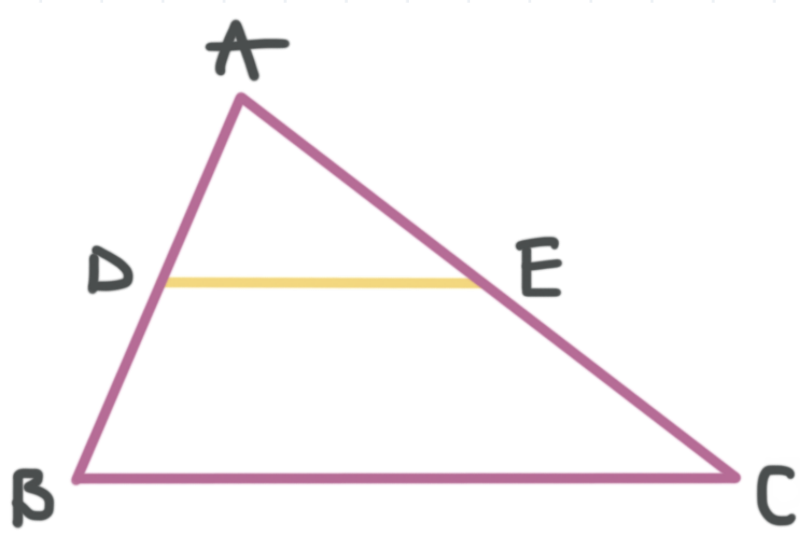
Midsegments of triangles

In this lesson we'll define midsegment of a triangle and show how it's used in finding the length of a side (or part of a side) of a triangle.

Midsegment of a triangle

Like the side-splitting segments we talked about in the previous lesson, a **midsegment** of a triangle is a line segment that intersects two sides of a triangle and is parallel to the third side of the triangle (the side it doesn't intersect).

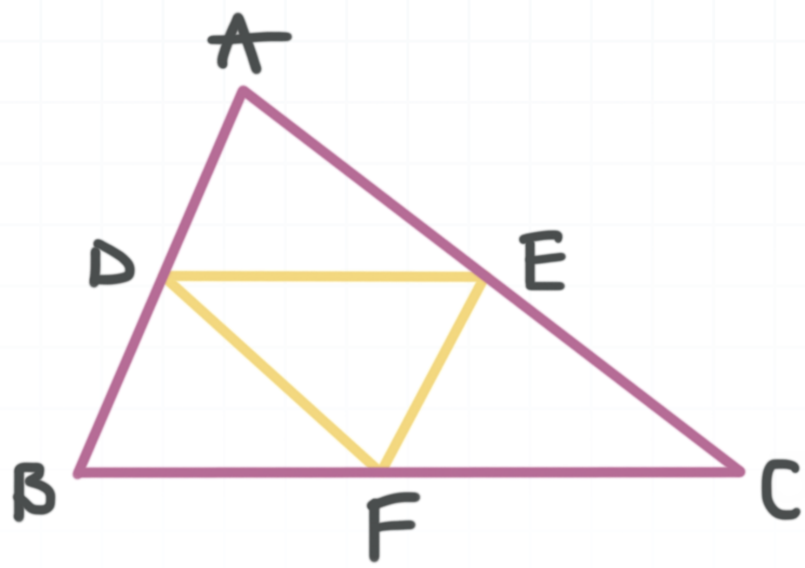
The difference between a midsegment and any other side-splitting segment, is that a midsegment intersects the triangle at the midpoints of the two sides that it splits, so it cuts them in half. So in the figure below, $\overline{DE}$ cuts $\overline{AB}$ and $\overline{AC}$ in half.



Remember that the midpoint of any side of a triangle divides that side into two parts of equal length, which means that $\overline{AD}=\overline{DB}$ and $\overline{AE}=\overline{EC}$.

