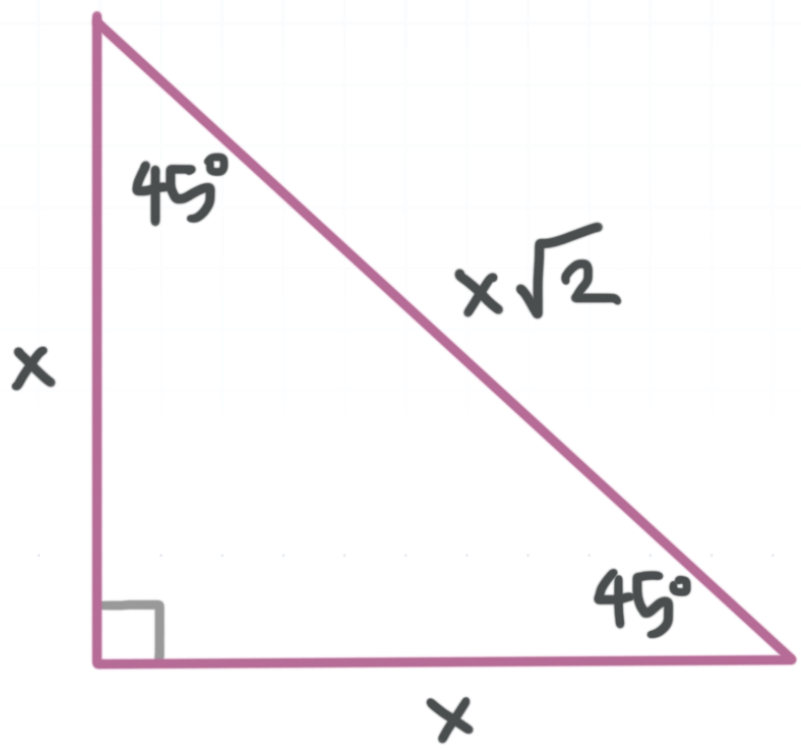
45-45-90 triangles

A 45-45-90 triangle is a special kind of right triangle, because it's isosceles with two congruent sides and two congruent angles. Since it's a right triangle, the length of the hypotenuse has to be greater than the length of each leg, so the congruent sides are the legs of the triangle.

In this figure, the legs are labeled x, and the hypotenuse is labeled $x\sqrt{2}$, because in a 45-45-90 triangle the ratio of the length of the hypotenuse to the length of each leg is equal to $\sqrt{2}$.

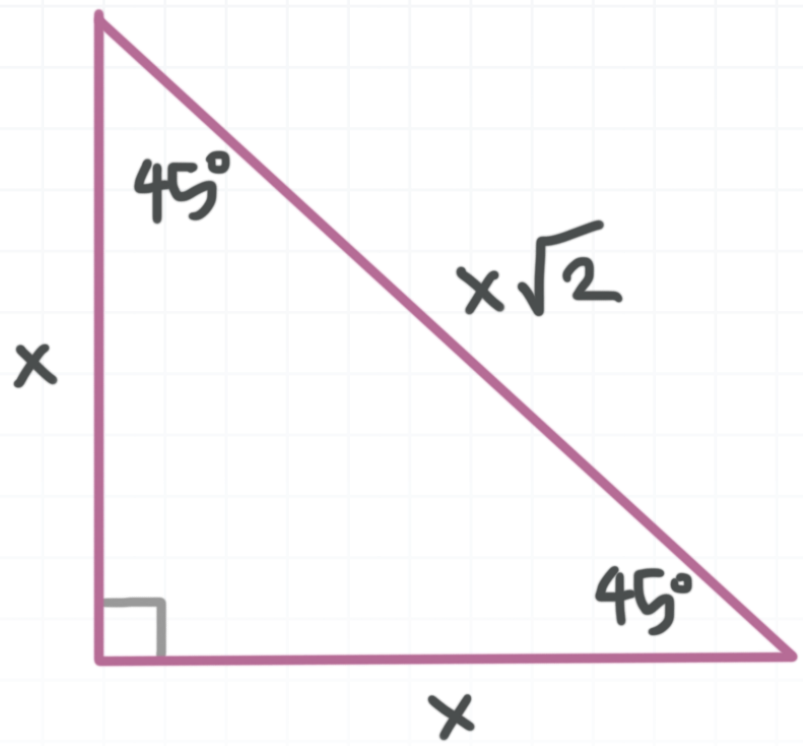


You can use this ratio to find the length of a leg of any 45-45-90 triangle if you know the length of the hypotenuse, or to find the length of the hypotenuse if you know the length of a leg.

Let's start by working through an example.

Example

If $x = 12$, what is the length of the hypotenuse?

