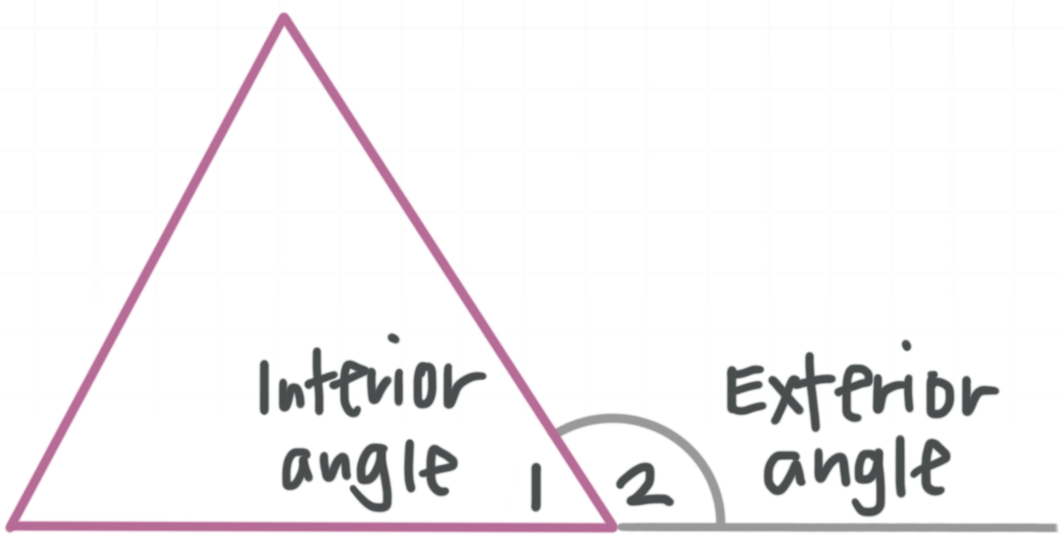
Exterior angles of polygons

In this lesson we'll look at exterior angles of polygons and the relationship between those and their corresponding interior angles.

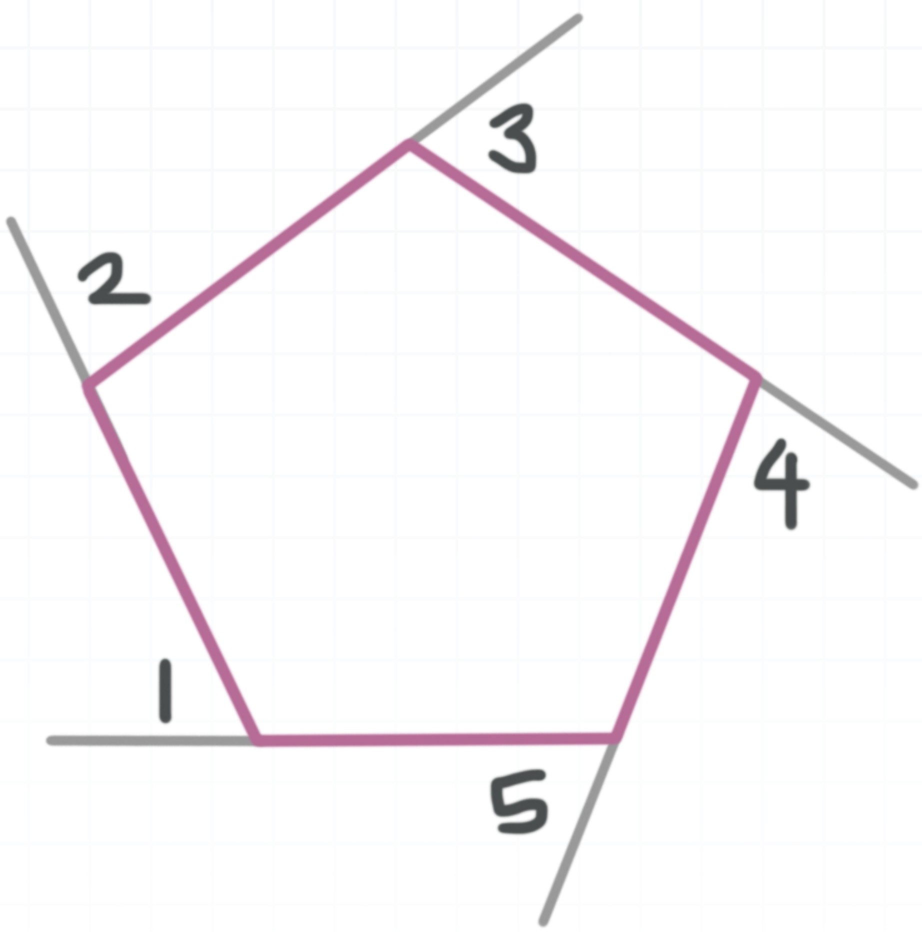
An exterior angle of a polygon is an angle that's supplementary to one of the interior angles of the polygon, has its vertex at the vertex of that interior angle, and is formed by extending one of the two sides of the polygon (at that vertex) in the direction opposite (180° away from) that side.



