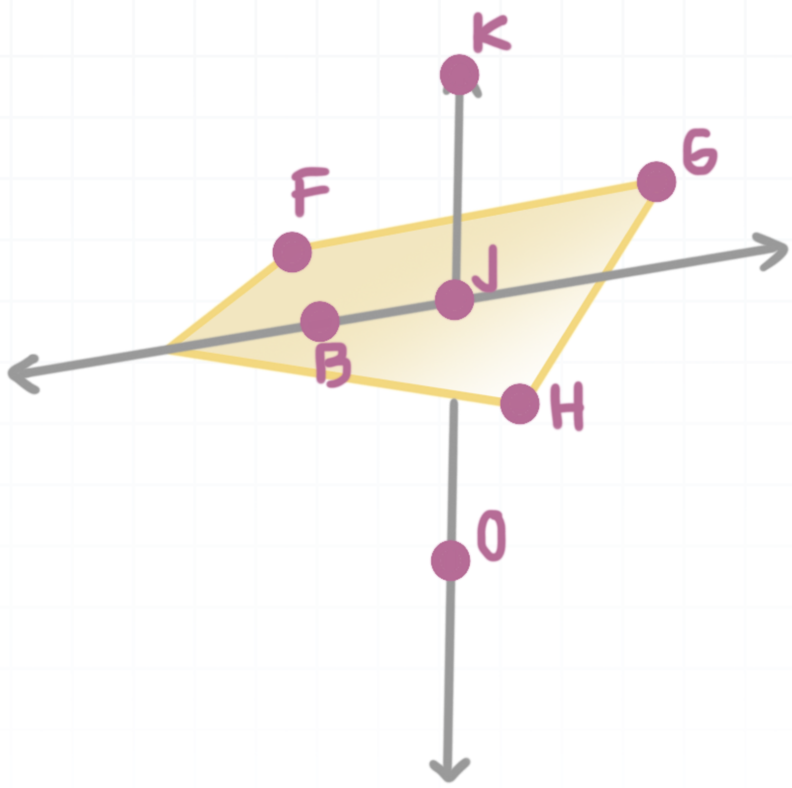
Geometry Workbook Solutions

krista king MATH

NAMING SIMPLE GEOMETRIC FIGURES

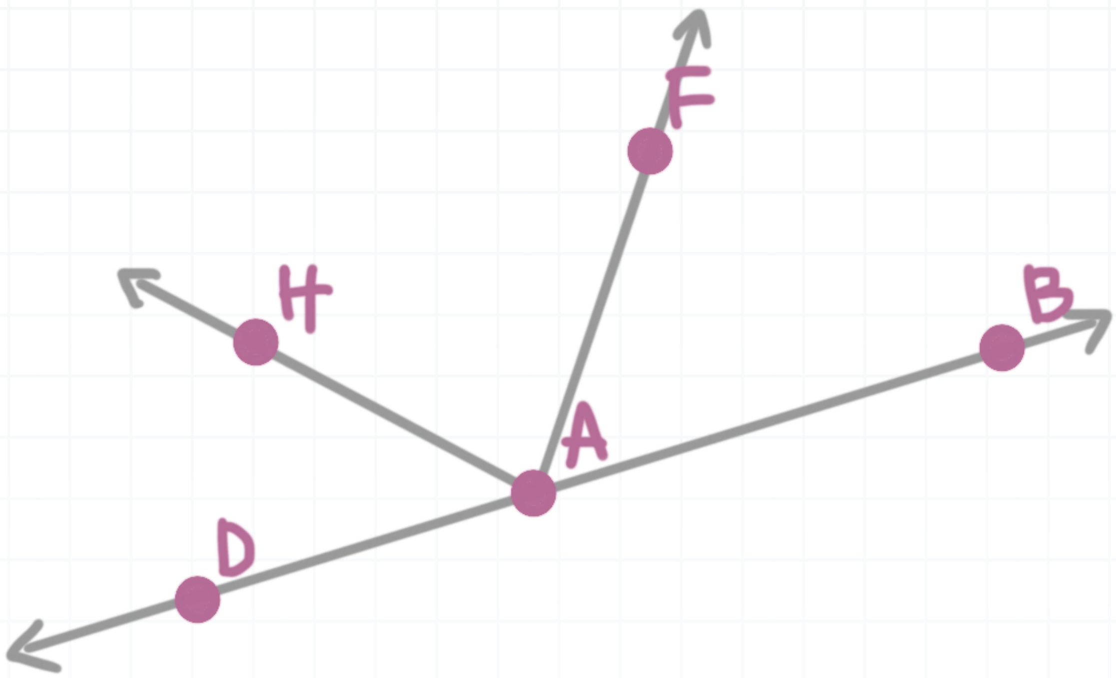
■ 1. Name the intersection of $\overline{BJ}$ and $\overline{KO}$.



Solution:

The intersection is the point J. The intersection of a two lines or line segments is always a point. A line is made up an infinite number of points. To find the intersection of two lines, we need to find the point that lies in both.

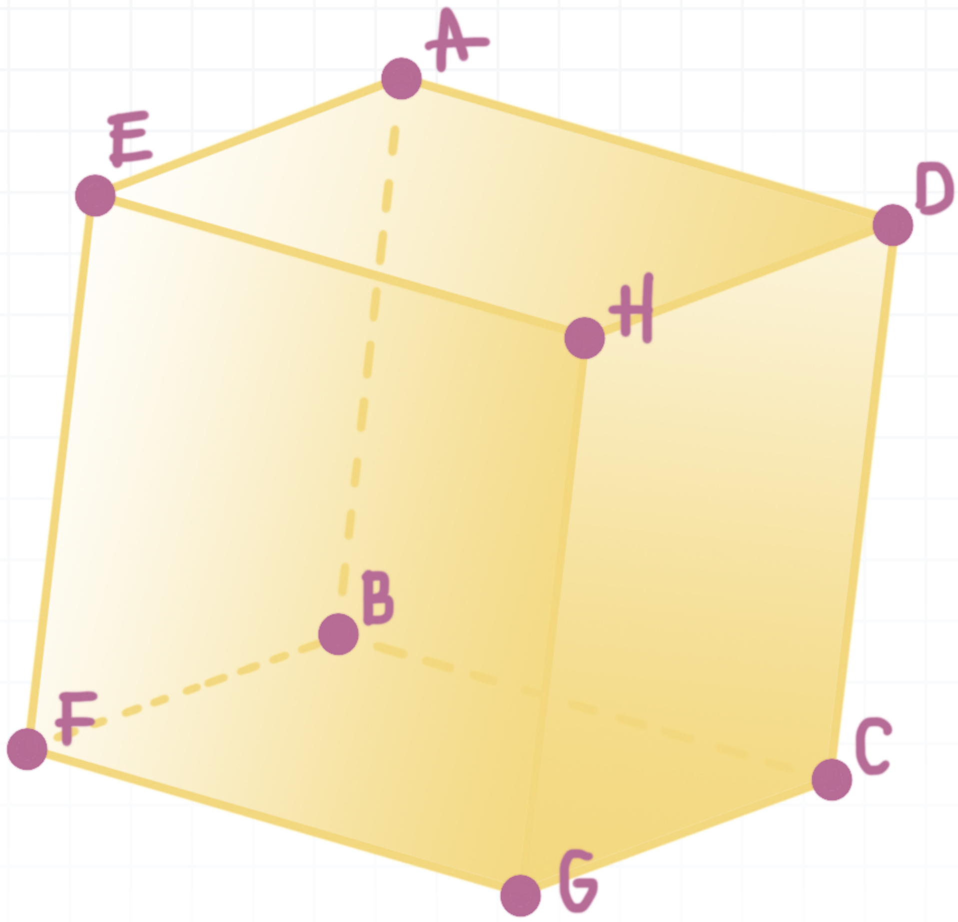
■ 2. Name the angle that forms a linear pair with $\angle DAF$.



Solution:

$\angle FAB$ (can also be named $\angle BAF$). Angles form a linear pair when they're adjacent, which means they share a common side and are supplementary, which means the degree measures of the pair have a sum of 180°.

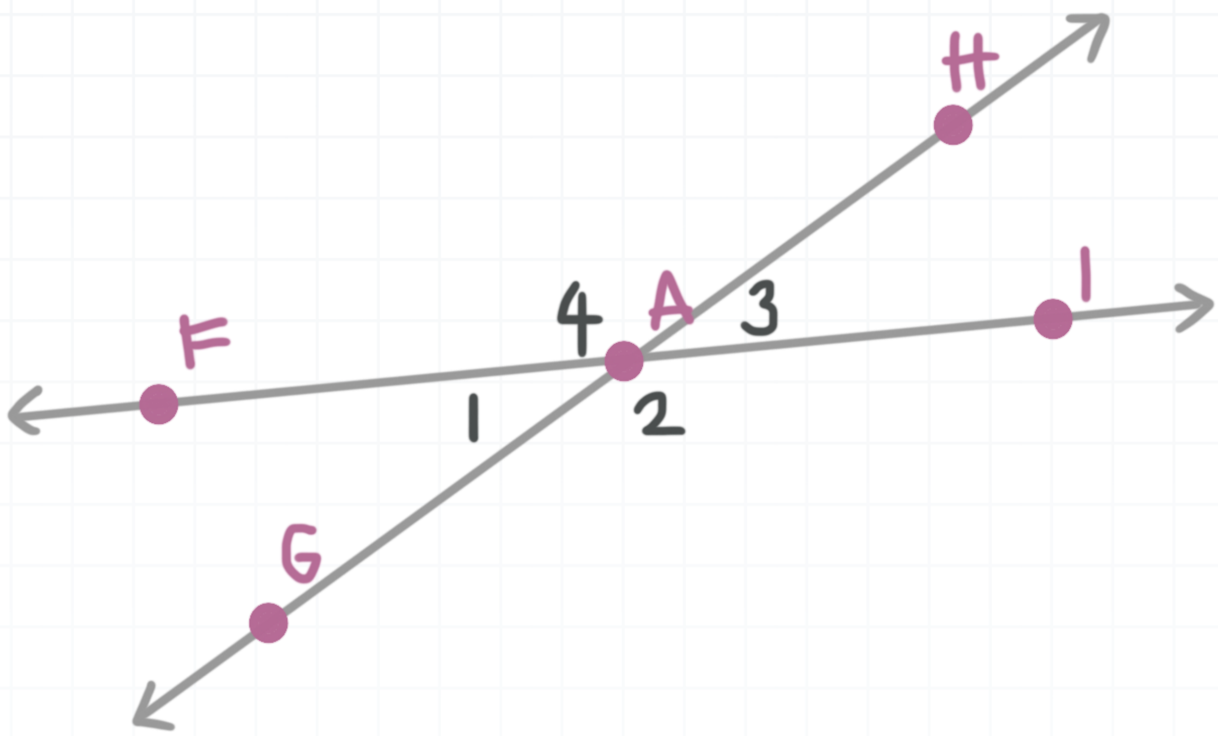
■ 3. Name three non-collinear points.



Solution:

There are many correct answers, one of which could be points A, B, and G. Collinear points all lie on the same line. Non-collinear points are points that do not all lie on the same line. So choose any three points that are not on the same line.

■ 4. Name a pair of vertical angles.



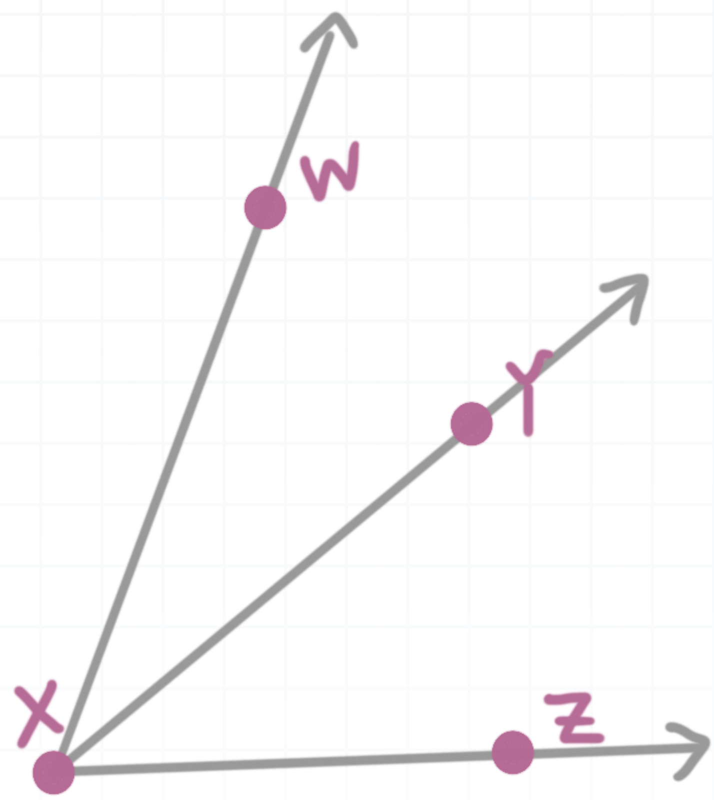
Solution:

$\angle 1$ and $\angle 3$, or $\angle 2$ and $\angle 4$. Vertical angles are formed when two lines intersect. They are the angles across from one another and are always congruent.

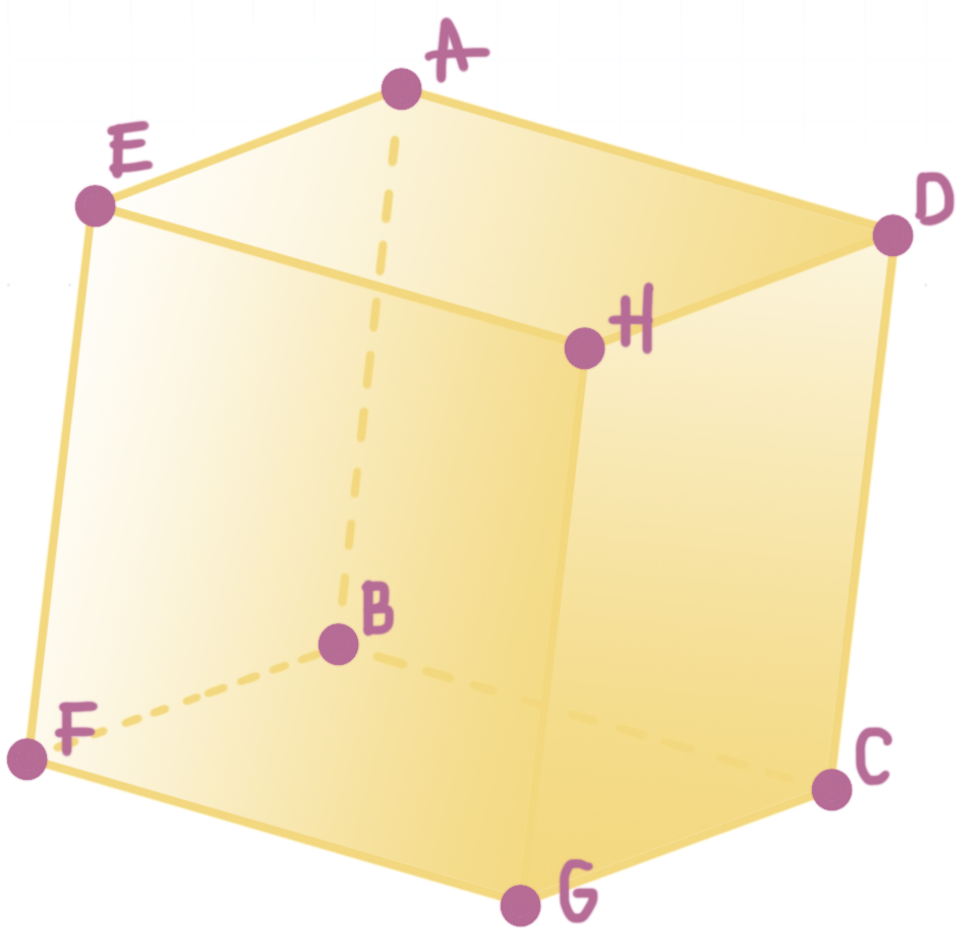
■ 5. $\overline{XY}$ is an angle bisector of $\angle WXZ$. Write the congruence statement that follows.

Solution:

$\angle WXY \cong \angle ZXY$



■ 6. Name the intersection of plane AEH and plane GCD.

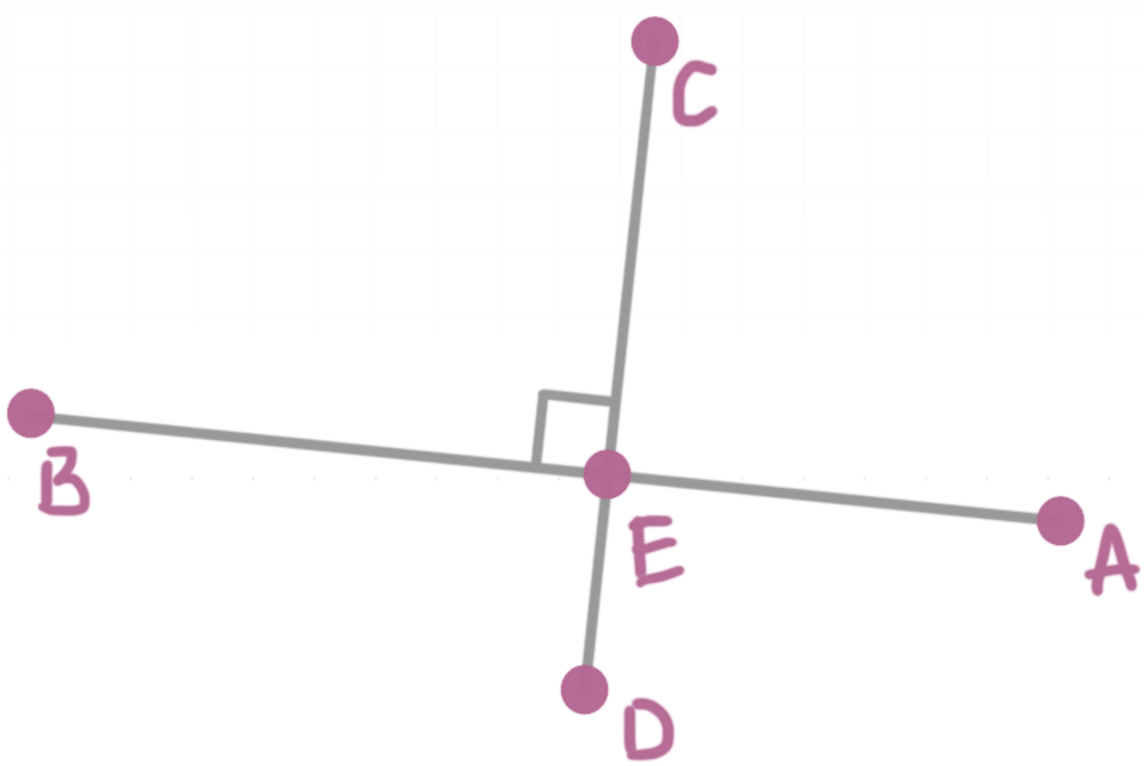


Solution:

$\overline{HD}$. The intersection of these two planes is a line. They share the segment $\overline{HD}$.

■ 7. $\overline{AB} \perp \overline{CD}$ and they intersect at E. Draw a sketch of this and include all necessary labels on your diagram.

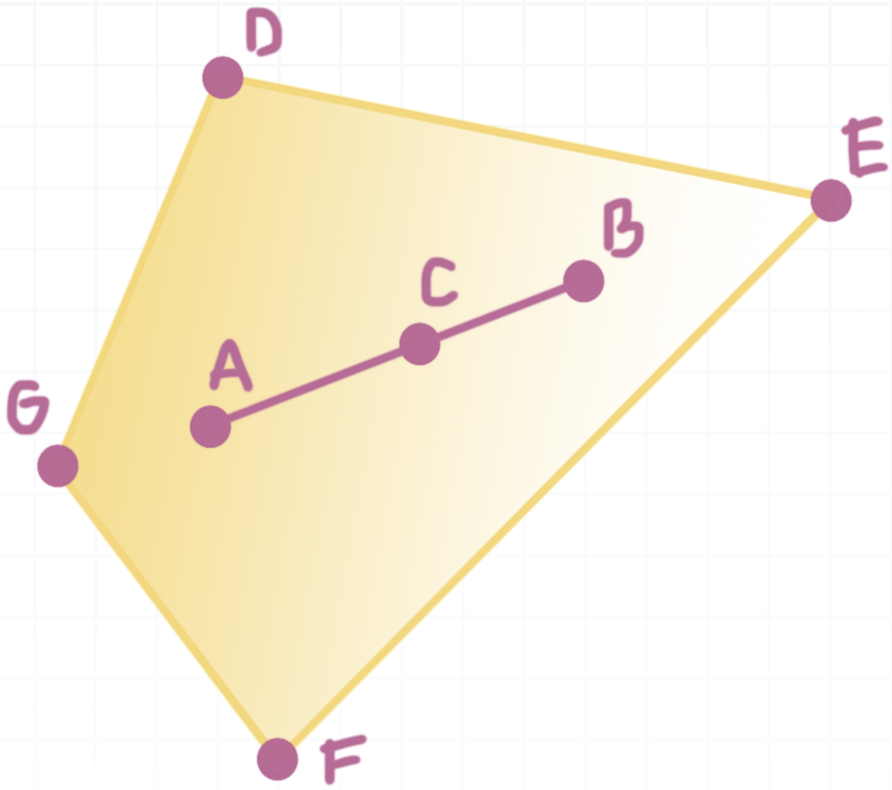
Solution:



■ 8. Sketch the following: $\overline{AB}$ lies on plane DEF and C is contained in $\overline{AB}$.

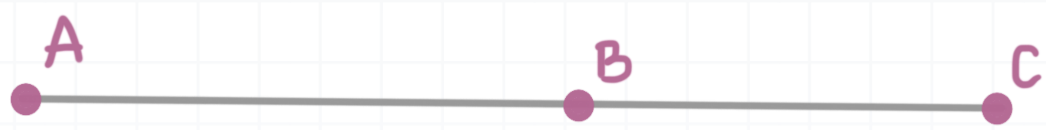
Solution:

There are many possible solutions, but one example might be



LENGTH OF A LINE SEGMENT

■ 1. In the line segment, $AB=14$ and $BC=10$. Find AC.



Solution:

$AC=24$ because

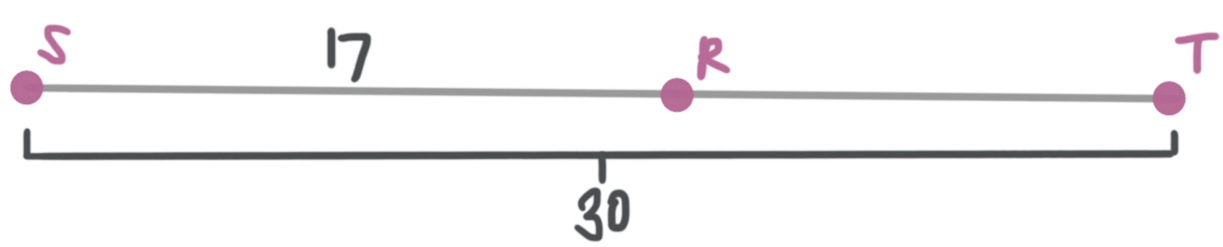
$$AB+BC=AC$$

$$14+10=24$$

■ 2. R lies between S and T. $ST=30$ and $SR=17$. Find RT.

Solution:

$RT=13$. Make a diagram of $\overline{ST}$ and place point R on the line segment. Label the lengths of ST and SR on your diagram.



Then we can say

$$SR + RT = ST$$

$$17 + RT = 30$$

$$RT = 13$$

■ 3. $JM = 2MP$ and $JP = 30$. Find JM and MP.

Solution:

$JM = 20$ and $MP = 10$. We know that $JM + MP = JP$ and we can substitute $2MP$ in for JM to get $2MP + MP = JP$. We can further substitute 30 to get $3MP = 30$. Therefore $MP = 10$ and $JM = 20$.

■ 4. B lies between L and N. $LB = x$, $BN = 2x + 5$, and $LN = 17$. Write an equation that can be used to find the value of x. Then find x.

Solution:

The equation $x + 2x + 5 = 17$ can be used to find the value of x, and $x = 4$. Make a diagram of $\overline{LN}$ and place point B between L and N. Label the given expressions on your diagram and note that $LB + BN = LN$. Substitute your expressions to get $x + 2x + 5 = 17$.

$$x + 2x + 5 = 17$$

$$3x = 12$$

$$x = 4$$

■ 5. $\overline{AB}$ bisects $\overline{DC}$ at E. $DC = 8$ cm, $AB = 10$ cm, and $AE = 4$ cm. Find DE and EB.

Solution:

$DE = 4$ cm and $EB = 6$ cm. Make a diagram of AB bisecting DC at E. Since $\overline{AB}$ is a bisector of $\overline{DC}$, we know that $DE = EC$. Since the whole length of DC is 8 cm, we know that $DE = 4$ because it's half of 8. Then we know that

$$AE + EB = AB$$

$$4 + EB = 10$$

$$EB = 6$$

■ 6. P lies between M and O. $MP = 3x - 4$, $PO = 2x + 2$, and $MO = 3x + 12$. Find x and MO.

Solution:

$x = 7$ and $MO = 33$. Make a diagram of $\overline{MO}$ and place point P between M and O. Label the given expressions on your diagram and note that

$$MP + PO = MO$$

$$3x - 4 + 2x + 2 = 3x + 12$$

$$5x - 2 = 3x + 12$$

$$2x = 14$$

$$x = 7$$

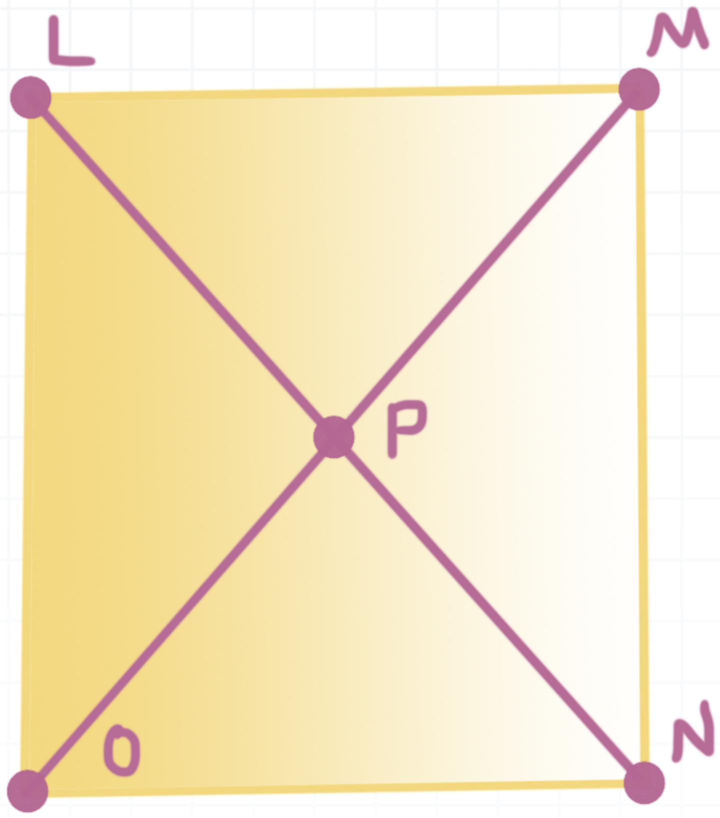
Then plug 7 into the following equation.

$$MO = 3x + 12$$

$$MO = 3(7) + 12$$

$$MO = 33$$

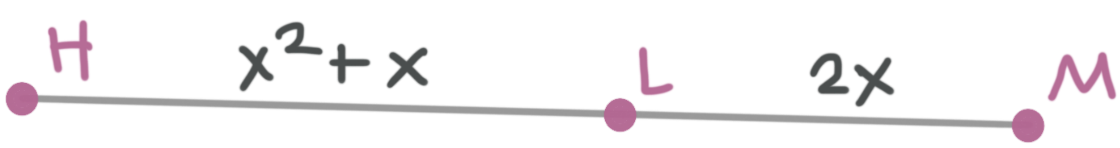
■ 7. The diagonals of a square bisect each other and are also congruent. The diagram below show diagonals $\overline{LN}$ and $\overline{MO}$ intersecting at P. Because they are bisectors, P is the midpoint of each segment. If $LP = 4.5$ inches, find MO.



Solution:

$MO = 9$ inches. Since $LP = 4.5$ inches, PN also equals 4.5 inches. This makes $LN = 9$ inches. And because the diagonals are congruent, $LN = MO$. This make $MO = 9$ inches as well.

■ 8. $HM = 10$. Use the diagram to find x and HL.



Solution:

$x = 2$ and $HL = 6$. We know that

$$HL + LM = HM$$

$$x^2 + x + 2x = 10$$

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

$$x = -5, 2$$

Because $x = -5$ results in negative segment lengths, $x = 2$ is the only possible solution. We then substitute $x = 2$ into the expression for HL and get

$$HL = x^2 + x$$

$$HL = (2)^2 + (2)$$

$$HL = 6$$

SLOPE AND MIDPOINT OF A LINE SEGMENT

■ 1. Find the length of $\overline{AB}$ given $A(-2,3)$ and $B(4,3)$.

Solution:

$AB=6$. The distance between points can be found using the distance formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(4-(-2))^2+(3-3)^2}$$

$$d=\sqrt{6^2+0^2}$$

$$d=\sqrt{36}$$

$$d=6$$

■ 2. Find the length of $\overline{EF}$ given $E(-3,-2)$ and $F(1,1)$.

Solution:

$EF=5$. The distance between points can be found using the distance formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(1-(-3))^2+(1-(-2))^2}$$

$$d=\sqrt{4^2+3^2}$$

$$d=\sqrt{25}$$

$$d=5$$

■ 3. Find the length of $\overline{JK}$ given $J(0,6)$ and $K(2,-4)$.

Solution:

$JK=2\sqrt{26}$. The distance between points can be found using the distance formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

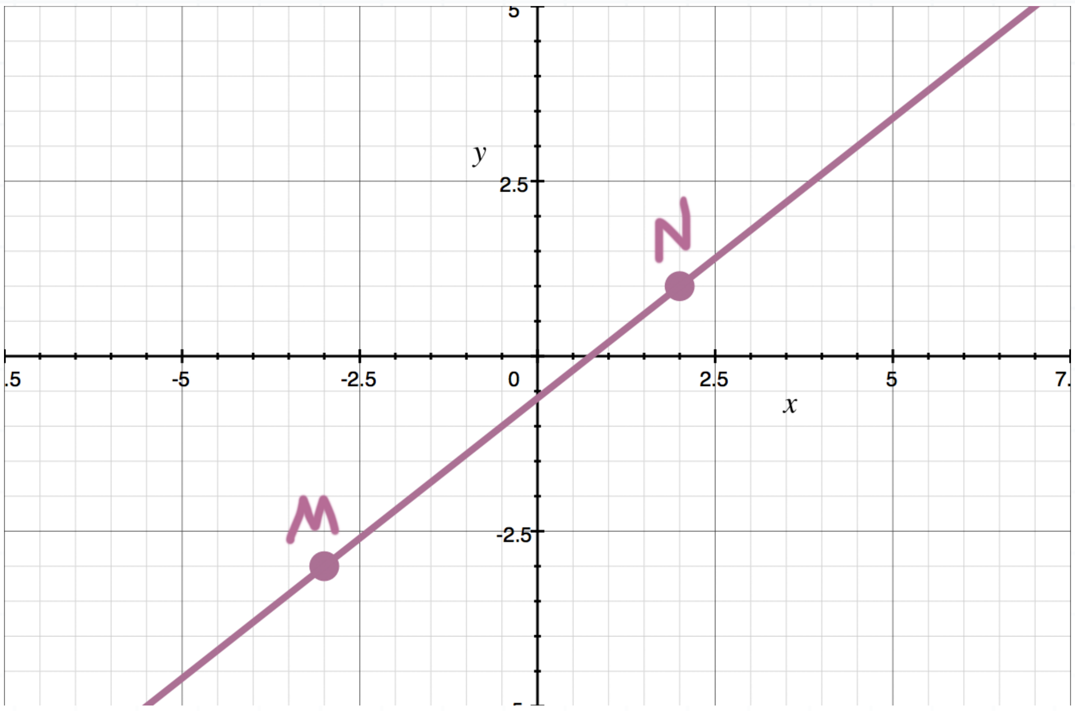
$$d=\sqrt{(2-0)^2+(-4-6)^2}$$

$$d=\sqrt{2^2+(-10)^2}$$

$$d=\sqrt{104}$$

$$d=2\sqrt{26}$$

■ 4. Find the slope of line MN.



Solution:

$m = 4/5$. Plug both points from the graph into the formula for the slope of a line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-3)}{2 - (-3)} = \frac{4}{5}$$

■ 5. Find the slope of the line passing through $S(-6,6)$ and $T(2, -4)$.

Solution:

$m = -5/4$. Plug both points into the formula for the slope of a line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 6}{2 - (-6)} = -\frac{10}{8} = -\frac{5}{4}$$

■ 6. J is the midpoint of $\overline{RF}$. Find the coordinates of J if $R(-4,6)$ and $F(0,-2)$.

Solution:

$J(-2,2)$. Use the midpoint formula and plug in the given points.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{-4 + 0}{2}, \frac{6 + (-2)}{2}\right)$$

$$(-2,2)$$

■ 7. P is the midpoint of $\overline{XY}$. Find the coordinates of X if $P(-3,6)$ and $Y(0,2)$.

Solution:

$X(-6,10)$. Using the midpoint formula, we can plug in what we know.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{x_1 + 0}{2} = -3, \frac{y_1 + 2}{2} = 6\right)$$

$$(x_1 = -6, y_1 + 2 = 12)$$

$$(x_1 = -6, y_1 = 10)$$

Therefore, the coordinates of X are $X(-6,10)$.

■ 8. E is a midpoint of $\overline{LM}$. $LE = 2x + 3$ and $LM = 6x - 4$. Find x and LM.

Solution:

$x = 5$ and $LM = 26$. Draw a diagram with E as the midpoint of $\overline{LM}$. Label your diagram with the given expressions. Because E is the midpoint,

$$LE = EM = 2x + 3$$

$$2(2x + 3) = 6x - 4$$

$$4x + 6 = 6x - 4$$

$$x = 5$$

Use substitution to find $LM = 6(5) - 4 = 26$.

PARALLEL, PERPENDICULAR, OR NEITHER

■ 1. $\overline{AB} \perp \overline{CD}$. The slope of $\overline{AB}$ is $2/3$. Find the slope of $\overline{CD}$.

Solution:

The slope of $\overline{CD}$ is $-3/2$. The symbol $\perp$ means the two line segments are perpendicular. Perpendicular lines intersect and form a right angle. The slopes of perpendicular lines are always opposite reciprocals of one another. Since the slope of $\overline{AB}$ is $2/3$, the slope of $\overline{CD}$ must be $-3/2$.

■ 2. $\overline{MN} \parallel \overline{ST}$, and the slope of $\overline{MN}$ is -2. Find the slope of $\overline{ST}$.

Solution:

$\overline{MN}$ has slope of -2. Parallel lines will never intersect and will always have the same slope.

■ 3. Are $\overline{XY}$ and $\overline{AB}$ parallel, perpendicular, or neither? $X(4,-3)$, $Y(-2,1)$, $A(1,3)$, and $B(3,6)$. Use the slopes of the lines to justify your answer.

Solution:

Perpendicular. Calculate both slopes.

$$m_{\overline{XY}} = \frac{1-(-3)}{-2-4} = -\frac{4}{6} = -\frac{2}{3}$$

$$m_{\overline{AB}} = \frac{6-3}{3-1} = \frac{3}{2}$$

Since these slopes are opposite reciprocals, the two lines are perpendicular.

■ 4. Are $\overline{EF}$ and $\overline{GH}$ parallel, perpendicular, or neither? $E(-1,4)$, $F(0,2)$, $G(-1,0)$, and $H(1,4)$. Use the slope of the lines to justify your answer.

Solution:

Neither. Calculate both slopes.

$$m_{\overline{EF}} = \frac{2-4}{0-(-1)} = \frac{-2}{1} = -2$$

$$m_{\overline{GH}} = \frac{4-0}{1-(-1)} = \frac{4}{2} = 2$$

-2 and 2 are not equal, nor are they opposite reciprocals. Therefore these lines are neither parallel nor perpendicular.

■ 5. Write the equation of a line in slope-intercept form that's perpendicular to the given line and passes through $(2,3)$.

$$y = \frac{1}{2}x + 2$$

Solution:

$y = -2x + 7$. The slope of the given line is $m = 1/2$. The slope of any line perpendicular to this one is $m = -2$. Use point slope with $m = -2$ and given point $(2,3)$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - 2)$$

$$y - 3 = -2x + 4$$

$$y = -2x + 7$$

■ 6. Write the equation of a line parallel to $y = 3x - 2$ that passes through $(0,3)$.

Solution:

$y = 3x + 3$. The slope of the given line is $m = 3$. The slope of any line parallel to this one is $m = 3$. We are also given the y-intercept of $(0,3)$. Expressed in slope-intercept form, we get $y = 3x + 3$.

■ 7. A square has opposite sides parallel and consecutive sides perpendicular and all sides are congruent. Quadrilateral $SQRE$ has coordinates $S(0,3)$, $Q(4,0)$, $R(1, -4)$, and $E(-3, -1)$. Determine whether or not $SQRE$ is a square by showing that the opposite sides are parallel and consecutive sides are perpendicular and that all sides are congruent.

Solution:

$SQRE$ is a square because the slopes of $\overline{SQ}$, $\overline{QR}$, $\overline{RE}$, and $\overline{ES}$ are $4/3$, $-3/4$, $4/3$, and $-3/4$ respectively, and the length of each side is 5.

■ 8. A square has opposite sides parallel and consecutive sides perpendicular and all sides are congruent. Quadrilateral $SQRE$ has coordinates $S(0,3)$, $Q(4,0)$, $R(1, -4)$, and $E(-3, -1)$. Determine if the diagonals of the square are perpendicular. Determine if the diagonals are congruent.

Solution:

The diagonals are both perpendicular and congruent. $\overline{SR}$ has slope -7 and $\overline{EQ}$ has slope $1/7$. The diagonals are also congruent with $SR = 5\sqrt{2} = EQ$.

DISTANCE BETWEEN TWO POINTS IN TWO DIMENSIONS

■ 1. Find the length of $\overline{GH}$ given $G(-2,1)$ and $H(4,1)$.

Solution:

$GH = 6$. Use the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-2 - 4)^2 + (1 - 1)^2}$$

$$d = \sqrt{(-6)^2 + 0^2}$$

$$d = \sqrt{36}$$

$$d = 6$$

■ 2. Find the length of $\overline{XY}$ given $X(-4,1)$ and $Y(0,2)$.

Solution:

$XY = \sqrt{17}$. Use the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-4 - 0)^2 + (1 - 2)^2}$$

$$d = \sqrt{16 + 1}$$

$$d = \sqrt{17}$$

■ 3. Find the perimeter of $\triangle EFG$ if $E(1,1)$, $F(1,6)$, and $G(5,4)$.

Solution:

The perimeter of $\triangle EFG = 10 + 2\sqrt{5}$. Using the distance formula, we can find the length of each side.

$$EF = \sqrt{(1 - 1)^2 + (1 - 6)^2} = \sqrt{25} = 5$$

$$FG = \sqrt{(6 - 4)^2 + (1 - 5)^2} = \sqrt{20} = 2\sqrt{5}$$

$$GE = \sqrt{(4 - 1)^2 + (5 - 1)^2} = \sqrt{25} = 5$$

Then the perimeter is

$$EF + FG + GE = 10 + 2\sqrt{5}$$

■ 4. Find the area of square $ABCD$ given $A(-8,0)$, $B(0,6)$, $C(6,-2)$, and $D(-2,-8)$.

Solution:

The area of square $ABCD$ is 100 units^2. Calculate the length of two adjacent sides.

$$AB = \sqrt{(6-0)^2 + (0-(-8))^2} = \sqrt{100} = 10$$

$$BC = \sqrt{((-2)-6)^2 + (6-0)^2} = \sqrt{100} = 10$$

$$CD = \sqrt{((-2)-(-8))^2 + (6-((-2))^2} = \sqrt{100} = 10$$

$$DA = \sqrt{(0-(-8))^2 + ((-8)-(-2))^2} = \sqrt{100} = 10$$

Then the area of the square is

$$A = lw = (10)(10) = 100$$

DISTANCE BETWEEN TWO POINTS IN THREE DIMENSIONS

■ 1. Find the distance between points with coordinates $(3,8,0)$ and $(3,8,6)$.

Solution:

$d = 6$. Use the distance formula for three dimensions.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(3 - 3)^2 + (8 - 8)^2 + (0 - 6)^2}$$

$$d = \sqrt{36}$$

$$d = 6$$

■ 2. Find the distance between points with coordinates $(2,5,-3)$ and $(2,8,1)$.

Solution:

$d = 5$. Use the distance formula for three dimensions.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(2-2)^2 + (5-8)^2 + (-3-1)^2}$$

$$d = \sqrt{25}$$

$$d = 5$$

■ 3. Find the distance between points with coordinates $(1,1,1)$ and $(5,5,5)$.

Solution:

$d = \sqrt{48} = 4\sqrt{3} \approx 6.93$. Use the distance formula for three dimensions.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(1-5)^2 + (1-5)^2 + (1-5)^2}$$

$$d = \sqrt{48}$$

$$d = 4\sqrt{3}$$

$$d \approx 6.93$$

■ 4. Find the distance between points with coordinates $(9,6,3)$ and $(-9,-6,-3)$.

Solution:

$d = \sqrt{504} = 6\sqrt{14} \approx 22.45$. Use the distance formula for three dimensions.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{((-9) - 9)^2 + ((-6) - 6)^2 + ((-3) - 3)^2}$$

$$d = \sqrt{324 + 144 + 36}$$

$$d = \sqrt{504}$$

$$d = 6\sqrt{14}$$

$$d \approx 22.45$$

MIDPOINT OF A LINE SEGMENT IN THREE DIMENSIONS

■ 1. Find the midpoint between points with coordinates $(3,8,0)$ and $(3,8,6)$.

Solution:

$(3,8,3)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{3+3}{2},\frac{8+8}{2},\frac{0+6}{2}\right)$$

$$(a,b,c)=(3,8,3)$$

■ 2. Find the midpoint between points with coordinates $(2,5,-3)$ and $(2,8,1)$.

Solution:

$(2,6.5,-1)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{2+2}{2},\frac{5+8}{2},\frac{-3+1}{2}\right)$$

$$(a,b,c)=(2,6.5,-1)$$

■ 3. Find the midpoint between points with coordinates $(1,1,1)$ and $(5,5,5)$.

Solution:

$(3,3,3)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{1+5}{2},\frac{1+5}{2},\frac{1+5}{2}\right)$$

$$(a,b,c)=(3,3,3)$$

4. Find the midpoint between points with coordinates $(9,6,3)$ and $(-9,-6,-3)$.

Solution:

$(0,0,0)$. The formula for the midpoint of two points in three dimensions uses the mean of three sets of numbers: the x-values, the y-values, and the z-values.

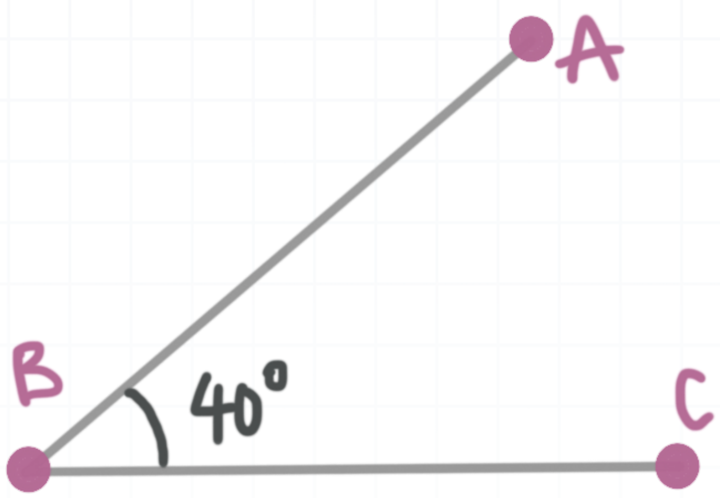
$$(a,b,c)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$$

$$(a,b,c)=\left(\frac{9+(-9)}{2},\frac{6+(-6)}{2},\frac{3+(-3)}{2}\right)$$

$$(a,b,c)=(0,0,0)$$

MEASURES OF ANGLES

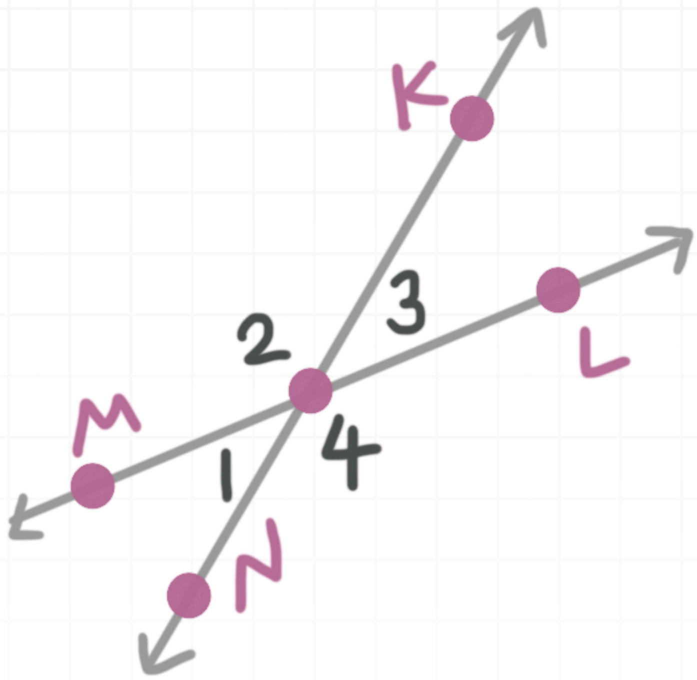
■ 1. Determine whether $\angle ABC$ is obtuse, acute, or right. Then find its supplement.



Solution:

$\angle ABC$ is acute because it has a degree measure less than 90°. Its supplement is 140° degrees.

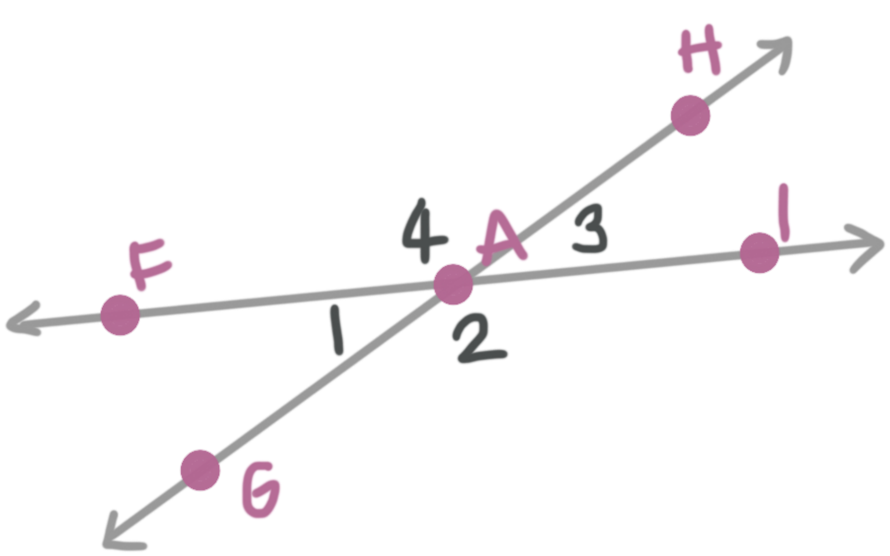
■ 2. $m\angle 1 = 35$. Find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



Solution:

$m\angle 2 = 145$, $m\angle 3 = 35$, and $m\angle 4 = 145$. Because $\angle 1$ and $\angle 2$ are supplementary angles, $m\angle 2 = 180 - 35 = 145$. $\angle 1 \cong \angle 3$ because they are vertical angles, and $\angle 2 \cong \angle 4$ because they are vertical angles.

■ 3. Find x, y, and z if $m\angle 1 = 3x - 2$, $m\angle 2 = 2y$, $m\angle 3 = 2x + 8$, and $m\angle 4 = 4z$.



Solution:

$x = 10$, $y = 76$, and $z = 38$. Since $m\angle 1 = m\angle 3$,

$$3x - 2 = 2x + 8$$

$$x = 10$$

Since $m\angle 1 + m\angle 2 = 180$,

$$(3(10) - 2) + m\angle 2 = 180$$

$$28 + m\angle 2 = 180$$

$$m\angle 2 = 152$$

Because $m\angle 2 = m\angle 4$, $m\angle 4 = 152$. Then

$$2y = 152$$

$$y = 76$$

and

$$4z = 152$$

$$z = 38$$

■ 4. $\angle 5$ and $\angle 6$ are complementary angles. $m\angle 5 = 3x - 6$ and $m\angle 6 = 2x - 14$. Find the measures of $\angle 5$ and $\angle 6$.

Solution:

$m\angle 5 = 60$ and $m\angle 6 = 30$. Because the angles are complementary, we know

$$m\angle 5 + m\angle 6 = 90$$

$$3x - 6 + 2x - 14 = 90$$

$$x = 22$$

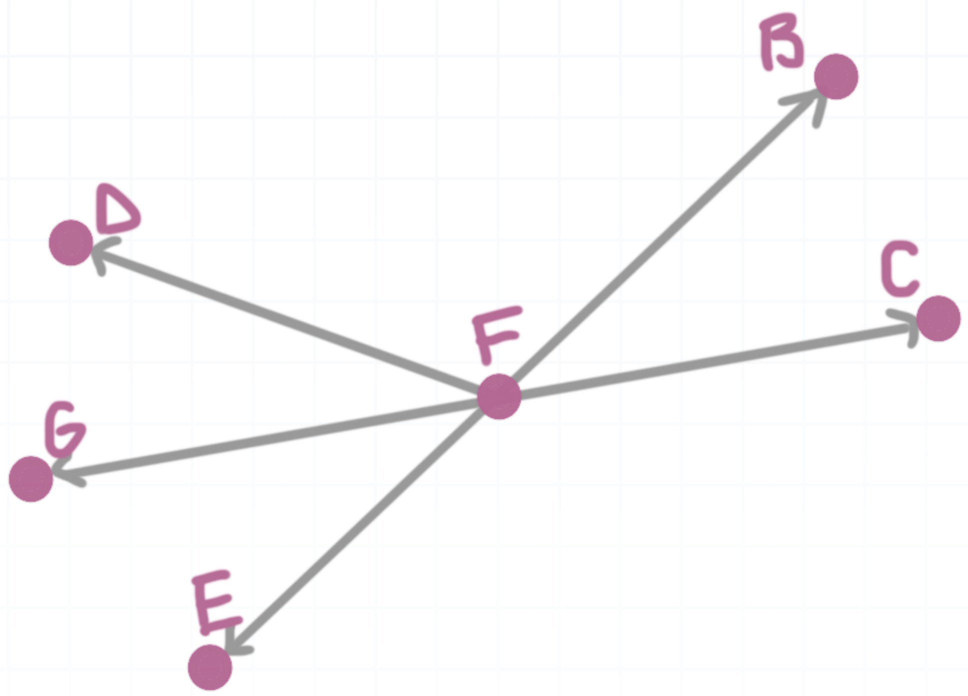
Then we can solve for the measures of each angle.

$$m\angle 5 = 3x - 6 = 3(22) - 6 = 60$$

$$m\angle 6 = 2x - 14 = 2(22) - 14 = 30$$

ADJACENT AND NONADJACENT ANGLES

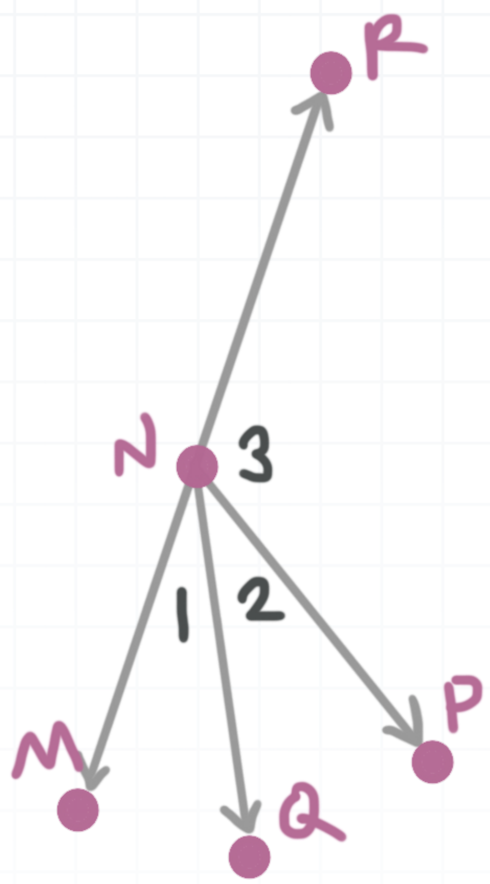
■ 1. Name the angle adjacent to $\angle EFG$.



Solution:

$\angle DFG$ and $\angle CFE$ are both adjacent to $\angle EFG$ because they share a common side.

■ 2. $m\angle 1 = 3x - 10$, $m\angle 2 = 2x - 20$, and $m\angle MNP = 60$. Find the value of x and $m\angle 1$, $m\angle 2$, and $m\angle 3$, given that $\overline{NR}$ and $\overline{NM}$ are opposite rays.



Solution:

$x = 18$, $m\angle 1 = 44$, $m\angle 2 = 16$, and $m\angle 3 = 120$. From the figure, we know that

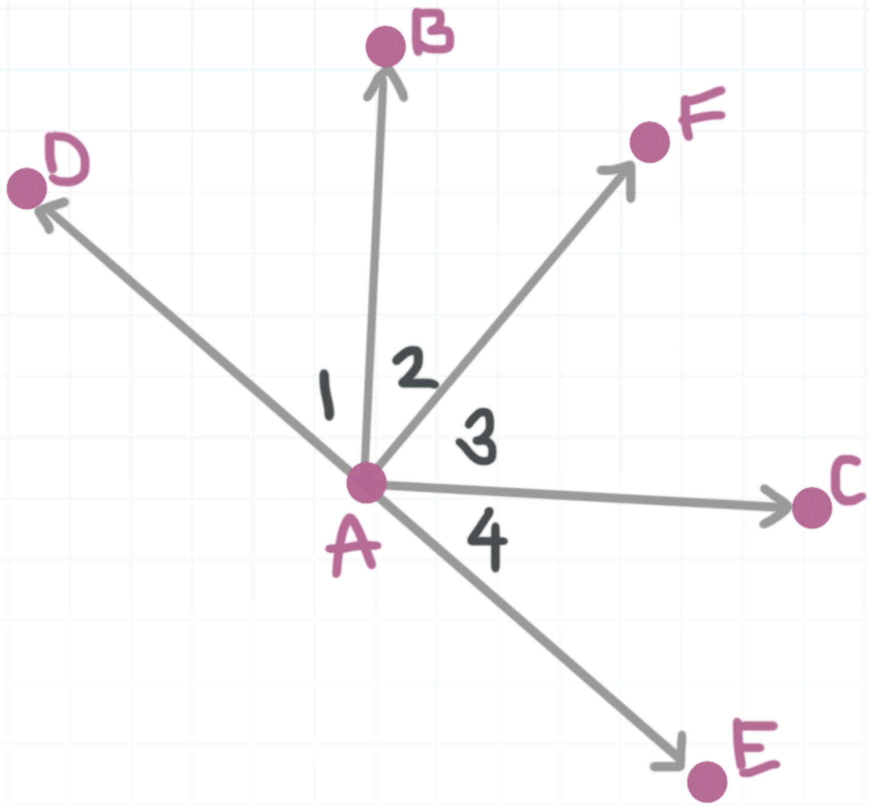
$$m\angle 1 + m\angle 2 = m\angle MNP$$

$$3x - 10 + 2x - 20 = 60$$

$$x = 18$$

Substituting $x = 18$ into each expression gives $m\angle 1 = 44$, $m\angle 2 = 16$, and $m\angle 3 = 120$, because it forms a linear pair with $\angle MNP$.

■ 3. $m\angle 2 = 42$, $\angle 3 \cong \angle 4$, $\angle FAE$ is a right angle, and $\angle DAE$ is a straight angle. Find $m\angle 1$, $m\angle 3$, and $m\angle 4$.



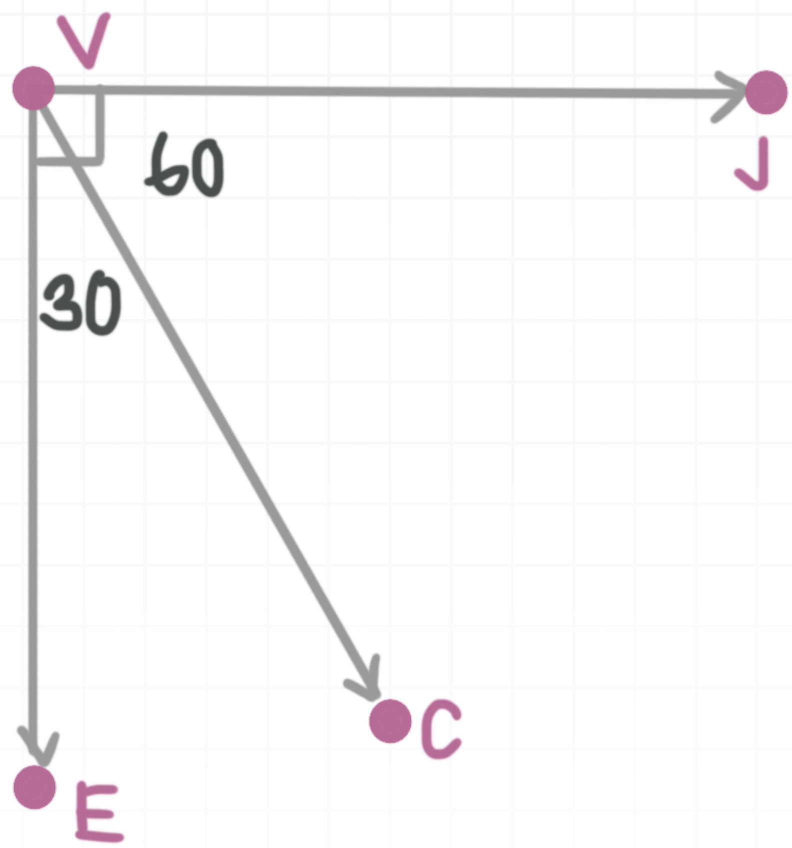
Solution:

$m\angle 1 = 48$, $m\angle 3 = 45$, and $m\angle 4 = 45$. Since $\angle FAE$ is a right angle, $m\angle 3 + m\angle 4 = 90^\circ$. And because $\angle 3$ and $\angle 4$ are congruent, they must both have a measure of 45°. This leaves $m\angle 1 = 48$, so all angles sum to 180°.

■ 4. $\angle JVC$ and $\angle EVC$ are adjacent and complementary. Further, suppose $m\angle JVC = 2m\angle EVC$. Sketch a diagram of this situation and find the measure of each angle.

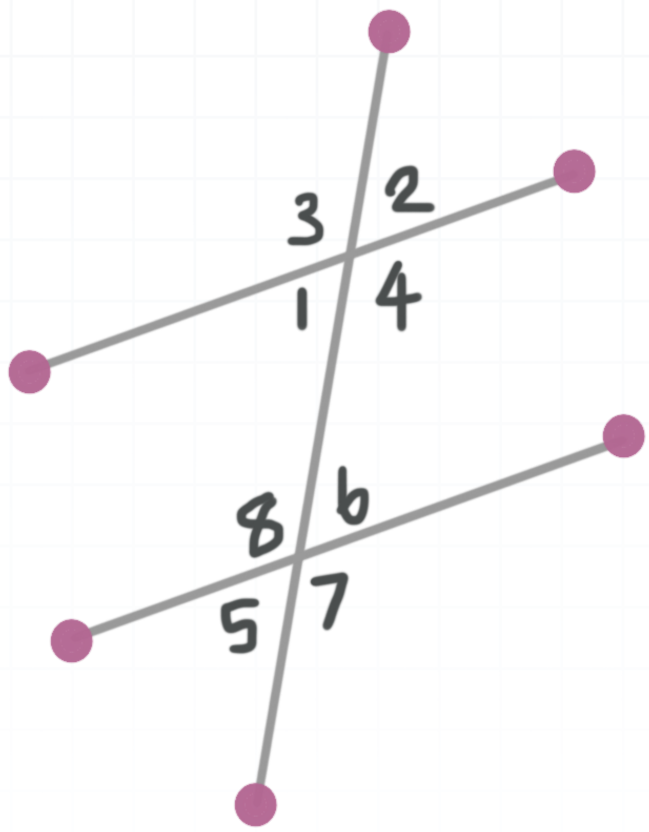
Solution:

$m\angle JVC = 60$ and $m\angle EVC = 30$. A diagram of the figure looks like this:


V
J
60
30
C
E

ANGLES AND TRANSVERSALS

■ 1. Name a pair of corresponding angles.



Solution:

There are four possible correct answers:

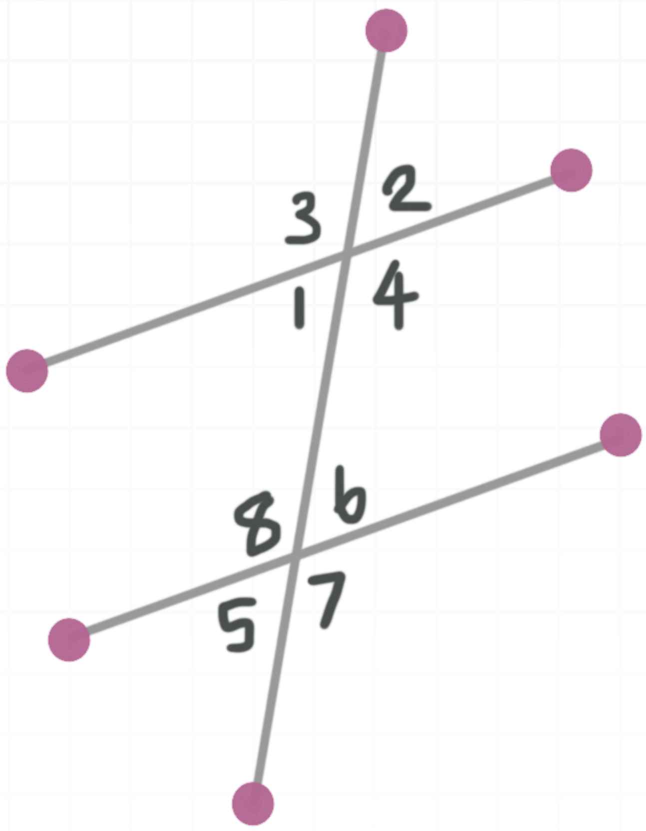
$\angle 7$ and $\angle 4$

$\angle 6$ and $\angle 2$

$\angle 5$ and $\angle 1$

$\angle 8$ and $\angle 3$

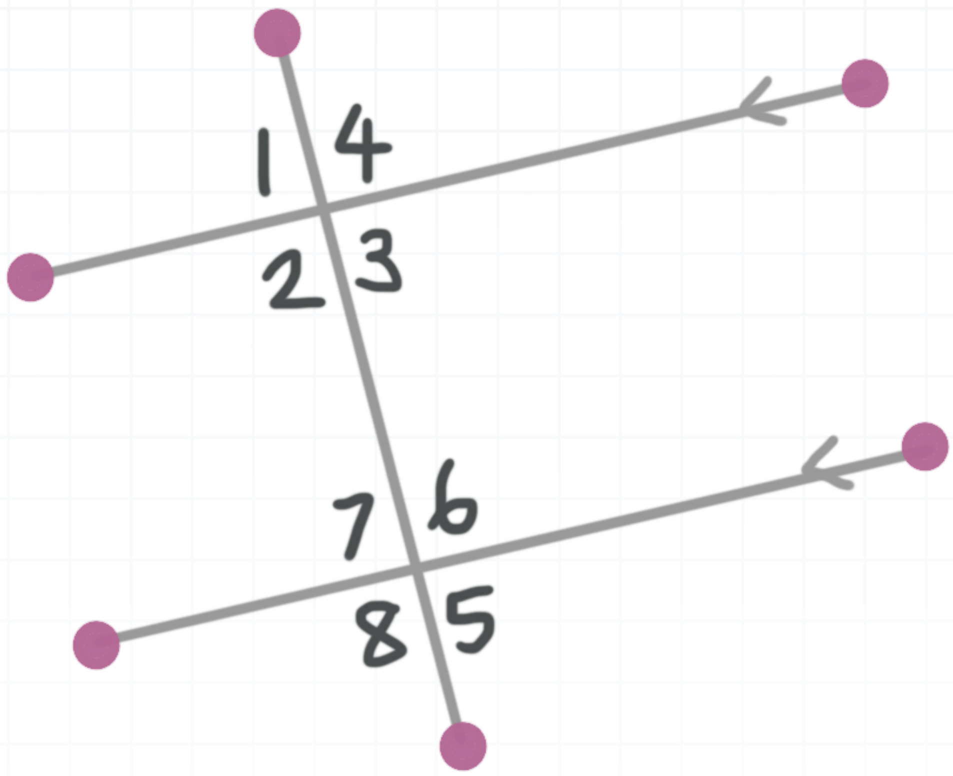
■ 2. Find $m\angle 2$, $m\angle 6$, and $m\angle 5$ if $m\angle 3 = 105$.



