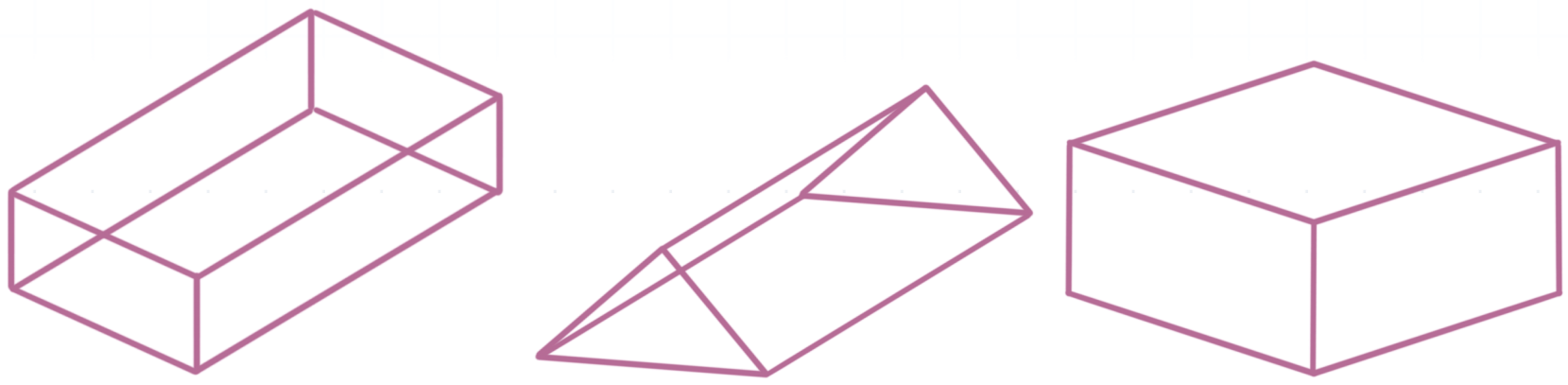
Nets/volume/surface area of prisms

In this lesson we'll look at an introduction to three-dimensional geometric figures, specifically nets, volume, and surface area of prisms.

Prism

A **polyhedron** is a closed three-dimensional shape that's made up of polygons (which are the **faces** of the polyhedron). A **prism** is a polyhedron that has a pair of parallel congruent faces of any shape, and whose other faces are parallelograms. In this lesson, we'll be dealing only with prisms in which all the faces other than the bases are rectangles. Here are some examples of prisms:



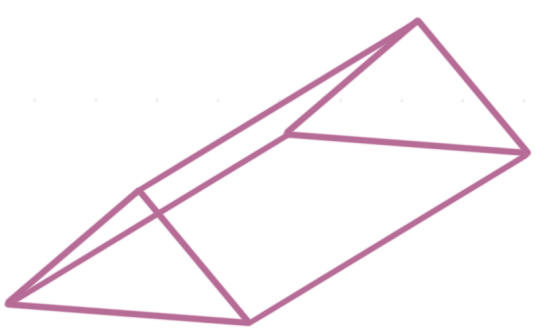
The height of a prism is the length of any line segment that has one endpoint on each base and is perpendicular to both bases. A prism whose bases are triangles is called a **triangular prism**, a prism whose bases are rectangles is called a **rectangular prism**, a prism whose bases are pentagons is called a **pentagonal prism**, etc.

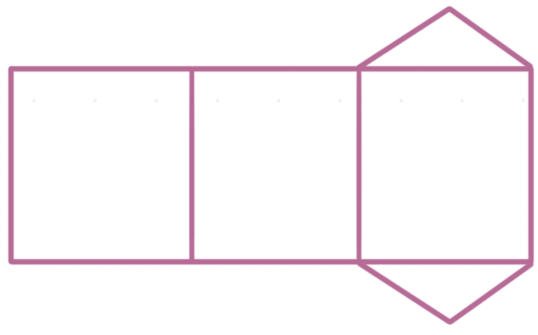
In a rectangular prism in which all the faces (not just the bases) are rectangles, any pair of parallel, congruent rectangles can be used as the bases.

Net

A **net** of a polyhedron is a two-dimensional flattened out version of it. We make the net by cutting the polyhedron along one or more of the edges until we can lay out the whole thing flat in a plane.

