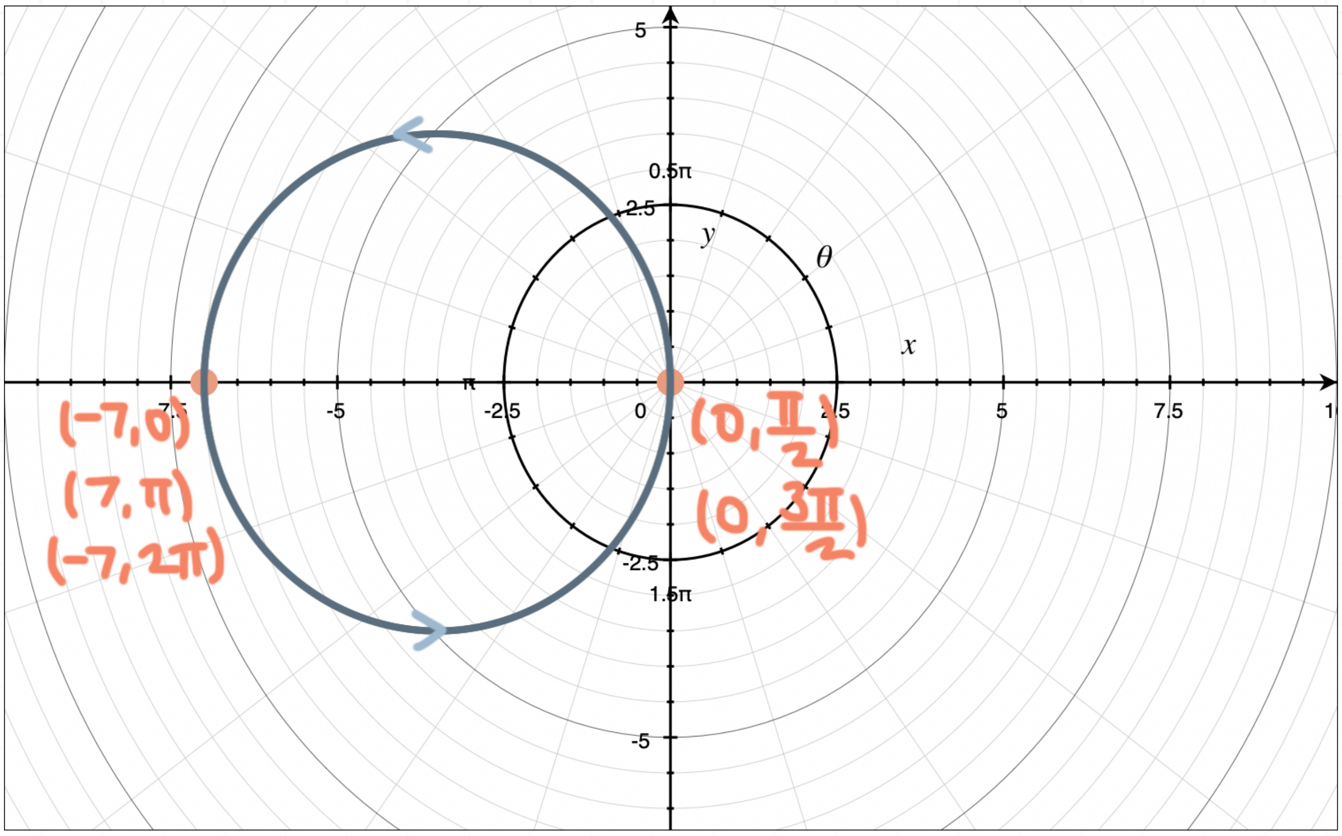
θ	0	$\pi/2$	π	$3\pi/2$	2π

r	-7	0	7	0	-7

Plotting these points on the polar graph gives



And if we connect these points with a smooth curve, in order, we see the graph of the circle. We start at $(-7,0)$, loop down around to the origin at $(0,\pi/2)$, then loop back up around to $(7,\pi)$, which is actually the same point as $(-7,0)$. From there on, we're retracing the same pieces of the circle over and over.



Now let's look at an example with an equation in the form $r = c\sin\theta$ instead of $r = c\cos\theta$. We'll see that the form $r = c\sin\theta$ also represents a circle.

