Midpoint of a line segment in three dimensions

In this lesson we'll look at how to find the midpoint of a line segment in three dimensions when we're given the endpoints of the line segment as coordinates in three-dimensional space.

Midpoint formula

We can use the midpoint formula for three dimensions to find the middle of the line segment that has endpoints $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$, which is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Let's work through an example.

Example

Find the midpoint of the line segment with endpoints P_1 and P_2.

$$P_1 = (4, -6, 8)$$

$$P_2 = (4, 3, -5)$$

We'll use the formula for the midpoint M of a line segment in three dimensions. We'll plug in the coordinates of the given points, $P_1 = (4, -6, 8)$ and $P_2 = (4, 3, -5)$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

$$M = \left(\frac{4 + 4}{2}, \frac{-6 + 3}{2}, \frac{8 + -5}{2} \right)$$

$$M = \left(\frac{8}{2}, \frac{-3}{2}, \frac{3}{2} \right)$$

$$M = \left(4, \frac{-3}{2}, \frac{3}{2} \right)$$

Let's work through a different type of example.

Example

Find the coordinates of point A if M is the midpoint of $\overline{AB}$.

$$M = (4.5, -3.5, 3)$$

$$B = (2, -4, 8)$$

Let's use (x_1, y_1, z_1) for A and (x_2, y_2, z_2) for B, and then use the midpoint formula and plug in what we know.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

$$(4.5, -3.5, 3) = \left(\frac{x_1 + 2}{2}, \frac{y_1 + (-4)}{2}, \frac{z_1 + 8}{2} \right)$$

Now we'll equate the numbers on the left-hand side to the corresponding expressions on the right-hand side.

$$4.5 = \frac{x_1 + 2}{2}$$

$$-3.5 = \frac{y_1 + (-4)}{2}$$

$$3 = \frac{z_1 + 8}{2}$$

Finally, we'll solve these three equations separately, and we'll get

$$4.5 = \frac{x_1 + 2}{2}$$

$$2(4.5) = x_1 + 2$$

$$9 = x_1 + 2$$

$$7 = x_1$$

and

$$-3.5 = \frac{y_1 + (-4)}{2}$$

$$2(-3.5) = y_1 - 4$$

$$-7 = y_1 - 4$$

$$-3 = y_1$$

and

$$3 = \frac{z_1 + 8}{2}$$

$$2(3) = z_1 + 8$$

$$6 = z_1 + 8$$

$$-2 = z_1$$

So the coordinates of point A are $(7, -3, -2)$.