Basic Concepts in Propositional Logic

Contents

Critical Thinker Academy

Preface

You can’t get very far in argument analysis without learning some basic concepts of **propositional** (also known as “**sentential**”) **logic**. This is where, for example, you learn the proper meaning of terms like "contradiction" and "consistency", which should be part of everyone's logical vocabulary.

In this set of lectures I review these basic concepts, but I don’t spend time working out formal proofs in propositional logic, as you would in a full course in symbolic logic. **My interest here is in presenting and explaining those concepts that are important for doing ordinary, everyday argument analysis.**

In particular, many of these concepts are central to discussions of **formal and informal fallacies**. Many fallacies can only be understood if you first understand how they are grounded in basic concepts of propositional logic.

The logic of **conditional statements** (statements of the form “If A then B”) is treated thoroughly in Part 4. The concepts covered in this section frequently show up in tests of analytical reasoning.

I don't have a separate course on **categorical logic** (or "**Aristotelian**" **logic**, as it's sometimes called), but the Appendix to this course covers the basic difference between categorical and propositional logic, and how to interpret the basic categorical statement forms, "All A are B", "Some A are B" and "Only A are B". These are closely related to propositional statement forms covered earlier in the course.

Introduction: What is Propositional Logic?

In this introduction I'm going to say a few things about what propositional logic is and why the basic concepts of propositional logic are so important for logical thinking and argument analysis.

Propositional logic is sometimes called "sentential" logic or "statement" logic. We saw in the **Basic Concepts in Logic and Argumentation** course that an argument is a set of claims, and the term "claim" is synonymous with the terms "proposition", "statement" or "assertion".

A **claim**, or **proposition**, is a bit of language that has the following defining feature: **it's the sort of thing that can be true or false**.

So, propositional logic deals with logical relationships between propositions.

But more importantly, it deals with logical relationships between propositions *taken as wholes*.

What does this mean? In propositional logic, **the fundamental unit of analysis is the whole proposition**, the thing that can be true or false. You're not interested in the parts of speech that make up the proposition, like subject terms and predicate terms.

In the proposition "**John is wearing a red coat**", for example, "**John**" is the subject term and "**is wearing a red coat**" is the predicate.

But the subject term "John", by itself, is neither true nor false, and neither is the predicate phrase, "is wearing a red coat". Only when you put them together do you get a whole proposition that can be true or false.

In propositional logic all we care about is the "**truth-value**" of a given proposition, whether it's true or false. This is why, in propositional logic, **we use single letters to symbolize a proposition**.

So, we might symbolize this proposition with the single letter "J", which will stand for the whole proposition, "John is wearing a red coat".

Once you've done this then you can ask how the truth value of this proposition relates to the truth value of other propositions.

Or, you can ask how the truth value of a compound claim relates to the truth value of the individual component claims that make it up.

An Example

"John is wearing a red coat and he's stolen a jeep."

This is an example of a **compound claim**, or compound proposition. It's composed of two claims:

"**John is wearing a red coat**." and "**John has stolen a jeep.**"

Each of these component claims is a proposition that can be true or false. We can ask, "is John wearing a red coat?". If he is, it's true, if he's not it's false. Similarly for whether John has stolen a jeep.

Now, from the standpoint of propositional logic, what's really interesting about this example is that **the compound claim *as a whole* is also a proposition that can be true or false**.

In this case it turns out that the compound claim as a WHOLE is true just in case BOTH of the component claims turn out to be true. If either one is false, then the compound claim as a whole will be false.

So, if it turns out that John has stolen a jeep but he's actually wearing a BLUE coat, then the compound claim as a whole is false.

This is an illustration of the general point that propositional logic is concerned with the way that the truth value of compound claims is a function of the truth value of the individual component claims.

Compound Claims and Logical Connectives

In propositional logic you construct compound claims out of a small number of basic logical connectives.

These are the basic types:

- You form **conjunctions** using “and”.
- You form **disjunctions** using “or”.
- You form **conditionals** using “if __ then __”,
- and you form **contradictories** or **negations** using “not-”.

Once you know the rules for how the truth value of these compound claims is a function of the truth values of the component claims, then you can evaluate the truth value of more complex claims like this:

“If A or B is true, then C is true and D is not true.”

Given the truth values of all of the component claims, you can work out the truth value of the compound claim.

This is the kind of exercise you might do in a formal logic course but it’s really not what we’re going to focus on in this course. **We’re going to focus on understanding the basic logical concepts that are derived from propositional logic.**

Outline of the Course

In **Part 1** we’re going to look at the definitions of the basic compound claims: What is a **conjunction**? What is a **disjunction**? What is a **conditional**?

In **Part 2**, we’ll look at the definition of the “**contradictory**” of a claim, and use this to define a set of related logical concepts, **contradiction** and **consistency**. These are very important concepts in logic and argument analysis.

In **Part 3** we’ll look at how to write and interpret the **contradictories** of the basic compound claims,

- not-(A and B)
- not-(A or B)
- not-(if A then B)

And finally in **Part 4** we'll look at some **different ways of writing and expressing conditionals and conditional relationships in ordinary language**.

There are a couple of reasons for spending extra time on this topic. The first is that conditional claims are used all the time in everyday argumentative contexts. But they can be expressed in all sorts of different ways, and it's sometimes hard to know how to evaluate arguments that employ them.

Second, for those practicing for the LSAT (Law School Admission Test) or other tests that focus on analytical reasoning, if there's one bit of logic that these tests requires you to understand above all others, its the logic of conditional claims and conditional arguments. If you don't understand how to interpret language that expresses conditional relationships, or you can't distinguish between valid and invalid argument forms that use conditionals, then your chances of doing well on a test like LSAT are slim at best.

Note: I've got a separate tutorial course that focuses on valid and invalid argument forms that use the conditional (see "Common Valid and Invalid Argument Forms"). In this course we're going to focus mainly on the logic of conditional claims and how to interpret conditional language.

Part 1: Compound Claims

1. Conjunctions (A and B)

A **conjunction** is a compound claim formed by, as the name suggestions, conjoining two or more component claims. The component claims are called the "conjuncts".

The logic of conjunctions is pretty straightforward.

Here's a simple conjunction pertaining to my preferences for pizza toppings:

Let A stand for the claim "**I love pepperoni**".

Let B stand for the claim "**I hate anchovies**".

Then the conjunction of A and B is the compound claim

"**I love pepperoni AND I hate anchovies**".

We want to know the conditions under which the conjunction as a whole is true or false.

In this case it's pretty obvious. The conjunction "A and B" is true just in case each of the conjuncts is true. If either one is false then the conjunction as a whole is false.

Truth Table for Conjunctions

It's sometimes handy to represent the logic of compound claims with a table that gives the truth value for the compound claim for every possible combination of truth values of the component claims.

For conjunctions the "**truth table**" looks like this:

A	and	B
T	T	T
T	F	F
F	F	T
F	F	F

Under each of the conjuncts we list all the possible truth values in such a way that each row represents a distinct logical combination of truth values.

In the first row, A is true and B is true. In the second row, A is true and B is false.

In the third row, A is false and B is true, and in the last row, A is false and B is false.

This exhausts all the possible combinations of truth values.

The middle column under the "and" represents the truth value of the conjunction taken as a whole, "A and B", as a function of the truth values for A and B in the adjacent row.

A	and	B
T	**T**	T
T	**F**	F
F	**F**	T
F	**F**	F

So, in the first row, we see that if both A and B are true, then the conjunction as a whole is true.

But for every other combination of truth values, where at least one of the conjunctions is false, then the conjunction as a whole is false.

We'll use truth tables like this one to represent the logic of all the compound forms we'll be looking at.

Conjunctions in Ordinary Language

Knowing the logic of the conjunction doesn't help much if you can't recognize when a conjunction is being used in ordinary language. Here are a few things to look out for.

"John is a Rolling Stones fan and John is a teacher."

"John is a Rolling Stones fan and a teacher."

In the first sentence the conjunctive form is transparent. Each of the conjuncts, "John is a Rolling Stones fan" and "John is a teacher", show up as complete sentences on either side of the "and".

But the second sentence represents the very same conjunction as the first sentence. The syntax is different, but from the standpoint of propositional logic, the semantics, the meaning of the sentence, is exactly the same as the first sentence. It's implicit that "a teacher" is a predicate term that takes "John" as the subject. Don't make the mistake of reading a sentence like this as a simple, non-compound claim.

Also, conjunctions don't always use the word "and" to flag the conjunction. In ordinary language there may be a subtle difference in meaning between this sentence using "**but**" ...

"John is a Rolling Stones fan BUT he doesn't like The Who."

... and the same sentence using "and", but from the standpoint of propositional logic, where all we care about is whether the sentence is true or not, and how the truth of the sentence depends on the truth of any component claims, this sentence still represents a conjunction. The "but" doesn't make any difference from this perspective. This sentence is still true just in case John is a Rolling Stones fan AND John doesn't like The Who.

There are other words that sometimes function to conjoin claims together, like "**although**", "**however**", and "**yet**". You can substitute all of these for "but" in this sentence and you'll get slight variations in the sense of what's being said, but **from the standpoint of propositional logic all of these represent the same conjunction, a claim of the form "A and B"**.

