

Surface area to volume ratio of prisms

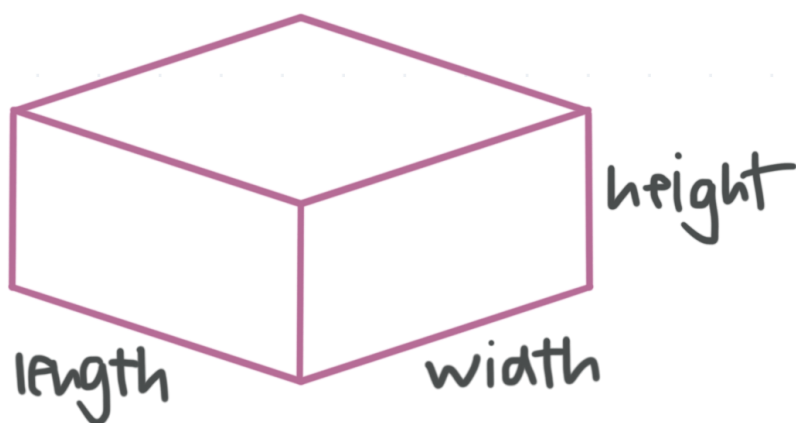
Remember that the formulas for the areas of the faces of a right rectangular prism (a prism in which all the faces are rectangles) are given as

Top and bottom	length x width
Left and right	width x height
Front and back	length x height

The formula for surface area of a rectangular prism (we'll call that area S , for "surface") is

$$S = 2lw + 2wh + 2lh$$

And the volume of a rectangular prism is the product of the length, the width, and the height.



$$V = lwh$$

Surface area to volume ratio



The ratio of the surface area, S , to the volume, V , can be expressed as a fraction S/V , or converted to a decimal. Since area is in units of length^2 , and volume is in units of length^3 , the ratio S/V will be in units of $(\text{length}^2)/(\text{length}^3) = \text{length}^{-1}$.

Let's start by working through an example.

Example

Calculate the surface area to volume ratio of a right rectangular prism that measures 4 cm high, 6 cm wide, and 8 cm long. Express your answer as a decimal rounded to the nearest tenth.

Use $S = 2lw + 2wh + 2lh$ to find the surface area.

$$S = 2(8 \text{ cm} \cdot 6 \text{ cm}) + 2(6 \text{ cm} \cdot 4 \text{ cm}) + 2(8 \text{ cm} \cdot 4 \text{ cm})$$

$$S = 2(48 \text{ cm}^2) + 2(24 \text{ cm}^2) + 2(32 \text{ cm}^2)$$

$$S = 96 \text{ cm}^2 + 48 \text{ cm}^2 + 64 \text{ cm}^2$$

$$S = 208 \text{ cm}^2$$

Use $V = lwh$ to find the volume.

$$V = (8 \text{ cm}) \cdot (6 \text{ cm}) \cdot (4 \text{ cm})$$

$$V = 192 \text{ cm}^3$$

Now find the ratio of surface area to volume.



$$\frac{S}{V} = \frac{208 \text{ cm}^2}{192 \text{ cm}^3} = \frac{13}{12} \text{ cm}^{-1}$$

As a decimal rounded to the nearest tenth, the ratio is 1.1 cm^{-1} .

Let's do another example.

Example

The surface area to volume ratio of a right rectangular prism is $(11/20) \text{ feet}^{-1}$. The volume of the prism is $1,140 \text{ feet}^3$. What is the prism's surface area?

We know that

$$\frac{S}{V} = \frac{11}{20} \text{ feet}^{-1}$$

and that the volume is $V = 1,140 \text{ feet}^3$. Therefore,

$$\frac{S}{1,140 \text{ feet}^3} = \frac{11}{20} \text{ feet}^{-1}$$

$$\left(\frac{S}{1,140 \text{ feet}^3} \right) (1,140 \text{ feet}^3) = \left(\frac{11}{20} \text{ feet}^{-1} \right) (1,140 \text{ feet}^3)$$

$$S = 627 \text{ feet}^2$$

