Angles and transversals

In this lesson we'll look at the angles formed when a pair of parallel lines is crossed by another line, called a "transversal."

Transversals

A **transversal** is a line that crosses at least two other lines. In the figure on the left, the transversal is crossing two non-parallel lines, and in the figure on the right, the transversal is crossing two parallel lines.

Special angle pairs

When a transversal crosses a pair of parallel lines, pairs of angles with special angle relationships are formed. In terms of angle measure, there are two types of angle pairs with these special relationships.

Congruent angles, which have the same measure.

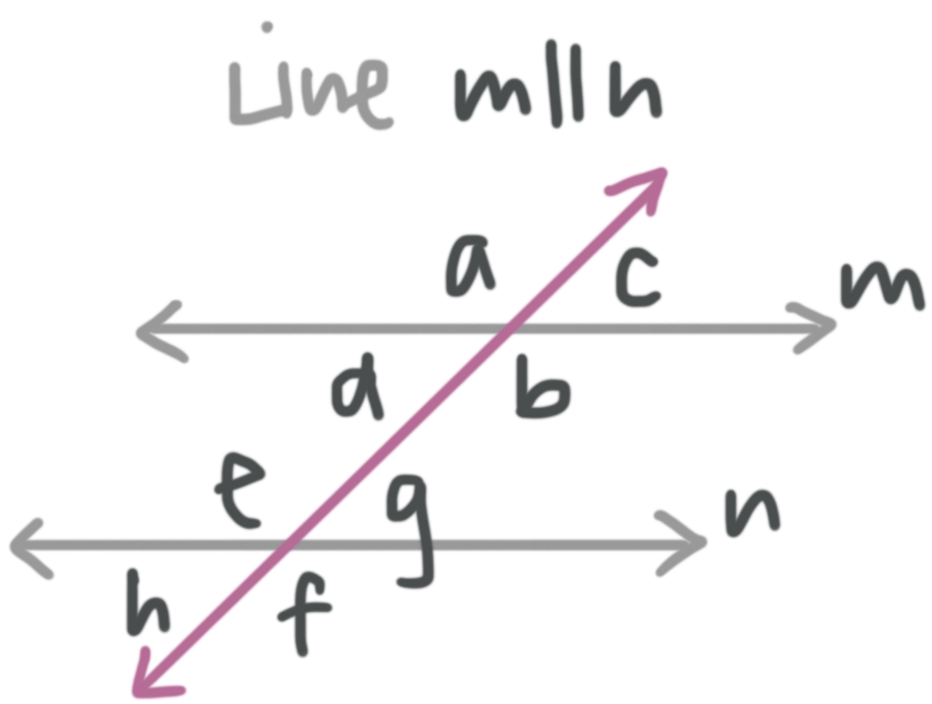
Supplementary angles, which have measures that add to 180°.

Actually, any pair of angles that have the same measure are congruent, not just pairs of angles that are formed when a transversal crosses a pair of parallel lines. And any pair of angles whose measures add up to 180° are supplementary.

Types of special angle pairs

Vertical angles share a vertex (which is the point of intersection of the transversal and one of the lines in the pair of parallel lines crossed by the transversal) but no ray, and they lie on opposite sides of the transversal. Vertical angle pairs are congruent.

There are four pairs of vertical angles in the figure below: (a, b), (c, d), (e, f), and (g, h). If a pair of lines are parallel (and we call them, say, m and n), we denote that by $m \parallel n$.



