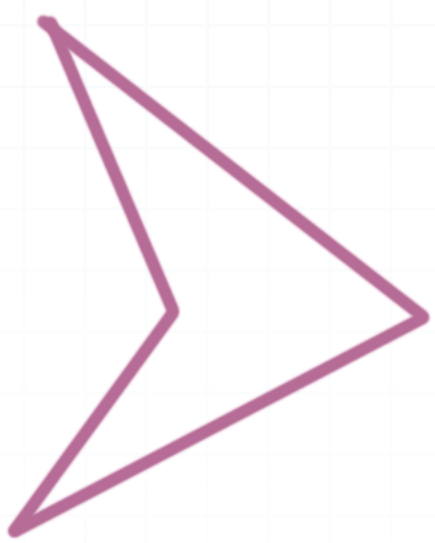
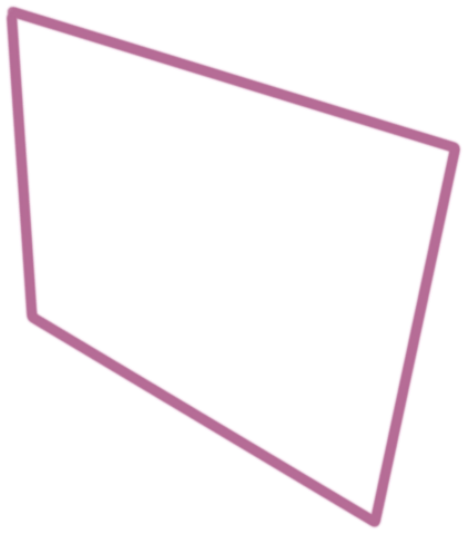
Measures of quadrilaterals

A quadrilateral is any closed four-sided figure. There are two types of quadrilaterals: concave and convex.

A concave quadrilateral has an interior angle with measure greater than $180°$



A convex quadrilateral has interior angles that are all of measure less than $180°$

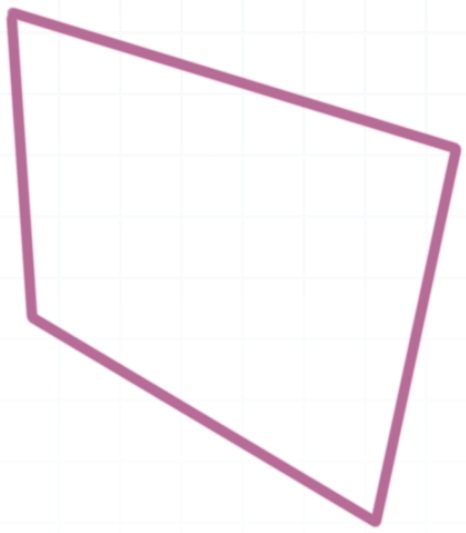


All quadrilaterals have four sides (edges), four corners (vertices), and four interior angles whose measures add up to $(4-2)180° = 360°$.

Here are some special types of convex quadrilaterals and their properties:

Trapezium

No pair of parallel sides and no pair of congruent sides (no pair of sides that have the same length)

