Measures of parallelograms

A **parallelogram** is a quadrilateral that has opposite sides that are parallel. The parallel sides let you know a lot about a parallelogram. Here are the special properties of parallelograms:

Parallelogram

Two pairs of opposite parallel sides

Opposite sides are congruent

Opposite angles are congruent

$$m\angle 1 = m\angle 3$$

$$m\angle 2 = m\angle 4$$

Consecutive angles are supplementary

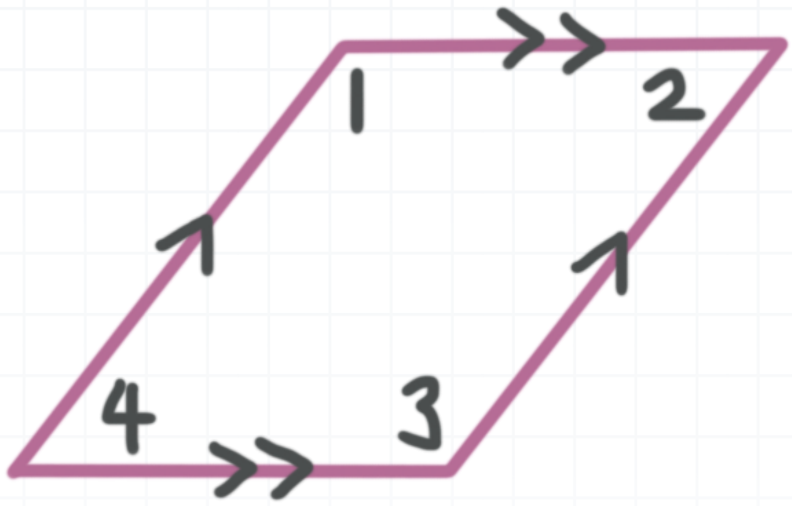
$$m\angle 1 + m\angle 2 = 180^\circ$$

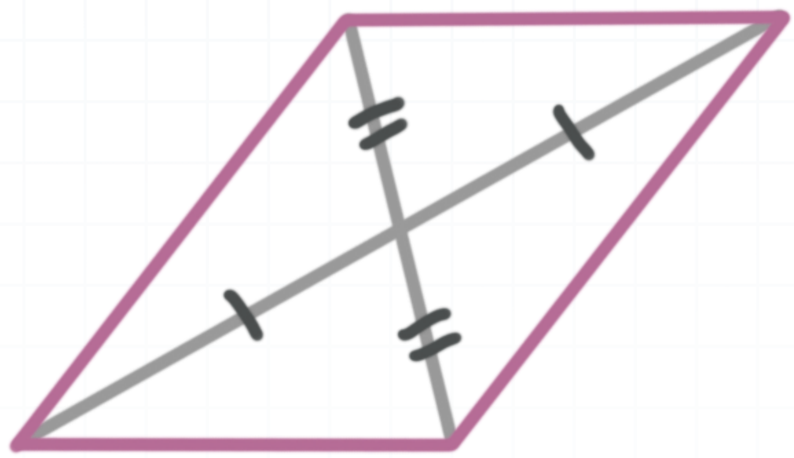
$$m\angle 2 + m\angle 3 = 180^\circ$$

$$m\angle 3 + m\angle 4 = 180^\circ$$

$$m\angle 4 + m\angle 1 = 180^\circ$$

Diagonals bisect each other (cut each other in half)

