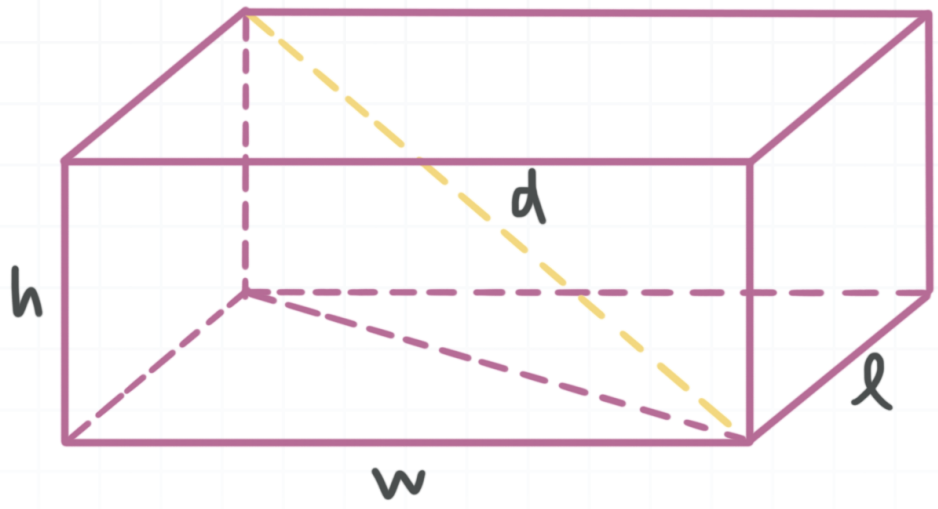
Diagonal of a right rectangular prism

The diagonal of a right rectangular prism goes from one corner of the prism, across the interior of it, all the way to the opposite corner.



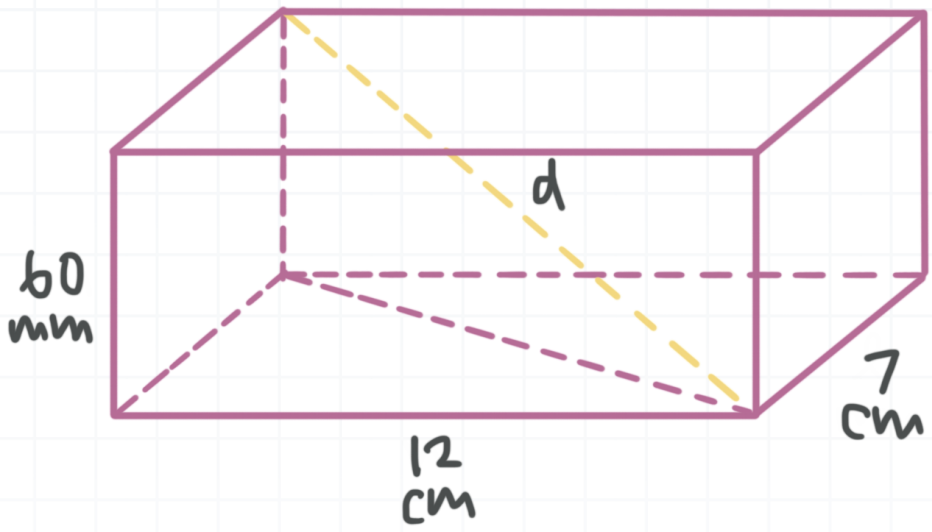
You can find the length of a diagonal of a right rectangular prism using

$$d = \sqrt{l^2 + w^2 + h^2}$$

where d is the length of the diagonal, and l, w, and h are the length, width, and height, respectively. Let's start by working through an example.

Example

What is the length of the diagonal of the right rectangular prism?



Not all of the dimensions here are given in the same units. Change 60 mm to centimeters first.