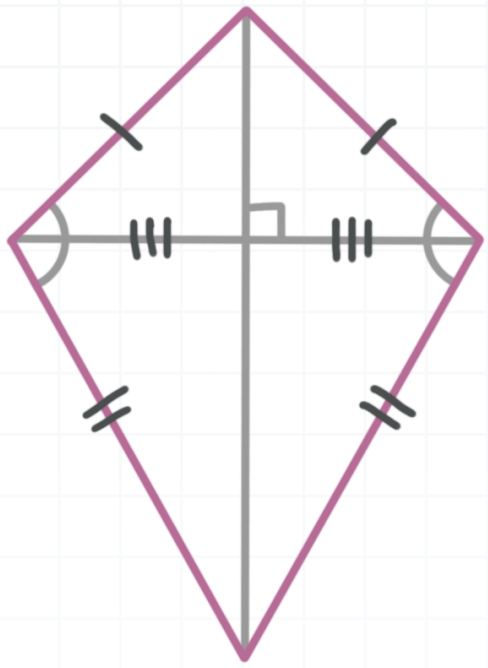


Kite

Has two pairs of adjacent congruent sides

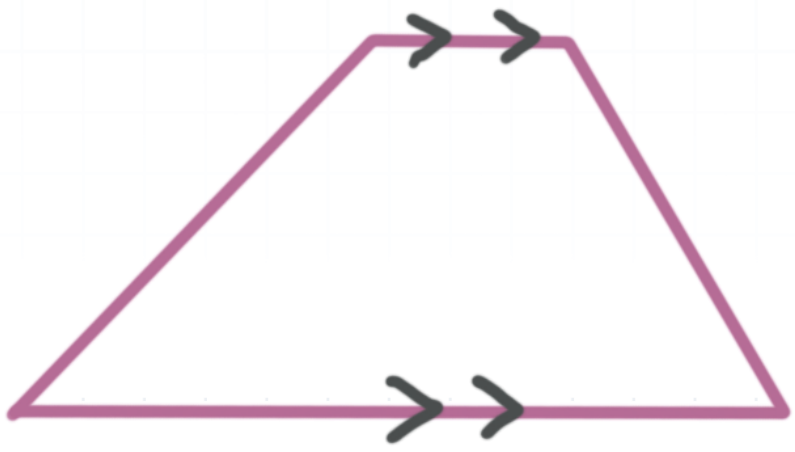
Has a pair of opposite congruent angles

Diagonals (line segments whose endpoints are a pair of opposite corners/vertices) cross to form right angles, and one of the diagonals bisects the other (cuts it in half)



