# Interior angles of polygons

In this lesson we'll look at how to find the measures of the interior angles of polygons.

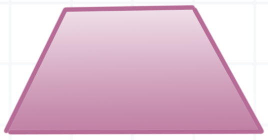
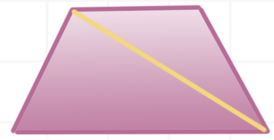
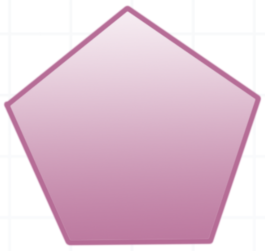
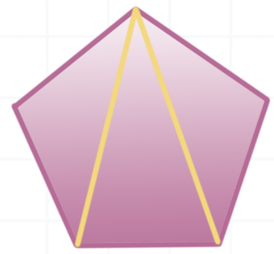
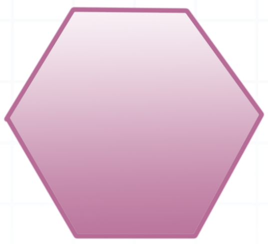
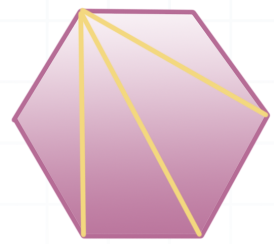
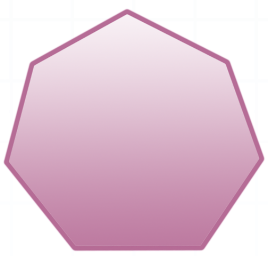
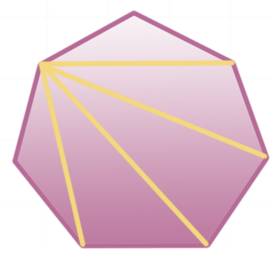
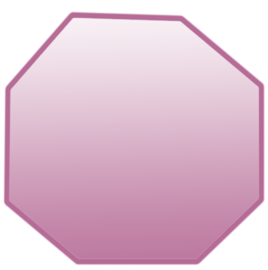
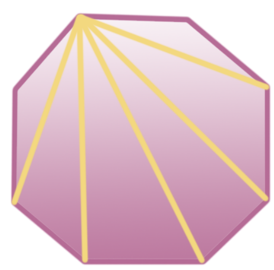
## Triangles

Triangles are $3$-sided polygons. The measures of the three interior angles of any triangle (the three angles inside the triangle) add up to $180^\circ$. For instance, in this figure, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.


## Polygons

The word "polygon" means "many-sided figure." A polygon has the same number of interior angles as it has sides, and a regular polygon has equal angles and equal sides.

Any polygon can be divided into triangles.

| Picture | Name | Sides | Triangles | Degrees inside |
|---|---|---|---|---|
|  | Quadrilateral | 4 | 2:  | $2(180^\circ) = 360^\circ$ |
|  | Pentagon | 5 | 3:  | $3(180^\circ) = 540^\circ$ |
|  | Hexagon | 6 | 4:  | $4(180^\circ) = 720^\circ$ |
|  | Heptagon | 7 | 5:  | $5(180^\circ) = 900^\circ$ |
|  | Octagon | 8 | 6:  | $6(180^\circ) = 1{,}080^\circ$ |
| | ... | ... | ... | ... |
| | $n$-gon | $n$ | $n-2$ | $(n-2)180^\circ$ |

Let's start by working through an example.

### Example

What is the measure of each interior angle in a regular icosagon (a $20$-sided figure)?

The sum of the measures of the interior angles in a polygon is $(n-2)180^\circ$, where $n$ is the number of sides in the polygon. For an icosagon, which is a $20$-sided figure, that would be

$$(20-2)180^\circ = 3{,}240^\circ$$

There are $20$ congruent interior angles because the shape is regular, so each interior angle measures

$$3{,}240^\circ \div 20 = 162^\circ$$

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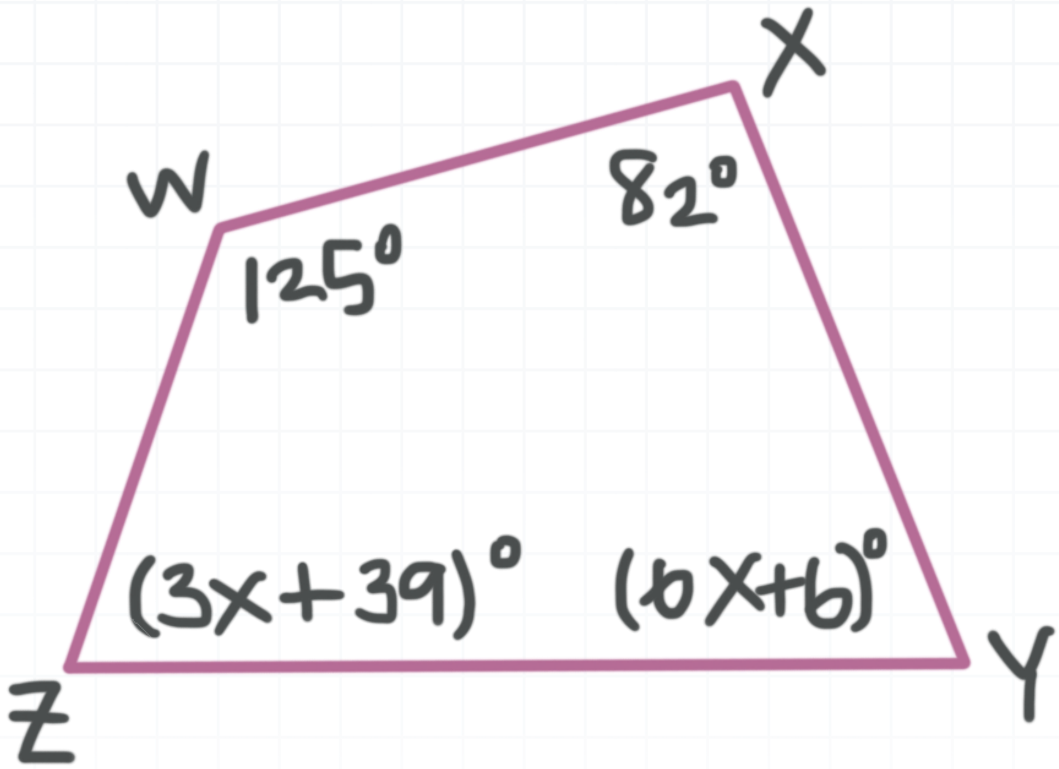
If the shape is not regular, then we can't assume that all of the angles are congruent. Let's look at an example of a non-regular quadrilateral in which the angles aren't equal.

**Example**

What is the measure of $\angle Z$?




The sum of the measures of the interior angles in a polygon with $n$ sides is $(n-2)180^\circ$. For a quadrilateral, that would be $(4-2)180^\circ = 360^\circ$. Set the sum of the four angles equal to $360^\circ$ and then solve for $x$.

$$125^\circ + 82^\circ + (3x+39)^\circ + (6x+6)^\circ = 360^\circ$$

$$(125 + 82 + 39 + 6)^\circ + (3x + 6x)^\circ = 360^\circ$$

$$252^\circ + 9x^\circ = 360^\circ$$

$$9x^\circ = 108^\circ$$

$$x = 12$$

Substitute $12$ for $x$ in $(3x+39)^\circ$ to find $m\angle Z$.

$$m\angle Z = (3 \cdot 12 + 39)^\circ$$

$$m\angle Z = (36 + 39)^\circ$$

$$m\angle Z = 75^\circ$$