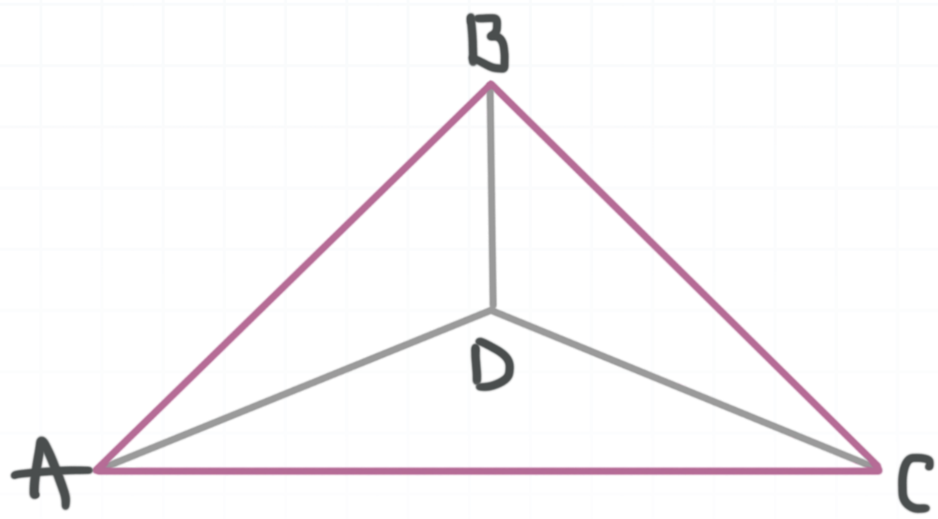
Topic: Circumscribed and inscribed circles of a triangle

Question: $\overline{AD}$, $\overline{BD}$, and $\overline{CD}$ are bisectors of $\angle CAB$, $\angle ABC$, and $\angle BCA$, respectively. Their point of intersection, D, is the incenter of $\triangle ABC$. Assuming the figure is drawn to scale, which of the following statements is true?



Answer choices:

A Point D is equidistant from A, B, and C but not equidistant from the sides of $\triangle ABC$.

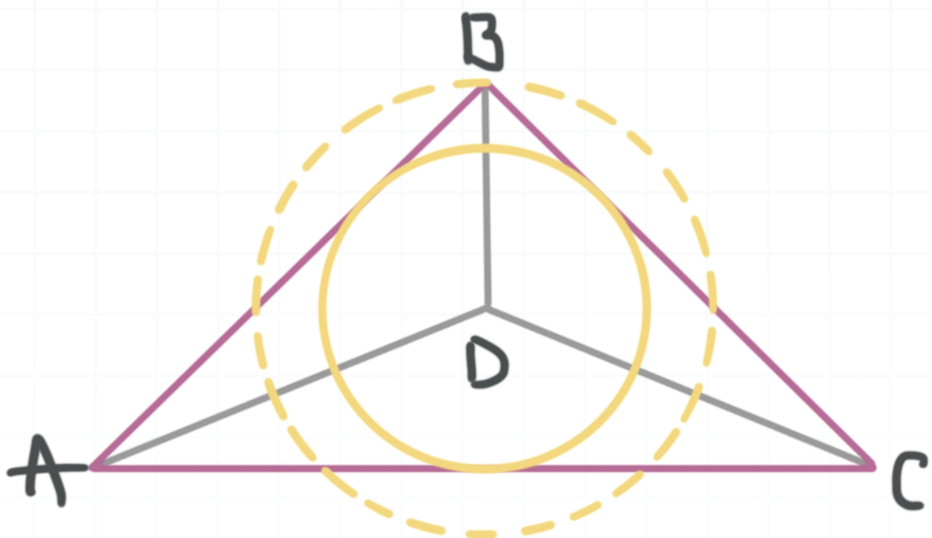
B Point D is equidistant from the sides of $\triangle ABC$ but not equidistant from A, B, and C.

C Point D is equidistant from A, B, and C and equidistant from the sides of $\triangle ABC$.

D Point D is neither equidistant from A, B, and C nor equidistant from the sides of $\triangle ABC$.

Solution: B

The incenter of any triangle, which is the point of intersection of the angle bisectors of the interior angles of the triangle, is equidistant from the sides of the triangle. So point D, which is the incenter of $\triangle ABC$, is equidistant from the sides of $\triangle ABC$. This means you can rule out answer choice D.