Solution:

$m\angle 2 = 75$, $m\angle 6 = 75$, $m\angle 5 = 75$. We know from the figure that $\angle 2$ and $\angle 3$ form a linear pair, making them supplementary. $\angle 2 \cong \angle 6$ because they are corresponding angles, and $\angle 6 \cong \angle 5$ because they are vertical angles.

■ 3. Find x and $m\angle 3$ if $m\angle 2 = 5x + 2$ and $m\angle 7 = 3x + 14$.



Solution:

$x = 20.5$ and $m\angle 3 = 75.5$. From the figure we know that $\angle 2$ and $\angle 7$ are consecutive interior angles. Consecutive interior angles are supplementary.

$$m\angle 2 + m\angle 7 = 180$$

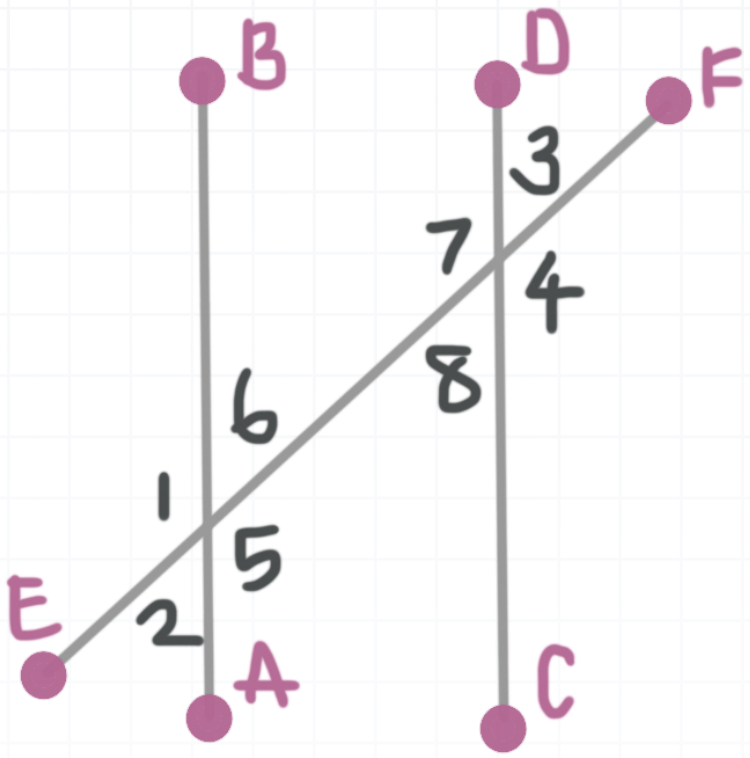
$$5x + 2 + 3x + 14 = 180$$

$$x = 20.5$$

Therefore,

$$m\angle 3 = 75.5$$

■ 4. Find the values of x and y if $\overline{AB}$ and $\overline{DC}$ are parallel lines, and if $m\angle 1 = 2x + y$, $m\angle 2 = 28$, and $m\angle 3 = x + 10$.



Solution:

$x = 18$ and $y = 116$. We know from the figure that $\angle 2 \cong \angle 3$ because they are alternate exterior angles. So we get

$$28 = x + 10$$

$$x = 18$$

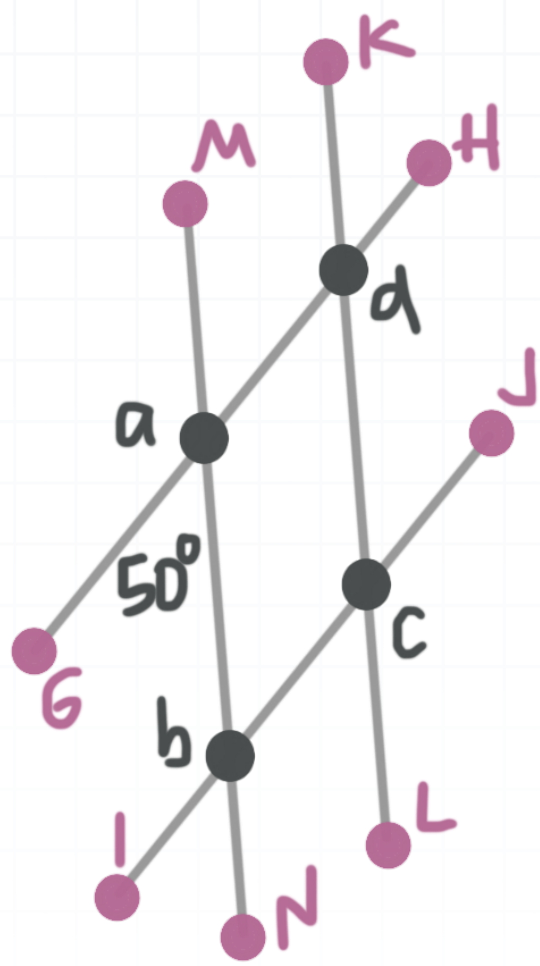
and

$$2x + y = 152$$

$$2(18) + y = 152$$

$$y = 116$$

■ 5. $\overline{MN}$ and $\overline{KL}$ are parallel. $\overline{GH}$ and $\overline{IJ}$ are parallel. Find the values of a, b, c, and d.



Solution:

Given the angle measure of 50°, we know that $m\angle a = 130^\circ$, because angle a is supplementary to the 50° angle. Angles b, c, and d are congruent to angle a, which means that $m\angle a = m\angle b = m\angle c = m\angle d = 130^\circ$.

INTERIOR ANGLES OF POLYGONS

■ 1. Find the sum of the interior angles of a hexagon.

Solution:

720. Using the formula for the sum of interior angles, and the fact that there are 6 sides in a hexagon, we get

$$(n-2)180$$

$$(6-2)180$$

$$(4)180$$

$$720$$

■ 2. Find the measure of each interior angle of a regular 15-gon.

Solution:

156. Using the formula for the sum of interior angles, and the fact that there are 15 sides in a 15-gon, we get

$$(n-2)180$$

$$(15 - 2)180$$

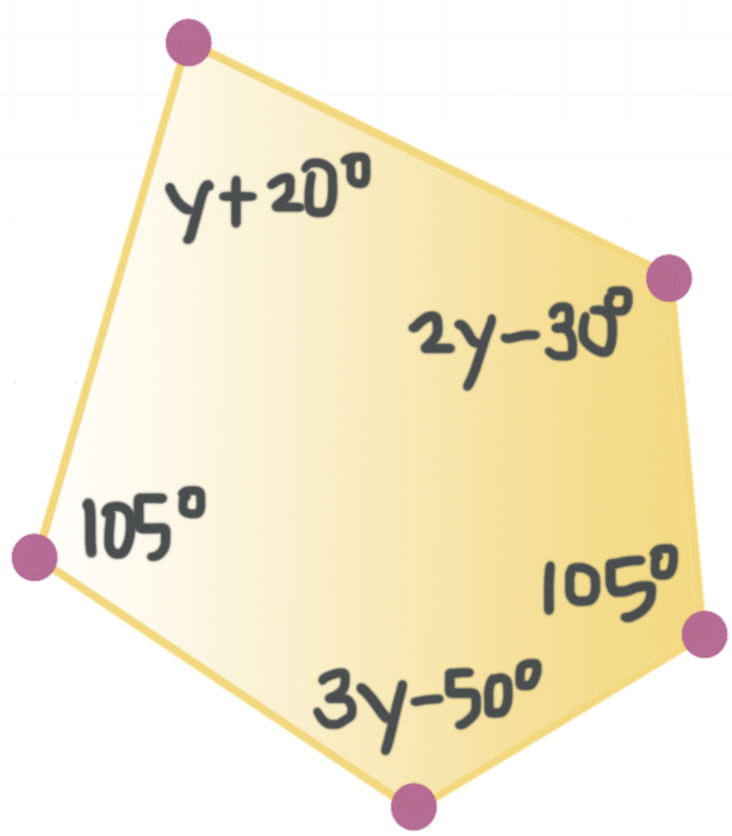
$$(13)180$$

$$2{,}340$$

To find the measure of one interior angle, we divide the sum of all interior angles $2{,}340^\circ$ by the number of interior angles 15.

$$\frac{2{,}340}{15} = 156^\circ$$

■ 3. Find the value of y. Then determine whether this a regular polygon.



Solution:

$y = 65$. This is not a regular polygon because the angle measures are $105, 85, 100, 105, 145$.

The interior angles of a pentagon have a sum of 540°.

$$105 + y + 20 + 2y - 30 + 105 + 3y - 50 = 540$$

$$6y + 150 = 540$$

$$y = 65$$

■ 4. Each interior angle measure of a regular polygon is 160°. Find the number of sides of this polygon.

Solution:

18 sides. From the formula for the measure of a single interior angle of a regular polygon, we get

$$\frac{(n-2)180}{n} = 160$$

$$(n-2)180 = 160n$$

$$180n - 360 = 160n$$

$$20n = 360$$

$$n = 18$$

EXTERIOR ANGLES OF POLYGONS

■ 1. Find the sum of the exterior angles of a decagon.

Solution:

360°. By the Exterior Angle Theorem, we know that the sum of the exterior angles of a polygon is always 360°.

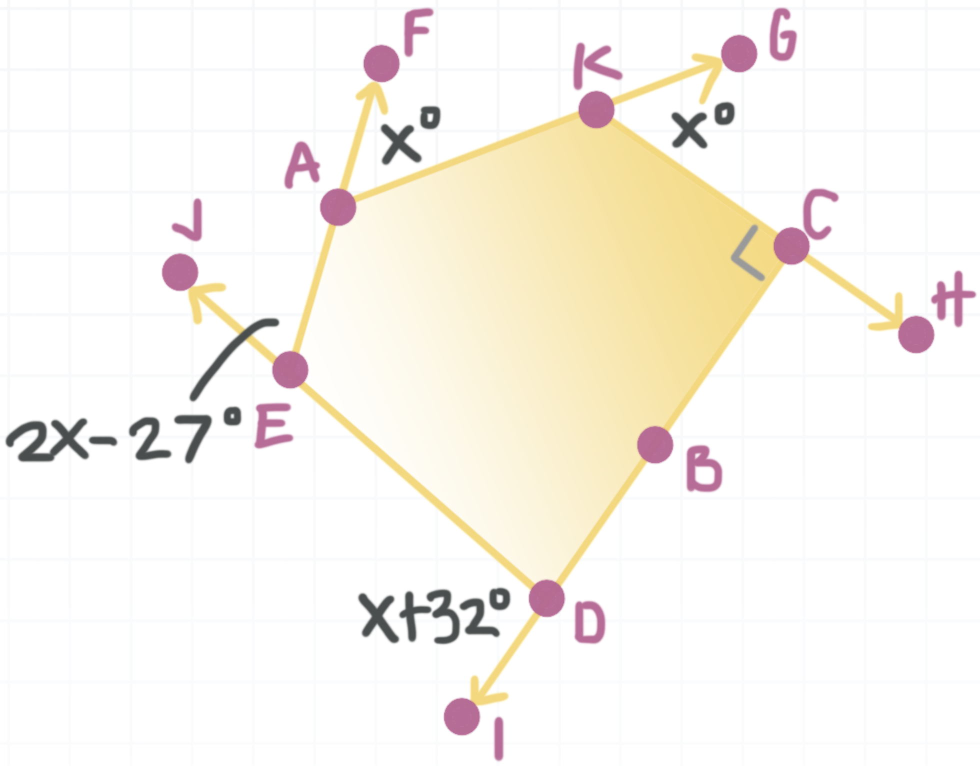
■ 2. Each exterior angle of a regular polygon has measure of 30°. Find the number of sides of this polygon.

Solution:

12 sides. Since the exterior angles of any polygon sum to 360°, the number of sides must be given by

$$\frac{360^\circ}{n}=\frac{360^\circ}{30^\circ}=12$$

■ 3. Find the value of x.



Solution:

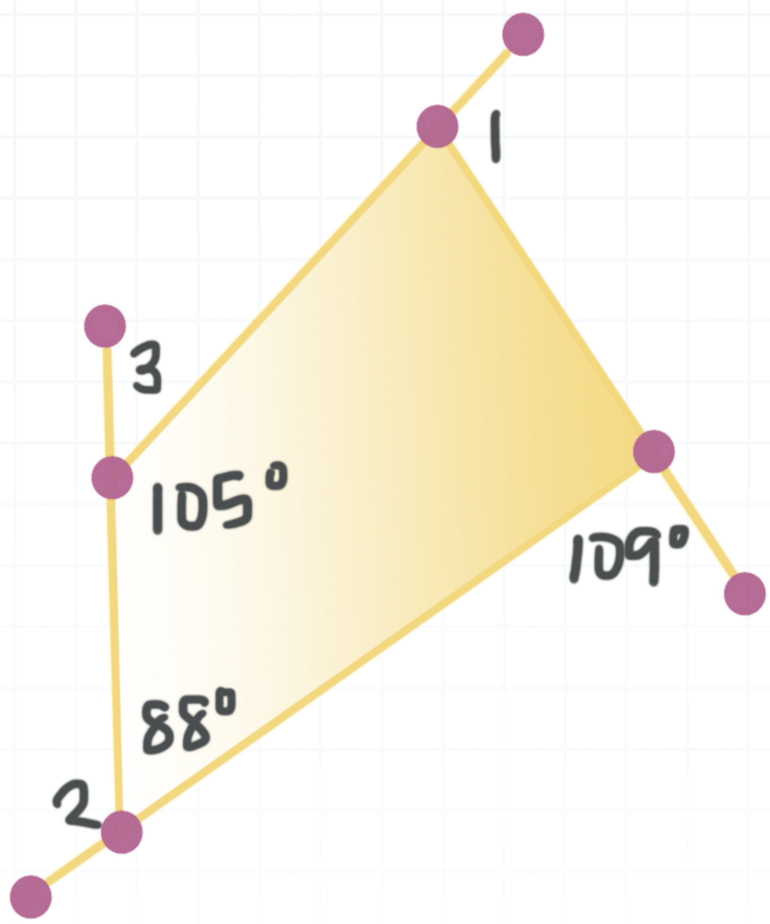
$x = 53$. All exterior angle measures are given, and the exterior angle at C must be 90°, since the interior angle there is also 90°. Because the exterior angles of any polygon always sum to 360°, we get

$$x + x + 90 + x + 32 + 2x - 27 = 360$$

$$5x + 95 = 360$$

$$x = 53$$

■ 4. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ based on the figure.

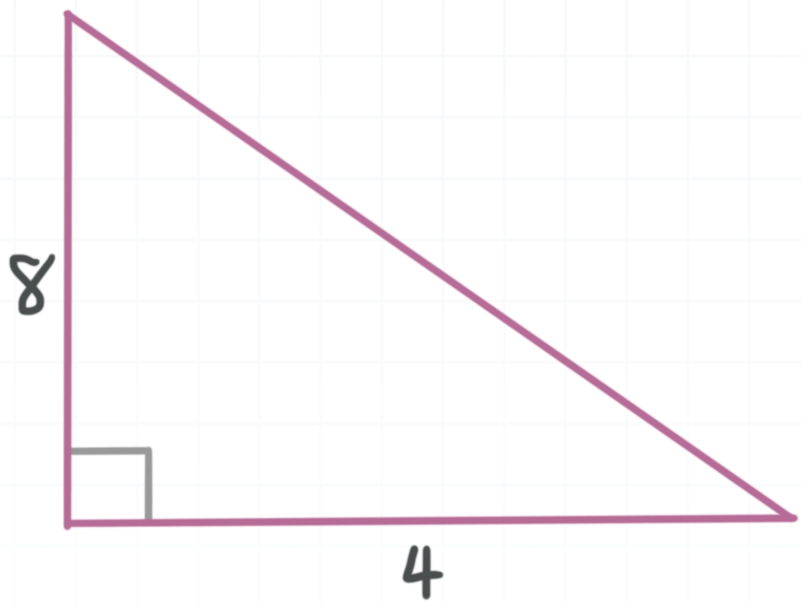


Solution:

$m\angle 1 = 84$, $m\angle 2 = 92$, and $m\angle 3 = 75$. Find $m\angle 2$ and $m\angle 3$ first because they form a linear pair with their adjacent angle. Then further find $m\angle 1$ by setting the sum of all exterior angles equal to 360°.

PYTHAGOREAN THEOREM

■ 1. Find the exact length of the hypotenuse.



Solution:

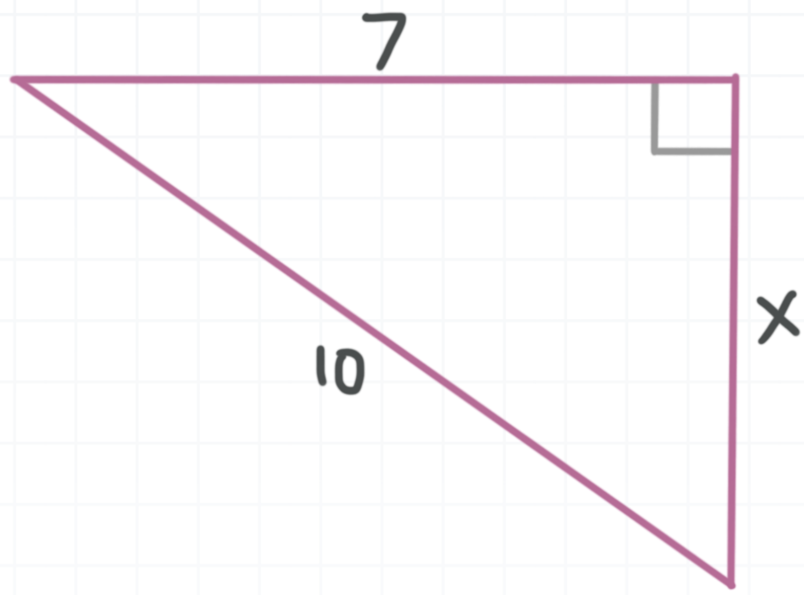
$4\sqrt{5}$. By the Pythagorean Theorem,

$$8^2 + 4^2 = c^2$$

$$c^2 = 80$$

$$c = \sqrt{80} = 4\sqrt{5}$$

■ 2. Find the exact length of the missing leg.



Solution:

$\sqrt{51}$. By the Pythagorean Theorem,

$$7^2 + x^2 = 10^2$$

$$x^2 = 10^2 - 7^2 = 51$$

$$x = \sqrt{51}$$

■ 3. Find the length of the diagonal of a rectangle with length 14 and width 8.

Solution:

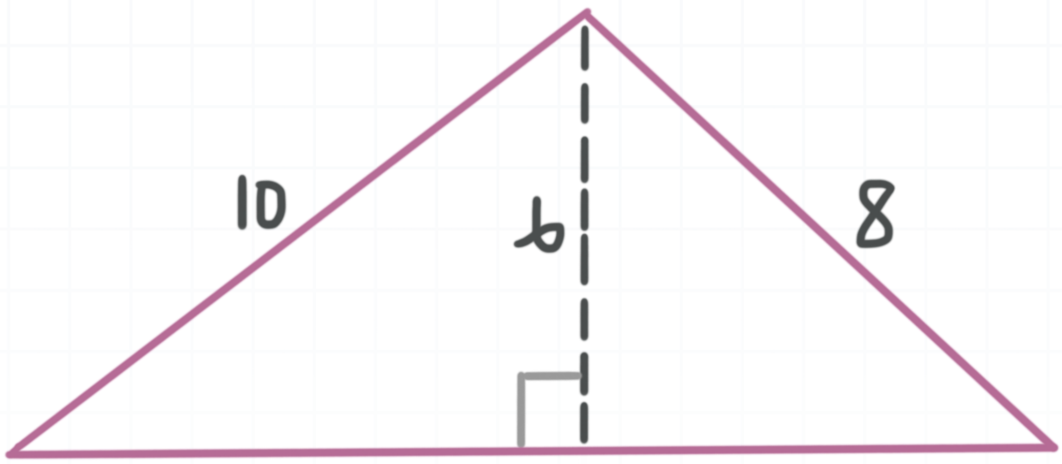
$2\sqrt{65}$. By the Pythagorean theorem,

$$14^2 + 8^2 = d^2$$

$$d^2 = 260$$

$$d = \sqrt{260} = 2\sqrt{65}$$

■ 4. Find the perimeter of the triangle to the nearest tenth.



Solution:

31.3. Separate your diagram into two right triangles, then use the Pythagorean Theorem to find each leg of those triangles, which together make the length of the missing side of the whole triangle. For the triangle on the left,

$$x^2 + 6^2 = 10^2$$

$$x^2 = 10^2 - 6^2$$

$$x = \sqrt{10^2 - 6^2}$$

For the triangle on the right,

$$y^2 + 6^2 = 8^2$$

$$y^2 = 8^2 - 6^2$$

$$y = \sqrt{8^2 - 6^2}$$

The length of the missing side of the triangle is therefore

$$z = \sqrt{10^2 - 6^2} + \sqrt{8^2 - 6^2}$$

$$z = \sqrt{100 - 36} + \sqrt{64 - 36}$$

$$z = \sqrt{64} + \sqrt{28}$$

$$z = 8 + 2\sqrt{7}$$

$$z \approx 13.3$$

The perimeter of the triangle is $10 + 8 + 13.3 = 31.3$.

PYTHAGOREAN INEQUALITIES

■ 1. The side lengths of a triangle are 10, 14, and 15. Determine whether the triangle is obtuse, acute, or right.

Solution:

The triangle is acute. The sum of the squared legs is $10^2 + 14^2 = 296$, and the square of the hypotenuse is $15^2 = 225$. Since $296 > 225$, the triangle is acute.

■ 2. The side lengths of a triangle are 7, 18, and 12. Determine whether this triangle is obtuse, acute, or right.

Solution:

The triangle is obtuse. The sum of the squared legs is $7^2 + 12^2 = 193$, and the square of the hypotenuse is $18^2 = 324$. Since $324 > 193$, the triangle is obtuse.

■ 3. A triangle's two shortest sides have lengths 8 and 6. Let x be the length of the third side. Give a compound inequality that represents all possible lengths of the third side, ensuring that the triangle is acute.

Solution:

$8 < c < 10$. If the triangle is acute, then $c^2 < a^2 + b^2$. We know that $8^2 + 6^2 = 100$, which means that $c^2 < 100$ will make this an acute triangle. Therefore $c < 10$ will work for its side length. However since 8 must be shorter, $c > 8$.

■ 4. The side lengths of a triangle in ascending order are x, $x + 2$, and 10. Find the value of x such that this is a right triangle.

Solution:

6. If we plug the given side lengths into the Pythagorean theorem, we get

$$x^2 + (x + 2)^2 = 10^2$$

$$x^2 + x^2 + 4x + 4 = 100$$

$$2x^2 + 4x - 96 = 0$$

$$x^2 + 2x - 48 = 0$$

$$(x + 8)(x - 6) = 0$$

$$x = -8, 6$$

The side length can't be negative, so $x = -8$ can't be a solution. Therefore, $x = 6$.

EQUATION OF A CIRCLE

■ 1. A circle has a radius of 4 and center at $(-2,5)$. Write the equation for this circle.

Solution:

The general equation for a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius. $(-2,5)$ is the center, so $h=-2$ and $k=5$. The radius is given as $r=4$. Substitute these values into the general equation and get $(x+2)^2+(y-5)^2=16$.

■ 2. Find the center and diameter of the circle given by $(x-3)^2+(y+2)^2=9$.

Solution:

The general equation for a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius. In this equation, $h=3$ and $k=-2$ making the center $(3,-2)$. The radius is $\sqrt{9}=3$. Therefore, the diameter of the circle is 6.

■ 3. A circle has a diameter with endpoints at $(-3, -1)$ and $(3,7)$. Find the equation of the circle.

Solution:

The diameter of the circle can be found by finding the distance between the points given. Use the distance formula to get

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(7 - (-1))^2 + (3 - (-3)^2}$$

$$d = \sqrt{100}$$

$$d = 10$$

Use the midpoint formula to find the center of the circle.

$$(a, b) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(a, b) = \left(\frac{(-3) + 3}{2}, \frac{(-1) + 7}{2}\right)$$

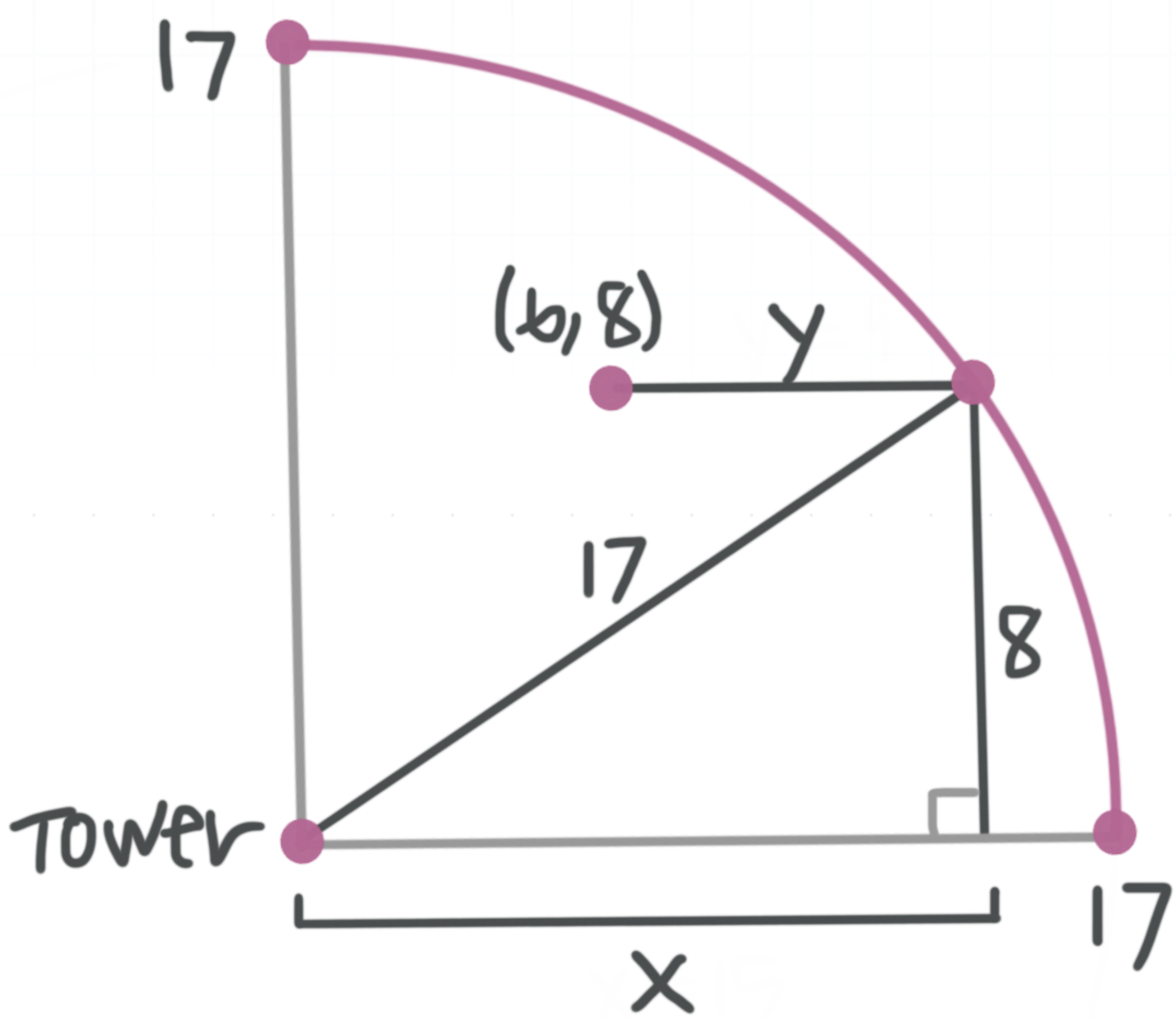
$$(a, b) = (0,3)$$

The general equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius. Substitute $h = 0$, $k = 3$, and $r = 5$ into the equation to get $x^2 + (y - 3)^2 = 25$.

■ 4. A cellphone tower services a 17 mile radius. A rest stop on the highway is 6 miles east and 8 miles north of the tower. If you continue to travel due east from the rest stop, for how many more miles will you be in range of the tower?

Solution:

Sketch the following diagram.



Find the horizontal length of the right triangle shown and get $x^2 + 8^2 = 17^2$. Solve for x.

$$x^2 + 8^2 = 17^2$$

$$x^2 + 64 = 289$$

$$x^2 = 225$$

$$x = 15$$

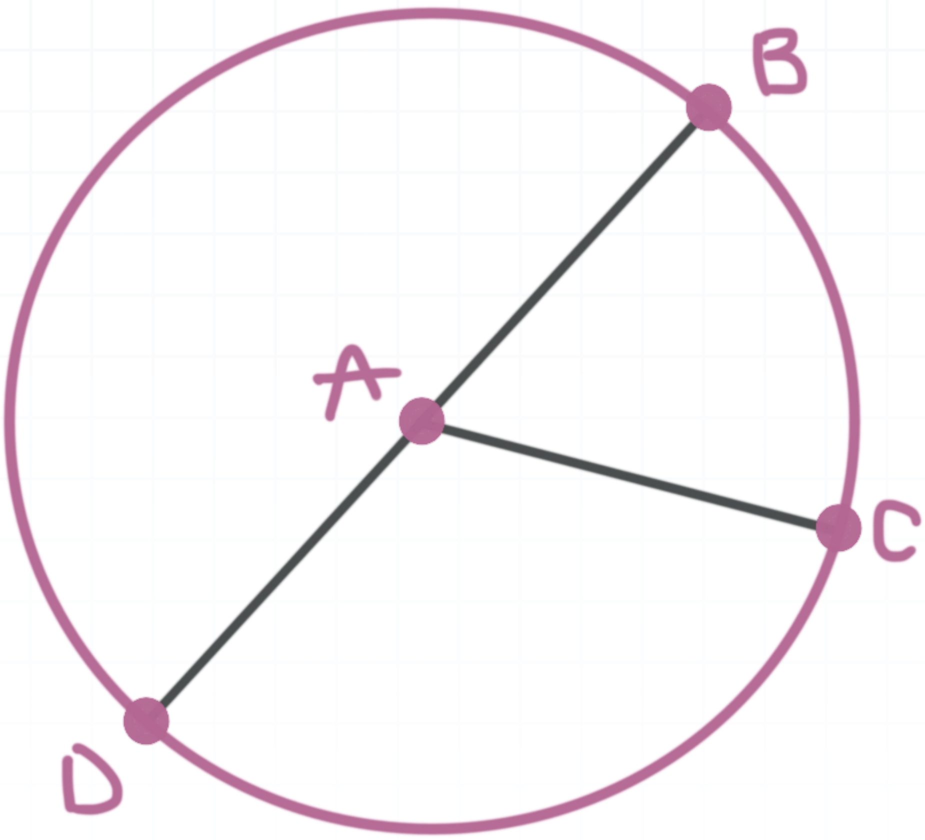
Now find y.

$$y = x - 6$$

$$y = 9$$

DEGREE MEASURE OF AN ARC

■ 1. In $\odot A$, $m\angle BAC = 65^\circ$ and $\overline{BD}$ is a diameter. Find the measure of arc DC.



Solution:

$\angle BAC$ is a central angle of $\odot A$, so arc BC is given by the measure of $\angle BAC$, and $m\angle BAC = 65^\circ$. Since $\angle BAC$ and $\angle CAD$ are supplementary,

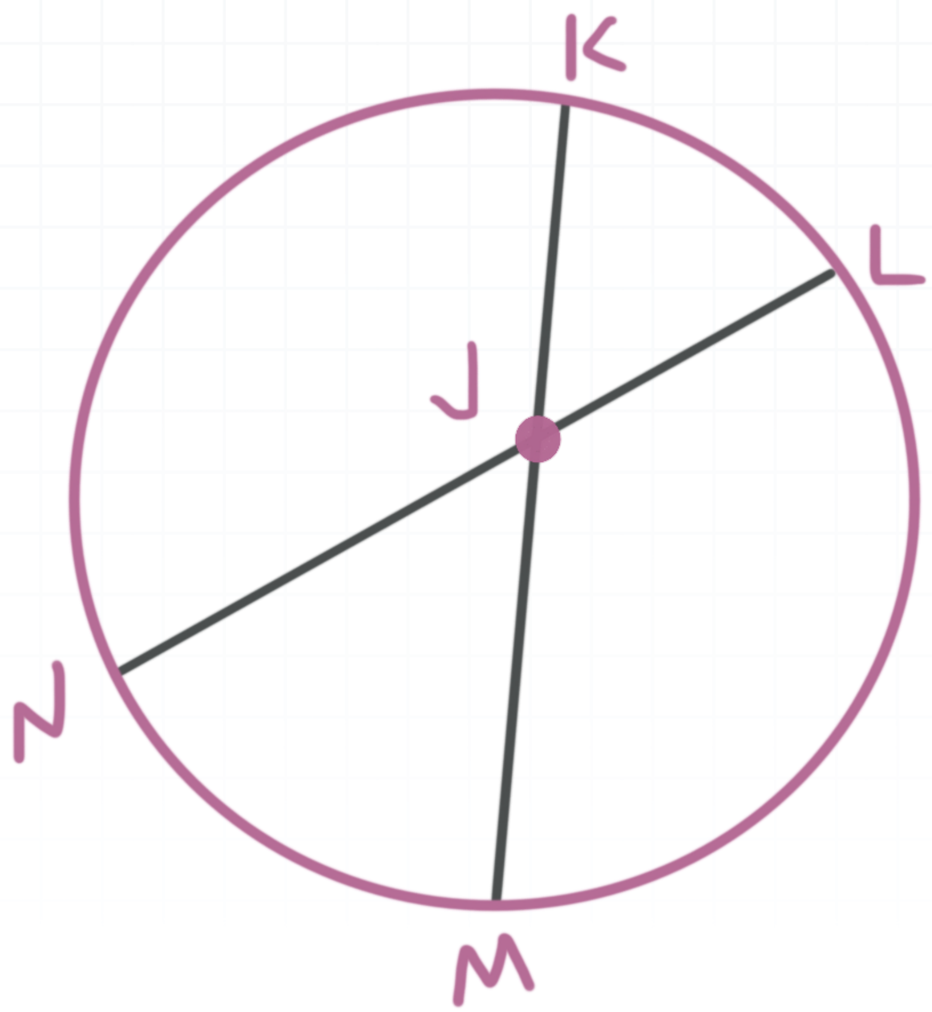
$$m\angle BAC + m\angle CAD = 180^\circ$$

$$65^\circ + m\angle CAD = 180^\circ$$

$$m\angle CAD = 115^\circ$$

Therefore, the measure of arc DC must be 115°.

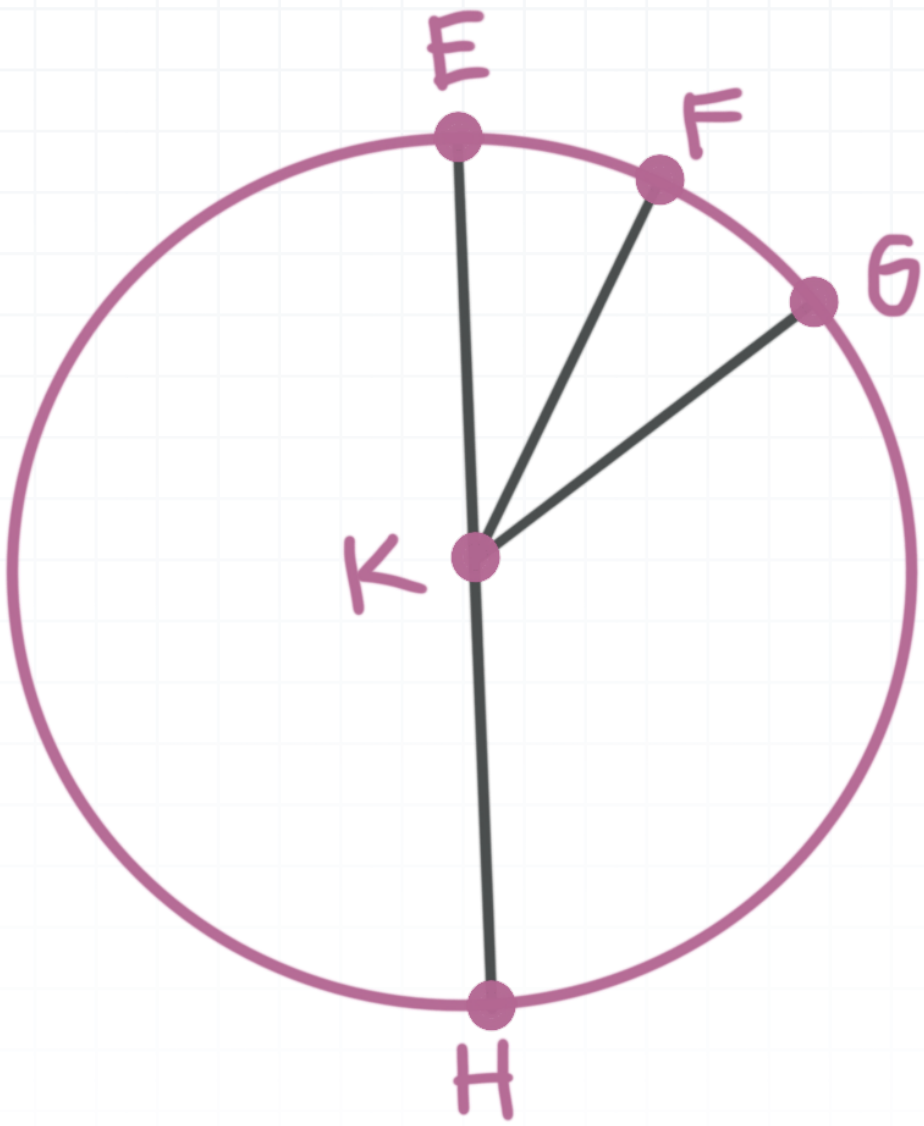
■ 2. In $\odot J$, $m\angle KJL = 54^\circ$ and $\overline{KM}$ and $\overline{LN}$ are diameters. Find the measure of arc MN.



Solution:

$\angle KJL$ is a central angle of $\odot J$. Because they are vertical angles, $m\angle KJL = m\angle NJM$. Because $m\angle KJL = 54^\circ$, that means $m\angle NJM = 54^\circ$, and therefore that the measure of arc MN is also 54°.

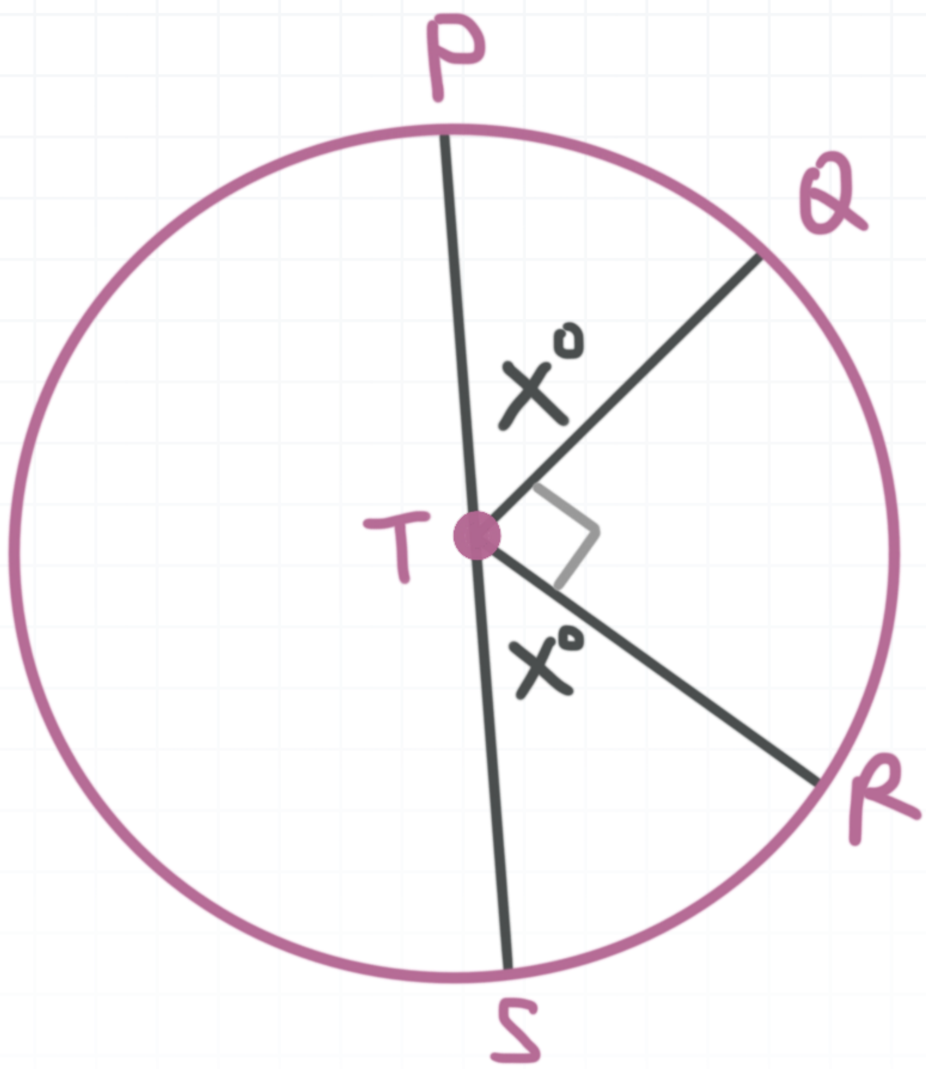
■ 3. In $\odot K$, $m\angle EKG = 70^\circ$, $\overline{EH}$ is a diameter, and $\overline{KF}$ bisects $\angle EKG$. Find the measure of arc FEH.



Solution:

$\angle EKG = 70^\circ$ and is bisected by KF, so $m\angle EKF$ and the measure of arc EF are both equal to 35°. Arc FEH is the sum of arc EF and arc EH, so arc FEH must have a measure of $35^\circ + 180^\circ = 215^\circ$.

■ 4. Find the measure of arc PR, if $\overline{PS}$ is the diameter of $\odot T$.



Solution:

Because $\overline{PS}$ is a diameter,

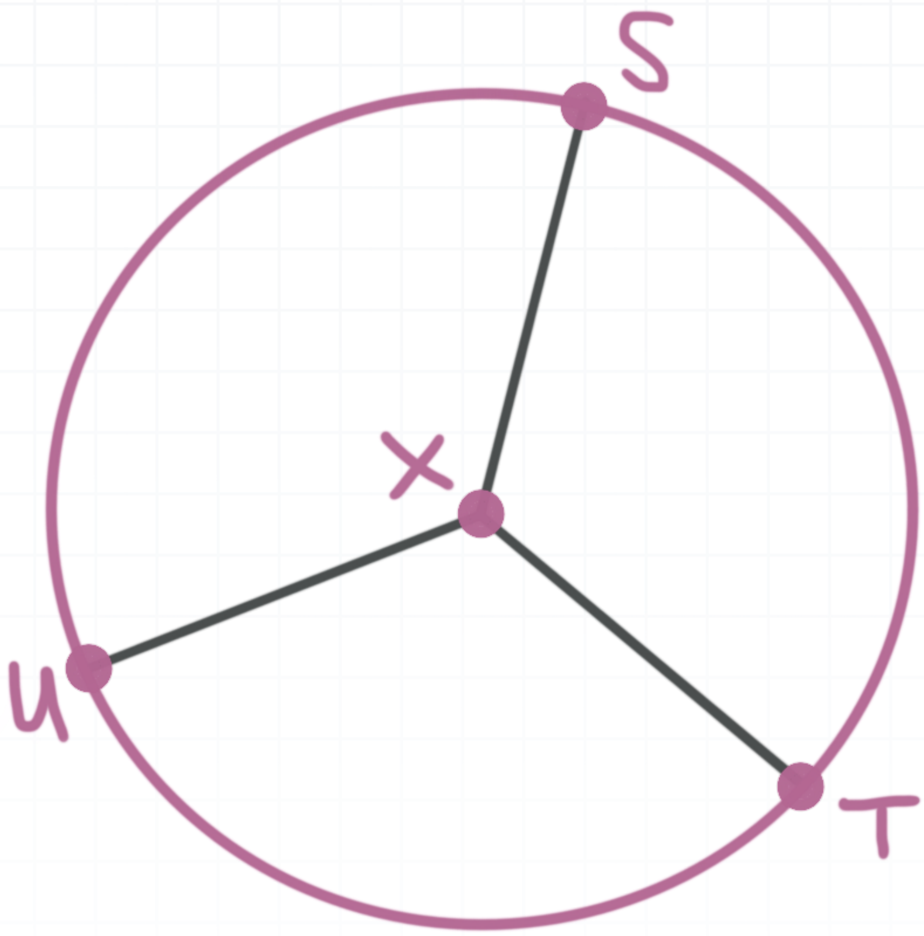
$$m\angle PTQ + m\angle QTR + m\angle RTS = 180^\circ$$

$$x + 90 + x = 180^\circ$$

$$x = 45^\circ$$

Then we know that the measure of arc PQ is 45° and the measure of arc QR is 90°, so the measure of arc PR is $45^\circ + 90^\circ = 135^\circ$.

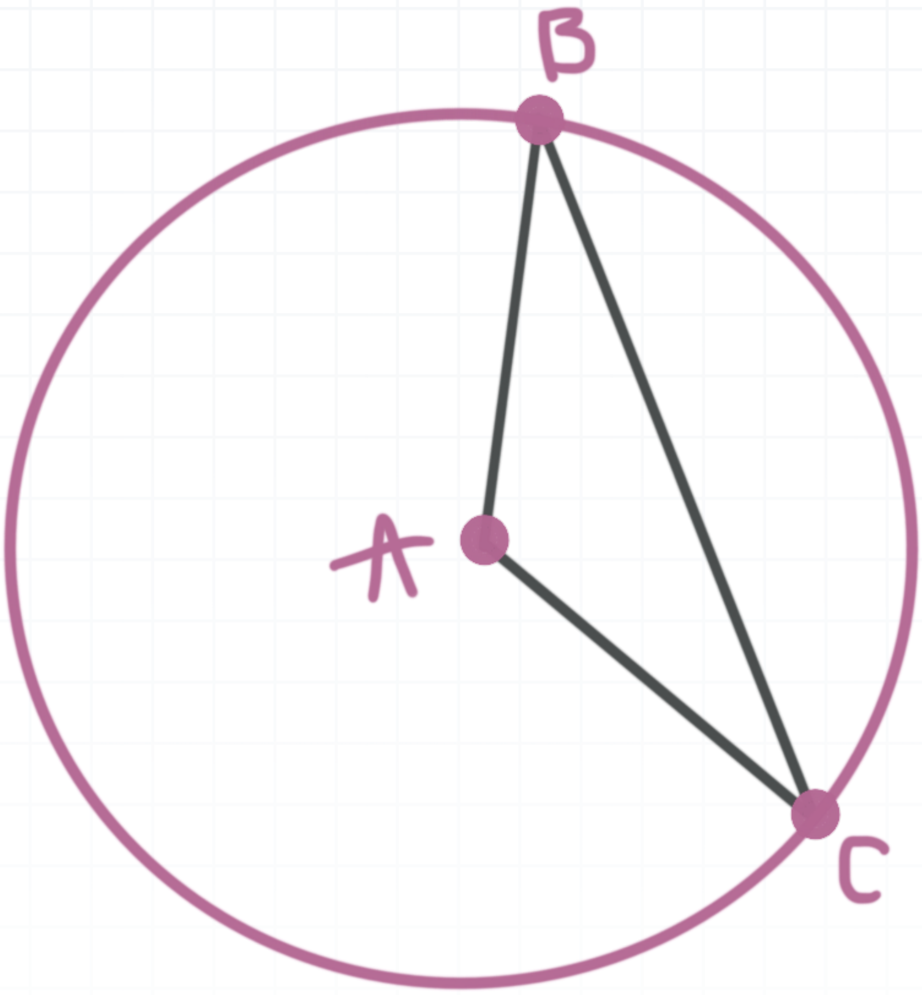
■ 5. In $\odot X$, $\angle UXS \cong \angle SXT \cong \angle UXT$. Find the measure of arc STU.



Solution:

The measures of arcs ST, TU, and US are congruent, and together they form the full circle, so each of those arcs has measure $360^\circ/3 = 120^\circ$. Therefore, arc STU measures $ST + TU = 120^\circ + 120^\circ = 240^\circ$.

■ 6. In $\odot A$, $m\angle ABC = 15^\circ$. Find the measure of arc BC.



Solution:

$\triangle ABC$ is isosceles because $\overline{AB}$ and $\overline{AC}$ are radii of $\odot A$.

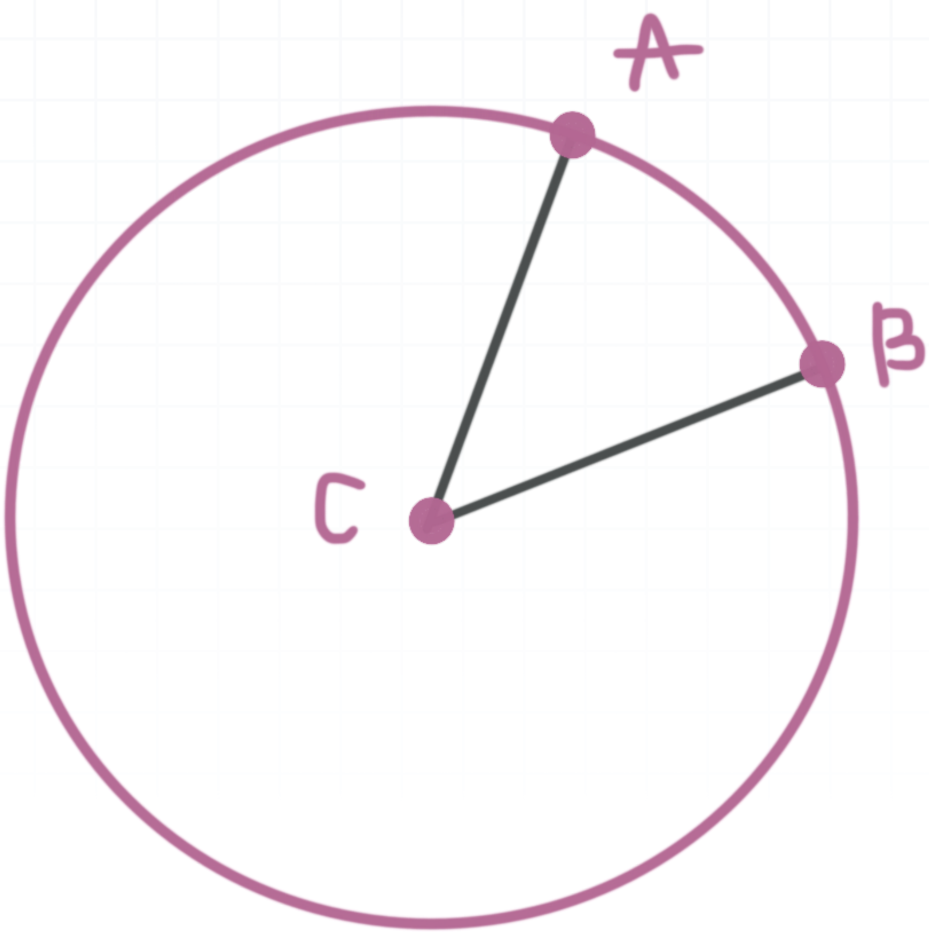
$$m\angle BAC + m\angle ABC + m\angle BCA = 180^\circ$$

$$m\angle BAC + 15^\circ + 15^\circ = 180^\circ$$

The measure of angle BAC is $m\angle BAC = 150^\circ$, which means the measure of arc BC is also 150°.

ARC LENGTH

■ 1. In $\odot C$, $m\angle ACB = 50^\circ$. Find the length of arc AB if $CA = 14$. Round your answer to the nearest hundredth.



Solution:

The circumference of a circle is

$$C = 2\pi r$$

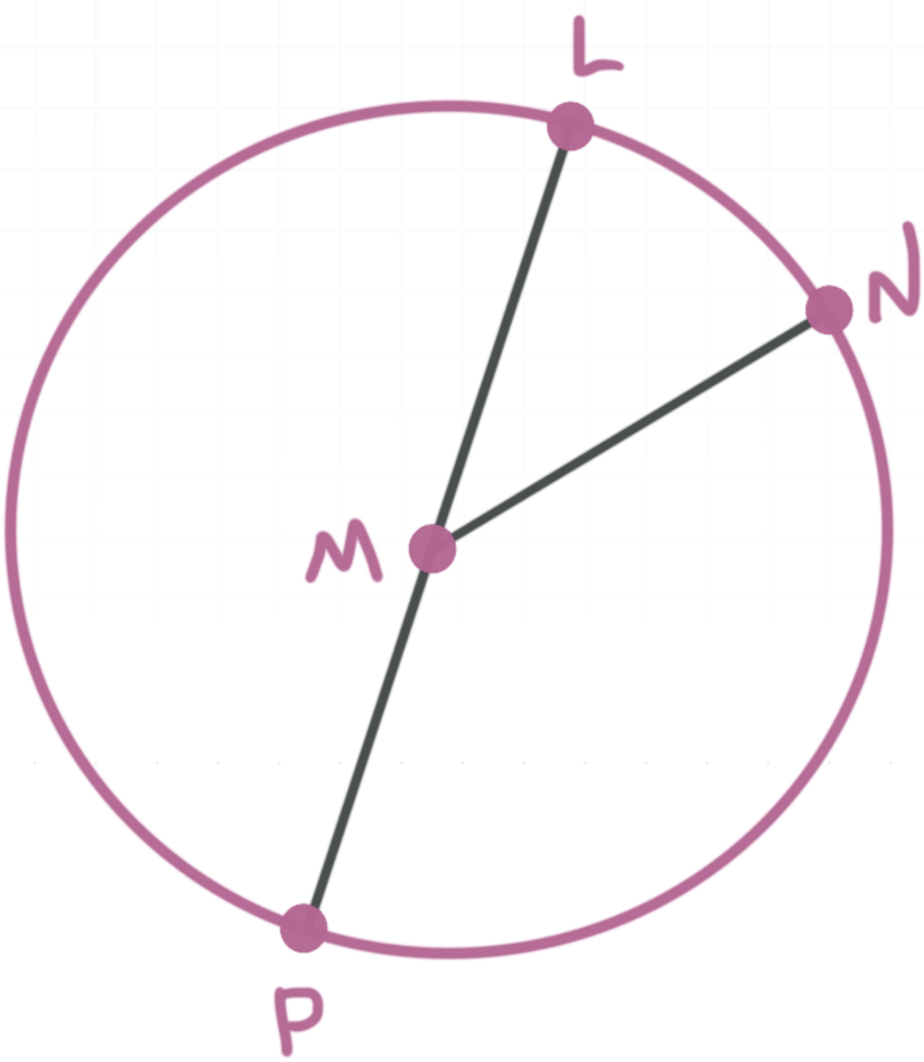
$$C = 2\pi(14)$$

$$C = 87.96$$

Then the measure of arc AB is 50°. The arc length of AB is therefore

$$\frac{50^\circ}{360^\circ}(87.96) = 12.2$$

■ 2. In $\odot M$, $m\angle LMN = 60^\circ$ and $\overline{LP}$ is a diameter. Find the length of arc LPN if $LP = 24$. Round your answer to the nearest hundredth.



Solution:

The measure of arc LN is 60°, so the measure of arc LPN is $360^\circ - 60^\circ = 300^\circ$. The circumference is

$$C = 2\pi r$$

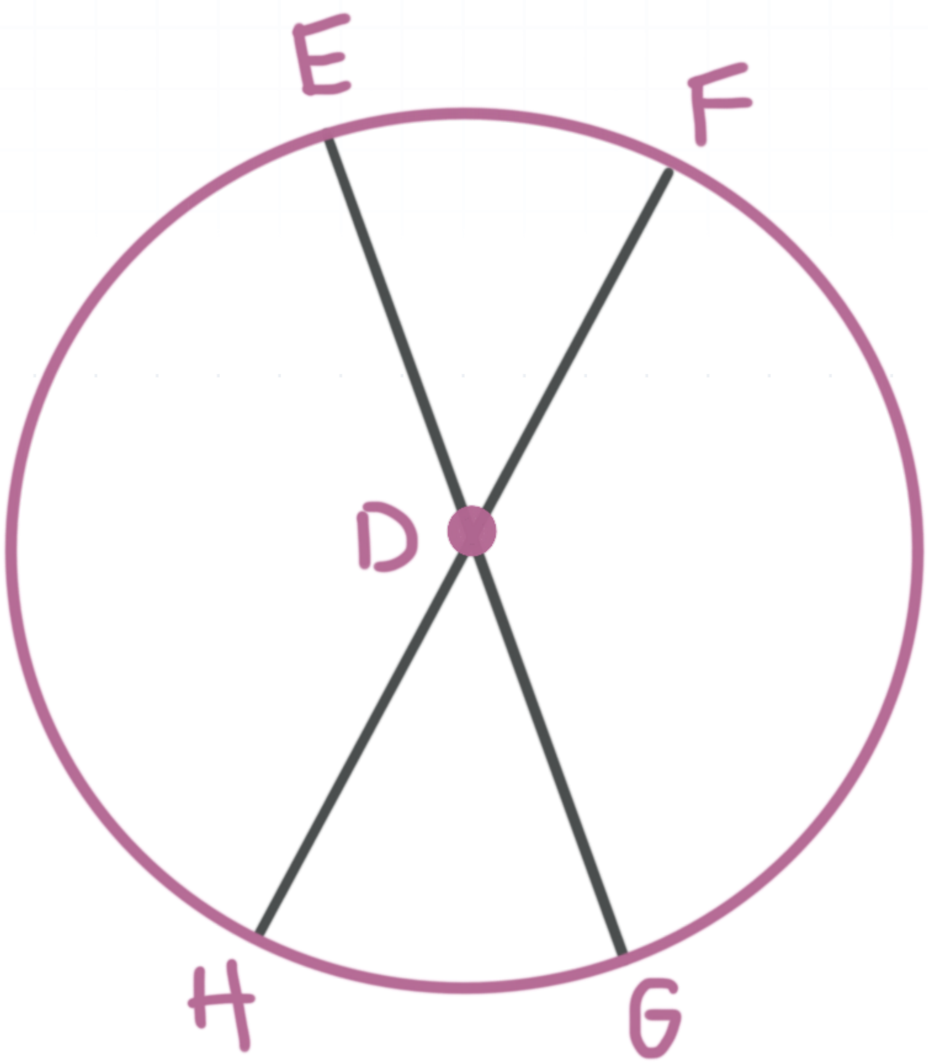
$$C = 24\pi$$

$$C = 75.40$$

So the arc length of LPN is

$$\frac{300^\circ}{360^\circ}(75.40) = 62.83$$

■ 3. $\overline{EG}$ and $\overline{FH}$ are diameters of $\odot D$. Find the length of arc HG if $m\angle EDF = 45^\circ$ and $ED = 16$. Write the exact value.



Solution:

The circumference of a circle is

$$C = 2\pi r$$

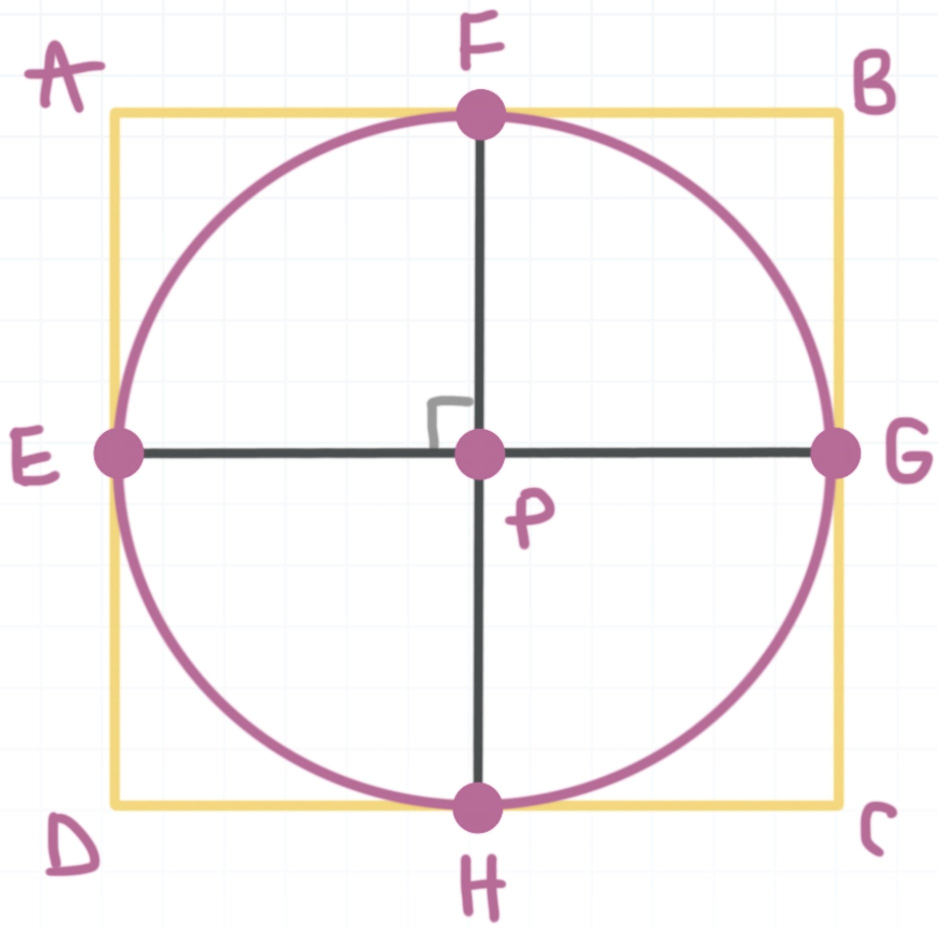
$$C = 2\pi(16)$$

$$C = 32\pi$$

Arcs EF and HG both have a measure of 45° since $\angle EDF$ and $\angle HDG$ are vertical angles. Therefore, the length of arc HG is

$$\frac{45^\circ}{360^\circ}(32)\pi = 4\pi$$

■ 4. The area of square $ABCD$ is 144 cm^2 and circle P is inscribed in the square. $\overline{EG}$ and $\overline{FH}$ are perpendicular to one another, and both are diameters of $\odot P$. E, F, G, and H are midpoints of each side of the square. Find the length of arc EF, rounded to the nearest hundredth.



