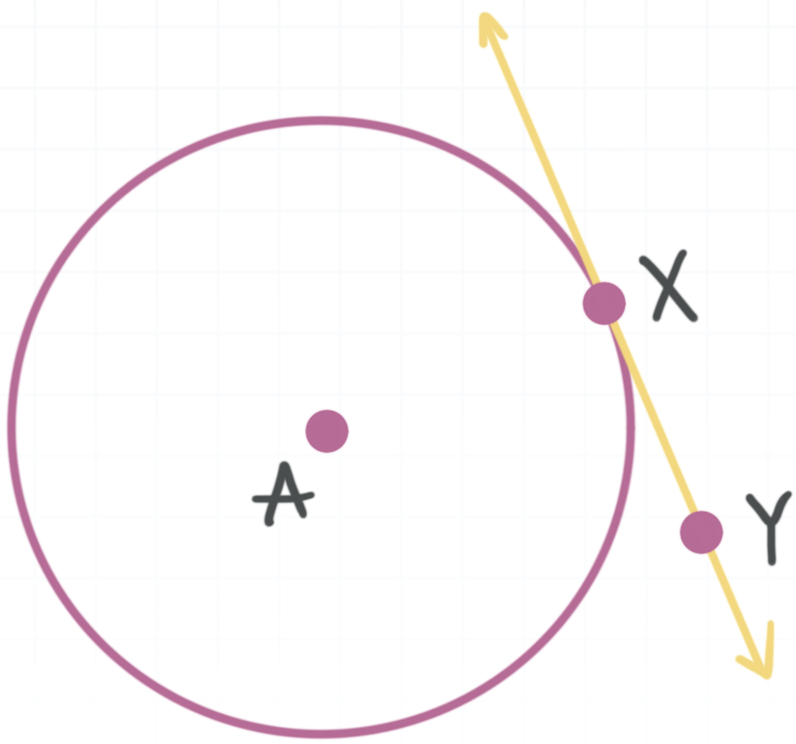
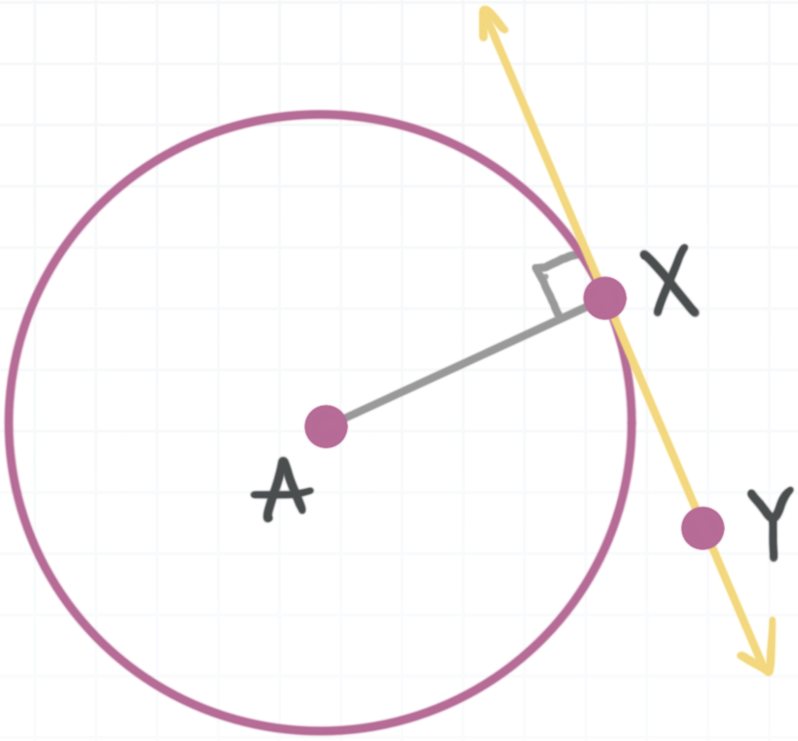
Tangent lines of circles

A **tangent line** of a circle is a line that intersects the circle at exactly one point. In the circle in the figure (with A at its center), line XY is tangent to the circle at point X. Point X is called the **point of tangency**.



