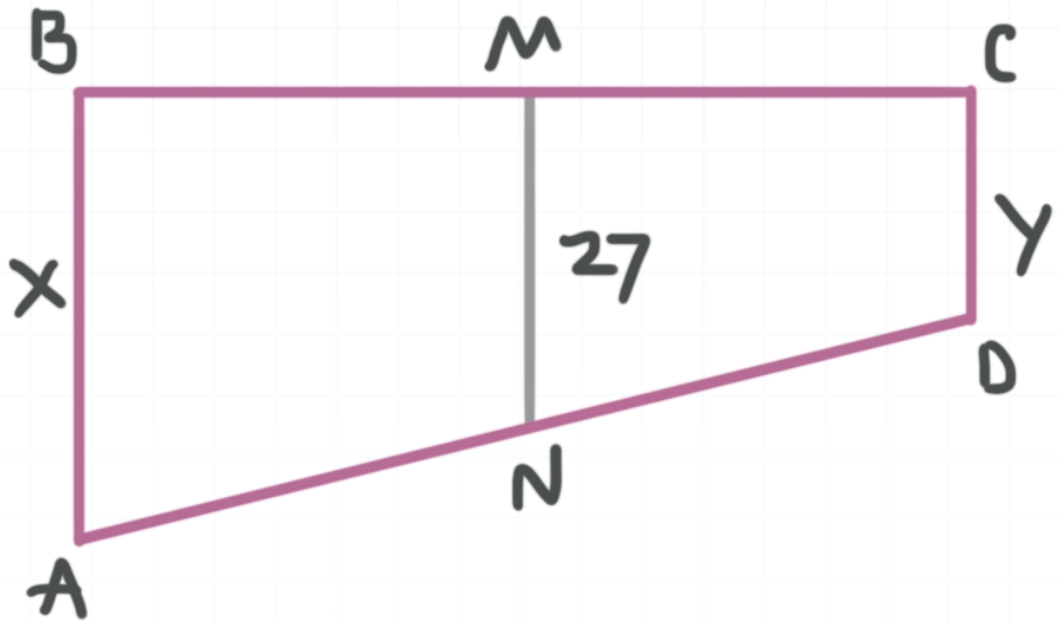
Topic: Midsegments of trapezoids

Question: In the figure, $ABCD$ is a trapezoid and M and N are midpoints of the opposite non-parallel sides. Find the value of $x + y$.



Answer choices:

A 27

B 34

C 45

D 54

Solution: D

You don't actually have to know the value of x or y to do this problem.

Because M and N are the midpoints of the opposite non-parallel sides of the trapezoid, $\overline{MN}$ is the midsegment, so

$$\overline{MN} = \frac{1}{2}(\overline{AB} + \overline{CD})$$

$$27 = \frac{1}{2}(x + y)$$

$$54 = x + y$$

Topic: Midsegments of trapezoids

Question: In the coordinate plane, a trapezoid $ABCD$ has vertices at $A = (0,8)$, $B = (12,8)$, $C = (9,1)$, and $D = (7,1)$. M and N are the midpoints of the opposite non-parallel sides. What is the length of $\overline{MN}$?

Answer choices:

A 2

B 5

C 7

D 12

Solution: C

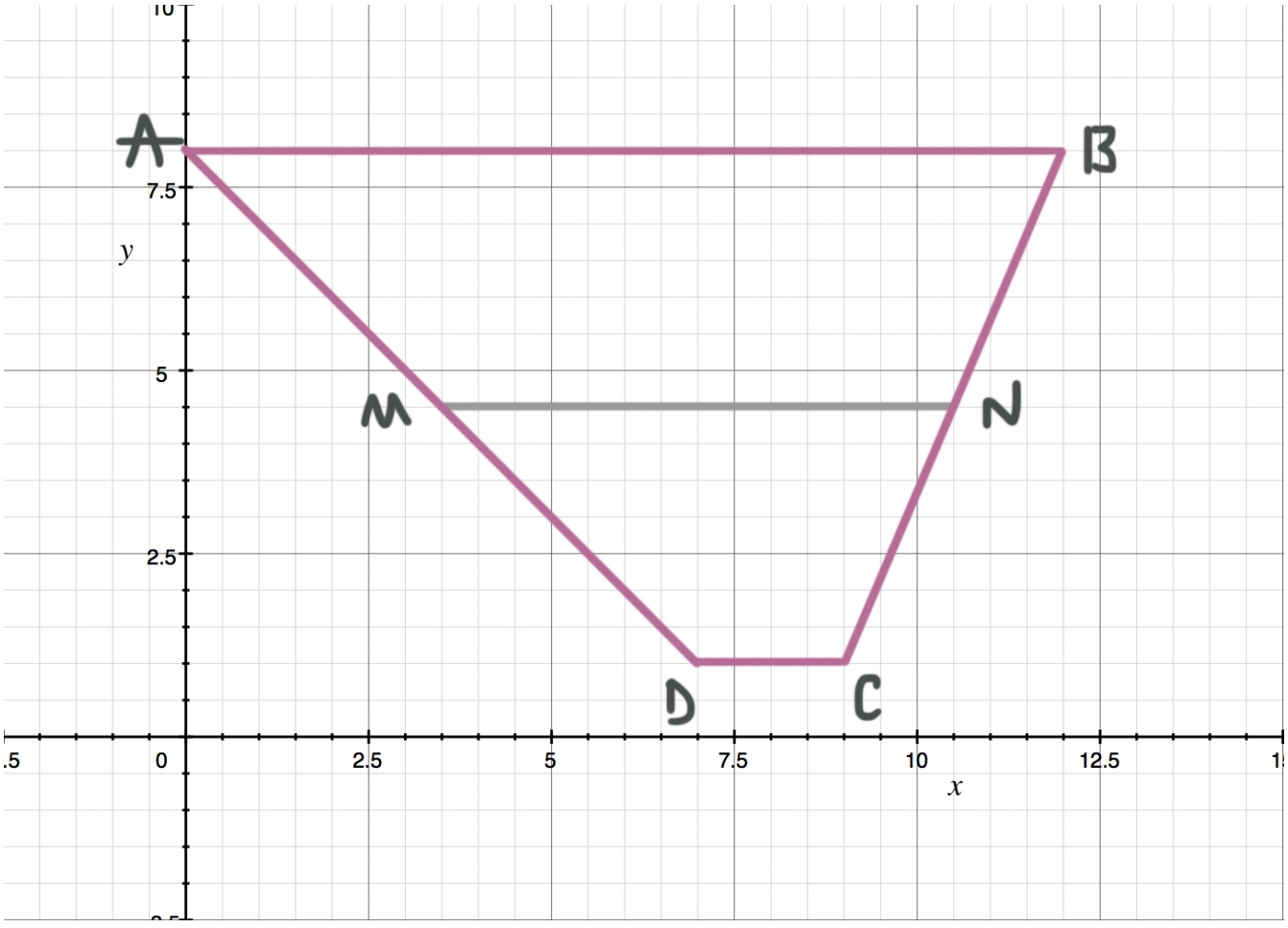
Notice that vertices A and B have the same y-coordinate of 8, and that vertices D and C have the same y-coordinate of 1. That means that $\overline{AB} \parallel \overline{CD}$ ($\overline{AB}$ and $\overline{DC}$ are the bases of the trapezoid).