$$m\angle a = m\angle b$$

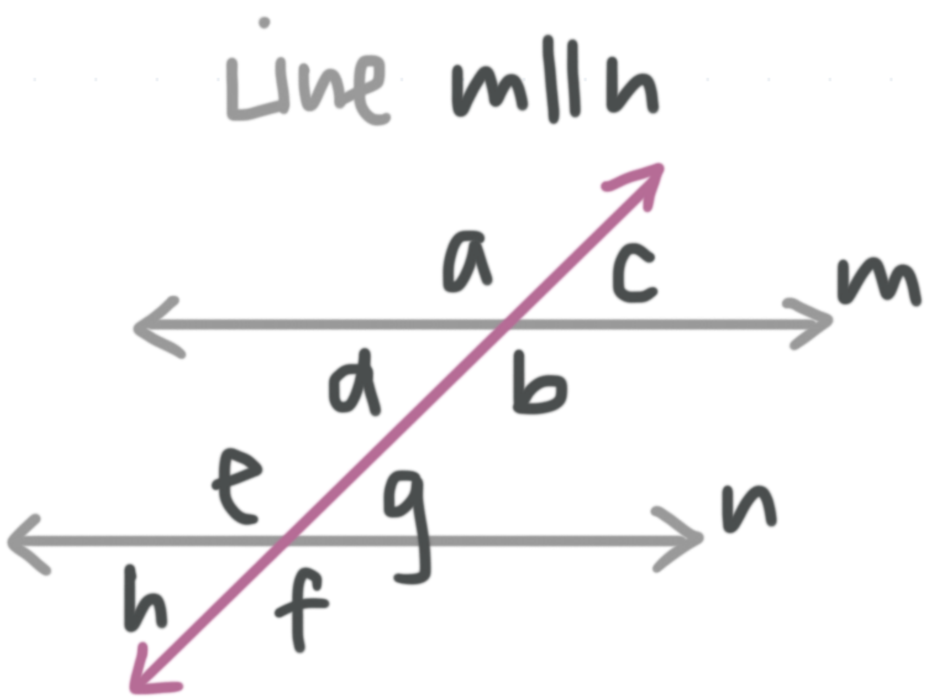
$$m\angle c = m\angle d$$

$$m\angle e = m\angle f$$

$$m\angle g = m\angle h$$

Corresponding angles are angles that lie on the same side of the transversal, their vertices are on opposite lines (in the pair of parallel lines crossed by the transversal), the interior of one of the angles in the pair lies partially inside the region between the parallel lines, and the interior of the other angle lies entirely outside the region between the parallel lines but entirely within the interior of the first angle. Corresponding angle pairs are congruent.

There are four pairs of corresponding angles in the figure below: (a, e), (b, f), (c, g), and (d, h).



$$m\angle a = m\angle e$$

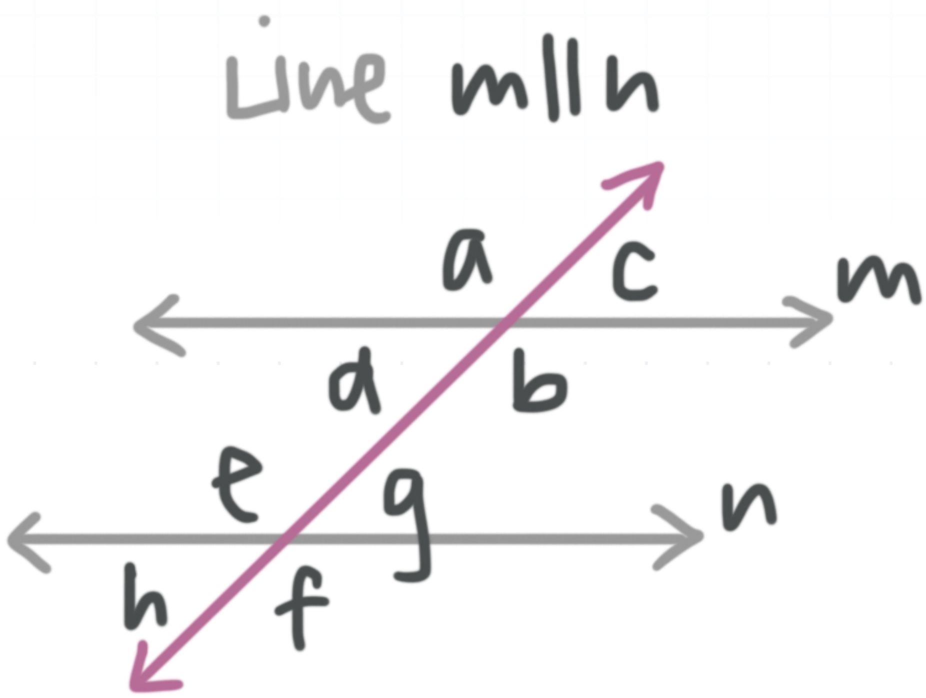
$$m\angle d = m\angle h$$

$$m\angle c = m\angle g$$

$$m\angle b = m\angle f$$

Alternate interior angles are angles that lie on opposite sides of the transversal, their vertices are on opposite lines (in the pair of parallel lines crossed by the transversal), and the interior of each angle in the pair lies partially inside the region between the parallel lines. Alternate interior angle pairs are congruent.