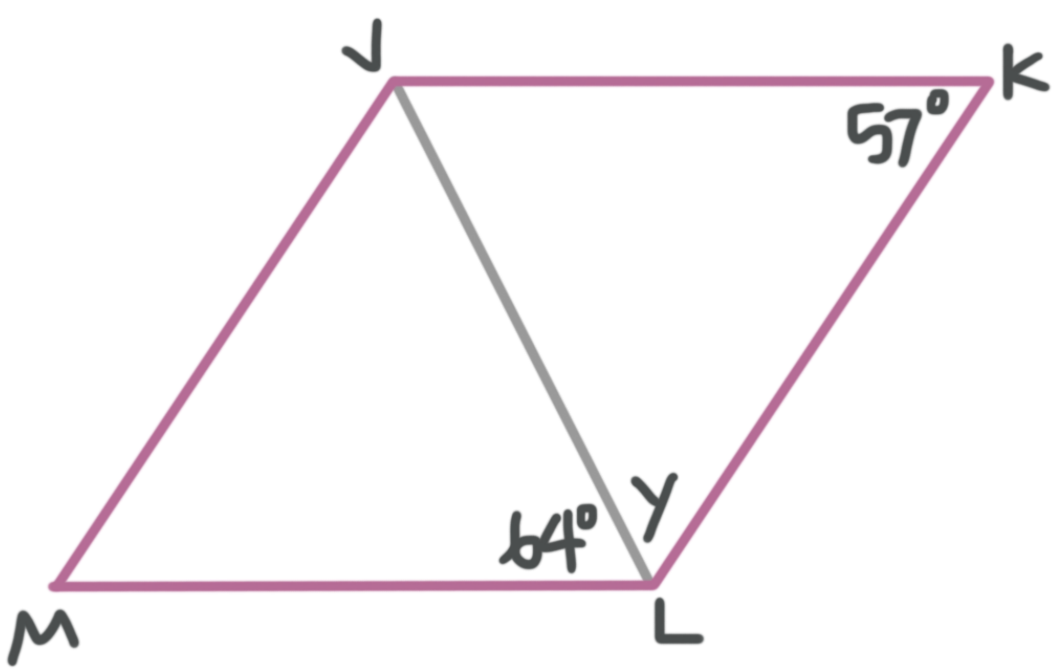




Let's look at a few examples.

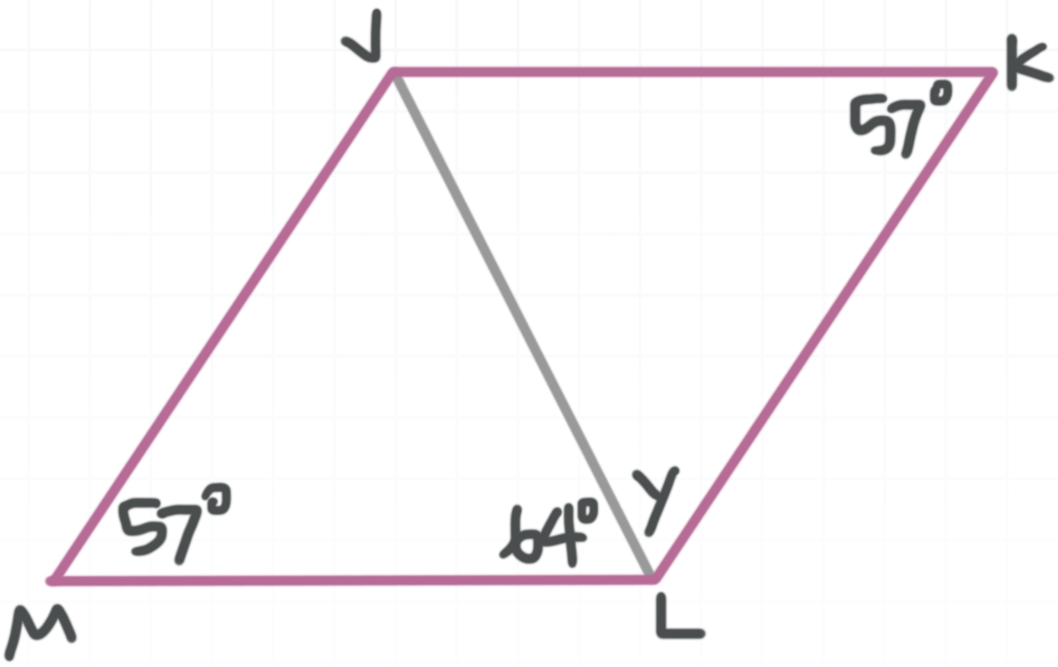
Example

The quadrilateral $JKLM$ is a parallelogram. Find the the value of y.