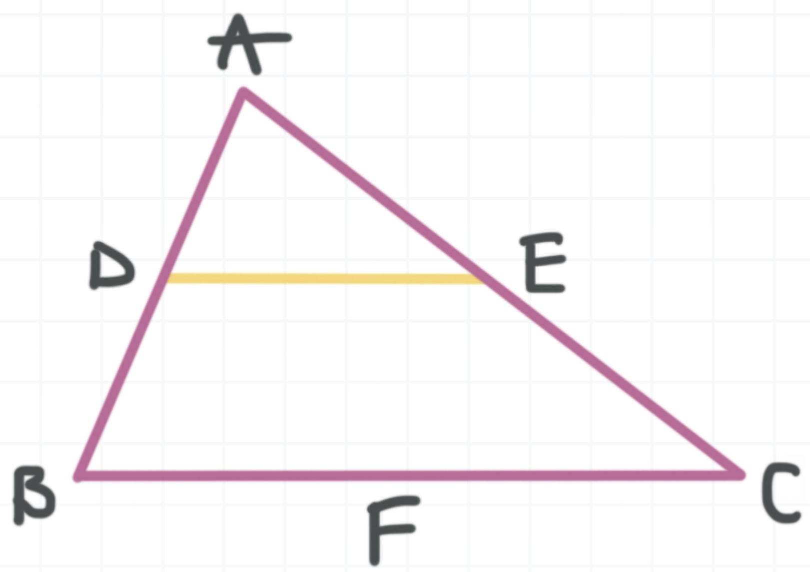
Triangles have three midsegments

Since a triangle has three sides, it also has three midsegments. Each midsegment is parallel to a different side of the triangle. If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, and F is the midpoint of $\overline{BC}$, then $\overline{DE}$, $\overline{DF}$, and $\overline{EF}$ are all midsegments of triangle ABC.



Midsegment of a triangle theorem

A midsegment of a triangle is parallel to the third side of the triangle (the side that the midsegment doesn't intersect), and the length of the midsegment is equal to half of the length of the third side. This means that if $\overline{DE}$ is a midsegment of this triangle,



then

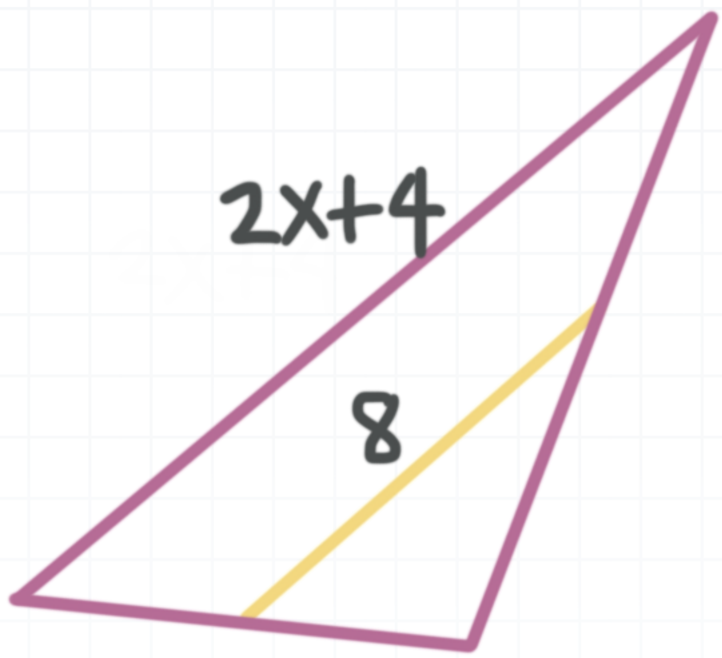
$$\overline{DE} \parallel \overline{BC} \text{ and } \overline{DE} = (1/2)\overline{BC}$$

Then it's also true that, if F is the midpoint of $\overline{BC}$, then $\overline{DE} = \overline{BF} = \overline{FC}$.

Let's work through some examples with the midsegment of a triangle theorem.

Example

If the segment of length 8 is a midsegment of the triangle in the figure, what's the value of x?



Because the segment of length 8 is a midsegment of this triangle, we know that its length is half that of the side of length $2x + 4$, so