Once we have the net of a polyhedron, we should be able to reconstruct it by folding the pieces of the net, using the polygons in the net as faces, and using each line segment in the net as the boundary between some pair of faces of the polyedron. Here is a triangular prism (on the left) and its net (on the right).





Surface area of a rectangular prism

The surface area of any prism (in fact, of any polyhedron) is the sum of the areas of all of its faces. Since the bases of a rectangular prism are

rectangles, and all the other faces are rectangles, we already have all the tools we'll need to find the surface area of a rectangular prism.

The areas of the faces of the rectangular prism are given in the table:

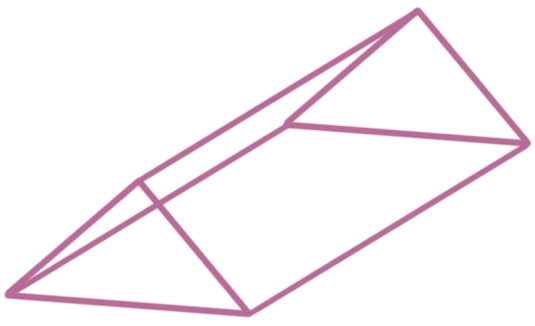
Top and bottom	length x width
Left and right	width x height
Front and back	length x height

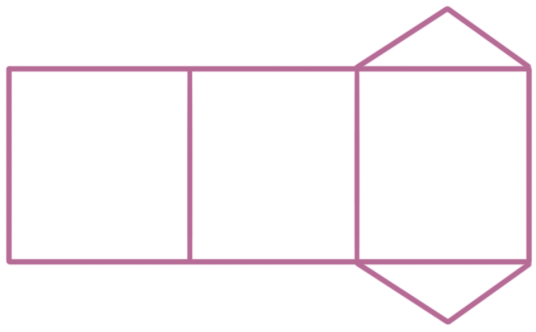
The formula for surface area of a rectangular prism is

$$A = 2lw + 2wh + 2lh$$

Surface area of a prism

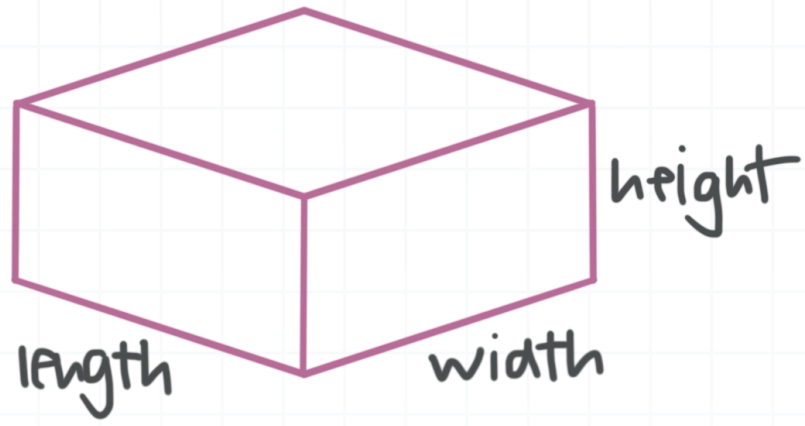
To find the surface area of any prism, it can be helpful to sketch its net, find the area of each shape in the net, and then add those areas. To find the surface area of this triangular prism, which has a pair of (congruent) triangular bases and three rectangular faces, find the areas of the three rectangles and two triangles in this net and add them.





Volume of a rectangular prism

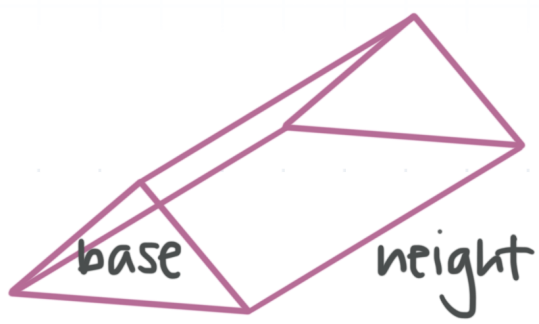
The volume of the type of rectangular prism we're studying is the product of its length, its width, and its height.



$$V = lwh$$

Volume of a prism

The volume of any prism is the product of its base and its height.



