Pythagorean inequalities

In this lesson we'll look at different types of triangles and how to use Pythagorean inequalities to determine what kind of triangle we have based on their angle measures and side lengths.

Types of triangles by angle sizes

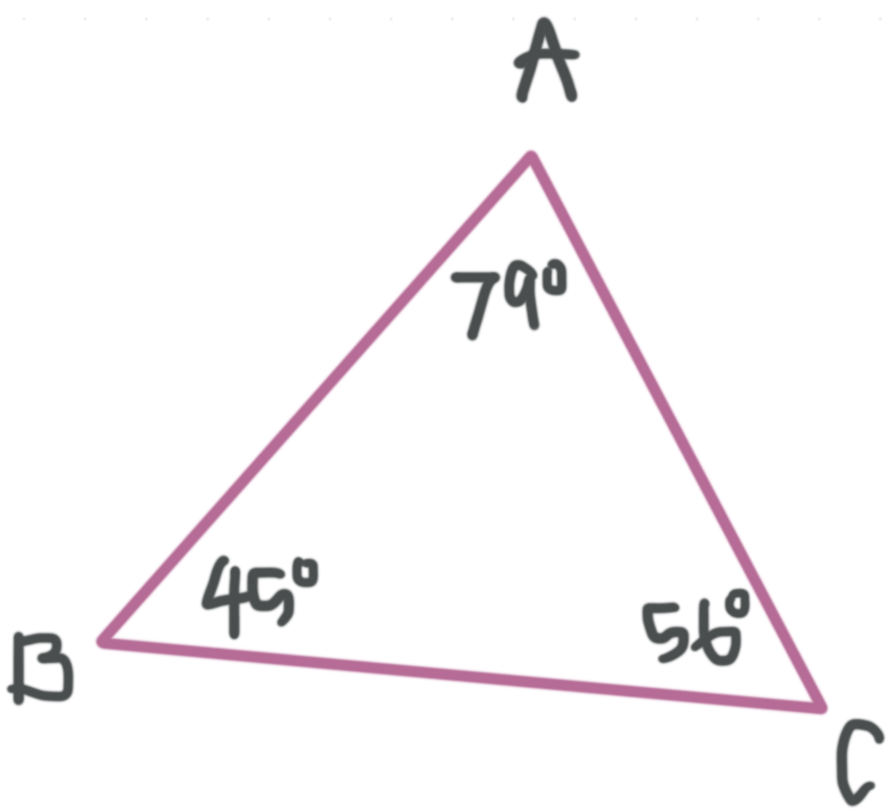
Acute triangle

All of the angles are smaller than 90°.

$m\angle A = 79^\circ$

$m\angle B = 45^\circ$

$m\angle C = 56^\circ$



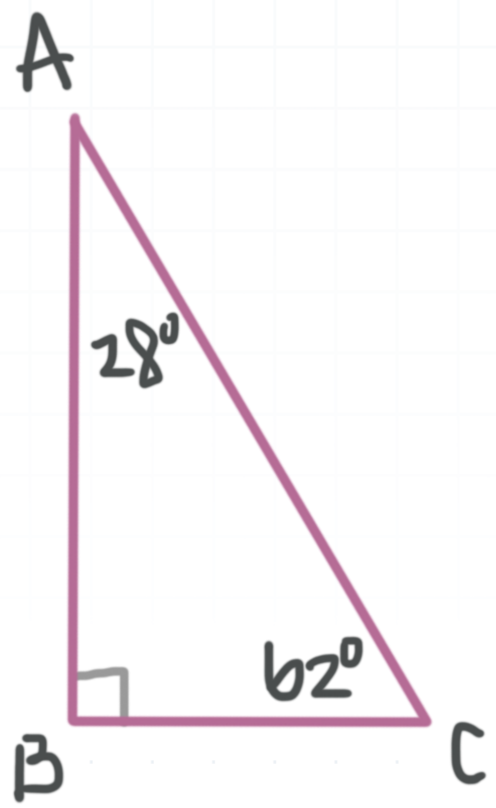
Right triangle

The triangle has a right angle.

