Arc length

In this lesson we'll look at how to find the length of an arc when we're given the radius and the measure of the corresponding central angle in degrees.

The arc length formula

An arc can be measured in degrees, but it can also be measured in units of length.

The circumference of a circle is the total length of the circle (the "distance around the circle"). An arc is part of a circle. In fact, the ratio of the measure m of an arc of a circle (in degrees) to the measure of the entire circle (in degrees), that is, the ratio of (the numerical value of) m to 360, is equal to the ratio of the arc length L to the circumference (total length) C of the circle:

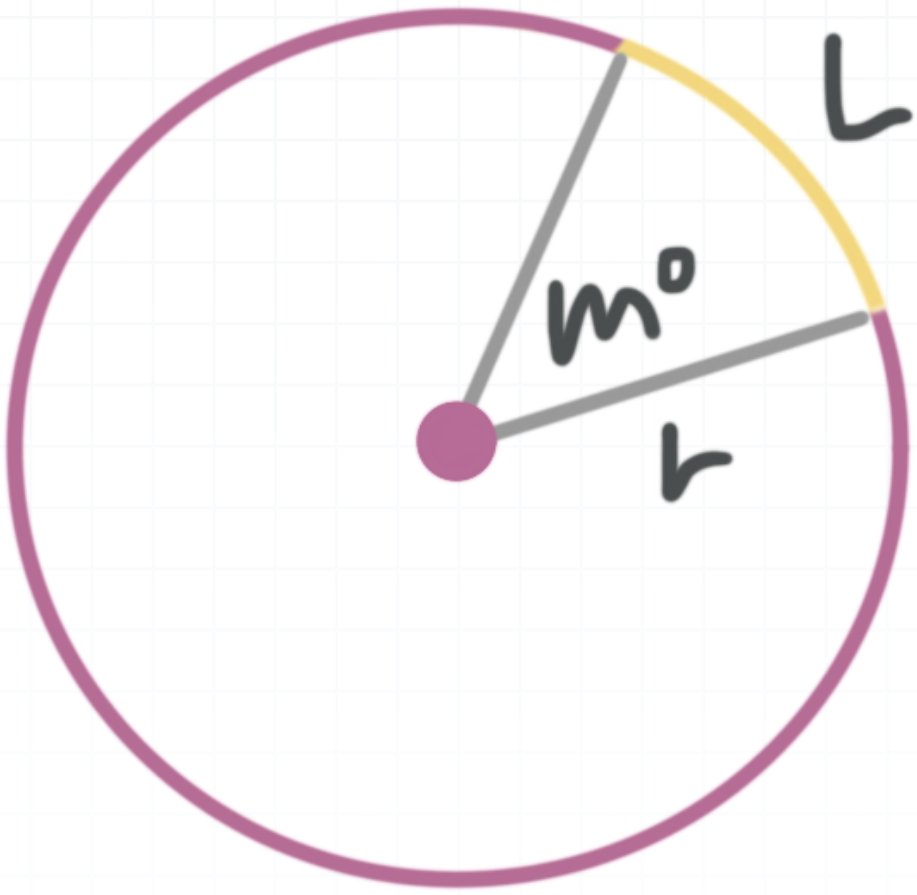
$$\frac{m}{360} = \frac{L}{C}$$

The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius of the circle. Therefore,

$$\frac{m}{360} = \frac{L}{2\pi r}$$

Solving for L, we get the arc length formula:

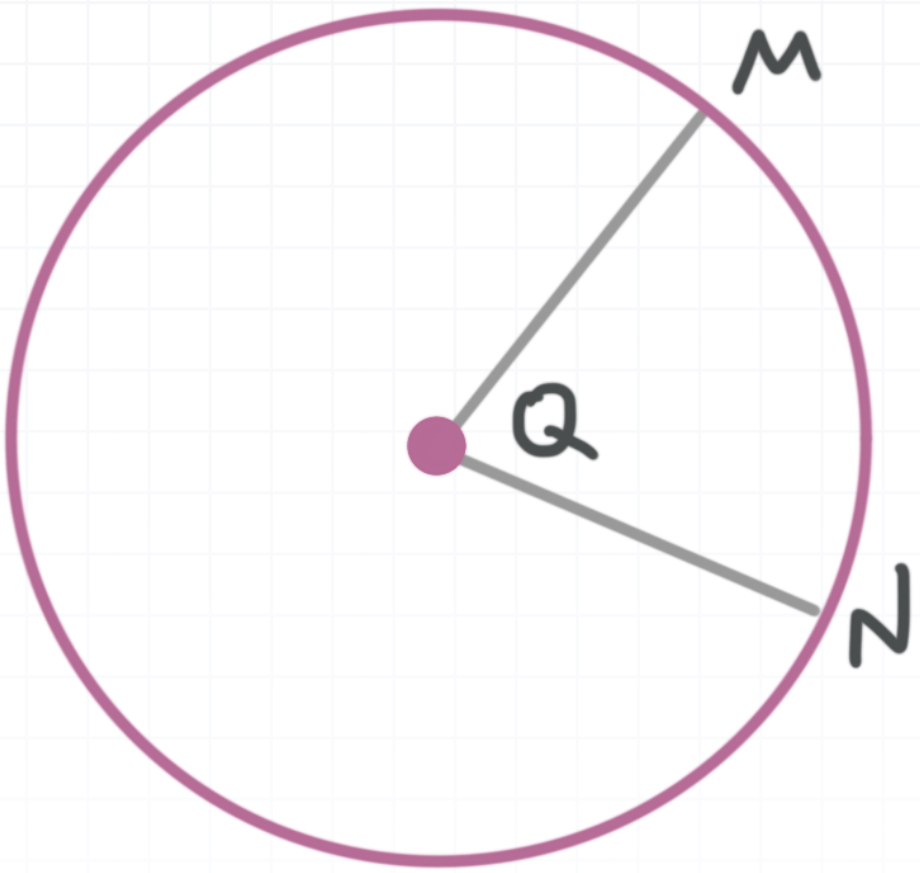
$$L = \frac{m}{360} \cdot 2\pi r$$



Many textbooks will refer to the arc length with the lowercase letter s. Let's start by working through an example.

Example

The radius of the circle (with center at Q) in the figure is 12 cm, and the measure of angle NQM is $86°$. What is the length of $\overset{\frown}{NM}$?



We know that the radius of the circle is 12 cm and the central angle that corresponds to $\overset{\frown}{NM}$ measures $86°$, so $m\overset{\frown}{NM} = 86°$. Now we can use the arc length formula.

$$L = \frac{m}{360} \cdot 2\pi r$$

$$L = \frac{86}{360} \cdot 2\pi(12)$$

$$L = \frac{43}{180} \cdot 24\pi$$

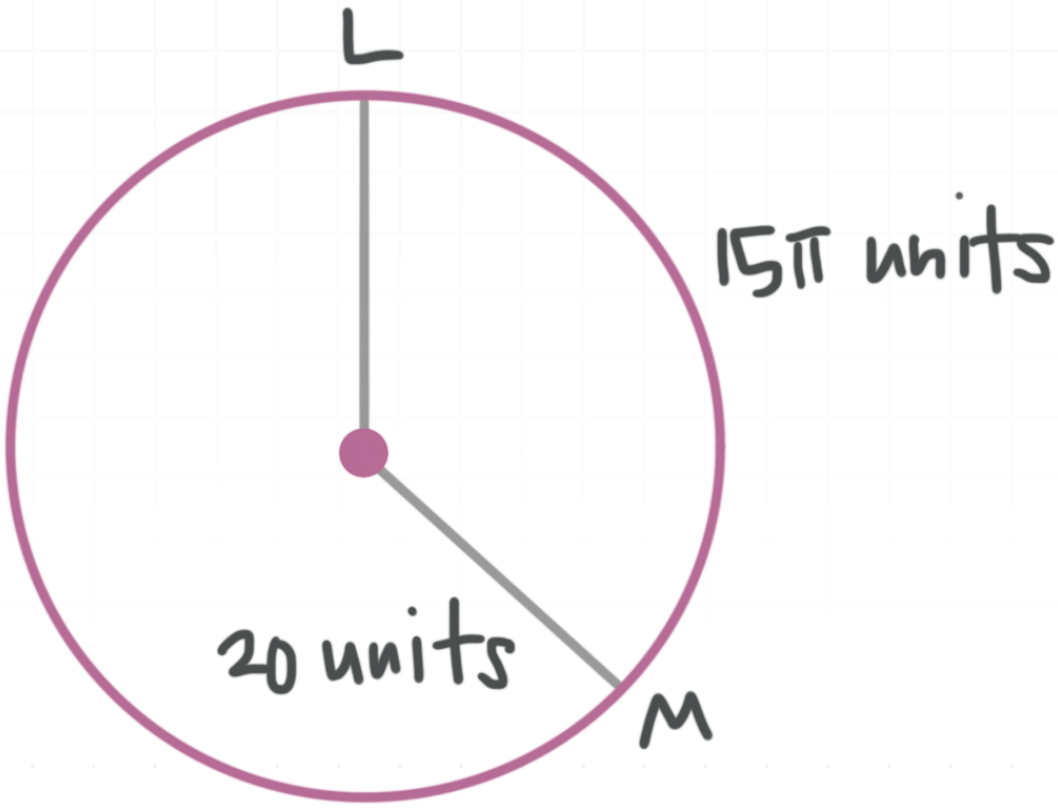
$$L = \frac{86}{15}\pi$$

$$L \approx 18.01$$

Sometimes you will be given the arc length and asked to find the degree measure of the arc or the measure of the corresponding central angle.

Example

Find the measure of the central angle (in degrees) that corresponds to $\overset{\frown}{ML}$.



Remember that the degree measure m of the arc is equal to the measure of the central angle that corresponds to the arc. We'll use the arc length formula

$$L = \frac{m}{360} \cdot 2\pi r$$

We know that $L = 15\pi$ and $r = 20$, so we'll solve for m.

$$L = \frac{m}{360} \cdot 2\pi r$$

$$15\pi = \frac{m}{360} \cdot 2\pi(20)$$

$$15\pi = \frac{m}{360} \cdot 40\pi$$

$$\frac{15\pi}{40\pi} = \frac{m}{360}$$

$$\frac{3}{8} = \frac{m}{360}$$

$$\frac{3}{8} \cdot 360 = m$$

$$m = 135$$

Therefore, the measure of the central angle that corresponds to $\overset{\frown}{ML}$ is $135°$.