$$8 = \frac{1}{2}(2x + 4)$$

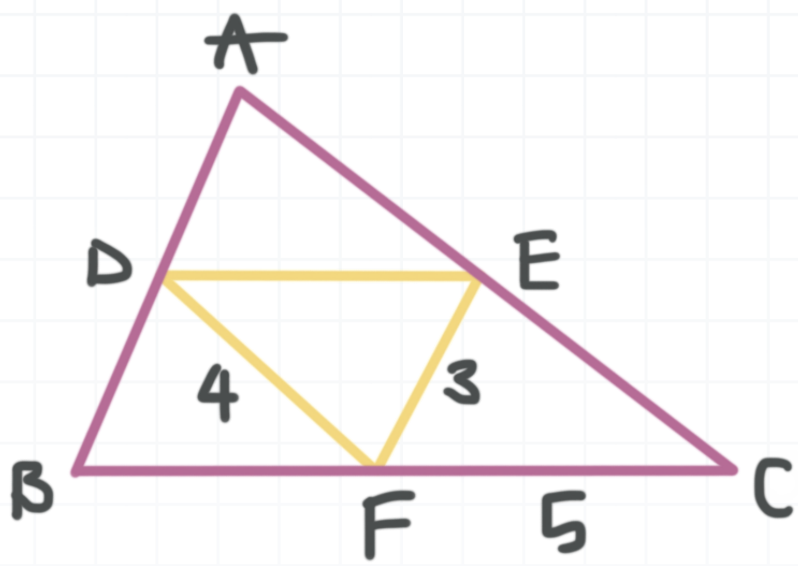
$$8 = x + 2$$

$$6 = x$$

Let's try one with a few more steps.

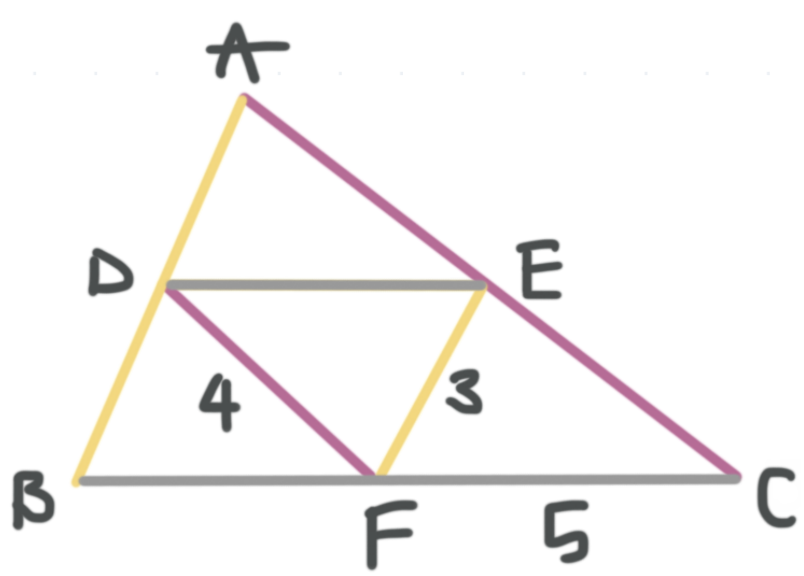
Example

If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, and F is the midpoint of $\overline{BC}$, find the perimeter of triangle ABC.

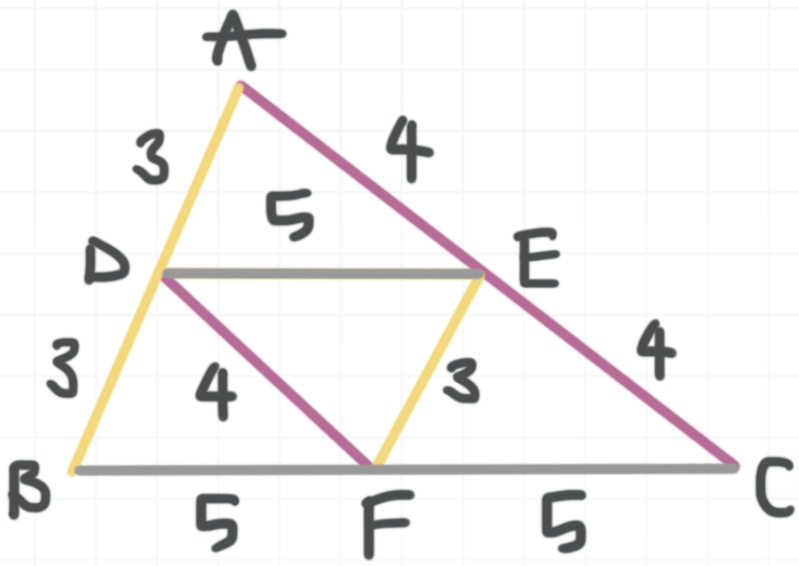


$\overline{DE}$, $\overline{DF}$, and $\overline{EF}$ are all midsegments of triangle ABC, which means we can determine the lengths of all three sides of this triangle by using the fact that the length of a midsegment of a triangle is half the length of the third side of the triangle (the side that the midsegment doesn't intersect).

Let's color each midsegment the same as the corresponding "third side of the triangle."



Now we can fill in what we know.



To find the perimeter, we'll just add the lengths of the two halves of each side of the triangle.

$$P = (3 + 3) + (4 + 4) + (5 + 5)$$

$$P = 6 + 8 + 10$$

$$P = 24$$