There are two pairs of alternate interior angles in the figure below: (d, g) and (b, e).

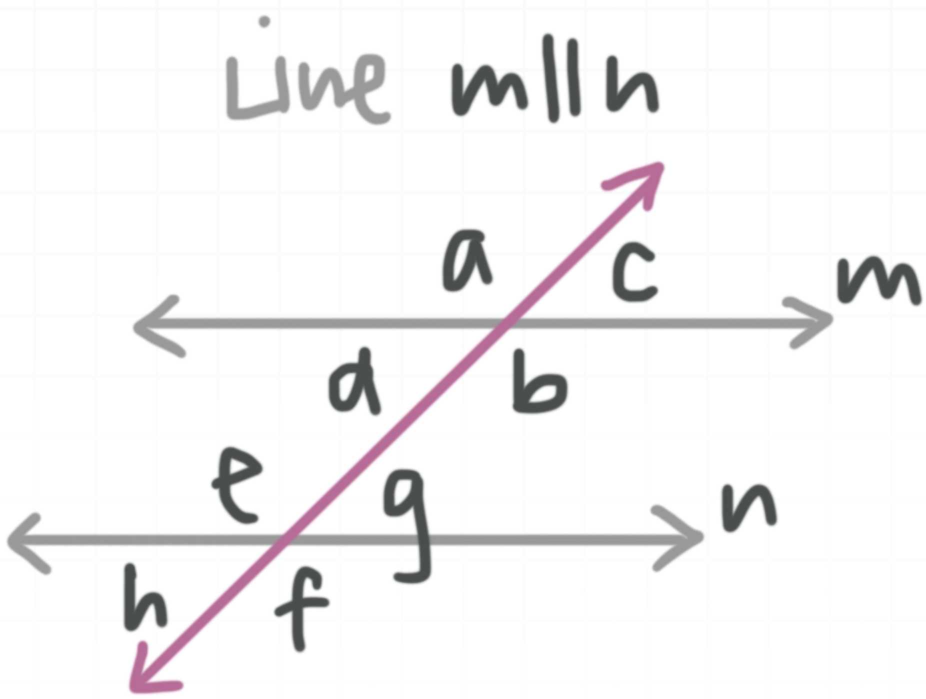


$$m\angle d = m\angle g$$

$$m\angle b = m\angle e$$

Alternate exterior angles are angles that lie on opposite sides of the transversal, their vertices are on opposite lines (in the pair of parallel lines crossed by the transversal), and the interior of each angle in the pair lies entirely outside the region between the parallel lines. Alternate exterior angle pairs are congruent.

There are two pairs of alternate exterior angles in the figure below: (a, f) and (c, h).