Realize that at each vertex of a polygon there are two exterior angles: one formed by extending one of the two sides at that vertex, and one formed by extending the other side at that vertex. Since both of the exterior angles at a given vertex are supplementary to the same interior angle, the two exterior angles always have equal measure.

This means that, in the figure above, $m\angle 1 + m\angle 2 = 180^\circ$.

The sum of the measures of the exterior angles in any polygon is 360° if we include only one of the two exterior angles at each vertex, which is what we'll mean going forward when we talk about a polygon's exterior angles. Here is an example of the exterior angles of a pentagon adding to 360°.

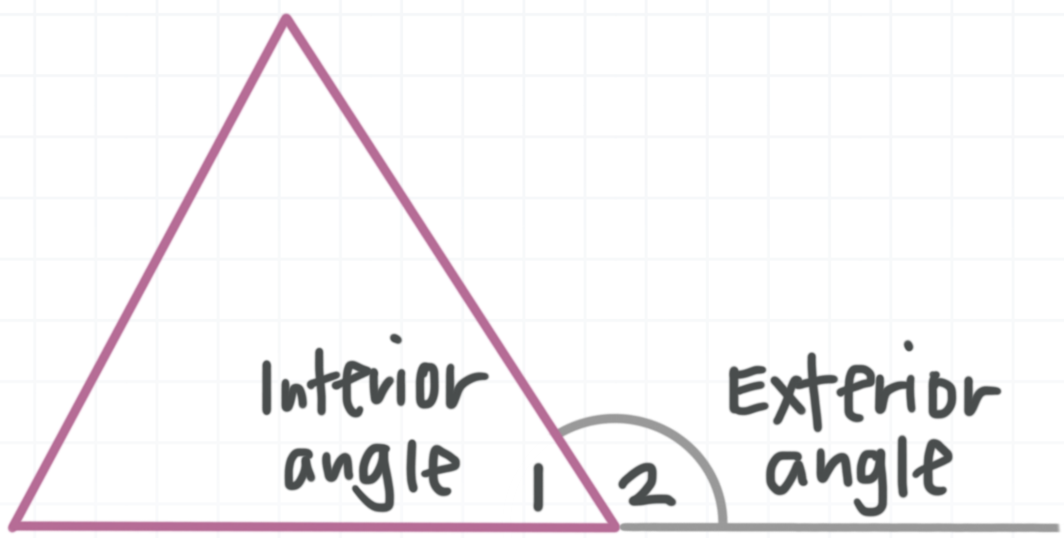


$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360^\circ$$

Remember also that the sum of the measures of the interior angles of a polygon with n sides is $(n-2)180^\circ$.