Trapezoid

Has exactly one pair of opposite parallel sides



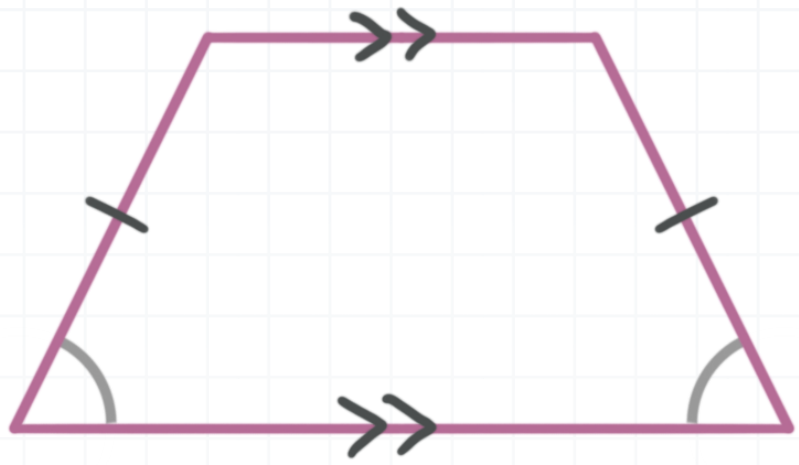
Isosceles trapezoid

Has exactly one pair of opposite parallel sides

Opposite non-parallel sides are congruent

Base angles are congruent

Diagonals are congruent



Parallelogram

Two pairs of opposite parallel sides

Opposite sides are congruent

Opposite angles are congruent

$$m\angle 1 = m\angle 3$$

$$m\angle 2 = m\angle 4$$

Consecutive angles are supplementary

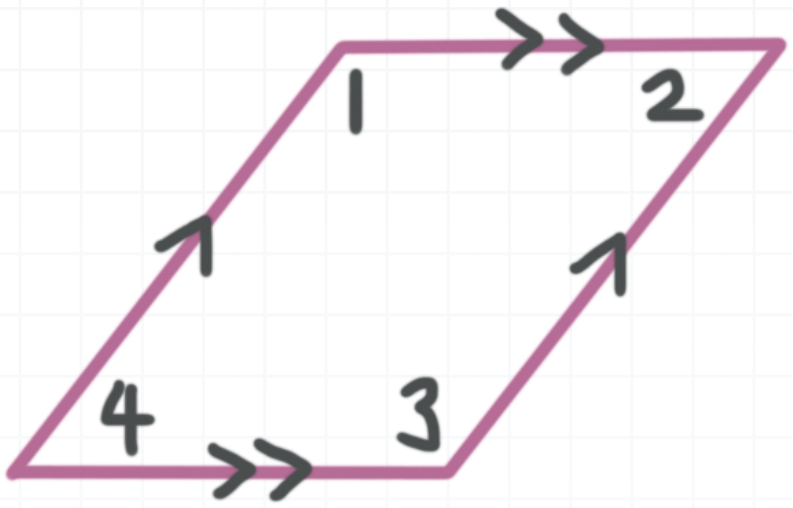
$$m\angle 1 + m\angle 2 = 180^\circ$$

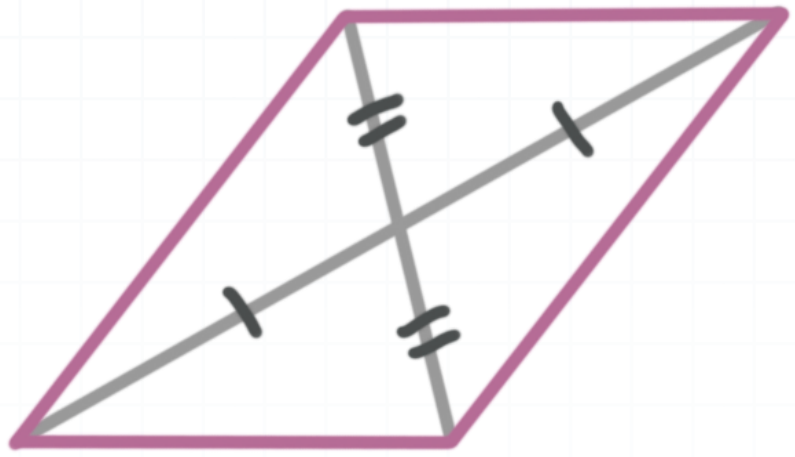
$$m\angle 2 + m\angle 3 = 180^\circ$$

$$m\angle 3 + m\angle 4 = 180^\circ$$

$$m\angle 4 + m\angle 1 = 180^\circ$$

Diagonals bisect each other (cut each other in half)





Rectangle

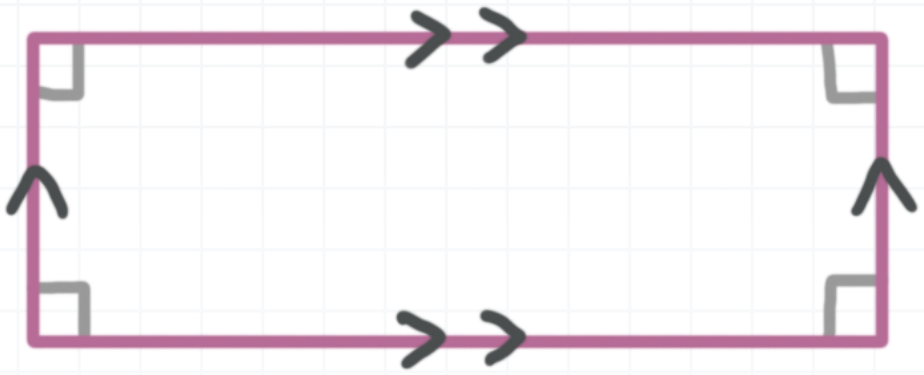
Two pairs of opposite parallel sides

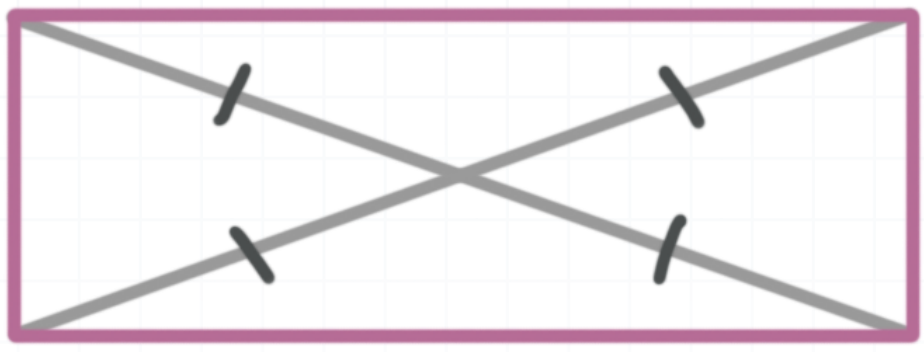
Opposite sides are congruent

All angles are right angles ($90°$)

Diagonals bisect each other (cut each other in half)