One last point. Conjunctions can have more than two component claims. A claim like

"A and B and C and D and E"

might represent a compound claim like,

"John is a writer and a director and a producer of The Simpson's tv show, but he's also a stand-up comic and an accomplished violinist".

This is still a conjunction, and it follows the same rules as any conjunction, namely, it's true as a whole just in case all those component claims are true, and false otherwise.

This is about all you need to know about the logic of conjunctions and how conjunctions are expressed in ordinary language.

2. Disjunctions (A or B)

You form a conjunction when you assert that two or more claims are all true at the same time. You form a **disjunction** when you assert that AT LEAST ONE of a set of claims is true.

"John is at the movies or John is at the library."

"In this case we've got two claims, "**John is at the movies**" and "**John is at the library**". The disjunction asserts that one of these is true, John is *either* at the movies *or* he's at the library.

The individual claims that make up a disjunction are called the "*disjuncts*". So, you'd say that in this case "A" and "B" are the disjuncts, and the disjunction is the whole claim, "A or B".

We're interested in the conditions in which the disjunction as a whole counts as true or false.

Inclusive and Exclusive Disjunctions

There are two kinds of cases we need to distinguish. Both of the sentences below express disjunctions. You can represent these as claims of the form "A or B".

1. **"A triangle can be defined as a polygon with three sides or as a polygon with three vertices."**
2. **"The coin landed either heads or tails."**

But there's a difference between the first sentence and the second sentence. With the first sentence, **both disjuncts can be true at the same time, they're not mutually exclusive**. You can define a triangle as a polygon with three sides, or as a polygon with three vertices, but both are equally good definitions.

With the second sentence it's different. A coin toss is either heads or tails, it can't be both. So the "or" expressed in the bottom sentence is more restrictive than the "or" expressed in the top sentence.

- The "or" in the first sentence is called an **"inclusive OR"**. It *includes* the case where both conjuncts can be true.
- The "or" on the second sentence is called an **"exclusive OR"**. It *excludes* the case where both conjuncts can be true.

When examining arguments that use "OR" you need to know what kind of OR you're dealing with, an inclusive OR or an exclusive OR, because the logic is different.

Truth Tables for Disjunctions

Here are the truth tables for the inclusive OR and the exclusive OR:

A	OR	B
T	**T**	T
T	**T**	F
F	**T**	T
F	**F**	F

Inclusive OR

A	and	B
T	**F**	T
T	**T**	F
F	**T**	T
F	**F**	F

Exclusive OR

With the inclusive OR, if either A or B is true, then the disjunction as a whole is true. The only case where it's false is if both A and B are false.

The truth table for the exclusive OR is exactly the same except for the first row, where both A and B are true. The exclusive OR says that A and B can't both be true at the same time, so for this combination the disjunction is false.

You're using an exclusive OR when you say things like, "The dice rolled either a six or a two", "The door is either open or shut", "I'm either pregnant or I'm not pregnant", "I either passed the course or I failed the course.".

On the other hand, if a psychic predicts that you will either come into some money or meet a significant new person in the next month, that's probably an inclusive OR, since they're probably not excluding the possibility that both might happen.

But sometimes it's hard to know whether an OR is intended to be inclusive or exclusive, and in those cases you might need to ask for clarification if an argument turns on how you read the "OR".

I'll finish here with the same point I made about conjunctions, namely that a string of disjunctions like this — **"A or B or C or D or E"** — is still a disjunction.

When I see strings like this I think of detective work, where you're given a set of clues and you've got to figure out who did it, and the string is a list of suspects, and with each new clue or bit of evidence you systematically eliminate one of the options until you're left with the killer. You see this reasoning with medical diagnosis or forensic research, and hypothesis testing in general; they all exploit the logic of disjunctive statements.

That's it for the logic of disjunctions!

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3. Conditionals (If A then B)

Conditionals are claims of the form **"If A is true, then B is true"**. We use and reason with conditionals all the time.

There is a lot that can be said about conditions. In this lecture we're just going to give the basic definition and the truth table for the conditional. In Part 4 of this course I'll come back to conditionals and say a few more things about the different ways in which we express conditional relationships in language.

The Parts of a Conditional: Antecedent and Consequent

Here's a conditional:

"If I miss the bus then I'll be late for work".

It's composed of two separate claims, **"I miss the bus"**, and **"I'll be late for work."**

The conditional claim is telling us that if the first claim is true, then the second claim is also true.

We have names for the component parts of a conditional. The first part, the claim that comes after the "if", is called the **antecedent** of the conditional. The second part, the claim that becomes after the "then", is called the **consequent** of the conditional.

"If (I miss the bus) then (I'll be late for work)".
antecedent *consequent*

The names are a bit obscure but they do convey a sense of the role that the claims are playing. What "antecedes" is what "comes before". The "consequent" is a "consequence" of what has come before.

The names are handy to know because they're used in translation exercises where you're asked to express a bit of natural language as a conditional, and they're used to to identify the most common logical fallacies that are associated with conditional arguments.

One of these fallacies, for example, is called **affirming the consequent**. You commit this fallacy when you're given a conditional like this and assume, from the fact that I was late for work, that I must have missed the bus. You're *affirming* the consequent and trying to *infer* the antecedent. This is an invalid inference, and the name for the fallacy, which you'll find in any logic or critical thinking textbook, is "affirming the consequent".

The Logic of Conditionals

It's pretty easy to understand what conjunctions and disjunctions assert, it's not quite as easy seeing exactly what it is that you're asserting when you assert a conditional.

Here's a conditional: **"If I drive drunk then I'll get into a car accident"**.

Question: If I assert that this conditional claim is true, *am I asserting that I'm driving drunk*?

No, I'm not.

Am I asserting t*hat I'm going to get into a car accident*?

No, I'm not asserting that either.

When I assert "if A then B", I'm *not* asserting A, and I'm *not* asserting B. What I'm asserting is a conditional relationship, a relationship of logical dependency between A and B. I'm saying that if A were true, then B would also be true. But I can say that *without* asserting that either A or B is *in fact* true.

It follows that **a conditional can be true even when both the antecedent and the consequent are false**. Here's an example:

"If I live in Beijing then I live in China."

I don't live in Beijing, and I don't live in China, so both the antecedent and the consequent are false, but this conditional is clearly true: if I *did* live in the city of Beijing, I *would* live in China.

Setting up the Truth Table

Shortly we're going to look at the truth table for the conditional, which gives you the truth value of the conditional for every possible combination of truth values of A and B. **The easiest way to understand that truth table is to think about the case where we would judge a conditional to be false.**

Here's a conditional:

"If I study hard then I'll pass the test."

Under what conditions would we say that this conditional claim is false?

Let's consider some possibilities. Let's say I didn't study hard, but I still passed the test. Here the antecedent is false but the consequent is true. In this case, would the conditional have been false?

Well, no. The conditional could still be true in this case. What it says is that if I study hard then I'll pass. It doesn't say that the *only* way I'll pass is if I study hard. So, my failing to study hard and still passing doesn't falsify the conditional.

So, this combination of truth values (antecedent false, consequent true) does *not* make the conditional false.

Now, what about this case? I didn't study hard and I didn't pass the test. Here, both the antecedent and the consequent are false.

This clearly doesn't falsify the conditional. If anything this is what you might expect would happen if the conditional was true and the test was hard!

So this combination of truth values (antecedent false, consequent false) doesn't make the conditional false either.

Now let's look at a third case: **I studied hard but I didn't pass the test**. Here, the antecedent is true but the consequent is false.

Under these conditions, could the conditional still be true?

No, it can't. THESE are the conditions under which a conditional is false, when the antecedent is true but the consequent turns out to be false.

If I studied hard and didn't pass, this is precisely what the conditional says won't happen.

So, in general, **a conditional is false just in case the antecedent is true and the consequent is false.**

This turns out to be the ONLY case when we want to say with certainty that a conditional claim is false. All other combinations of truth values are consistent with the conditional being true.

This, now, gives us the truth table for the conditional.

Truth Table for Conditionals

A	→	B
T	**T**	T
T	**F**	F
F	**T**	T
F	**T**	F

For the sake of consistency and familiarity with the truth tables for the conjunction and the disjunction, I've placed the arrow symbol in between the A and the B to represent the conditional operator, where "A → B" means "If A then B". Actually in formal symbolic logic an arrow is often used as the symbol for the conditional. It's also common in formal logic to use a sideways horseshoe symbol for the conditional, but there's no need to complicate things more than they are.

The arrow has the virtue that it gives a nice visual cue about the direction of the logical dependency between the antecedent and the consequent. The conditional asserts that if A is true then you can infer B, it doesn't go the other way, it doesn't say that if B is true then you can infer A.

Notice that the second row gives the only combination of truth values that makes the conditional false, when A is true and B is false. For all other combinations the conditional counts as true. This definition doesn't always give intuitive results about how to interpret language that uses conditionals, but for purposes of doing propositional logic it gets it right in all the cases that matter.

That's it for now for the logic of the conditional. We'll come back to conditionals when we talk about contradictories, and in Part 4 of this course, where we look at different ways we express conditionals in ordinary language.