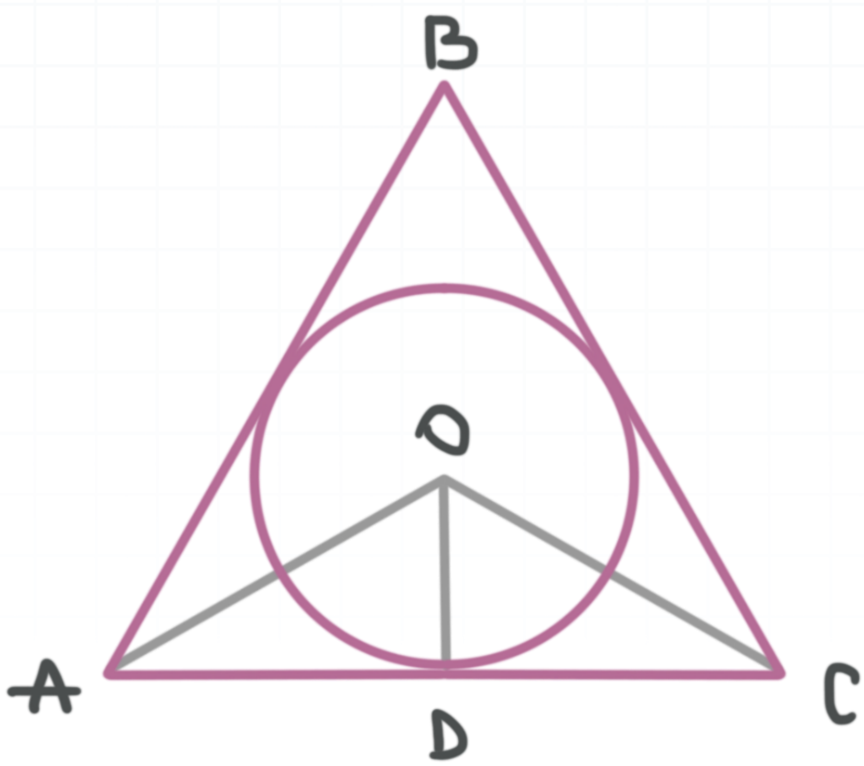


Notice that if you were to draw a circle of larger radius (with center at D), you couldn't make it hit all three vertices. That tells you that D isn't equidistant from A, B, and C, so you can also rule out answer choices A and C.

The only statement that's true is the one in answer choice B.

Topic: Circumscribed and inscribed circles of a triangle

Question: In equilateral triangle ABC, the sides are of length 8, $\overline{OA}$ and $\overline{OC}$ are bisectors of $\angle DAB$ and $\angle BCD$, respectively. The circle (with center at point O) is inscribed in $\triangle ABC$. If $\overline{DC} = \overline{OD} \cdot \sqrt{3}$, what is the radius of the circle (the length of $\overline{OD}$)?



Answer choices:

A $\quad 4$

B $\quad 3$

C $\quad 4\sqrt{3}$

D $\quad \dfrac{4\sqrt{3}}{3}$

Solution: D

Let r be the radius of the inscribed circle of $\triangle ABC$ (r is the length of $\overline{OD}$). We've been given that $\overline{DC} = \overline{OD} \cdot \sqrt{3}$, which means that

$$4 = r\sqrt{3}$$

$$r = \frac{4}{\sqrt{3}}$$

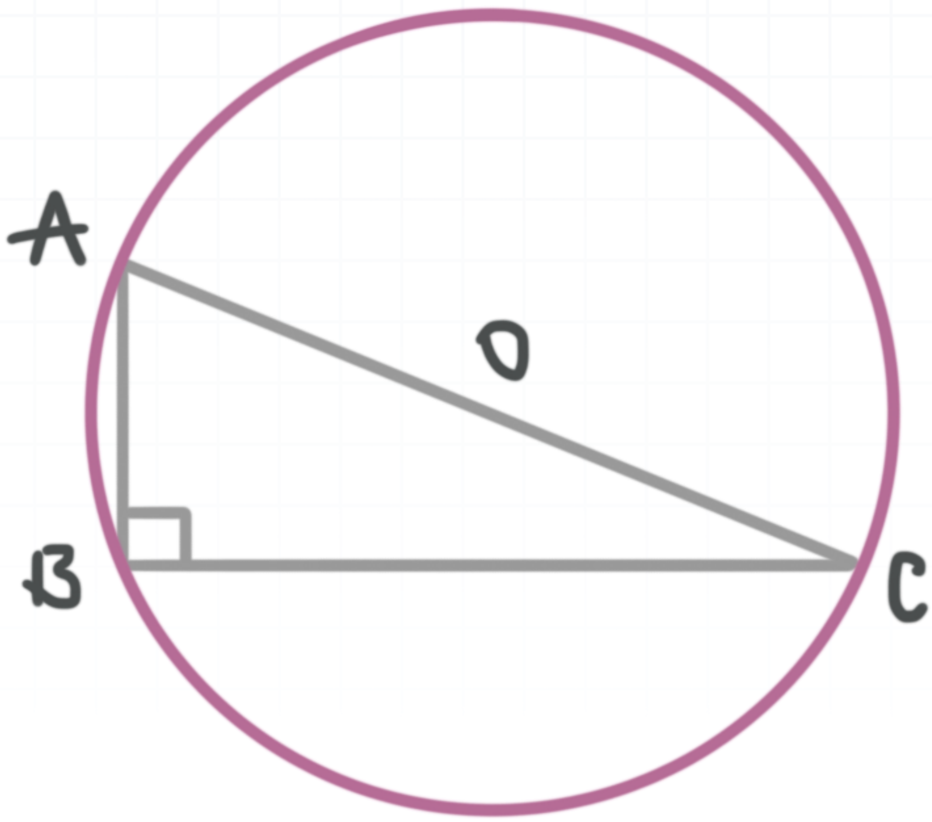
We'll rationalize the denominator by multiplying both the numerator and denominator by $\sqrt{3}$.