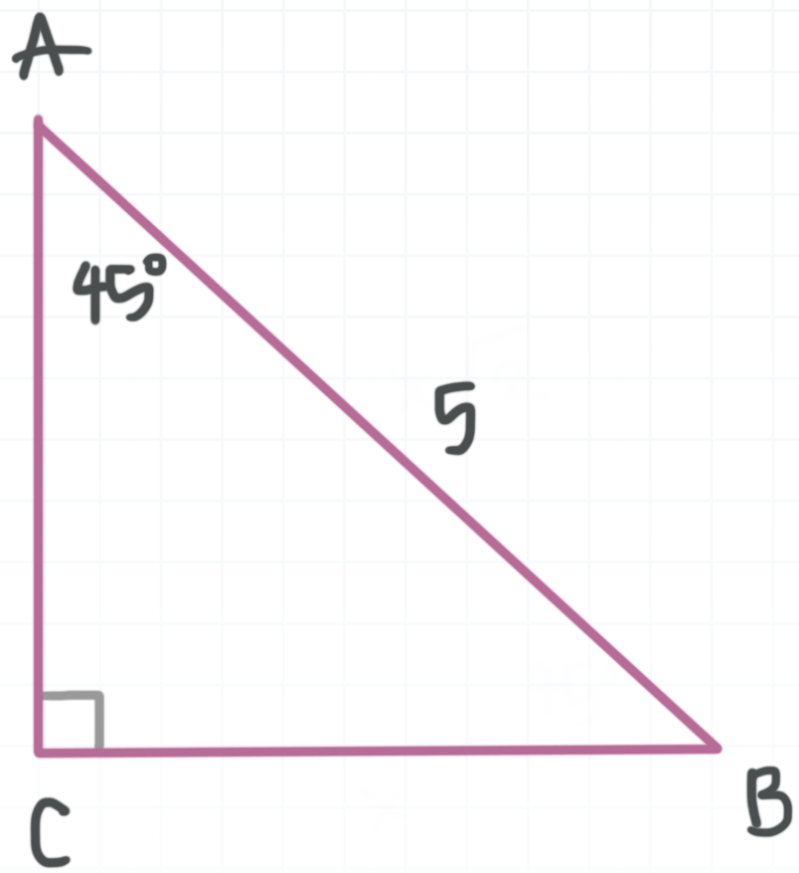


The length of the hypotenuse is $x\sqrt{2}$. We know $x = 12$, so the length of the hypotenuse is $x\sqrt{2}$, or $12\sqrt{2}$.

Let's do another example.

Example

What are the lengths of side $\overline{AC}$ and side $\overline{CB}$?



We can see that $\triangle ABC$ is a 45-45-90 triangle, because $m\angle A = 45°$ and $m\angle C = 90°$, which means the measure of $\angle B$ must be

$$m\angle B = 180° - (m\angle A + m\angle C)$$

$$m\angle B = 180° - (45° + 90°)$$

$$m\angle B = 180° - 135°$$

$$m\angle B = 45°$$

Therefore, angles A and B are congruent, which means that the lengths of the sides opposite angles A and B (sides $\overline{CB}$ and $\overline{AC}$, respectively) are congruent.

So let x be the length of $\overline{AC}$. This tells us that the length of the hypotenuse ($\overline{AB} = 5$) is represented by $x\sqrt{2}$. We want to set up an equation that we can use to find x.

$$5 = x\sqrt{2}$$

$$x = \frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

This means sides $\overline{AC}$ and $\overline{CB}$ both have length $5\sqrt{2}/2$. We could also have used the Pythagorean theorem to find x.

$$a^2 + b^2 = c^2$$

$$x^2 + x^2 = 5^2$$

$$2x^2 = 25$$

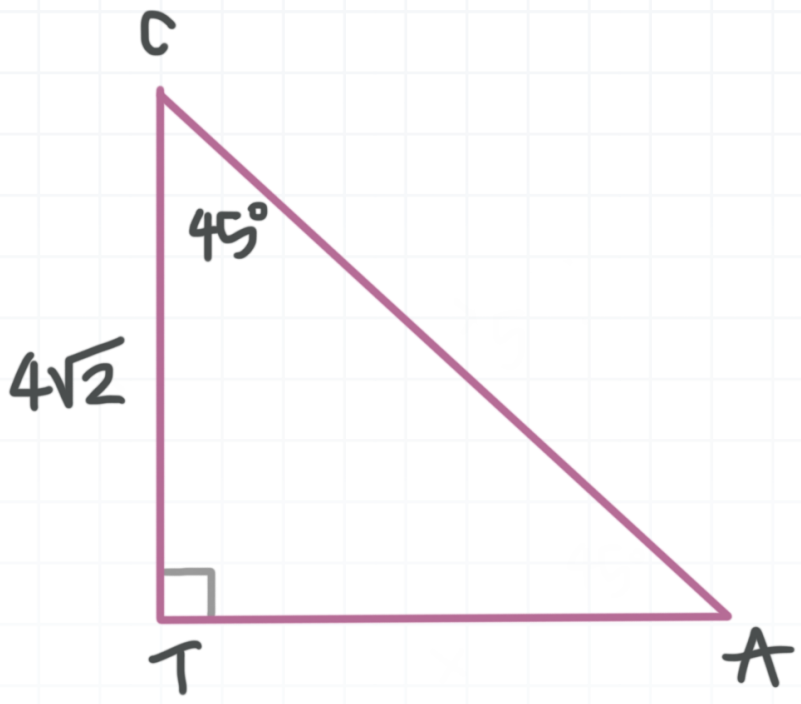
$$x^2 = \frac{25}{2}$$

$$x = \frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

Let's try another example.

Example

What is the length of side $\overline{CA}$?



Because the measures of two of the interior angles of this right triangle are $45°$ and $90°$, we immediately see that the measure of the remaining interior angle is $45°$, so this is a 45-45-90 triangle.

The pattern for the lengths of the sides of a 45-45-90 triangle is x, x, and $x\sqrt{2}$, where x is the length of each leg. In this case, $x = 4\sqrt{2}$. The length of the hypotenuse is $x\sqrt{2}$, so the length of side $\overline{CA}$, which is the hypotenuse, is

$$x\sqrt{2} = (4\sqrt{2})(\sqrt{2}) = 4(\sqrt{2} \cdot \sqrt{2}) = 4(2) = 8$$