Example

Sketch the graph of $r = -10\sin\theta$.

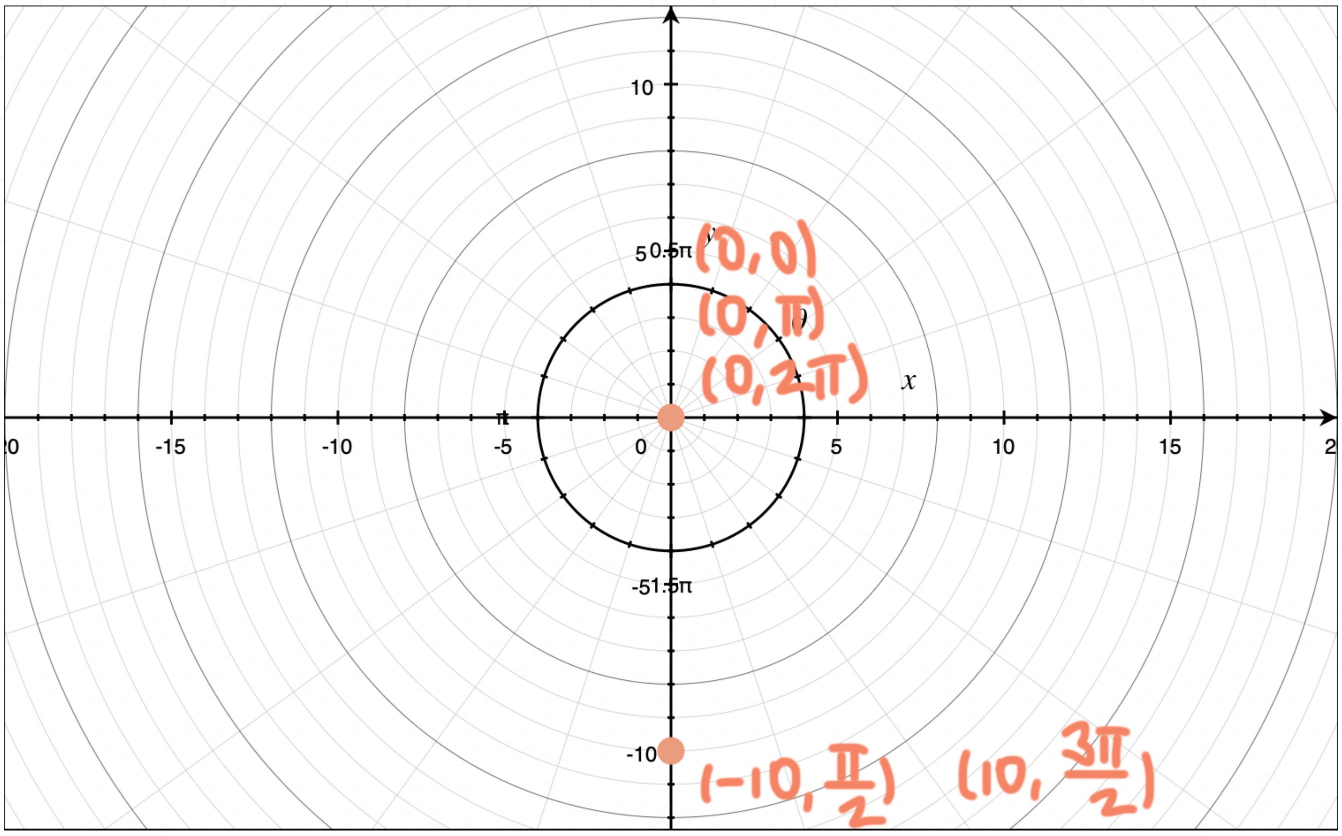
The trigonometric function in this polar equation is $\sin\theta$, and its argument (the angle at which sine is evaluated) is θ. So we'll set

