

# Triangle side-splitting theorem

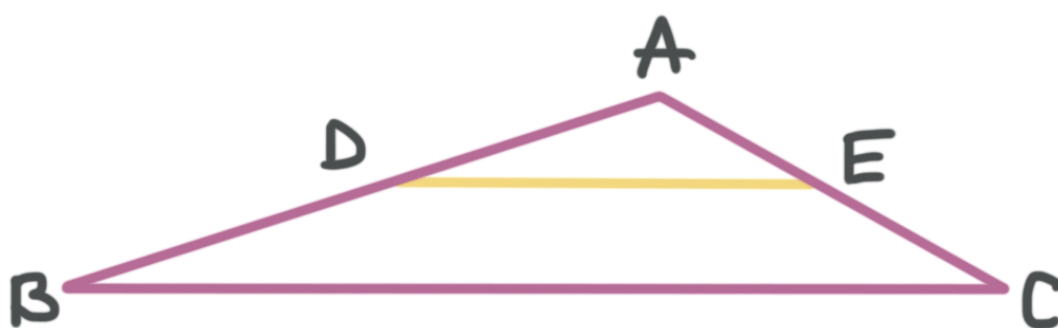
In this lesson we'll look at the triangle side-splitting theorem and how it's used in finding the length of part of a side of a triangle.

## Triangle side-splitting theorem

If a line segment intersects two sides of a triangle, and is parallel to the third side of the triangle, then the two sides intersected by the segment are split proportionally (in terms of their lengths).

In the triangle below, segment  $\overline{DE}$  is parallel to side  $\overline{BC}$ ,  $\overline{DE} \parallel \overline{BC}$ , so the segment splits the lengths of sides  $\overline{AB}$  and  $\overline{AC}$  of the triangle proportionally:

$$\frac{\overline{AD}}{\overline{DB}} = \frac{\overline{AE}}{\overline{EC}}$$

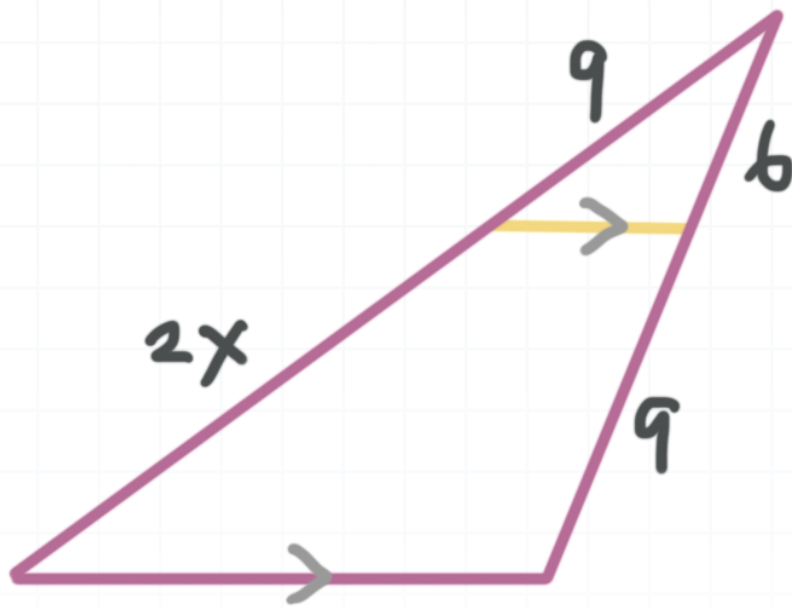


Let's start by working through an example of how to find the length of part of one side of a triangle using a segment that splits two sides of the triangle (including the side in question) proportionally.

### Example



In the figure, two sides of the triangle are split by a segment that's parallel to the third side. Find the value of  $x$ .



Since the segment is parallel to the third side, we can use the triangle side-splitting theorem to find the value of the variable.

The ratio  $9/2x$  has to be equal to  $6/9$ .

$$\frac{9}{2x} = \frac{6}{9}$$

Cross multiply.