Diagonals are congruent





Rhombus

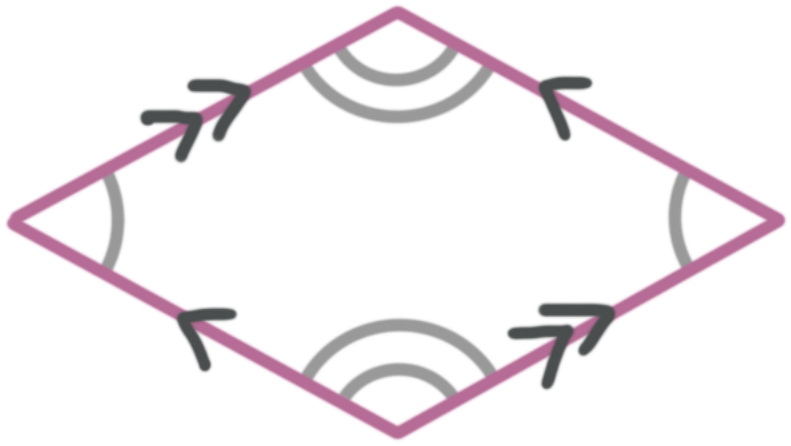
Two pairs of opposite parallel sides

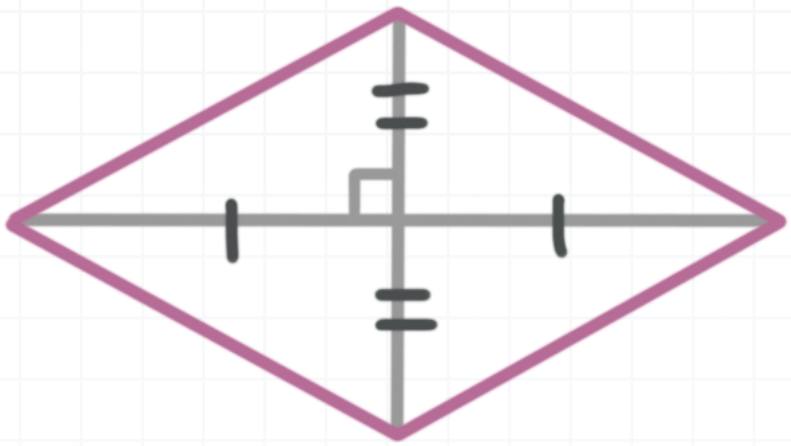
All sides are congruent

Opposite angles are congruent

Consecutive angles are supplementary

Diagonals are perpendicular bisectors of each other (cut each other in half and form right angles)





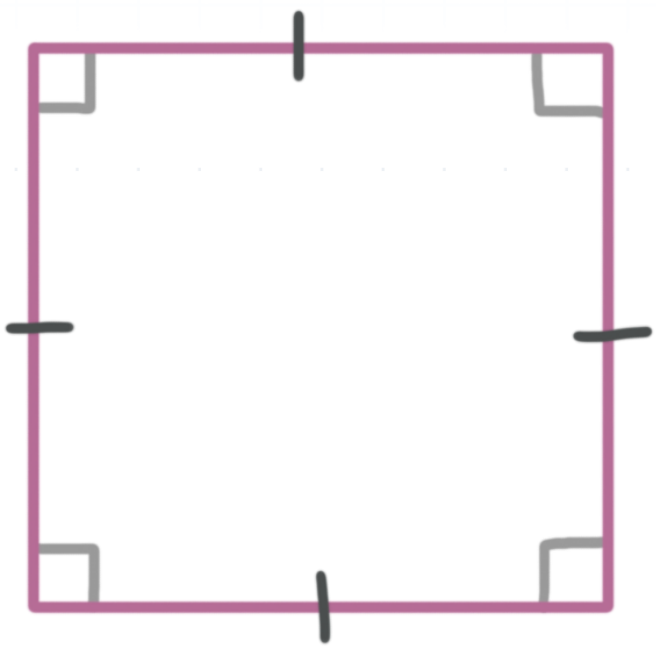
Square

Two pairs of opposite parallel sides

All angles are right angles

All sides are congruent

Diagonals are perpendicular bisectors of each other (cut each other in half and form right angles)



Notice the following:

Every rectangle is a parallelogram, but not every parallelogram is a rectangle.

Every square is a rectangle (and therefore also a parallelogram), but not every rectangle is a square.

