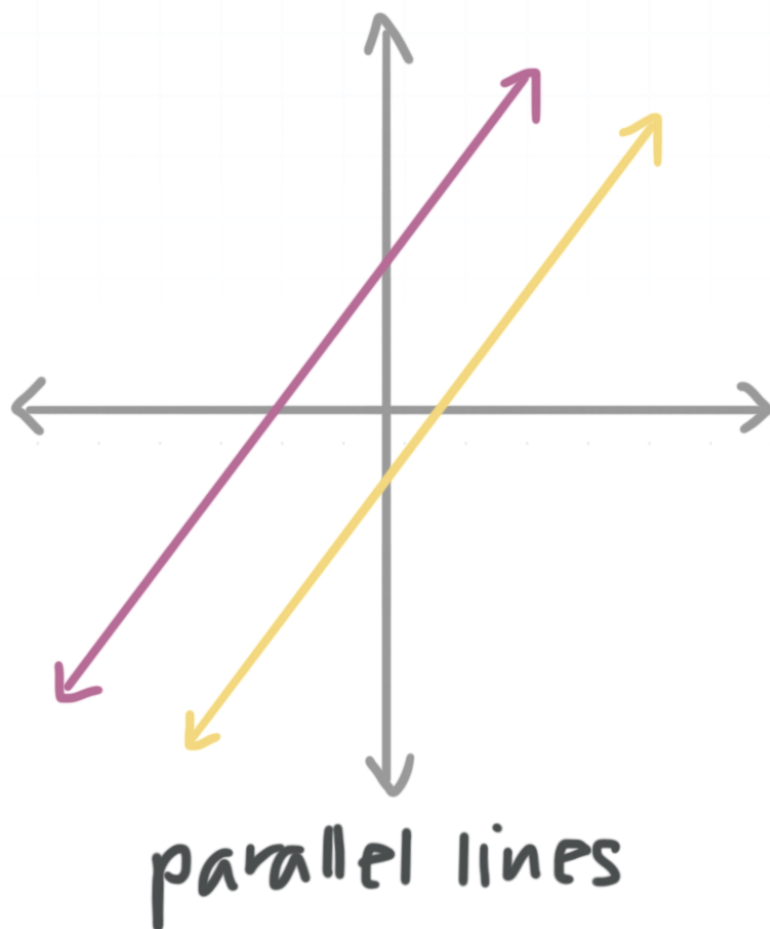# Parallel, perpendicular, or neither

In this lesson we'll look at how to use the slopes of two lines in the Cartesian plane (the xy-plane) to see if the lines are perpendicular, parallel, or neither.
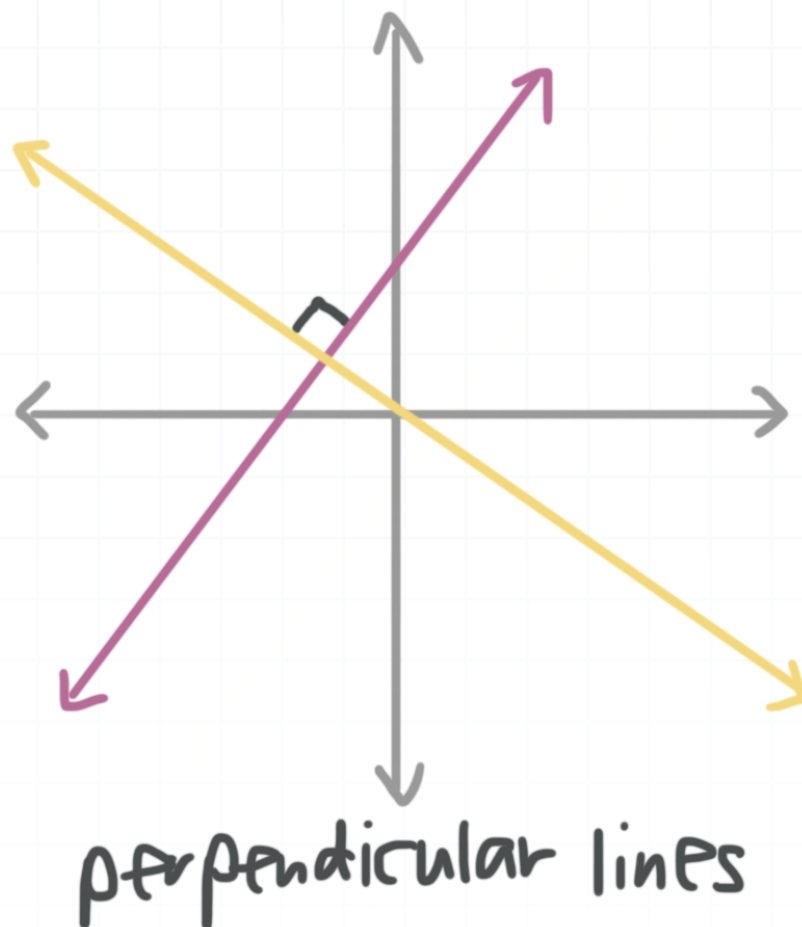
Parallel lines

Parallel lines are lines with equal slopes. Parallel lines will never intersect each other, because they'll always be the same distance apart.

parallel lines

Perpendicular lines

Perpendicular lines have slopes that are negative reciprocals of each other, and they intersect to form $90°$ angles.

perpendicular lines

Let's do a few examples.

Example

Each pair of points in the table below are points that lie on the given line. Which lines are parallel to each other, and which lines are perpendicular?

Line	Point 1	Point 2
Line AB	(3,1)	(-3,11)
Line CD	(0,2)	(5,5)
Line EF	(-5,-5)	(0,-2)

Use the slope formula for each line.

Slope of line $\overleftrightarrow{AB}$:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 1}{-3 - 3} = \frac{10}{-6} = -\frac{5}{3}$$

Slope of line $\overleftrightarrow{CD}$:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{5 - 0} = \frac{3}{5}$$

Slope of line $\overleftrightarrow{EF}$:
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-5)}{0 - (-5)} = \frac{-2 + 5}{0 + 5} = \frac{3}{5}$$

Lines $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ have the same slope, $3/5$, so these two lines are parallel, unless they're one and the same line. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ have slopes that are negative reciprocals of each other, so these two lines are perpendicular. Lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ are also perpendicular, for the same reason.

Let's see how we can find the slope of any line that's parallel to a given line.

Example

What is the slope of any line that's parallel to $\overleftrightarrow{CD}$, if $\overleftrightarrow{CD}$ passes through the points $(4,5)$ and $(-2,8)$?

Parallel lines have the same slope, so we need to find the slope of $\overleftrightarrow{CD}$. Using the slope formula, we get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{-2 - 4} = \frac{3}{-6} = -\frac{1}{2}$$

Any line that's parallel to $\overleftrightarrow{CD}$ will have a slope of $-1/2$.

Now let's look at how to find the slope of any line that's perpendicular to a given line.

Example

What is the slope of any line that's perpendicular to $\overleftrightarrow{WX}$, if $\overleftrightarrow{WX}$ passes through the points $(-3, 5)$ and $(2, -6)$?

Perpendicular lines have slopes that are negative reciprocals of each other, so we first need to find the slope of $\overleftrightarrow{WX}$. Using the slope formula, we get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 5}{2 - (-3)} = \frac{-11}{5} = -\frac{11}{5}$$

Now we find the negative reciprocal of $-11/5$ by turning it upside down and multiplying by -1, which gives $5/11$. The slope of any line that's perpendicular to $\overleftrightarrow{WX}$ is $5/11$.