$$\theta = \frac{\pi}{2}$$

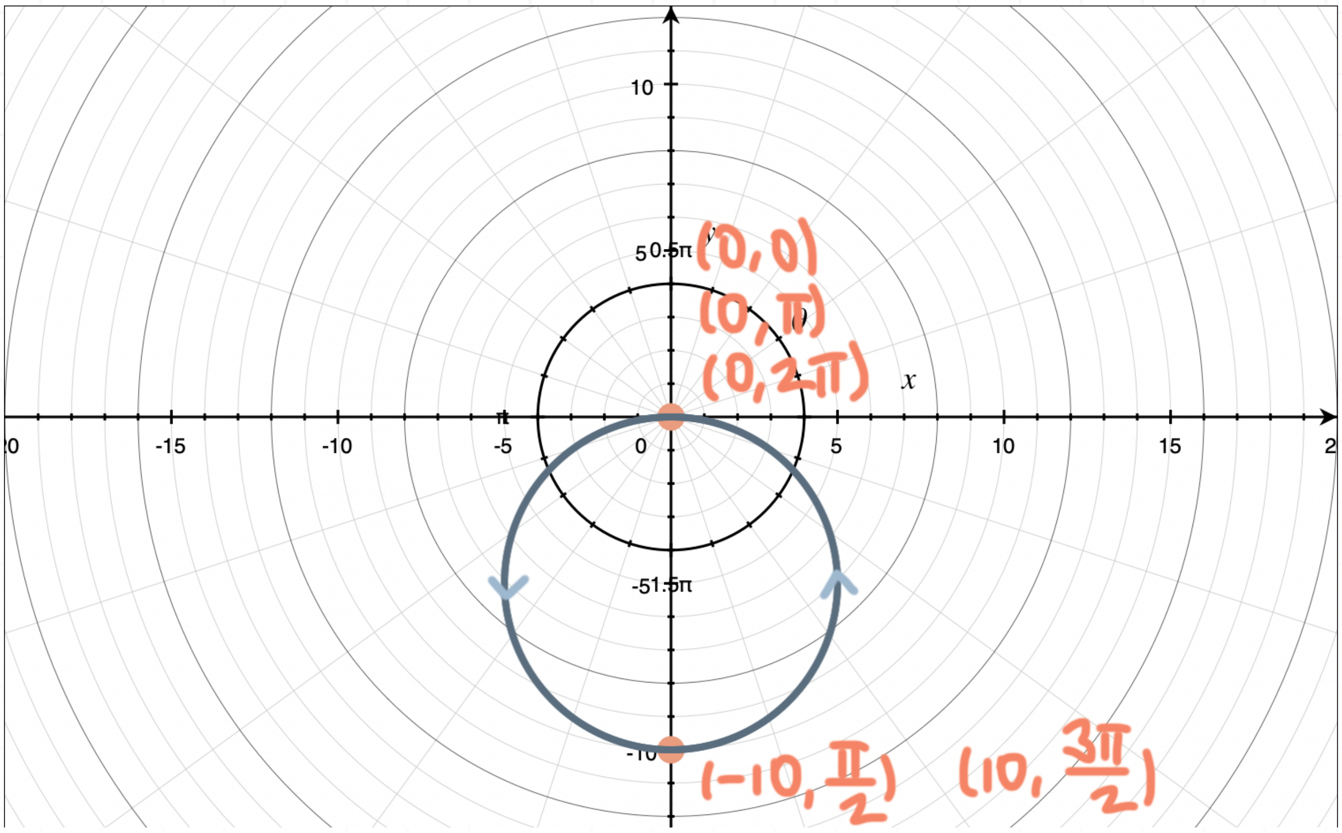
Now we'll make a table with multiples of $\pi/2$, like $\theta = 0$, $\pi/2$, π, $3\pi/2$, 2π, etc., and include the values of r that correspond to each of these θ-values.

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	0	-10	0	10	0

Plotting these points on the polar graph gives



And if we connect these points with a smooth curve, in order, we see the graph of the circle. We start at $(0,0)$, loop down around to $(-10,\pi/2)$, then loop back up around to $(0,\pi)$, which is actually the same point as $(0,0)$. From there on, we're retracing the same pieces of the circle over and over.


10
5
-5
-10
-15
-10
-5
0
5
10
15
x
0.5π
1.5π
(0, 0)
(0, π)
(0, 2π)
(−10, π/2)
(10, 3π/2)