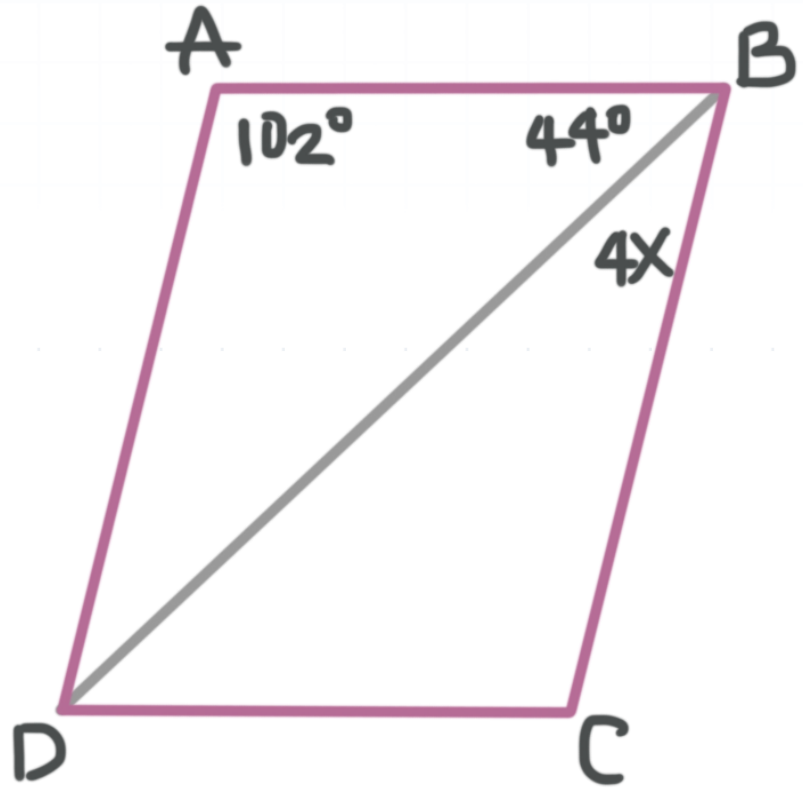
Every rhombus is a parallelogram, but not every parallelogram is a rhombus.

Every square is a rhombus, but not every rhombus is a square. A rhombus that isn't a square is in the shape of a diamond.

Let's start by working through an example.

Example

The figure below is a parallelogram. What is the value of x?



Angles DAB and ABC are consecutive angles in this parallelogram (they're next to each other, not across the figure from each other), so they're supplementary.

$$m\angle DAB + m\angle ABC = 180^\circ$$

$$102^\circ + (44^\circ + 4x) = 180^\circ$$

$$146^\circ + 4x = 180^\circ$$

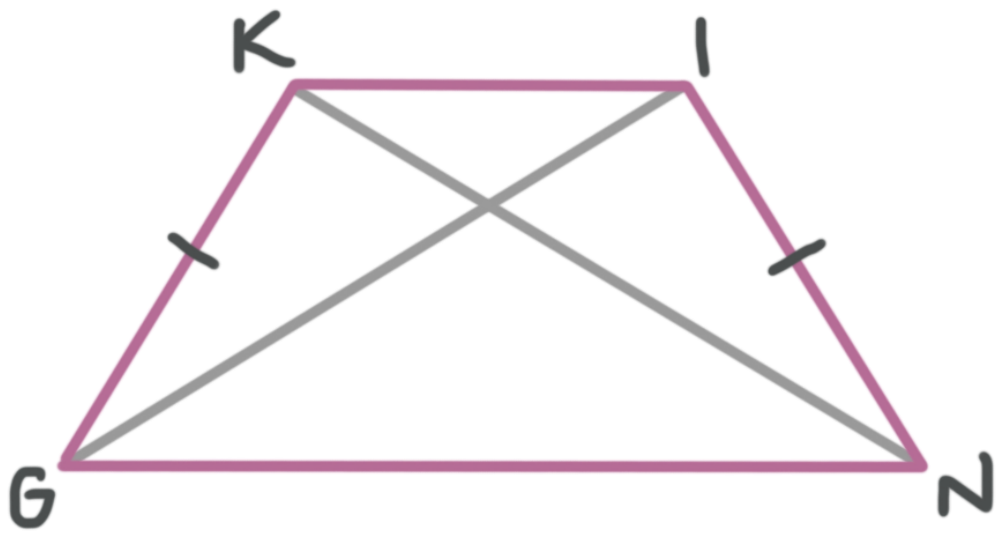
$$4x = 34^\circ$$

$$x = 8.5^\circ$$

Let's look at one more example.

Example

The figure below is a trapezoid. What is the length of $\overline{KN}$ if $\overline{KN} = 5x + 2$ and $\overline{IG} = 4x + 20$?



The side lengths of $\overline{KG}$ and $\overline{IN}$ are marked as being congruent, which means this is an isosceles trapezoid. The diagonals of an isosceles trapezoid are congruent, which means that $\overline{KN} = \overline{IG}$. Therefore,

$$5x + 2 = 4x + 20$$

$$5x = 4x + 18$$

$$x = 18$$

So we see that

$$\overline{KN} = 5x + 2$$

$$\overline{KN} = 5(18) + 2$$

$$\overline{KN} = 92$$