Also, since the y-coordinates of vertices A and B are equal, the length of $\overline{AB}$ is the difference in their x-coordinates (which are 12 and 0), so

$$\overline{AB} = 12 - 0 = 12$$

Similarly, since the y-coordinate of vertices D and C are equal, the length of $\overline{DC}$ is the difference in their x-coordinates (which are 9 and 7), so

$$\overline{DC} = 9 - 7 = 2$$



Since M and N are the midpoints of the opposite non-parallel sides of this trapezoid, $\overline{MN}$ is the midsegment, so

$$\overline{MN} = \frac{1}{2}(\overline{AB} + \overline{DC})$$

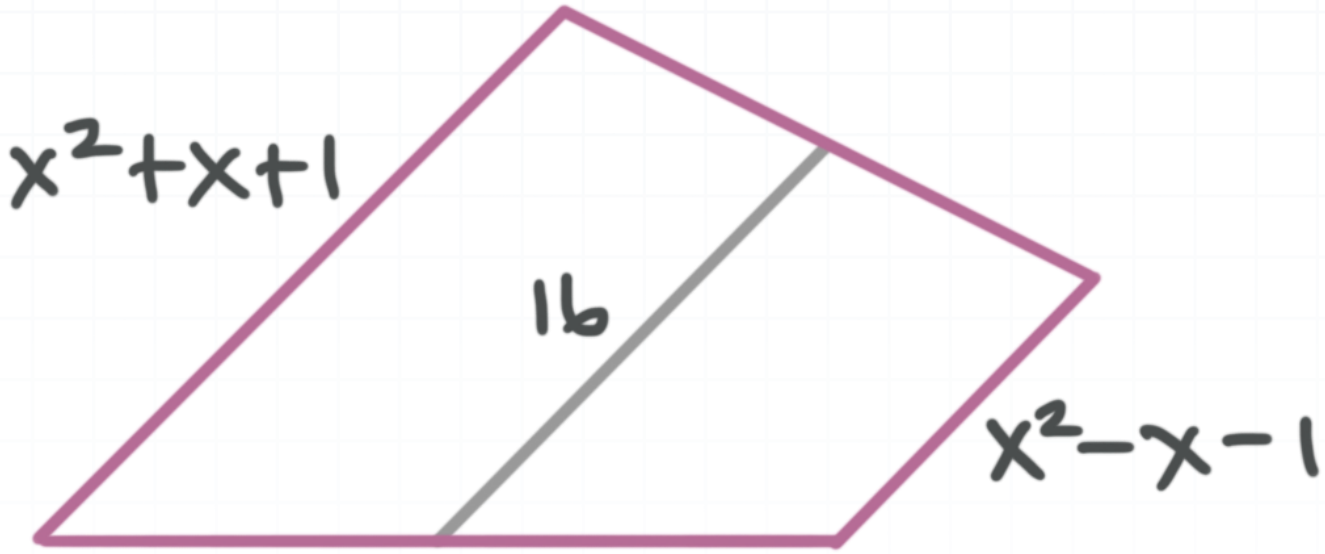
$$\overline{MN} = \frac{1}{2}(12 + 2)$$

$$\overline{MN} = \frac{1}{2}(14)$$

$$\overline{MN} = 7$$

Topic: Midsegments of trapezoids

Question: In the trapezoid the length of the midsegment is 16, the length of the longer base is $x^2 + x + 1$, and the length of the shorter base is $x^2 - x - 1$. Calculate the length of the shorter base.



Answer choices:

A 11

B 12

C 13

D 14

Solution: A

The length of the midsegment is half the sum of the lengths of the bases. Therefore,

$$16 = \frac{1}{2}\left[(x^2 + x + 1) + (x^2 - x - 1)\right]$$

$$16 = \frac{1}{2}(2x^2)$$

$$16 = x^2$$

$$\pm 4 = x$$

First, use $x = -4$ to calculate the lengths of the bases.

Length of the longer base: $(-4)^2 + (-4) + 1 = 13$

Length of the shorter base: $(-4)^2 - (-4) - 1 = 19$

These results are contradictory (they indicate that the length of the longer base is less than the length of the shorter base), so rule out $x = -4$. That leaves only $x = 4$. So we get

Length of the longer base: $4^2 + 4 + 1 = 21$

Length of the shorter base: $4^2 - 4 - 1 = 11$