Part 2: Contradiction and Consistency

In Part 2 of this series of tutorials we're going to look at the concept of the **contradictory** of a claim, distinguish it from the **contrary** of a claim, define what a **contradiction** is, and introduce the concepts of **consistency** and **inconsistency** when applied to a set of claims.

1. Contradictories (not-A)

Let A be the claim **"John is at the movies"**.

The **contradictory of A** is defined as a claim that **always has the opposite truth value of A**. So, whenever A is true, the contradictory of A is false, and whenever A is false, the contradictory of A is true.

There are couple of different ways that people write the contradictory of A. We're going to write it in English as **"not-A"**. But in textbooks and websites that treat logic in a more formal way you'll likely see "not-A" written with a tilde (~A) or as a corner-of-a-rectangle shape (¬ A).

What does the contradictory assert? It asserts that the claim A is false. There are couple of ways of saying this, some more natural than others.

You can read **not-A** as **"A is false"**, i.e.

"'John is at the movies' is false",

or

"It is not the case that John is at the movies".

But the most natural formulation is obviously

"John is NOT at the movies".

For simple claims like this it's not too hard to find a natural way of expressing the contradictory. For compound claims, like conjunctions or disjunctions or conditionals, finding the contradictory isn't so simple, and sometimes we have to revert to more formal language to make sure we're expressing the contradictory accurately. In Part 3 we'll spend some time looking at the contradictories of compound claims.

Here’s the truth table for the contradictory. Pretty simple. Whenever A is true, not-A is false, and vice versa.

A	not-A
T	**F**
F	**T**

The definition is simple, but the concept is important, and it isn’t trivial when you’re looking at real-world arguments involving more complex claims.

For example, when you’re debating an issue it’s important that all parties understand what it means for the claim at issue to be true and what it means for it to be false, so that everyone understands what sorts of evidence would count for or against the claim. And this requires that you understand the logic of contradictories.

So as I said, the concept may be simple but from a logical literacy standpoint it’s not trivial.

2. Contradictories vs Contraries

One of the problems that people have with identifying contradictories is that they sometimes confuse them with **contraries**. In this lecture we'll clear up the distinction.

Here's our claim,

A = **"The ball is black."**

Now consider the claim **"The ball is white"**. Is this the contradictory of A?

It's tempting to say this. There's a natural sense in which "the ball is white" is opposite in meaning to "the ball is black", since black and white are regarded as opposites in some sense. And it's also true that they both can't be true at the same time: if the ball is black then it's not white, and vice versa.

But this is NOT the contradictory of A.

Why not?

Well, what if the ball we're dealing with is a GREY ball? The ball isn't black, and it isn't white — it's grey.

This possibility is relevant because it's a counterexample to the basic definition of a contradictory.

If "the ball is white" is the contradictory of "the ball is black", then these are supposed to have opposite truth values in all possible worlds; whenever one is true the other is false, and vice versa.

But if the ball is grey, then A, "The ball is black" is false, since grey is not black.

But if A is false, then the contradictory of A must be TRUE. But "The ball is white" is NOT true, it's false as well. *Both of these claims are false*. And that's not supposed to be possible if these are genuine contradictories of one another.

We do have a word to describe pairs of claims like these, though. They're called **contraries**.

Two claims are *contraries* of one another if they can't both be true at the same time, but they can both be false.

So, the ball can't be both black and white, but if it's grey, or red, or blue, then it's neither black nor white. *These are contraries, not contradictories*.

Now, how do we formulate the contradictory of "The ball is black" so that it always has the opposite truth value?

Like so: you say "**The ball is NOT black**".

Now, the ball being grey doesn't violate the definition of a contradictory. In this world, A is false, but not-A is true — it's true that the ball is not black.

* * *

Examples like this illustrate why it's sometimes helpful to have the formal language in the back of your head. A more formal way of stating the contradictory is "it is not the case that A" — "it is not the case that the ball is black" — which is equivalent to "the ball is not black".

The language is pretty stiff but if you stick "it is not the case that ..." in front of the claim, you're guaranteed not to make the mistake we've seen here, of mistaking contrary properties for contradictory properties. In some cases, when dealing with more complex, compound claims, it's the only way to formulate the contradictory.

3. Contradictions (A and not-A)

The concept of a **contradiction** is very important in logic. In this lecture we'll look at the standard logical definition of a contradiction.

Here's the standard definition. **A contradiction is a conjunction of the form "A and not-A", where not-A is the contradictory of A.**

So, a contradiction is a compound claim, where you're simultaneously asserting that a proposition is both true and false.

Given the logic of the conjunction and the contradictory that we've looked at in this course, we can see that the defining feature of a contradiction is that **for all possible combinations of truth values, the conjunction comes out false**, since a conjunction is only true when both of the conjuncts are true, but by definition, if the conjuncts are contradictories, they can never be true at the same time:

A	A and (not-A)	not-A
T	**F**	F
F	**F**	T

ALWAYS FALSE

So, propositional logic requires that all contradictions be interpreted as false. It's logically impossible for a claim to be both true and false in the same sense at the same time.

This is known as the "**principle of non-contradiction**", and some people have argued that this is the most fundamental principle of logical reasoning, in that no argument could be rationally persuasive to anyone if they were consciously willing to embrace contradictory beliefs.

There's a minor subtlety in the definition of a contradiction that I want to mention.

Here's a pair of claims:

"**John is at the movies.**" and "**John is not at the movies.**"

This is clearly a contradiction, since these are contradictories of one another. John can't be both at the movies and not at the movies at the same time.

Now, what about this pair?

"**John is at the movies.**" and "**John is at the store.**"

Recall, now, that these are *contraries* of one another, not contradictories. They can't both be true at the same time, but they can both be false at the same time.

Our question is: *Does this form a contradiction*?

This is actually an interesting case from a formal point of view. Let's assume that being at the store implies that you're not at the movies (so we're excluding the odd possibility where a movie theater might actually be in a store).

Then, it seems appropriate to say that, since they both can't be true at the same time, it would be contradictory to assert that John is both at the movies and at the store. And that's the way most logicians would interpret this. They'd say that the law of non-contradiction applies to this conjunction even though, strictly speaking, these aren't logical contradictories of one another. **The key property that it has, is that it's a claim that is false for all possible truth values**.

Here's another way to look at it.

This is the truth table for the conjunction:

John is at the movies.	and	John is at the store.
T	**T**	T
T	**F**	F
F	**F**	T
F	**F**	F

A conjunction is true only when both conjuncts are true. For all other truth values it's false.

But in our case, the top line of the truth table doesn't apply, since our two claims are contraries — they can't both be true at the same time. So this case never applies.

John is at the movies.	and	John is at the store.
T	T	T
T	**F**	F
F	**F**	T
F	**F**	F

The remaining three lines give you all the possible truth values for contraries, and now we see that the conjunction comes out false for all of them.

This kind of example raises a question that logicians might debate — whether, on the one hand, a contradiction should be defined as a conjunction of contradictory claims, or, on the other hand, whether it should be defined as any claim that is false in all logically possible worlds.

Examples like these suggest to some people that it is this latter definition which is more fundamental, that's it more fundamental to say that a contradiction is a claim that is logically false, false in all possible worlds.

This issue isn't something you'll have to worry about, though. If you're a philosopher or a logician this may be interesting, but for solving logic problems and analyzing arguments, it doesn't make any difference.

4. Consistent vs Inconsistent Sets of Claims

Like the terms "valid" and "invalid", the most common use of "**consistency**" and "**inconsistency**" in everyday language is different from its use in logic.

In everyday language, something is "consistent" of it's predictable or reliable. So, a "consistent A student" is a student who regularly and predictably gets As. A consistent athlete is one who reliably performs at a certain level regardless of the circumstances. An inconsistent athlete performs well sometimes and not so-well other times, and their performance is hard to predict.

This **isn't** how we use the terms "consistent" and "inconsistent" in logic.

In logic, "consistency" is a property of sets of claims. **We say that a set of claims is *consistent* if it's logically possible for all of them to be true at the same time.**

What does "logically possible" mean here? ***Logically possible*** **means that the set of claims doesn't entail a logical contradiction.**

A contradiction is a claim that is false in all logically possible worlds, and we usually write the general form of a contradiction as a claim of the form "A and not-A".

"not-A" is usually interpreted as the contradictory of A, but as we saw in the last tutorial, this is can also be the contrary of A.

So, if it's logically impossible for a set of claims to be true at the same time, then we say that the set is logically inconsistent.

Examples

Let's look at some examples:

"All humans are mortal."

"Some humans are not mortal."

These clearly form an inconsistent set, since these are logical contradictories of one another. “Mortal” means you will some day die. “Not mortal” means you’ll never die, you’re “immortal”. If one is true then the other must be false, and vice versa.

Now what about this set?

“All humans are mortal.”

“Simon is immortal.”

Can both of these be true at the same time?

In this case, the answer is “yes”, both of these can be true. *They’re only inconsistent if you assume that Simon is human* —if that were true, then these would be inconsistent. But “Simon” is just a name for an individual, so Simon could be a robot or an angel or an alien, and if so then the claim about all humans being mortal wouldn’t apply.

Now, if you added the assumption about Simon being human as a claim to this set, like so ...

“All humans are mortal.”

“Simon is immortal.”