Solution:

The length of each side of the square is $\sqrt{144} = 12$. Because $\odot P$ is inscribed in square $ABCD$, the diameter of the circle is 12. The circumference is

$$C = 2\pi r$$

$$C = 12\pi$$

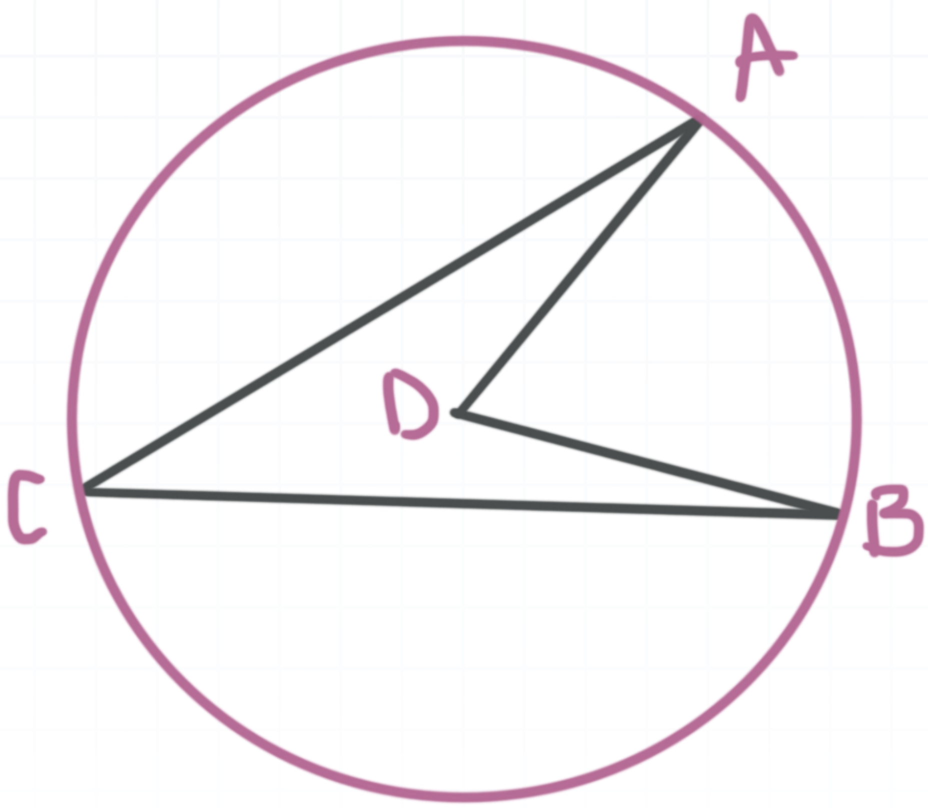
$$C = 37.70$$

$m\angle EPF = 90^\circ$. So the measure of arc EF is also 90°. So the length of arc EF is

$$\frac{90^\circ}{360^\circ}(37.70) = 9.43$$

INSCRIBED ANGLES OF CIRCLES

1. In $\odot D$, $m\angle ADB = 88^\circ$. Find $m\angle ACB$.

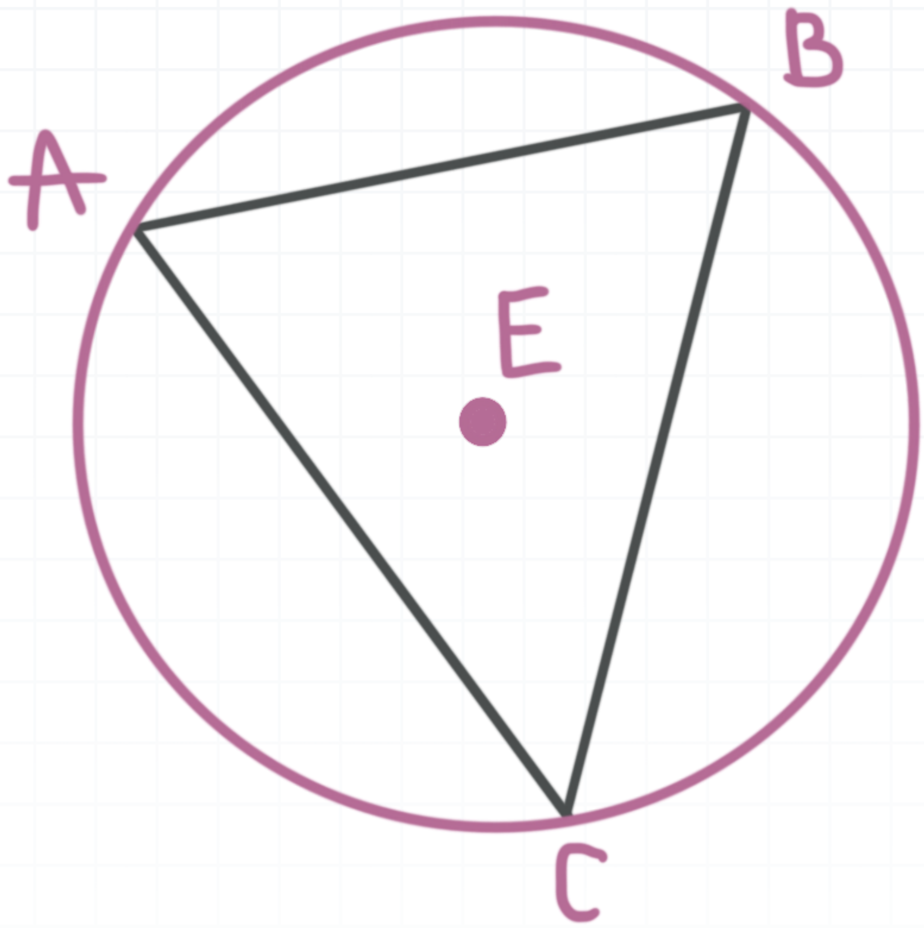


Solution:

$m\angle ADB = 88^\circ$, which means the measure of arc AB is 88°. Which means $m\angle ACB$ is

$$\angle ACB = \frac{1}{2}88^\circ = 44^\circ$$

2. In $\odot E$, $\overline{AC} \cong \overline{CB}$ and $m\angle ABC = 55^\circ$. Find the measure of arc AB.



Solution:

$\triangle ABC$ is isosceles with $m\angle A = m\angle B = 55^\circ$.

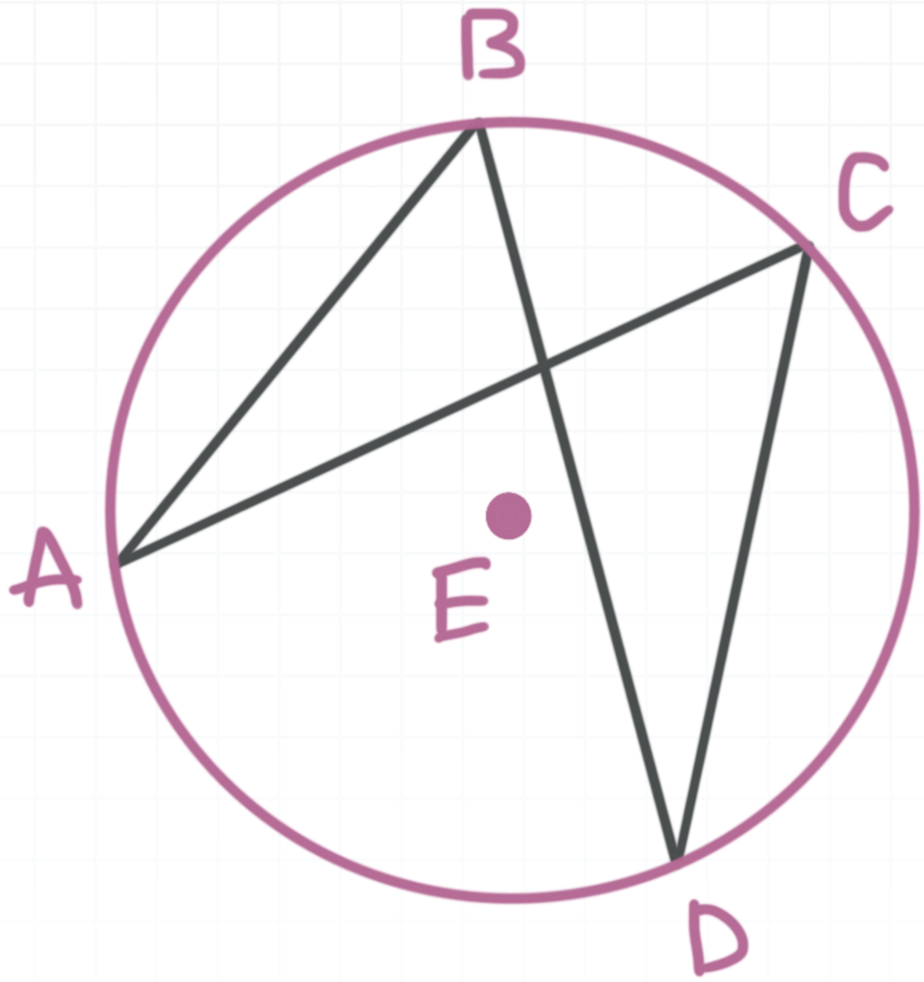
$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$55^\circ + 55^\circ + m\angle C = 180^\circ$$

$$m\angle C = 70^\circ$$

Then the measure of arc AB is $2m\angle C = 2(70^\circ) = 140^\circ$.

■ 3. In $\odot E$ the measure of arc AB is 100°, the measure of arc BC is 40°, and the measure of CD is 110°. Find $m\angle ABD$.



Solution:

Arcs AB, BC, CD, and DA form the full circle, so they sum to 360°.

$$AB + BC + CD + DA = 360^\circ$$

$$100^\circ + 40^\circ + 110^\circ + DA = 360^\circ$$

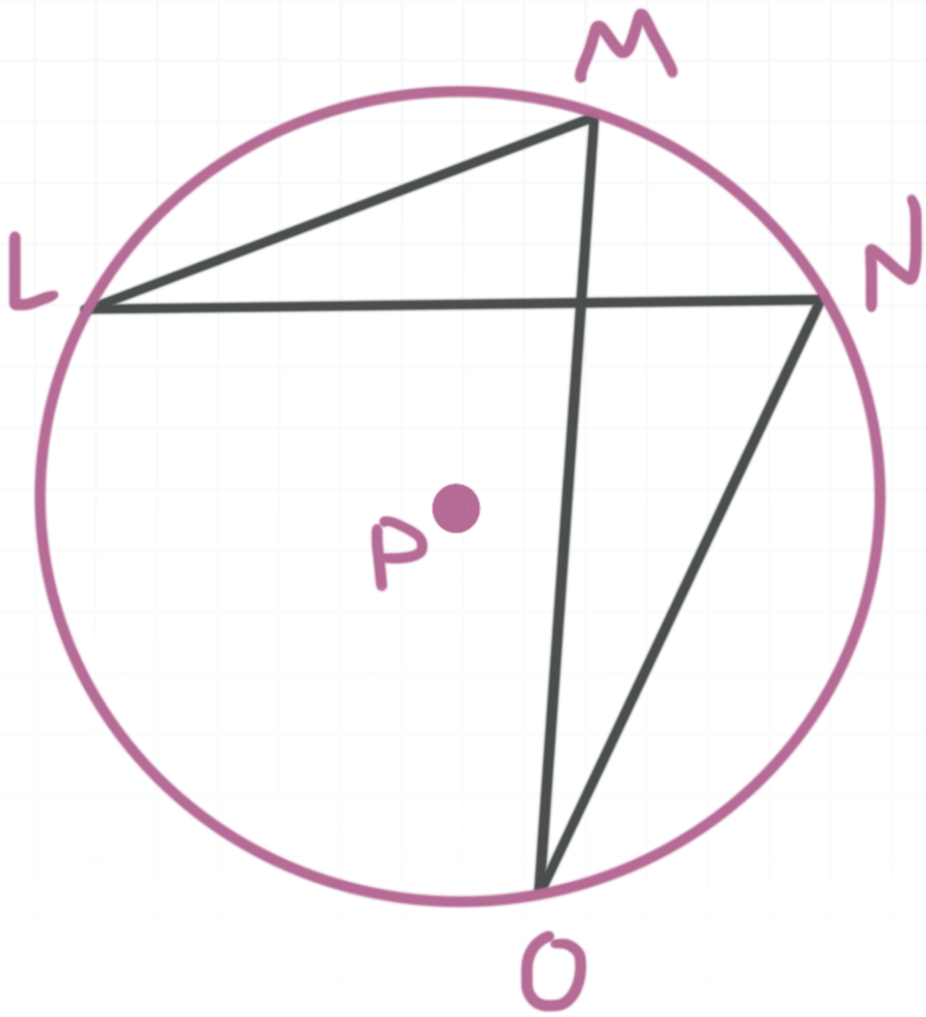
$$250^\circ + DA = 360^\circ$$

$$DA = 110^\circ$$

Therefore, the measure of angle ABD is

$$m\angle ABD = \frac{1}{2}110^\circ = 55^\circ$$

■ 4. In $\odot P$, $m\angle LMO = 2x - 18$ and the measure of arc $LO = 88^\circ$. Find x.



Solution:

The measure of angle LMO, if the measure of arc LO is 88°, is

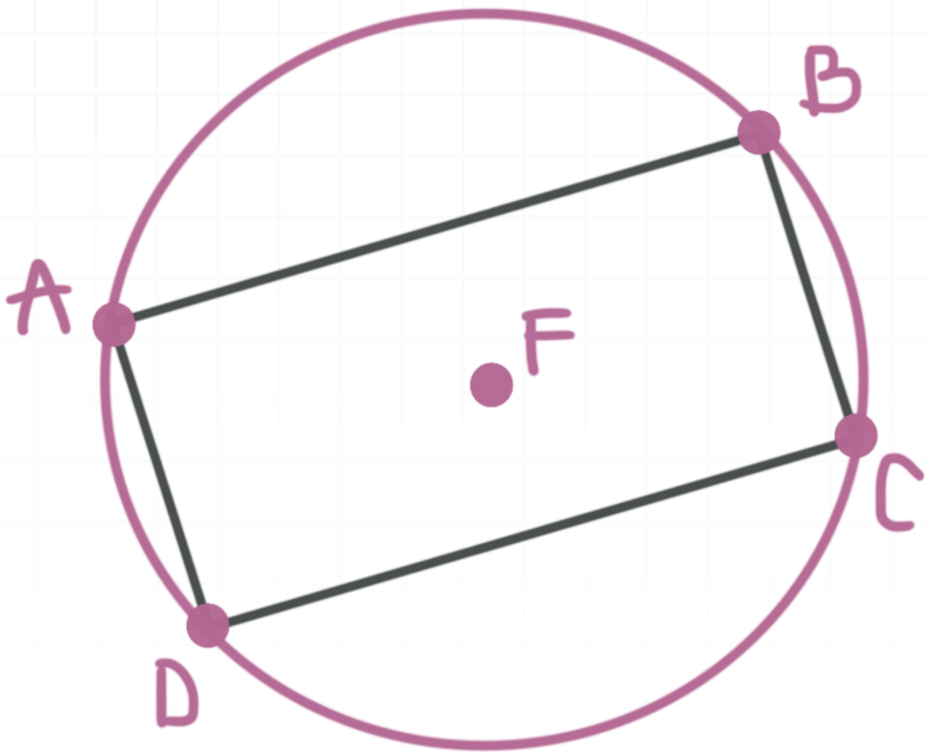
$$\angle LMO = \frac{1}{2}88^\circ$$

$$2x - 18 = \frac{1}{2}88^\circ$$

$$2x - 18 = 44^\circ$$

$x = 31^\circ$

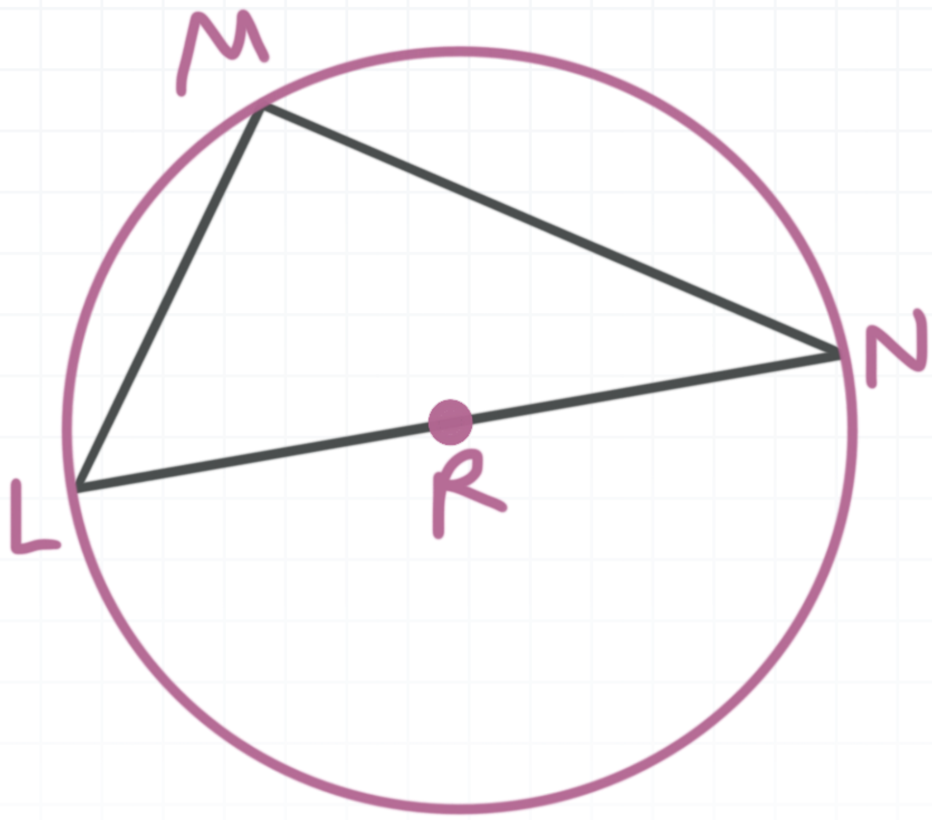
■ 5. Rectangle $ABCD$ is inscribed in $\odot F$ and the measure of arc DAC is 230°. Find the measure of arc AB.



Solution:

All of the angles of the rectangle measure 90°. AC and BD must be diameters of the circle. So the measure of arc DC is $360^\circ - 230^\circ = 130^\circ$. Arcs AB and DC are congruent, so they both have a measure of 130°.

■ 6. In $\odot R$, $\overline{LN}$ is a diameter, $m\angle MLN = 4x + 20$, and $m\angle LNM = 5x - 38$. Find the measure of arc LM.



Solution:

We know that

$$m\angle LMN = \frac{1}{2}(180^\circ) = 90^\circ$$

And since the three interior angles of a triangle always sum to 180°,

$$m\angle LMN + m\angle LNM + m\angle MLN = 180^\circ$$

$$90^\circ + 4x + 20 + 5x - 38 = 180^\circ$$

$$9x + 72 = 180^\circ$$

$$x = 12$$

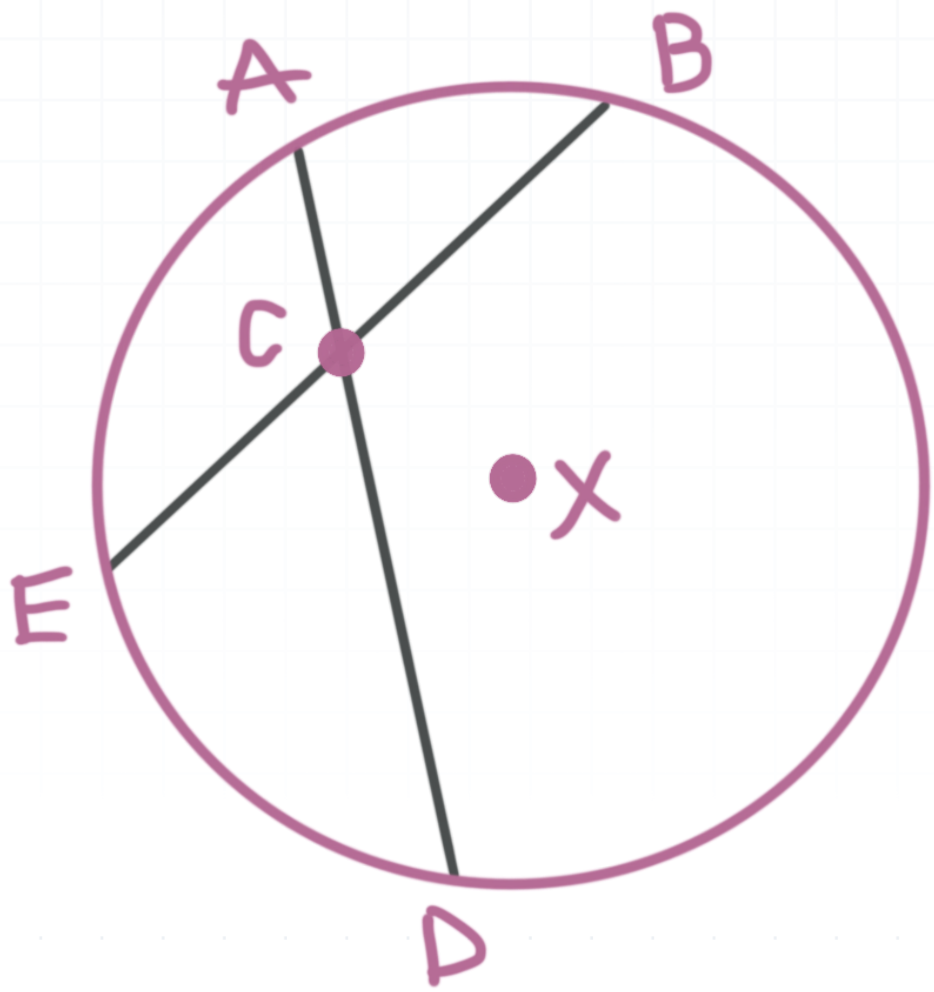
Therefore, $m\angle LNM = 5(12) - 38 = 22^\circ$, so

$$22^\circ = \frac{1}{2}(\text{arc } LM)$$

$$LM = 44^\circ$$

VERTEX ON, INSIDE AND OUTSIDE THE CIRCLE

■ 1. $\overline{AD}$ and $\overline{EB}$ are chords of $\odot X$. The measure of arc AB is 35° and the measure of arc ED is 85°. Find $m\angle ECD$.



Solution:

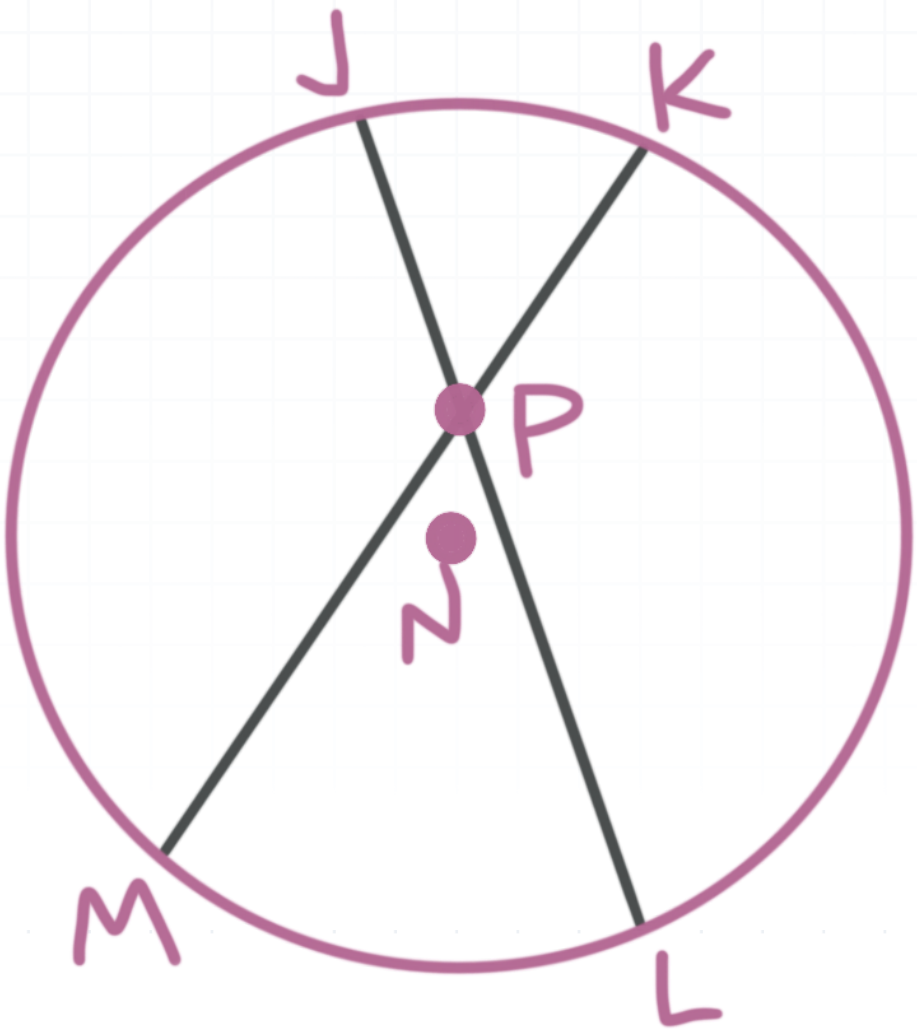
$m\angle ECD$ is given by

$$m\angle ECD = \frac{1}{2}(\text{arc } ED + \text{arc } AB)$$

$$m\angle ECD = \frac{1}{2}(85^\circ + 35^\circ)$$

$$m\angle ECD = 60^\circ$$

■ 2. $\overline{JL}$ and $\overline{KM}$ are chords of $\odot N$. The measure of arc JK is 25° and $m\angle JPK = 40^\circ$. Find the measure of arc ML.



Solution:

The measure of $\angle JPK$ is given by

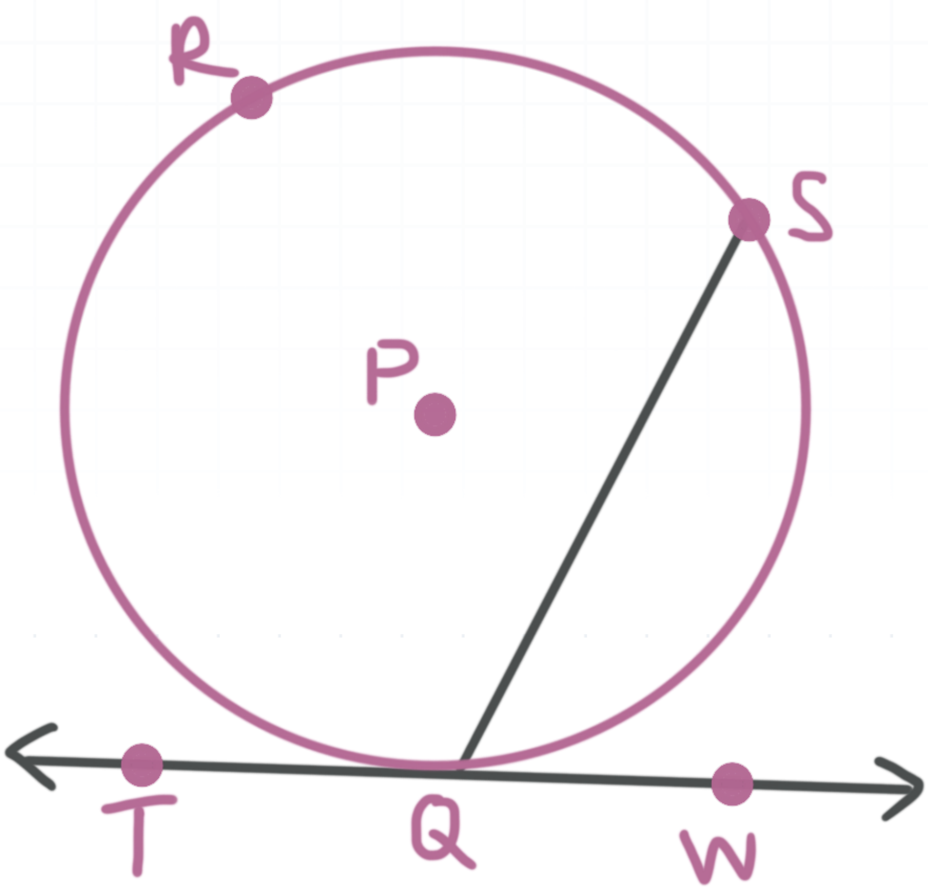
$$m\angle JPK = \frac{1}{2}(\text{arc } ML + \text{arc } JK)$$

$$40^\circ = \frac{1}{2}(\text{arc } ML + 25^\circ)$$

$$80^\circ = \text{arc } ML + 25^\circ$$

$$\text{arc } ML = 55^\circ$$

■ 3. $\overline{SQ}$ is a chord and $\overline{TW}$ is a tangent line of $\odot P$. The measure of arc SRQ is 194°. Find $m\angle SQW$.



Solution:

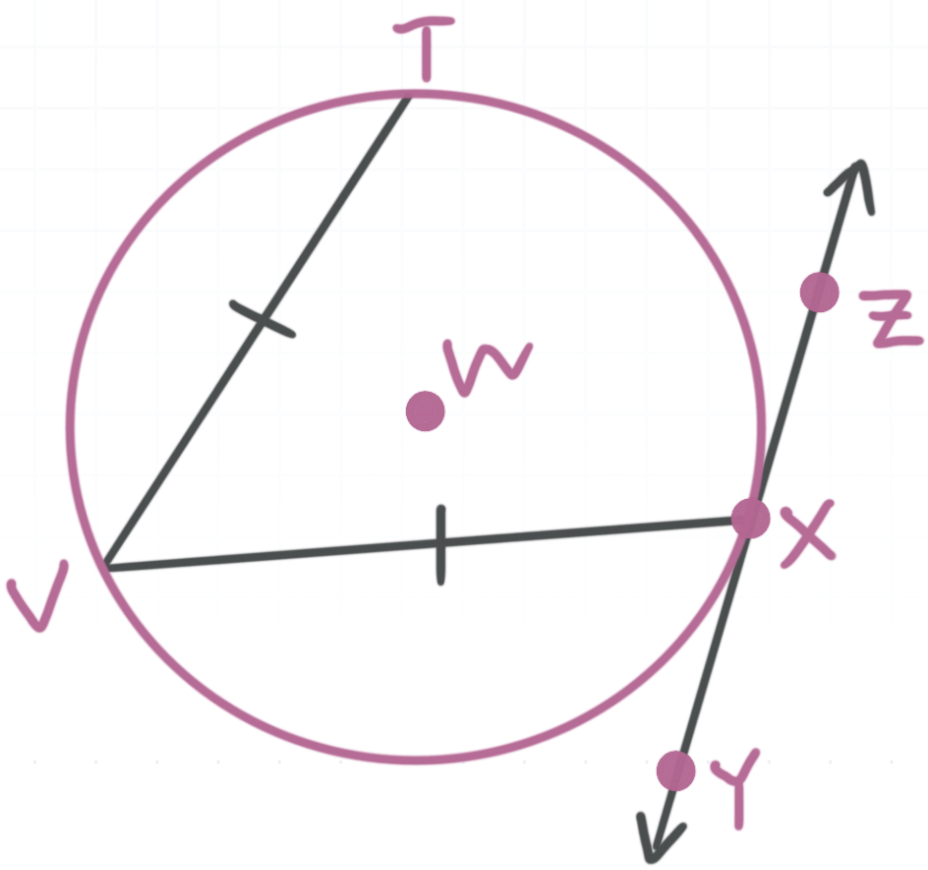
We can find $m\angle SQW$ as

$$m\angle SQW = \frac{1}{2}(\text{arc } SQ)$$

$$\text{arc } SQ = 360^\circ - \text{arc } SRQ = 360^\circ - 194^\circ = 166^\circ$$

$$m\angle SQW = \frac{1}{2}(166^\circ) = 83^\circ$$

■ 4. $\overline{TV}$ and $\overline{VX}$ are congruent chords, and $\overline{ZY}$ is a tangent line of $\odot W$. If $m\angle TVX = 48^\circ$, find $m\angle VXY$.



Solution:

Find the length of different arcs in the circle.

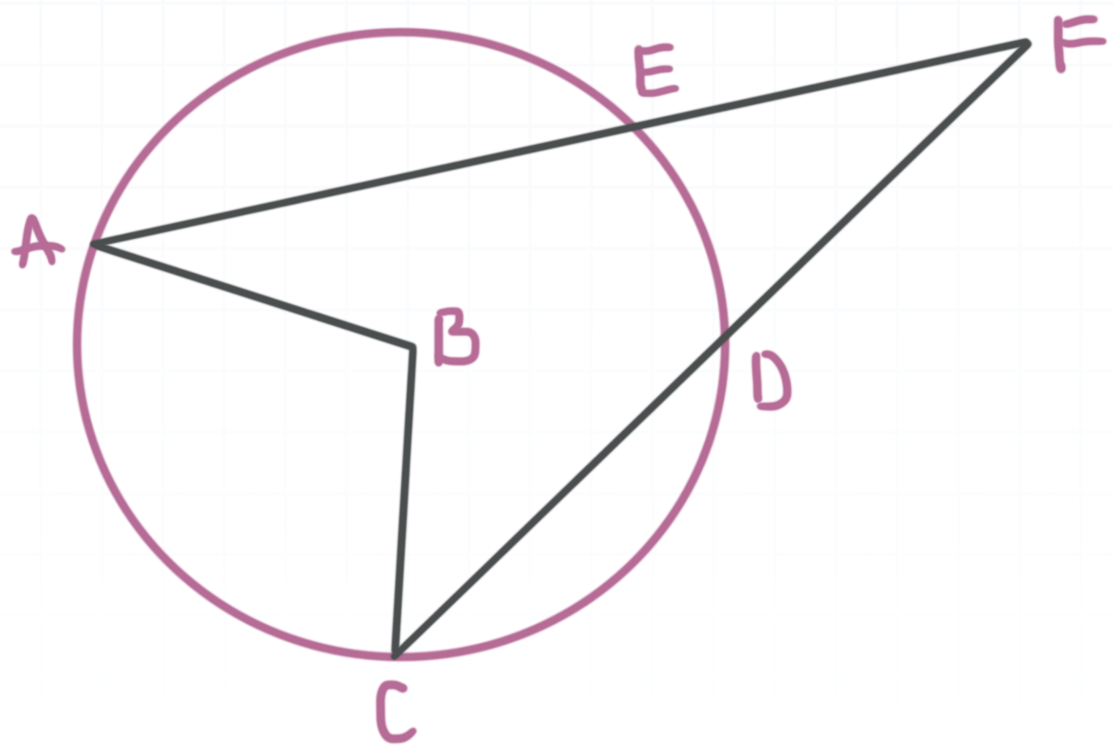
$$\text{arc } TX = 2(m\angle TVX) = 2(48) = 96$$

$$\text{arc } VX = \frac{1}{2}(360^\circ - \text{arc } TX) = \frac{1}{2}(360^\circ - 96^\circ) = 132^\circ$$

Then $m\angle VXY$ is given by

$$m\angle VXY = \frac{1}{2}(132^\circ) = 66^\circ$$

■ 5. arc $AC = 98^\circ$ and arc $ED = 54^\circ$ in $\odot B$. Find $m\angle AFC$.



Solution:

We can find $m\angle AFC$ as

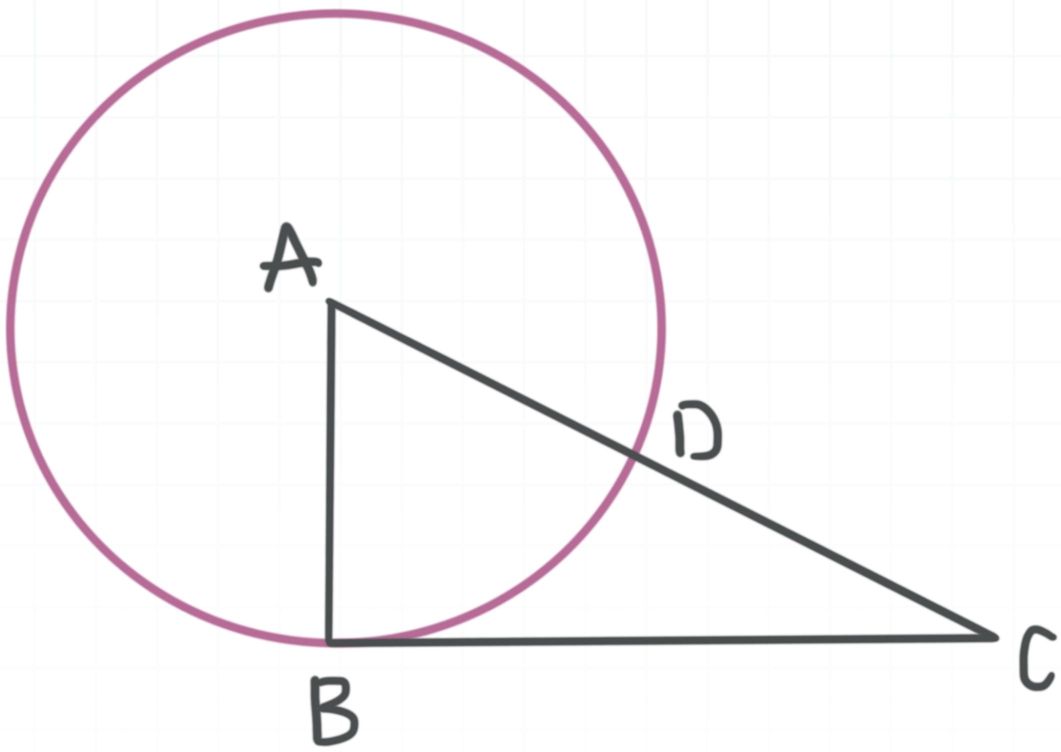
$$m\angle AFC = \frac{1}{2}(\text{arc } AC - \text{arc } ED)$$

$$m\angle AFC = \frac{1}{2}(98^\circ - 54^\circ)$$

$$m\angle AFC = 22^\circ$$

TANGENT LINES OF CIRCLES

■ 1. $\odot A$ has radius AB and tangent line $\overline{BC}$. If $AB = 6$ and $BC = 8$, find DC.



Solution:

We can plug some lengths from the triangle into the Pythagorean Theorem.

$$AB^2 + BC^2 = AC^2$$

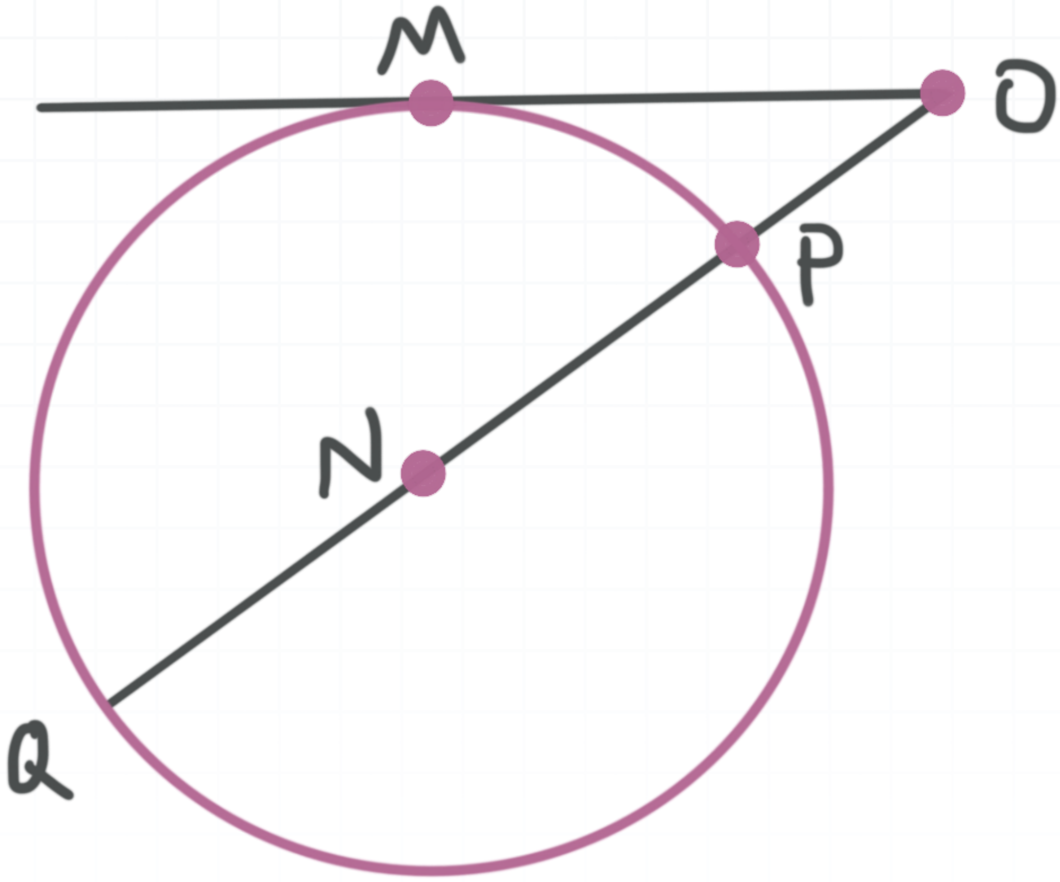
$$6^2 + 8^2 = AC^2$$

$$100 = AC^2$$

$$10 = AC$$

Then $DC = 10 - AD = 10 - 6 = 4$.

■ 2. $\overline{MO}$ is a tangent line of $\odot N$. If $MO = 12$ and $PO = 8$, find the length of the radius.



Solution:

Let the length of the radius be x. Then use the Pythagorean Theorem.

$$MN^2 + MO^2 = ON^2$$

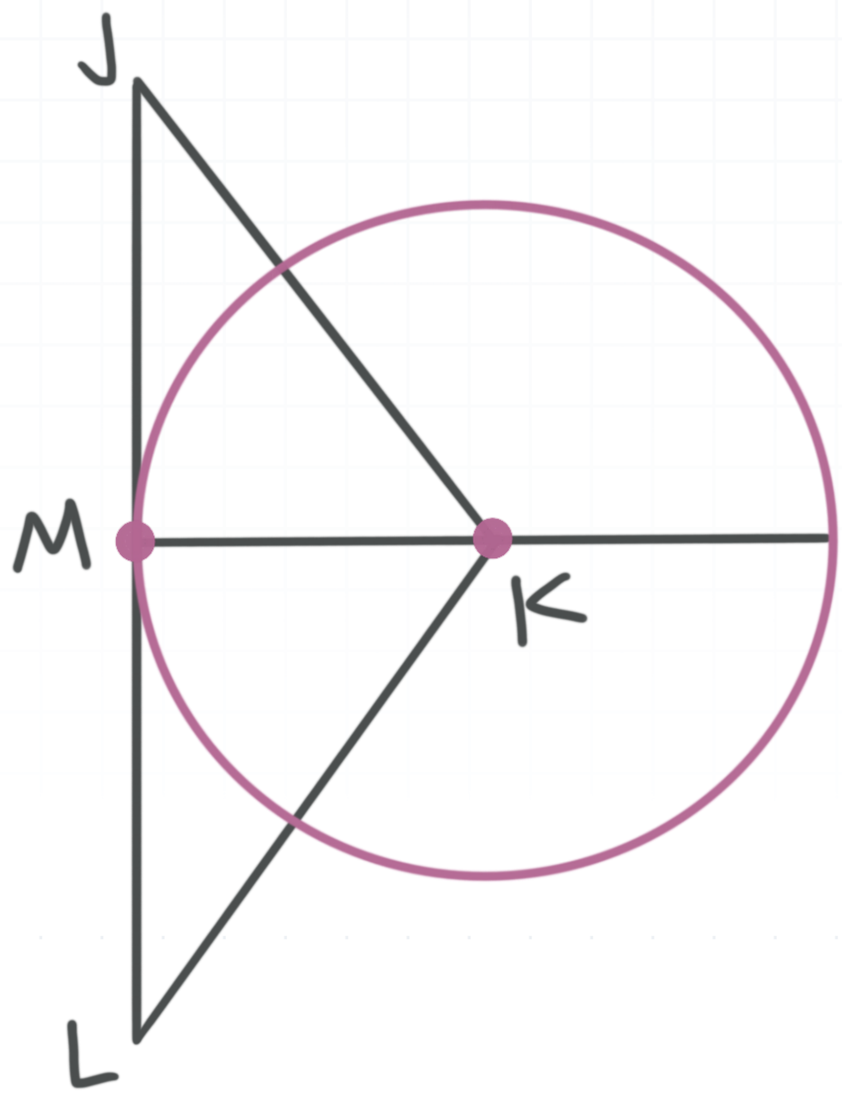
$$x^2 + 12^2 = (x + 8)^2$$

$$x^2 + 144 = x^2 + 16x + 64$$

$$16x = 80$$

$x = 5$

■ 3. $\triangle JKL$ is isosceles, $\overline{JL}$ is a tangent line, $JM = LM$ and $m\angle JKL = 120^\circ$. If $MK = 8$, find JL.



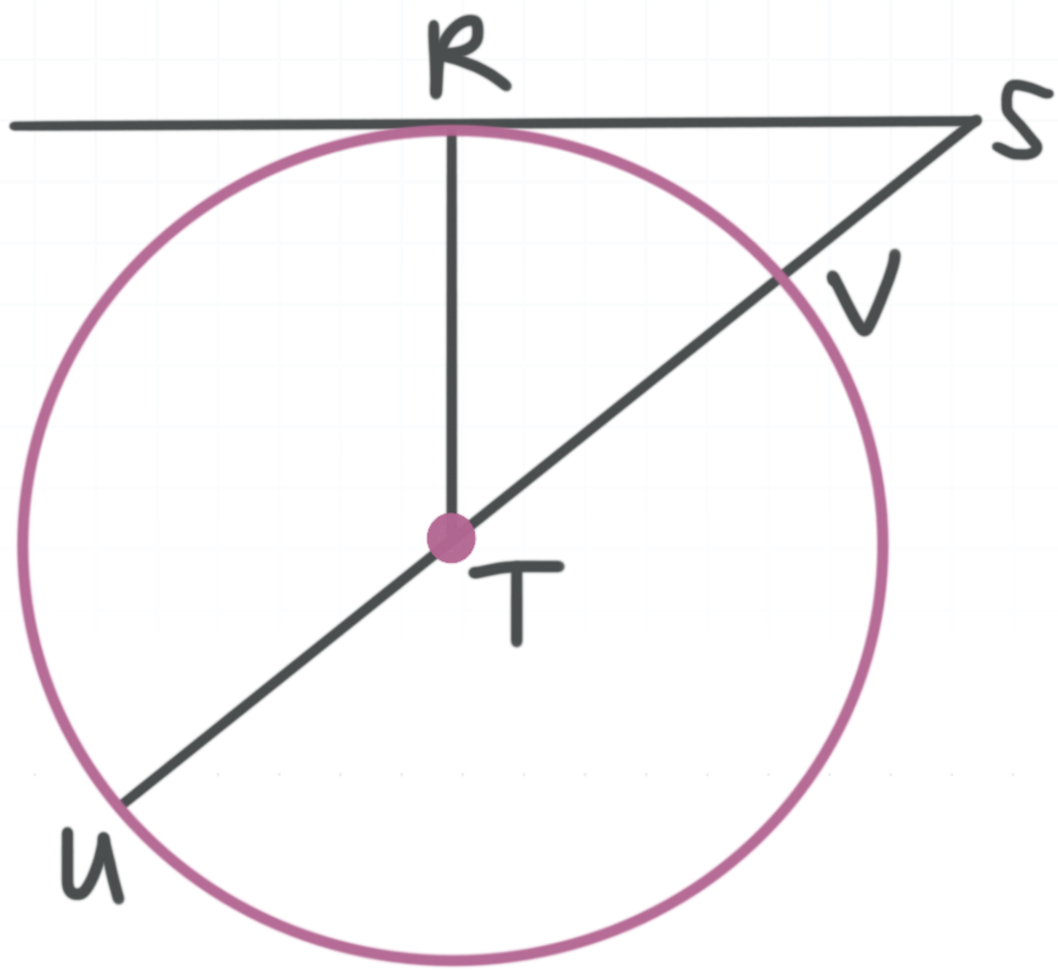
Solution:

The triangles $\triangle JMK$ and $\triangle LMK$ are congruent and $\overline{MK}$ bisects $\angle JKL$. We know $m\angle JKL = 120^\circ$, so $m\angle JKM = m\angle LKM = 60^\circ$. $m\angle KMJ$ must be 90° because $\overline{MK}$ is a radius and $\overline{JL}$ is a tangent line. Use the rules for $30-60-90$ triangles to find JM.

$$JM = MK\sqrt{3} = 8\sqrt{3}$$

$$JL = 2JM = 2\left(8\sqrt{3}\right) = 16\sqrt{3}$$

■ 4. In $\odot T$, $\overline{RS}$ is a tangent line and the diameter $\overline{UV}$ has length of 6. Find VS if $RS = 4$.



Solution:

We know that $TV = RT = 3$ because they are both radii and are half the length of the diameter. From the Pythagorean Theorem,

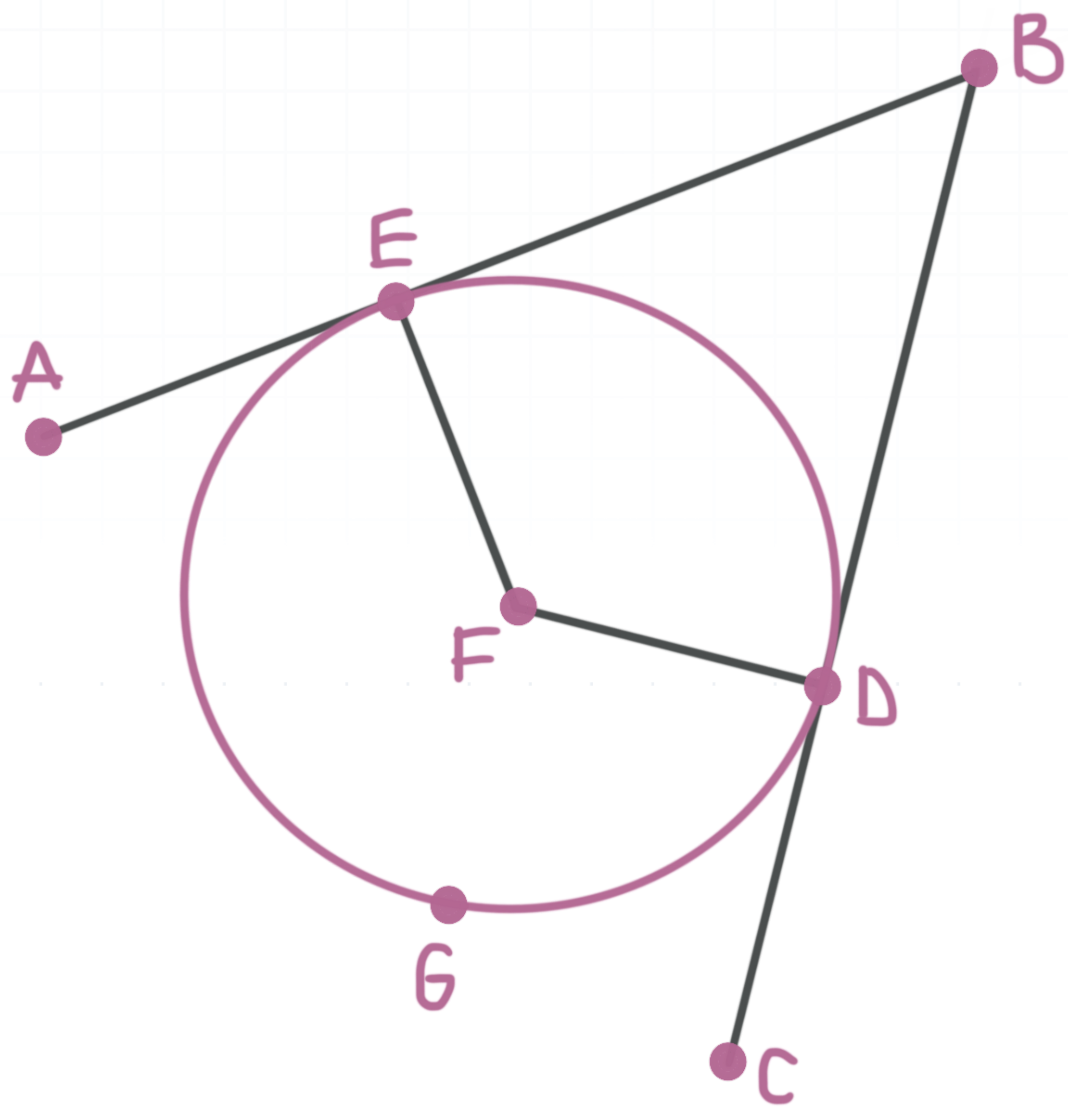
$$RT^2 + RS^2 = TS^2$$

$$3^2 + 4^2 = TS^2$$

$$TS = 5$$

Which means the length of VS is $VS = TS - TV = 5 - 3 = 2$.

■ 5. arc $EGD = 240^\circ$ and $\overline{BF}$ bisects $\angle EFD$. Find the length of the radius of $\odot F$ if $FB = 14$.

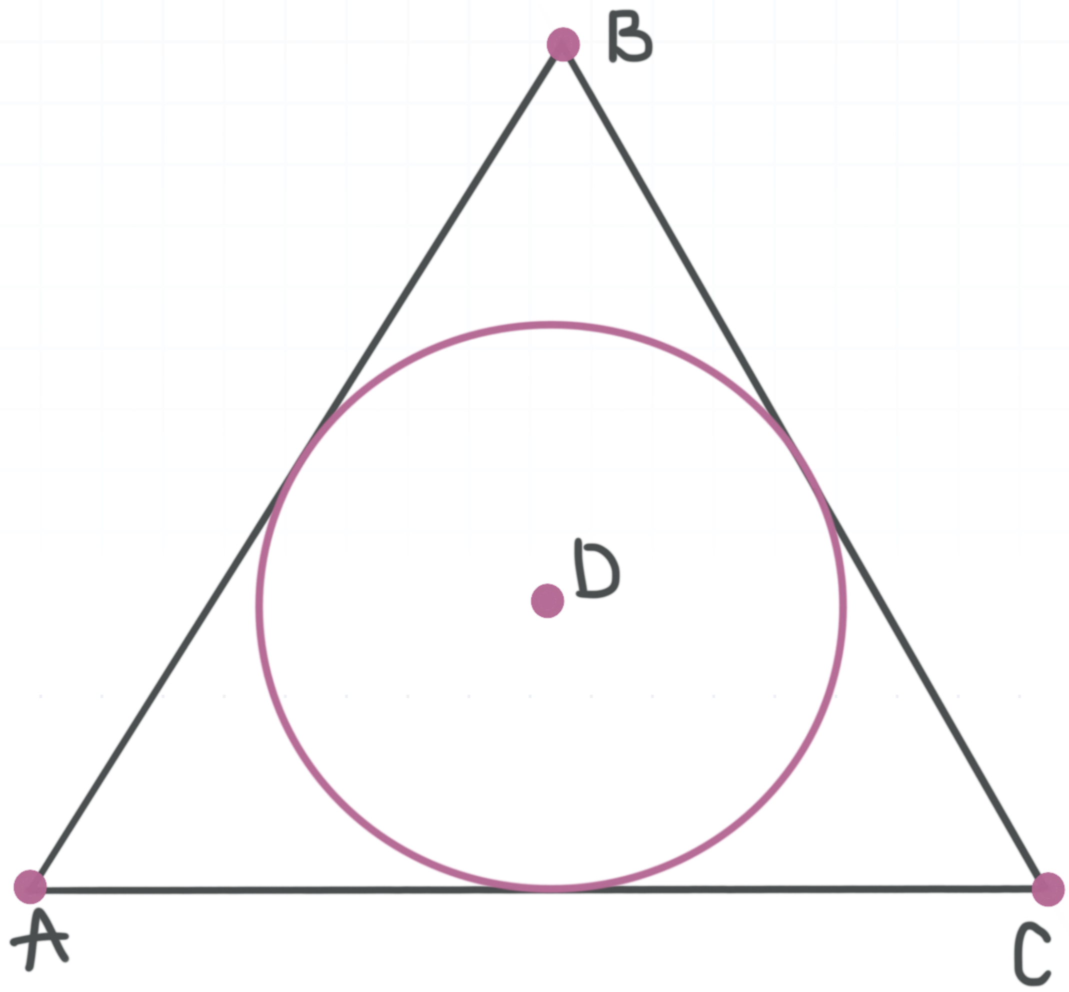


Solution:

We know that $m\angle EFD = 360^\circ - 240^\circ = 120^\circ$.

Draw in $\overline{FB}$ and note $\triangle FEB$ and $\triangle DFB$ are congruent $30-60-90$ triangles. $\overline{FB}$ is a hypotenuse of these triangles and has a measure of 14. $\overline{EF}$ and $\overline{FD}$ are the shortest legs of these triangles and have a measure of $14/2=7$.

■ 6. Find the perimeter of $\triangle ABC$ if the radius of $\odot D$ is 10 feet and $\triangle ABC$ is equilateral.



Solution:

We know that $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are congruent since $\triangle ABC$ is equilateral. Which means $\angle A=\angle B=\angle C=60^\circ$.

Since the diameter is 10, the length of half of each side must be $10\sqrt{3}$ if you draw segments at $\overline{AD}$, $\overline{DB}$, and $\overline{DC}$. Each side must have a length of

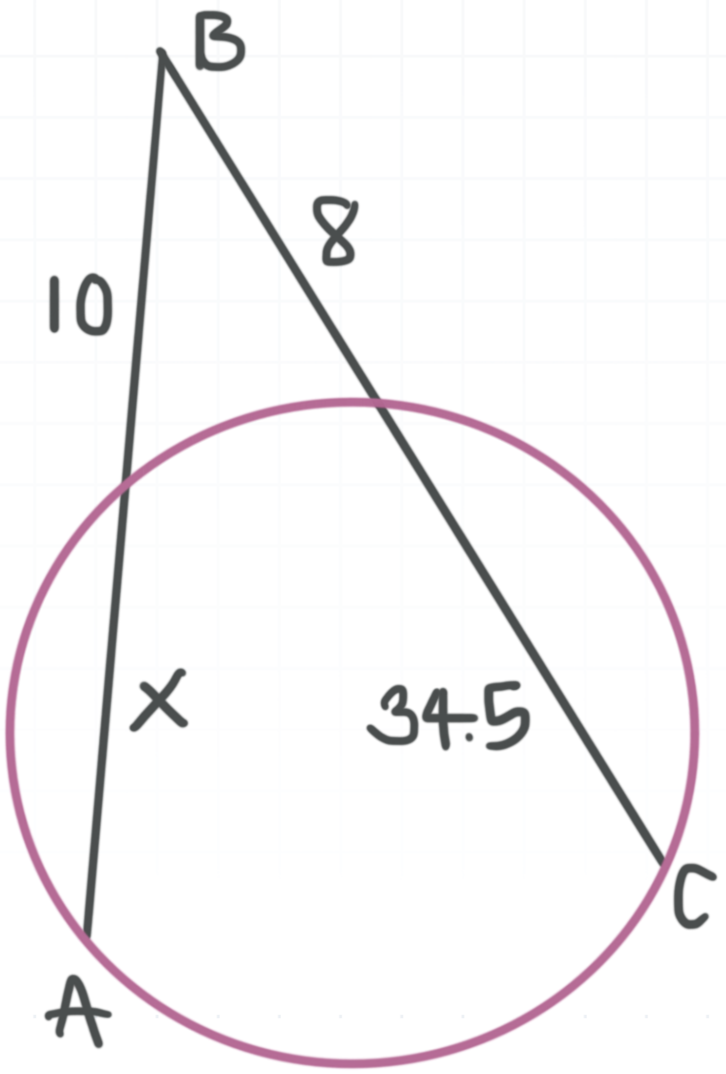
$$2\left(10\sqrt{3}\right)=20\sqrt{3}$$

Find the perimeter by adding up the three sides to get

$$P=20\sqrt{3}+20\sqrt{3}+20\sqrt{3}=60\sqrt{3}$$

INTERSECTING TANGENTS AND SECANTS

■ 1. $\overline{AB}$ and $\overline{CB}$ are secants and intersect at B. Find the value of x.



Solution:

From the figure, we know that

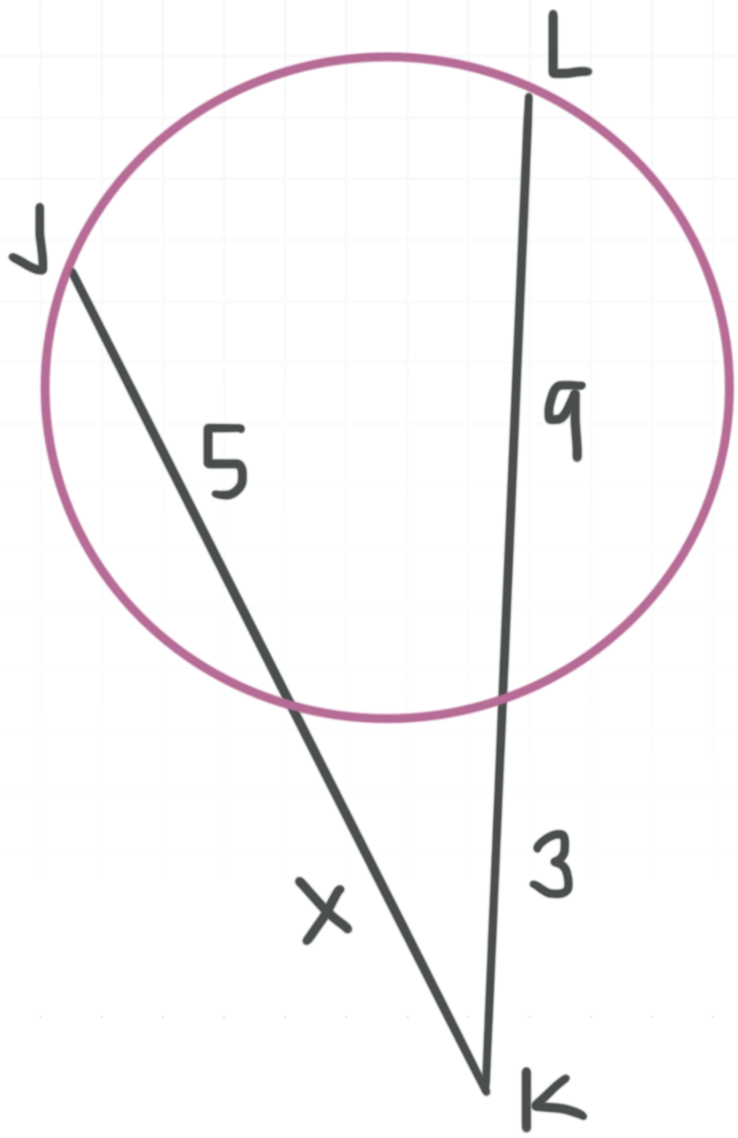
$$10(10 + x) = 8(8 + 34.5)$$

$$100 + 10x = 340$$

$$10x = 240$$

$$x = 24$$

■ 2. $\overline{JK}$ and $\overline{LK}$ are secants and intersect at K. Find the value of x.



Solution:

From the figure, we know that

$$x(x + 5) = 3(3 + 9)$$

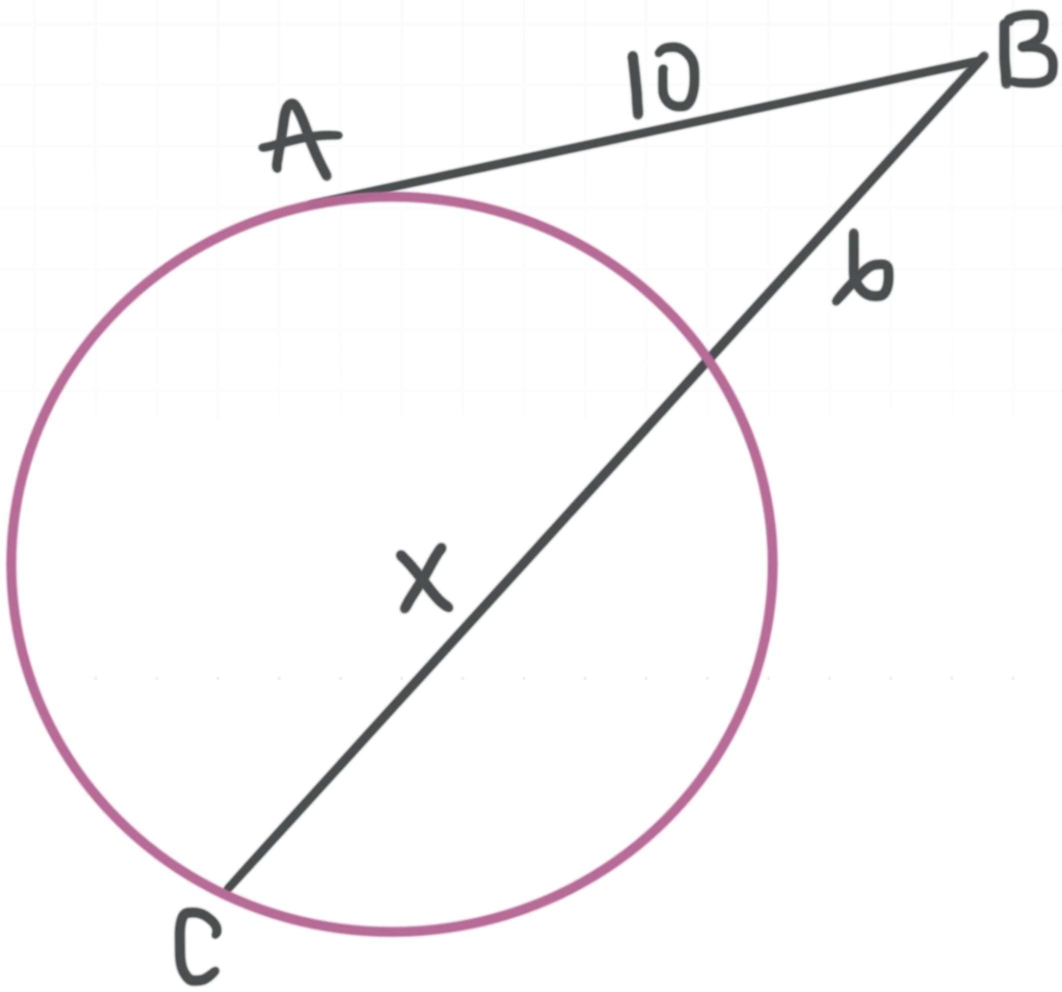
$$x^2 + 5x = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x + 9)(x - 4) = 0$$

Either $x = -9$ or $x = 4$, but x represents a physical distance, which means it can't have a negative value, so $x = 4$.

