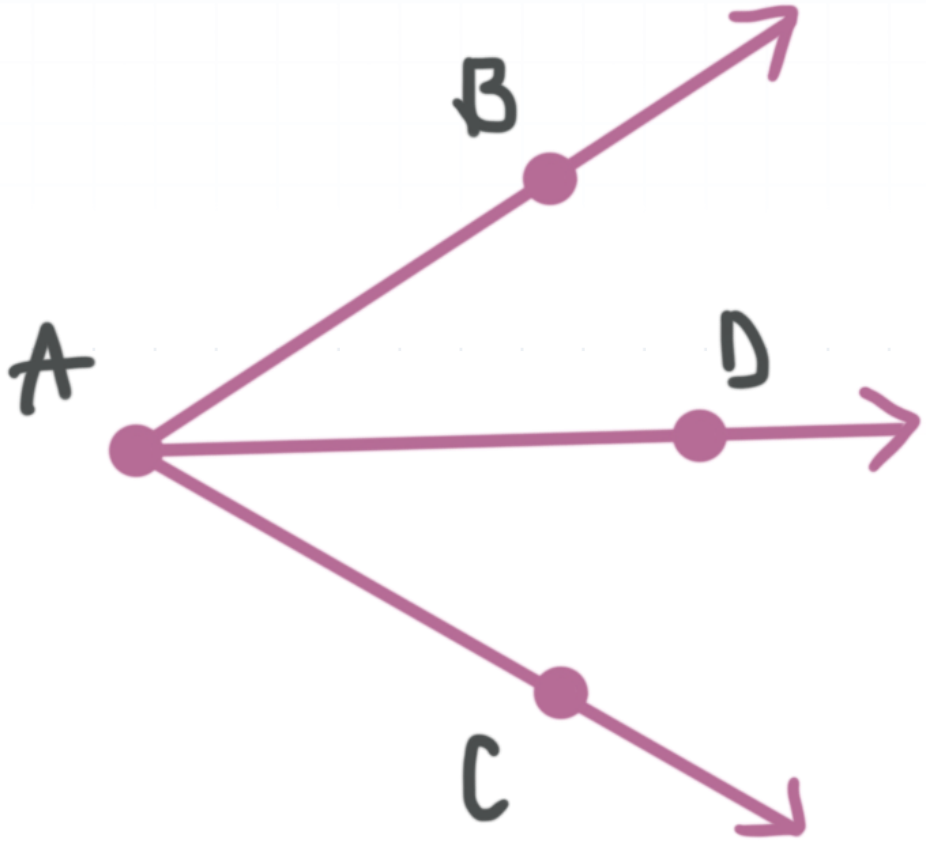
Perpendicular and angle bisectors

In this lesson we'll look at how to use the properties of perpendicular and angle bisectors to get information about geometric figures.

Angle bisectors

An angle bisector is a line, line segment, or ray that goes through the vertex of an angle and divides the angle into two congruent angles that each have measure equal to half that of the original angle. If $\overrightarrow{AD}$ bisects $\angle CAB$,



then

$$m\angle DAB = m\angle CAD$$

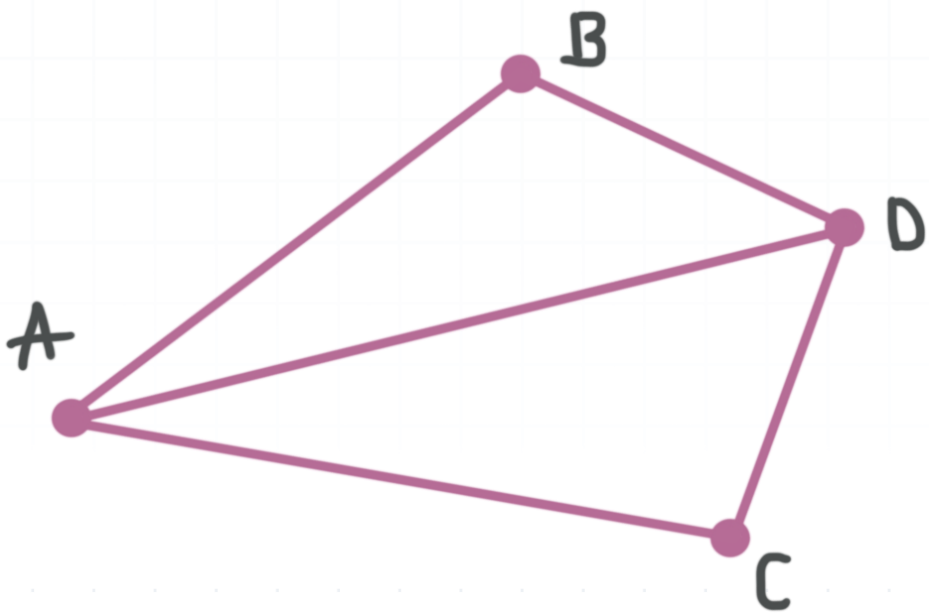
$$m\angle DAB = \frac{1}{2}m\angle CAB = m\angle CAD$$

$$2m\angle DAB = m\angle CAB = 2m\angle CAD$$

Let's look at an example.