$m\angle A = 28^\circ$

$m\angle B = 90^\circ$

$m\angle C = 62^\circ$



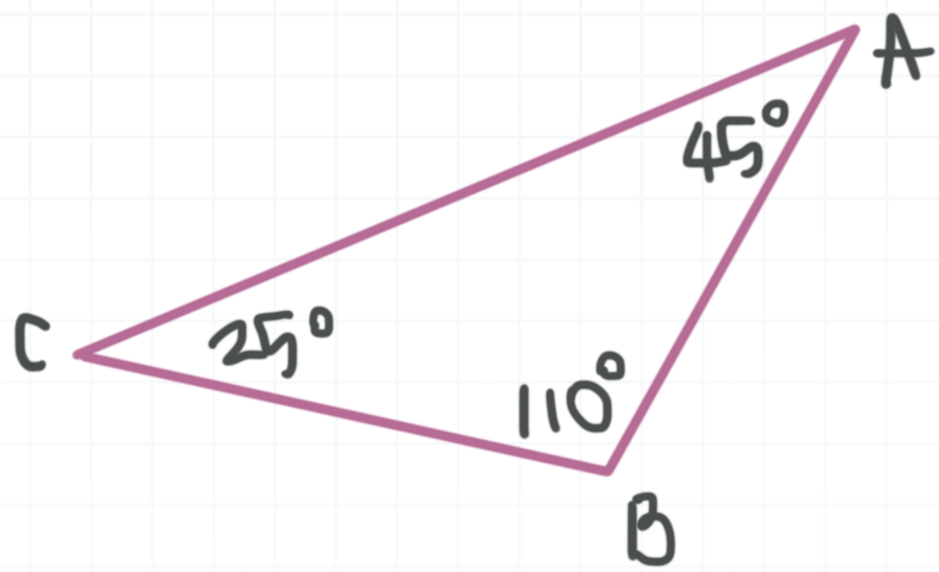
Obtuse triangle

The triangle has an angle greater than 90°.

$m\angle A = 45^\circ$

$m\angle B = 110^\circ$

$m\angle C = 25^\circ$

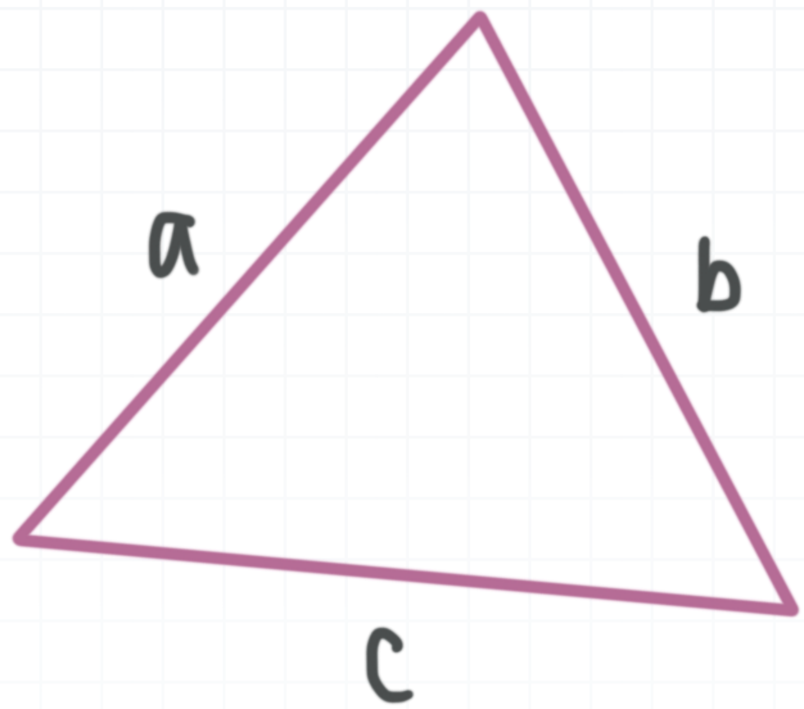


Pythagorean inequalities

There is a relationship between the lengths of the two shortest sides of a triangle and the length of its longest side. If the triangle is not a right triangle, then the relationship is an inequality. Just like in the Pythagorean theorem, we call the short sides a and b and the long side c.