$V =$ (area of base)(height)

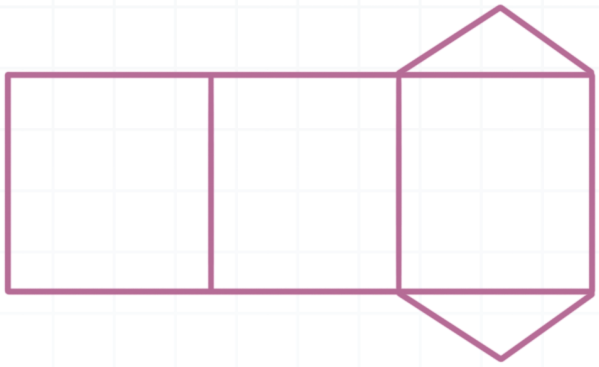
The volume of a triangular prism is the product of the area of one of its bases (one of its triangles) and its height (the length of any line segment that has one endpoint on each base and is perpendicular to both bases).

Let's start by working through an example.

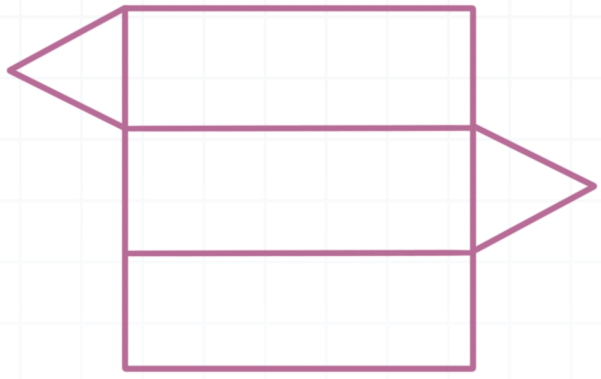
Example

Which net does not belong to a prism?

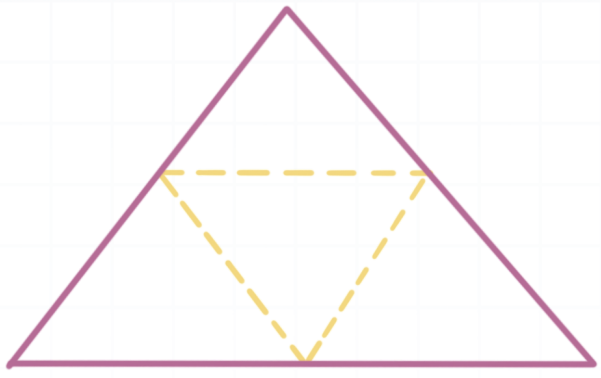
A.



B.



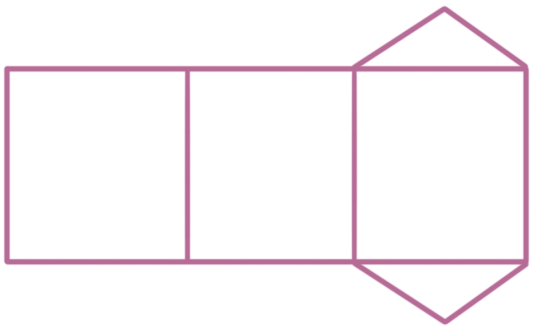
C.

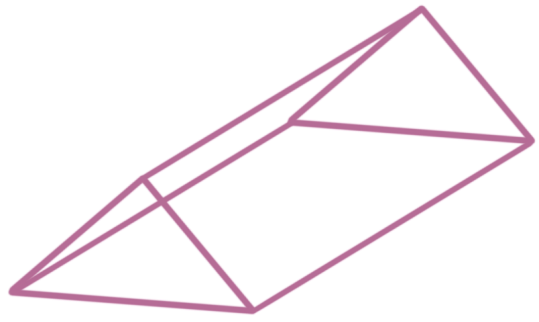


D.

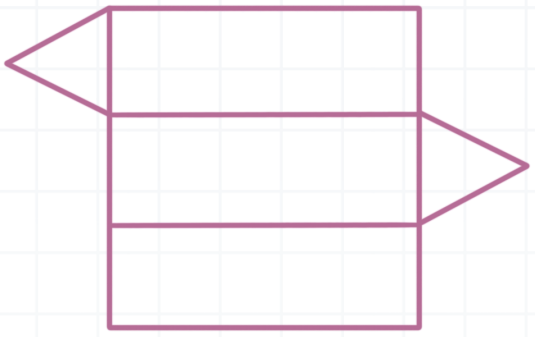
Net C is the net of a triangular pyramid. All of the other nets are nets of prisms, because they have one pair of congruent bases and all the other faces are rectangles.