■ 3. $\overline{AB}$ is a tangent line and $\overline{BC}$ is a secant of the circle. Find the value of x.



Solution:

From the figure, we know that

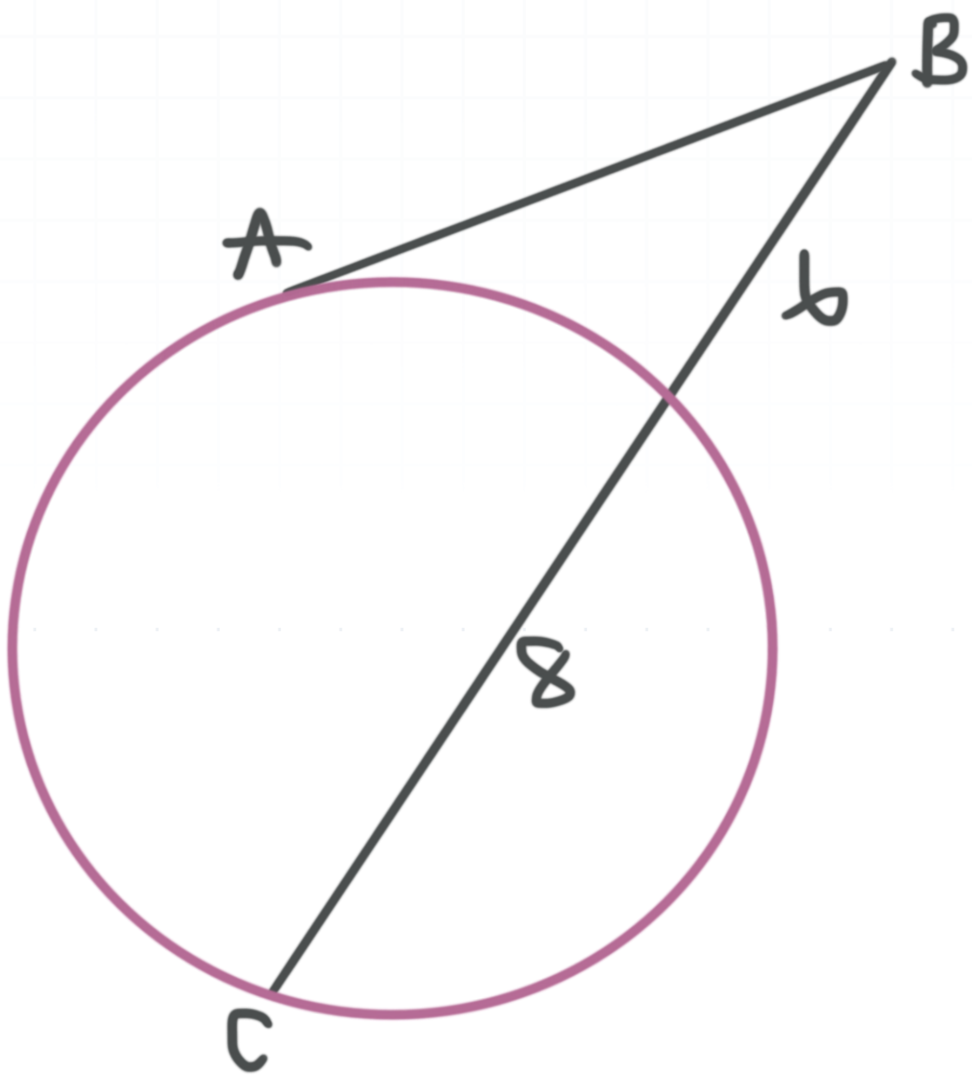
$$10^2 = 6(6 + x)$$

$$100 = 36 + 6x$$

$$6x = 64$$

$$x = \frac{32}{3}$$

■ 4. $\overline{AB}$ is a tangent line and $\overline{CB}$ is a secant of the circle. Find the length of AB.



Solution:

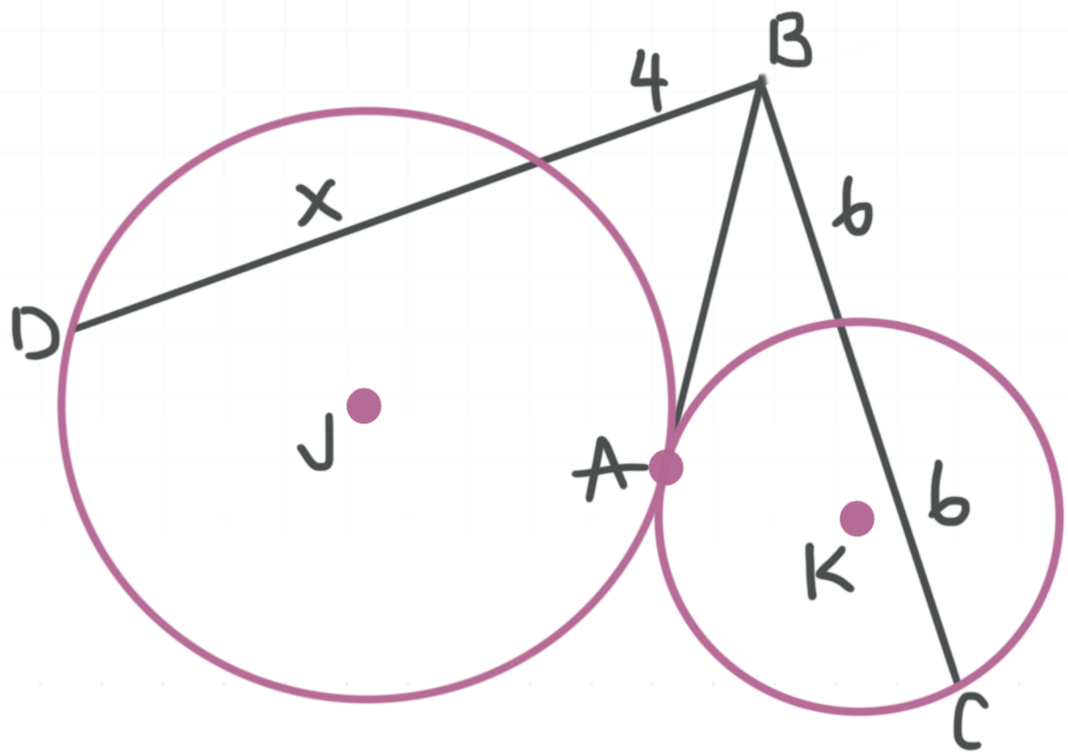
From the figure, the length of AB is

$$x^2 = 6(6+8)$$

$$x^2 = 84$$

$$x = \sqrt{84} = 2\sqrt{21}$$

■ 5. $\overline{DB}$ is a secant of $\odot J$ and $\overline{CB}$ is a secant of $\odot K$. $\overline{AB}$ is a tangent for both circles. Find x.



Solution:

From the figure, we know that

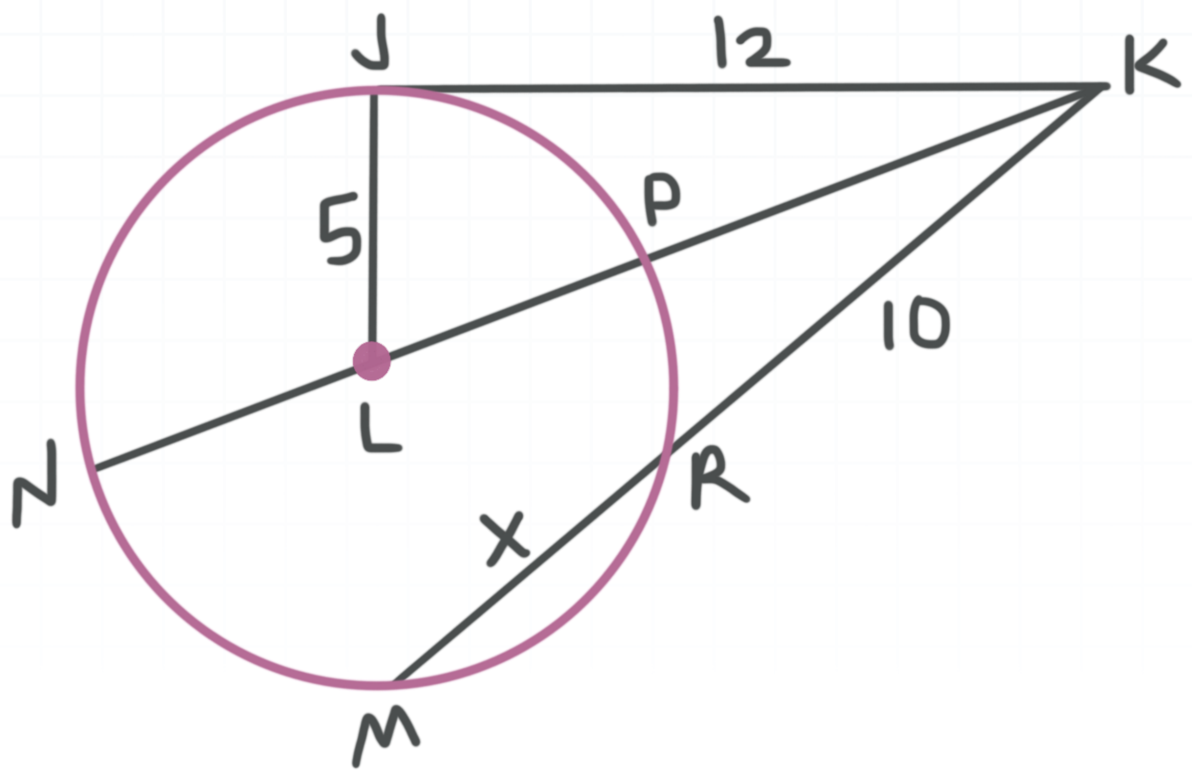
$$AB^2 = 6(6+6) = 72$$

$$AB^2 = 4(4+x)$$

$$72 = 4(4 + x)$$

$$x = 14$$

■ 6. $\overline{JK}$ is a tangent line, $\overline{KN}$ and $\overline{KM}$ are secants, and $\overline{LJ}$ and $\overline{LP}$ are radii of $\odot L$. Find x.



Solution:

We can use the Pythagorean Theorem.

$$LK^2 = JK^2 + LJ^2$$

$$LK^2 = 12^2 + 5^2 = 169$$

$$LK = 13$$

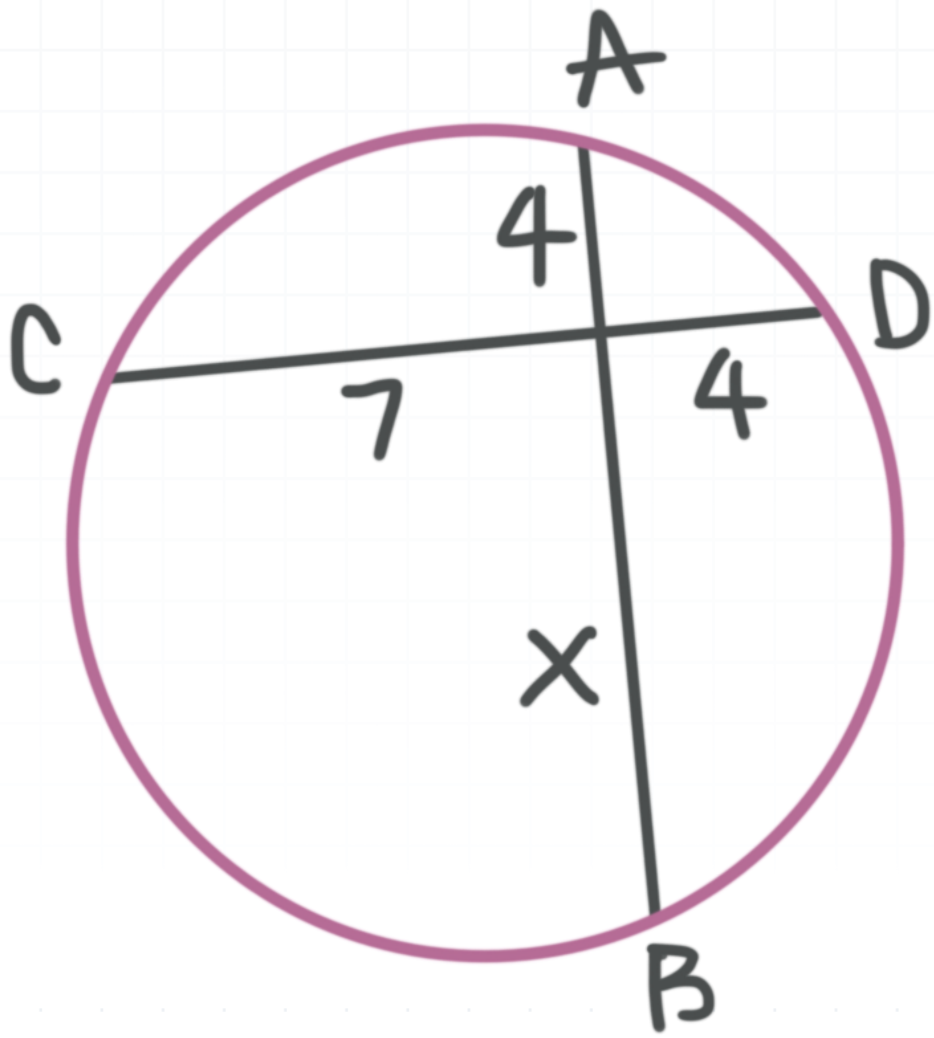
Therefore, $PK = 8$ and $NP = 10$. Which means that we can set up an equation to solve for x.

$$8(8 + 10) = 10(10 + x)$$

$$x = 4.4$$

INTERSECTING CHORDS

■ 1. $\overline{AB}$ and $\overline{CD}$ are intersecting chords of the circle. Find x.



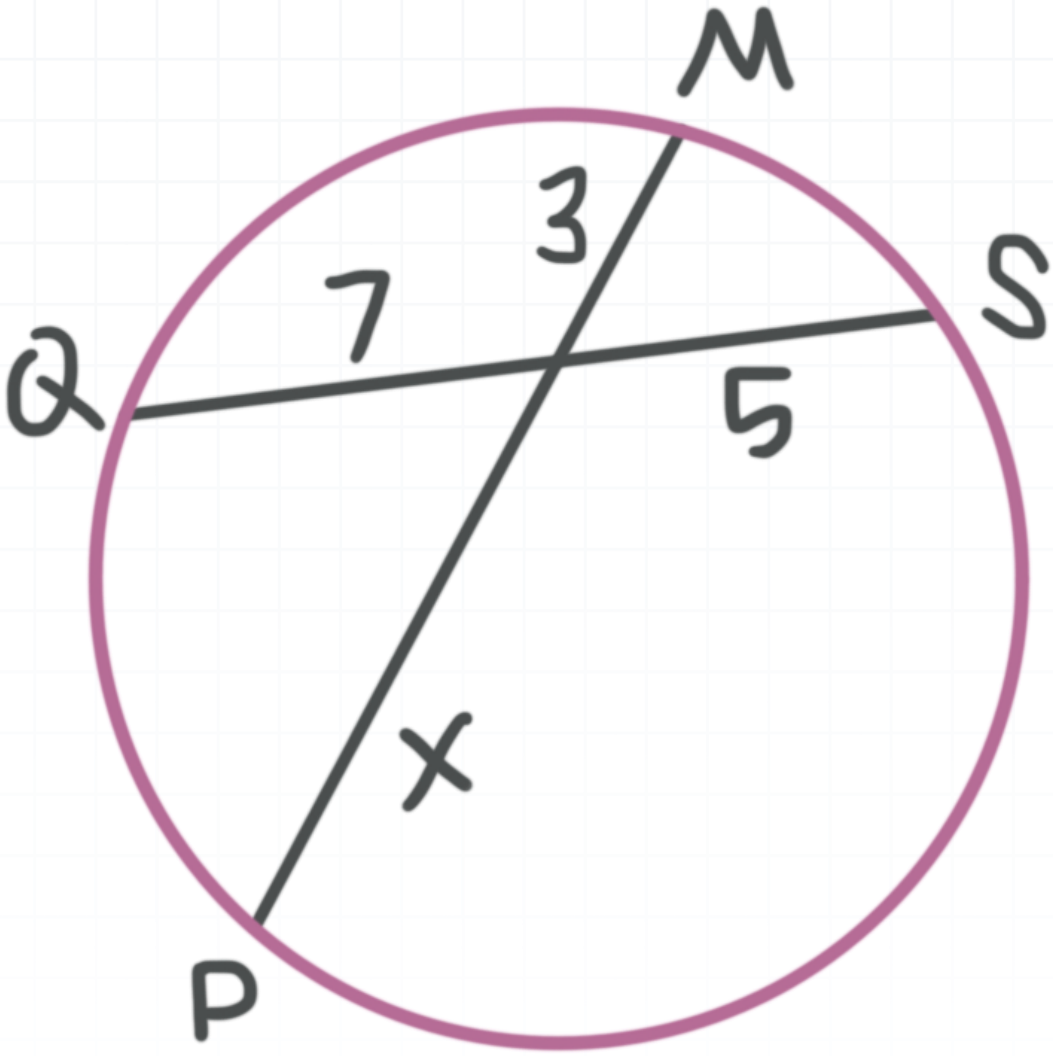
Solution:

From the figure, we know that

$$4x = 7(4)$$

$$x = 7$$

■ 2. $\overline{MP}$ and $\overline{QS}$ are intersecting chords of the circle. Find x.



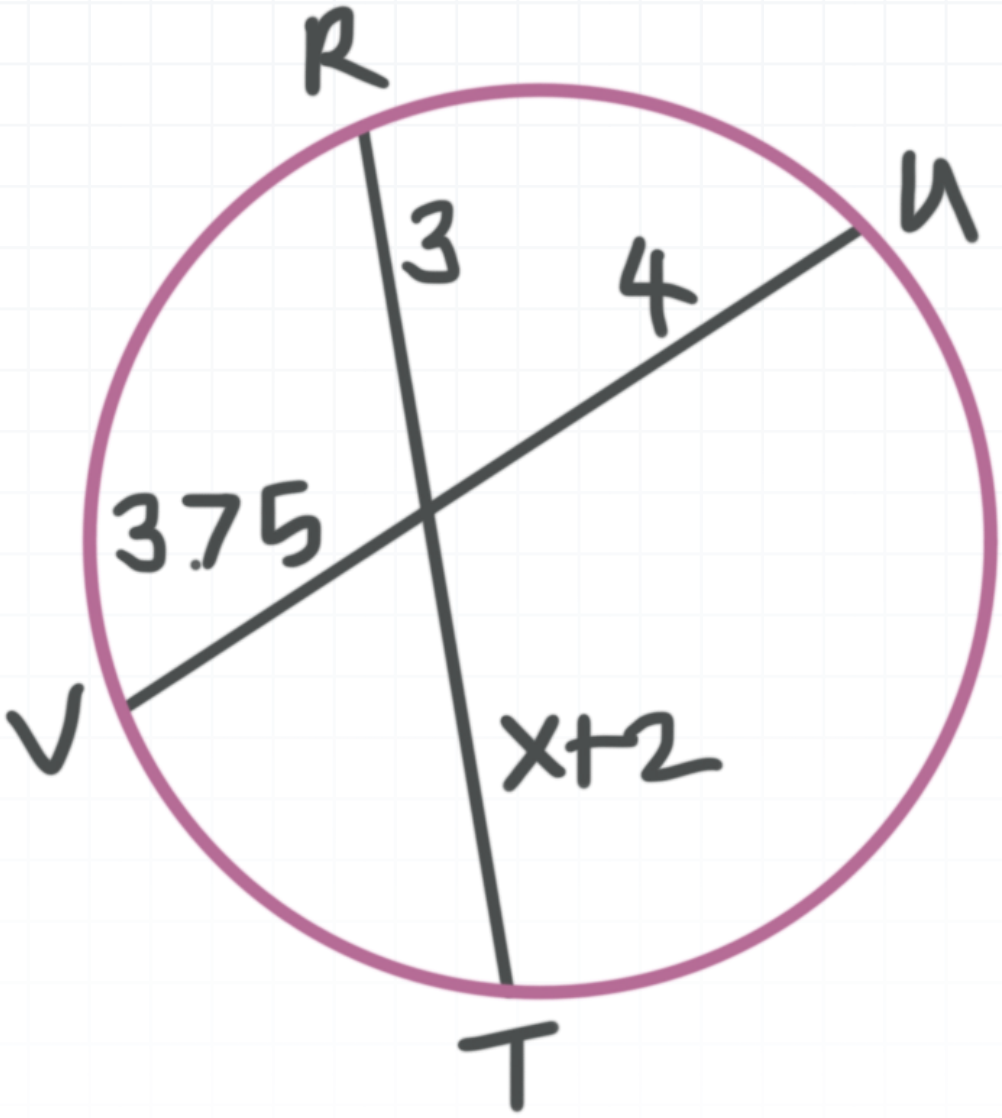
Solution:

From the figure, we know that

$$3x = (5)(7)$$

$$x = \frac{35}{3}$$

■ 3. $\overline{RT}$ and $\overline{UV}$ are intersecting chords of the circle. Find x.



Solution:

From the figure, we can say

$$x(x+2) = (3.75)(4)$$

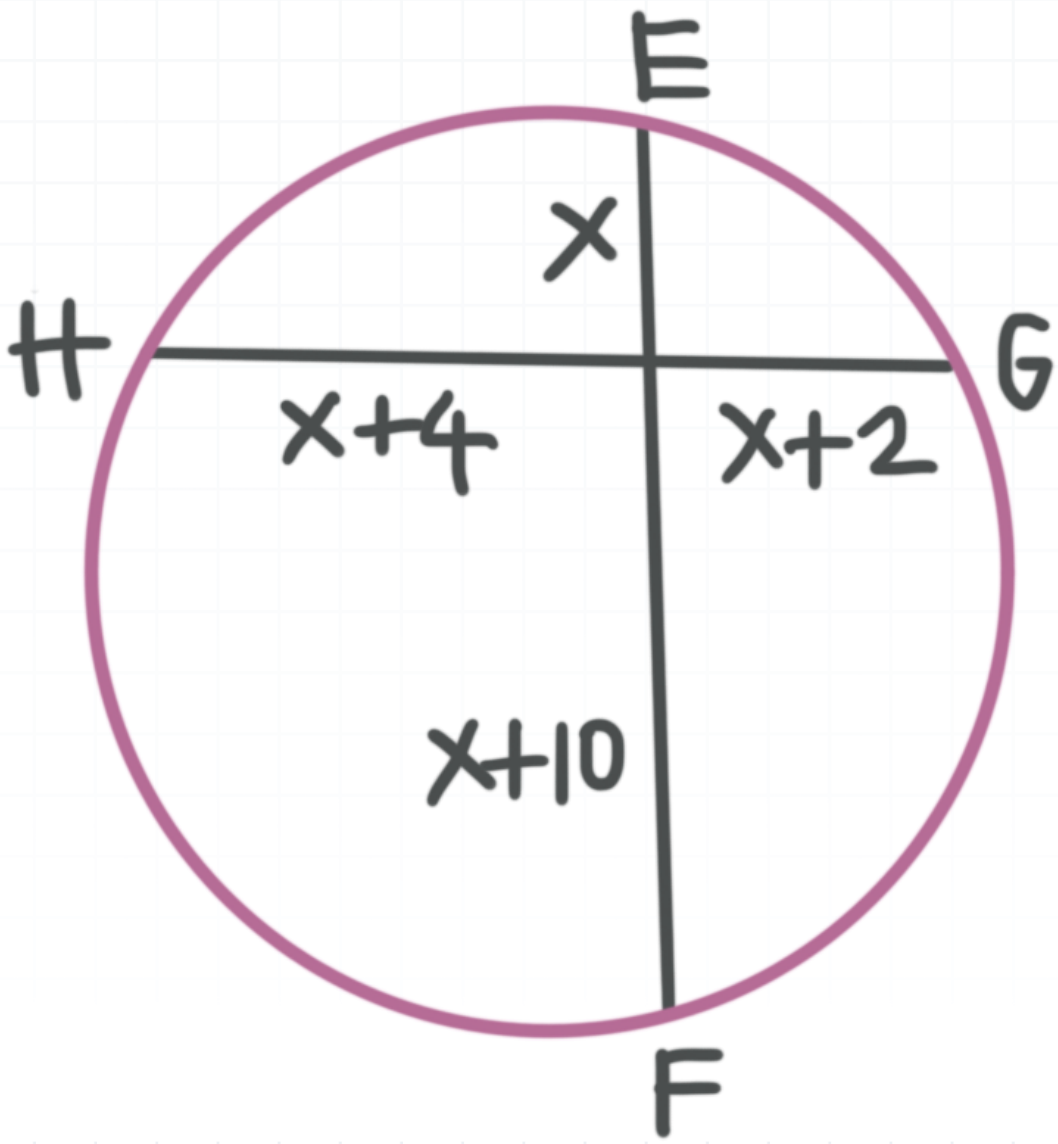
$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$x = 3$$

■ 4. $\overline{EF}$ and $\overline{HG}$ are intersecting chords of the circle. Find x.



Solution:

From the figure, we know that

$$x(x+10) = (x+2)(x+4)$$

$$x^2 + 10x = x^2 + 6x + 8$$

$$x = 2$$

INTERIOR ANGLES OF TRIANGLES

■ 1. $\triangle LMN$ is a right, isosceles triangle where $\angle M$ is the vertex angle. Find $m\angle L$, $m\angle M$, and $m\angle N$.

Solution:

$m\angle L = 45$, $m\angle M = 90$, and $m\angle N = 45$. If M is the vertex angle, it's where the legs of the isosceles triangle intersect. This must be our 90° angle. Because it's isosceles, the two base angles must be congruent. They must both be 45°.

■ 2. $\triangle ABC$ has $m\angle A = 3x + 5$, $m\angle B = 10x + 5$, and $m\angle C = 4x$. Find the value of x and determine whether this is an obtuse, acute, or right triangle.

Solution:

$x = 10$ such that $m\angle A = 35$, $m\angle B = 105$, and $m\angle C = 40$. $\triangle ABC$ is an obtuse triangle because it has one obtuse angle.

$$m\angle A + m\angle B + m\angle C = 180$$

$$3x + 5 + 10x + 5 + 4x = 180$$

$$17x + 10 = 180$$

$$17x = 170$$

$$x = 10$$

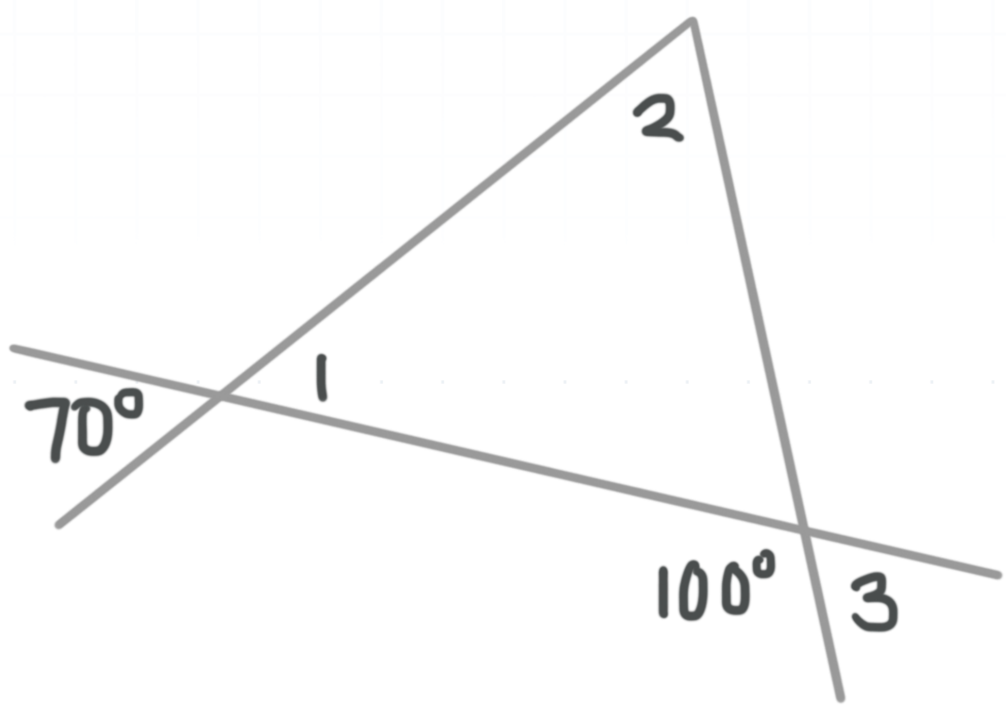
Substitute x to get the actual angle measures.

$$m\angle A = 3x + 5 = 3(10) + 5 = 30 + 5 = 35^\circ$$

$$m\angle B = 10x + 5 = 10(10) + 5 = 100 + 5 = 105^\circ$$

$$m\angle C = 4x = 4(10) = 40^\circ$$

■ 3. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ from the figure.



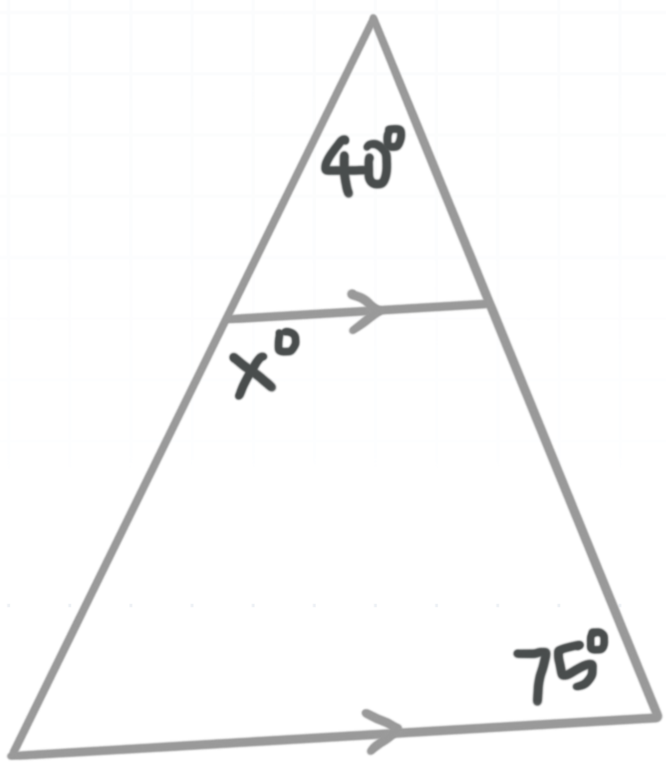
Solution:

$m\angle 1 = 70$, $m\angle 2 = 30$, and $m\angle 80$. $m\angle 1 = 70$ because vertical angles are formed. $m\angle 3 = 80$ because a linear pair is formed. The sum of the angles in the triangle is 180, so

$$m\angle 2 = 180 - 70 - 80$$

$$m\angle 2 = 30$$

4. Find the value of x from the figure.



Solution:

$x = 115$. The larger triangle and the smaller triangle share the measure 40°. The smaller triangle has a sum of angles of 180° and we know the corresponding angles in the figure are congruent, making one of the angles of this small triangle 75°. So the third angle in the small triangle is

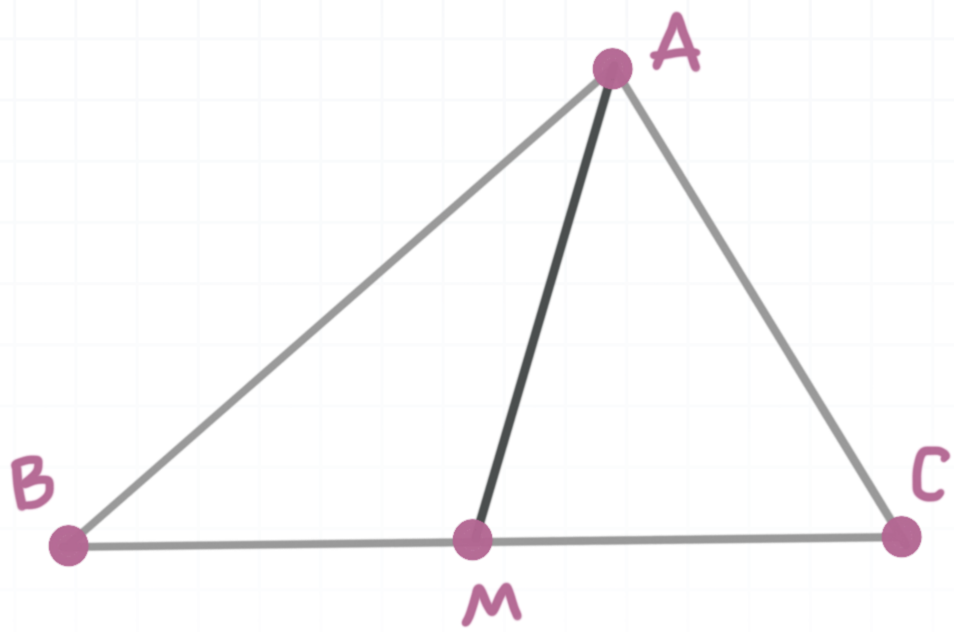
$$180 - 40 - 75 = 65^\circ$$

This angle and x form a linear pair, which means

$$x = 180 = 65 = 115^\circ$$

PERPENDICULAR AND ANGLE BISECTORS

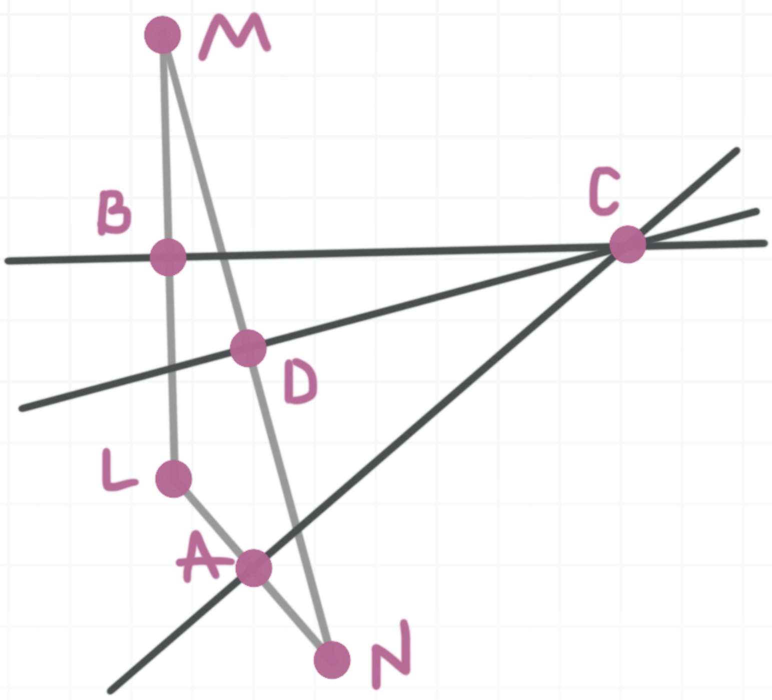
1. $\overline{AM}$ is an angle bisector of $\triangle ABC$. $m\angle BMA = 108$ and $m\angle MBA = 40$. Find x if $m\angle CAM = 2x + 12$.



Solution:

$x = 10$. $\overline{AM}$ is an angle bisector, therefore $m\angle BAM = m\angle CAM$. The interior angles of a triangle always sum to 180°. We can find $m\angle BAM = 32$ and because $m\angle BAM = m\angle CAM$, $2x + 12 = 32$ and $x = 10$.

2. $\overline{AC}$, $\overline{DC}$, and $\overline{BC}$ are perpendicular bisectors of $\triangle NLM$. Give the special name for C and find the length of ND if $NM = 14x - 22$ and $DM = 3x + 1$.



Solution:

C is called a circumcenter. If $\overline{DC}$ is a perpendicular bisector of $\overline{NM}$, then $ND = DM$, and $ND + DM = NM$.

$$3x + 1 + 3x + 1 = 14x - 22$$

$$6x + 2 = 14x - 22$$

$$x = 3$$

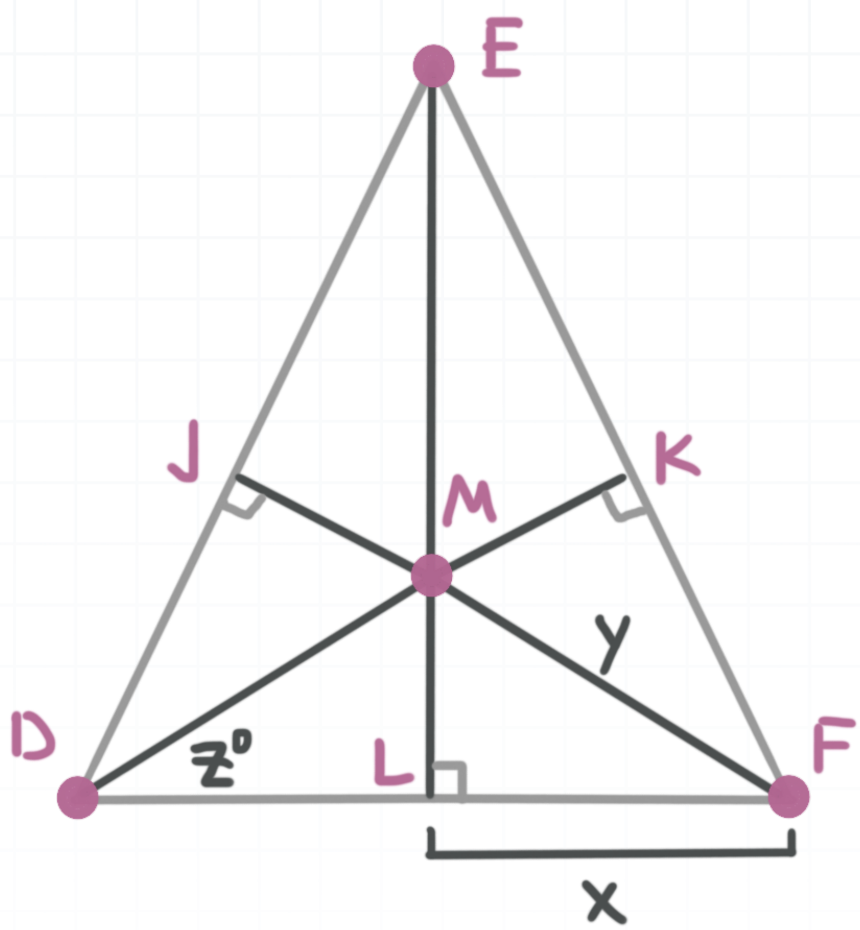
Substitute $x = 3$ and get

$$ND = 3x + 1$$

$$ND = 3(3) + 1$$

$$ND = 10$$

■ 3. Find the values of x, y, and z, given M is an incenter, $MK = 6$, $FK = 8$, and $m\angle EDF = 80$.



Solution:

$x = 8$, $y = 10$, and $z = 40$. $FL = FK$ because $\triangle LFM \cong \triangle KFM$. So $x = FK = FL = 8$ and $KM = LM = 6$. Using the Pythagorean theorem,

$$FK^2 + KM^2 = FM^2$$

$$8^2 + 6^2 = y^2$$

$$100 = y^2$$

$$10 = y$$

Because M is an incenter, we know that DM is a perpendicular bisector of $\angle EDF$. And because we were told that $m\angle EDF = 80$, we can say

$$z = \frac{80^\circ}{2} = 40^\circ$$

■ 4. $\triangle ABC$ has coordinates $A(-3,1)$, $B(3,3)$, and $C(2,-2)$. Write the equation for the perpendicular bisector of $\overline{AB}$.

Solution:

$y = -3x + 2$. The slope of $\overline{AB}$ is

$$m = \frac{3-1}{3-(-3)} = \frac{1}{3}$$

The midpoint of $\overline{AB}$ is

$$\left(\frac{-3+3}{2}, \frac{1+3}{2}\right) = \left(\frac{0}{2}, \frac{4}{2}\right) = (0,2)$$

The perpendicular bisector of $\overline{AB}$ passes through $(0,2)$ and has a slope of -3. The equation for the line must be $y = -3x + 2$.

CIRCUMSCRIBED AND INSCRIBED CIRCLES OF A TRIANGLE

■ 1. Equilateral triangle ABC is inscribed in $\odot D$. Find $m\angle ADC$.

Solution:

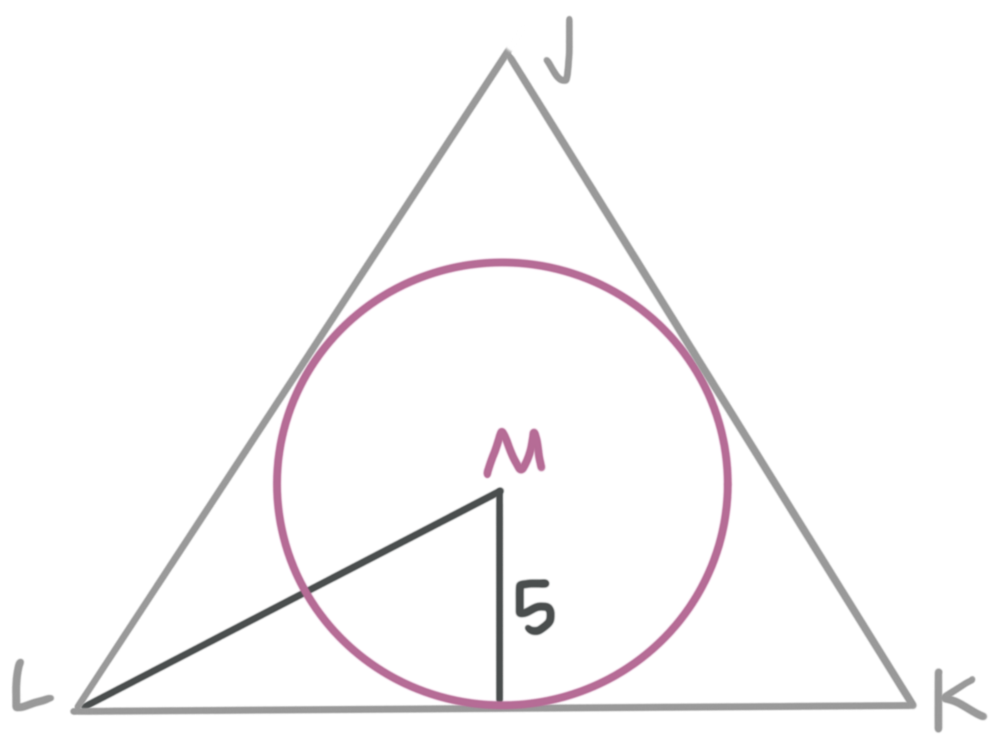
120. $m\angle ABC = 60$ because the triangle is equilateral.

$$m\angle ADC = 2(m\angle ABC)$$

$$m\angle ADC = 2(60)$$

$$m\angle ADC = 120$$

■ 2. $\triangle JKL$ is equilateral and is circumscribed about $\odot M$. The radius of $\odot M$ is 5. Find the perimeter of $\triangle JKL$.

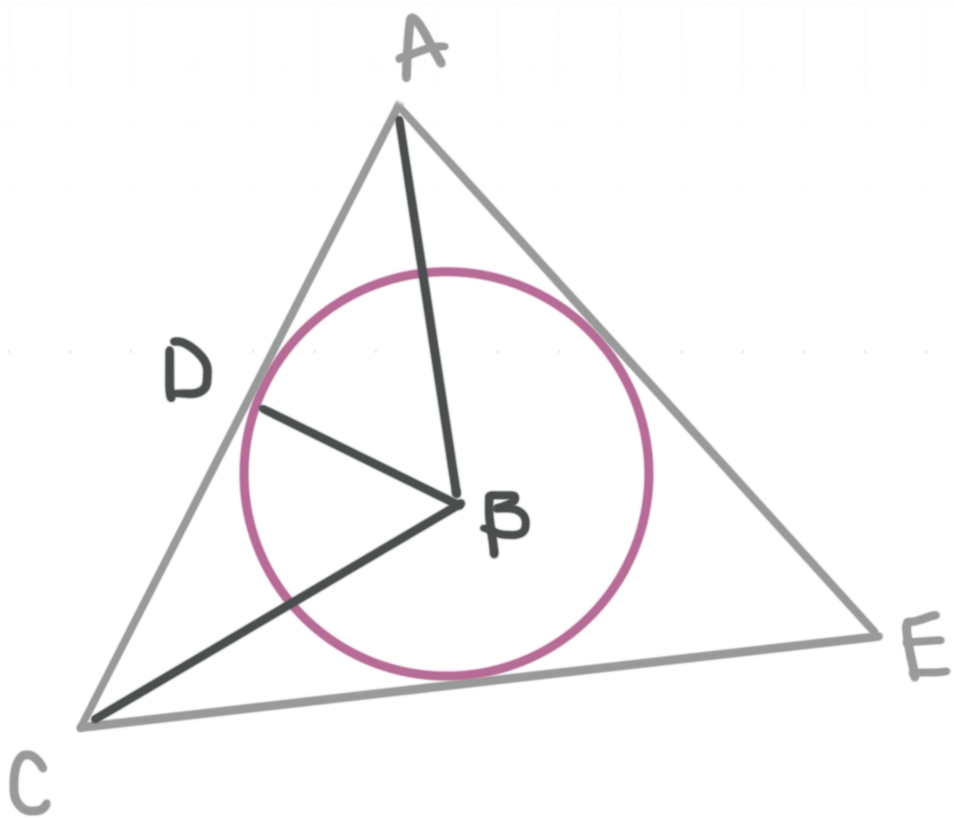


Solution:

$30\sqrt{3}$. The angles of the triangle are 60. Draw a line segment from M to L which bisects $\angle L$. Use the $30-60-90$ rule to find half the length of one of the sides of the triangle to be $5\sqrt{3}$. Double this to find the length of one side and get $10\sqrt{3}$. The perimeter is

$$3\left(10\sqrt{3}\right)=30\sqrt{3}$$

3. If $AB=12$, find the length of the radius of $\odot B$.

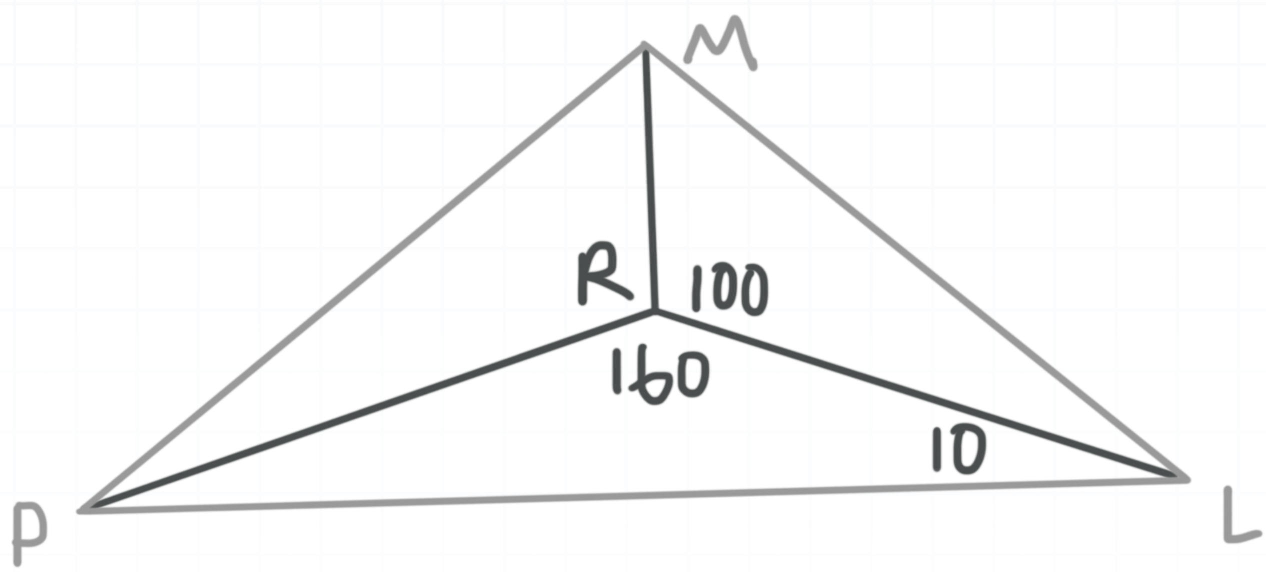


Solution:

$\triangle ADC$ is an equilateral triangle, and using the $30-60-90$ rules we know

$$AD = \frac{1}{2}(12) = 6$$

■ 4. R is the incenter of $\triangle PML$. Find $m\angle PMR$.



Solution:

70. $\overline{RP}$, $\overline{RM}$, and $\overline{RL}$ bisect $\angle P$, $\angle M$, and $\angle L$ respectively. Use the Triangle Sum Theorem to find that $m\angle PMR = 70$.

MEASURES OF QUADRILATERALS

■ 1. A rectangle has a width of 6 inches and diagonal with length 10 inches. Find the perimeter of the rectangle.

Solution:

28 inches. Rectangles have four right angles. When a diagonal is drawn in the rectangle, a right triangle is formed. The Pythagorean Theorem is used to find the missing side length of the triangle.

$$a^2 + b^2 = c^2$$

$$6^2 + b^2 = 10^2$$

$$b^2 = 100 - 36 = 64$$

$$b = 8$$

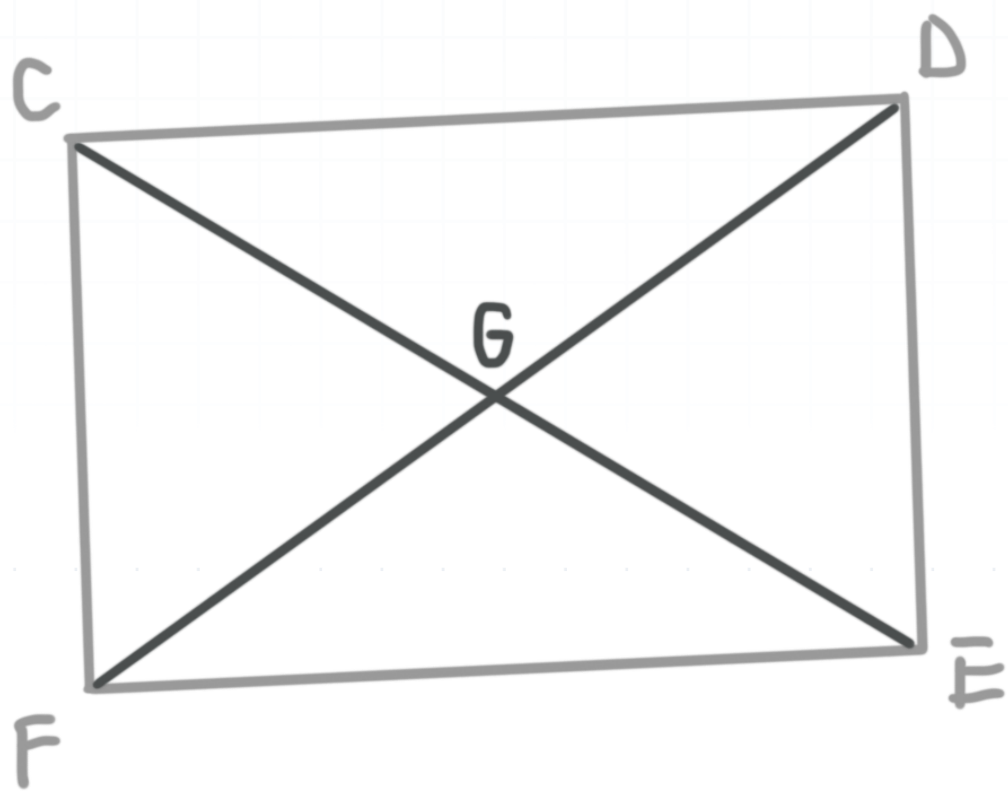
The missing length of the rectangle is 8 inches. Then the perimeter is the sum of the side lengths: $2(6) + 2(8) = 28$ inches.

■ 2. Classify quadrilateral $ABCD$ with vertices at $A(1,-3)$, $B(5,0)$, $C(10,0)$, and $D(6,-3)$.

Solution:

$ABCD$ is a parallelogram and a rhombus with $AB = BC = CD = DA = 5$ and $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$. A parallelogram must have two sets of parallel line segments by definition, and a rhombus is a special type of parallelogram having four congruent sides.

■ 3. $CDEF$ is a rectangle with diagonals intersecting at G. $CG = 2x + 1$, $DG = x + 4$, $FG = 4y - 1$, and $EG = y + 5$. Find FD.



Solution:

$FD = 14$. The diagonals of a rectangle are congruent and they bisect each other. So $CE = FD$ and $CG = EG = FG = DG$. By substitution,

$$2x + 1 = x + 4$$

$$x = 3$$

In addition,

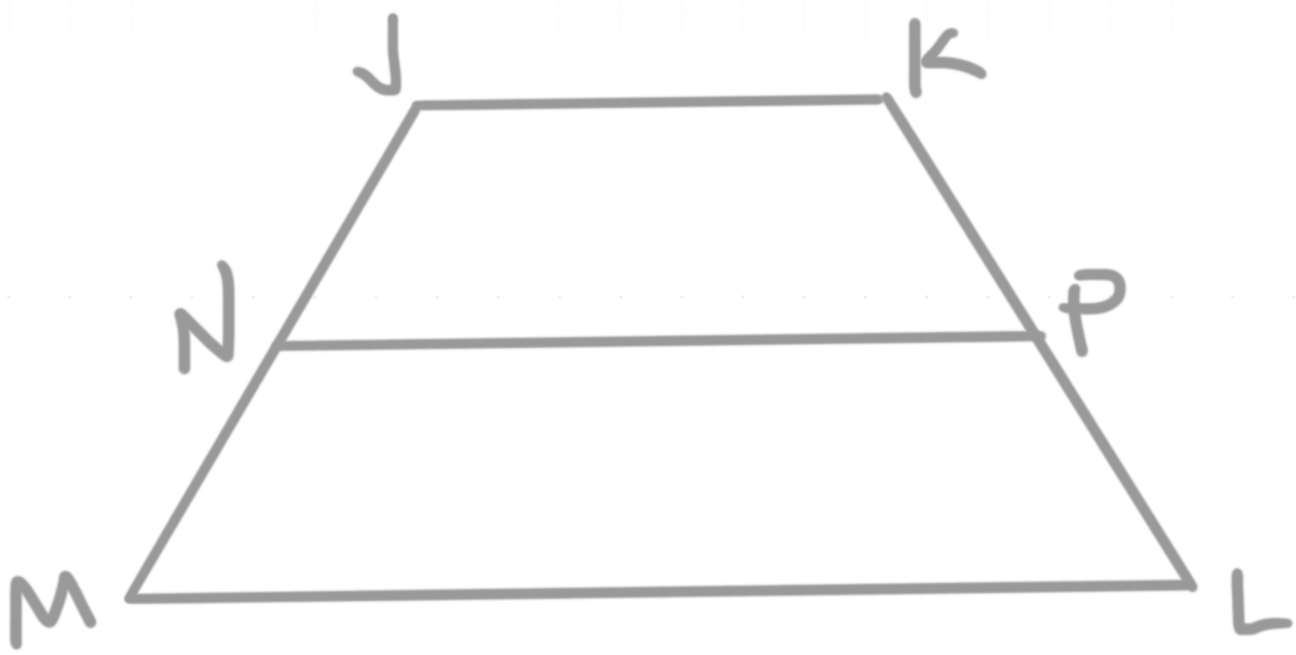
$$4y - 1 = y + 5$$

$$3y = 6$$

$$y = 2$$

So $CG = EG = FG = DG = 7$, and therefore $FD = 14$.

■ 4. $JKLM$ is an isosceles trapezoid with median $\overline{NP}$. $MJ = 14$, $m\angle MLP = 72$, $NP = 16$, and $ML = 20$. Find KP, $m\angle MJK$, and JK.



Solution:

$KP = 7$, $m\angle MJK = 108$, and $JK = 12$. Isosceles triangles have congruent legs so $MJ = LK = 14$. The median bisects these legs, making $KP = 7$. The base angles $\angle M$ and $\angle L$ are both 72, and $m\angle M + m\angle J = 180$, so $m\angle MJK = 108$. The median is half the sum of the bases, so

$$NP = \frac{1}{2}(JK + ML)$$

$$16 = \frac{1}{2}(JK + 20)$$

$$32 = JK + 20$$

$$JK = 12$$

MEASURES OF PARALLELOGRAMS

■ 1. $ABCD$ is a parallelogram with $m\angle A = 2x + 10$, $m\angle B = y - 5$, and $\angle C = 100$. Find the values of x and y.

Solution:

$x = 45$ and $y = 85$. We know that $m\angle A = m\angle C$ and $m\angle B = m\angle D$ because the opposite angles of a parallelogram are congruent. $m\angle C + m\angle D = 180$ because consecutive angles of a parallelogram are supplementary. This gives the following equations.

$$2x + 10 = 100$$

$$x = 45$$

and

$$y - 5 = 80$$

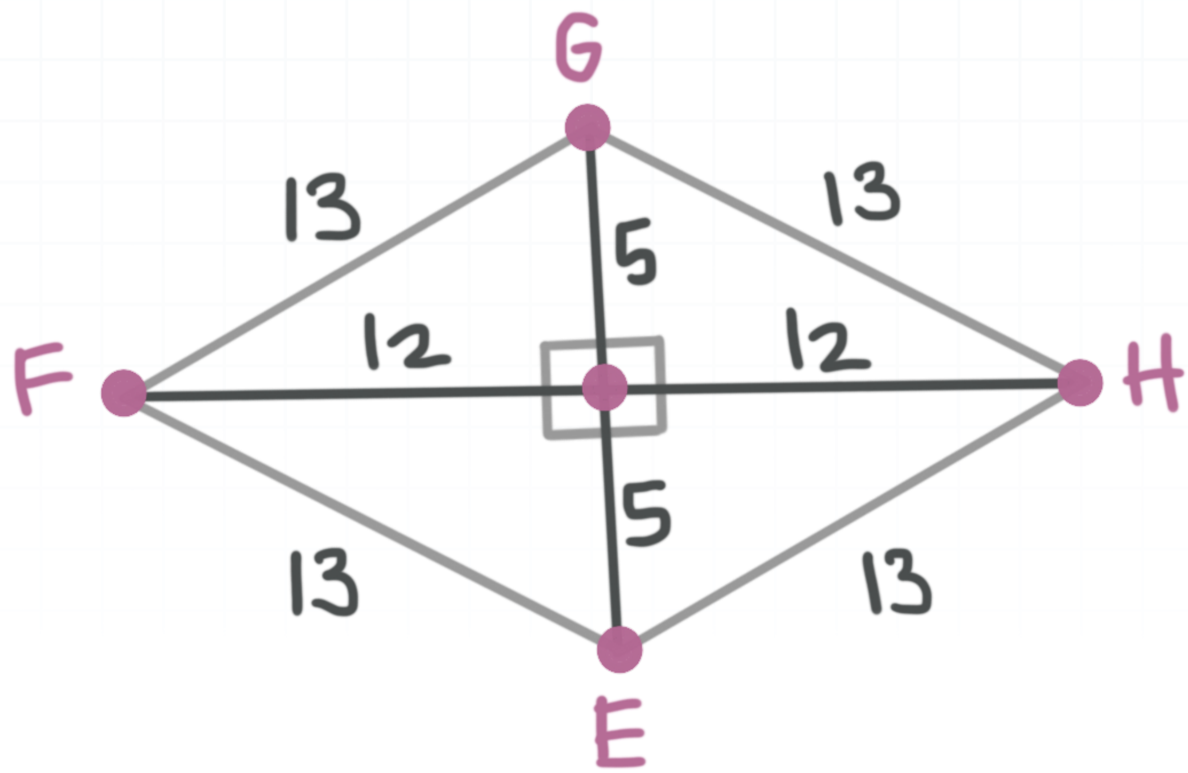
$$y = 85$$

■ 2. $EFGH$ is a rhombus with $FH = 24$ and $GE = 10$. Find the perimeter of $EFGH$.

Solution:

The perimeter is 52. In a rhombus, the diagonals bisect each other and are also perpendicular. Four congruent right triangles are formed, and the Pythagorean Theorem can be used to find the length of the hypotenuse of each triangle.

$$12^2 + 5^2 = 13^2$$



This gives side length of 13. The perimeter is then found by finding the sum of the side lengths of the rhombus.

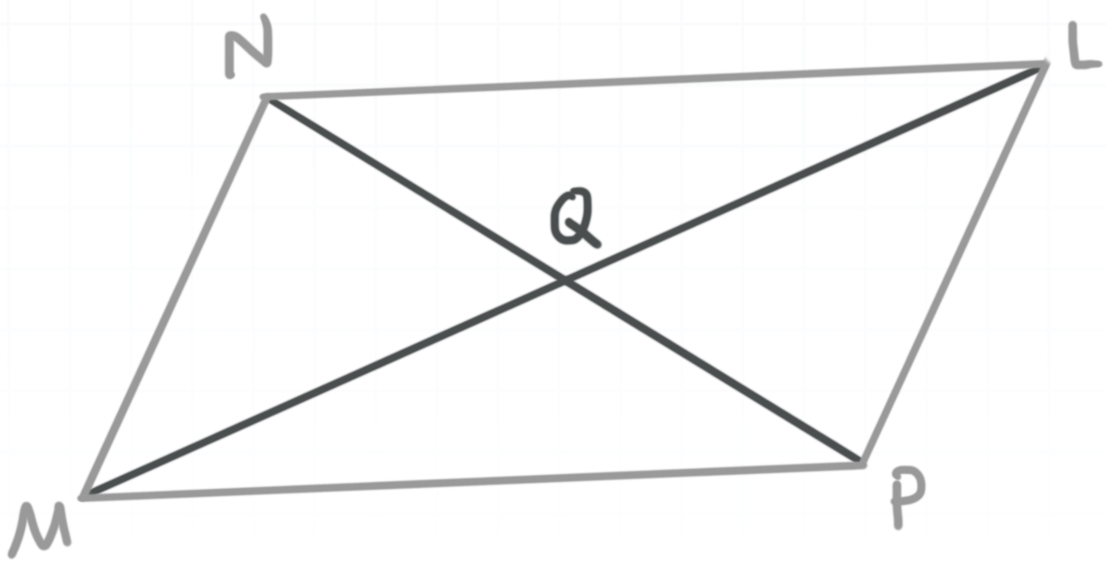
$$13 + 13 + 13 + 13 = 52$$

■ 3. $JKLM$ has vertices $J(-3,2)$, $K(3,0)$, $L(3,-6)$, and $M(-3,-4)$. Determine whether $JKLM$ is a parallelogram by checking if it has two sets up opposite sides that are congruent.

Solution:

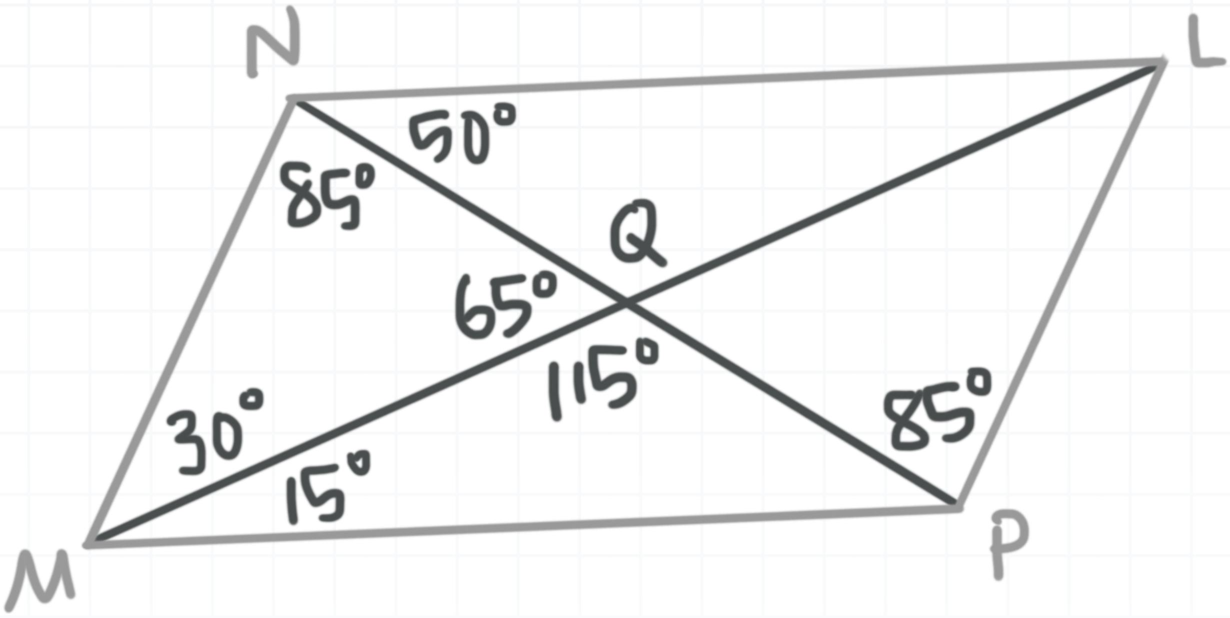
$JKLM$ is a parallelogram because $JM = KL = 6$ and $JK = ML = 2\sqrt{10}$.