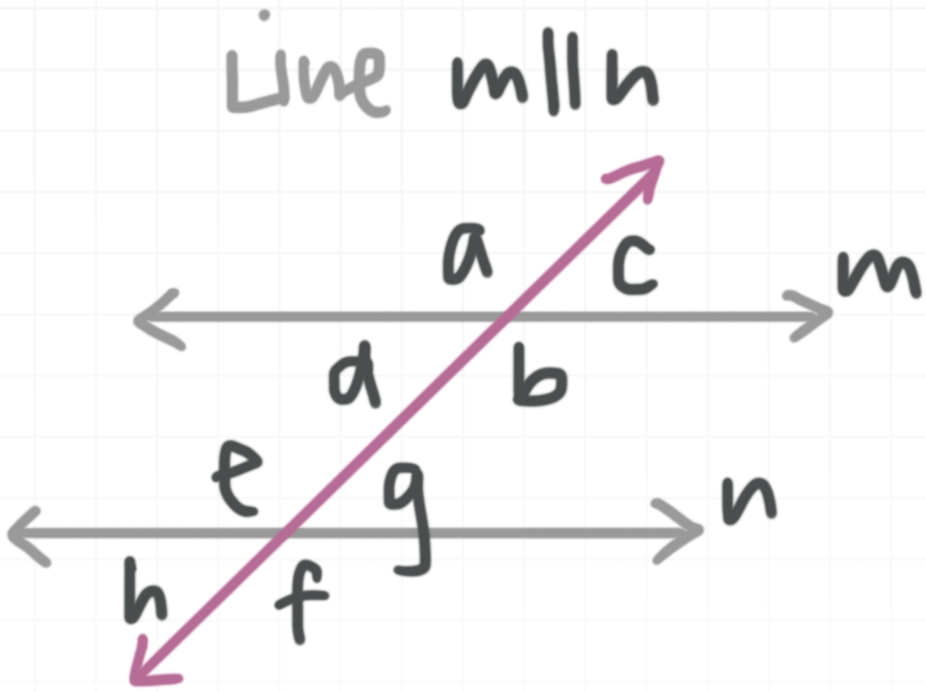


$$m\angle a = m\angle f$$

$$m\angle c = m\angle h$$

Consecutive interior angles are angles that lie on the same side of the transversal, their vertices are on opposite lines (in the pair of parallel lines crossed by the transversal), the interior of each angle in the pair lies partially inside the region between the parallel lines, and the interiors of the angles overlap in the region between the parallel lines. Consecutive interior angle pairs are supplementary.

There are two pairs of consecutive interior angles in the figure below: (b, g) and (d, e).

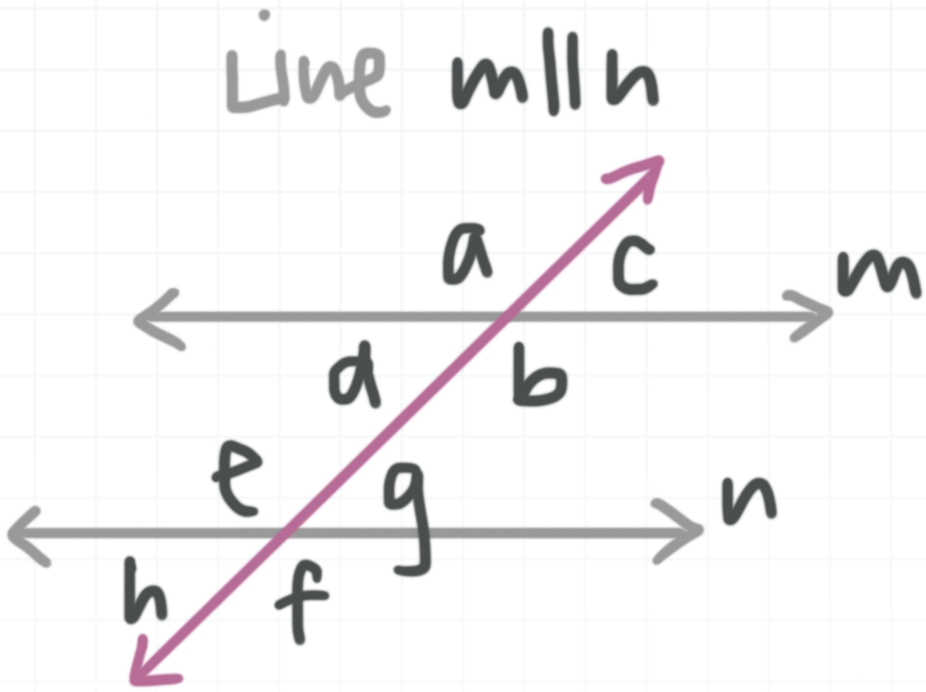


$$m\angle d + m\angle e = 180^\circ$$

$$m\angle b + m\angle g = 180^\circ$$

Adjacent angles are angles that share a vertex and one ray, and they can be on the same side of the transversal or on opposite sides of it, but (like all pairs of adjacent angles, not just those that are formed when a transversal crosses a pair of parallel lines) their interiors do not overlap. Adjacent angle pairs are supplementary.

There are eight pairs of adjacent angles in the figure below: four pairs that lie on the same side of the transversal ((a, d), (b, c), (e, h), and (f, g)) and four pairs that lie on opposite sides of the transversal ((a, c), (b, d), (e, g), and (f, h)).