“Simon is human.”

... then you’d have an inconsistent set. Here you have three claims where, if any two of them are true, the third has to be false.

Let’s take a moment and look at this at little closer.

If all humans are mortal, and if Simon is immortal, then it logically follows that Simon can’t be human.

By “logically follows” I mean that you can construct a valid argument from these premises for the conclusion that Simon is not human, like so:

1. All humans are mortal.
2. Simon is immortal.
Therefore, Simon is not human.

This is what it means to say that the set entails a logical contradiction. From the set one can deduce a claim that is either the contradictory or the contrary of one of the other claims in the set.

Here's another way to represent this. The first two claims entail a claim that is the contradictory of the third claim. And from this it becomes evident that, to assert that all the claims in the set are true is to assert a formal contradiction.

Now, we can run this with any pair of claims in the set. If we set the second and third claims as true, for example, then we can infer that the first must be false. If Simon is immortal and if Simon is human, then it must be the case that not all humans are mortal, which contradicts the first claim.

The only remaining pair to check is the first and the third claims. If it's true that all humans are mortal, and it's true that Simon is human, we can validly infer that Simon is mortal, which contradicts the second claim.

An Important Fact About Inconsistent Claims

This example helps to illustrate another important fact about inconsistent sets of claims. IF we're given a set of claims that we know is inconsistent, then we know that at least one of the claims in the set must be FALSE.

So, if we want re-establish consistency, we need to abandon or modify at least one of these claims.

We used logic to establish that the set is inconsistent, but it's important to understand that logic alone can't tell us which of these claims to modify.

Logic tells us is that you can't consistently believe all of these claims at the same time, but it doesn't tell us how in any particular case to resolve the inconsistency.

Nevertheless, it can be very helpful in argumentation to have a group of people come to agree that a set of claims is inconsistent. In the end they may disagree about how to resolve the inconsistency, but it's still an achievement to get everyone to realize that they can't accept everything on the table, that something has to go.

Part 3: Contradictories of Compound Claims

1. not-(not-A)

In this series of lectures we'll be looking at how to write and interpret the **contradictories of the basic compound claim forms** — conjunctions, disjunctions, and conditionals.

But before we do that we should first talk about the **contradictory of a contradictory**. Contradictories are sometimes called "negations", so this rule is commonly called "double negation".

The rule is straightforward. If you take a claim and negate it, and then negate the negation, you recover the original claim:

not-(not-A) = A

Here's a simple example:

"Sarah makes good coffee."

The contradictory of this is "It is not the case that Sarah makes good coffee", or, more naturally, "Sarah does not make good coffee".

If we now take the contradictory of this, we get an awkward expression. If you were being very formal you'd say

"It is not the case that it is not the case that Sarah makes good coffee."

or

"It is false that it is false that Sarah makes good coffee."

These are pretty unnatural. "It is not the case that Sarah does not make good coffee" is also pretty unnatural. But you don't have to say this. With double negation you recover the original claim, so you can just say

"Sarah makes good coffee".

Double negation is mostly used as a *simplification rule* in formal logic, but we use it intuitively in ordinary speech all the time.

One word of caution. To use double negation correctly you need to know how to construct and recognize the contradictories of different kinds of claims.

For example, let's say someone wants to say

"It's false that Sarah and Tom didn't go bike riding".

If I want to simplify this using double negation I need to know how to interpret the contradictory of "Sarah and Tom didn't go bike riding".

But does this mean that "*Both* Sarah *and* Tom went bike riding"? Or does it mean "*Either* Sarah *or* Tom went bike riding"?

To be sure about this you need to know how to interpret the contradictory of a conjunction where each of the conjuncts is already negated — "Sarah didn't go bike riding AND Tom didn't go bike riding.

The correct answer is the second one, "Either Sarah or Tom went bike riding". But you'll need to check out the lecture on negating conjunctions to see why.

2. not-(A and B)

Let's look at how to construct the contradictory of a conjunction.

Here's our claim:

"Dan got an A in physics and a B in history."

When I say that this claim is false, what am I saying?

The conjunction says that both of these are true. The conjunction is false when either one or the other of the conjuncts is false, or both are false.

This gives us the form of contradictory:

"Either Dan didn't get an A in physics OR he didn't get a B in history".

The contradictory of a conjunction is a DISJUNCTION, an "OR" claim. You construct it by changing the AND to an OR and negating each of the disjuncts.

If you wanted to be really formal about it could write a derivation of the contradictory but the rule is fairly simple to remember. When you have a "not-" sign in front of a conjunction, you "push" the "not-" inside the brackets, distribute the "not-" across both of the conjuncts, and then switch the "AND" to an "OR".

The basic rule looks like this:

not-(A and B) = (not-A) or (not-B)

If this formula looks "algebraic" to you, that's because it is. This is a formal equivalence in propositional logic. It's also a basic formula of *Boolean logic*, which computer scientists will be familiar with since it's the logic that governs the functioning of digital electronic devices.

Let's do some problems with it.

Question: What's the contradictory of

"John and Mary went to the zoo"?

Answer:

"Either John didn't go to the zoo or Mary didn't go to the zoo."

You need to recognize the individual conjuncts, negate them, and put an "OR" in between. Note that the "either" is just a stylistic choice — "either A or B" is equivalent to "A or B".

You can also write the answer like this, of course:

"Either John or Mary didn't go to the zoo".

Let's try another one.

Question: What's the contradictory of

"Sam loves hotdogs but he doesn't like relish"?

With this one you have to remember that, from the standpoint of propositional logic, the "but" is functioning just like "and", and the whole thing is still a conjunction.

You'll also need to pay attention to the negation, "doesn't like relish", because you're going to end up negating this negation, which gives us an opportunity to use the double-negation rule.

Here's the answer:

"Either Sam doesn't like hotdogs or he likes relish".

You replace the conjunction with a disjunction, and you negate the disjuncts. Note that we've used double negation on the second disjunct. It's much easier to write "he likes relish" than "it's not the case that he doesn't like relish".

Well, that's about all you need to know about negating conjunctions. The basic rule is easy to remember:

"not-(A and B)" = "not-A OR not-B"

We'll see in the next lecture that the rule for the contradictory of a disjunction is very similar.

3. not-(A or B)

Now that we've done the contradictory of a conjunction, the **contradictory of a disjunction** will be no problem.

"Dan will either go to law school or become a priest".

This is a disjunction. What am I saying when I say that this disjunction is false?

The disjunction says that either one, or the other, or both of these are true — Dan will either go to law school, or he'll become a priest, or both.

If this is false, that means that *Dan doesn't do any of these things*. He doesn't go to law school, *and* he doesn't become a priest.

So the contradictory looks like this:

"Dan will not go to law school AND Dan will not become a priest."

The disjunction has become a conjunction, with each of the conjuncts negated. This is structurally identical to the rule we saw in the previous video, with the "OR" and the "AND" switched.

In English we have a natural construction that is equivalent to this conjunction:

"Dan will *neither* go to law school *nor* become a priest."

Remember this translation rule for "neither … nor …":

"(not-A) and (not-B)" is the same as "neither A nor B".

Don't be fooled by the "or" in "nor" — this not a disjunction, it's a conjunction.

Let's put the rules for the contradictory of the conjunction and the disjunction side-by-side, so we can appreciate the formal similarity:

not-(A and B) = (not-A) or (not-B)

not-(A or B) = (not-A) and (not-B)

In propositional logic these together are known as **DeMorgan's Rules** or **DeMorgan's Laws**, named after Augustus DeMorgan who formalized these rules in propositional logic in the 19th century.

They're also part of Boolean logic, and are used all the time in computer science and electrical engineering in the design of digital circuits.

That's it. These are the rules you need to know to construct the contradictories of conjunctions and disjunctions.

4. not-(If A then B)

The **contradictory of the conditional** is probably the least intuitive of all the contradictory forms that we'll look at. But we've already discussed this topic when we introduced the conditional and presented the truth table for the conditional, so this should be review.

Here's a conditional:

"If I pay for dinner then you'll pay for drinks."

What does it mean to say that this conditional claim is false?

When we first introduced the conditional we looked at this question. We determined that the only condition under which we would certainly agree that this claim is false is when the **antecedent is true but the consequent is false**.

This gives us the form for the contradictory. The most natural way to say it is **"I pay for dinner but you don't pay for drinks"**. I'm affirming the antecedent and denying the consequent.

Recall from our discussion of the conjunction that "but" just means "and", and that this is a conjunction, not a conditional.

Let me repeat that. **The contradictory of a conditional is not itself a conditional, it's a conjunction.**

Here's the general rule that makes this clear:

not-(If A then B) = A and not-B = A but not-B

The contradictory of a conditional is a conjunction that affirms the antecedent of the conditional but denies the consequent. Almost always, though, it's more natural to phrase the contradictory as "A but not-B", as in "I pay for dinner but you don't pay for drinks".

These are the conditions under which, if they obtained, we'd say that the original conditional was false.

Why not "If A then not-B"?

The **most common mistake** that students make when solving problems that require taking the contradictory of a conditional is to interpret the contradictory as a conditional of this form:

"If A then not-B"

This is a tempting interpretation of the contradictory, but it just doesn't work. There are a couple of ways of seeing why this is so. One way uses truth tables.