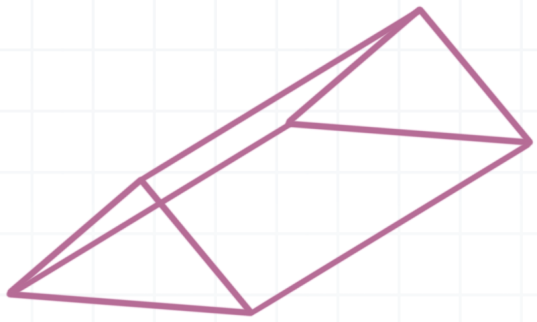
Net A is a net of a triangular prism because it has two triangular bases.



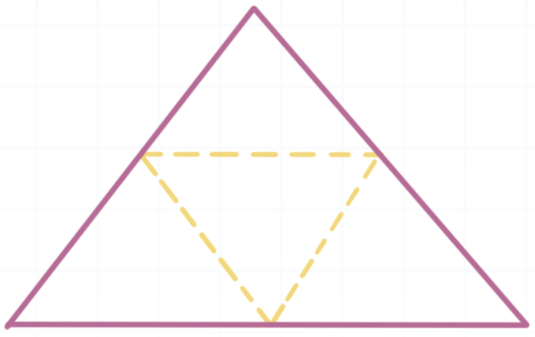


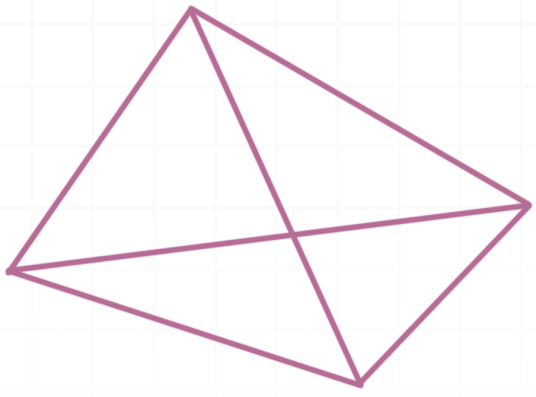
Net B is another example of a net of a triangular prism.



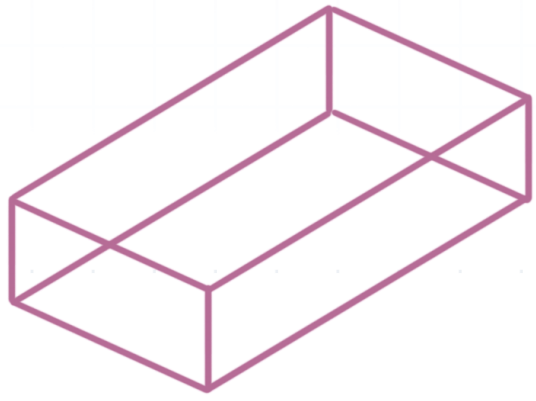


Net C is a net of a triangular pyramid and not a prism.





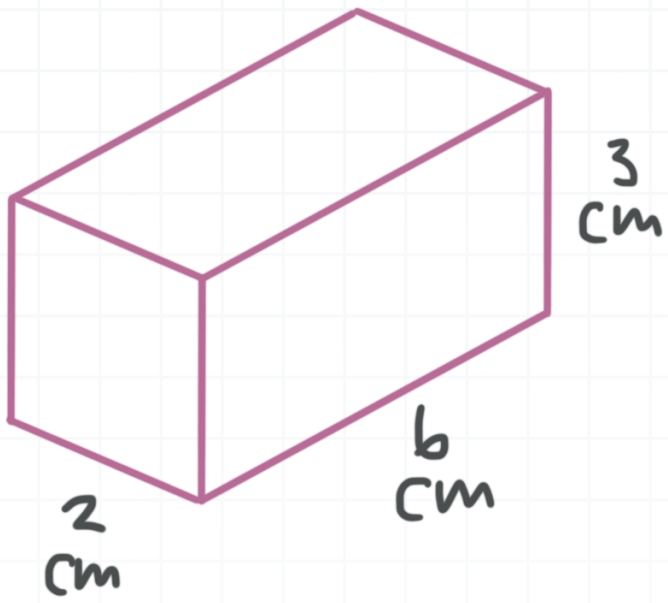
Net D is a net of a rectangular prism.