$m\angle a + m\angle d = 180^\circ$

$m\angle a + m\angle c = 180^\circ$

$m\angle b + m\angle c = 180^\circ$

$m\angle d + m\angle b = 180^\circ$

$m\angle e + m\angle g = 180^\circ$

$m\angle e + m\angle h = 180^\circ$

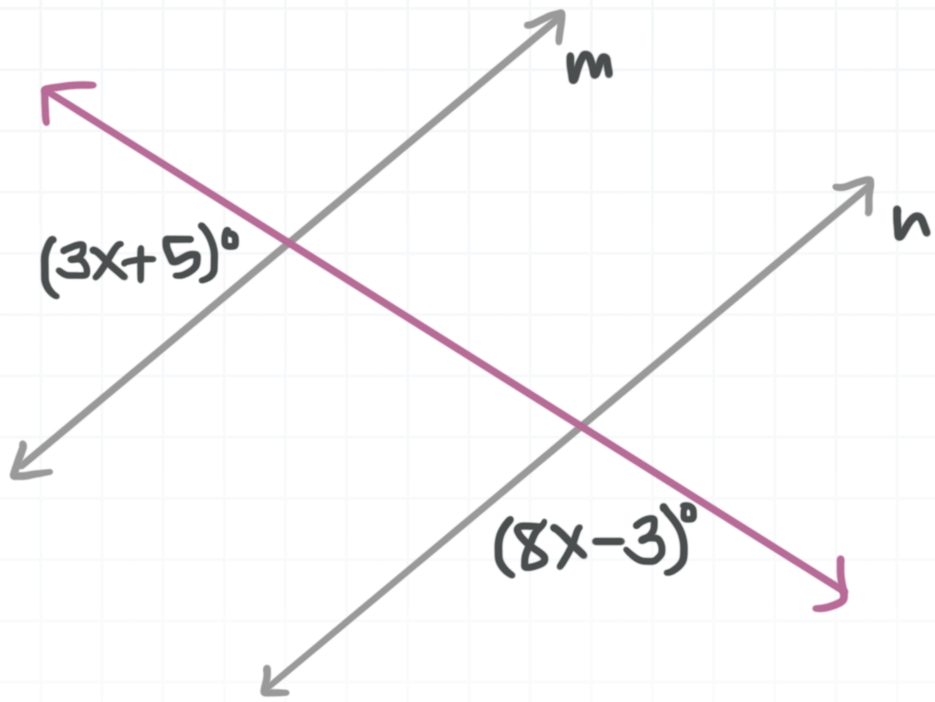
$m\angle g + m\angle f = 180^\circ$

$m\angle f + m\angle h = 180^\circ$

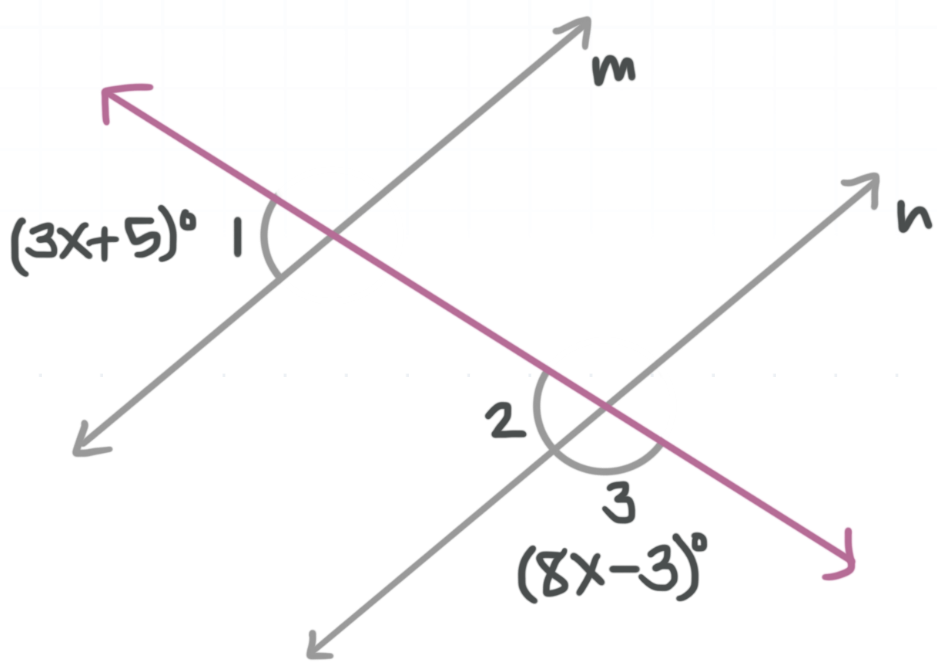
We often use these angle pair relationships to solve problems.

Example

Solve for the variable. Find the value of x to the nearest tenth, given that $m \parallel n$.