Conditional

A	→	B
T	**T**	T
T	**F**	F
F	**T**	T
F	**T**	F

Contradictory of the Conditional

A	?	B
T	**F**	T
T	**T**	F
F	**F**	T
F	**F**	F

On the left is the truth table for the conditional. The conditional is true for all truth values of A and B except when A is true and B is false.

The contradictory of the conditional is, by definition, a claim that is true whenever the conditional is false, and vice versa. So the middle column has the opposite truth value of the conditional, for the same values of A and B.

This, we know, *must* be the truth table for the contradictory of the conditional. The question is, ***what operations on A and B will yield this truth table?*** That's the question represented by the question mark in between A and B.

We can see right away that a truth table formed by simply negating the consequent won't do. On the left is the truth table for the contradictory of the conditional that we know must be true. On the right is the truth table where the only change is negating the consequent.

A	?	B
T	**F**	T
T	**T**	F
F	**F**	T
F	**F**	F

A	→	not-B
T	**F**	T
T	**T**	F
F	**T**	T
F	**T**	F

In the truth table on the right I've switched the truth values in the B-column, and I've evaluated the truth value of the conditional in the middle column according to the rule that the conditional as a whole is true except when A is true and B is false.

You can see that the truth values for this new conditional don't match up with the truth values for the contradictory of the conditional. They match for the cases where A is true, but not where A false.

From this alone we can rule this out as a candidate for the contradictory. Whatever functions as the contradictory of the conditional has to be more restrictive in its truth value, so that it comes out false whenever A is false.

Now let's look at the truth table for the conjunction with B negated.

A	?	B
T	**F**	T
T	**T**	F
F	**F**	T
F	**F**	F

A	and	not-B
T	**F**	T
T	**T**	F
F	**F**	T
F	**F**	F

As you can see, this gives us *exactly* what we need — the truth tables match. What we've just done confirms our rule, that the contradictory of a conditional is a conjunction with the B-term negated.

* * *

For the sake of having them all in one place, here are the formulas we introduced that give the contradictories for the basic compound claims of propositional logic:

not-(not-A) = A

not-(A and B) = (not-A) or (not-B)

not-(A or B) = (not-A) and (not-B)

not-(If A then B) = A and (not-B)

Part 4: Ways of Saying "If A then B"

1. A if B

In this section of the course we'll be looking at various different ways that we express conditional relationships in language.

The basic syntax for the conditional is "if A then B", but in ordinary language we have lots of ways of expressing conditionals that don't use this form.

We'll start with the form **"A if B".**

"If I pay for dinner then you'll pay for drinks."

This is written in standard conditional form. The antecedent is "I pay for dinner". The consequent is "you'll pay for drinks".

The "if" is what flags the antecedent. In standard form, the antecedent comes immediately after the "if".

Now, *I can write the same claim like this*:

"You'll pay for drinks if I pay for dinner."

Here the consequent is now at the beginning of the sentence and the antecedent is at the end. **But the antecedent is still "I pay for dinner"**. The "if" flags the antecedent just as as it does when the conditional is written in standard form.

Here's the **general translation rule:**

B if A = If A then B

I want to mention something here that might be a source of confusion. I've written it as a "B if A" rather than "A if B", so that the As and Bs correspond when compared with the conditional in standard form. So the same symbols represent the antecedent and the conditional in both versions.

But you shouldn't expect the same letter to always represent the antecedent of the conditional. The symbols are arbitrary. I could write the same rule in all these different ways,

A if B = If B then A

Q if P = If P then Q

$ if @ = If @ then $

and it would still represent the same rule.

What matters is that in standard form, whatever follows the "if' is the antecedent. The trick in interpreting different versions of the conditional is to *identify the claim that is functioning as the antecedent*, so that you can then re-write the conditional in standard form.

This is actually a very useful skill when analyzing ordinary arguments. We'll eventually cover the valid and invalid argument forms that use the conditional, and these are always expressed using the conditional in standard form, so in order to apply your knowledge of valid and invalid argument forms you need to be able to translate conditionals into standard form.

Let's finish with a couple examples. The exercise is to write these conditionals in standard form:

"David will be late if he misses the bus."

and

"You'll gain weight if you don't exercise."

The answers looks like this:

"David will be late if he misses the bus."
=
"If David misses the bus then he'll be late."

"You'll gain weight if you don't exercise."
=
"If you don't exercise then you'll gain weight."

The rule is that you look for the “if”, and whatever follows the “if” is the antecedent of the conditional.

This is the simplest alternative form for the conditional. As we’ll see, there are other forms, and they can be trickier to translate.

2. A only if B

There's a big difference between saying that A is true IF B is true, and A is true ONLY IF B is true. Let's look at this difference.

Here's a conditional expressed using "only if":

"The match is burning only if there's oxygen in the room."

We need to figure which of these is the antecedent, "the match is burning" or "there's oxygen in the room".

Given what we did in the last lecture, it's tempting to just look for the "if" and apply the rule, whatever comes after the "if" is the antecedent, and conclude that "there's oxygen in the room" is the antecedent.

But that's wrong. "There's oxygen in the room" **isn't** the antecedent.

If this was the antecedent, then the sentence would be saying that if there's oxygen in the room then the match will be burning. But if you're saying that then you're saying that *the presence of oxygen is enough to guarantee* that the match is burning.

That's not what's being said. What's being said is that the presence of oxygen in the room is *necessary* for the match to be burning, it doesn't say that it *will* be burning.

This sentence expresses a conditional, but **the antecedent of the conditional is, in fact, "the match is burning".**

The "only if" makes a dramatic difference. This sentence is equivalent to the following conditional written in standard form:

"If the match is burning then there's oxygen in the room."

The "only if" actually reverses the direction of logical dependency. When you have "only if", the claim that precedes the "only if' is antecedent, what follows it is the consequent.

Here's the "only if" rule:

"A only if B" = "If A then B"

The antecedent doesn't come after the "if", the consequent comes after the "if".

Let's take away the symbols and compare the "if" and "only if' rules.

___ if ___

___ only if ___

When you're given a conditional that uses "if" or "only if", you look for the "if", and if the "if" is all by itself, then the antecedent is what immediately follows the "if".

If the "if" is preceded by "only" then you do the opposite, what follows the "only if" is the consequent, what precedes the "only if" is the antecedent. Once you've got that, then the rest is easy:

(consequent) if (antecedent)

(antecedent) only if (consequent)

From here you can easily write the conditional in standard form, "If A then B".

Examples

Let's look at some examples. Here are two sentences:

"Our team will kick off if the coin lands heads."

"I'll buy you a puppy only if you promise to take care of it."

They both express conditionals. You need to write these in standard form, in the form "If A then B". That requires that you identify the antecedent and the conditional in each sentence.

In the first sentence, "Our team will kick off if he coin lands heads", the "if" appears by itself, so we know that what immediately follows the 'if" is the antecedent.

So we write the conditional in standard form as follows:

"If the coin lands heads then our team will kick off."

For the second sentence we have an "only if", so we know to do precisely the opposite of what we did in the previous case. The antecedent is what precedes the "only if". So, you write the conditional in standard form like this:

"If I buy you a puppy then you promise to take care of it."

This conditional doesn't say that if you promise to take care of it I'll buy you a puppy. It's saying that if I buy you a puppy, then you can be sure that you promised to take care of it, because that was a necessary condition for buying the puppy. But merely promising to take care of the puppy doesn't guarantee that I'll buy it.

It might be a bit more natural to write it like this: "If I *bought* you a puppy then you *promised* to take care of it".

Sometimes shifting the tenses around can be helpful in expressing conditionals like this in a more natural way. For our purposes they mean the same thing.

Here's the general rule once again:

"A only if B" = "If A then B"

In the next lecture we'll look at what happens when you combine the "if" and "only if".

3. A if and only if B

You may have heard the expression “**if and only if”** in a math class or some other context. In this tutorial we’ll look at what this means as a conditional relationship.

As the name suggests, “A if and only if B” is formed by conjoining two conditionals using the “if” rule and the “only if” rule:

“A if B” and “A only if B”

So it asserts two things, that A is true if B is true, AND that A is true ONLY IF B is true.

You can use the “if” rule and the “only if” rule to translate these into standard conditionals, and when you do the expression looks like this:

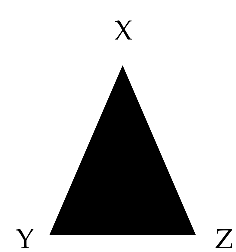
“If B then A” and “if A then B”

This asserts that *the conditional relationship runs both ways*. Given A you’re allowed to infer B, and, given B you’re allowed to infer A.

It’s not surprising, then, this is also called a **biconditional**. You might encounter the biconditional written in different ways, but they all mean the same thing, that A implies B and B implies A.

Biconditionals show up a lot in formal logic and mathematics. **They’re used to demonstrate the logical equivalence of two different expressions**. From a propositional logic standpoint, the defining feature of a biconditional is that the claims, A and B, always have the same truth value — if A is true, then B is true, and vice versa.

Here’s an example of a biconditional relationship whose truth is obvious:



Let A = “The triangle XYZ has two equal sides.”