There are $1{,}000$ mm in 1 m, and there are 100 cm in 1 m. Since both 100 cm and $1{,}000$ mm are equal to 1 m, they're equal to each other, so

$$100 \text{ cm} = 1{,}000 \text{ mm}$$

Dividing both sides of this equation by $1{,}000$ mm, we get

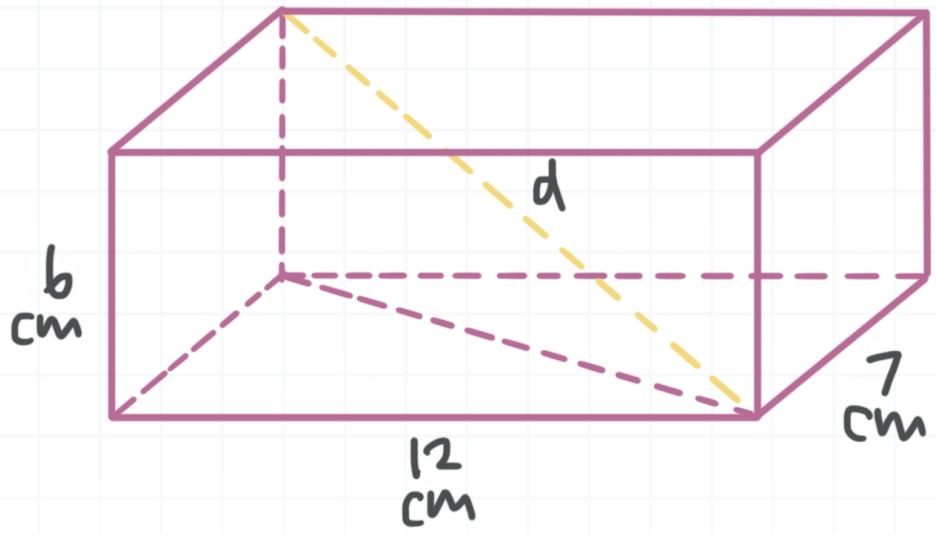
$$\frac{100 \text{ cm}}{1{,}000 \text{ mm}} = \frac{1{,}000 \text{ mm}}{1{,}000 \text{ mm}}$$

$$\frac{1 \text{ cm}}{10 \text{ mm}} = 1$$

Therefore, the conversion factor is going to be $(1 \text{ cm})/(10 \text{ mm})$, so

$$60 \text{ mm} = 60 \text{ mm} \cdot \frac{1 \text{ cm}}{10 \text{ mm}} = 6 \text{ cm}$$

Then the dimensions are



Plugging these into the formula for the diagonal, we get

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{(7\text{ cm})^2 + (12\text{ cm})^2 + (6\text{ cm})^2}$$

$$d = \sqrt{49\text{ cm}^2 + 144\text{ cm}^2 + 36\text{ cm}^2}$$

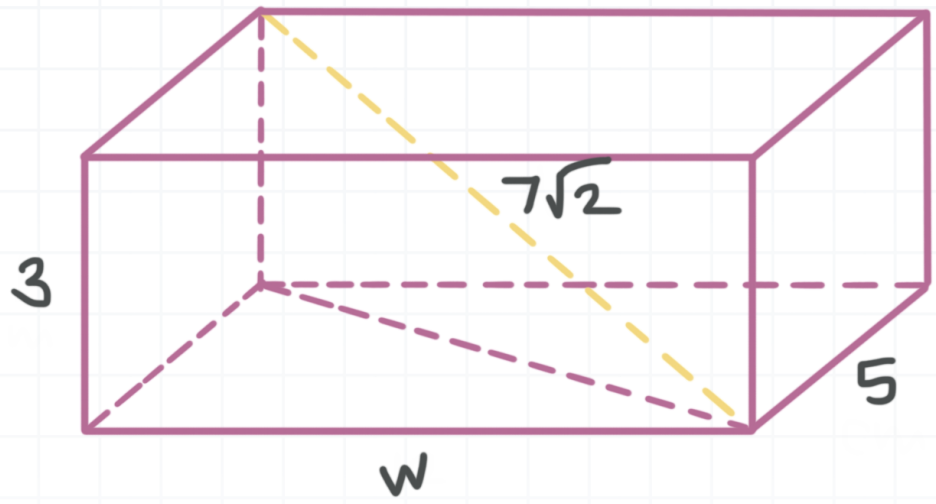
$$d = \sqrt{229\text{ cm}^2}$$

$$d = \sqrt{229}\text{ cm}$$

Let’s try another one.

Example

Find the width of the right rectangular prism.



We just need to plug the dimensions we've been given into the formula for the diagonal.

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$7\sqrt{2} = \sqrt{5^2 + w^2 + 3^2}$$

Manipulate the equation to solve for w, starting with squaring both sides.

$$(7\sqrt{2})^2 = (\sqrt{5^2 + w^2 + 3^2})^2$$

$$(49)(2) = 5^2 + w^2 + 3^2$$

$$98 = 25 + w^2 + 9$$

$$98 = 34 + w^2$$

$$64 = w^2$$

$$8 = w$$