Let's think about which angle pair relationships to use.



Angles 1 and 2 are congruent because they are a pair of corresponding angles. Angles 2 and 3 are supplementary because they are adjacent angles. Combining these facts, we see that angles 1 and 3 are supplementary, so

$$(3x+5)^\circ + (8x-3)^\circ = 180^\circ$$

$3x^\circ + 5^\circ + 8x^\circ - 3^\circ = 180^\circ$

$11x^\circ + 2^\circ = 180^\circ$

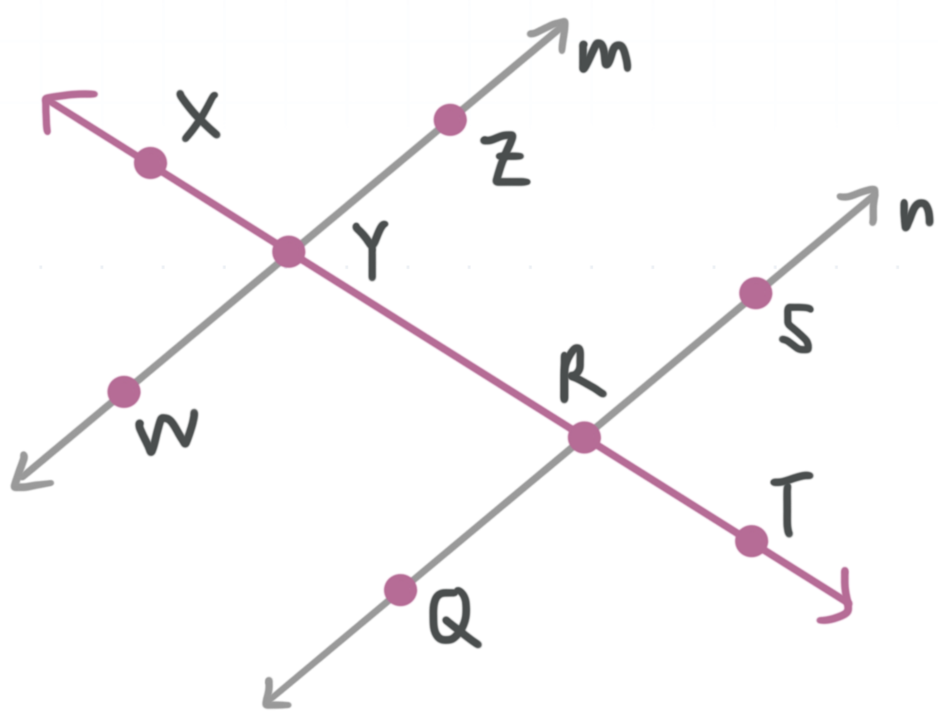
$11x^\circ = 178^\circ$

$x \approx 16.2$

Let's do one more.

Example

What is the measure of $\angle XYZ$, given that $m \parallel n$, $m\angle ZYR = (5x + 35)^\circ$, and $m\angle QRY = (15x - 5)^\circ$?



Looking at the diagram, we can see that $\angle XYZ$ and $\angle ZYR$ are adjacent angles, so they're supplementary. $\angle ZYR$ and $\angle QRY$ are alternate interior angles, so they're congruent.

Now we can use these facts to find $m\angle XYZ$. Let's begin by solving for x. $m\angle ZYR = m\angle QRY$ because they are congruent angles.

$$m\angle ZYR = m\angle QRY$$

$$(5x + 35)^\circ = (15x - 5)^\circ$$

$$35^\circ = 10x^\circ - 5^\circ$$

$$40^\circ = 10x^\circ$$

$$x = 4$$

Now we can find $m\angle ZYR$ by substituting 4 for x.

$$m\angle ZYR = (5x + 35)^\circ$$

$$m\angle ZYR = (5 \cdot 4 + 35)^\circ$$

$$m\angle ZYR = (20 + 35)^\circ$$

$$m\angle ZYR = 55^\circ$$

And because $\angle XYZ$ and $\angle ZYR$ are supplementary angles,

$$m\angle XYZ + m\angle ZYR = 180^\circ$$

$$m\angle XYZ + 55^\circ = 180^\circ$$

$$m\angle XYZ + 55^\circ - 55^\circ = 180^\circ - 55^\circ$$

$m\angle XYZ = 125^\circ$