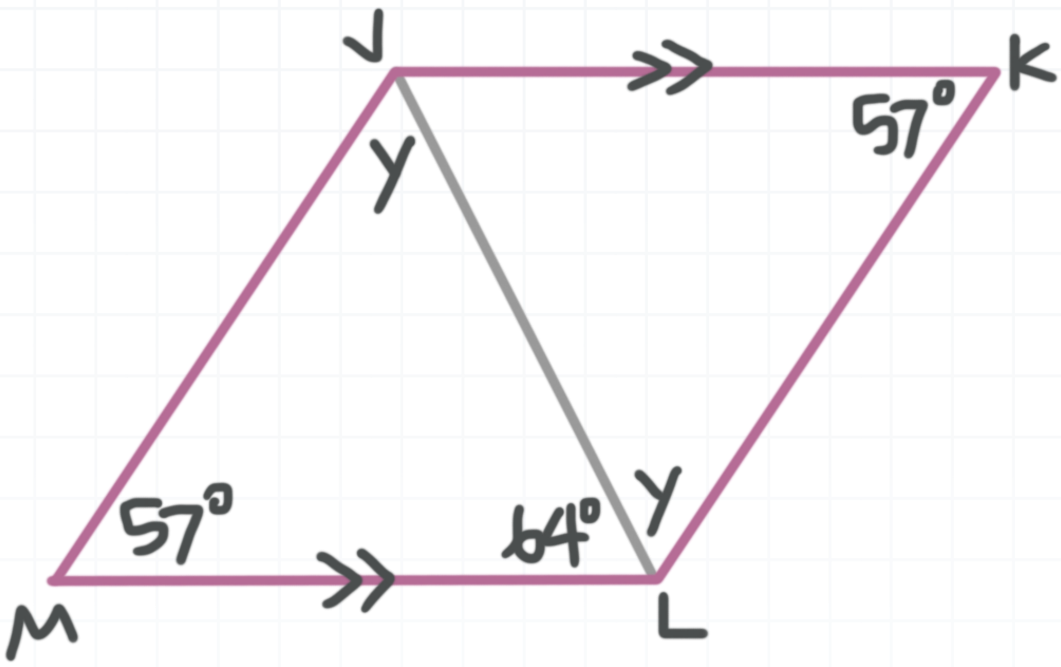


Opposite angles of a parallelogram are congruent, so

$$m\angle LMJ = m\angle JKL = 57^\circ$$



Now we can use the fact that opposite sides of a parallelogram are parallel to state that $\overline{JK} \parallel \overline{ML}$. This means that the diagonal $\overline{JL}$ is also a transversal that crosses a pair of parallel lines (the extensions of $\overline{JK}$ and $\overline{ML}$ to infinity in both direction). This means that $\angle KLJ$ and $\angle MJL$ are a pair of alternate interior angles. Alternate interior angles are congruent, so $m\angle MJL = m\angle KLJ = y$.



The measures of the three interior angles of a triangle add up to 180°, so we can set up an equation for the sum of the interior angles of $\triangle JML$ and solve for y.