Let B = “The triangle XYZ has two equal angles.”

It’s clear that if A is true then B is also true. The sides XY and XZ are equal, and from the diagram you can see this requires that the angles at Y and Z must also be equal.

And it's also clear that converse is true as well, that if a triangle has two equal angles then it also has two equal sides. So "if B then A" is also true.

But if both of these conditionals are true, then we can say that A is true if and only if B is true, and vice versa.

One of the helpful things about learning about the biconditional as a concept is that it helps us to remember that *ordinary* conditionals are only *half* a biconditiona*l, they only go one way*. If A implies B it doesn't follow that you can go backwards and say that B implies A. It reminds us that you need to argue or demonstrate that you can run the inference in the other direction as well.

4. A unless B

There are a lot of ways of saying “if A then B”. We can even say it without using the words “if” or “then”.

Here’s an example:

“Jennifer won’t go to the party unless Lauren goes too.”

The word “unless” is acting like a conditional operator in this sentence.

If you were asked to rewrite this as a standard conditional, you would probably translate this as

“If Lauren doesn’t to the party then Jennifer won’t go to the party.”

This exactly right.

We’ve done two things here. First, we recognize that the antecedent of the conditional is what comes immediately after the “unless”.

Second, we recognize that we need to take the *contradictory* of this claim to get the meaning of the conditional right.

So, we look for the “unless”, take what immediately follows, negate it, and make that the antecedent of the conditional.

This gives us the form of the general rule:

B unless A = If not-A then B

If you say that B is true unless A is true, then you’re saying that if A is false then B is true.

Once again, don’t be too fixated on which letters we’re using to represent the antecedent and the consequent.

For me, I like simple translation rules that are easy to remember, so I usually say to myself, **read “unless” as “if not-”**. This is probably the easiest way to remember this rule.

Here’s a final example that puts a small spin on things. The claim is

“Unless you pay us one million dollars, you’ll never see your pet goldfish again.”

Here the “unless” is at the beginning of the sentence rather than in the middle, but the rule still applies. You should **read “unless” as “if not-”**.

So the translation is

“If you don’t pay us one million dollars, then you’ll never see your pet goldfish again”.

It doesn’t matter where the “unless” shows up in the sentence, the translation rule still applies.

5. The Contrapositive (If not-B then not-A)

The **contrapositive** is a very important translation rule. It's mainly used for simplifying conditionals that use negations, but it's used extensively in LSAT logic games and other tests of analytical reasoning.

The contrapositive is easy to illustrate.

"If I live in Paris then I live in France."

This is a conditional claim, and it happens to be true (assuming we're talking about Paris, the city with the Eiffel Tower, and not some other city with the same name).

Now, if this is true, then this is also true:

"If I don't live in France then I don't live in Paris".

This is the **contrapositive** of the conditional above.

The contrapositive is a conditional formed by switching the antecedent and the consequent and negating them.

Here's the general rule:

"If A then B" = "If not-B then not-A".

Here are few more examples:

Conditional:

"If we win the game then we'll win the championship."

Contrapositive:

"If we didn't win the championship then we didn't win the game."

If the first conditional is true, then if we didn't win the championship, we can be sure that we didn't win the game.

The rule applies to all conditionals, even false or nonsensical conditionals like this one:

Conditional:

"If George Bush is a robot then Jackie Chan is Vice-President."

This is absurd, of course, but if this conditional was true — if George Bush's being a robot actually entailed that Jackie Chan was the Vice President" — then the contrapositive would also be true:

Contrapositive:

"If Jackie Chan is not Vice President, then George Bush is not a robot."

For our last example let's mix it up.

Conditional:

"You won't become a good player if you don't practice."

It's written with the "if" in the middle, so to write the contrapositive you have to make sure you've got the antecedent and the consequent right.

Well, the "if" rule says that whatever follows the "if" is the antecedent, so we know the antecedent is "You don't practice", and the consequent is "you won't become a good player".

Now, to write the contrapositive, you switch the antecedent and the consequent and negate both parts.

The consequent of the original is "you won't become a good player". Negating this you get "You will become a good player". This becomes the antecedent of the contrapositive.

Contrapositive:

"If you become a good player than you practiced."

Sometimes you may want to shift tenses a bit to make a claim sound more natural. In this case I've written it as "you become a good player" rather than "you will become a good player", but it doesn't make much difference. I could have also written it as "you became a good player".

Once you've got the antecedent of the contrapositive it's easy to write the consequent — "if you become a good player then you must have practiced."

The challenge with problems like these is to not get turned around and mistake an antecedent for a consequent. In this case, *the most common error* would be to interpret the contrapositive as “If you practice then you’ll become a good player”. This is very tempting, but it’s not entailed by the original claim.

The original claim doesn’t say that if you practice you’re guaranteed to become a good player. All it says is that if don’t practice then you’re certainly not going to become a good player. So, what we can infer is that if you end up becoming a good player, then we can be sure of one thing, that you practiced.

The value of these rules is that they keep you from assuming that you know more than you do, based on the information given.

6. (not-A) or B

Here's another translation rule for conditionals that doesn't use "if" or "then". This one is useful for interpreting disjunctions as conditionals, or rewriting conditionals in the form of a disjunction.

It turns out **you can write any conditional as a disjunction, a claim of the form "A or B"**.

Consider this conditional:

"If live in Paris, then I live in France."

There are only three possibilities for the way the disjunction can be phrased:

- A or B
- A or (not-B)
- (not-A) or B

From the title of this lecture you already know the answer to this question, but for the sake of demonstrating why this answer is correct let's work through these.

A or B = I live in Paris or I live in France.

If the original conditional is true, is this disjunction true? No, this disjunction doesn't have to be true. The original conditional is consistent with me living in New York, say. There's no reason why I have to live in Paris or France. So this won't work. Let's try the next one.

A or (not-B) = I live in Paris or I don't live in France.

If the original conditional is true, does it follow that either I live in Paris or I don't live in France? That would be an odd inference, wouldn't it? This entails that Paris is the only city in France that I'm allowed to live in. This doesn't follow, so strike that one out.

(not-A) or B = I don't live in Paris or I live in France.

Now, if the original conditional is true, does it imply that either I don't live in Paris or I live in France?

It does. Let's see why.

Recall that what the "OR" means is that one or the other of these must be true, they can't both be false. It's easy to see that they can't both be false. If they were both false, I'd be saying that I live in Paris but I don't live in France. That's impossible, since Paris is in France. So both of these disjuncts can't be false at the same time; one of them must be true.

Now let's assume that the first disjunct is false: "I do live in Paris." Does it then follow that I live in France? Yes it does — Paris is in France.

Now assume that the second disjunction is false: "I don't live in France." Does it then follow that I don't live in Paris? Yes it does, for the same reason. This is indeed the correct translation.

If you're not convinced you can show that these are logically equivalent with truth tables:

A	B
T	T
T	F
F	T
F	F

Disjunction

not-A	OR	B
F	T	T
F	F	F
T	T	T
T	T	F

Conditional

A	→	B
T	T	T
T	F	F
F	T	T
F	T	F

On the left is all the possible truth values of A and B. In the middle is the truth table for the disjunction when you negate A. On the right is the truth table for the conditional.

You can see that the truth values for the disjunction and the conditional, highlighted in grey, match up.

Some people find these kinds of explanations helpful, and some don't. Either way, the general rule is easy to remember:

If A then B = (not-A) or B

It's helpful to have the brackets around "not-A" so that you don't confuse this expression with the contradictory of a disjunction, "not-(A or B)". Brackets in logic function like they do in math. They clarify the order of operations when it might otherwise be unclear.

Let's look at a few examples.

Conditional:

If we win the game then we'll win the championship.

Disjunction:

We won't win the game or we'll win the championship.

I admit that these translations don't always sound very natural, but if you think about the semantics of disjunctions and work through the reasoning you'll see that they get the logic right.

And sometimes you can express them in a more natural way, like this:

"Either we lost the game or we won the championship."

Regardless, you won't go wrong if you trust the translation rule.

Here's one more example:

Conditional:

"If there's no gas in the car then the car won't run."

Disjunction:

"There's gas in the car or the car won't run."

This one sounds pretty natural as it is.

This translation rule can be handy for working through certain kinds of LSAT logic problems where you have to represent conditional rules on a diagram. Sometimes it's easier to do this when the rule is expressed as an "either __ or ___" proposition.

7. Necessary and Sufficient

The last set of terms we'll look at for expressing conditional relationships involve the concepts of "**necessity**" and "**sufficiency**".

"If I become rich, then I'll be happy."

Here's a question: When I say this, am I saying that becoming rich is *necessary* for me to be happy?

If I say that becoming rich is necessary for my happiness, then I'm saying that there's *no way* for me to be happy *unless* I'm rich.

That doesn't seem right.

What does seem right is to say that my becoming rich is *sufficient* for my being happy.

"Sufficient" means that it's *enough to guarantee* that I'll be happy. But it *doesn't* imply that becoming rich is the *only* way that I can be happy. My becoming rich is sufficient for my happiness, but it's not necessary for it.

In terms of the antecedent and the consequent of the original conditional, we can say that

"If A then B" = "A is sufficient for B".