■ 4. $NLPM$ is a parallelogram with diagonals intersecting at point Q. $m\angle MNP = 85$, $m\angle MQP = 115$, and $m\angle MNL = 135$. Find $m\angle PML$.



Solution:

$m\angle PML = 15$. Consecutive interior angles are supplementary in a parallelogram. Triangles are also formed and we can find missing angle measures by using the Triangle Sum Theorem, vertical angles, and linear pairs within our diagram.


N
L
Q
M
P
50°
85°
65°
115°
30°
15°
85°

AREA OF A RECTANGLE

■ 1. The base of a rectangle is 8 feet. Find its height if the area of the rectangle is 80 ft^2.

Solution:

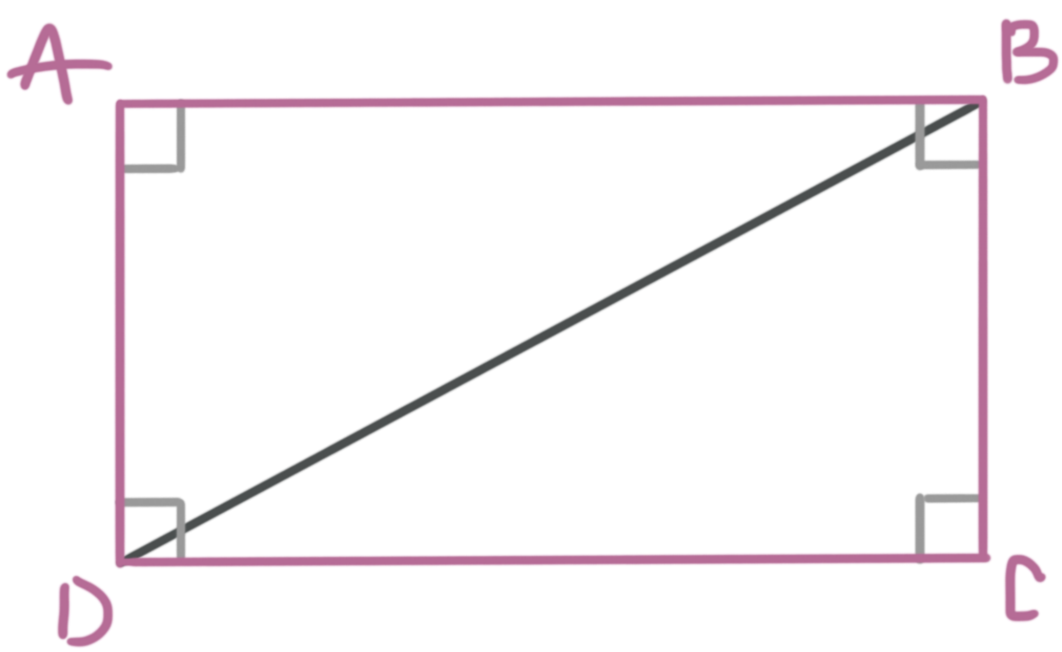
We can use the formula for the area of the rectangle to plug in everything we know, and then solve for the height.

$$A = bh$$

$$80 = 8h$$

$$h = 10 \text{ feet}$$

■ 2. In rectangle $ABCD$, $BD = 13$ and $AB = 12$. Find the area of this rectangle.



Solution:

Use the Pythagorean Theorem to find the length of $\overline{AD}$.

$$AD^2 + AB^2 = BD^2$$

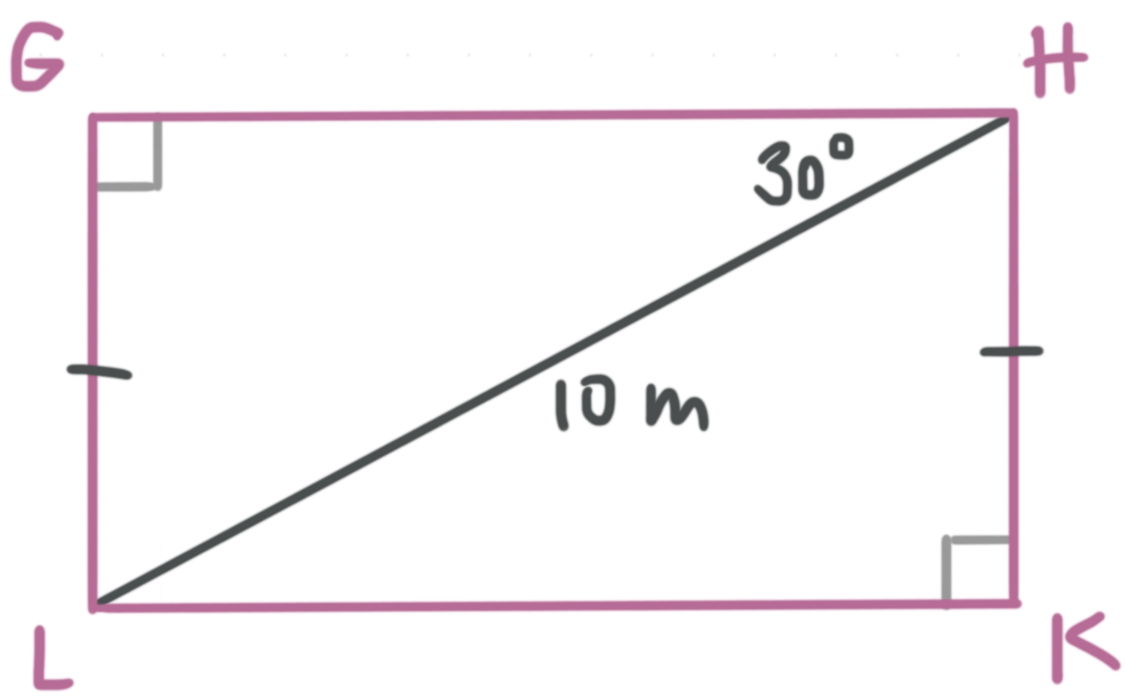
$$AD^2 + 12^2 = 13^2$$

$$AD^2 = 25$$

$$AD = 5$$

Therefore, the area of the rectangle is $A = bh = (5)(12) = 60$.

■ 3. In rectangle $GHKL$, $LH = 10$ and $m\angle GHL = 30$. Find the exact area of the rectangle.



Solution:

$\triangle GHL$ is a special right triangle with degree measures $30 - 60 - 90$. The diagonal of the rectangle is 10 and is the hypotenuse of $\triangle GHL$. The shortest leg of the triangle is $\overline{GL}$ and this side is half the length of the hypotenuse. $GL = 5$ and $\overline{GH}$ is the product of the shorter leg and $\sqrt{3}$. Therefore, $GH = 5\sqrt{3}$. The area of the rectangle is

$$A = bh = (5)\left(5\sqrt{3}\right) = 25\sqrt{3}$$

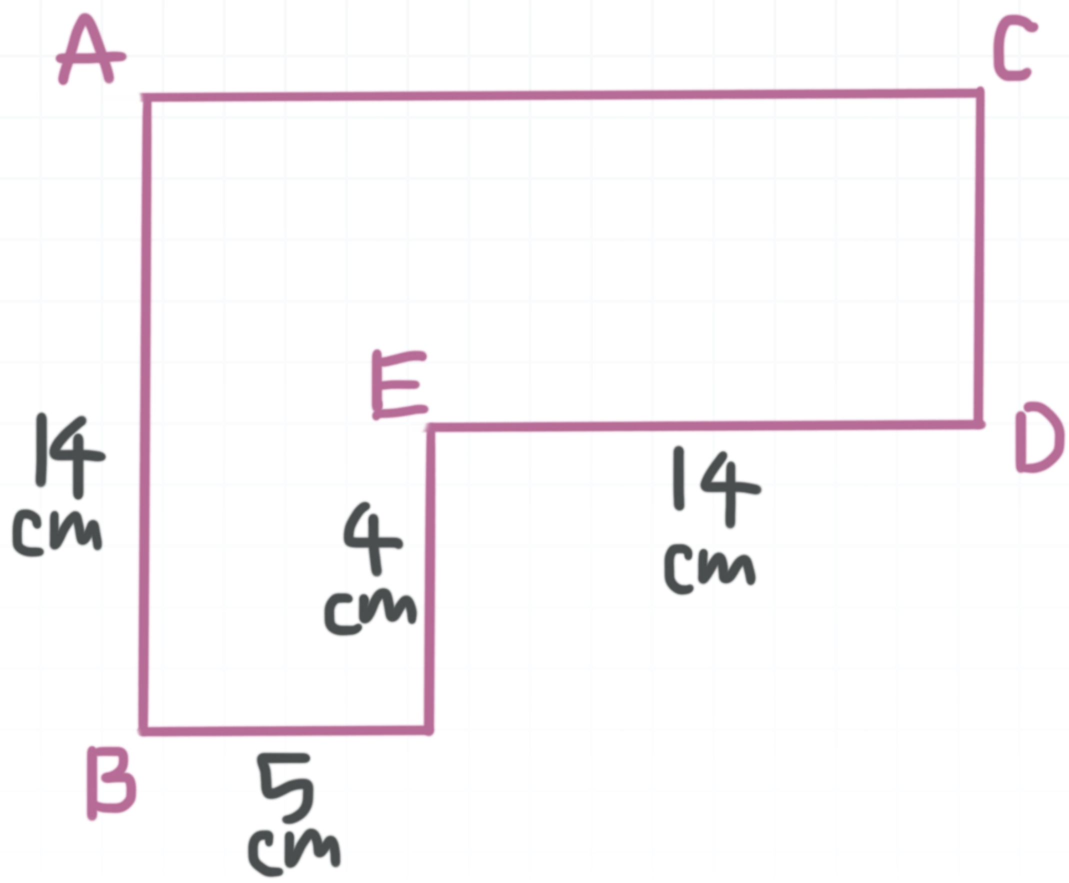
■ 4. The area of a small square flower garden is 49 ft^2. Suppose we wish to make the garden bigger by adding 6 feet to one of the sides. How much more square footage is available in this new rectangular garden?

Solution:

The original square garden has dimensions 7 feet by 7 feet. By adding 6 feet onto one of the sides, we get a new rectangle with dimensions 13 feet by 7 feet. The new garden has an area of $(13)(7) = 91$ ft^2. To find the area gained, take $91 - 49 = 42$ ft^2.

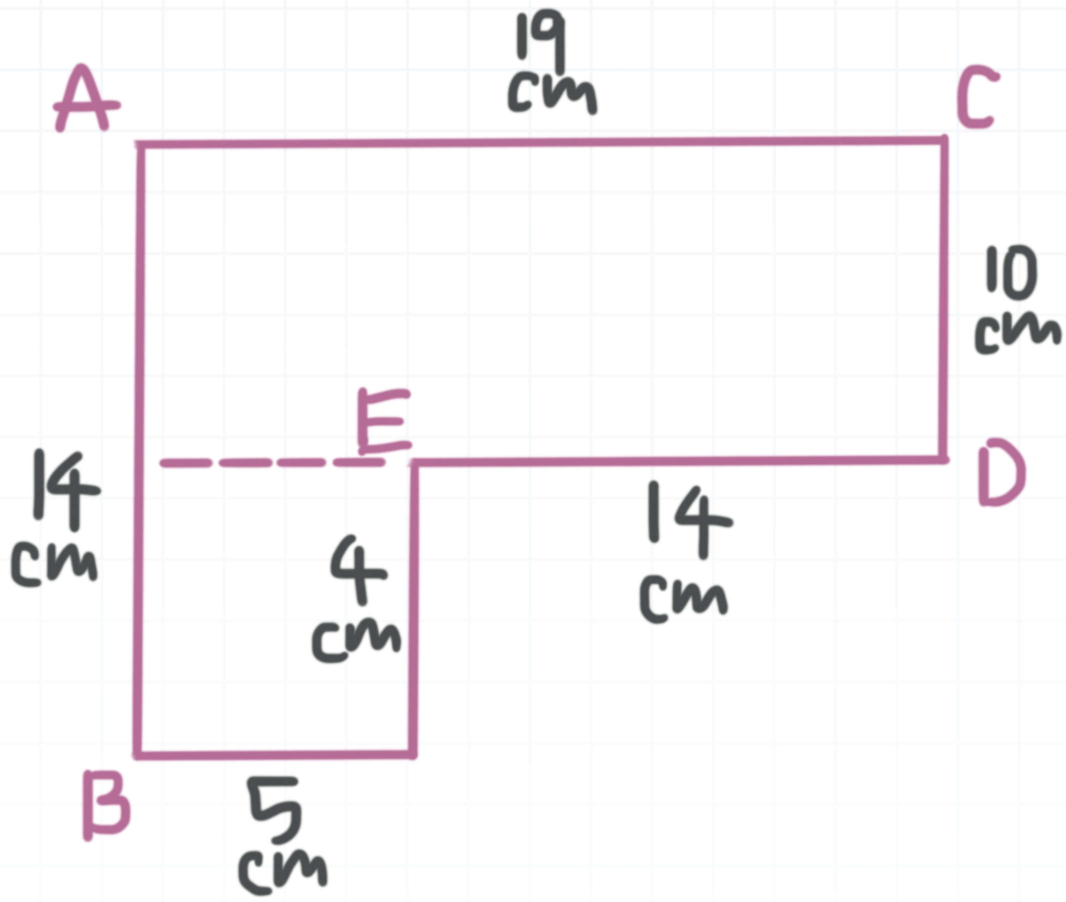
AREA OF A RECTANGLE USING SUMS AND DIFFERENCES

1. Find the area of the figure.



Solution:

Segment the figure into two rectangles, and fill out the rest of the figure.



The area of the larger rectangle is $A_1 = lw = (19)(10) = 190$ cm^2.

The area of the smaller rectangle is $A_2 = lw = (5)(4) = 20$ cm^2.

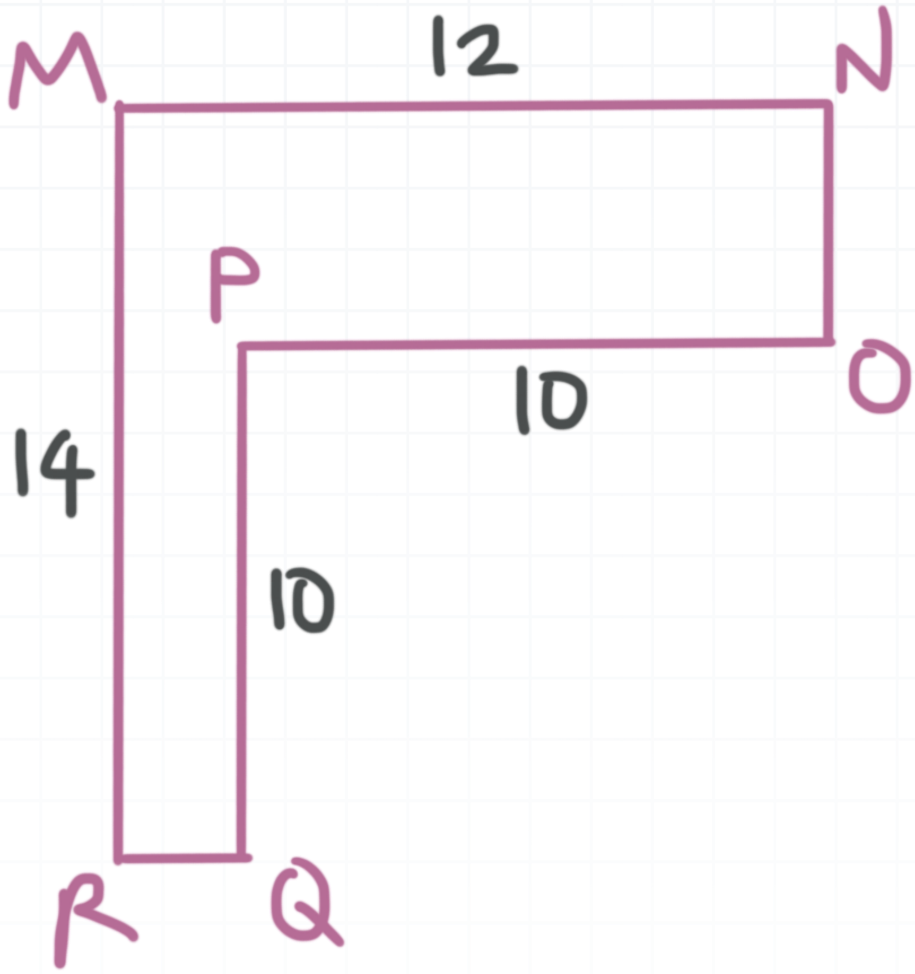
Then the area of the whole figure is

$$A = A_1 + A_2$$

$$A = 190 \text{ cm}^2 + 20 \text{ cm}^2$$

$$A = 210 \text{ cm}^2$$

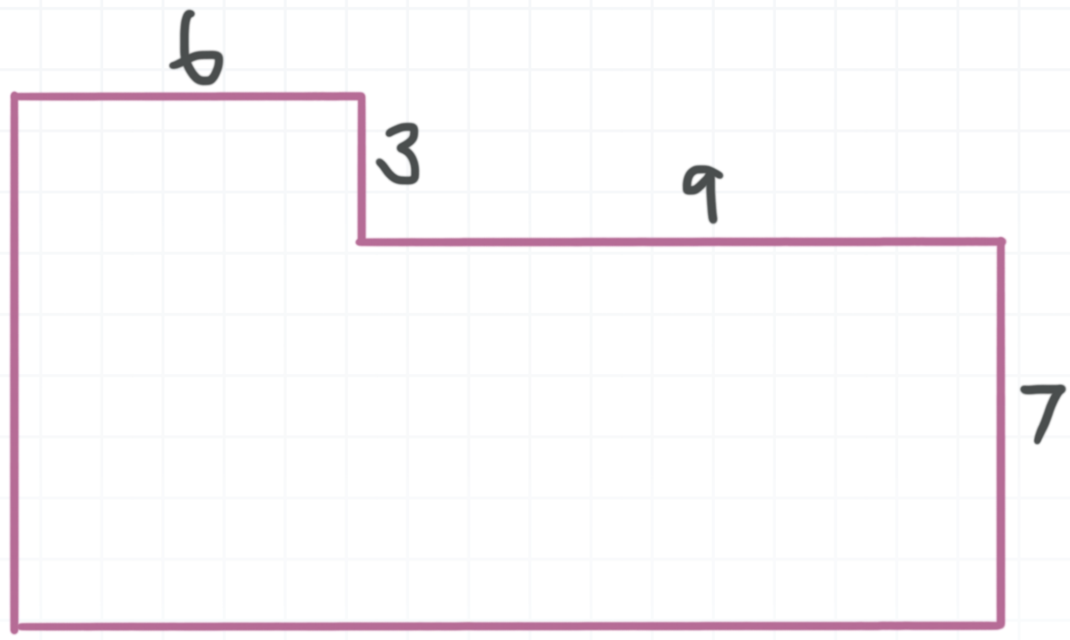
■ 2. Find the area of the figure.



Solution:

The area of larger rectangle with three vertices at M, N, and R is $A = (14)(12) = 168$. The area of the smaller rectangle with three vertices at P, O, and Q is $A = (10)(10) = 100$. Using the difference method, the area of the figure is $168 - 100 = 68$.

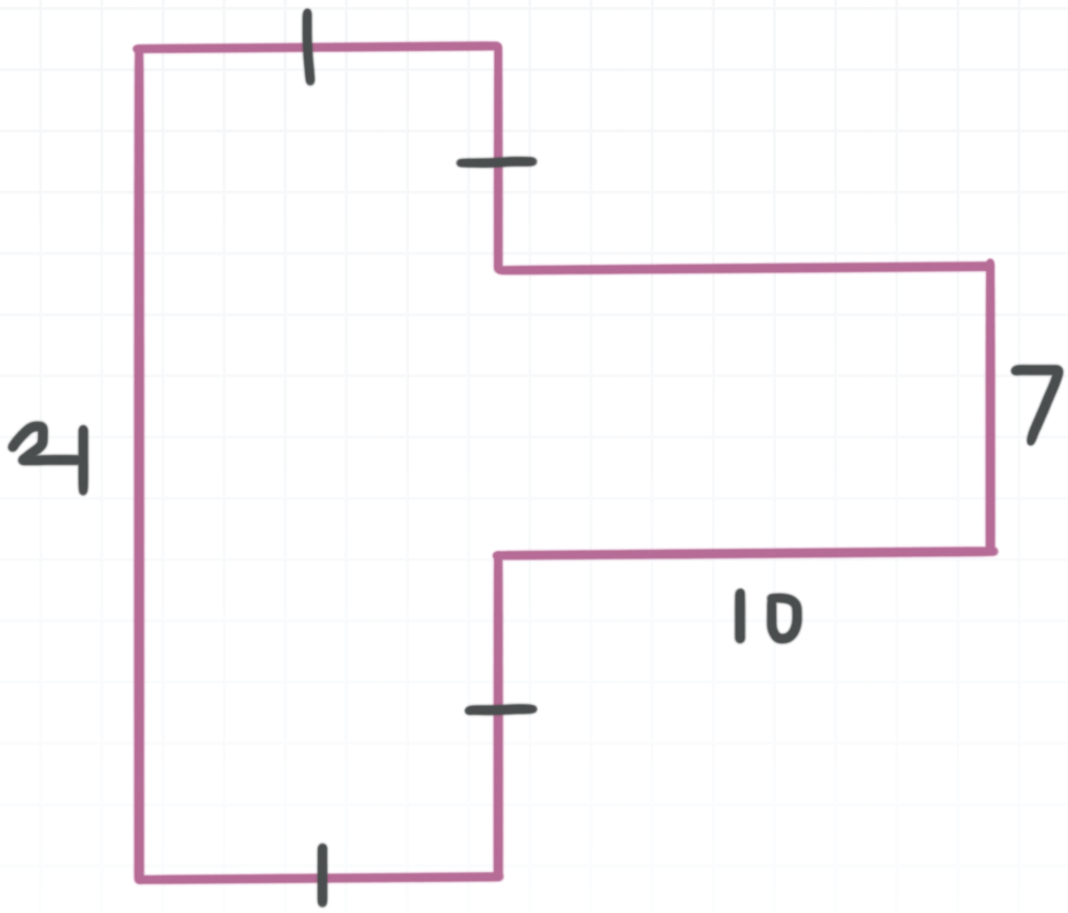
■ 3. Find the area of the figure.



Solution:

The area of the little rectangle in the upper left is $A = (6)(3) = 18$. The area of the larger rectangle at the bottom is $A = (6 + 9)(7) = 105$. Using the sum method, the area of the figure is $18 + 105 = 123$.

■ 4. Find the area of the figure.



Solution:

The area of the rectangle on the left is $A = (21)(7) = 147$. The area of the rectangle on the right is $A = 10(7) = 70$. Using the sum method, the area of the figure is $A = 147 + 70 = 217$.

PERIMETER OF A RECTANGLE

■ 1. A rectangle has a base of 10 meters. The height is 4 meters greater than the base. Find the perimeter of this rectangle.

Solution:

By the formula for the perimeter of a rectangle, we get

$$P = 2b + 2h$$

$$P = 2(10) + 2(10 + 4)$$

$$P = 20 + 18$$

$$P = 48$$

■ 2. The area of a rectangle is 40 ft^2. Find the perimeter of this rectangle if the length of the rectangle is 3 feet longer than the width.

Solution:

First, we'll write the equation for the area and plug in what we know.

$$A = bh$$

$$40 = b(b + 3)$$

$$40 = b^2 + 3b$$

$$0 = b^2 + 3b - 40$$

$$0 = (b + 8)(b - 5)$$

$$b = -8, 5$$

The base of the rectangle can't be defined by a negative number, so the base must be 5 feet long. The height is therefore $h = 5 + 3 = 8$ feet, and the perimeter is

$$p = 2b + 2h$$

$$p = 2(5) + 2(8)$$

$$p = 10 + 16$$

$$p = 26$$

■ 3. Find the perimeter of a rectangle with vertices at $A(-3,0)$, $B(0,4)$, $C(4,1)$, and $D(1, -3)$.

Solution:

We need to use the distance formula to calculate the distance between adjacent points, which will give us the length of each side of the rectangle.

$$d_{AB} = \sqrt{(4-0)^2 + (0-(-3))^2} = 5$$

$$d_{BC} = \sqrt{(1-4)^2 + (4-0))^2} = 5$$

$$d_{CD} = \sqrt{((-3)-1)^2 + (1-4)^2} = 5$$

$$d_{AD} = \sqrt{(-3)-0)^2 + (1-(-3))^2} = 5$$

Therefore, the perimeter is

$$p = AB + BC + CD + AD$$

$$p = 5 + 5 + 5 + 5$$

$$p = 20$$

■ 4. Find the value of x if the base of the rectangle has length $x + 4$, the height of the rectangle is x, and the perimeter of a rectangle is 20 units.

Solution:

Plug what you know into the formula for the perimeter of a rectangle.

$$P = 2b + 2h$$

$$20 = 2(x + 4) + 2(x)$$

$$20 = 2x + 8 + 2x$$

$$20 = 4x + 8$$

$$12 = 4x$$

$$x = 3$$

AREA OF A PARALLELOGRAM

■ 1. Find the area of a parallelogram with $b = 14$ yards and $h = 10$ yards.

Solution:

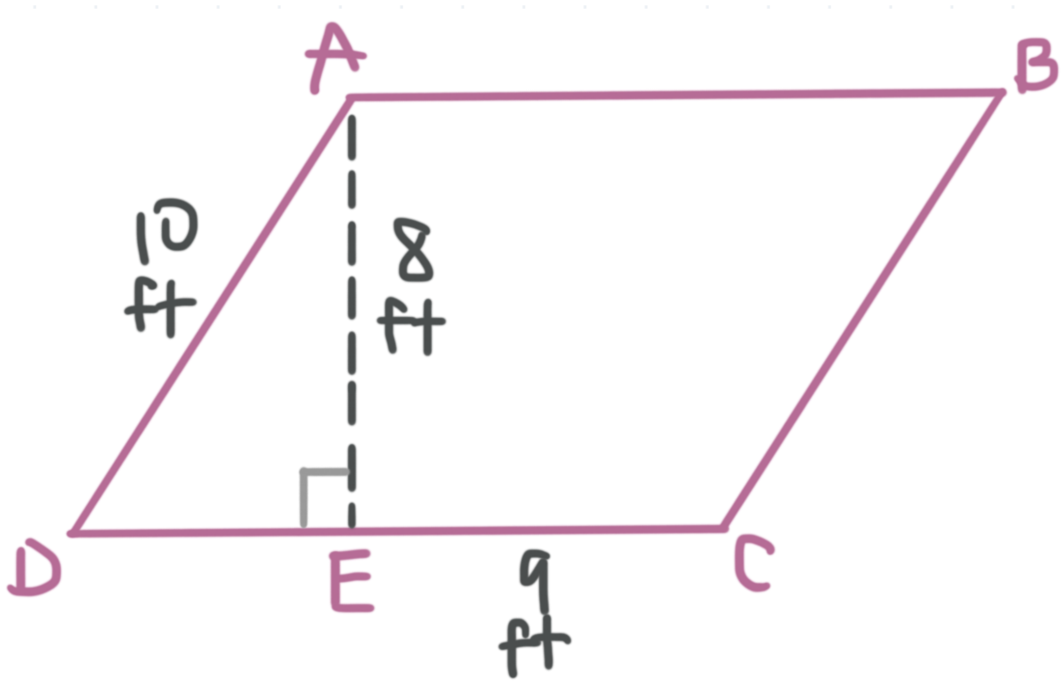
The area of a parallelogram is given by the product of its base and height.

$$A = bh$$

$$A = (14)(10)$$

$$A = 140 \text{ yd}^2$$

■ 2. Find the area of the parallelogram.



Solution:

Find the missing side, ED, of the right triangle using Pythagorean Theorem.

$$ED^2 + 8^2 = 10^2$$

$$ED = 6$$

Find the length of the base of the parallelogram, $\overline{DC}$.

$$DC = 6 + 9$$

Then the area is

$$A = bh = (15)(8) = 120$$

■ 3. Find the area of parallelogram $JKLM$, if $J(0,0)$, $K(1,3)$, $L(-5,3)$, and $M(-6,0)$.

Solution:

Graph the parallelogram and find the base and height. The base is $b = 6$ and the height is $h = 3$. Then the area is

$$A = bh = (6)(3) = 18$$

■ 4. A parallelogram has a base that is 3 feet longer than it is tall. The area of the parallelogram is 88 square feet. Find the height of the parallelogram.

Solution:

Using the equation for area, we can find the height.

$$A = bh$$

$$88 = (h+3)(h)$$

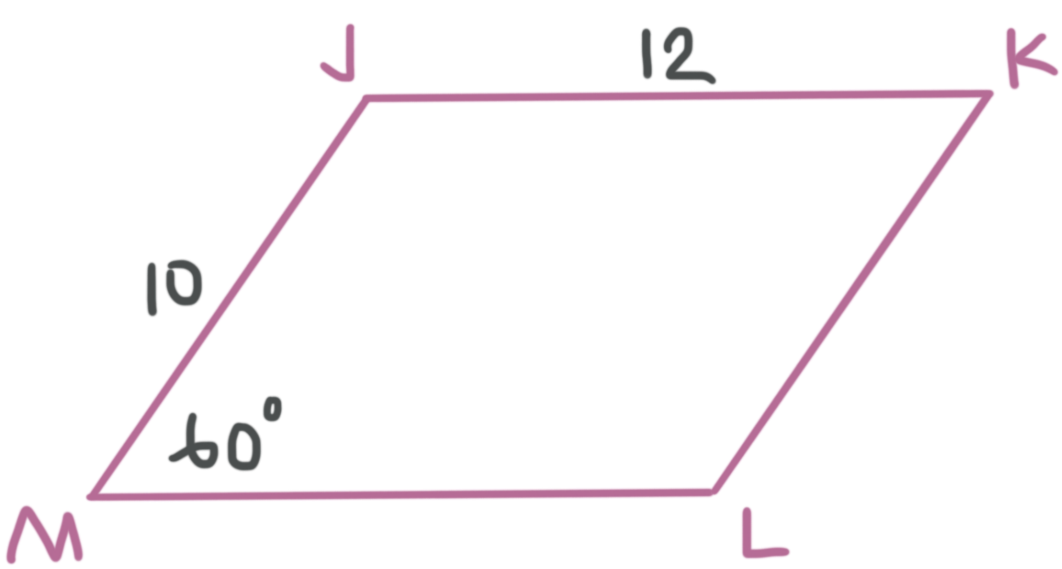
$$88 = h^2 + 3h$$

$$0 = h^2 + 3h - 88$$

$$0 = (h+11)(h-8)$$

$$h = 8$$

■ 5. Find the exact area of the parallelogram.



Solution:

The height forms a right angle with the base. A $30-60-90$ triangle is formed with 10 as its hypotenuse. The height can be found by applying $30-60-90$ rules to get $h=5\sqrt{3}$. Then the area is the parallelogram is

$$A=bh=12(5\sqrt{3})=60\sqrt{3}$$

AREA OF A TRAPEZOID

■ 1. Find the area of a trapezoid with base lengths 16 and 18, and height 10.

Solution:

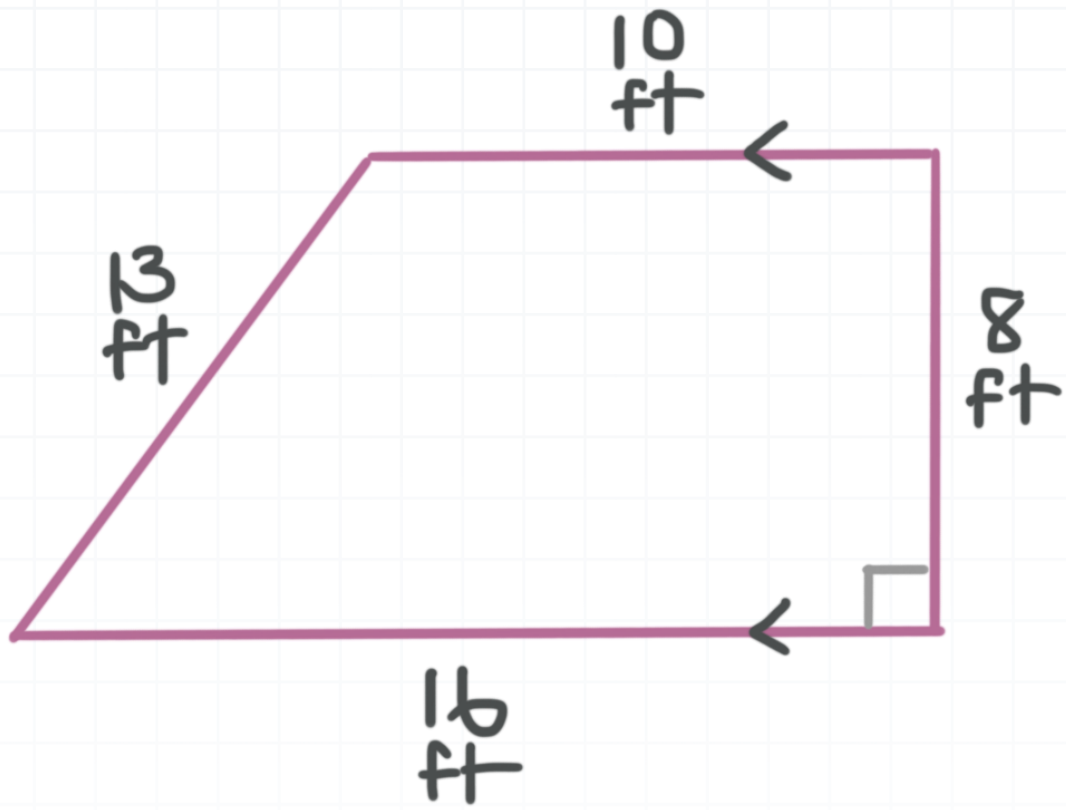
If we plug what we've been given into the formula for the area of a trapezoid, we get

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(10)(16 + 18)$$

$$A = 170$$

■ 2. Find the area of the trapezoid.



Solution:

If we plug what we've been given into the formula for the area of a trapezoid, we get

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(8)(16 + 10)$$

$$A = 104$$

■ 3. Find the exact area of the trapezoid that has congruent 2-meter bases and a height of 4 meters.

Solution:

The area of the trapezoid is

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(4)(2 + 2)$$

$$A = \frac{1}{2}(16)$$

$$A = 8$$

■ 4. The area of a trapezoid is 60 m^2. One of the bases has a measure of 7 m and the height of the trapezoid is 10 m. Find the length of the other base.

Solution:

We can plug what we know into the formula for the area of a trapezoid, and then solve for the length of the second base.

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$60 = \frac{1}{2}(10)(7 + \text{base}_2)$$

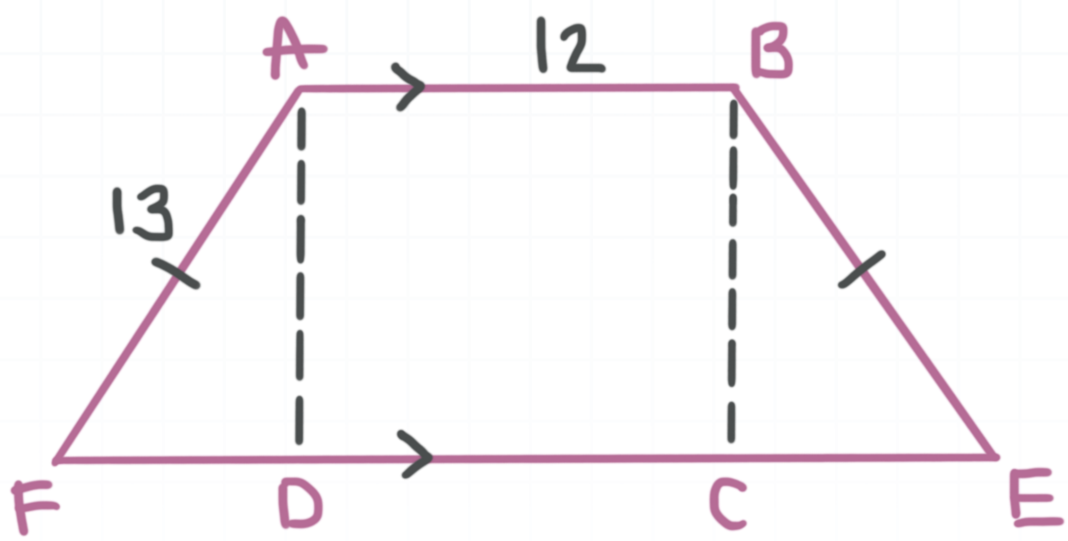
$$120 = 10(7 + \text{base}_2)$$

$$120 = 70 + 10\text{base}_2$$

$$50 = 10\text{base}_2$$

$$\text{base}_2 = 5$$

5. Find the area of trapezoid $ABEF$, if $ABCD$ is a square.



Solution:

We know that $AB = AD$ because $ABCD$ is a square. The height of the trapezoid is 12. Use the Pythagorean Theorem to find FD and CE.

$$FD^2 + AD^2 = AF^2$$

$$FD^2 + 12^2 = 13^2$$

$$FD = 5 = CE$$

Then $\overline{FE} = 5 + 12 + 5 = 22$. Therefore, the area of the trapezoid is

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(12)(22 + 12)$$

$$A = 204$$

AREA OF A TRIANGLE

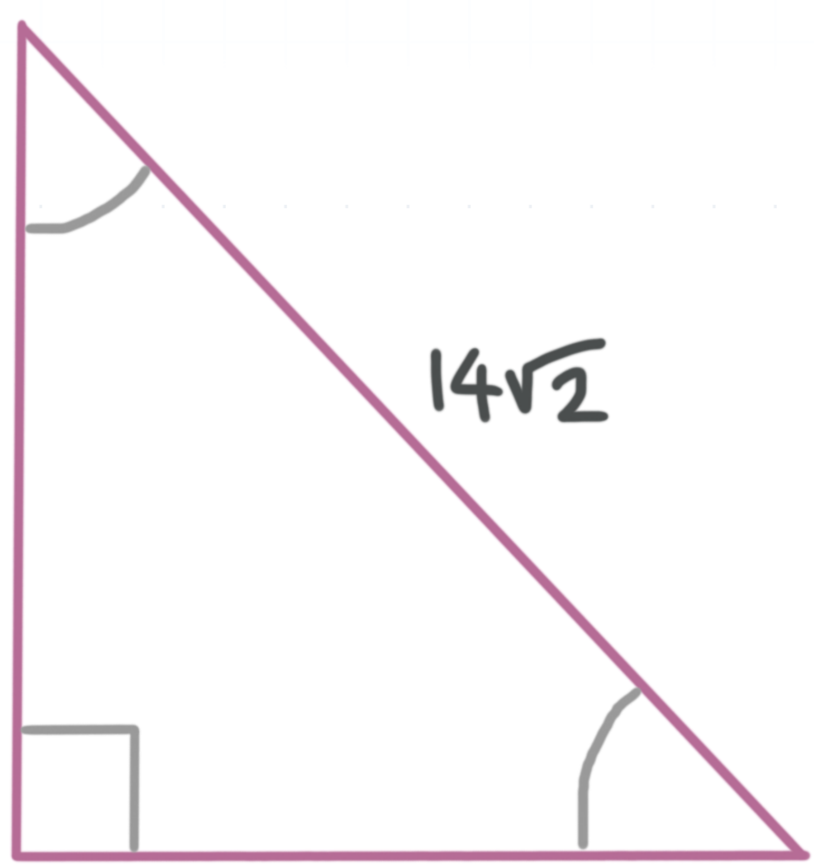
■ 1. Find the area of a triangle that has base length 16 and height 14.

Solution:

The area of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(16)(14) = 112$$

■ 2. Find the area of the triangle.

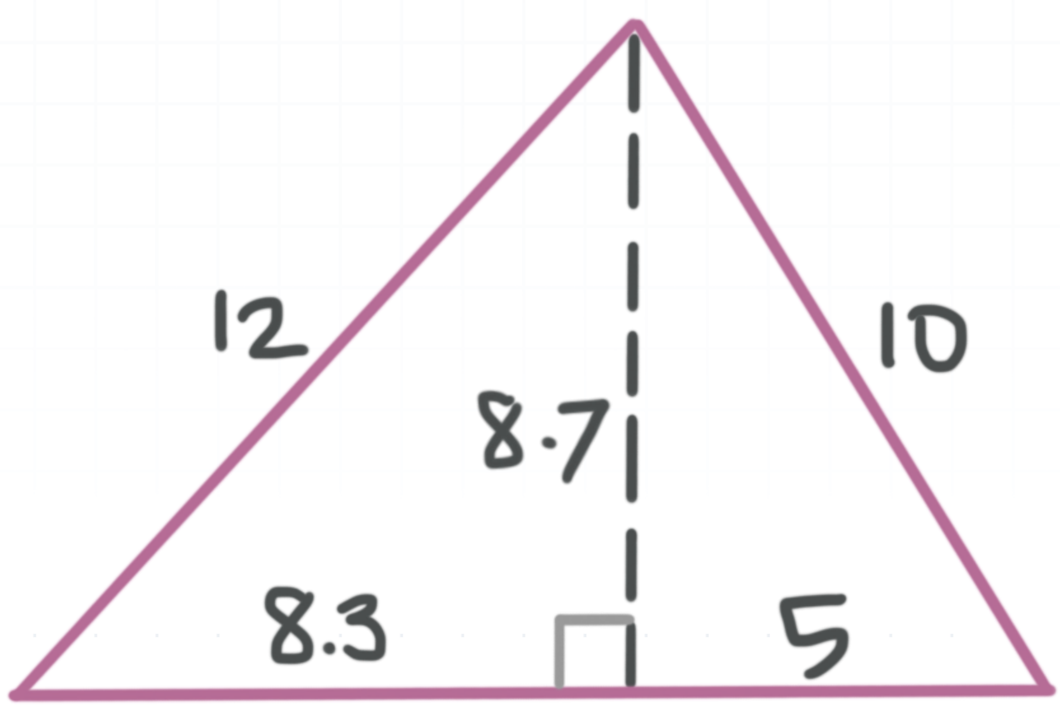


Solution:

Using $45-45-90$ rules, we find that the base of the triangle has length $b=14$ and height $h=14$. Then the area of the triangle is

$$A=\frac{1}{2}bh=\frac{1}{2}(14)(14)=98$$

■ 3. Find the area of the triangle.

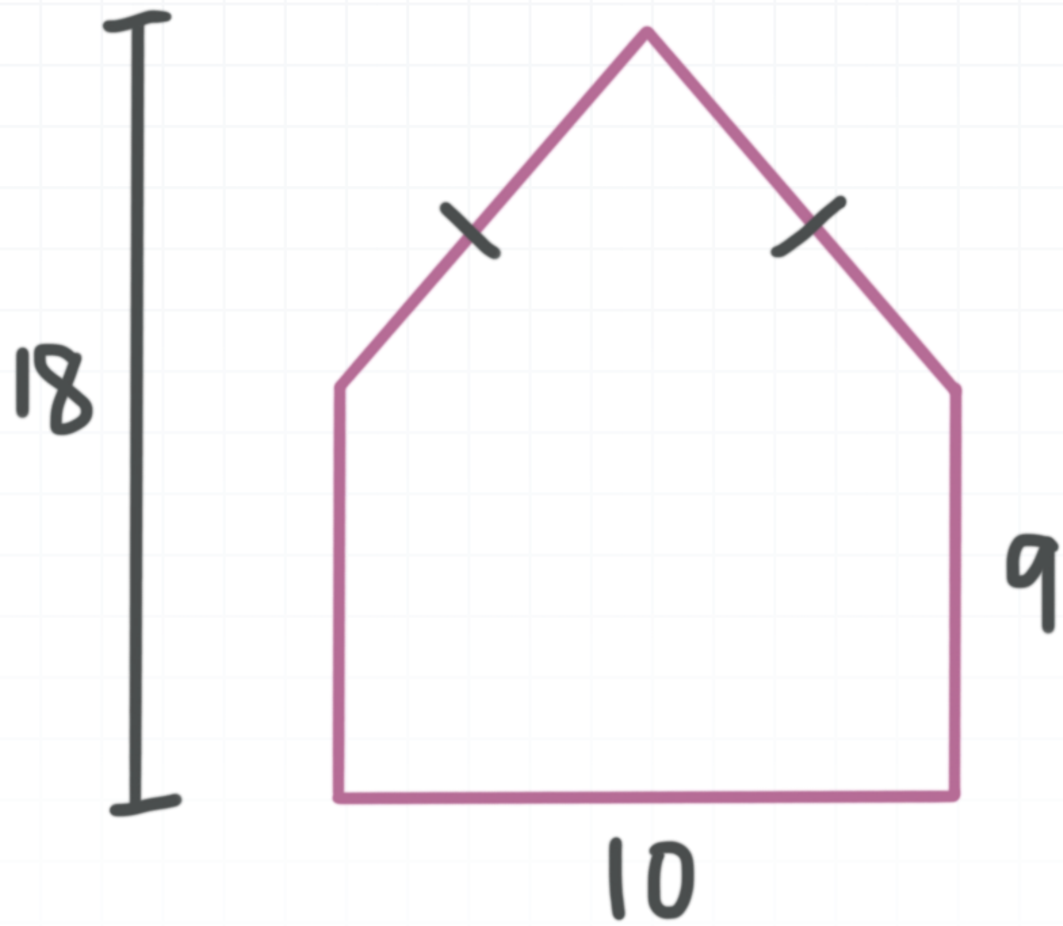


Solution:

The area of the triangle is

$$A=\frac{1}{2}bh=\frac{1}{2}(13.3)(8.7)=57.855$$

■ 4. Find the area of the figure below.



Solution:

The area of the rectangle is

$$A_R = bh = (10)(9) = 90$$

The area of the triangle is

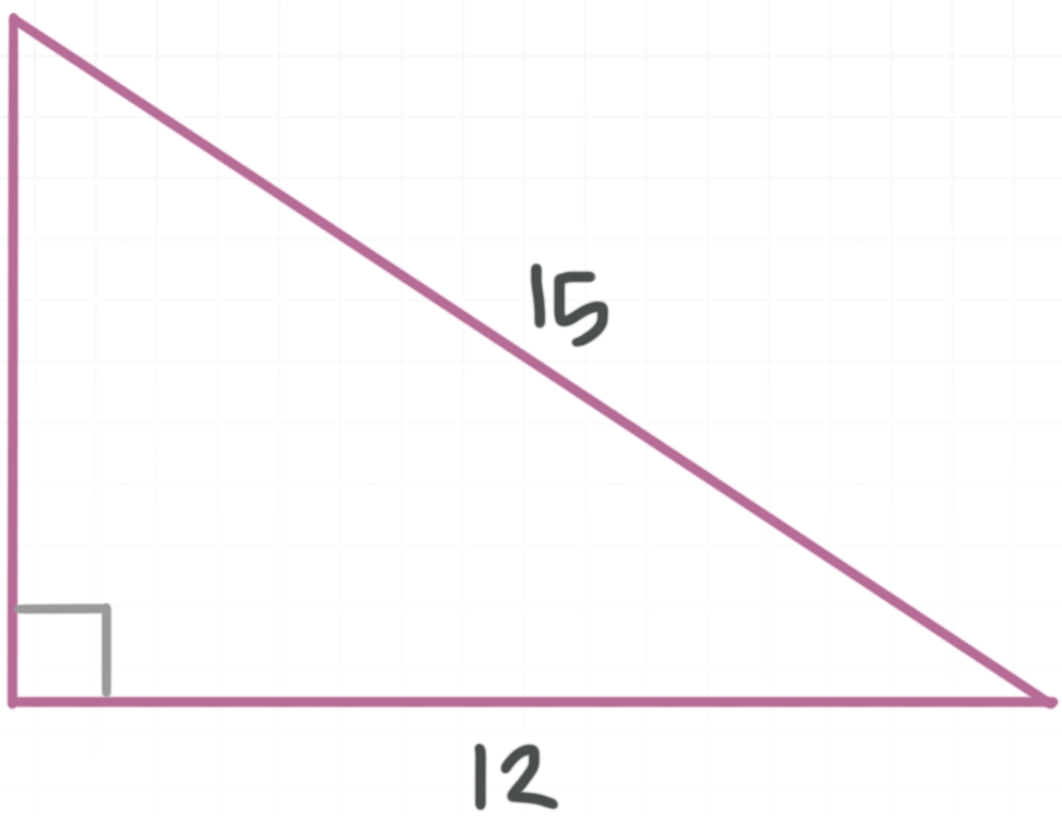
$$A_T = \frac{1}{2}(bh) = \frac{1}{2}(10)(9) = 45$$

Therefore, the area of the entire region is

$$A = A_R + A_T = 90 + 45 = 135$$

PERIMETER OF A TRIANGLE

1. Find the perimeter of the triangle.



Solution:

Let the missing side be x.

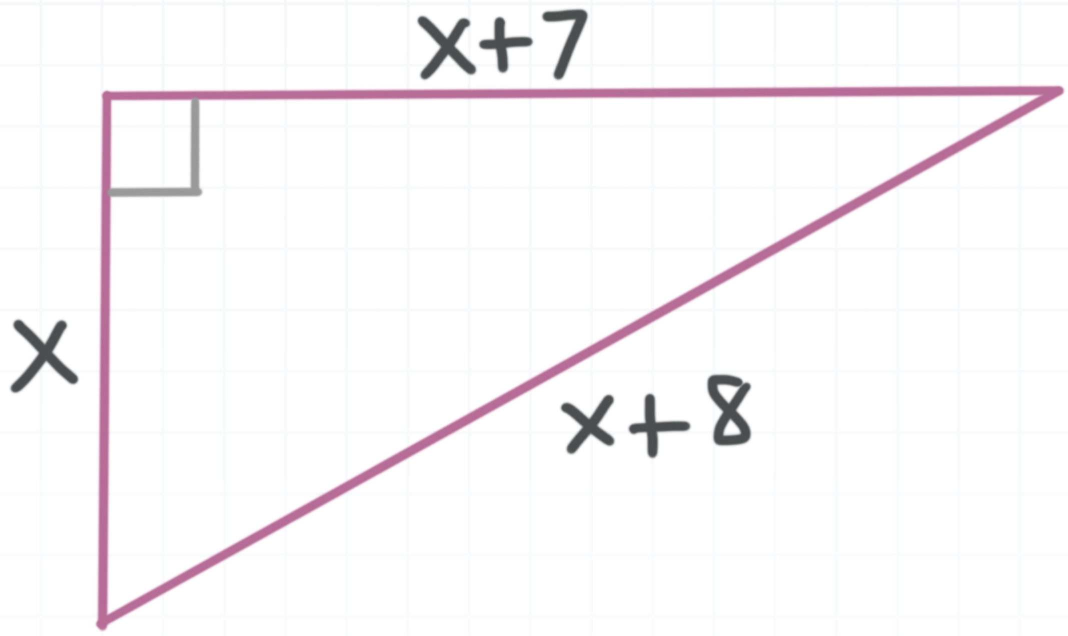
$$x^2 + 12^2 = 15^2$$

$$x^2 = 15^2 - 12^2 = 81$$

$$x = \sqrt{81} = 9$$

The perimeter of the triangle is $9 + 12 + 15 = 36$.

2. Find the perimeter of the triangle.



Solution:

The perimeter can be found by plugging the side lengths into the Pythagorean Theorem.

$$x^2 + (x+7)^2 = (x+8)^2$$

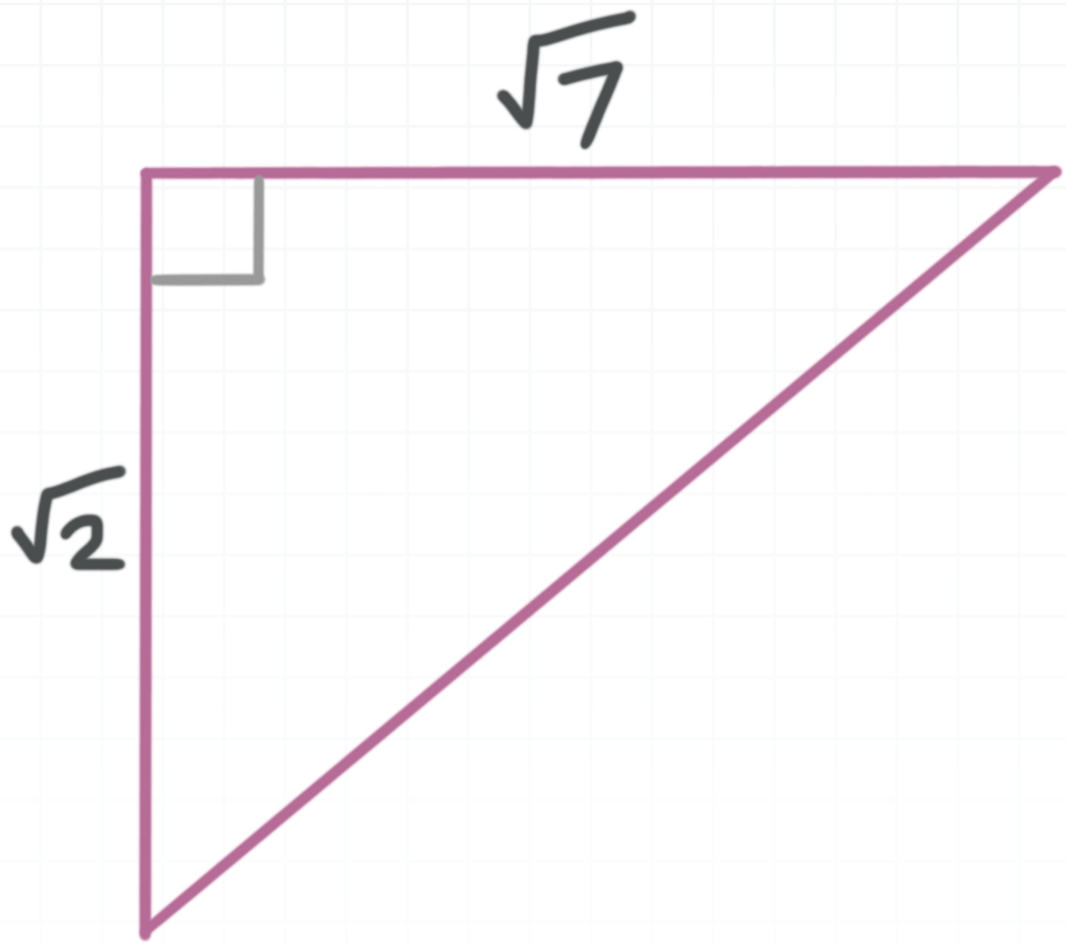
$$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

So $x = 5$ or $x = -3$. But one of the legs of the triangle is x, which means x cannot have a negative value, because that would mean we'd have a negative side length. Therefore, $x = 5$ and the side lengths must be 5, 12, and 13. Which means the perimeter of the triangle is $5 + 12 + 13 = 30$.

3. Find the exact perimeter of the triangle.



Solution:

Plug the side lengths into the Pythagorean Theorem to find the length of the hypotenuse.

$$(\sqrt{2})^2 + (\sqrt{7})^2 = c^2$$

$$2 + 7 = c^2$$

$$c^2 = 9$$

$$c = \sqrt{9} = 3$$

Therefore, the perimeter of the triangle is $\sqrt{2} + \sqrt{7} + 3$.

■ 4. Find the perimeter of a right, isosceles triangle, to the nearest hundredth, in which one of the legs measures 5 inches.

Solution:

Draw a right, isosceles triangle and note that both legs must be 5 inches long. Find the hypotenuse using the Pythagorean Theorem.

$$c^2 = 5^2 + 5^2$$

$$c^2 = 50$$

$$c = \sqrt{50} = 5\sqrt{2}$$

Then the perimeter is

$$P = 5 + 5 + 5\sqrt{2} = 10 + 5\sqrt{2} = 10 + 7.07 \approx 17.07 \text{ inches}$$

AREA OF A CIRCLE

1. Find the area of a circle to the nearest hundredth with a diameter of 44 inches.

Solution:

If the diameter is 44 inches, then the radius is half that: 22 inches. Plug the radius into the formula for the area of a circle.

$$A = \pi r^2$$

$$A = \pi (22)^2$$

$$A = 1{,}520.53$$

2. The area of a circle is 300 cm^2. Find the length of the radius to the nearest tenth of a centimeter.

Solution:

Plug the area into the formula for the area of a circle, and then solve for the radius, r.

$$A = \pi r^2$$

$$300 = \pi r^2$$

$$r^2 = \frac{300}{\pi} = 95.5$$

$$r = \sqrt{95.5} = \approx 9.8$$

■ 3. Find the exact area of a circle with a circumference of 18π.

Solution:

Plug the circumference into the formula for the circumference of a circle.

$$C = d\pi$$

$$18\pi = d\pi$$

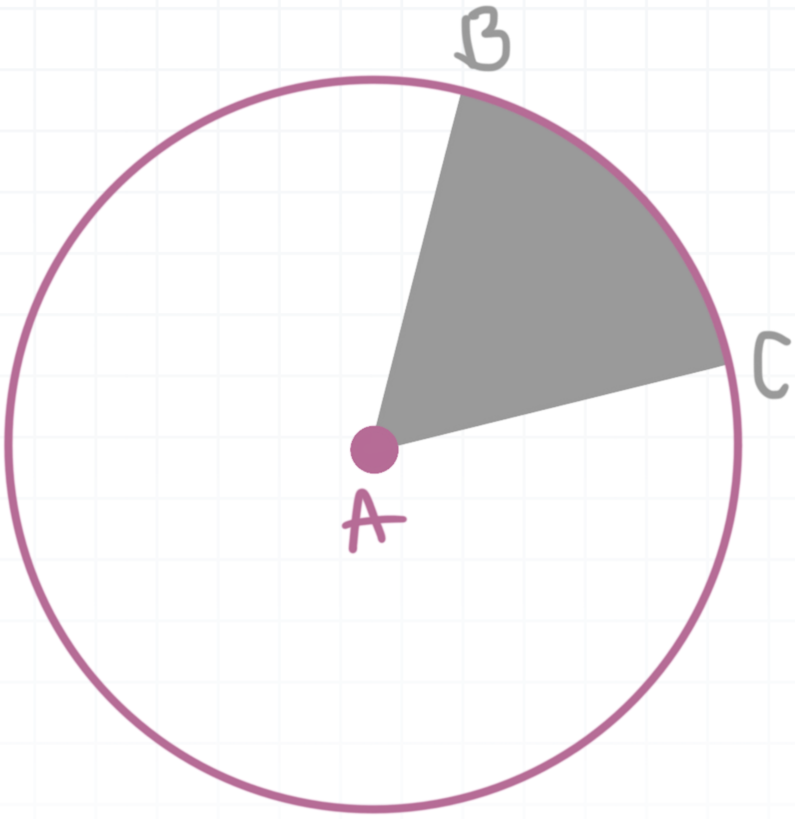
$$d = 18$$

Because the diameter has length 18, the length of the radius is $r = 9$. Therefore, the area of a circle is

$$A = \pi(9)^2$$

$$A = 81\pi$$

■ 4. Find the area of the shaded region to the nearest tenth if $m\angle BAC = 60^\circ$ and $AC = 16$ feet.



Solution:

The shaded area represents $60/360$, or $1/6$ of the total area. The area of the full circle is

$$A = \pi r^2 = \pi(16)^2 \approx 804.2$$

so the area of the shaded region, which is $1/6$th of the circle, is

$$A = \frac{1}{6}(804.2) \approx 134.0$$

CIRCUMFERENCE OF A CIRCLE

■ 1. To the nearest hundredth, find the circumference of a circle that has a radius of 14 feet.

Solution:

The circumference of the circle is

$$C = 2\pi r = 2\pi(14) = 28\pi \approx 87.96$$

■ 2. Find the area of a circle with a circumference of 400 ft.

Solution:

We can use the formula for circumference.

$$C = 2\pi r$$

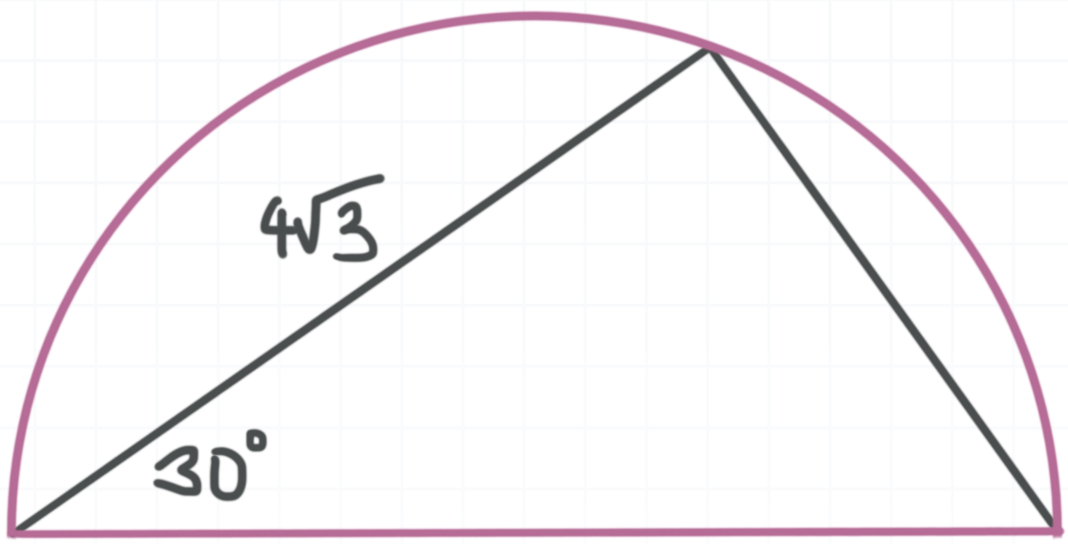
$$400 = 2\pi r$$

$$r = \frac{200}{\pi} \approx 63.66$$

Then the area of the circle is

$$A = \pi(63.66)^2 \approx 12{,}731.61$$

3. Find the exact circumference of the semicircle.



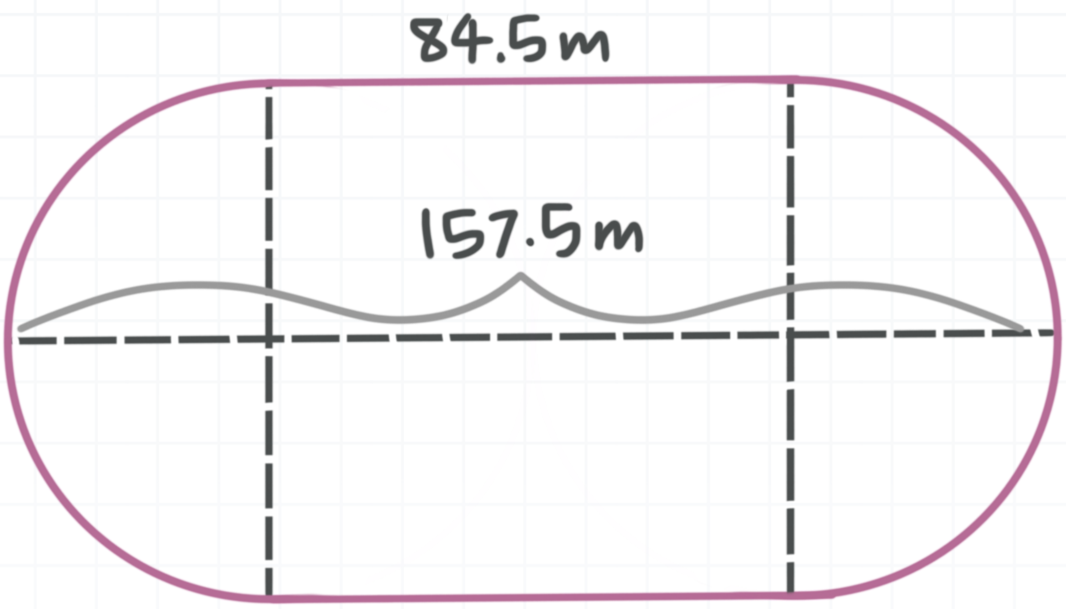
Solution:

Find the diameter of the circle using $30-60-90$ rules to find $d=8$.

$$C=2\pi r=d\pi=8\pi$$

The semicircle has a circumference that is half of the overall circumference of the circle.