Example

If $m\angle DAB = 31°$ and $m\angle BDC = 66°$, and $\overline{AD}$ is a bisector of both $\angle CAB$ and $\angle BDC$, what is $m\angle DCA$?



Using what we already know, we see that

$$m\angle CAD = m\angle DAB = 31°$$

and

$$m\angle ADC = \frac{1}{2}m\angle BDC = \frac{1}{2} \cdot 66° = 33°$$

The measures of the three interior angles of any triangle add up to $180°$. Applying that to $\triangle ACD$, we have

$$m\angle CAD + m\angle ADC + m\angle DCA = 180°$$

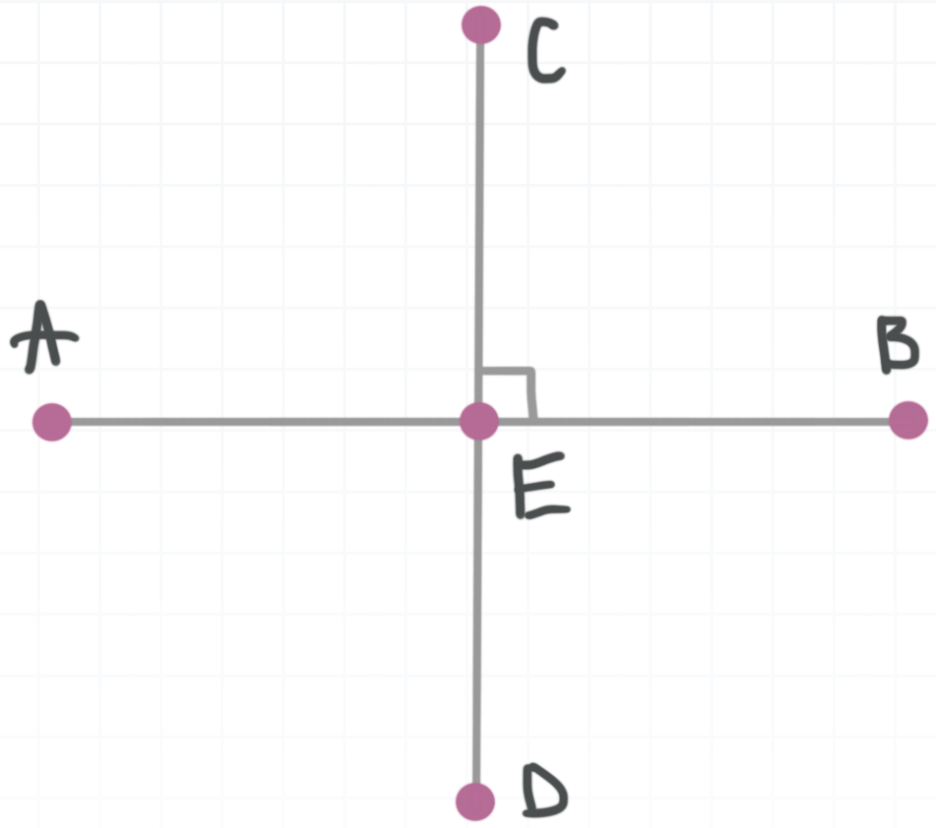
$$31° + 33° + m\angle DCA = 180°$$

$$64° + m\angle DCA = 180°$$

$$m\angle DCA = 116°$$

Perpendicular bisectors

A perpendicular bisector is a line, line segment, or ray that crosses a line segment at its midpoint and forms a right angle where it crosses. $\overline{CD}$ is a perpendicular bisector of $\overline{AB}$ at point E.



