Converting rectangular equations to polar

Now that we can convert polar equations into rectangular equations, let's work backwards and convert rectangular equations into polar equations.

Converting this way it a little easier, because we can always use the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$ to make substitutions for x and y. Every time we see an x, we'll replace it with $r\cos\theta$, and every time we see a y, we'll replace it with $r\sin\theta$.

For some rectangular equations, the conversion formula $x^2 + y^2 = r^2$ can come in handy as well.

Let's do an example.

Example

Convert the rectangular equation $x^2 + y^2 = 64$ to polar coordinates.

If we use the conversion equation $x^2 + y^2 = r^2$, we can see right away that we get the polar curve $r^2 = 64$, or $r = 8$.

But we could have also used the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$ to arrive at the same answer.

$$(r\cos\theta)^2 + (r\sin\theta)^2 = 64$$

$$r^2\cos^2\theta + r^2\sin^2\theta = 64$$

$$r^2(\cos^2\theta + \sin^2\theta) = 64$$

If we remember the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ from Trigonometry, then we get

$$r^2(1) = 64$$

$$r^2 = 64$$

$$r = 8$$

Let's do another example where we can't use $x^2 + y^2 = r^2$.

Example

Convert the rectangular equation $y = 25x$ to polar coordinates.

Using the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$r\sin\theta = 25r\cos\theta$$

$$\frac{r\sin\theta}{r\cos\theta} = 25$$

$$\frac{\sin\theta}{\cos\theta} = 25$$

From the quotient identity for tangent, we can rewrite the left side of this equation to get

$$\tan\theta = 25$$

We'll do another one to get a little more practice.

Example

Convert the rectangular equation $y = -6x^2$ to polar coordinates.

Using the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$r\sin\theta = -6(r\cos\theta)^2$$

$$\frac{r\sin\theta}{(r\cos\theta)^2} = -6$$

$$\frac{r\sin\theta}{(r\cos\theta)(r\cos\theta)} = -6$$

$$\frac{r\sin\theta}{r\cos\theta}\left(\frac{1}{r\cos\theta}\right) = -6$$

$$\frac{\sin\theta}{\cos\theta}\left(\frac{1}{r\cos\theta}\right) = -6$$

The quotient identity for tangent from Trigonometry gives us

$$\tan\theta\left(\frac{1}{r\cos\theta}\right) = -6$$

$$\frac{\tan\theta}{r\cos\theta} = -6$$

$$\frac{\tan\theta}{\cos\theta} = -6r$$

This is a great time to point out that some equations are better expressed in polar coordinates, while others are better expressed in rectangular coordinates.

In this previous example, the rectangular $y = -6x^2$ is much easier to understand than its equivalent polar equation,

$$\frac{\tan\theta}{\cos\theta} = -6r$$

So this equation is probably best expressed in rectangular coordinates, if we get to choose which coordinate system to use. On the other hand, if we look back at the first example, the equation $r = 8$ is a simpler way to express $x^2 + y^2 = 64$, so polar coordinates might be better for that equation.

Let's do just a couple more.

Example

Convert the rectangular equation $(x+9)^2 + (y-13)^2 = 64$ to polar coordinates.

Using the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$(r\cos\theta + 9)^2 + (r\sin\theta - 13)^2 = 64$$

$$(r^2\cos^2\theta + 18r\cos\theta + 81) + (r^2\sin^2\theta - 26r\sin\theta + 169) = 64$$

$$(r^2\cos^2\theta + r^2\sin^2\theta) + (18r\cos\theta - 26r\sin\theta) + (81 + 169) = 64$$

$$r^2 + 18r\cos\theta - 26r\sin\theta + 250 = 64$$

$$r^2 + 18r\cos\theta - 26r\sin\theta = -186$$

We'll do one more.

Example

Convert the rectangular equation to polar coordinates.

$$\frac{(x+5)^2}{9} + \frac{(y-7)^2}{4} = 1$$

Using the conversion equations $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$\frac{(r\cos\theta + 5)^2}{9} + \frac{(r\sin\theta - 7)^2}{4} = 1$$

We can clear the fractions by multiplying both sides of this equation by 36.

$$4(r\cos\theta + 5)^2 + 9(r\sin\theta - 7)^2 = 36$$

$$4(r^2\cos^2\theta + 10r\cos\theta + 25) + 9(r^2\sin^2\theta - 14r\sin\theta + 49) = 36$$

$$(4r^2\cos^2\theta + 40r\cos\theta + 100) + (9r^2\sin^2\theta - 126r\sin\theta + 441) = 36$$