4. To the nearest tenth, find the distance around the following track.



Solution:

The length is comprised of two straight stretches and two semicircles. The length of the radius of each semicircle must be

$$\frac{(157.5-84.5)}{2}=36.5$$

The circumference of each semicircle is

$$\frac{1}{2}(2)(36.5)\pi \approx 114.67$$

$84.5+84.5+114.67+114.67=398.3$ meters

NETS/VOLUME/SURFACE AREA OF PRISMS

■ 1. Find the volume of a rectangular prism with length 14 feet, width 10 feet, and height 5 feet.

Solution:

Plugging the dimensions into the formula for volume of a rectangular prism, we get

$$V = lwh = (14)(10(5) = 700\text{ ft}^3$$

■ 2. Find the surface area of a rectangular prism with length 14 feet, width 10 feet, and height 5 feet.

Solution:

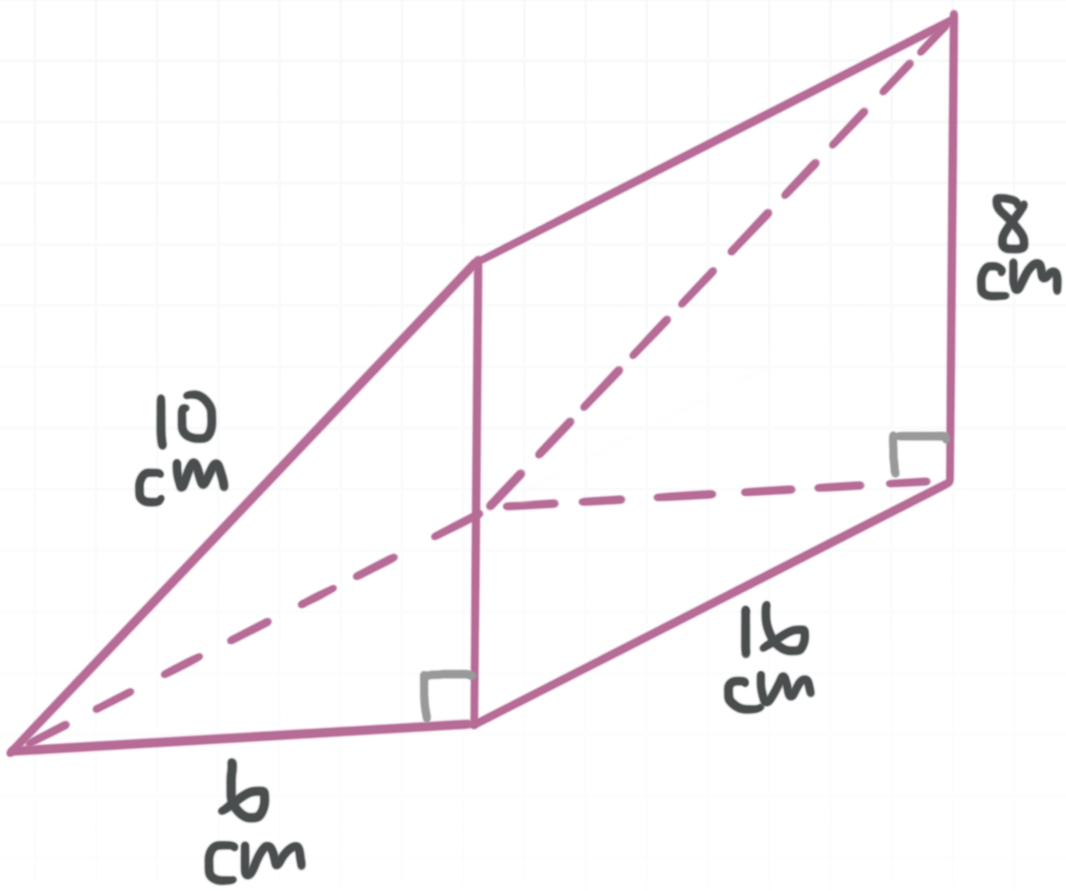
Plugging the dimensions into the formula for surface area of a rectangular prism, we get

$$SA = 2lw + 2wh + 2lh$$

$$SA = 2(14)(10) + 2(10)(5) + 2(14)(5)$$

$$SA = 520\text{ ft}^2$$

■ 3. Find the surface area of the triangular prism.



Solution:

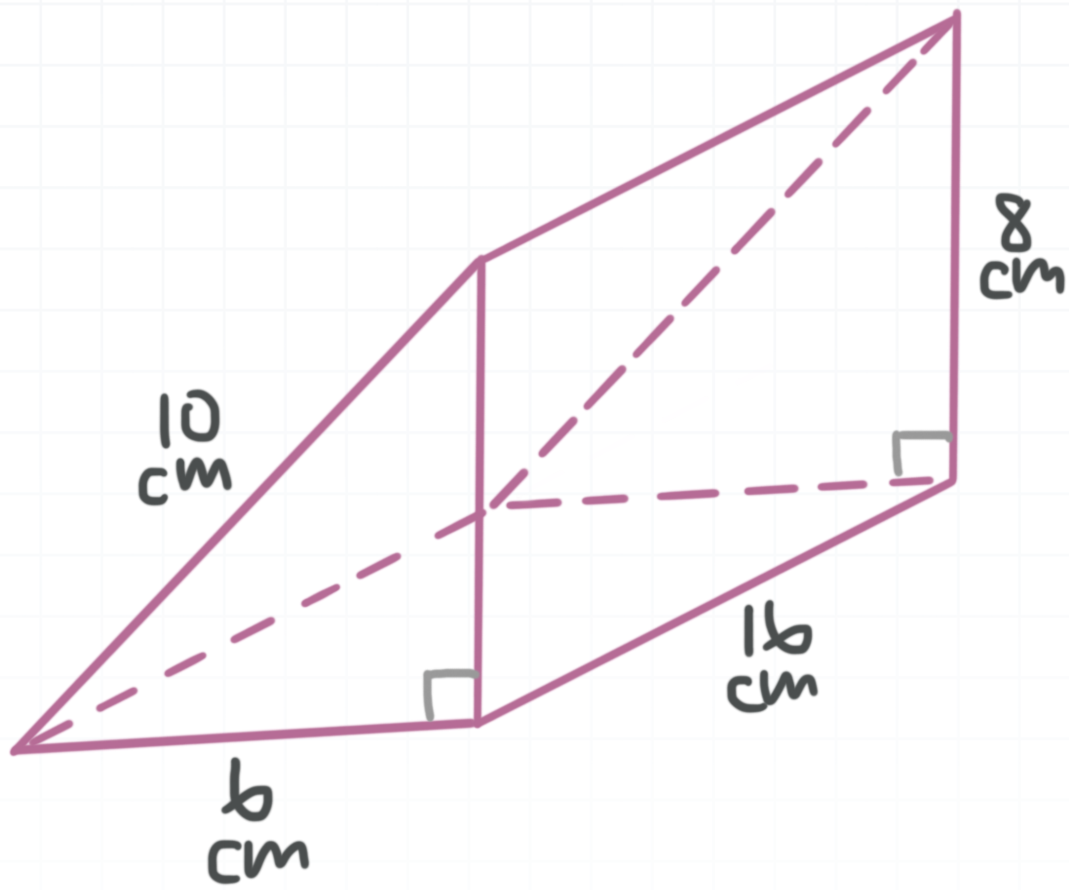
The surface area is given by

$$SA = 2\left(\frac{1}{2}\right)(6)(8) + (6)(16) + (16)(8) + (10)(16)$$

$$SA = 48 + 96 + 128 + 160$$

$$SA = 432\ \text{cm}^2$$

4. Find the volume of the triangular prism.



Solution:

The volume is given by

$$V = (\text{area of the end})(\text{length})$$

$$V = \frac{1}{2}(6)(8)(16)$$

$$V = 384 \text{ cm}^3$$

SURFACE AREA TO VOLUME RATIO OF PRISMS

■ 1. A rectangular prism has length, width, and height of 5 inches. Find the ratio of its surface area to its volume.

Solution:

The volume of the prism is

$$V = lwh = (5)(5)(5) = 125$$

and the surface area is

$$SA = 2lw + 2wh + 2lh$$

$$SA = 2(5)(5) + 2(5)(5) + 2(5)(5)$$

$$SA = 150$$

The ratio is $150/125 = 6/5$.

■ 2. A cube has a volume of 216 in^3. Suppose we double the length of each side of the cube. What is the ratio of the smaller cube to the larger cube?

Solution:

A cube has equal length, width, and height, so

$$V = lwh$$

$$V = l^3$$

$$216 = l^3$$

$$l = \sqrt[3]{216} = 6$$

Each side length is 6. Double each side length to get $6(2) = 12$. The new volume would be $(12)^3 = 1{,}728$. The ratio is

$$\frac{216}{1{,}728} = \frac{1}{8}$$

■ 3. In lowest terms, find the ratio of volume to surface area of a cube with side length x.

Solution:

The volume of the cube would be $V = lwh = x^3$, and the surface area would be

$$SA = 2lw + 2lh + 2wh$$

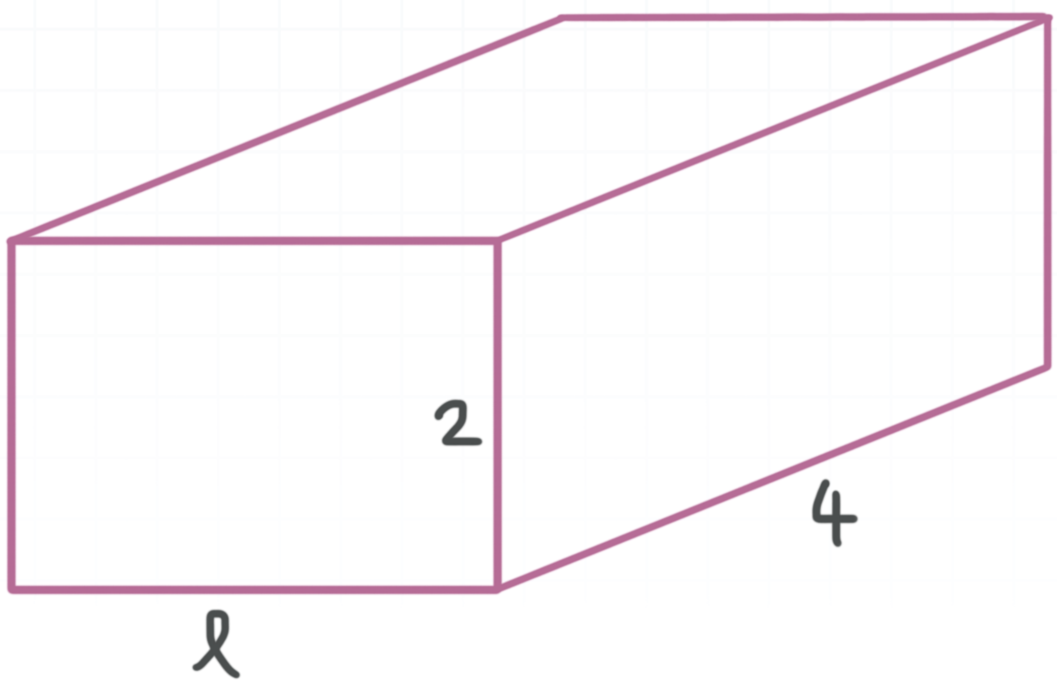
$$SA = 2(x^2) + 2(x^2) + 2(x^2)$$

$$SA = 6x^2$$

The ratio of volume to surface area is

$$\frac{x^3}{6x^2} = \frac{x}{6}$$

■ 4. The ratio of the volume to surface area for the following rectangular prism is $1:2$. Find the length of the prism.



Solution:

The volume of the rectangular prism is

$$V = lwh = (2)(4)l = 8l$$

and the surface area is

$$SA = 2(2)(4) + 2(2l) + 2(4l) = 16 + 12l$$

Setting up a proportion with the given ratio, we get

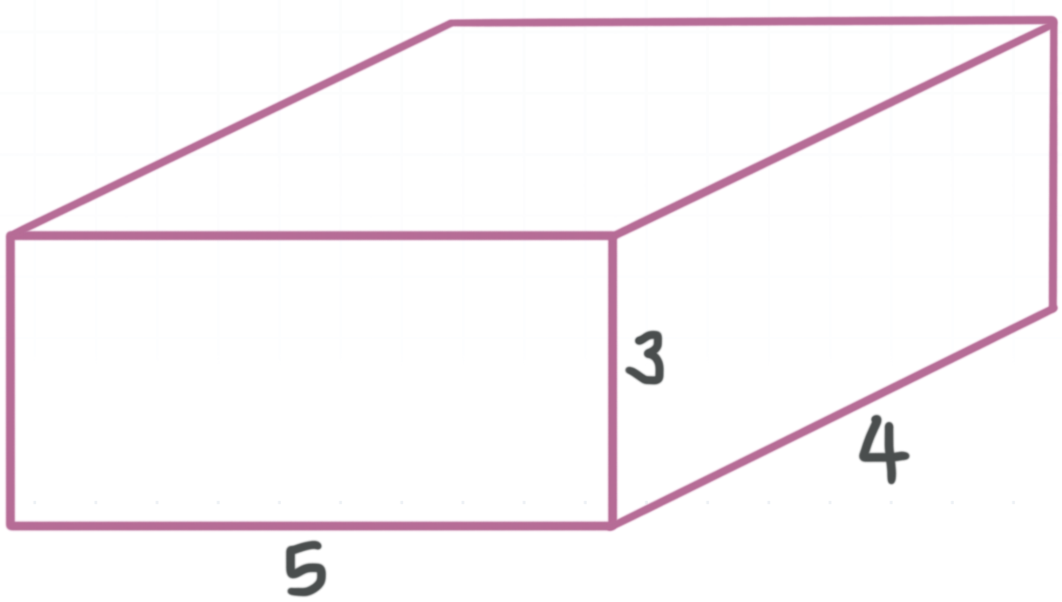
$$\frac{8l}{16+12l}=\frac{1}{2}$$

$$16l=12l+16$$

$$4l=16$$

$$l=4$$

■ 5. How many times greater will the surface area of this rectangular prism be if we double each side length?



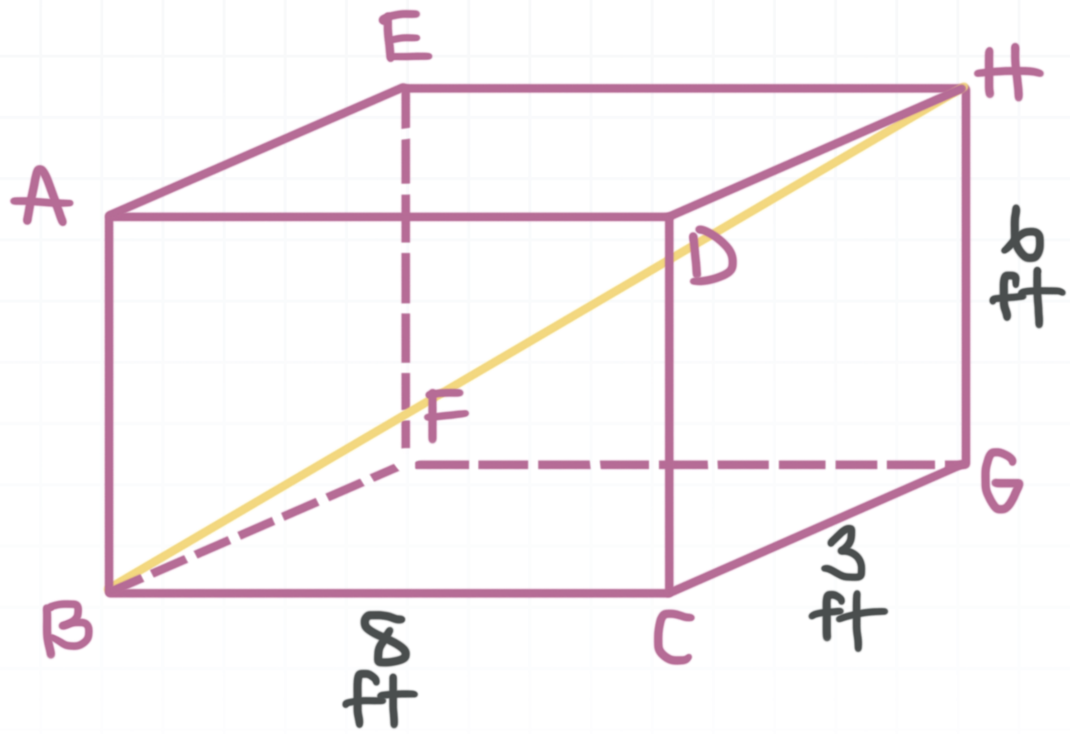
Solution:

The original surface area is $2(20)+2(12)+2(15)=94$. The new surface area would be $2(80)+2(48)+2(60)=376$. Therefore, the ratio gives

$$\frac{376}{94}=4 \text{ times greater}$$

DIAGONAL OF A RIGHT RECTANGULAR PRISM

■ 1. Find the length of BH in the right rectangular prism.



Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{8^2 + 3^2 + 6^2}$$

$$d = \sqrt{64 + 9 + 36}$$

$$d = \sqrt{109}$$

■ 2. Find the length of the diagonal of a cube with side length 10.

Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{10^2 + 10^2 + 10^2}$$

$$d = \sqrt{300}$$

$$d = 10\sqrt{3}$$

■ 3. If the length of the diagonal of a cube is $4\sqrt{3}$, find the length of each side of the cube.

Solution:

The diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$4\sqrt{3} = \sqrt{x^2 + x^2 + x^2} = \sqrt{3x^2}$$

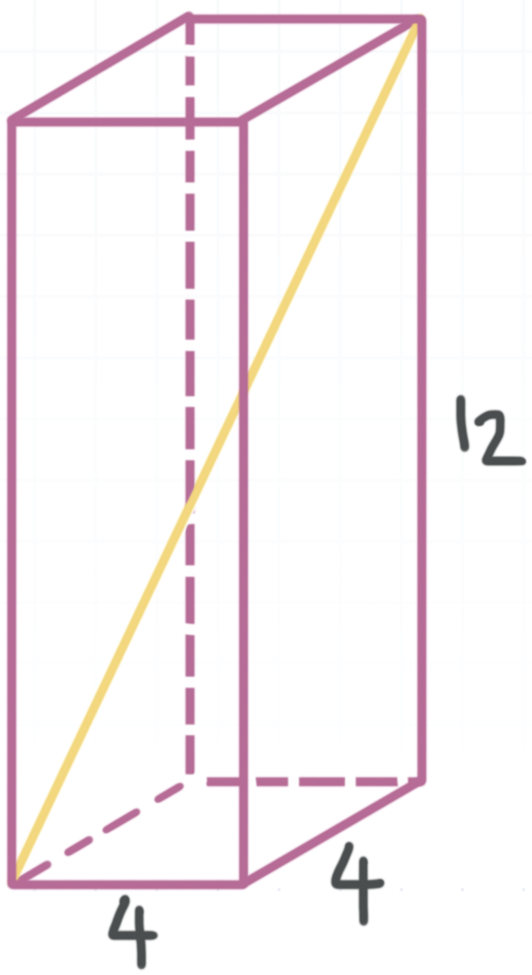
$$16(3) = 3x^2$$

$$x^2 = 16$$

$$x = 4$$

So the cube is $4 \times 4 \times 4$.

■ 4. Find the length of the diagonal of the right rectangular prism.



Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$d = \sqrt{4^2 + 4^2 + 12^2}$$

$$d = \sqrt{176}$$

$$d = 4\sqrt{11}$$

■ 5. A right, rectangular prism has dimensions $4 \times 5 \times x$. Find the value of x if the diagonal is $5\sqrt{2}$.

Solution:

The length of the diagonal is given by

$$d = \sqrt{l^2 + w^2 + h^2}$$

$$5\sqrt{2} = \sqrt{4^2 + 5^2 + x^2}$$

$$5\sqrt{2} = \sqrt{41 + x^2}$$

$$50 = 41 + x^2$$

$$x^2 = 9$$

$$x = 3$$

NETS/VOLUME/SURFACE AREA OF PYRAMIDS

■ 1. A pyramid has a square base with area 25 ft^2 and height 6 feet. Find the volume of this pyramid.

Solution:

The volume is given by

$$V = \frac{1}{3}(b)(h) = \frac{1}{3}(25)(6) = 50 \text{ ft}^3$$

■ 2. A pyramid has a square base with area 25 ft^2 and height 6 feet. Find the surface area of this pyramid.

Solution:

Find the height of each triangle using the Pythagorean Theorem. The height of the pyramid is one of the legs of the right triangle formed and so is half the length of a side of the base.

$$6^2 + 2.5^2 = 42.25$$

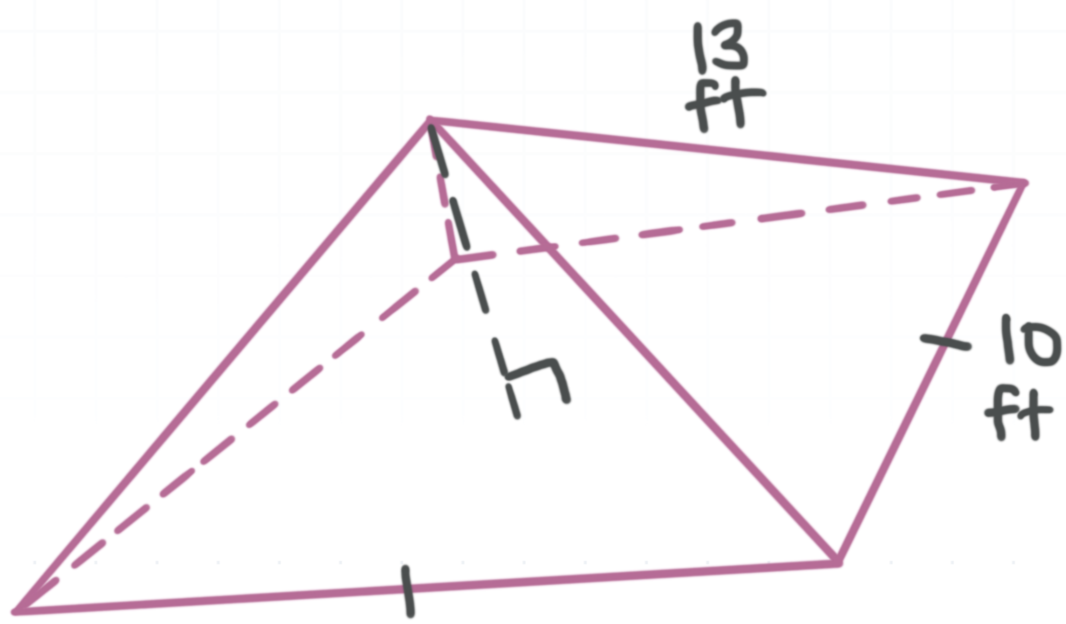
$$6^2 + 2.5^2 = 6.5^2$$

Find the area of one of the triangles of the pyramid using the formula for the area of a triangle.

$$A = \frac{1}{2}bh = \frac{1}{2}(5)(6.5) = 16.25$$

There are four triangles of equal area, so take $4(16.25) = 65$. Add on the area of the base of the pyramid to get $65 + 25 = 90$ ft^2.

■ 3. Find the surface area of the pyramid.



Solution:

Find the height of each triangle using the Pythagorean Theorem.

$$13^2 - 5^2 = 144$$

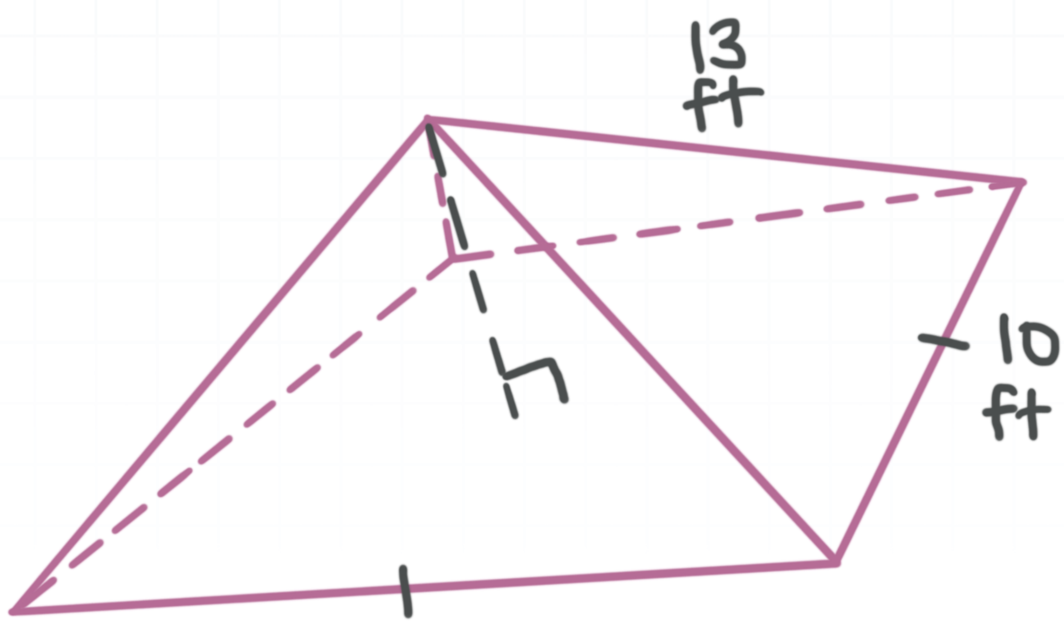
$$13^2 - 5^2 = 12^2$$

Find the area of each triangle.

$$A = \frac{1}{2}(10)(12) = 60$$

There are four triangles with this area, so take $4(60) = 240$. Now add on the base area of 100 to get $240 + 100 = 340$ ft^2.

■ 4. Find the height of the following pyramid to the nearest tenth. Then find its volume.



Solution:

Find the height using the Pythagorean Theorem.

$$12^2 - 5^2 = 119$$

$$12^2 - 5^2 = 10.9^2$$

The height of the pyramid is 10.9 ft. Then the volume of the pyramid is

$$V = \frac{1}{3}(b)(h) = \frac{1}{3}(10)(10)(10.9) = 363.3 \text{ ft}^3$$

NETS/VOLUME/SURFACE AREA OF CYLINDERS

■ 1. Find the volume of a cylinder with diameter 10 cm and height 12 cm.

Solution:

The volume of the cylinder is given by

$$V = \pi r^2 h$$

$$V = (5^2)\pi(12)$$

$$V = 942.5 \text{ cm}^3$$

■ 2. Find the height of a cylinder with volume $2{,}814.867$ in^3 and radius 8.

Solution:

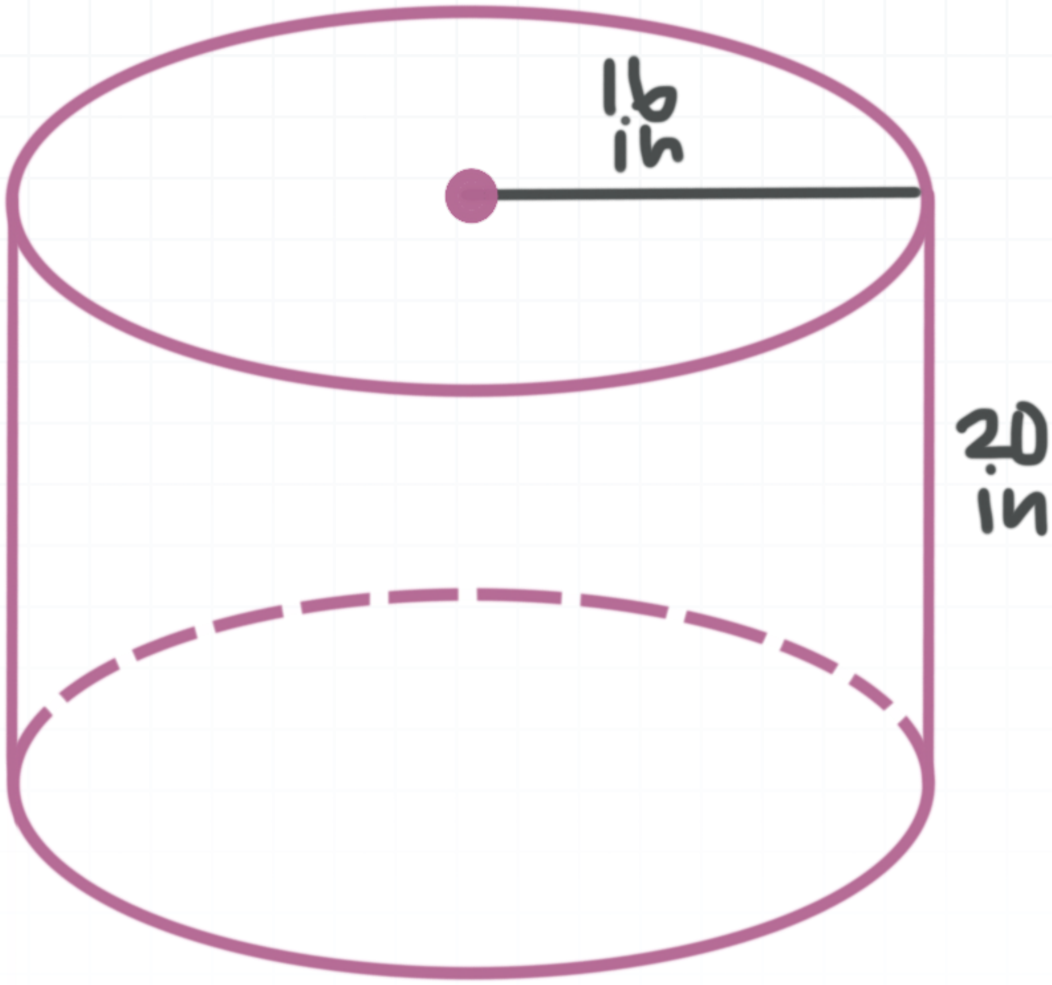
We'll plug into the volume equation, then solve it for height.

$$V = \pi r^2 h$$

$$2{,}814.867 = \pi(8^2)h$$

$$h = \frac{2{,}814.867}{64\pi} \approx 14 \text{ inches}$$

3. Find the surface area of the cylinder.



Solution:

Find the area of the circular bases.

$$A = \pi r^2 = \pi(16)^2 = 804.25$$

Double this to find the combined area of the top and bottom of the cylinder to get $804.25(2) = 1{,}608.50$. Now find the area of the rectangle by finding the circumference of the circle and multiplying it by the height of the cylinder.

$$C = 2\pi r = 2\pi(16) = 100.53$$

$$A = 100.53(20) = 2{,}010.62$$

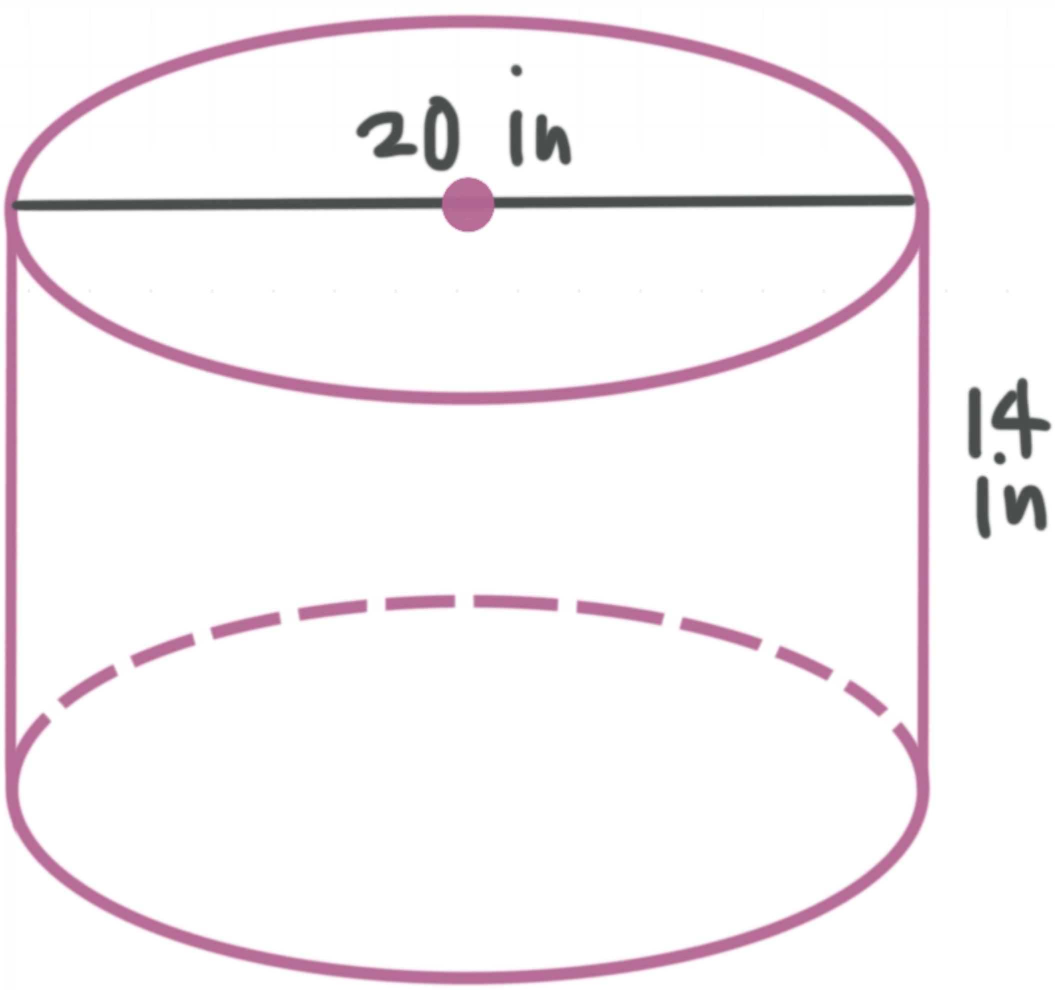
Then the total surface area of the cylinder is

$$A_{\text{Total}} = A_{\text{Bases}} + A_{\text{Side}}$$

$$A_{\text{Total}} = 1{,}608.50 + 2{,}010.62$$

$$A_{\text{Total}} = 3{,}619.12 \text{ in}^2$$

■ 4. The circumference of the base of the cylinder is 62.832 inches. Find its volume.



Solution:

The formula for the circumference of the circle gives

$$C = 2\pi r$$

$$62.832 = 2\pi r$$

$$r = 10$$

So the volume of the cylinder is

$$V = \pi r^2 h = \pi(10^2)(14) = 4{,}398.23 \text{ in}^3$$

NETS/VOLUME/SURFACE AREA OF CONES

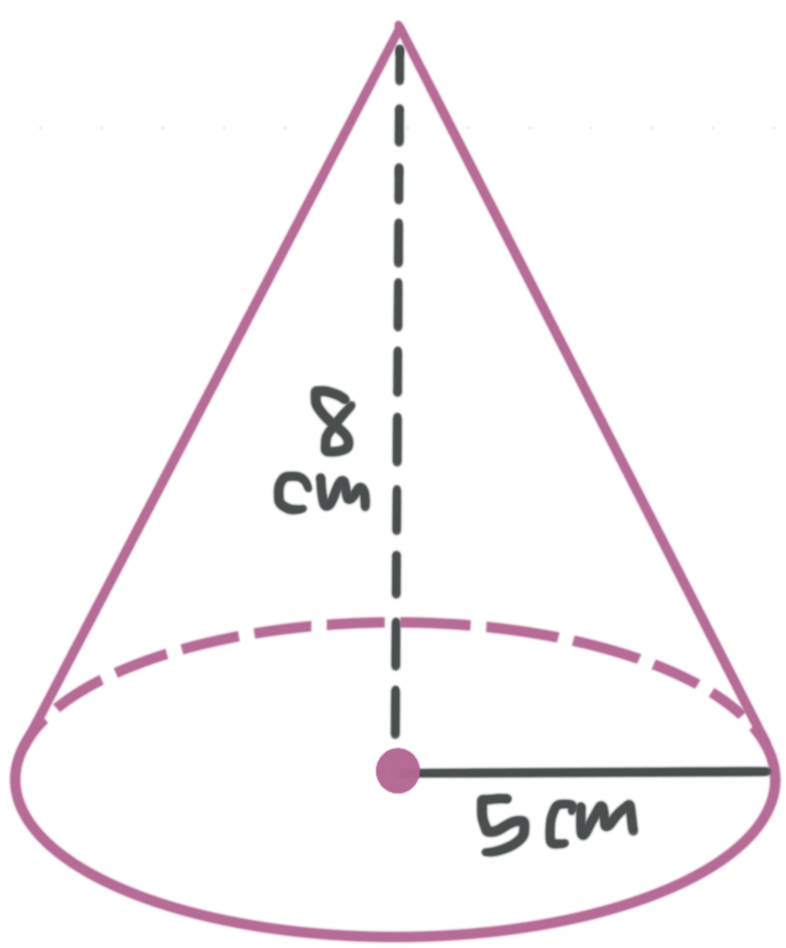
■ 1. Find the volume of a right cone with a height of 10.5 inches and a diameter of 8 inches at its base to the nearest hundredth.

Solution:

The volume of the cone is

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4^2)(10.5) = 175.93 \text{ in}^3$$

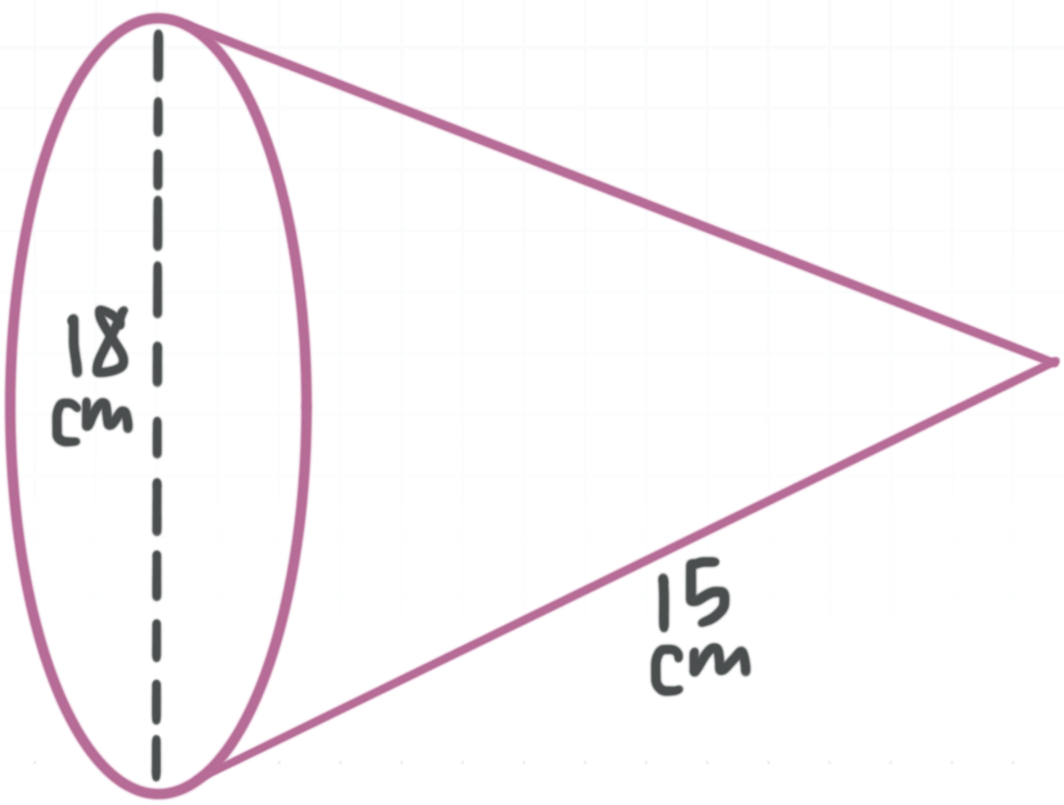
■ 2. Find the slant height of the cone.



Solution:

The slant height is $\sqrt{89} \approx 9.43$ cm.

■ 3. Find the surface area of the cone in terms of π.



Solution:

The volume of the cone is given by $V = L + \pi r^2$. Since $L = \pi rl = \pi(9)(15) = 135\pi$, we know the volume is

$$V = 135\pi + 81\pi = 216\pi$$

■ 4. The volume of a cone is 100π. Find the length of its radius if its height is 12.

Solution:

Using the formula for the volume of a cone, we get

$$V = \frac{1}{3}\pi r^2 h$$

$$100\pi = \frac{1}{3}\pi r^2(12)$$

$$100\pi = 4\pi r^2$$

$$25 = r^2$$

$$r = 5$$

VOLUME/SURFACE AREA OF SPHERES

■ 1. Find the volume to the nearest hundredth of a sphere with radius 15 inches.

Solution:

Plug what you know into the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(15)^3 = 14{,}137.17 \text{ in}^3$$

■ 2. A basketball has a diameter of 9.55 inches. Find its surface area to the nearest hundredth.

Solution:

Plug what you know into the formula for the surface area of a sphere.

$$S = 4\pi r^2 = 4\pi\left(\frac{9.55}{2}\right)^2 = 286.52 \text{ in}^2$$

■ 3. A sphere has radius 10. How much greater is the volume than the surface area in terms of π?

Solution:

Find the volume and the surface area, then find the difference between the two.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3 = \frac{4{,}000}{3}\pi$$

$$S = 4\pi r^2 = 4\pi(10)^2 = 400\pi$$

The difference is

$$\frac{4{,}000}{3}\pi - 400\pi = \frac{4{,}000}{3}\pi - \frac{1{,}200}{3}\pi = \frac{2{,}800}{3}\pi$$

■ 4. A sphere has a volume of 288π. Find its diameter.

Solution:

Plug what you know into the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

$$288\pi = \frac{4}{3}\pi r^3$$

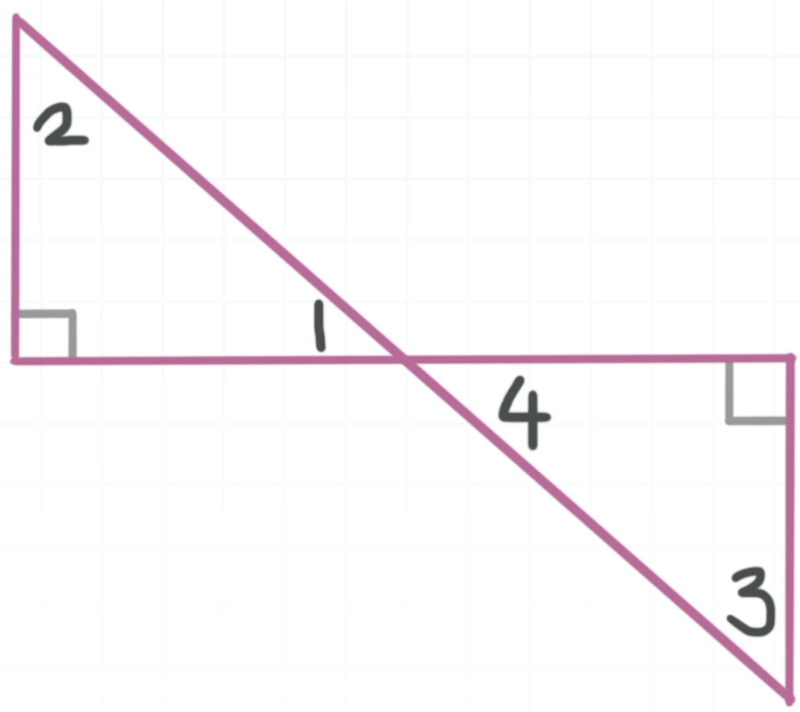
$$r^3 = 216$$

$$r = 6$$

Because the radius is 6 units long, the diameter must be 12 units long, and $d = 12$.

CONGRUENT ANGLES

■ 1. $m\angle 3 = 4x - 11$ and $m\angle 1 = 5x + 2$. Find $m\angle 2$.



Solution:

$m\angle 2 = 33^\circ$. $\angle 1 \cong \angle 4$ because they are vertical angles. And because the three interior angles of a triangle always sum to 180°,

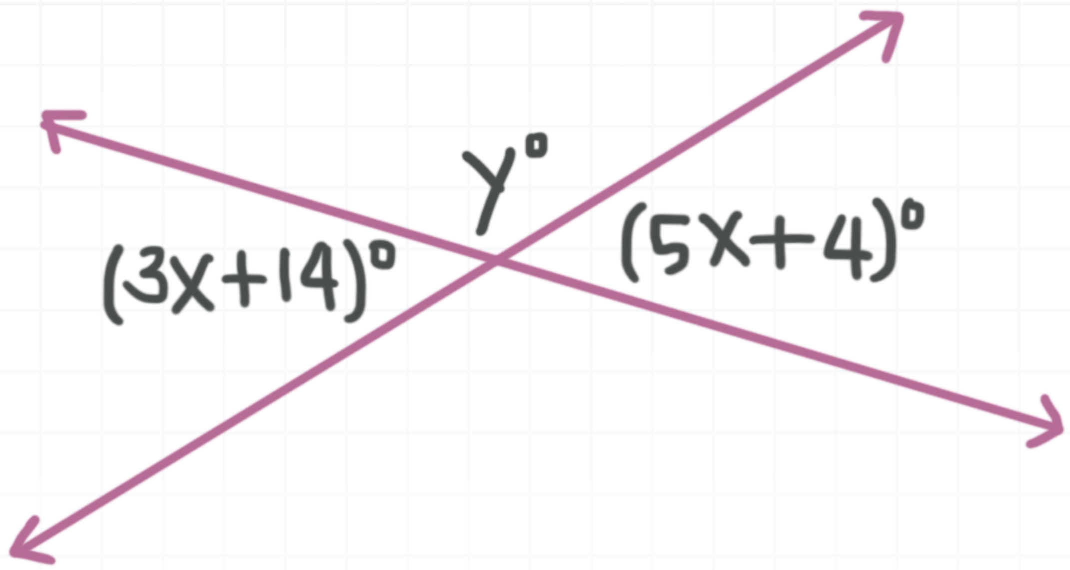
$$m\angle 3 + m\angle 4 + 90 = 180$$

$$4x - 11 + 5x + 2 + 90 = 180$$

$$x = 11$$

Then $m\angle 2 = m\angle 3 = 4(11) - 11 = 33^\circ$.

■ 2. Find the values of x and y.



Solution:

$x = 5$ and $y = 151$. Because they are vertical angles, we know that

$$3x + 14 = 5x + 4$$

$$10 = 2x$$

$$x = 5$$

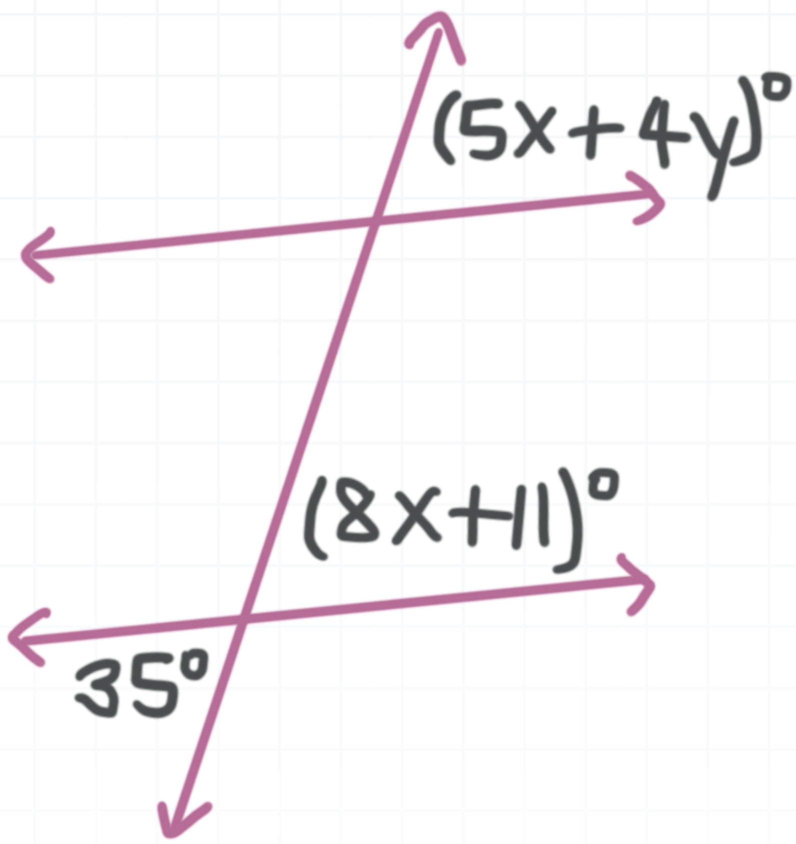
And because $(3x + 14)°$ and $y°$ are supplementary angles, we can say

$$3x + 14 + y = 180$$

$$3(5) + 14 + y = 180$$

$$y = 151$$

■ 3. Find the value of x and y.



Solution:

$x = 3$ and $y = 5$. Because they are vertical angles, we know that

$$35 = 8x + 11$$

$$24 = 8x$$

$$x = 3$$

And because they are alternate exterior angles, we can say

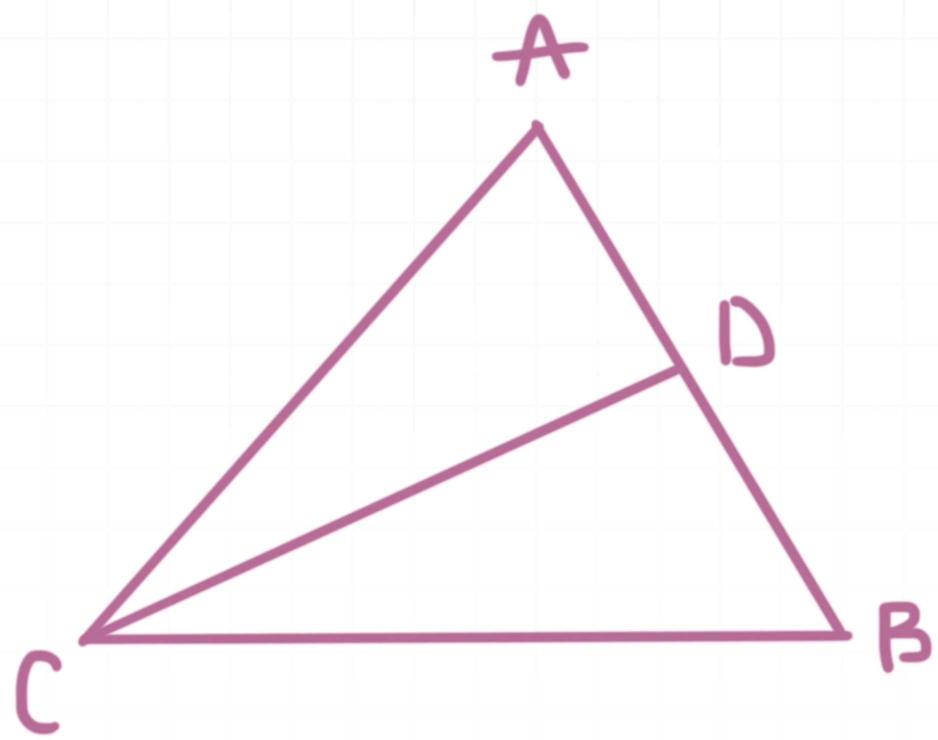
$$5x + 4y = 35$$

$$5(3) + 4y = 35$$

$$4y = 20$$

$$y = 5$$

4. $\overline{CD}$ is an angle bisector of the triangle and $\overline{CD} \perp \overline{AB}$. $m\angle CAD = 5x - 10$ and $m\angle BCD = 25$. Find x.



Solution:

$x = 15$. We know the interior angles of a triangle sum to 180°, so

$$m\angle DBC = 180^\circ - 90^\circ - 25^\circ = 65^\circ$$

And because $m\angle DBC = m\angle CAD$, we can say

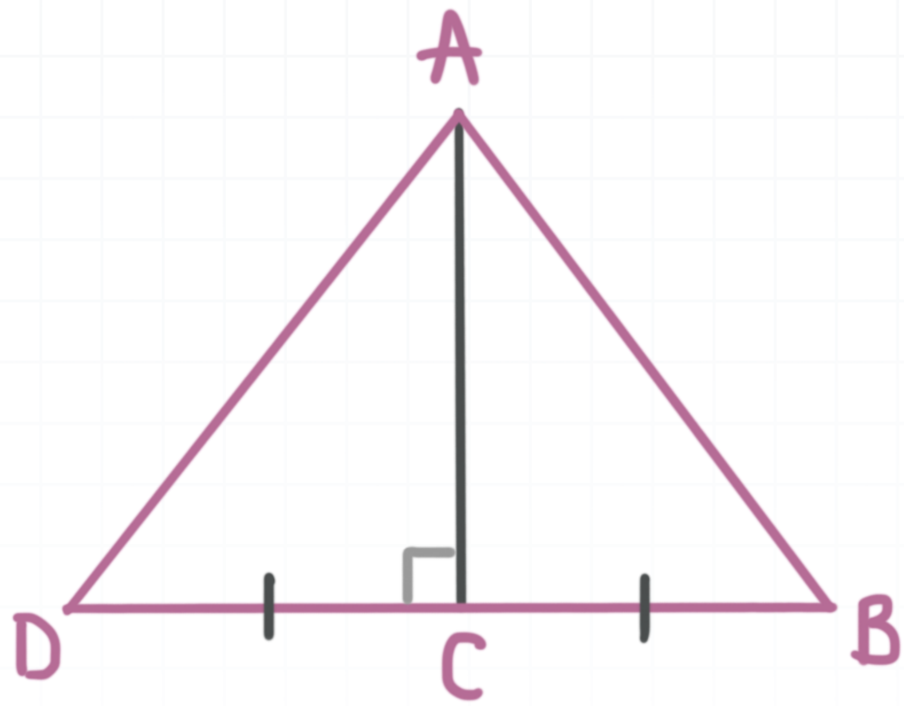
$$5x - 10 = 65$$

$$5x = 75$$

$$x = 15$$

TRIANGLE CONGRUENCE WITH SSS, ASA, SAS

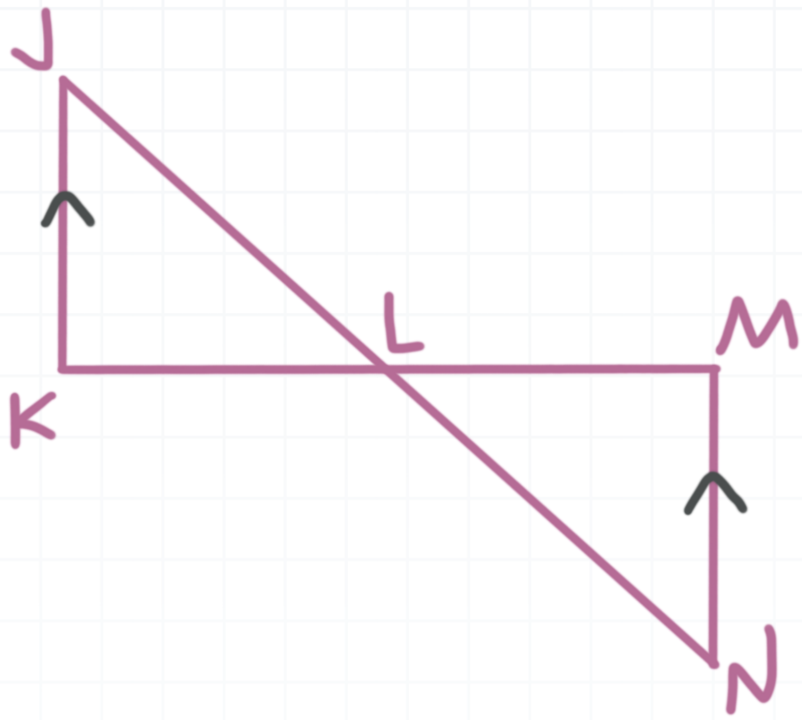
■ 1. Fill in the blank. $\triangle ABC \cong \triangle ADC$ by the ________________ Theorem.



Solution:

SAS (Side-Angle-Side) Theorem. We know $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. We know $\angle ACD \cong \angle ACB$ because they are both right angles. And we know $\overline{DC} \cong \overline{BC}$ because of the markings shown on the diagram.

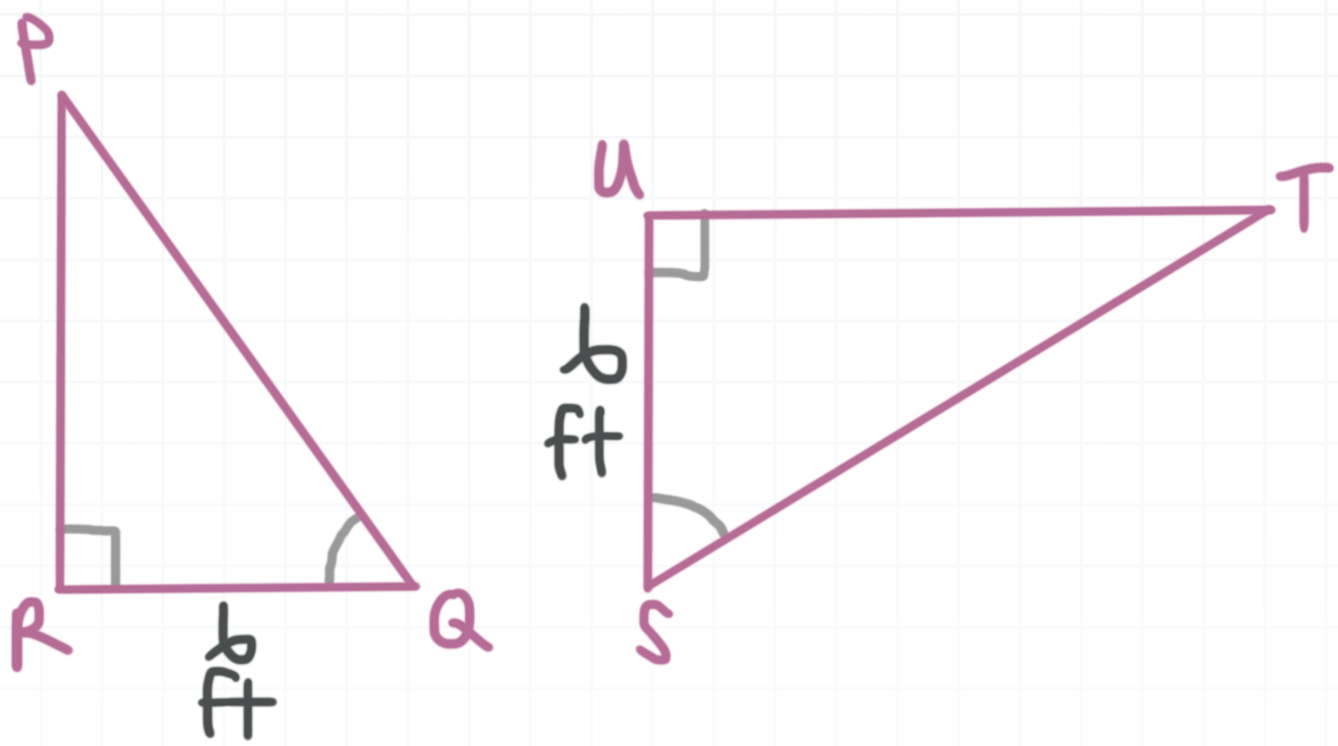
■ 2. Fill in the blank. L is a midpoint of $\overline{JN}$. $\triangle JKL \cong \triangle NML$ by the ________________ Theorem.



Solution:

ASA (Angle-Side-Angle) Theorem. $\angle JLK \cong \angle NLM$ because they are vertical angles. $\angle J \cong \angle N$ because they are alternate interior angles. $JL = NL$ because L is a midpoint of $\overline{JN}$.

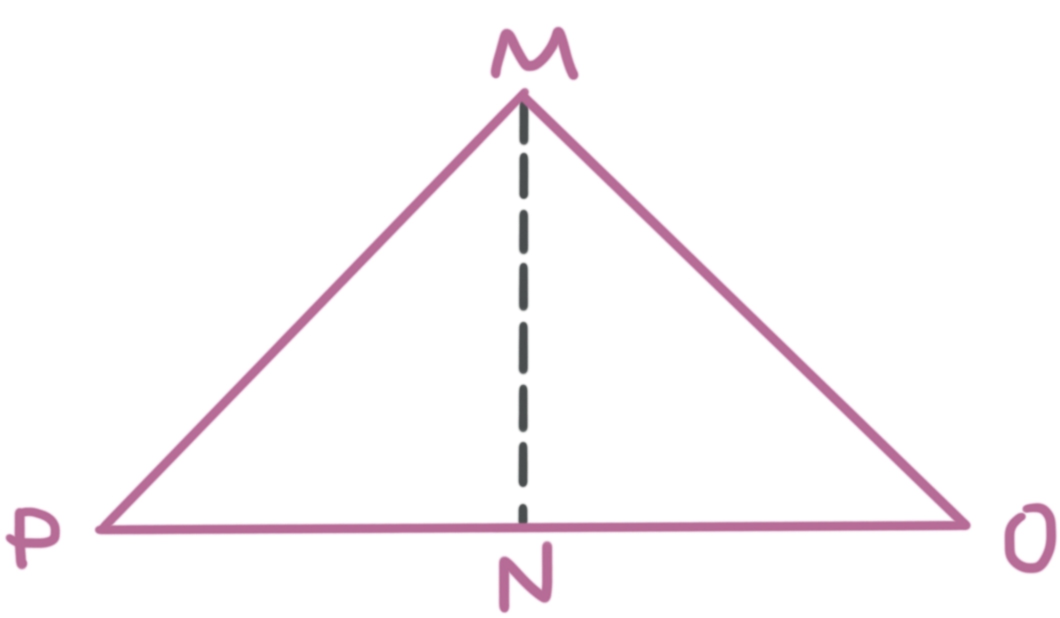
■ 3. $\triangle PRQ \cong \triangle$ ________________ by the ________________ Theorem.



Solution:

$\triangle TUS$ by the ASA (Angle-Side-Angle) Theorem. In the diagram, $\angle Q \cong \angle W$, $\angle R \cong \angle U$, and $RQ = 6 = US$.

■ 4. $\triangle PMD$ is an isosceles triangle with vertex angle at M. N is a midpoint of $\overline{PD}$. $\triangle PMN \cong \triangle DMN$ by the ________________ Theorem.

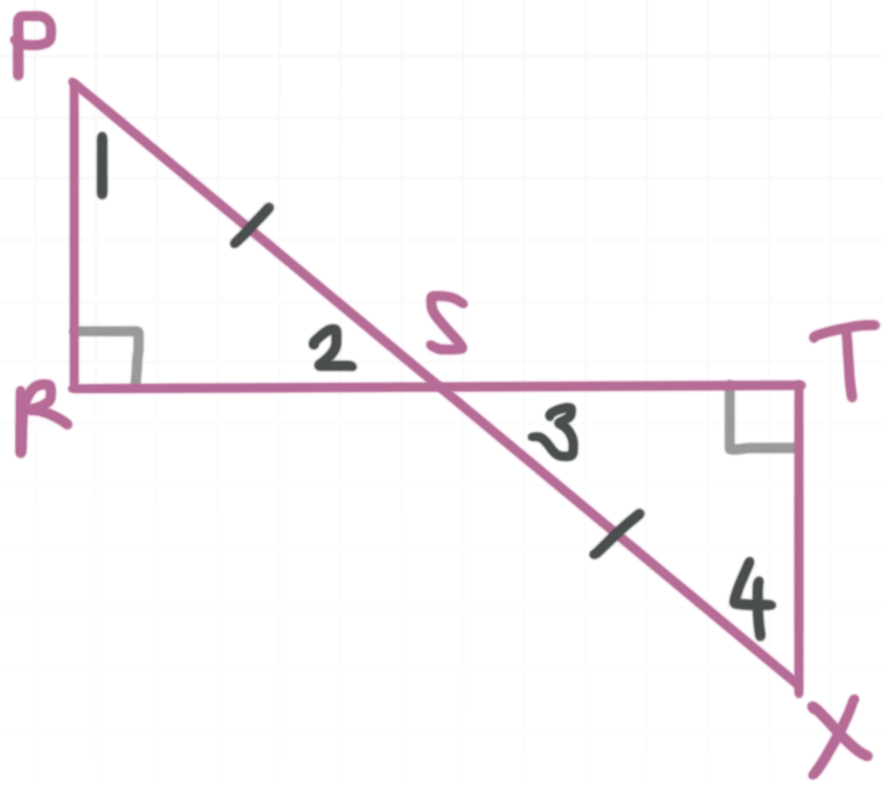


Solution:

SSS (Side-Side-Side) Theorem. We know that $\overline{MN} \cong \overline{MN}$ by the Reflexive Property of Congruence. $\overline{PM} \cong \overline{DM}$ because $\triangle PMD$ is isosceles. And $\overline{PN} \cong \overline{DN}$ because N is a midpoint.