$$9(9) = 6(2x)$$

$$81 = 12x$$

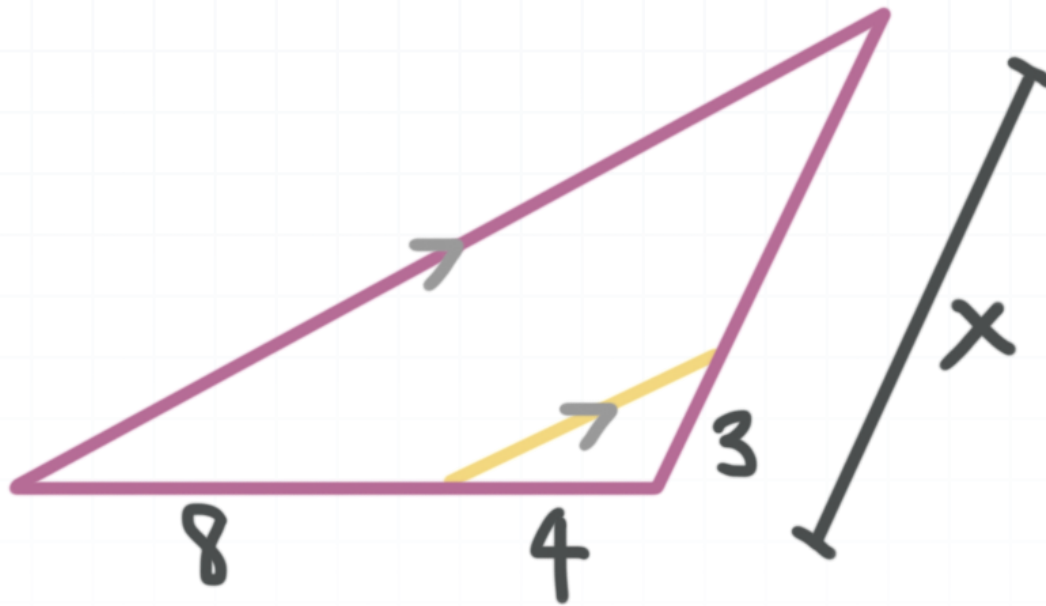
$$6.75 = x$$



Let's try another one.

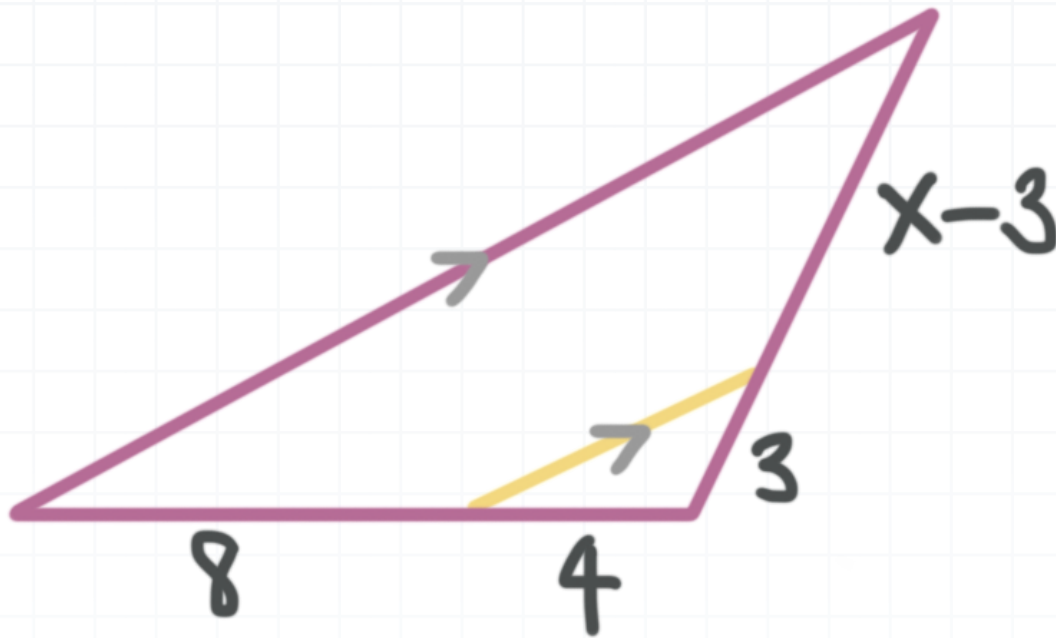
### Example

In the figure, two sides of the triangle are split by a segment that's parallel to the third side. Find the value of  $x$ .



Since the segment is parallel to the third side, we can use the triangle side-splitting theorem to find the value of the variable. We know that the length of one of the sides that are split is  $x$ , and that the length of one part of that split side is 3. That means that the length of the other part of that split side is  $x - 3$ .





The ratio  $3/(x - 3)$  has to be equal to  $4/8$ .

$$\frac{3}{x - 3} = \frac{4}{8}$$

Cross multiply.

$$8(3) = 4(x - 3)$$

$$24 = 4x - 12$$

$$36 = 4x$$

$$9 = x$$