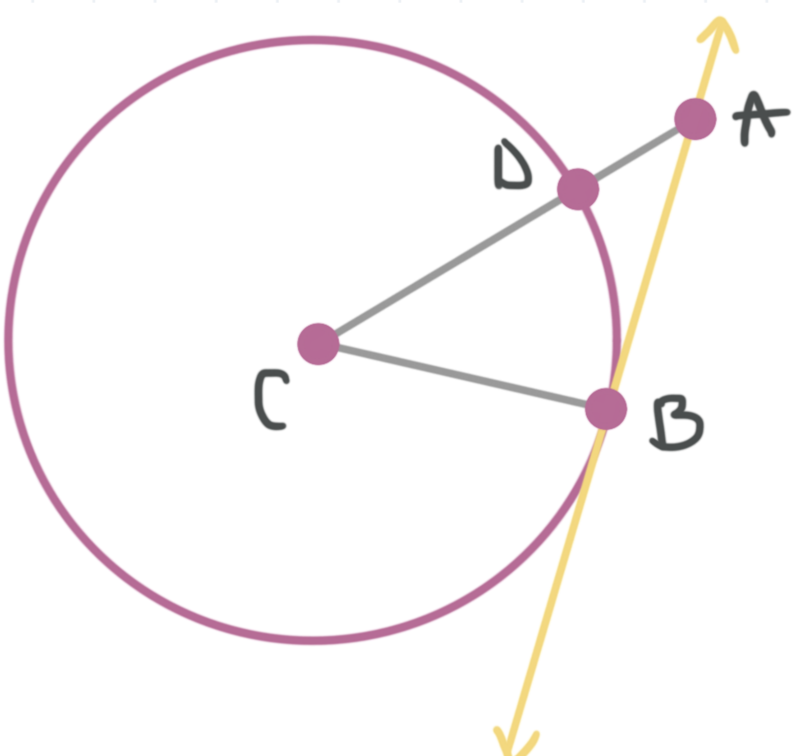
The radius drawn from the center of the circle to the point of tangency is always perpendicular to the tangent line. In the figure below, Radius $\overline{AX}$ is perpendicular to $\overleftrightarrow{XY}$.



Let's start by working through an example.

Example

The radius of the circle in the figure (with C at its center) is 5. Also, $\overline{AB} = 6$, $\overline{DA} = 3$, and line $\overleftrightarrow{AB}$ intersects the circle at B. Determine whether line $\overleftrightarrow{AB}$ is tangent to the circle.



If line $\overleftrightarrow{AB}$ is tangent to the circle, then radius $\overline{CB}$ will be perpendicular to line $\overleftrightarrow{AB}$, and $\angle ABC$ will be a right angle, so triangle ABC will be a right triangle. That will be true if and only if the Pythagorean theorem is satisfied for the triangle.

So we want to determine whether the following equation is true.

$$(\overline{CB})^2 + (\overline{AB})^2 = (\overline{CA})^2$$

Since $\overline{CB}$ is a radius, we know that $\overline{CB} = 5$. We also know that $\overline{CA} = \overline{CD} + \overline{DA}$, and that $\overline{CD}$ is a radius, so $\overline{CD} = 5$. Since $\overline{DA} = 3$, we see that $\overline{CA} = 5 + 3 = 8$. Now we can check the Pythagorean theorem.

