Geometry Workbook Solutions

Pythagorean theorem

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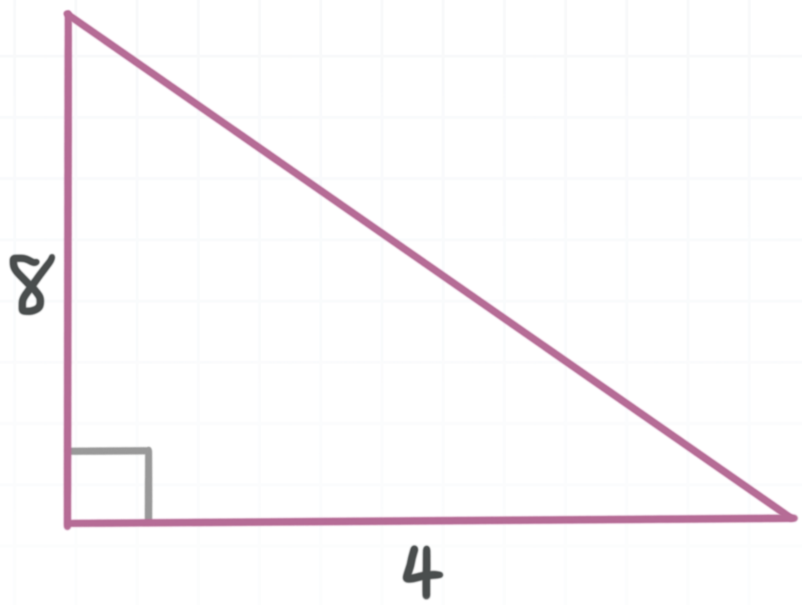
PYTHAGOREAN THEOREM

■ 1. Find the exact length of the hypotenuse.



Solution:

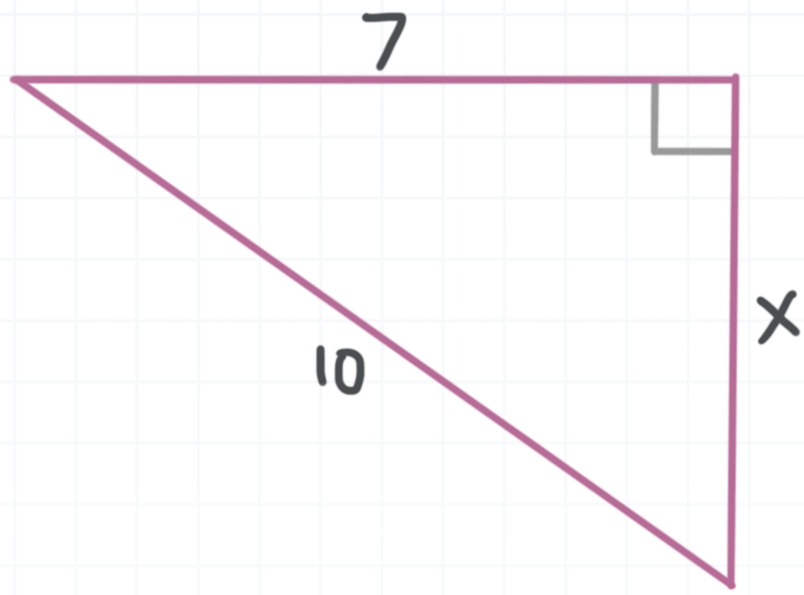
$4\sqrt{5}$. By the Pythagorean Theorem,

$$8^2 + 4^2 = c^2$$

$$c^2 = 80$$

$$c = \sqrt{80} = 4\sqrt{5}$$

■ 2. Find the exact length of the missing leg.



Solution:

$\sqrt{51}$. By the Pythagorean Theorem,

$$7^2 + x^2 = 10^2$$

$$x^2 = 10^2 - 7^2 = 51$$

$$x = \sqrt{51}$$

■ 3. Find the length of the diagonal of a rectangle with length 14 and width 8.