TRIANGLE CONGRUENCE WITH AAS, HL

■ 1. Which theorem could be used to prove $\triangle PRS \cong \triangle XTS$?

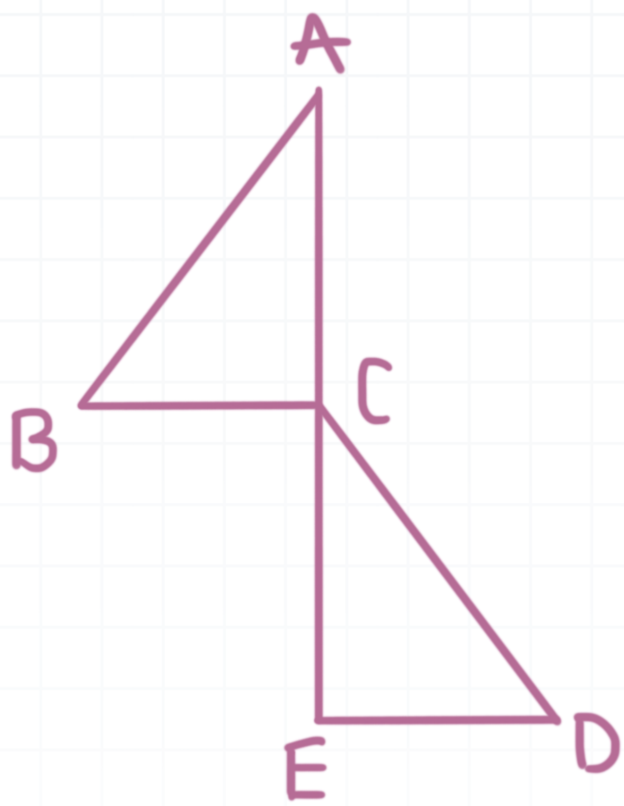


Solution:

AAS (Angle-Angle-Side) Theorem. We know that $\angle 2 \cong \angle 3$ because they are vertical angles, and that $\angle R \cong \angle T$ because they are both right angles, and the diagram shows $\overline{PS} \cong \overline{XS}$.

■ 2. Which theorem could be used to prove $\triangle ACB \cong \triangle ECD$? The following facts are given about the triangles.

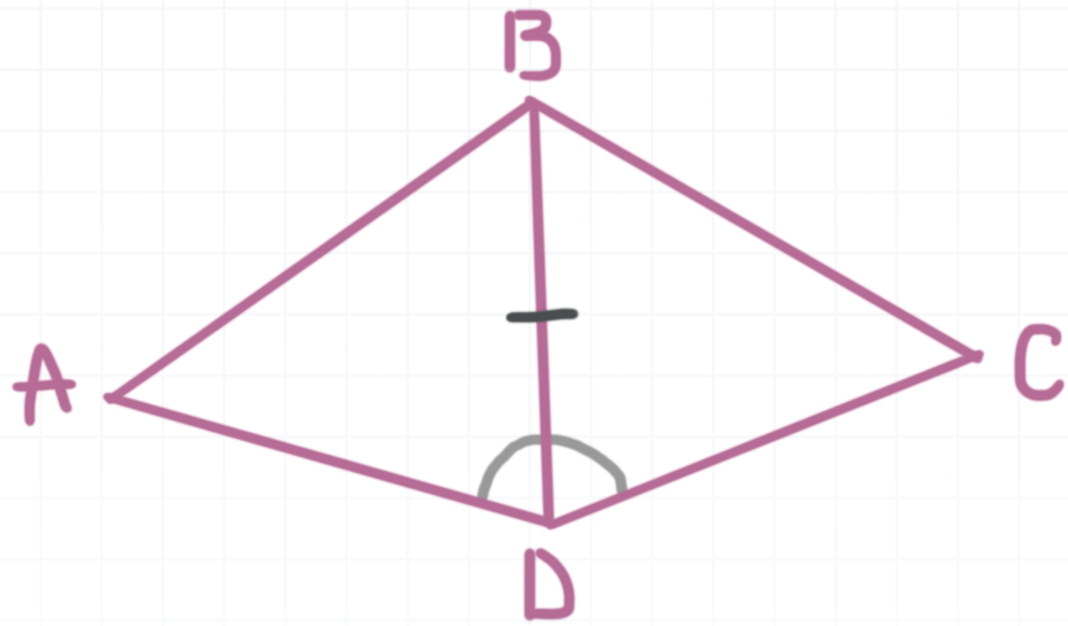
$\overline{AE} \perp \overline{BC}$, $BC || DE$, $\overline{AB} \cong \overline{DC}$, and C is a midpoint of $\overline{AE}$



Solution:

HL. We’re told that the hypotenuses are congruent. We also know that $AC = EC$, because C is a midpoint. This makes a set of legs congruent.

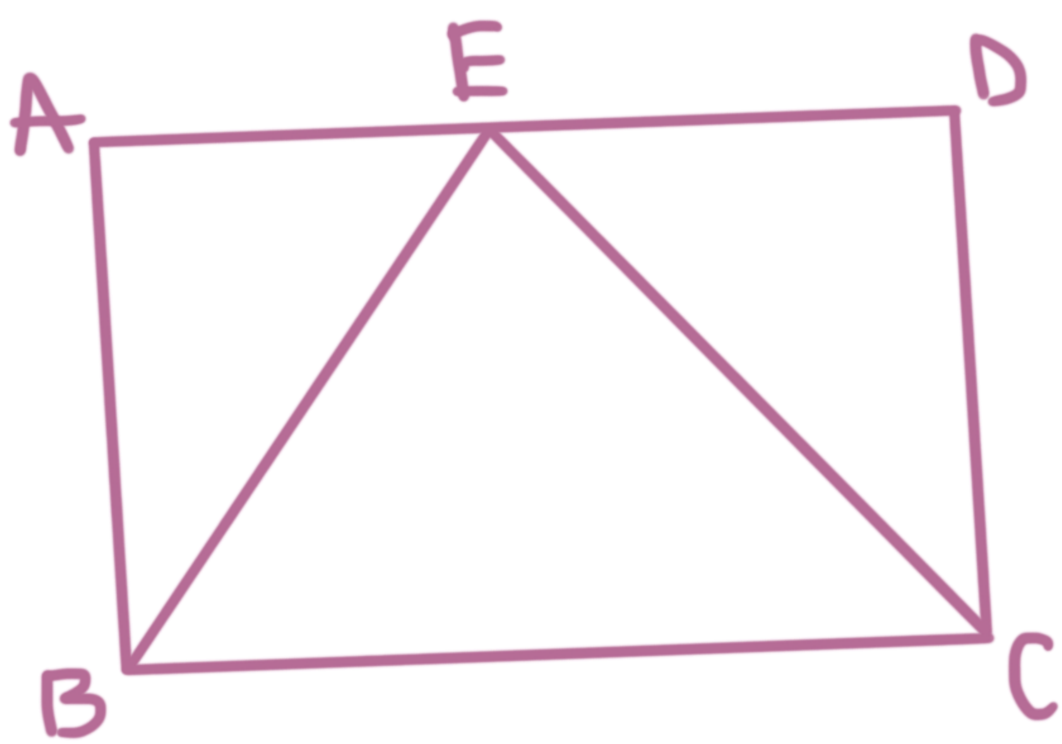
■ 3. What additional information would we need to prove these triangles are congruent using AAS Theorem?



Solution:

$\angle A \cong \angle C$

■ 4. $ABCD$ is a rectangle. BEC is an isosceles triangle with vertex angle at E. Write a proof to verify that $\triangle BAE \cong \triangle CDE$ by the HL Theorem.

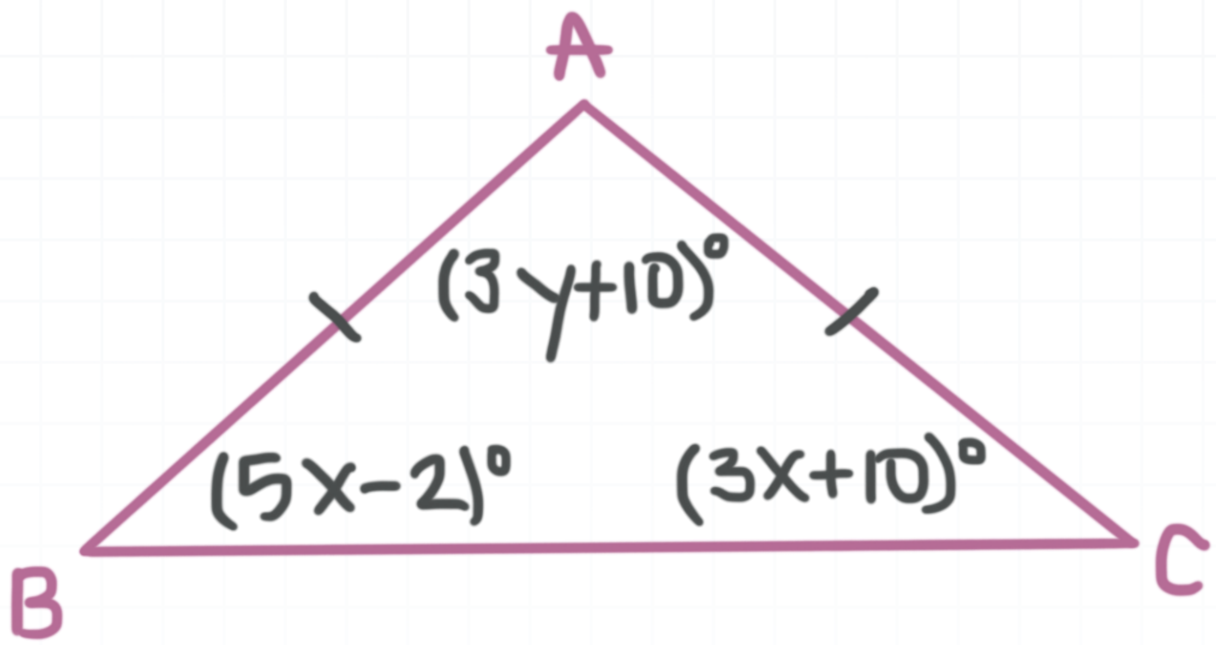


Solution:

$\angle A$ and $\angle D$ must be right angles, because $ABCD$ is a rectangle. $\triangle BAE$ and $\triangle CDE$ must be right triangles by definition of a right triangle. $\overline{AB} \cong \overline{DC}$ because opposite sides of rectangles are congruent, and $\overline{BE} \cong \overline{CE}$ because $\triangle BEC$ is an isosceles triangle. Therefore, $\triangle BAE \cong \triangle CDE$ by the HL Theorem.

ISOSCELES TRIANGLE THEOREM

■ 1. Find the values of x and y.



Solution:

$x = 6$ and $y = 38$. Because the triangle is isosceles, we get

$$5x - 2 = 3x + 10$$

$$2x = 12$$

$$x = 6$$

Therefore, the matching angles are

$$m\angle B = m\angle C = 5(6) - 2 = 28^\circ$$

Which means that $m\angle A$ must be $m\angle A = 180 - 28 = 124^\circ$, which means the value of y is

$$3y + 10 = 124$$

$$3y = 114$$

$$y = 38$$

■ 2. $\triangle JKL$ is isosceles with vertex angle K. $JK = 4x - 5$, $LK = 3x + 8$, and $m\angle J = 2x + 4$. Find $m\angle L$.

Solution:

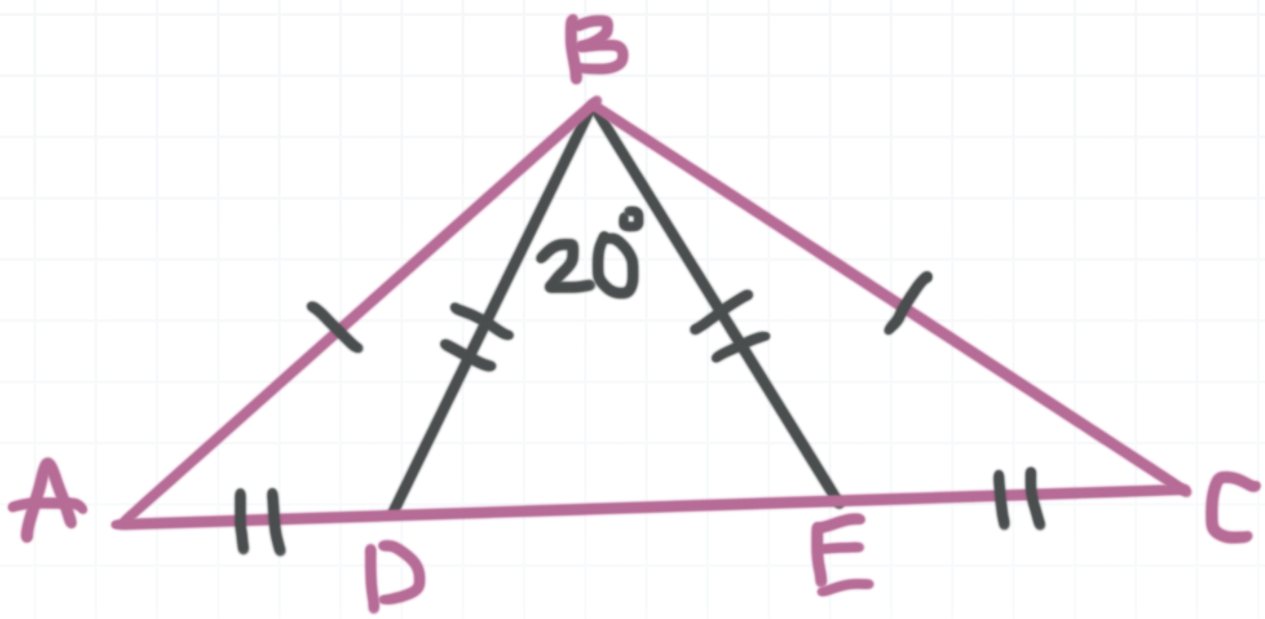
30°. $JK = LK$ because the triangle is isosceles.

$$4x - 5 = 3x + 8$$

$$x = 13$$

Then we can say $m\angle J = 2(13) + 4 = 30^\circ$, and therefore that $m\angle C = m\angle J = 30^\circ$ by the Isosceles Triangle Theorem.

■ 3. Find $m\angle ABC$.



Solution:

100°. By the Triangle Sum Theorem and the Isosceles Triangle Theorem,

$$m\angle BDE = m\angle BED = \frac{180 - 20}{2} = 80^\circ$$

Then because they form a linear pair,

$$m\angle ADB = 180 - 80 = 100^\circ$$

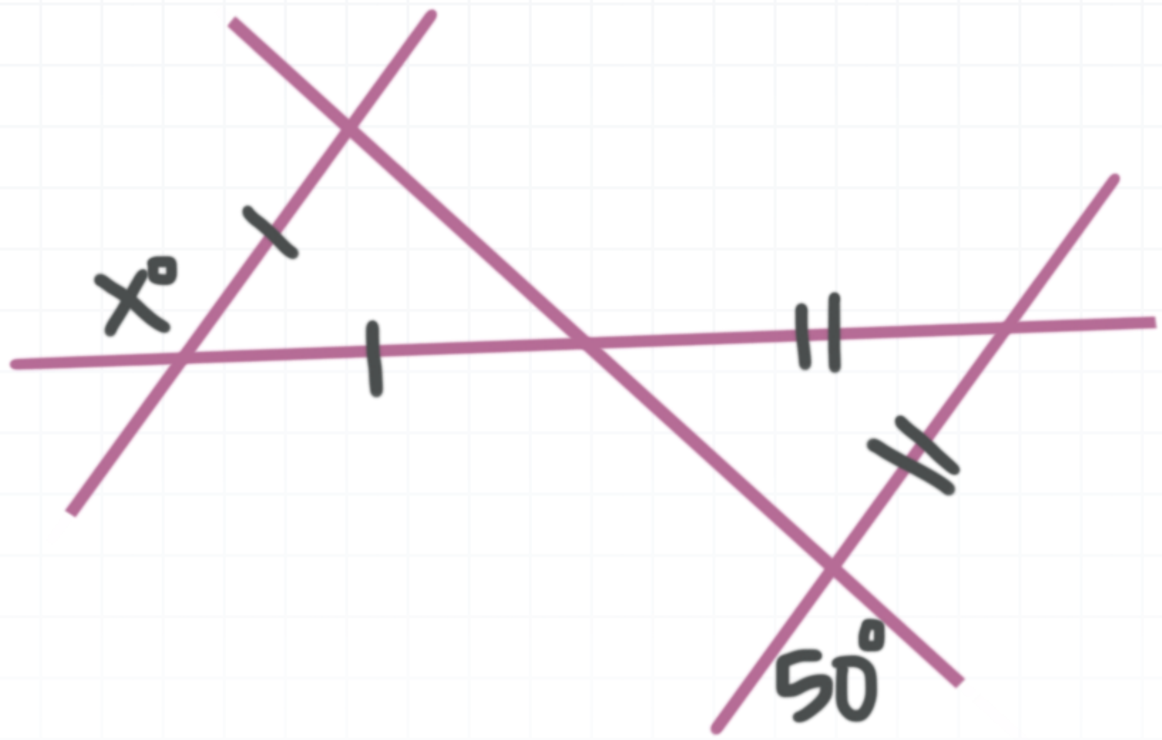
So

$$m\angle ABD = \frac{180 - 100}{2} = 40^\circ \text{ and } m\angle EBC = 40^\circ$$

Which means

$$m\angle ABC = 40 + 20 + 40 = 100^\circ$$

■ 4. Find x.



Solution:

100°. Use the Isosceles Triangle Theorem, vertical angles, and supplementary angles to find all the missing angles in the diagram.

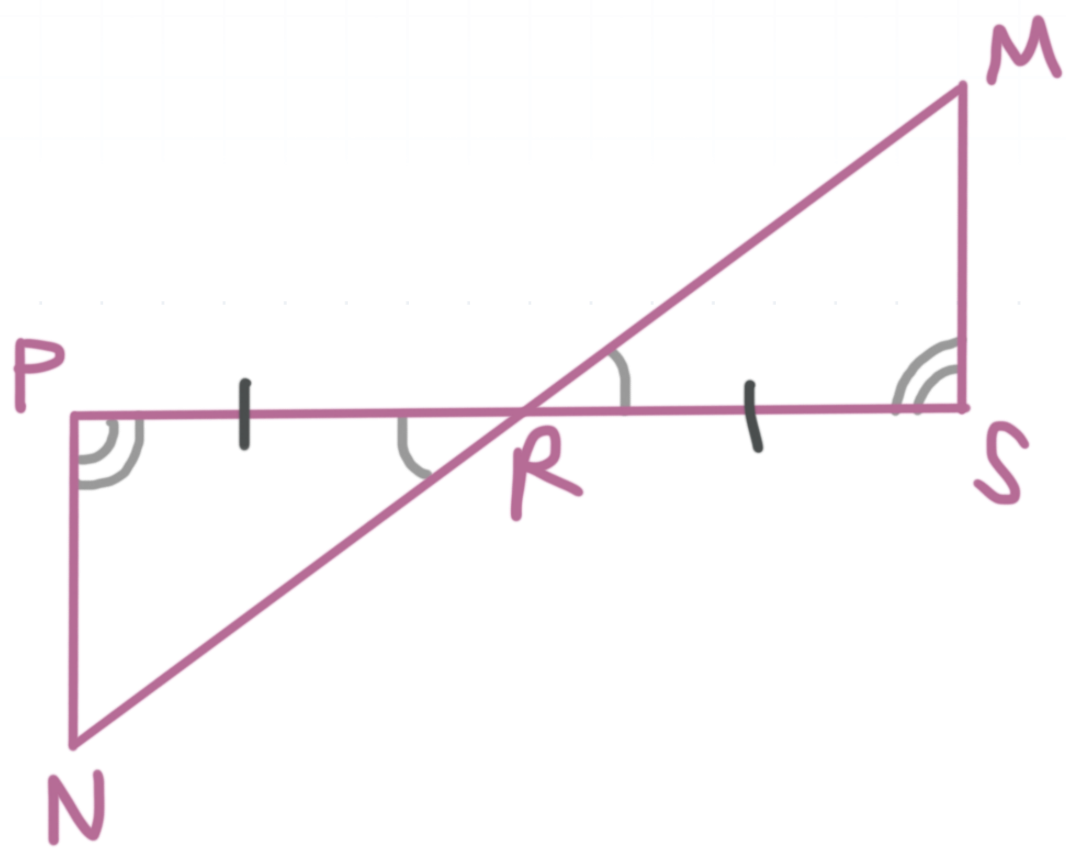
CPCTC

■ 1. Fill in the blank. Given $\triangle LMO \cong \triangle SQR$, $\overline{LO} \cong$ ________________.

Solution:

$\overline{SR}$. By CPCTC, these two line segments must be congruent if the triangles are congruent.

■ 2. Determine whether $\angle M \cong \angle N$. Justify your answer.



Solution:

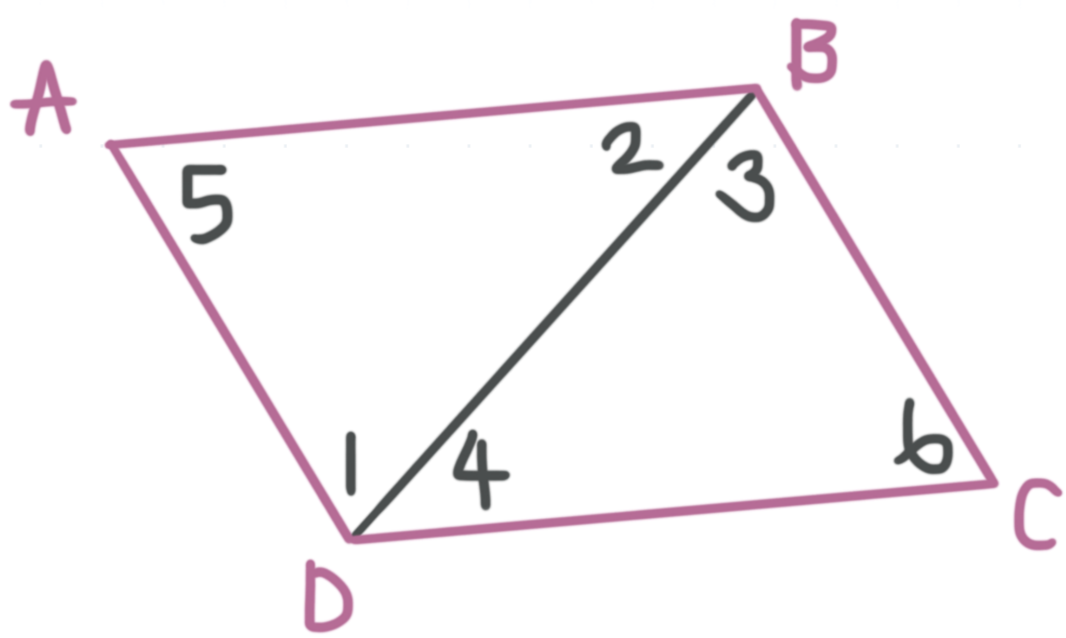
Yes, $\triangle PRN \cong \triangle SRM$ by the ASA Theorem. Therefore, $\angle M \cong \angle N$ by CPCTC.

■ 3. $\triangle DOG \cong \triangle TCA$ by SSS. What three conclusions can be drawn by CPCTC?

Solution:

$\angle D \cong \angle T$, $\angle O \cong \angle C$, and $\angle G \cong \angle A$. Congruent parts of congruent triangles are congruent (CPCTC), which makes each corresponding pair of angles congruent.

■ 4. Given $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$, prove $\overline{AB} \cong \overline{CD}$.



Solution:

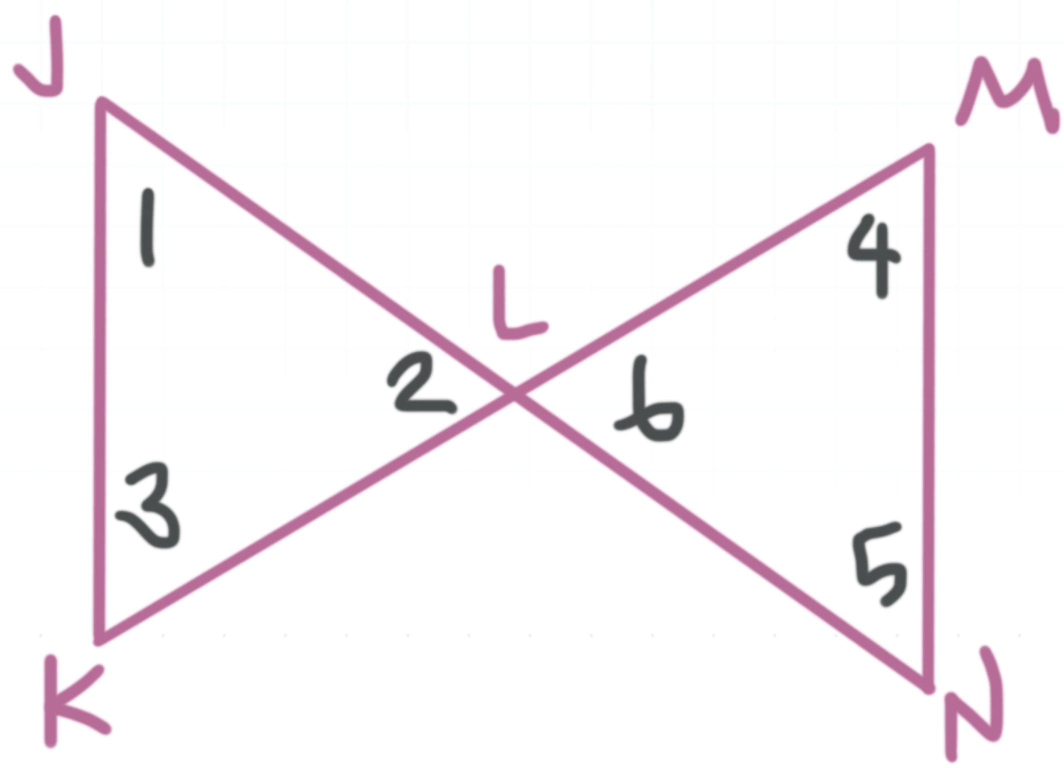
1. Given $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.

2. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence.

3. $\triangle ABD \cong \triangle CDB$ by the ASA Theorem.

4. $\overline{AB} \cong \overline{CD}$ by CPCTC.

■ 5. Given that L is the midpoint of $\overline{JN}$ and $\overline{KM}$, prove $\overline{JK} \cong \overline{NM}$.



Solution:

1. L is the midpoint of $\overline{JN}$ and $\overline{KM}$, so by definition of midpoint $JL = NL$ and $ML = KL$.

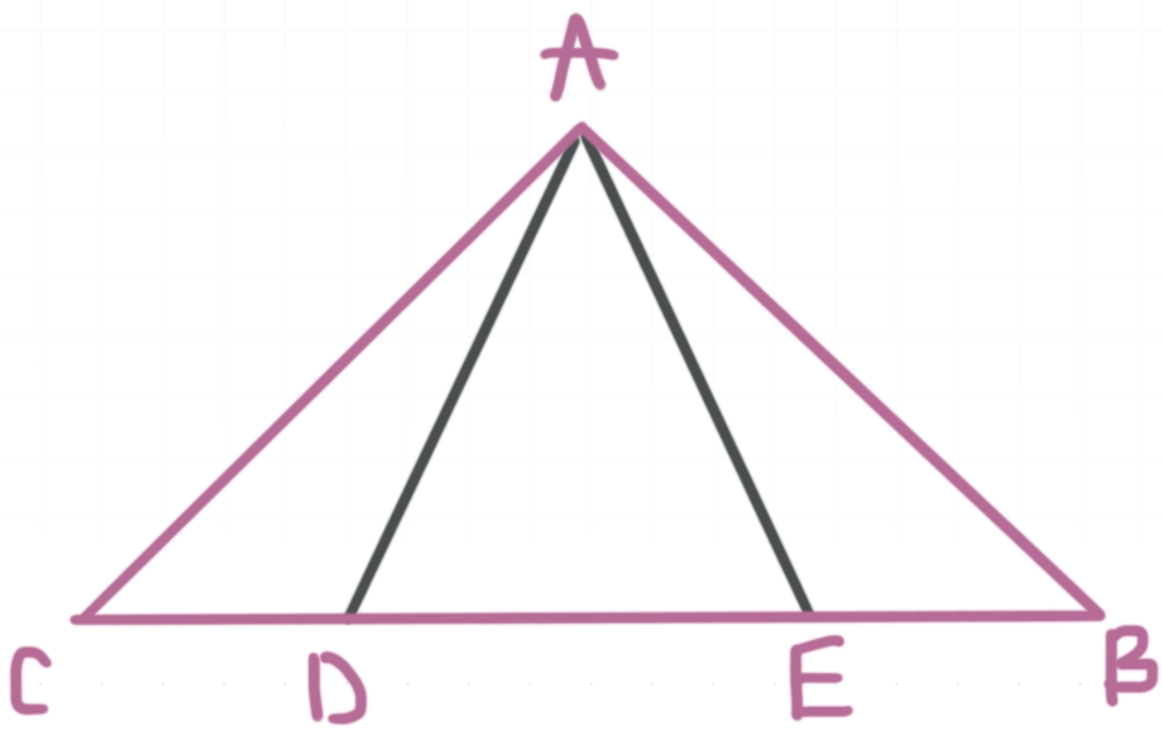
2. $\overline{JL} \cong \overline{NL}$ and $\overline{ML} \cong \overline{KL}$ by definition of congruent segments.

3. $\angle 2 \cong \angle 6$ by definition of vertical angles.

4. $\triangle JLK \cong \triangle NLM$ by SAS Theorem.

5. $\overline{JK} \cong \overline{NM}$ by CPCTC.

■ 6. Given that $\triangle CAB$ is an isosceles triangle, that D is the midpoint of $\overline{CE}$, and that E is the midpoint of $\overline{BD}$, prove that $\triangle DAE$ is isosceles.



Solution:

1. $\triangle CAB$ is an isosceles triangle, so by definition, $\overline{AC} \cong \overline{AB}$.

2. D is the midpoint of $\overline{CE}$, and E is the midpoint of $\overline{BD}$, so by definition $\overline{CD} = \overline{DE}$ and $\overline{DE} = \overline{EB}$.

3. $\overline{CD} \cong \overline{DE}$ and $\overline{DE} \cong \overline{EB}$ by definition of congruent segments.

4. $\overline{CD} \cong \overline{EB}$ by the Transitive Property of Congruence.

5. $\angle C \cong \angle B$ by the Isosceles Triangle Theorem.

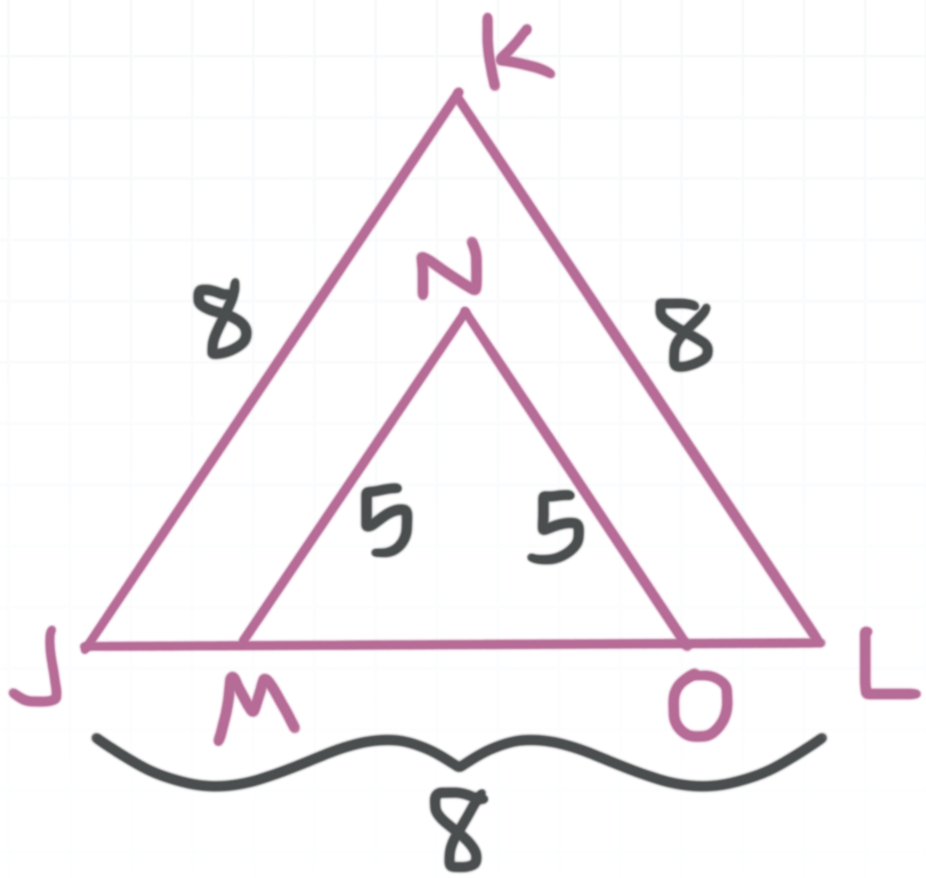
6. $\triangle ACD \cong \triangle ABE$ by the SAS Theorem.

7. $\overline{AD} \cong \overline{AE}$ by CPCTC.

8. $\triangle DAE$ is isosceles by the definition of an isosceles triangle.

SIMILAR TRIANGLES

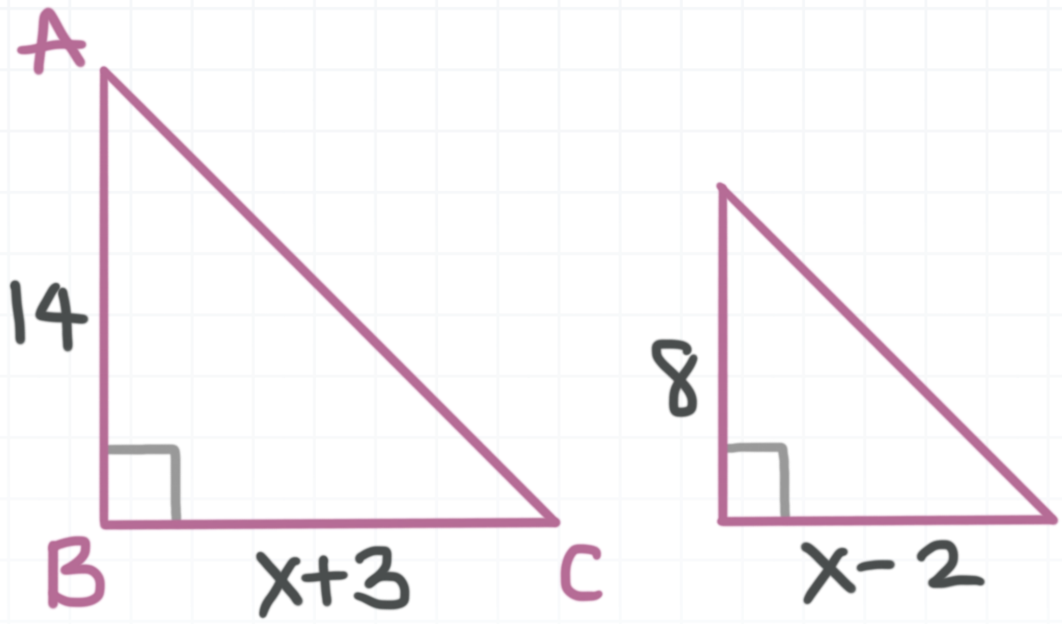
■ 1. $\triangle JKL$ is similar to $\triangle MNO$. Find MO.



Solution:

5. $8/5$ must be the proportion of all corresponding sides of the similar triangles. $\triangle JKL$ is equilateral and $\triangle MNO$ must also be equilateral.

■ 2. $\triangle ABC$ is similar to $\triangle DEF$. Set up a proportion to find the value of x.



Solution:

$x = 26/3$. Set up a proportion, the cross multiply to solve for x.

$$\frac{14}{8} = \frac{x+3}{x-2}$$

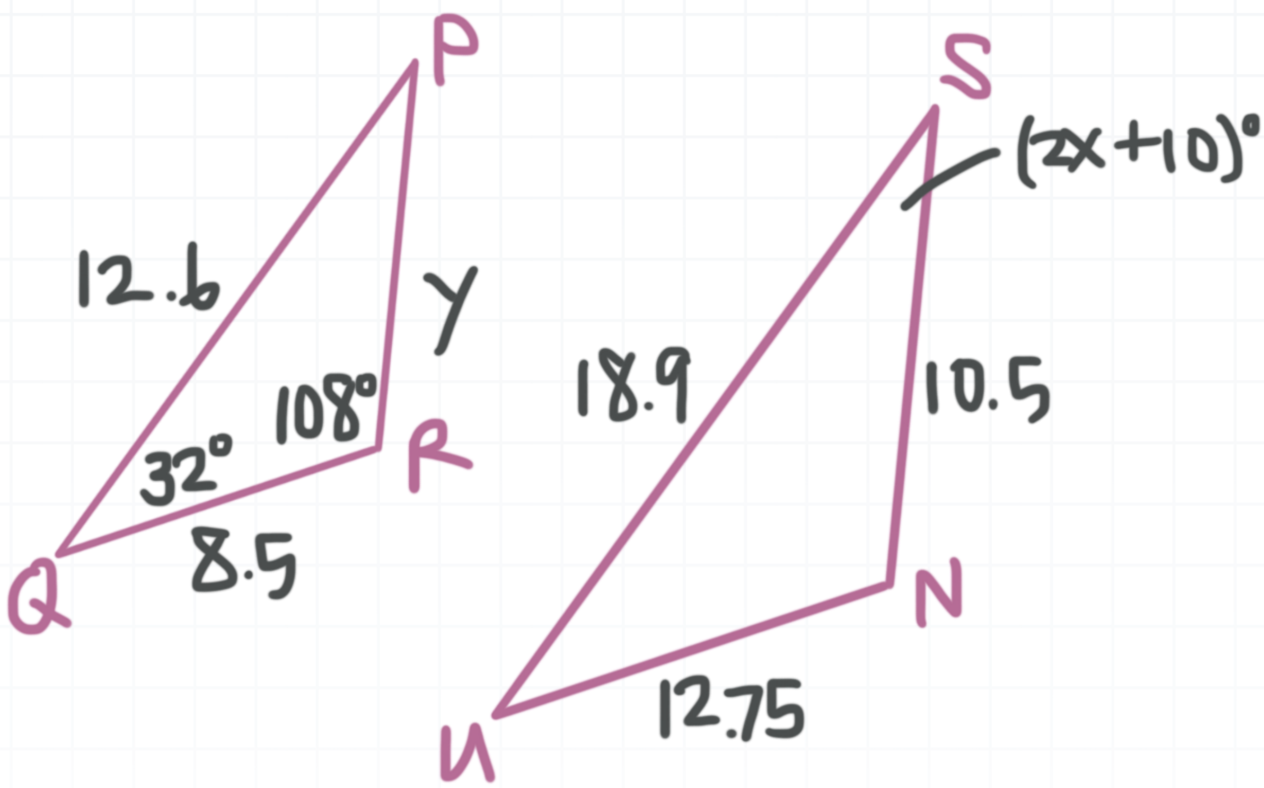
$$14(x-2) = 8(x+3)$$

$$14x - 28 = 8x + 24$$

$$6x = 52$$

$$x = \frac{26}{3}$$

■ 3. $\triangle PQR$ is similar to $\triangle SUN$. Find the values of x and y.



Solution:

$x = 15$ and $y = 7$. Set up a proportion to find y.

$$\frac{8.5}{12.75} = \frac{y}{10.5}$$

$$(8.5)(10.5) = 12.75y$$

$$12.75y = 89.25$$

$$y = 7$$

Then we can find $m\angle P$ as $m\angle P = 180^\circ - 32^\circ - 108^\circ = 40^\circ$. And since $m\angle P = m\angle S$, we can find x be setting up an equation.

$$40 = 2x + 10$$

$$30 = 2x$$

$$x = 15$$

■ 4. A 14-foot tree casts a 6-foot long shadow. A 3.5-foot tall child would have a shadow length of how many feet?

Solution:

0.75 feet. Set up the proportion.

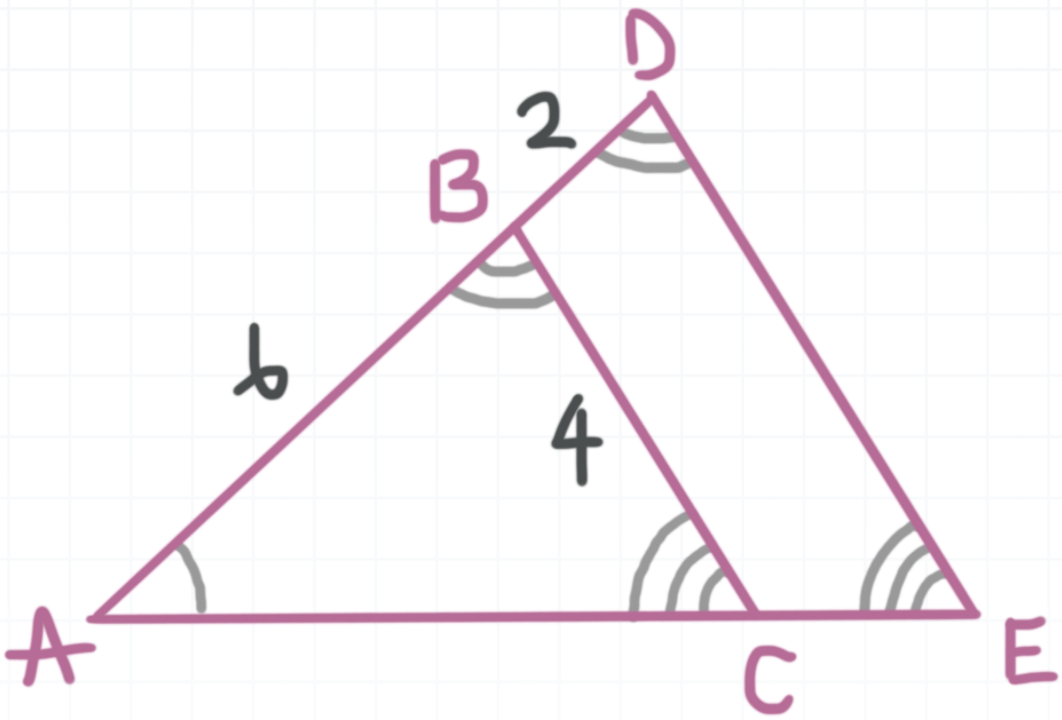
$$\frac{14}{3} = \frac{3.5}{x}$$

$$14x = (3)(3.5)$$

$$14x = 10.5$$

$$x = 0.75$$

■ 5. Find DE.



Solution:

$DE = 16/3$. $\triangle ABC$ is similar to $\triangle ADE$, so

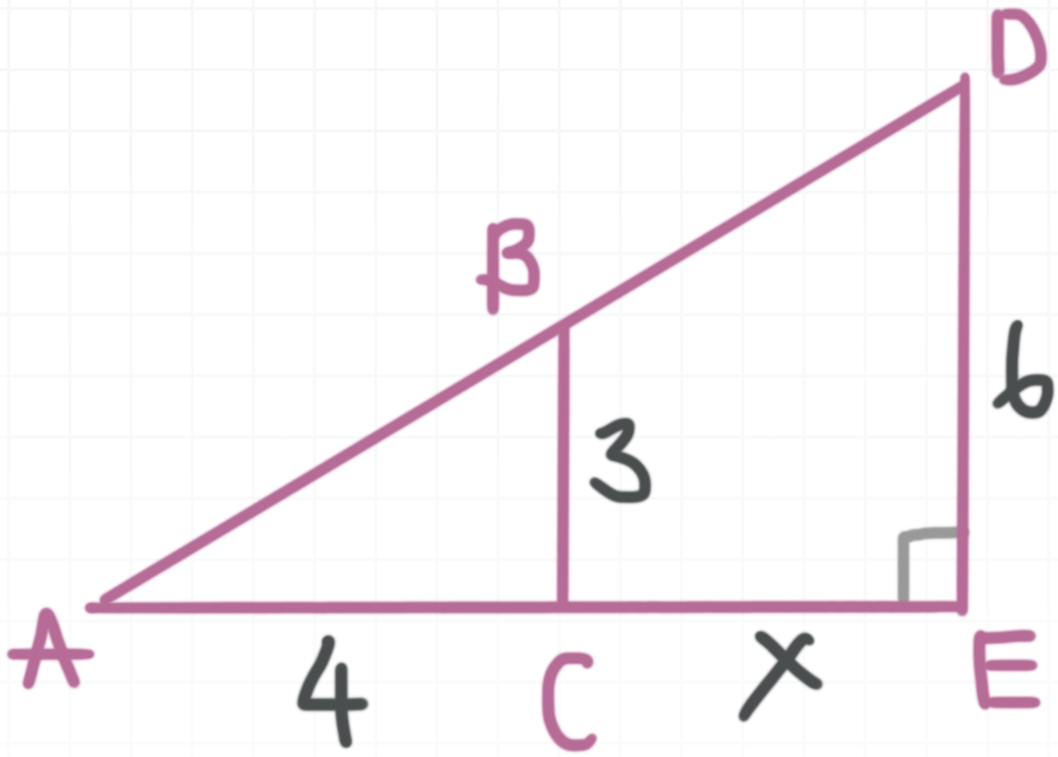
$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\frac{6}{4} = \frac{8}{DE}$$

$$6DE = 32$$

$$DE = \frac{16}{3}$$

■ 6. Find CE.



Solution:

$CE=4$. $\triangle ABC$ is similar to $\triangle ADE$, so

$$\frac{AC}{AE}=\frac{BC}{DE}$$

$$\frac{4}{4+x}=\frac{3}{6}$$

$$24=3(4+x)$$

$$24=12+3x$$

$$12=3x$$

$$x=4$$

45-45-90 TRIANGLES

■ 1. $\triangle PDX$ is an isosceles right triangle with vertex $\angle D$, and $PD = 4$. Find DX and XP.

Solution:

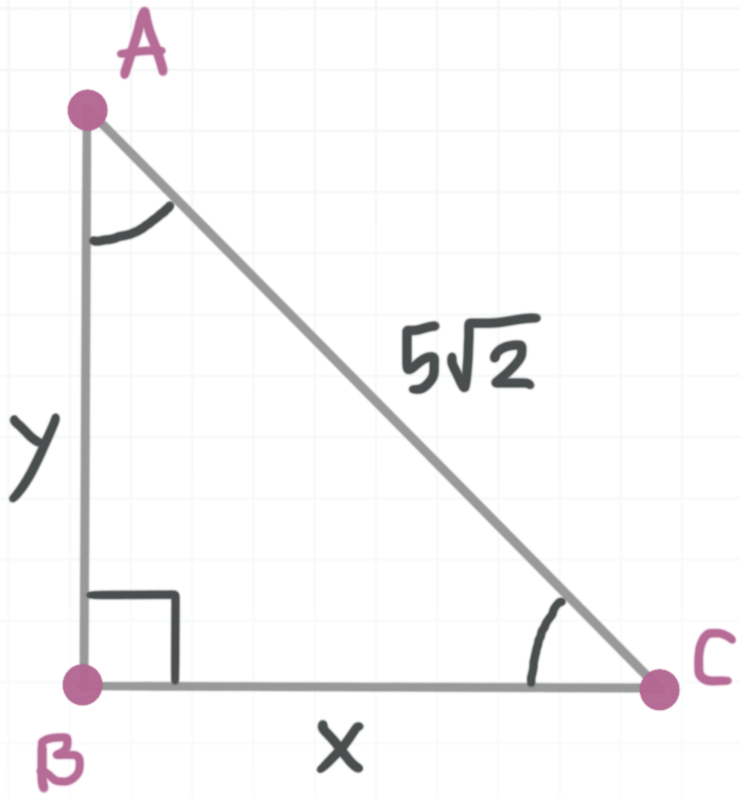
$DX = 4$ and $XP = 4\sqrt{2}$. By the $45-45-90$ rule of right triangles, legs of the triangle are congruent and the hypotenuse has a measure of $\sqrt{2}$. The legs are PD and DX and both have measures of 4. The hypotenuse is XP and has measure $4\sqrt{2}$.

■ 2. A square has a perimeter of 40 meters. Find the length of the diagonal of the square.

Solution:

$10\sqrt{2}$. Since the perimeter is 40, we know the length of each side is 10. Using the $45-45-90$ rule of right triangles, we get the length of the diagonal to be $10\sqrt{2}$.

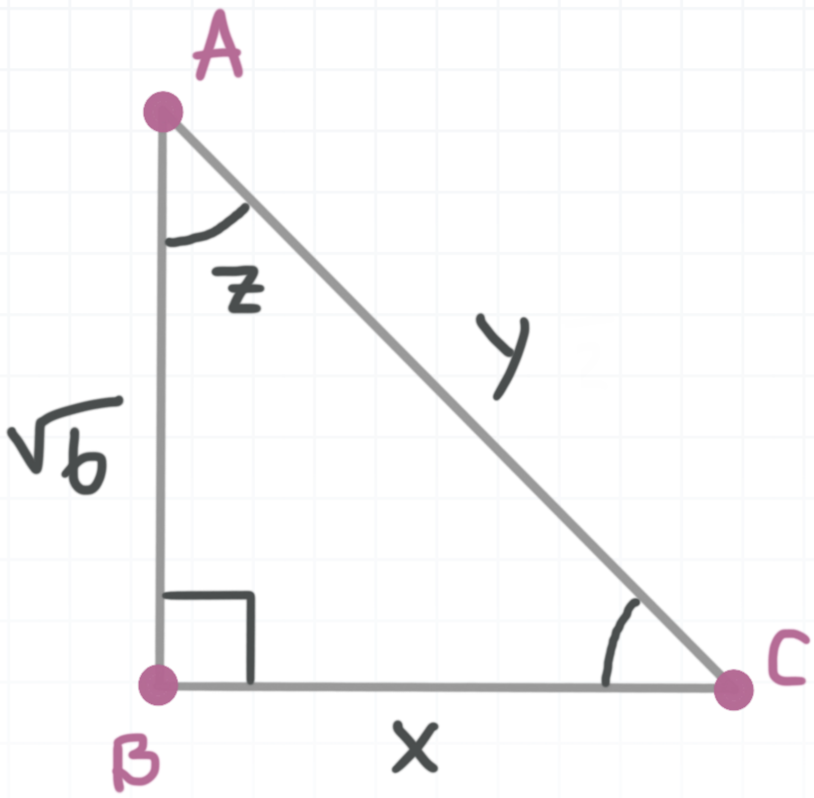
■ 3. Find the values of x and y.



Solution:

$x=5$ and $y=5$. By the $45-45-90$ rule of right triangles, legs of the triangle are congruent and the hypotenuse has a measure of $\sqrt{2}$. The value of each leg must equal 5.

■ 4. Find the values of x, y, and z.



Solution:

$x = \sqrt{6}$, $y = 2\sqrt{3}$, and $z = 45$. In our $45-45-90$ special right triangle, the legs are congruent. $\overline{AB} \cong \overline{CB}$, which means they both have a measure of $\sqrt{6}$. The length of the hypotenuse can be found by taking the measure of the leg and multiplying it by $\sqrt{2}$.

$$y = \sqrt{6}\sqrt{2}$$

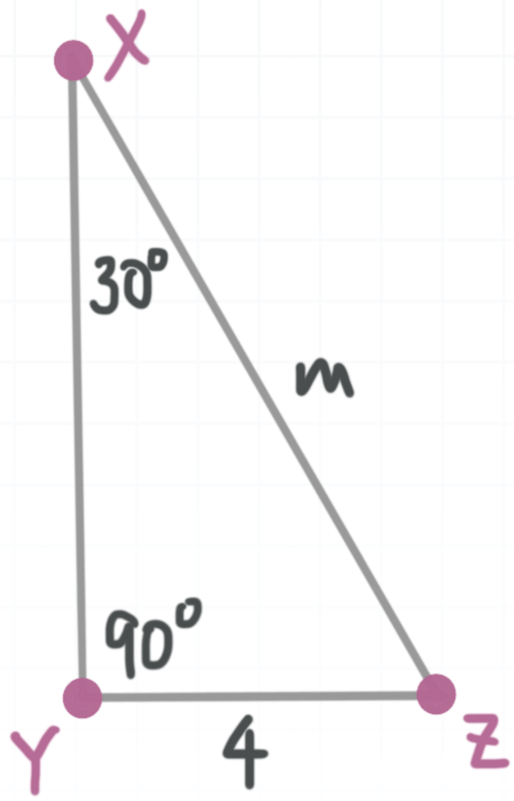
$$y = \sqrt{12}$$

$$y = \sqrt{4}\sqrt{3}$$

$$y = 2\sqrt{3}$$

30-60-90 TRIANGLES

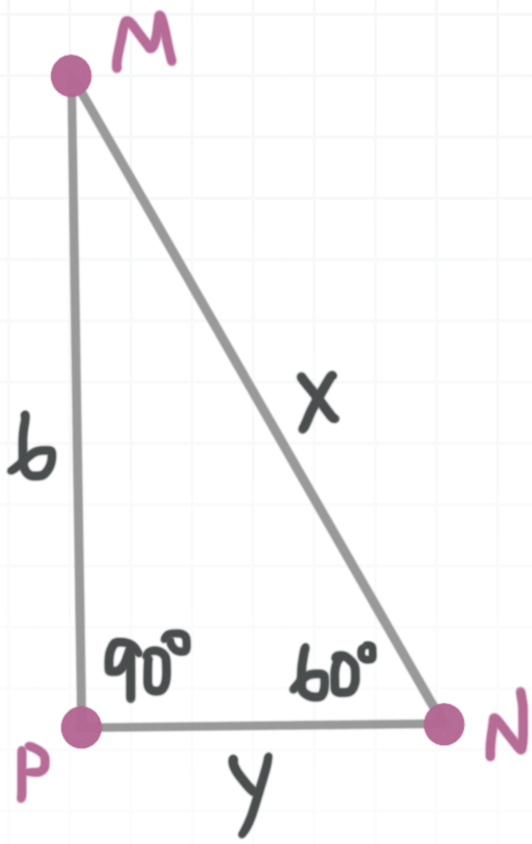
■ 1. Find the value of m in the given triangle.



Solution:

$m=8$. m represents the length of the hypotenuse of this $30-60-90$ triangle. The hypotenuse is always twice as long as the shortest leg, so $m=2(4)=8$.

■ 2. Find the values of x and y in the given triangle.



Solution:

$x = 4\sqrt{3}$ and $y = 2\sqrt{3}$. In a $30 - 60 - 90$ triangle, the length of the longer leg is always the product of the length of the shorter leg and $\sqrt{3}$.

$$6 = y\sqrt{3}$$

$$y = \frac{6\sqrt{3}}{3}$$

$$y = 2\sqrt{3}$$

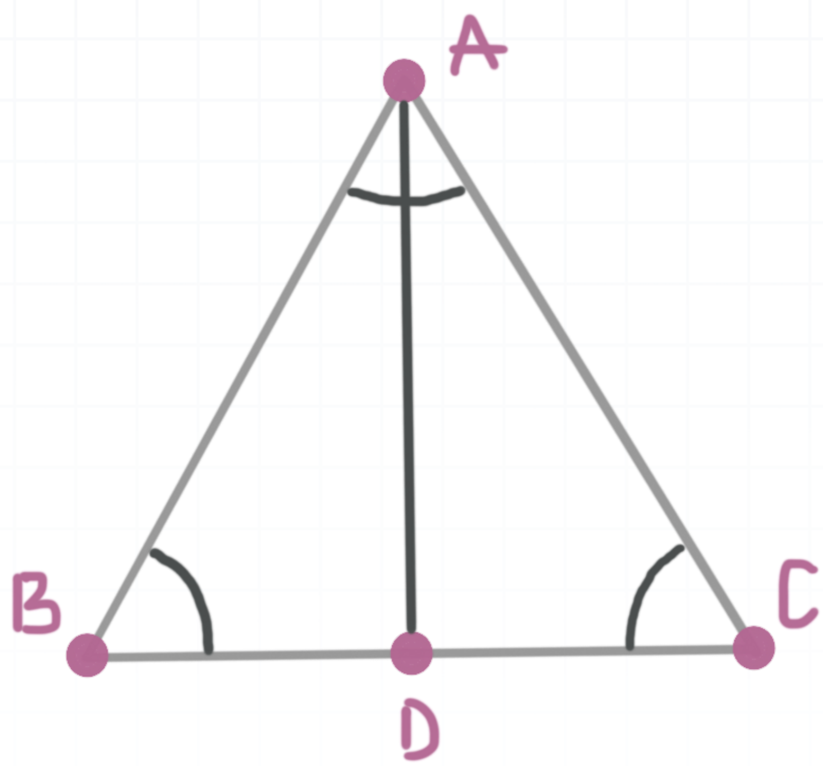
The length of the hypotenuse is twice the length of the shortest leg.

$$x = 2(y)$$

$$x = 2\left(2\sqrt{3}\right)$$

$$x = 4\sqrt{3}$$

■ 3. $\triangle BAC$ is an equilateral triangle. The perimeter is 42 cm and $m\angle ADC = 90$. Find AD.



Solution:

$AD = 7\sqrt{3}$. The perimeter is 42, therefore the length of each side of the triangle is 14. $\overline{AD}$ is an altitude of the triangle, so two $30-60-90$ triangles are formed. $BD = CD = 7$, which is the shortest leg of each of your special right triangles. The longer leg of a $30-60-90$ triangles is the product of the shortest leg and $\sqrt{3}$. So $AD = 7\sqrt{3}$.

■ 4. $\triangle XYZ$ is an equilateral triangle. $\overline{XM}$ is an altitude, median, and angle bisector of the triangle. If $XM = 9$, find the perimeter of the triangle.

Solution:

The perimeter is $18\sqrt{3}$. Draw an equilateral triangle and label $\overline{XM}$. Find the length of YM.

$$XM = YM\sqrt{3}$$

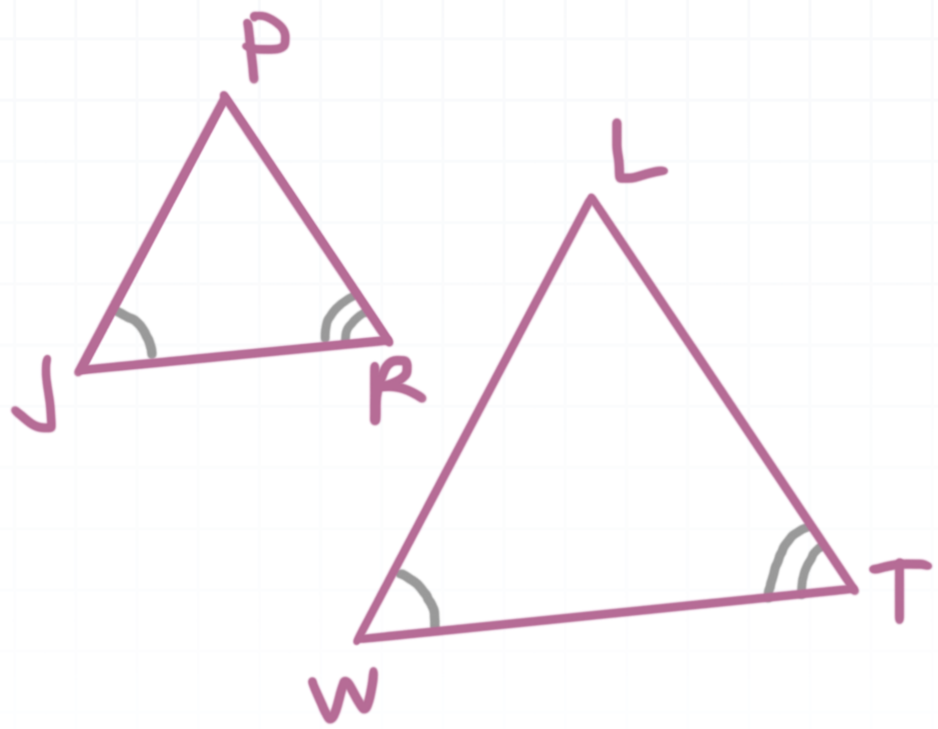
$$YM = \frac{XM}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

Now find the length of YZ. $YZ = 6\sqrt{3}$. So the perimeter is

$$3\left(6\sqrt{3}\right) = 18\sqrt{3}$$

TRIANGLE SIMILARITY THEOREMS

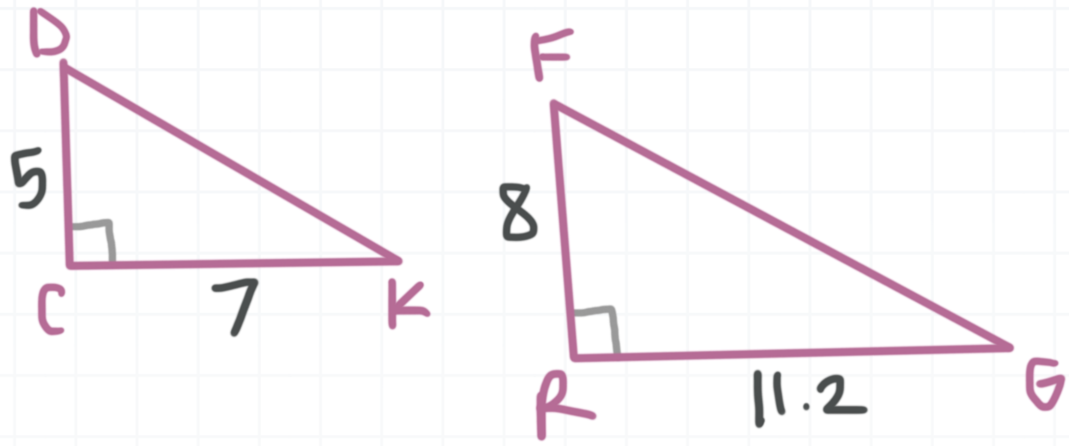
■ 1. Write a similarity statement for the triangles and provide the theorem that proves they're similar.



Solution:

$\triangle JPR \sim \triangle WLT$ by AA. If two angles of a triangle are congruent to two angles of another triangle, then the triangles must be similar.

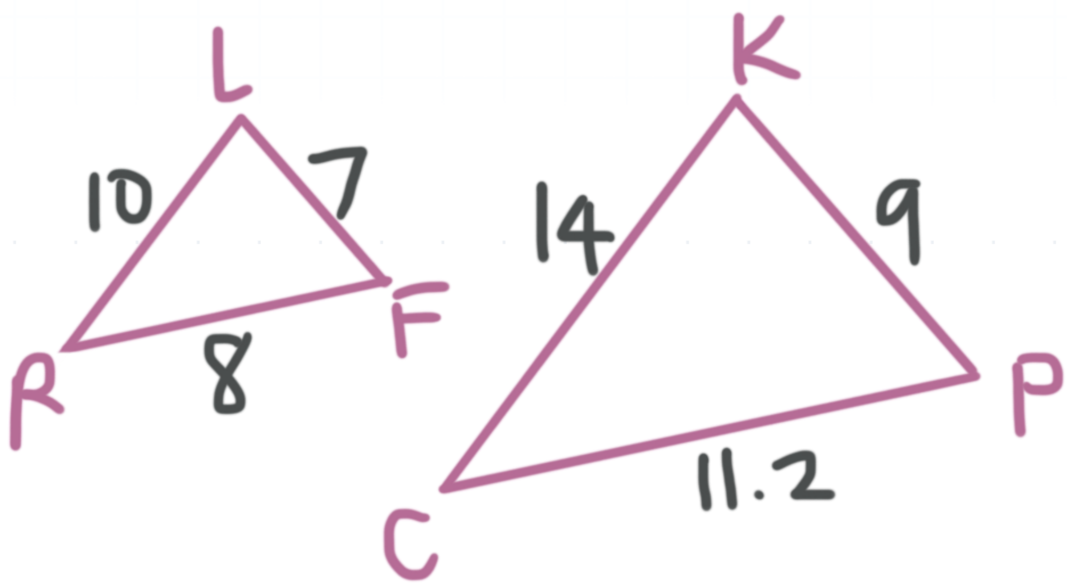
■ 2. Write a similarity statement for the triangles and provide the theorem that proves they're similar.



Solution:

$\triangle DCK \sim \triangle FRG$ by SAS. $\angle C \cong \angle R$ and $5/8 = 7/11.2$.

■ 3. Is $\triangle RLF \sim \triangle CKP$? Explain.