Or in other words, **the antecedent is sufficient to establish the consequent.**

This is the first general rule: A conditional can always be translated into a sentence stating that the truth of the antecedent is sufficient to ensure the truth of the consequent.

So how do interpret the language of "necessity"?

Well, let's go back to our original claim.

"If I become rich, then I'll be happy."

We can't say that A is necessary for B. But we *can* say that B is necessary for A.

In other words, we can't say that my being rich is necessary for my happiness. But we can say that my happiness is a necessary consequence of my being rich. In other words, if I end up rich then I'm necessarily happy.

So, *relationships of necessity and relationships of sufficiency are converses of one another*. If A is sufficient for B then B is necessary for A.

It's easier to see if you have the rules side by side:

A is sufficient for B = If A then B

A is necessary for B = If B then A

When you see a claim of the form "A is sufficient for B", then you read that as saying that if A is true then B is guaranteed; the truth of A is sufficient for the truth of B.

When you see a claim of the form "A is necessary for B", then you should imagine *flipping the conditional around*, because the B term is now playing the role of the antecedent.

Another way of stating the rule for "necessary" is to express it in terms of the contrapositive, "If not-A then not-B". The only way to make this clear is to look at examples.

"Oxygen is necessary for combustion."

This doesn't mean that if there's oxygen in the room then something is going to combust. Matches don't spontaneously burst into flame just because there's oxygen in the room.

What this statement says is that if there's combustion going on then you know that oxygen must be present. And you would write that like this:

"If there's combustion then there's oxygen."

Or, you could write it in the contrapositive form,

"If there's no oxygen then there's no combustion".

Either way will do.

Let's do an example working the other way. We're given the conditional,

"If I have a driver's license then I passed a driver's test."

How do we write this in terms of necessary and sufficient conditions?

How about this?

"Having a driver's license is necessary for passing a driver's test."

Does this work?

No, it doesn't. It would be very odd to say this, since it implies that you already have to have a driver's license in order to pass a driver's test!

We need to switch these around:

"Passing a driver's test is necessary for having a driver's license."

Using the language of sufficiency, you'll reverse these:

"Having a driver's license is sufficient for passing a driver's test."

This is a little awkward, but the logic is right. If you know that someone has a driver's license, that's sufficient to guarantee that at some point they passed a driver's test.

Finally, I want to draw attention to the parallels between the language of necessary and sufficient conditions and the language of "if and only if". These function in exactly the same way.

A is necessary for B = A if B = If B then A

A is sufficient for B = A only if B = If A then B

Both emphasize that a conditional relationship only goes one way, and that if you can establish that both are true then you've established biconditional relationship:

A is necessary and sufficient for B

means the same as

A if and only if B

which means the same as

If (B then A) and (If A then B)

That's it for this section on the different ways we express conditionals in ordinary language.

I know from experience that mastering the rules that we've been discussing in these last few lectures really does make you become aware of logical relationships, and by itself this will help you to detect errors in reasoning and help you to be clear and precise in your own reasoning.

Part 5: Categorical Claims and Their Contradictories

1. Categorical vs Propositional Logic

In any standard logic textbook you'll see separate chapters on both propositional logic and categorical logic. Sometimes categorical logic is called "Aristotelian" logic, since the key concepts in this branch of logic were first developed by the Greek philosopher Aristotle.

I'm not planning on doing a whole course on categorical logic at this stage, but there are a few concepts from this tradition that are important to have under your belt when doing very basic argument analysis, so in this next series of tutorials I'm going to introduce some of these basic concepts.

In this introduction I'm going to say a few a words about what the basic difference is between categorical logic and propositional logic.

In the course on "Basic Concepts in Logic and Argumentation" we saw a lot of arguments and argument forms that are basically categorical arguments and that use the formalism of categorical logic.

Here's a classic example.

1. All humans are mortal.
2. Simon is human.

Therefore, Simon is mortal.

This argument is valid. When you extract the form of this argument it looks like this:

1. All H are M
2. x is an H

Therefore, x is an M

The letters are arbitrary, but it's usually a good idea to pick them so they can help us remember what they represent.

Now, the thing I want to direct your attention to is **how different this symbolization is from the symbolization in propositional logic**.

When we use the expression “All H are M”, the “H” and the “M” DO NOT represent PROPOSITIONS, they don’t represent complete claims. In propositional logic each letter symbolizes a complete proposition, a bit of language that can be true or false. Here, the H and the M aren’t propositions.

So what are they?

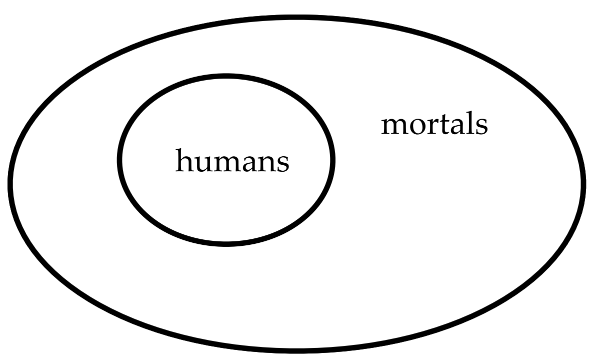
They’re **categories**, or **classes**. H stands for the *category* of human beings, M stands for the *category* of all things that are mortal, that don’t live forever.

These categories or classes are like buckets that contain all the things that satisfy the description of the category.

What they *don’t* represent is a complete claim that can be true or false. This is a fundamental difference in how you interpret symbolizations in categorical logic compared to how you interpret them in propositional logic.

In categorical logic, you get a complete claim by stating that there is a particular relationship between different categories of things.

In this case, when we say that **all humans are mortal**, you can visualize the relationship between the categories like this:



We’re saying that the category of mortals CONTAINS the category of humans. Humans are a SUBSET of the category of things that die. The category of mortals is larger than the category of humans because lots of other things can die besides human beings. This category includes all living things on earth.

Now, when you assert this *relationship* between these two *categories*, you have a complete proposition, a claim that makes an assertion that can be true or false.

This is the fundamental difference between symbolizations in propositional logic and categorical logic.

In propositional logic you use a single letter to represent a complete proposition.

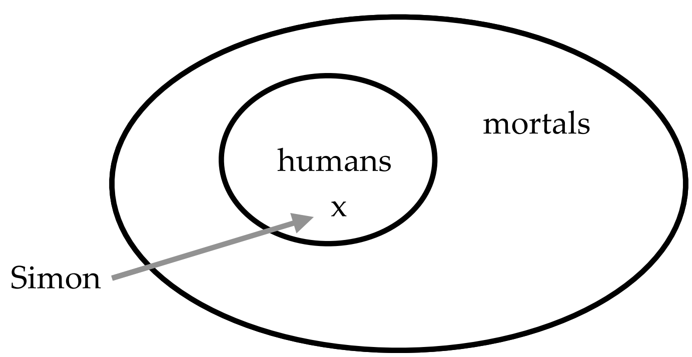
In categorical logic the analysis is more fine-grained. You're looking INSIDE a proposition and symbolizing the categories that represent the subject and predicate terms in the proposition, and you construct a proposition by showing how these categories relate to one another.

Now, what does that small "x" represent, in "x is an H"?

It represents an INDIVIDUAL human being, Simon.

In categorical logic you use capital letters to represent categories or classes of things, and you use lower-case letters to represent individual members of any particular category.

On a diagram like this you'd normally us a little x to represent Simon,



like this:

So, putting the X for Simon inside the category of humans is a way of representing the whole proposition, "Simon is human".

Notice, also, that from this diagram you can see at a glance why the argument is VALID. This diagram represents the first two premises of the argument. When judging validity you ask yourself, if these premises are true, could the conclusion possibly be false?

And you can see that it can't be false. If x is inside the category of humans, then it HAS to be inside the category of mortals, since humans are subset of mortals.

In a full course in categorical logic you would learn a whole set of diagramming techniques for representing and evaluating categorical arguments, but that's not something we're going to get into here.

What we're going to talk about is what sorts of claims lend themselves to categorical analysis. These are claims with the following forms:

- All A are B
- Some A are B
- No A are B
- All A are not-B
- Some A are not-B
- No A are not-B

Claims like "All humans are mortal", "Some men have brown hair", "No US President has been female", "All mammals do not have gills" and so on.

The Greek philosopher Aristotle worked out a general scheme for analyzing arguments that use premises of this form. In the tutorial course on "Common Valid and Invalid Argument Forms" we'll look at a few of the most common valid and invalid categorical argument forms.

In the remaining lectures in this section all I really want to do is look at the semantics of categorical claims, what they actually assert, and how to write the contradictory of these categorical claims.

2. All A are B

This is the classic universal generalization, “**All A are B**”.

Here some examples of claims with this form:

- **“All humans are mortal.”**
- **“All whales are mammals.”**
- **“All lawyers are decent people.”**

Two things to note about these sorts of generalizations:

- First, the “all” is strict; when you read “All”, it really means “ALL”, **no exceptions**.
- Second, we often don’t use the “all” to express a universal generalization. “Humans are mortal” means “humans in general are mortal”, it’s implied that you’re talking about all humans. Similarly, “whales are mammals” means “all whales are mammals”, and “lawyers are decent people” means “all lawyers are decent people.”