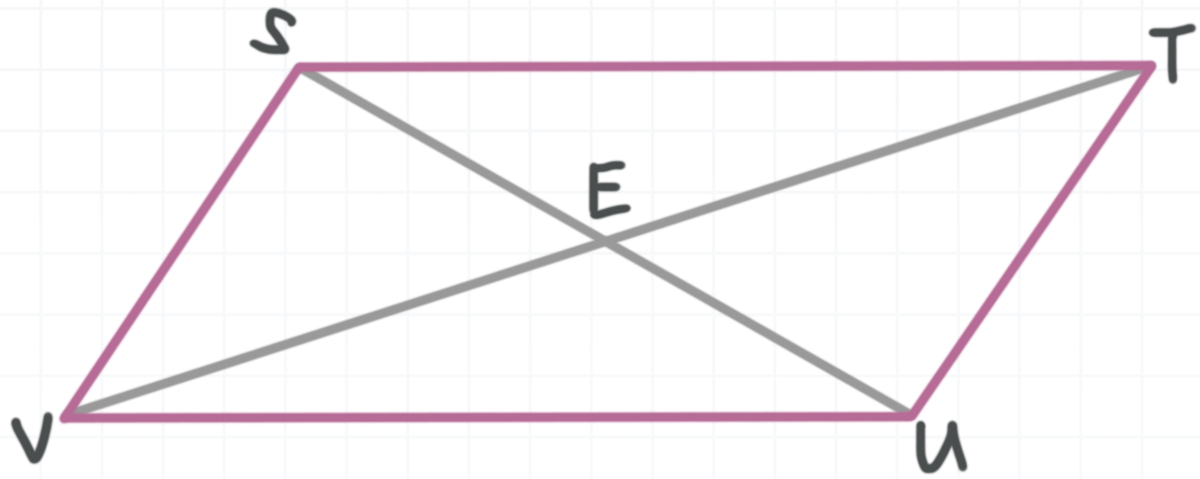
$$y + 57^\circ + 64^\circ = 180^\circ$$

$$y = 59^\circ$$

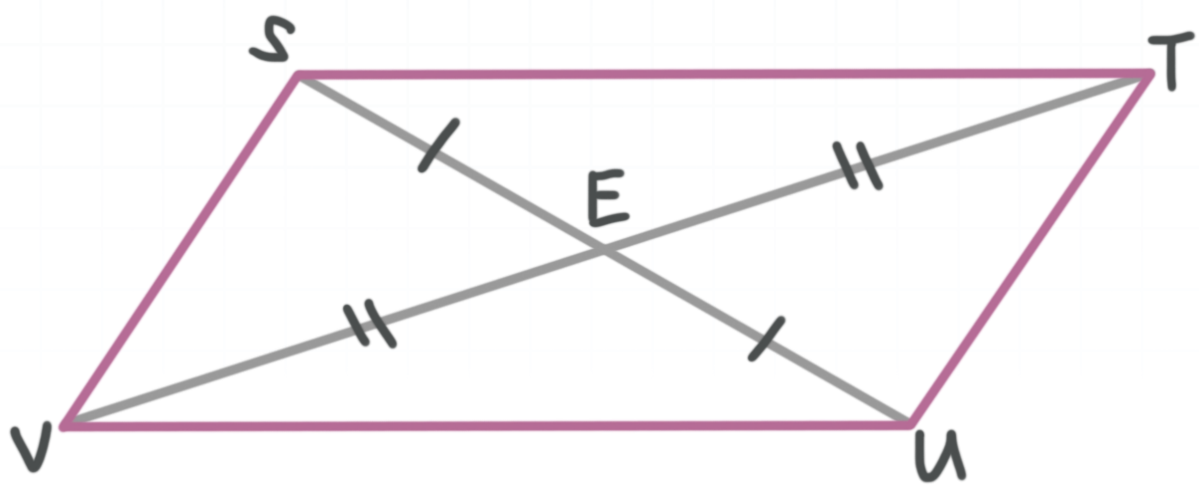
Let's do an example that involves the diagonals of a parallelogram.

Example

The quadrilateral $STUV$ in the figure below is a parallelogram. If $\overline{VT} = 4n + 34$ and $\overline{VE} = 7n - 3$, what is the length of $\overline{ET}$?



We know that the diagonals of a parallelogram bisect each other. Let's add this information into the diagram.



Now we can see the relationships we need. Because the diagonals bisect each other, $\overline{VE} = \overline{ET}$ and the length of $\overline{VE}$ is half that of $\overline{VT}$. We can use what we know to find the length of $\overline{VE}$, and then we'll know the length of $\overline{ET}$ as well.

$$\overline{VE} = \frac{1}{2}(\overline{VT})$$

$$7n - 3 = \frac{1}{2}(4n + 34)$$

$$7n - 3 = 2n + 17$$

$$5n = 20$$

$$n = 4$$

Therefore,

$$\overline{ET} = \overline{VE} = 7n - 3$$

$$\overline{ET} = \overline{VE} = 7(4) - 3$$

$$\overline{ET} = \overline{VE} = 25$$