Solution:

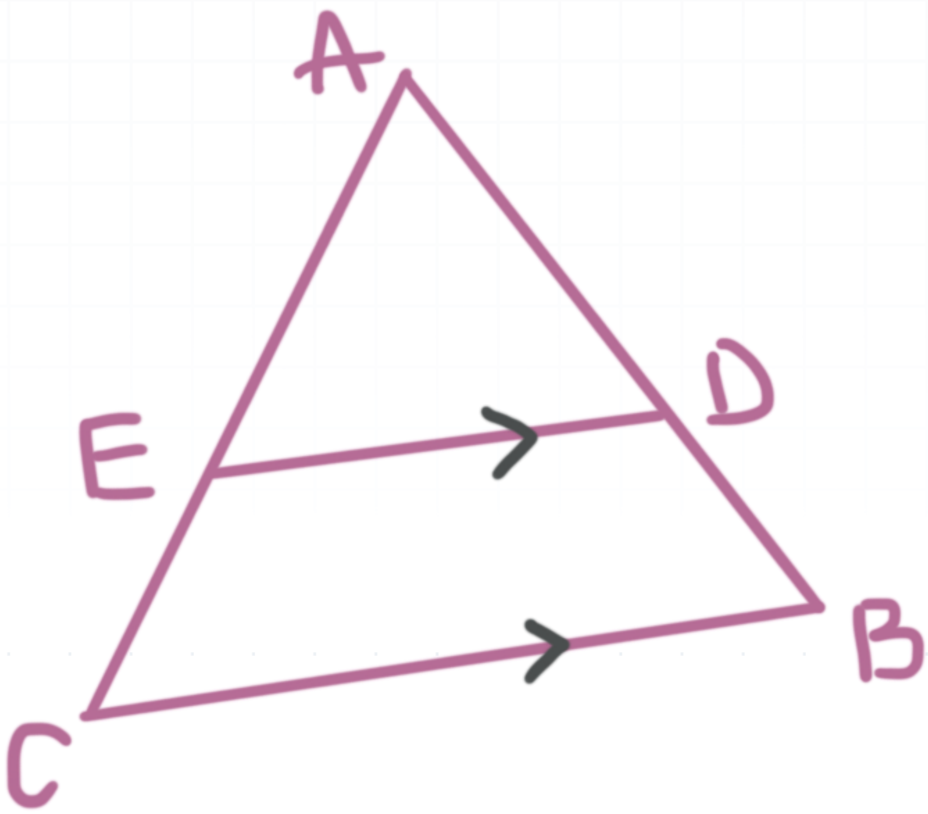
No, these triangles are not similar. Setting up proportions, it's true that

$$\frac{10}{14} = \frac{8}{11}$$

However, the triangles cannot be similar because

$$\frac{10}{14} \neq \frac{7}{9}$$

■ 4. Prove $\triangle AED \sim \triangle ACB$.

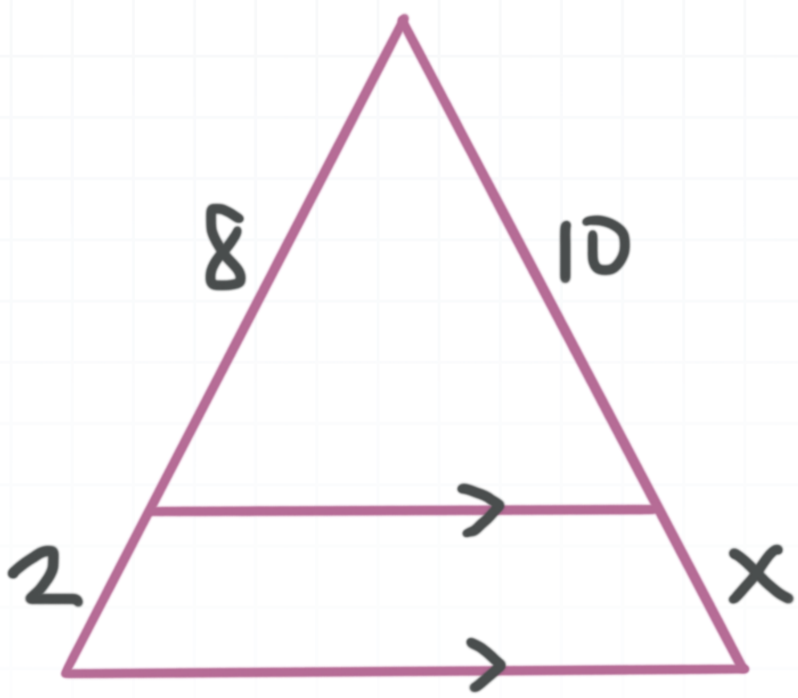


Solution:

$\triangle AED \sim \triangle ACB$ by AA. $\angle A \cong \angle A$ by the Reflexive Property of congruence, and $\angle EDA \cong \angle CBA$ because they are corresponding angles.

TRIANGLE SIDE-SPLITTING THEOREM

■ 1. Solve for x.



Solution:

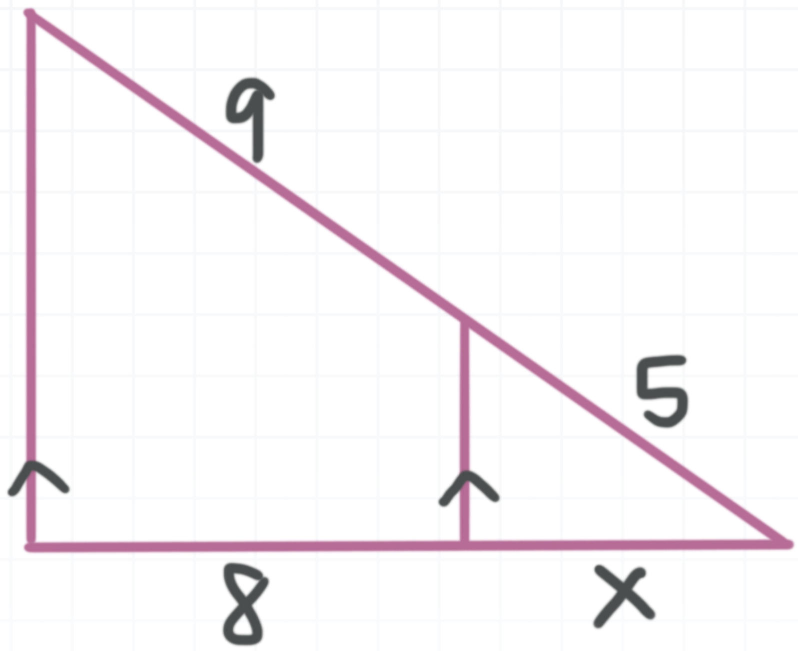
2.5. Set up a proportion.

$$\frac{8}{10}=\frac{2}{x}$$

$$8x=20$$

$$x=2.5$$

■ 2. Solve for x.



Solution:

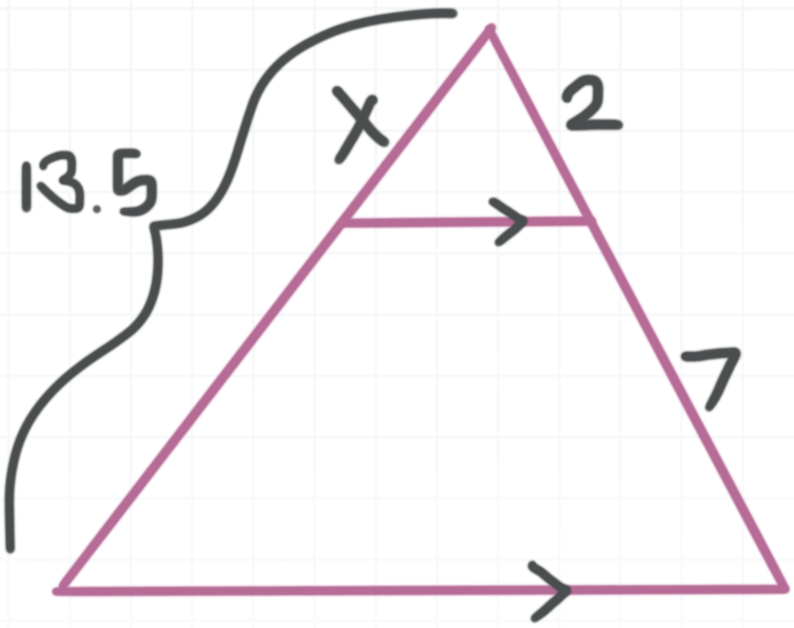
$40/9$. Set up a proportion.

$$\frac{9}{8} = \frac{5}{x}$$

$$9x = 40$$

$$x = \frac{40}{9}$$

■ 3. Solve for x.



Solution:

3. Set up a proportion.

$$\frac{2}{x} = \frac{7}{13.5 - x}$$

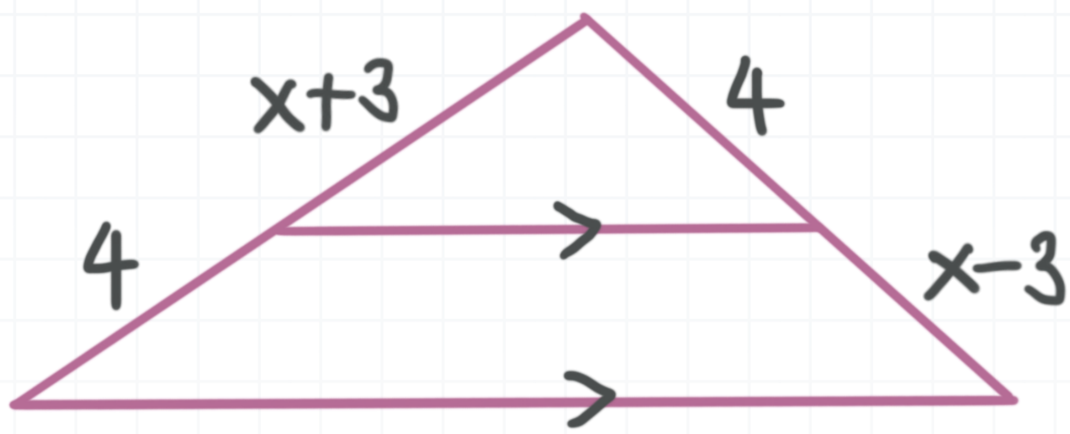
$$2(13.5 - x) = 7x$$

$$27 - 2x = 7x$$

$$27 = 9x$$

$$x = 3$$

■ 4. Solve for x.



Solution:

5. Set up a proportion.

$$\frac{x+3}{4} = \frac{4}{x-3}$$

$$(x+3)(x-3) = (4)(4)$$

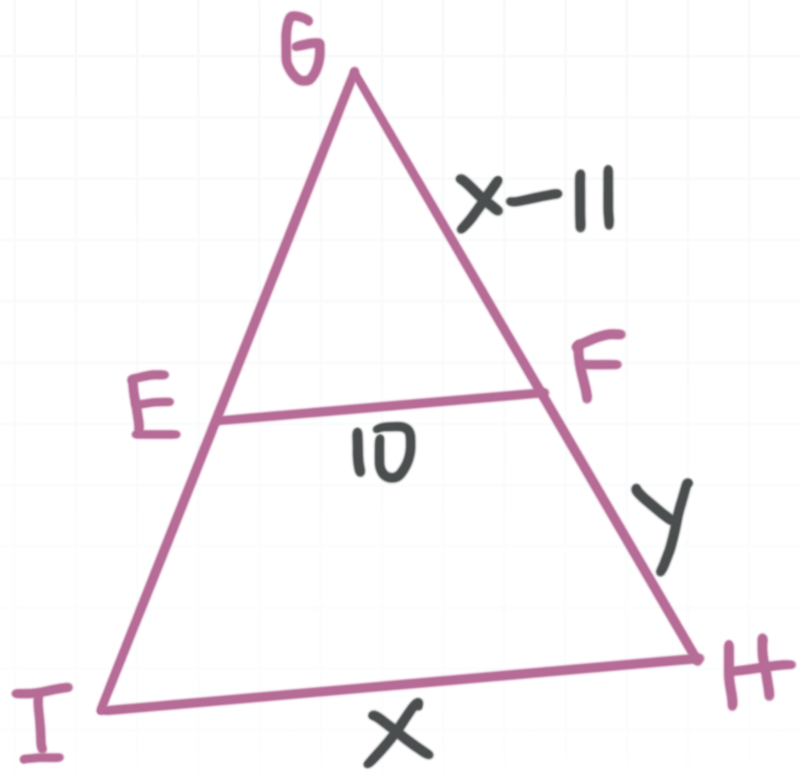
$$x^2 - 9 = 16$$

$$x^2 = 25$$

$$x = 5$$

MIDSEGMENTS OF TRIANGLES

1. $\overline{EF}$ is a midsegment of $\triangle IGH$. Find x and y.



Solution:

$x = 20$ and $y = 9$. Because EF is a midsegment of $\triangle IGH$,

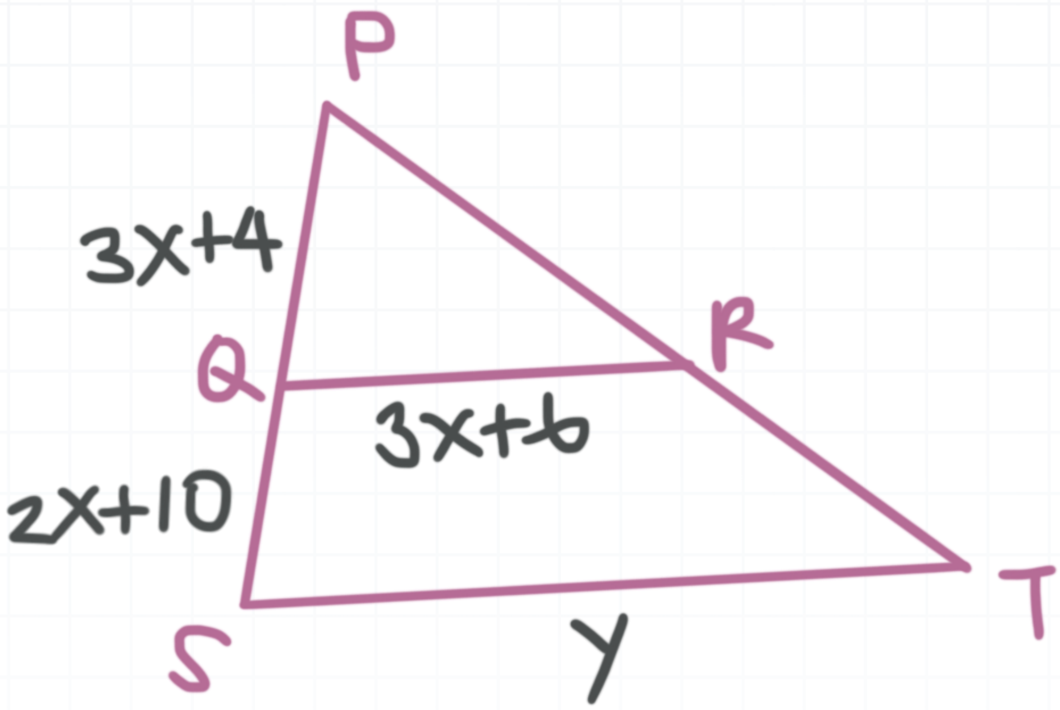
$$IH = 2(EF)$$

$$x = 2(10)$$

$$x = 20$$

Then $GF = 20 - 11 = 9$, we know that $y = 9$.

■ 2. $\overline{QR}$ is a midsegment of $\triangle SPT$. Find x and y.



Solution:

$x=6$ and $y=48$. Because $SQ=PQ$, we get

$$2x+10=3x+4$$

$$x=6$$

Then we can use $x=6$ to find the length of the midsegment.

$$QR=3x+6$$

$$QR=3(6)+6$$

$$QR=24$$

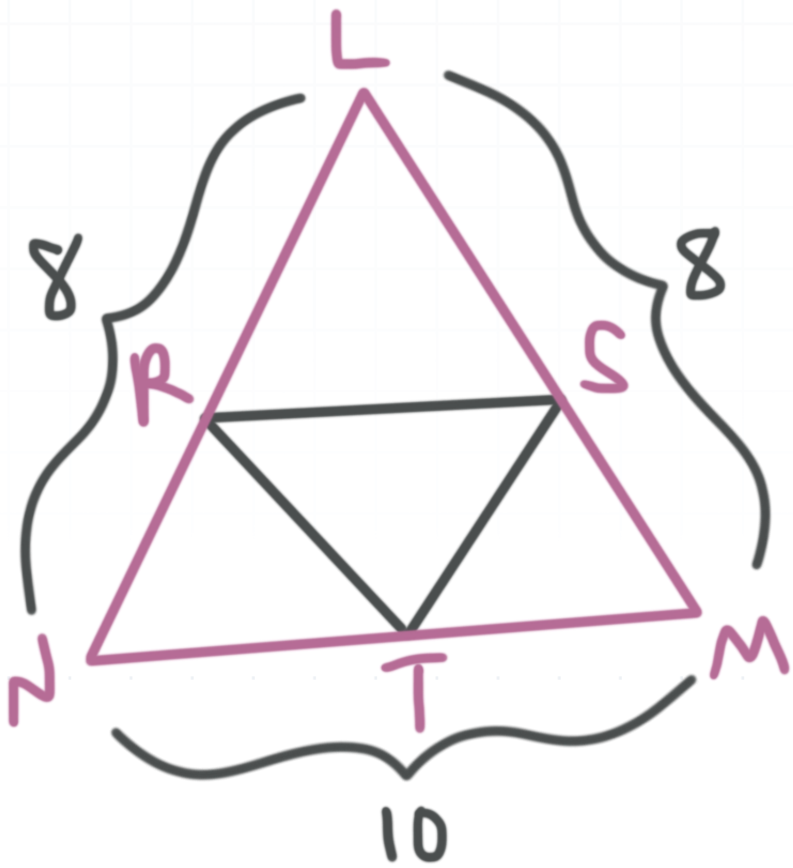
Then the value of y is given by

$$ST = 2QR$$

$$y = 2(24)$$

$$y = 48$$

■ 3. $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$ are midsegments of $\triangle NLM$. Find the perimeter of quadrilateral $RTMS$.



Solution:

18. The four side lengths are given by

$$RT = \frac{1}{2}(8) = 4$$

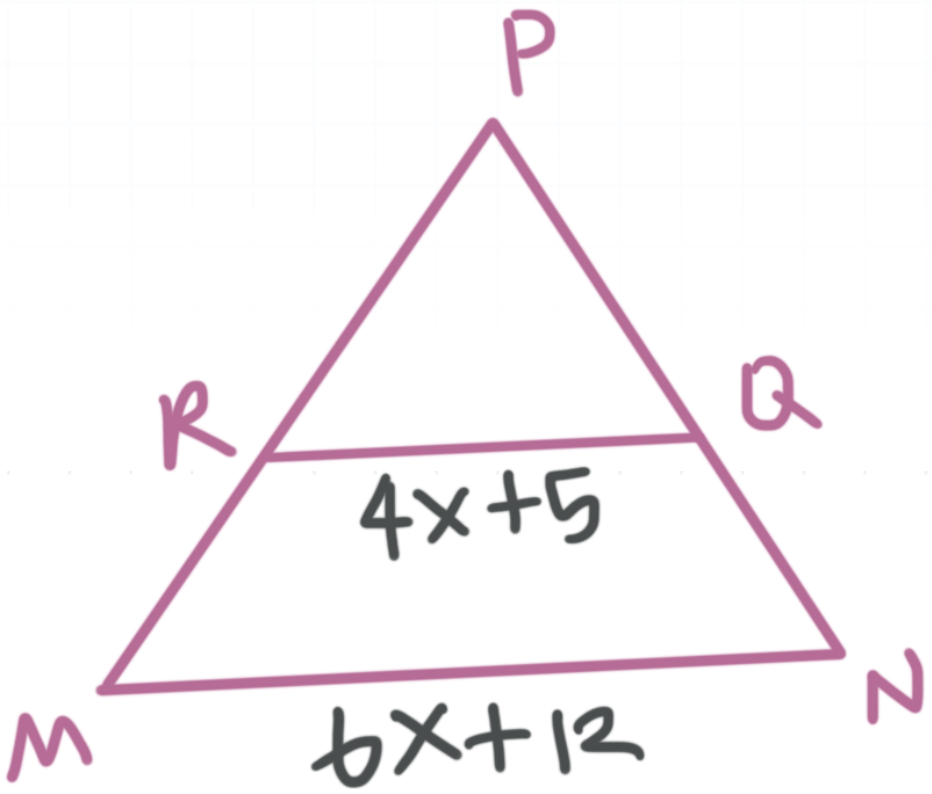
$$SM = \frac{1}{2}(8) = 4$$

$$TM = \frac{1}{2}(10) = 5$$

$$RS = \frac{1}{2}(10) = 5$$

Then the perimeter of $RTMS$ is $4 + 4 + 5 + 5 = 18$.

■ 4. $\overline{RQ}$ is a midsegment of $\triangle MPN$. Find x and MN.



Solution:

$x = 1$ and $MN = 18$. From the formula for the midsegment of a triangle, we get

$$RQ = \frac{1}{2}MN$$

$$4x + 5 = \frac{1}{2}(6x + 12)$$

$$8x + 10 = 6x + 12$$

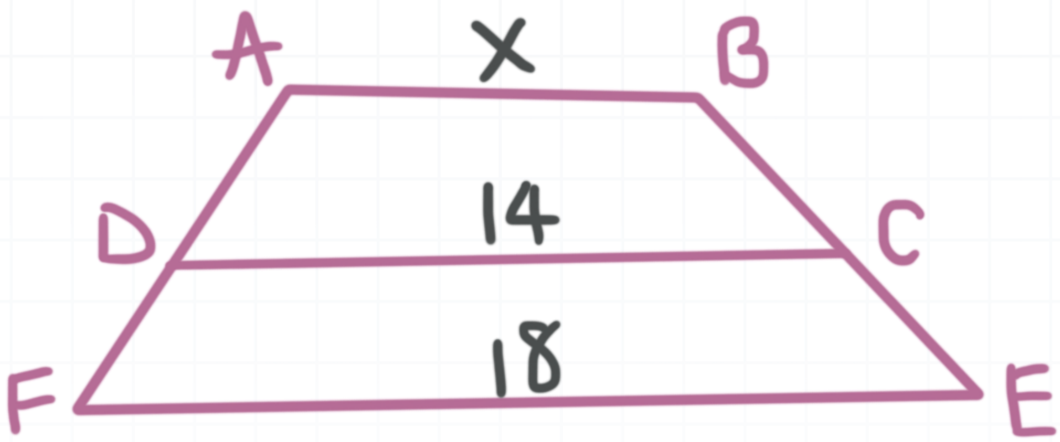
$$2x = 2$$

$$x = 1$$

Then MN is $MN = 6(1) + 12 = 18$.

MIDSEGMENTS OF TRAPEZOIDS

■ 1. The trapezoid has midsegment $\overline{DC}$. Find the value of x.



Solution:

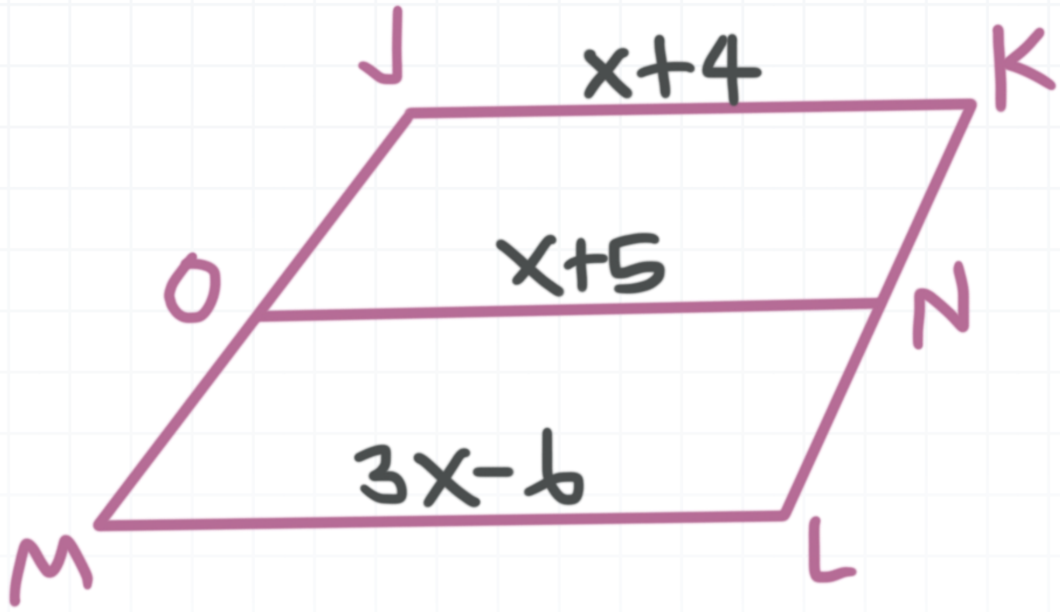
10. The length of the midsegment will be given by

$$DC = \frac{1}{2}(AB + FE)$$

$$14 = \frac{1}{2}(x + 18)$$

$$x = 10$$

■ 2. $\overline{ON}$ is a midsegment of trapezoid $JKLM$. Find JK, ON, and ML.



Solution:

10, 11, 12. The length of ON will be given by

$$ON = \frac{1}{2}(JK + ML)$$

$$x + 5 = \frac{1}{2}(x + 4 + 3x - 6)$$

$$x + 5 = 2x - 1$$

$$x = 6$$

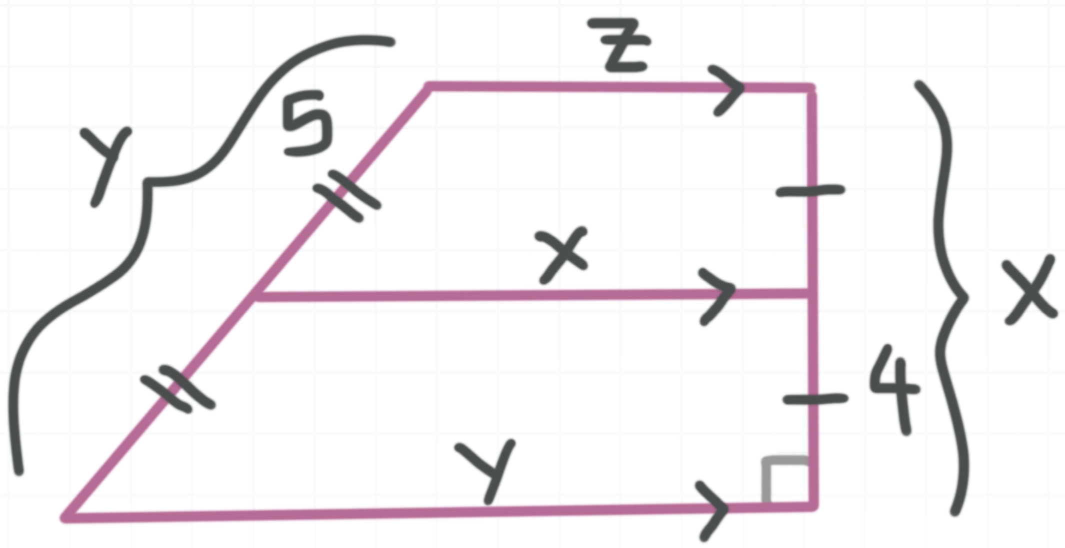
If $x = 6$, then the lengths of the line segments are

$$JK = 6 + 4 = 10$$

$$ON = 6 + 5 = 11$$

$$ML = 3(6) - 6 = 12$$

■ 3. Find x, y, and z.



Solution:

$8, 10, 6$. We know from the formula for the midsegment of a trapezoid, and the fact that $x = 2(4) = 8$ and $y = 2(5) = 10$, we get

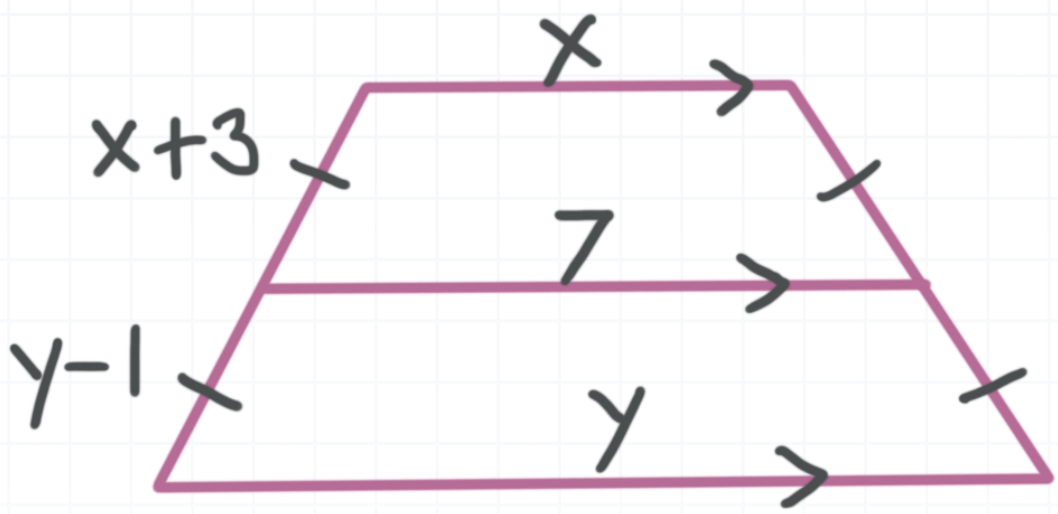
$$x = \frac{1}{2}(y + z)$$

$$8 = \frac{1}{2}(10 + z)$$

$$10 + z = 16$$

$$z = 6$$

■ 4. Find x and y.



Solution:

$x = 5$ and $y = 9$. Given congruent segments on the left side of the trapezoid, we get

$$x + 3 = y - 1$$

$$x - y = -4$$

And from the formula for the midsegment of a trapezoid, we get

$$\frac{1}{2}(x + y) = 7$$

$$x + y = 14$$

Use elimination to solve the system of equations by subtracting $x - y = -4$ from $x + y = 14$.

$$x + y - (x - y) = 14 - (-4)$$

$$x + y - x + y = 14 + 4$$

$$2y = 18$$

$$y = 9$$

Then

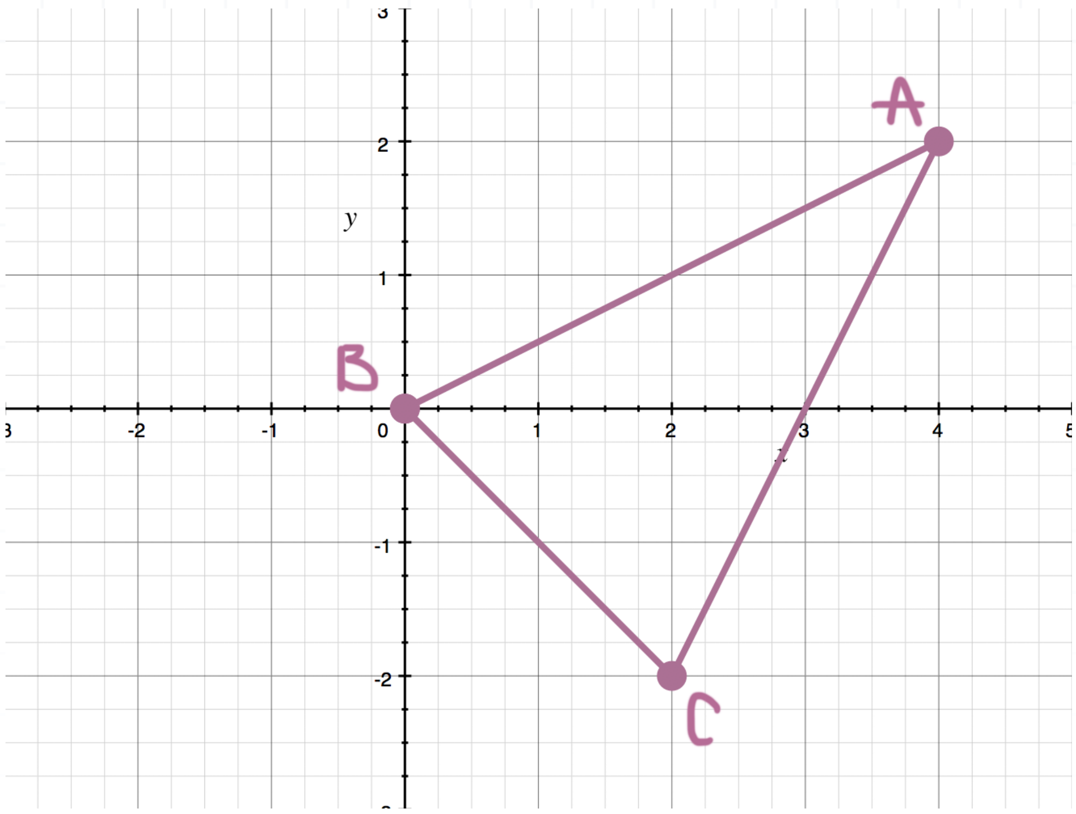
$$x + y = 14$$

$$x + 9 = 14$$

$$x = 5$$

TRANSLATING FIGURES IN COORDINATE SPACE

■ 1. Find the new coordinates of $\triangle ABC$ under a translation of $(x, y) \to (x+3, y-2)$.



Solution:

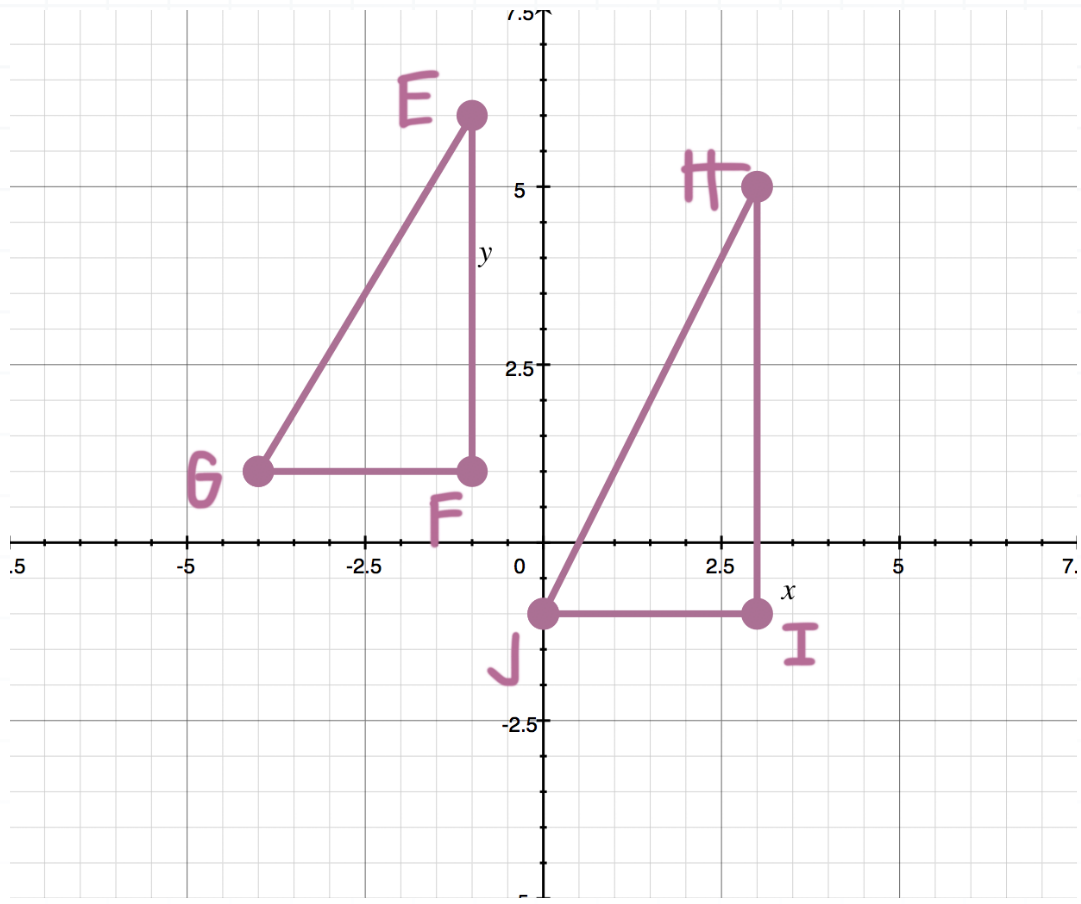
$A'(7,0)$, $B'(3,-2)$, $C'(5,-4)$. Translating each vertex of the triangle gives the vertices of the translated triangle.

$$A(4,2) \to (4+3, 2-2) \to (7,0)$$

$$B(0,0) \to (0+3, 0-2) \to (3,-2)$$

$$C(2,-2) \to (2+3, -2-2) \to (5,-4)$$

■ 2. Is $\triangle EFG$ is a translation of $\triangle HIJ$? Explain why or why not.



Solution:

No, $\triangle EFG$ is not a translation of $\triangle HIJ$. We can see that EF is shorter than HI. For this to be a translation, the triangles must be congruent.

■ 3. $\odot A$ has its center at the origin and radius 3. Find the equation of this circle under a translation of 2 units to the right and 4 units up on the coordinate plane.

Solution:

$(x-2)^2+(y-4)^2=3^2$

4. A rectangle has a diagonal with endpoints at $(5,1)$ and $(14,7)$. Find the area of this rectangle under the translation $(x,y) \to (x-5,y-4)$.

Solution:

54. The area of the original rectangle is $(9)(6)=54$. Because this is a translation, the rectangle will not change in size, but rather be moved to a new location in the plane. The area will remain the same.

ROTATING FIGURES IN COORDINATE SPACE

■ 1. $X(2,5)$ is rotated clockwise about the origin and its translated coordinate is $X'(-5,2)$. By how many degrees was this point rotated?

Solution:

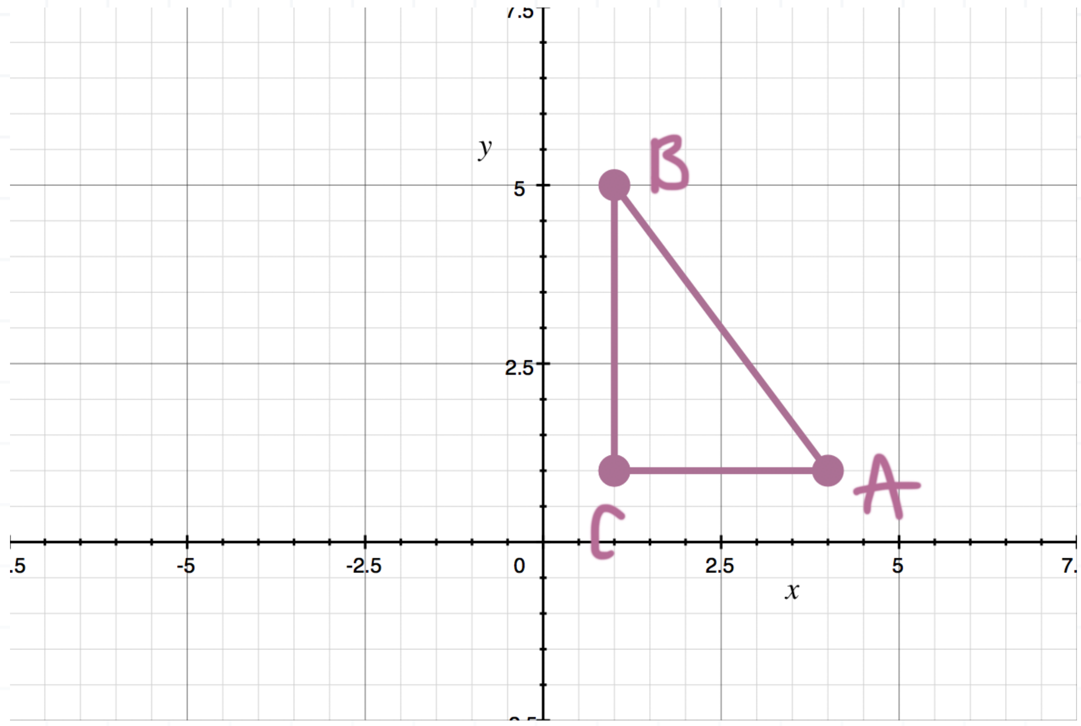
90°

■ 2. $B(-3-1)$ is rotated 180° counterclockwise about the origin. Find B'.

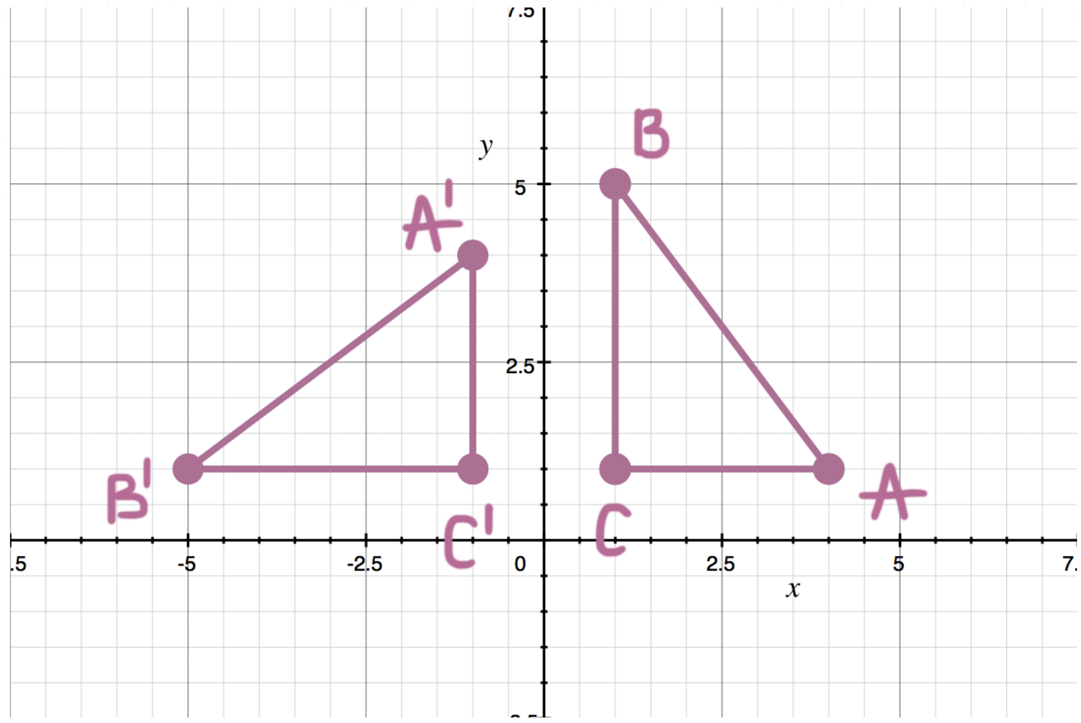
Solution:

$B'(3,1)$. B must remain the same distance from the origin and move counterclockwise. B is in quadrant III, but will rotate 180° and end up in quadrant I. Its coordinates will therefore be $(3,1)$.

■ 3. Graph $\triangle ABC$ under a rotation of 90° counterclockwise.



Solution:



■ 4. $G(-4,-6)$ is first translated 5 units to the right and 3 units up on the coordinate plane. Then this new coordinate is rotated 90° clockwise about the origin. Find its new coordinate.

Solution:

$(-3, -1)$. The translation gives us

$$G(-4, -6) \to (-4+5, -6+3) \to (1, -3)$$

Under a 90° rotation clockwise, the new coordinate will be $(-3, -1)$.

REFLECTING FIGURES IN COORDINATE SPACE

■ 1. Find the coordinates of $A(-4,5)$ under a reflection over the x-axis.

Solution:

$A'(-4,-5)$. Reflecting over the x-axis will move the point from quadrant II to quadrant III, but will keep the point 5 units from the x-axis.

■ 2. Find the coordinates of $J(3,4)$ under a reflection over the y-axis.

Solution:

$J'(-3,4)$. Reflecting over the y-axis will move the point from quadrant I to quadrant II, but will keep the point 3 units from the y-axis.

■ 3. Find the coordinates of $K(-1,4)$ under a reflection over the line $y=2$.

Solution:

$K'(-1,0)$. $y=2$ is a horizontal line running through $y=2$. K is 2 units above this line and therefore its reflection will be 2 units below this line.

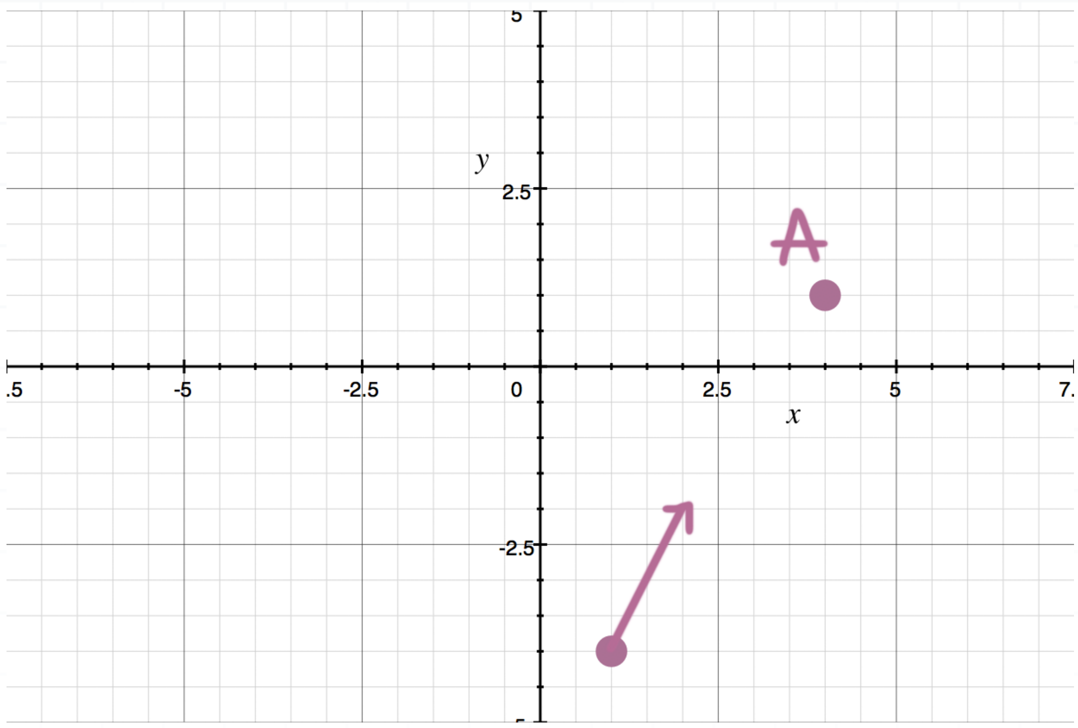
■ 4. Find the coordinates of $P(5,-2)$ under a reflection over the line $y=x$.

Solution:

$P'(-2,5)$. When a point is reflected over the line $y=x$, its transformation is $(x,y)\to(y,x)$.

TRANSLATION VECTORS

■ 1. Find A' as directed by the vector shown.

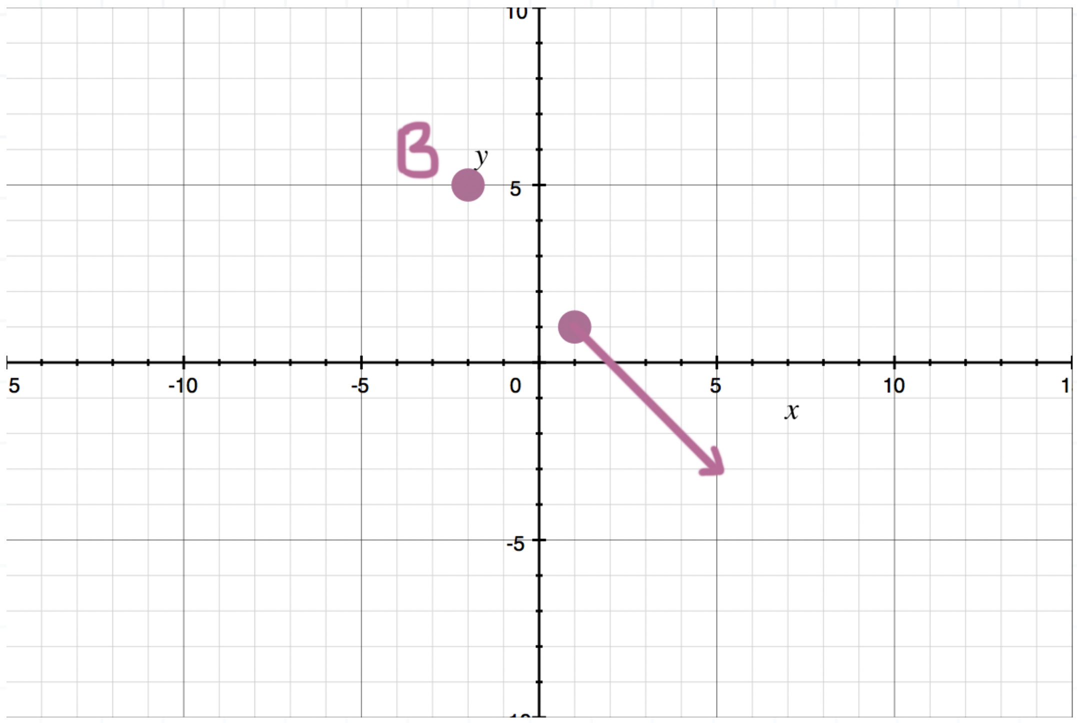


Solution:

$A'(5,3)$. The vector shows a translation of 1 unit in the x-direction and 2 units in the y-direction.

$$A(4,1) \to (4+1,1+2) \to (5,3)$$

■ 2. Find B' as directed by the vector shown.

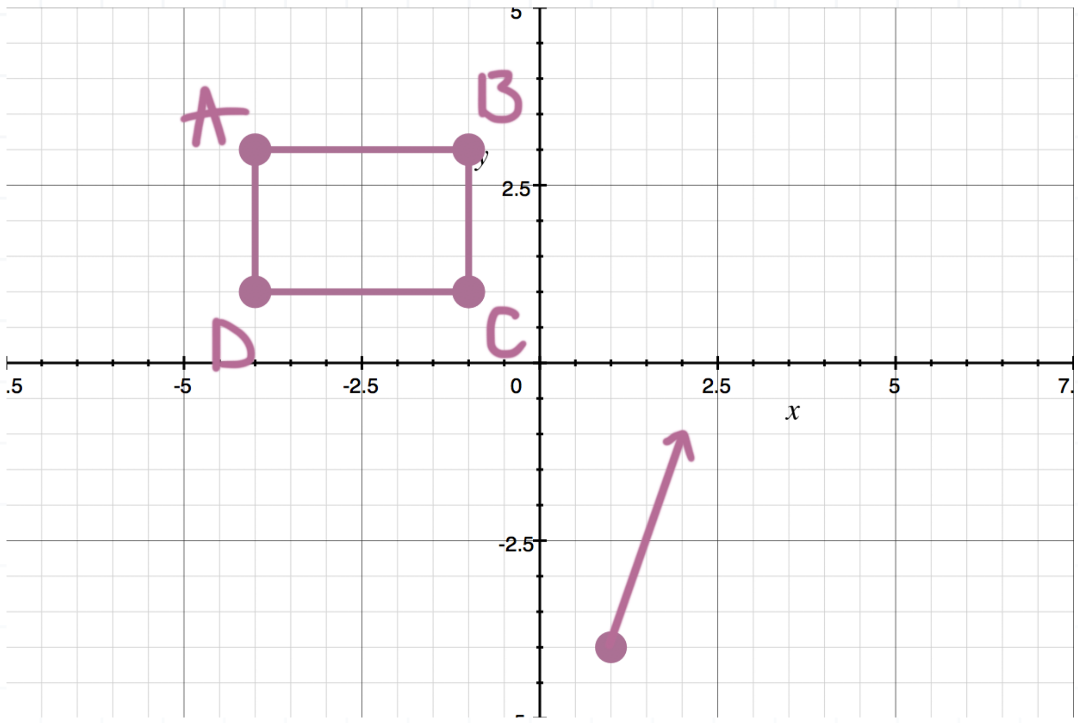


Solution:

$B'(2,1)$. The vector shows a translation of 4 units in the x-direction and -4 units in the y-direction.

$$B(-2,5) \to (-2+4,5-4) \to (2,1)$$

■ 3. Find D' as directed by the vector shown.

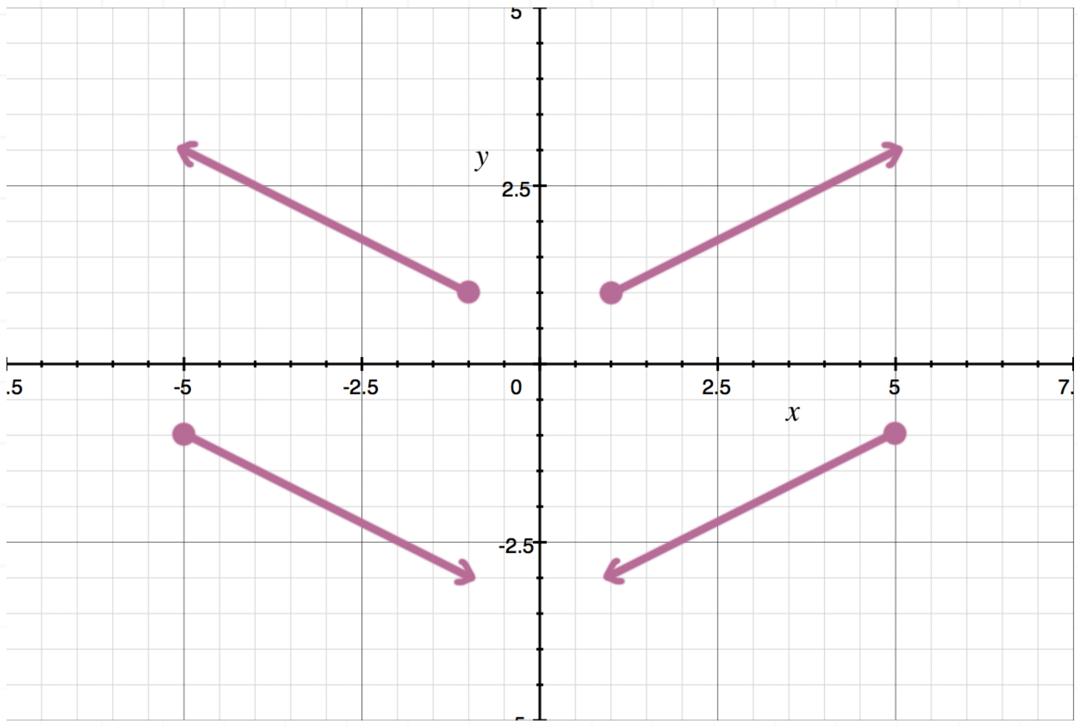


Solution:

$D'(-3,4)$. The vector shows a translation of 1 unit in the x-direction and 3 units in the y-direction.

$$D(-4,1) \to (-4+1,1+3) \to (-3,4)$$

■ 4. $M(3,1)$ is rotated 90° counterclockwise about the origin. Which translation vector (name the quadrant that contains the vector) would translate M to the correct location on the coordinate plane?

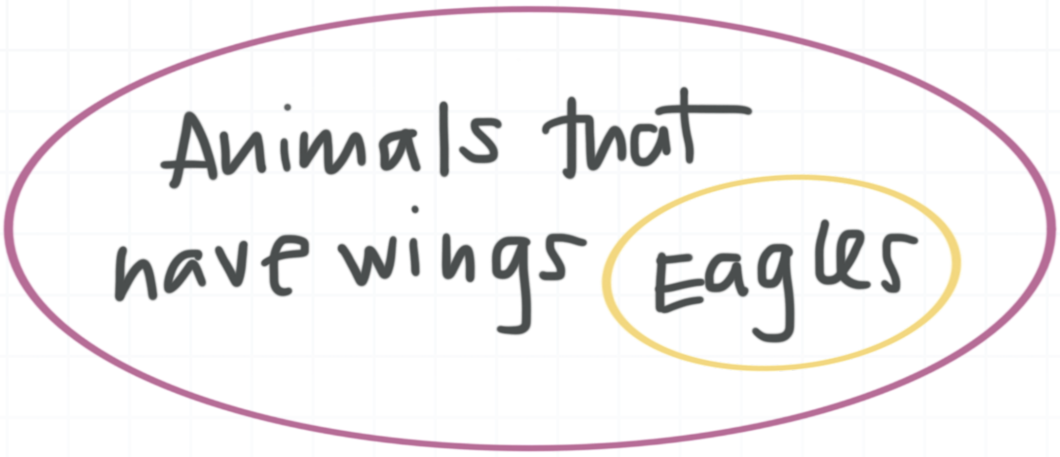


Solution:

The vector in the quadrant II. This vector is the one that translates the point $M(3,1)$ to a location 90° in the counterclockwise direction to the point $M'(-1,3)$. If we sketch a vector from $(3,1)$ to $(-1,3)$, we see that it has the same length and direction as the vector in quadrant II.

CONDITIONALS AND EULER DIAGRAMS

■ 1. Write the if-then statement that corresponds to the Euler diagram.

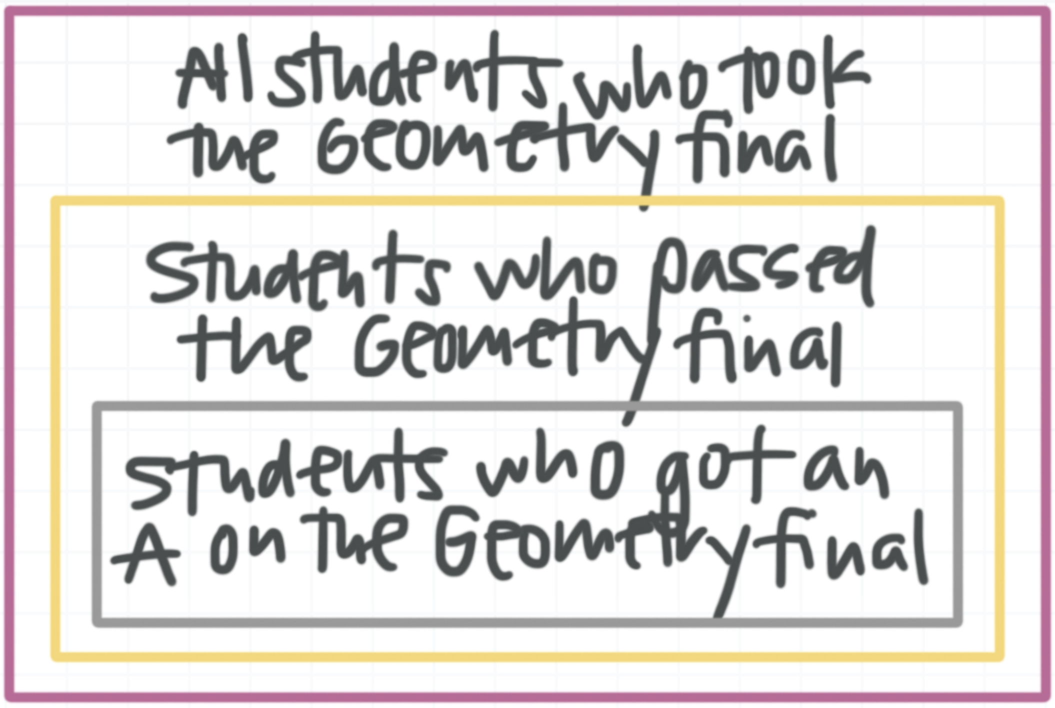


Solution:

"If an animal is an eagle, then it has wings."

■ 2. True or false? The if-then statement is true based on the Euler diagram.

"If a student passed the geometry final, then they got an A."

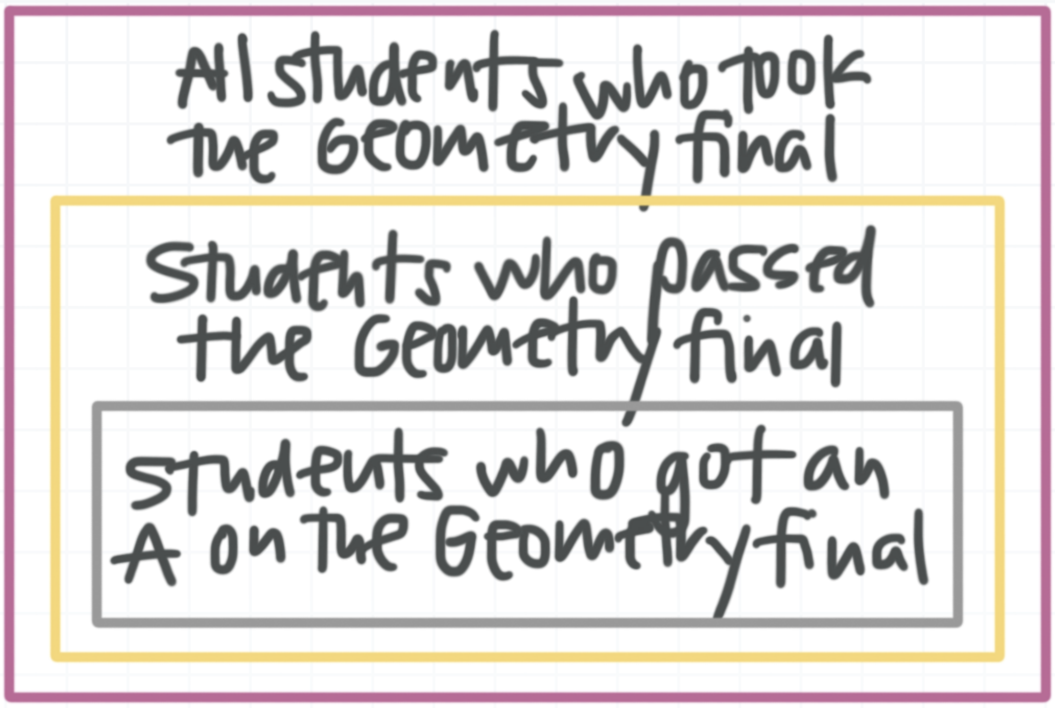


Solution:

False. The correct if-then statement would have been: "If a student got an A on the geometry final, then they passed." A student may have gotten a B and still passed.

■ 3. True or false? The statement is true based on the Euler diagram.

"If a student took the Geometry final exam, then they passed the test."



Solution:

False. People who passed the final exam are a subset of the people who took the final exam. Not all students who took the final exam passed it. The correct if-then statement would have been: If a student passed the Geometry final exam, then they took the exam.

■ 4. Draw a Euler diagram for the statement, "All quadrilaterals are polygons."

Solution:

All polygons

All quadrilaterals

CONVERSES OF CONDITIONALS

■ 1. Write the converse for the if-then statement.

"If M is a midpoint of $\overline{AB}$, then $AM = MB$."

Solution:

"If $AM = MB$, then M is a midpoint of $\overline{AB}$."

■ 2. Write the converse for the if-then statement.

"If a polygon is a triangle, the sum of its angles is 180°.

Solution:

"If the sum of the angles of a polygon is 180°, then it is a triangle."

■ 3. Write the converse of the if-then statement. Then determine if the converse is always, sometimes, or never true.

"If $\angle 1$ and $\angle 2$ are vertical angles, then they are congruent."

Solution:

“If $\angle 1$ and $\angle 2$ are congruent, then they are vertical angles.” The converse is sometimes true. $\angle 1$ and $\angle 2$ can be vertical, but they could also form a linear pair in which both angles have a degree measure of 90° and are congruent.

■ 4. Write the converse of the if-then statement. Determine if the converse is true or false. If it’s false, provide a counterexample.

“If an animal is a cow, then it has four legs.”

Solution:

“If an animal has four legs, then it’s a cow.” The converse is fall, because a four-legged animal could be many other kinds of animals, like, for example, a goat.

ARRANGING CONDITIONALS IN A LOGICAL CHAIN

1. Fill in the blank with a logical conclusion.

All parallelograms have four sides.

All four-sided figures are quadrilaterals.

All parallelograms ______________________.

Solution:

"are quadrilaterals." This follows by the Law of Syllogism.

2. If Jane's alarm does not go off, she will be late to school. If Jane is late to school, she will get in trouble. Jane got in trouble. Can a valid conclusion be drawn? Explain.

Solution:

No valid conclusion can be drawn. We know if Jane's alarm goes off she will get in trouble. But the converse may not be true. She may have gotten in trouble for another reason.

■ 3. Write the missing statement that will make the last statement true.

1. If a driver is going 60 mph, he is speeding.

2. ____

3. If a driver is going 60 mph, he will receive a speeding ticket.

Solution:

If a driver is speeding, he will get a speeding ticket.

■ 4. All squares are rectangles. Rewrite this statement in if-then form: $JKLM$ is a rectangle. Can a valid conclusion be drawn?

Solution:

If a polygon is a square, then it's a rectangle. We're given that $JKLM$ is a rectangle. No valid conclusion can be drawn, because the converse is not always true. We can only draw a valid conclusion if we're given the hypothesis part of the statement.

W W W . K R I S T A K I N G M A T H . C O M