$$(\overline{CB})^2 + (\overline{AB})^2 = (\overline{CA})^2$$

$$5^2 + 6^2 = 8^2$$

$$25 + 36 = 64$$

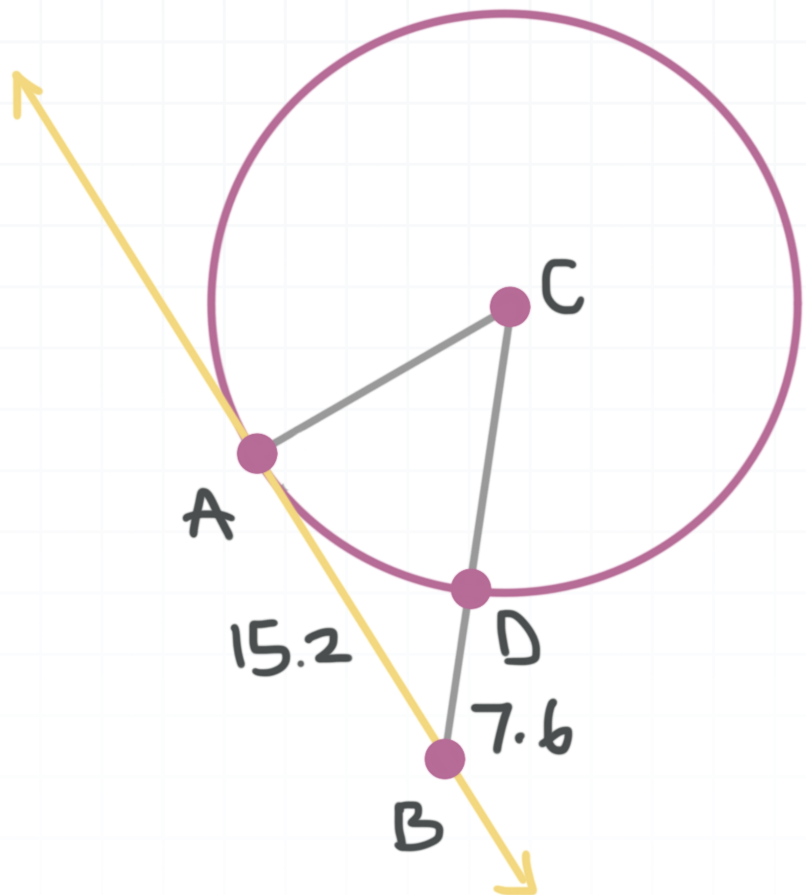
$$61 \neq 64$$

This means that $\angle ABC$ is not a right angle, and line AB is therefore not tangent to the circle.

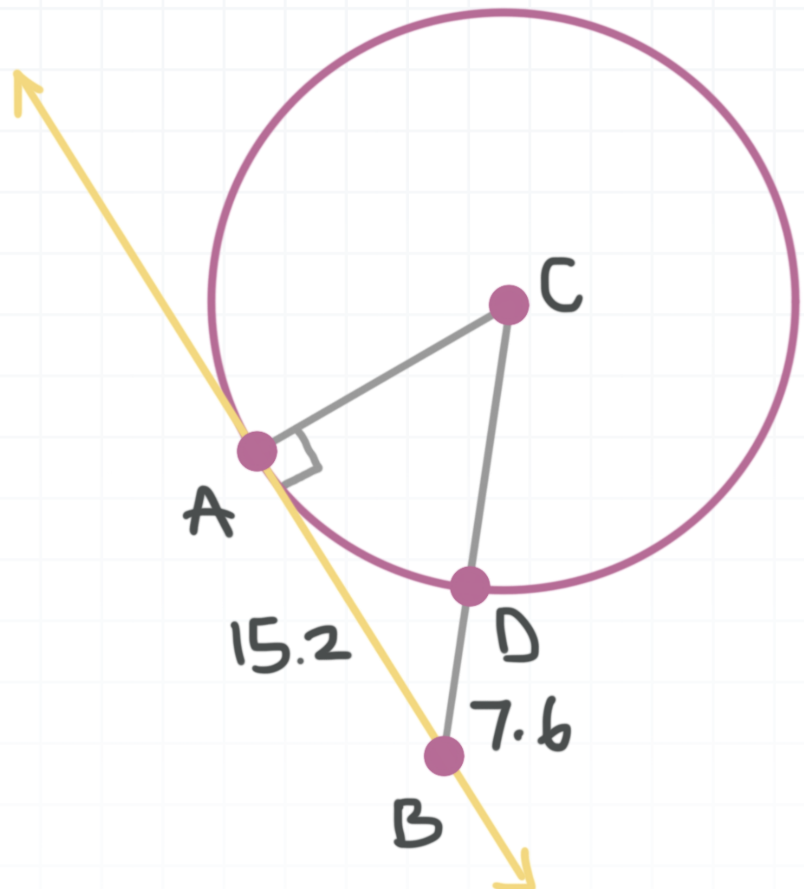
Let's do one more.

Example

Find the radius of the circle in the figure (with C at its center), given that $\overline{AB} = 15.2$, $\overline{DB} = 7.6$, and $\overleftrightarrow{AB}$ is tangent to the circle at A.



A radius drawn to point A will be perpendicular to $\overleftrightarrow{AB}$ and form right triangle BAC.



Let's call the radius x. Then $\overline{AC} = x$ and $\overline{CD} = x$. Now we can use the Pythagorean theorem to set up an equation and solve for x.

$$(\overline{AC})^2 + (\overline{AB})^2 = (\overline{CB})^2$$

$$x^2 + 15.2^2 = (x + 7.6)^2$$

$$x^2 + 231.04 = x^2 + 15.2x + 57.76$$

Subtract x^2 and 57.76 from both sides.

$$173.28 = 15.2x$$

$$x = 11.4$$

The radius of the circle is 11.4.