Slope and midpoint of a line segment

In this lesson we'll look at how to find the slope and midpoint of a line segment in the Cartesian plane (the xy-plane). We'll start with how to find the slope.

Slope

The **slope** of a line segment is the rate at which the line segment is increasing or decreasing.

In other words, if we graph the line segment in the xy-plane, the slope will tell us how fast the segment is going "uphill" (a positive slope) or "downhill" (a negative slope) from left to right. You can find the slope of a line segment if you have two points that lie on it.

Let's say we have points with coordinates (x_1, y_1) and (x_2, y_2). Then the formula for the slope, m, is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's start by working through an example.

Example

What is the slope of any line segment that passes through the points $(4, -5)$ and $(-3,6)$?

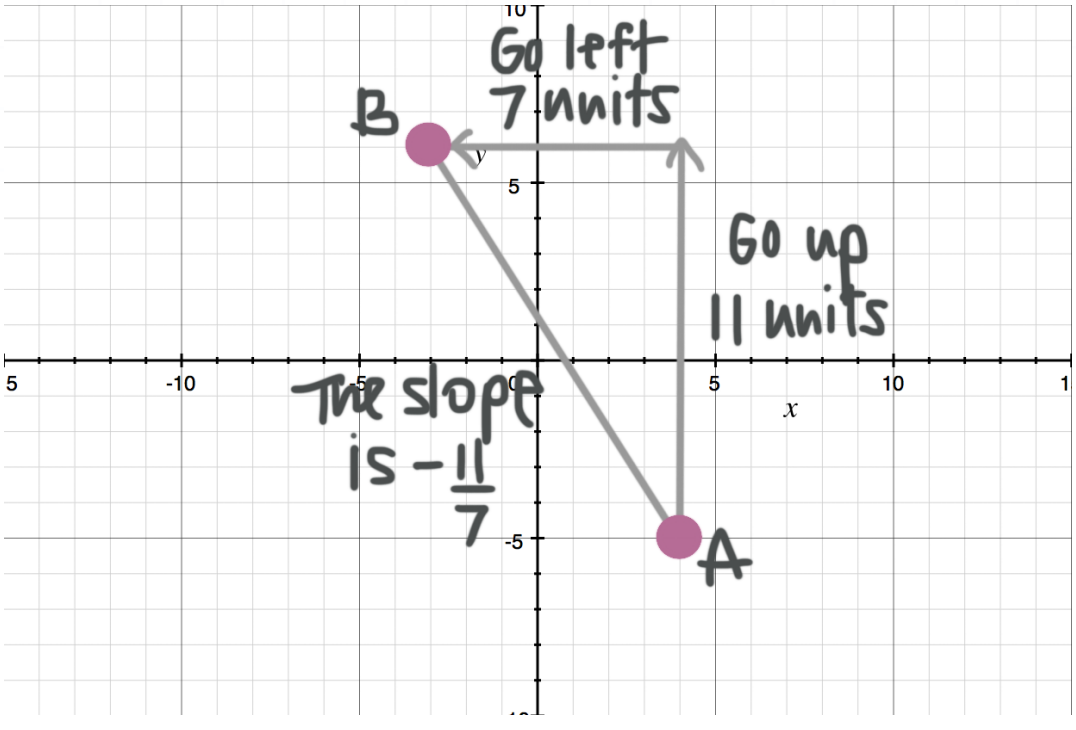
Let $(x_1, y_1) = (4, -5)$ and $(x_2, y_2) = (-3,6)$. We'll plug the coordinates of these points into the formula for the slope of a line segment.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-5)}{-3 - 4} = \frac{6 + 5}{-3 - 4} = \frac{11}{-7} = -\frac{11}{7}$$

It doesn't matter which point we use first. If we switch the points and let $(x_1, y_1) = (-3,6)$ and $(x_2, y_2) = (4, -5)$, we get the same slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 6}{4 - (-3)} = \frac{-5 - 6}{4 + 3} = \frac{-11}{7} = -\frac{11}{7}$$

We can also find the slope by plotting the points:



Midpoint

The midpoint of a line segment is the point that's halfway between the endpoints, so it divides the line segment into two equal parts.

The formula for the coordinates of the midpoint, M, of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Let's look at an example.

Example

What is the midpoint of the line segment with endpoints $(3, -8)$ and $(-5,7)$?

Let $(x_1, y_1) = (3, -8)$ and $(x_2, y_2) = (-5,7)$, then plug these coordinates into the formula for the midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + (-5)}{2}, \frac{-8 + 7}{2}\right) = \left(\frac{-2}{2}, \frac{-1}{2}\right) = \left(-1, -\frac{1}{2}\right)$$

We can also plot the endpoints, and the midpoint, on a graph.