Now, in the real world, people sometimes aren’t careful and will make a generalization that they will acknowledge has exceptions. They might say “All politicians are crooks”, but then they might admit that one or two are pretty decent. They’re not lying, they’re just not being precise, or maybe they’re exaggerating for the sake of dramatic effect. What they really mean is “Most” or “Almost all” politicians are crooks.

In logic it matters a great deal whether you mean “all” or “almost all”, so just be aware of the strictness of your language; if you don’t really mean “all”, then don’t say “all”.

The **contradictory of a universal generalization** is pretty straightforward, there’s just one thing to be on the lookout for.

If I say “All humans are mortal”, it’s tempting to think that the contradictory might be “No humans are mortal”.

But this is wrong. This isn’t the contradictory.

Remember that the contradictory has to have the opposite truth value in all possible worlds; if one is true then the other must be false, and vice versa.

But imagine a world in which half the people are mortal and the other half are immortal. In this world, BOTH of these statements would be FALSE. This shouldn't happen if they're genuine contradictories.

No, these are CONTRARIES. They can't both be true at the same time, but they can both be false.

The contradictory of "All humans are mortal" is

"SOME humans are NOT mortal".

or, "Some humans are immortal."

These two claims always have opposite truth values. If one is true the other has to be false.

Here's the general form:

not-(All A are B) = Some A are not-B

And here are some examples:

Claim: **"All dogs bark."**

Contradictory: "Some dogs don't bark."

Claim: "Canadians are funny."

Contradictory: "Some Canadians are not funny."

Note that here you need to remember that "Canadians are funny" makes a claim about all Canadians, logically you need to read it as "All Canadians are funny".

Claim: "All Michael Moore films are not good."

The translation rule works just the same, but you need to use double-negation on the "not good" part. So the contradictory looks like

Contradictory: "Some Michael Moore films are good".

You see how this works.

3. Some A are B

Let's look at this expression, **"Some A are B".**

- **"Some dogs have long hair."**
- **"Some people weigh over 200 pounds."**
- **"Some animals make good pets."**

It might seem that "some" is so vague that it's hard to know exactly what it means, but in logic "some" actually has a very precise meaning. **In logic, "some" means "at least one".**

"Some" is vague in one sense, but it sets a precise lower bound. If "some dogs have long hair", then you can be certain that *at least one dog* has long hair.

So, "some people weigh over 200 pounds" means "at least one person weighs over 200 lbs".

"Some animals make good pets" means "at least one animal makes a good pet".

There are couple of equivalent ways of saying this. If you want to say **"some dogs have long hair"**, then you could say

"At least one dog has long hair",

or

"There is a dog that has long hair",

or

"There exists a long-haired dog".

These are all different ways of saying "at least one".

Here's something to be aware of. The standard reading of "At least one A is B" is consistent with it being true that "All A are B".

So if I say "Some dogs have long hair", this doesn't rule out the possibility that all dogs in fact have long hair.

But sometimes, “some” is intended to rule out this possibility. Sometimes we want it to mean “at least one, *but not all*”. Like if I say, “some people will win money in next month’s lottery”, I mean “at least one person will win, but not everyone will win”. Which reading is correct — whether it means “at least one” or “at least one but not all” — will depend on the specific context.

Now, let’s look at t**he contradictory of “Some A are B”**.

“Some dogs have long hair.”

If this is false, what does this imply? Does it imply that ALL dogs have long hair?

No. At most, this would be a contrary, if we were reading “Some” as “At least one, but not all”.

No, the contradictory of “Some dogs have long hair” is

“No dogs have long hair.”

If no dogs have long hair then it’s always false that at least one dog has long hair, and vice versa.

So, the **general form of the contradictory** looks like this:

not-(Some A are B) = No A are B

Examples:

Claim: “Some movie stars are rich.”

Contradictory: “No movie stars are rich.”

Claim: “There is a bird in that tree.”

Note that this is equivalent to saying that there is at least one bird in that tree, or “Some bird is in that tree”. So the contradictory is

Contradictory: “No bird is in that tree.”

Another example:

Claim: “Some dogs don’t have long hair.”

Contradictory: “No dogs don’t have long hair.”

This is a bit awkward. The easiest way to say this is "**All dogs have long hair**". Here we're just applying the rule for writing the contradictory of a universal generalization:

not-(All are B) = Some A are not-B

which is the form the original claim has.

This is about all you need to know about the logic of "Some A are B".

4. Only A are B

Let’s take a look at **“ONLY A are B”**.

- **“Only dogs make good pets.”**
- **“Only Great White sharks are dangerous.”**
- **“Only postal employees deliver U.S. mail.”**

You won’t be surprised to learn that the logic of “Only A are B” parallels the use of “only if” in the logic of conditionals. There, “A if B” is equivalent to “B only if A”, where the antecedent is switched between “if” and “only if”.

Here, the switch is between “Only” and “All”.

- **“Only dogs make good pets”** means the same thing as **“All good pets are dogs”.**
- **“Only Great White sharks are dangerous”** means “**All dangerous sharks are Great Whites”**.
- **“Only postal employees deliver U.S. mail”** means **“All people who deliver U.S. mail are postal employees”**.

The **general translation rule** is this:

“Only A are B” can re-written as “All B are A”.

Note how similar this is to the translation rule for conditionals:

“A only if B” is equivalent to “B if A”.

The difference, of course, is that the As and Bs refer to very different things. In categorical logic the As and Bs refer to categories or classes of things. In propositional logic the As and Bs refer to whole claims, in this case either the antecedent or the consequent of a conditional claim.

It’s true, though, that the logical dependency relationships are very similar — in both cases when you do the translation you the switch the As and the Bs —and recognizing the analogy can be helpful when you’re doing argument analysis. We’ll come back to this in the course on “common valid and invalid argument forms”.

Now, let's look at **the contradictory of "Only A are B"**.

Someone says that "**only dogs make good pets"**. You say "no", that's not true. What's the contradictory?

The contradictory would be to say that **"Some good pets are not dogs"**.

How do we know this? How do we know that, for example, it's not **"Some dogs are not good pets**"?

Well, you can figure it out just by thinking about the semantics and knowing what a contradictory is, but there's a formal shortcut we can use to check the answer.

We can exploit the fact that we know that **"Only dogs make good pets"** is equivalent to **"All good pets are dogs"**.

Now, we have the original claim in the form "All A are B", and we already know how to write the contradictory of a universal generalization, it's "Some A are not B". And this gives us the form of the contradictory, "Some good pets are not dogs".

This is typical with these sorts of rules. Once you know some basic translation rules and some rules for writing contradictories, you can often rewrite an unfamiliar claim in a form that is more familiar and that allows you to apply the rules that you do know.

The **general form of the contradictory** looks like this:

not-(Only A are B) = Some B are not A

So, if our original claim is **"Only movie stars are rich"**, the contradictory is "**Some rich people are not movie stars"**. It's like the rule for "ALL", but you need to reverse the As and the Bs.

Here's another one: **"Only Starbucks makes good coffee"**.

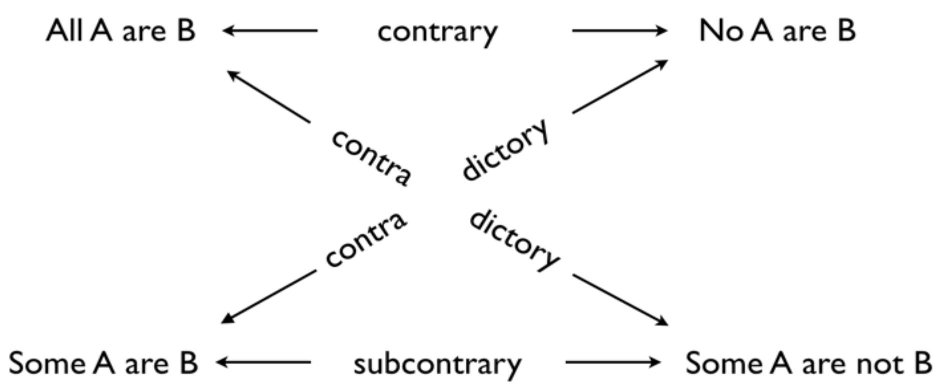
The **contradictory** is **"Some good coffee is not made by Starbucks"**.

You replace "only" with "some", switch the As and Bs, but make sure you take the negation of the predicate class.

This is all you need to know about the logic of "Only A are B".

5. The Square of Opposition

Here’s a handy diagram that might help some of you memorize the contradictories of the different categorical forms.



This diagram is sometimes called the “**Square of Opposition”**.

It’s not a complete version of the Square of Opposition that shows up in most textbooks. I’ve left off some relationships that appear in the complete diagram, since we haven’t talked about them. But it captures at a glance the contradictories of the categorical claims that use All, Some and No.

The contradictories are on the diagonal. At the top you have contraries. “All A are B” and “No A are B” can’t both be true, but they can both be false.

At the bottom you have what are called “subcontraries”. Can you guess what this is?

Well, as the name subjects, it’s a contrary relationship, but in this case the two claims can both be true at the same time, but they can’t both be false. That’s the opposite of a contrary, so it’s called a “subcontrary”.

Anyway, I thought I’d mention this in case anyone finds it useful.