This tells us that

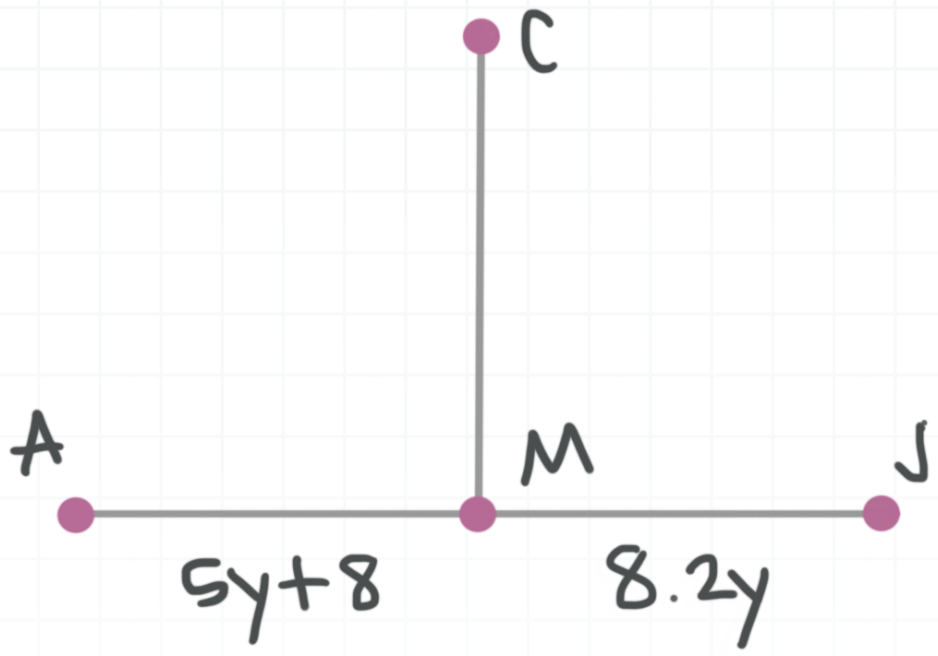
$$m\angle CEA = m\angle BEC = m\angle AED = m\angle DEB = 90°$$

$$\overline{AE} = \overline{EB}$$

Let's look at a few more example problems.

Example

Find the value of y if $\overline{CM}$ is a perpendicular bisector of $\overline{AJ}$.



Because $\overline{CM}$ is a perpendicular bisector of $\overline{AJ}$, we know that $\overline{AM} = \overline{MJ}$, so

$$5y + 8 = 8.2y$$

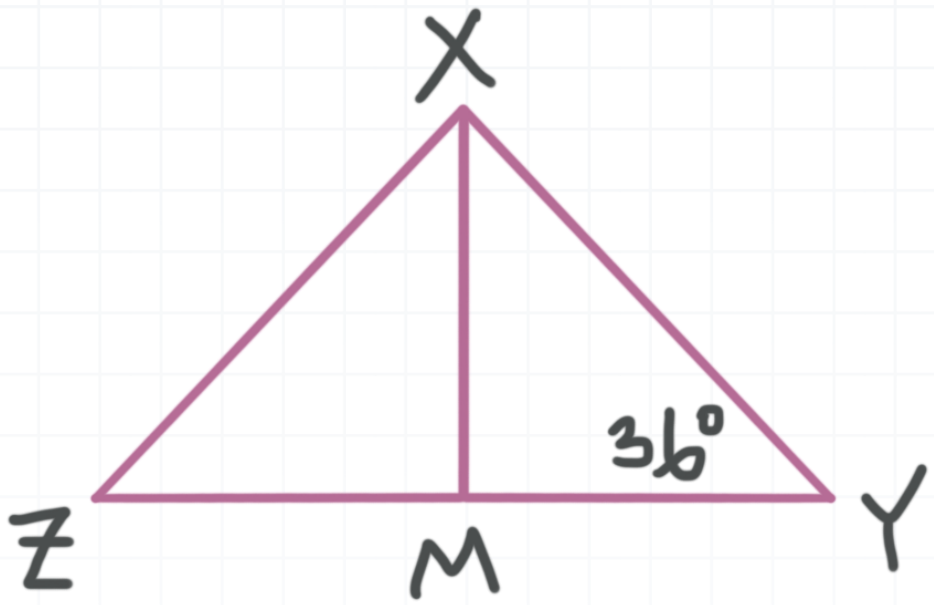
$$8 = 3.2y$$

$$y = 2.5$$

Let's look at one more problem.

Example

Find $m\angle MXY$ if $\overline{XM}$ is a perpendicular bisector of $\overline{ZY}$.



We know $\angle YMX$ is a right angle, so $m\angle YMX = 90°$. The measures of the three interior angles of any triangle add up to $180°$. Applying that to $\triangle XMY$, we have

$$m\angle YMX + m\angle XYM + m\angle MXY = 180°$$

$$90° + 36° + m\angle MXY = 180°$$

$$126° + m\angle MXY = 180°$$

$$m\angle MXY = 54°$$