Example

Given that the triangle in the diagram is equilateral, what is the measure of angle 2? ("Lateral" means "side," so an equilateral polygon is a polygon in which all sides have equal length.)



The measures of the interior angles in a triangle add to 180°. An equilateral triangle is also equiangular (which means that all of its interior angles have the same measure). That means that each interior angle measures $180^\circ \div 3 = 60^\circ$, so $m\angle 1 = 60^\circ$. $\angle 1$ and $\angle 2$ are supplementary, which means that

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$60^\circ + m\angle 2 = 180^\circ$$

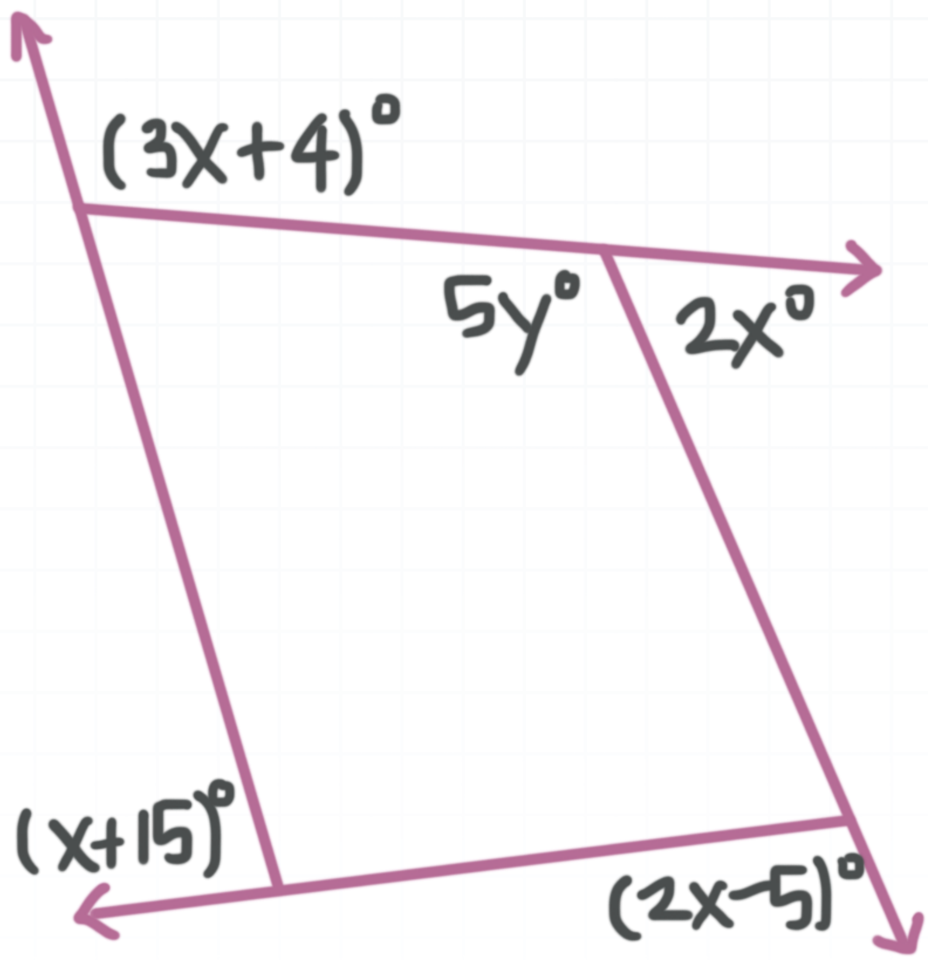
$$m\angle 2 = 120^\circ$$

We could also have solved this problem by using the fact that all of the exterior angles sum to 360°. A triangle has three interior angles, so it also has three exterior angles (if we include only one of the two exterior angles at each vertex). Since all of the interior angles are congruent, all of the exterior angles will also be congruent. This means that $m\angle 2 = 360^\circ \div 3 = 120^\circ$.