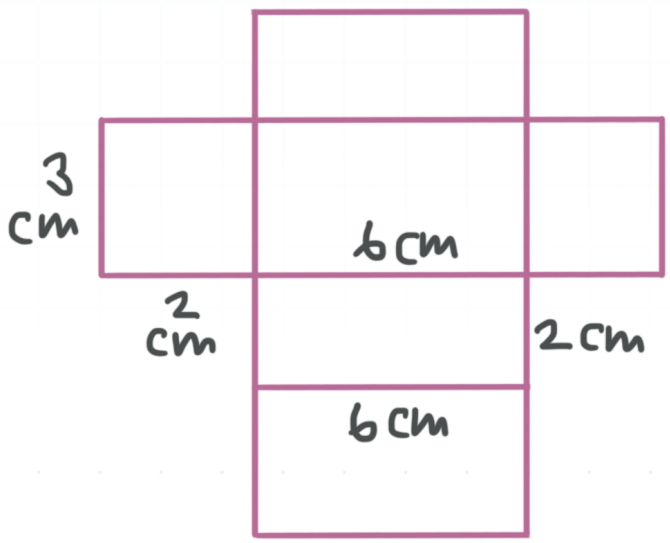
Let’s do an example of surface area.

Example

What is the surface area of the figure?



It can be helpful to draw a net of the figure to calculate the surface area.



Now we can see that we have three pairs of shapes. We can find the area of each and then add them.

$$A = 2[(2\text{ cm})(3\text{ cm})] + 2[(3\text{ cm})(6\text{ cm})] + 2[(2\text{ cm})(6\text{ cm})]$$

$$A = 2[(6\text{ cm}^2)] + 2[(18\text{ cm}^2)] + 2[(12\text{ cm}^2)]$$

$$A = 12\text{ cm}^2 + 36\text{ cm}^2 + 24\text{ cm}^2$$

$$A = 72\text{ cm}^2$$

You can also think of the surface area of a rectangular box as the sum of the areas of its six sides.

Top and bottom	length x width
Left and right	width x height
Front and back	length x height

We'll use the surface area formula.

$$A = 2lw + 2wh + 2lh$$

Plugging in 6 cm for l, 2 cm for w, and 3 cm for h, we get

$$A = 2[(6 \text{ cm})(2 \text{ cm})] + 2[(2 \text{ cm})(3 \text{ cm})] + 2[(6 \text{ cm})(3 \text{ cm})]$$

$$A = 2[(12 \text{ cm}^2)] + 2[(6 \text{ cm}^2)] + 2[(18 \text{ cm}^2)]$$

$$A = 24 \text{ cm}^2 + 12 \text{ cm}^2 + 36 \text{ cm}^2$$

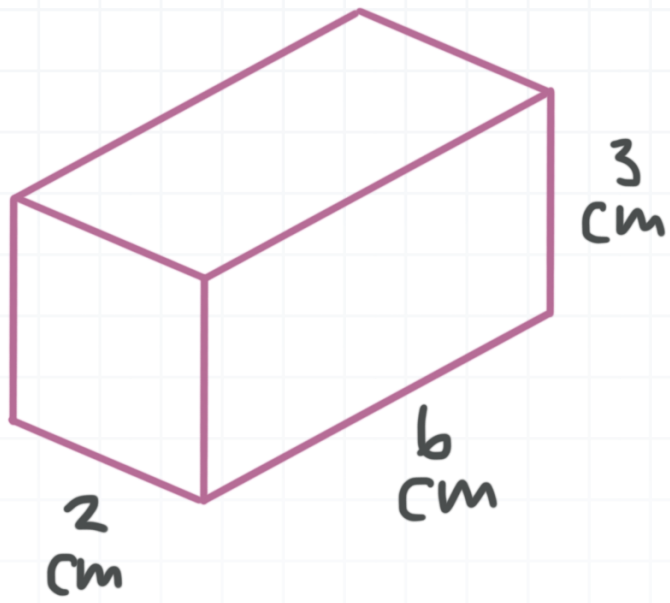
$$A = 72 \text{ cm}^2$$

Which is what we got by using the net.

Let's do an example with volume.

Example

What is the volume of the figure?



To find the volume of the rectangular prism, multiply the length, width, and height.

$$V = lwh$$

$$V = (6 \text{ cm})(2 \text{ cm})(3 \text{ cm})$$

$$V = 36 \text{ cm}^3$$