$$r = \frac{4}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$r = \frac{4\sqrt{3}}{3}$$

Topic: Circumscribed and inscribed circles of a triangle

Question: In the right triangle ABC, $\overline{AB} = 5$ and $\overline{BC} = 12$. The circle (with center at O) is circumscribed around $\triangle ABC$. What is the radius of the circle?



Answer choices:

A 13

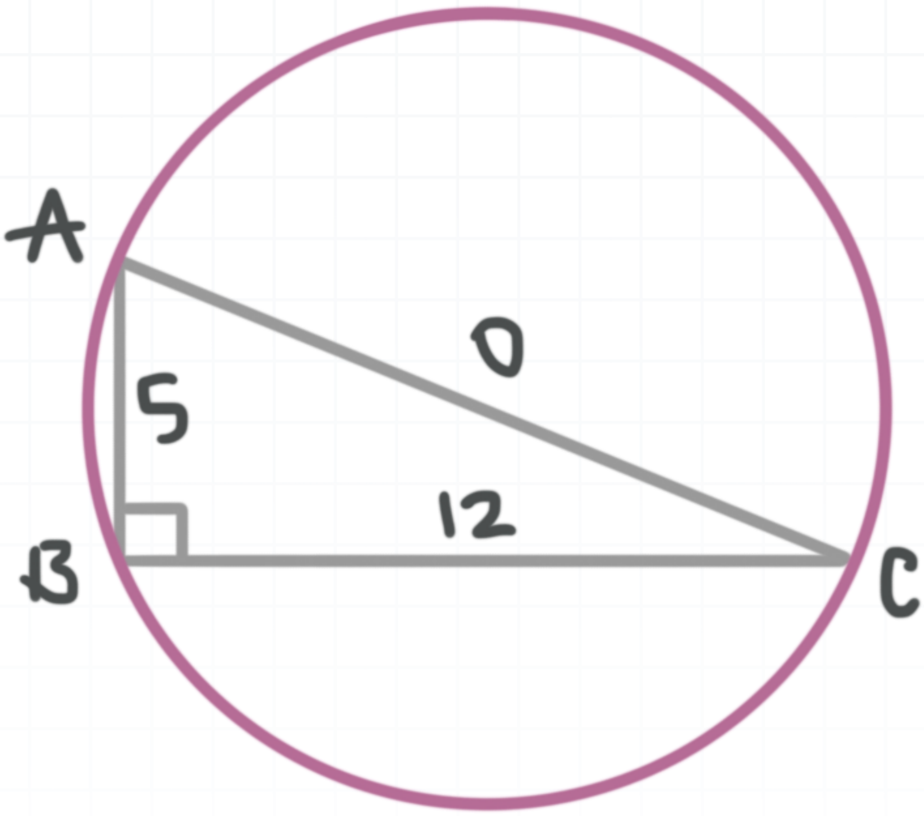
B 12.5

C 6.5

D 6

Solution: C

Fill in the figure with what we know.



Remember, the measure of an inscribed angle is half that of its intercepted arc. So because $\angle CBA$ is a 90° inscribed angle, its intercepted arc, $\overset{\frown}{CA}$, has measure 180°.

This guarantees that $\overline{AC}$ is a diameter of the circle.

Use the Pythagorean theorem to find $\overline{AC}$.

$$(\overline{AB})^2 + (\overline{BC})^2 = (\overline{AC})^2$$

$$5^2 + 12^2 = (\overline{AC})^2$$

$$169 = (\overline{AC})^2$$

$$\overline{AC} = 13$$

The radius is half the diameter (half the length of $\overline{AC}$).

$$\frac{1}{2}(\overline{AC}) = \frac{1}{2}(13) = 6.5$$