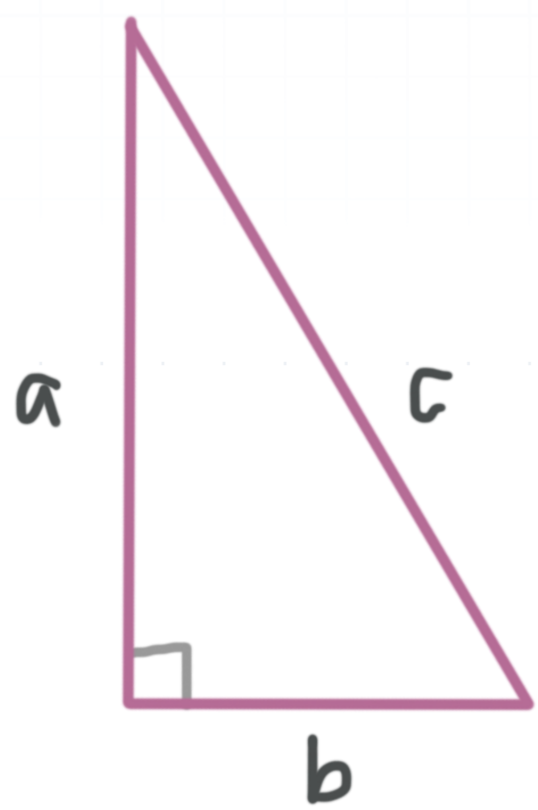
Acute triangle

$$a^2 + b^2 > c^2$$



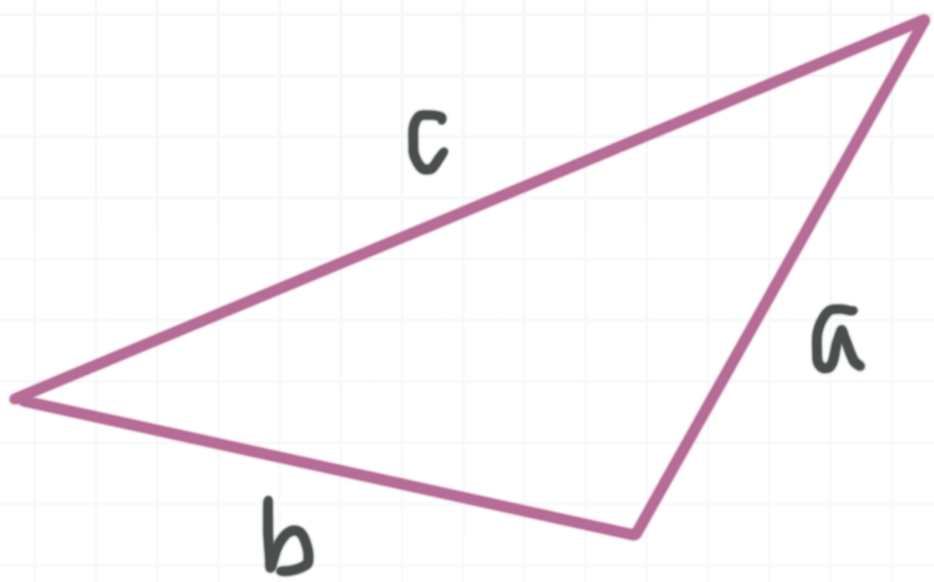
Right triangle

$$a^2 + b^2 = c^2$$



Obtuse triangle

$$a^2 + b^2 < c^2$$



Angles and sides in triangles

One thing to remember about triangles is that the smallest angle is always opposite the shortest side and the biggest angle is always opposite the longest side. Sometimes this can help you when you think through Pythagorean inequality problems.

Let's start by working through an example.

Example

Classify the triangle with sides of length 10, 5, and 9 as acute, obtuse, or right.

Use Pythagorean inequalities to classify the triangle. The two shortest sides are a and b and the longest side is c, so we can assign the letters a, b, and c as follows:

$a = 5$

$b = 9$

$c = 10$

Let's see how $a^2 + b^2$ compares with c^2.

$a^2 + b^2 \ \ ? \ \ c^2$

$5^2 + 9^2 \ \ ? \ \ 10^2$

$25 + 81 \ \ ? \ \ 100$

$106 \ \ ? \ \ 100$

$106 > 100$

Because $a^2 + b^2 > c^2$, this is an acute triangle.

Let's do one more like the first one.

Example

Classify the triangle with sides of length 9, 7, and 12 as acute, obtuse, or right.

Use Pythagorean inequalities to classify the triangle. The two shortest sides are a and b and the longest side is c, so we can assign the letters a, b, and c as follows:

$$a = 7$$

$$b = 9$$

$$c = 12$$

Let's see how $a^2 + b^2$ compares with c^2.

$$a^2 + b^2 \quad ? \quad c^2$$

$$7^2 + 9^2 \quad ? \quad 12^2$$

$$49 + 81 \quad ? \quad 144$$

$$130 \quad ? \quad 144$$

$$130 < 144$$

Because $a^2 + b^2 < c^2$, this is an obtuse triangle.

Let's try one with a bit more reasoning involved.

Example

The lengths of two sides of a certain triangle are 13 and 12. If the remaining side is the longest side, what is the smallest integer value its length can take that would make the triangle obtuse?

For a triangle to be obtuse, the length of its sides need to satisfy the inequality $a^2 + b^2 < c^2$. We want to find the smallest perfect square that's bigger than $a^2 + b^2$.

We can set $a = 13$ and $b = 12$, so we have

$$a^2 + b^2 = 13^2 + 12^2 = 169 + 144 = 313$$

We want 313 to be less than c^2, and we want c to be the smallest integer that satisfies $313 < c^2$, so let's take the square root of 313 and round up to the next integer.

$$\sqrt{313} \approx 17.692$$

This number rounds up to 18, so $c^2 = 18^2 = 324$, and the smallest integer value the length of the third side can take that makes the triangle obtuse is 18.