Solution:

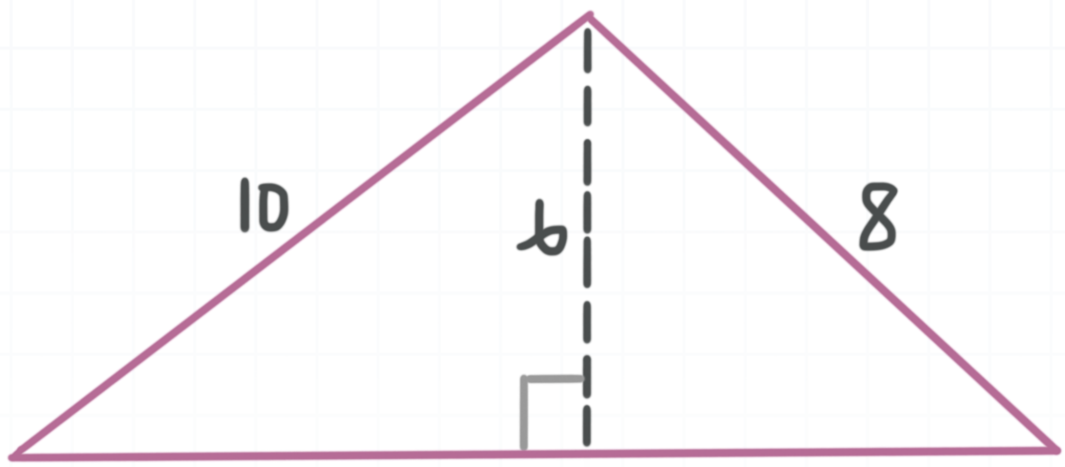
$2\sqrt{65}$. By the Pythagorean theorem,

$$14^2 + 8^2 = d^2$$

$$d^2 = 260$$

$$d = \sqrt{260} = 2\sqrt{65}$$

■ 4. Find the perimeter of the triangle to the nearest tenth.



Solution:

31.3. Separate your diagram into two right triangles, then use the Pythagorean Theorem to find each leg of those triangles, which together make the length of the missing side of the whole triangle. For the triangle on the left,

$$x^2 + 6^2 = 10^2$$

$$x^2 = 10^2 - 6^2$$

$$x = \sqrt{10^2 - 6^2}$$

For the triangle on the right,

$$y^2 + 6^2 = 8^2$$

$$y^2 = 8^2 - 6^2$$

$$y = \sqrt{8^2 - 6^2}$$

The length of the missing side of the triangle is therefore

$$z = \sqrt{10^2 - 6^2} + \sqrt{8^2 - 6^2}$$

$$z = \sqrt{100 - 36} + \sqrt{64 - 36}$$

$$z = \sqrt{64} + \sqrt{28}$$

$$z = 8 + 2\sqrt{7}$$

$$z \approx 13.3$$

The perimeter of the triangle is $10 + 8 + 13.3 = 31.3$.

PYTHAGOREAN INEQUALITIES

■ 1. The side lengths of a triangle are 10, 14, and 15. Determine whether the triangle is obtuse, acute, or right.

Solution:

The triangle is acute. The sum of the squared legs is $10^2 + 14^2 = 296$, and the square of the hypotenuse is $15^2 = 225$. Since $296 > 225$, the triangle is acute.

■ 2. The side lengths of a triangle are 7, 18, and 12. Determine whether this triangle is obtuse, acute, or right.

Solution:

The triangle is obtuse. The sum of the squared legs is $7^2 + 12^2 = 193$, and the square of the hypotenuse is $18^2 = 324$. Since $324 > 193$, the triangle is obtuse.

■ 3. A triangle's two shortest sides have lengths 8 and 6. Let x be the length of the third side. Give a compound inequality that represents all possible lengths of the third side, ensuring that the triangle is acute.

Solution:

$8 < c < 10$. If the triangle is acute, then $c^2 < a^2 + b^2$. We know that $8^2 + 6^2 = 100$, which means that $c^2 < 100$ will make this an acute triangle. Therefore $c < 10$ will work for its side length. However since 8 must be shorter, $c > 8$.

■ 4. The side lengths of a triangle in ascending order are x, $x + 2$, and 10. Find the value of x such that this is a right triangle.

Solution:

6. If we plug the given side lengths into the Pythagorean theorem, we get

$$x^2 + (x + 2)^2 = 10^2$$

$$x^2 + x^2 + 4x + 4 = 100$$

$$2x^2 + 4x - 96 = 0$$

$$x^2 + 2x - 48 = 0$$

$$(x + 8)(x - 6) = 0$$

$$x = -8, 6$$

The side length can't be negative, so $x = -8$ can't be a solution. Therefore, $x = 6$.

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