Let's look at another example.

Example

Find the value of y.



The sum of the measures of the quadrilateral's four exterior angles, shown in the figure, must be 360°. Therefore,

$$3x^\circ + 4^\circ + 2x^\circ + 2x^\circ - 5^\circ + x^\circ + 15^\circ = 360^\circ$$

$$3x^\circ + 2x^\circ + 2x^\circ + x^\circ + 4^\circ - 5^\circ + 15^\circ = 360^\circ$$

$$8x^\circ + 14^\circ = 360^\circ$$

$$8x^\circ = 346^\circ$$

$$x = 43.25^\circ$$

The interior angle of measure $5y^\circ$ and the exterior angle of measure $2x^\circ$ are supplementary.

$$5y^\circ + 2x^\circ = 180^\circ$$

Substitute 43.25 for x and solve for y.

$$5y^\circ + 2(43.25)^\circ = 180^\circ$$

$$5y^\circ + 86.5^\circ = 180^\circ$$

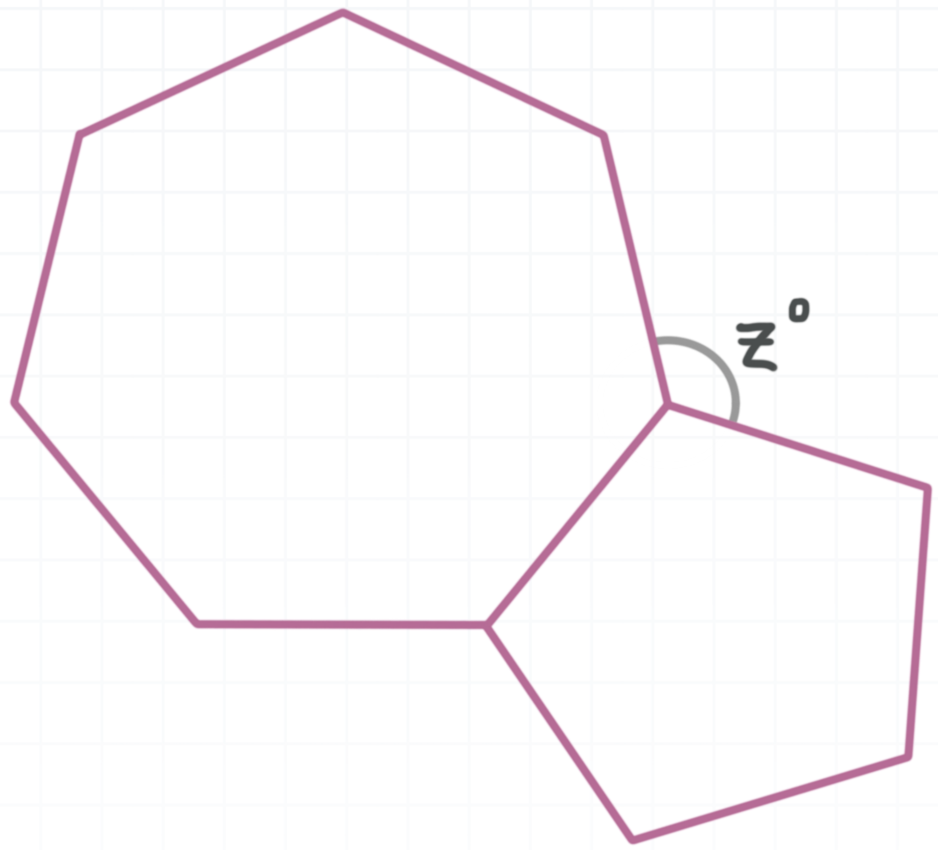
$$5y^\circ = 93.5^\circ$$

$$y = 18.7$$

Let's look at one more example.

Example

The figure shows a regular heptagon and a regular pentagon. Find the value of z to the nearest hundredth.



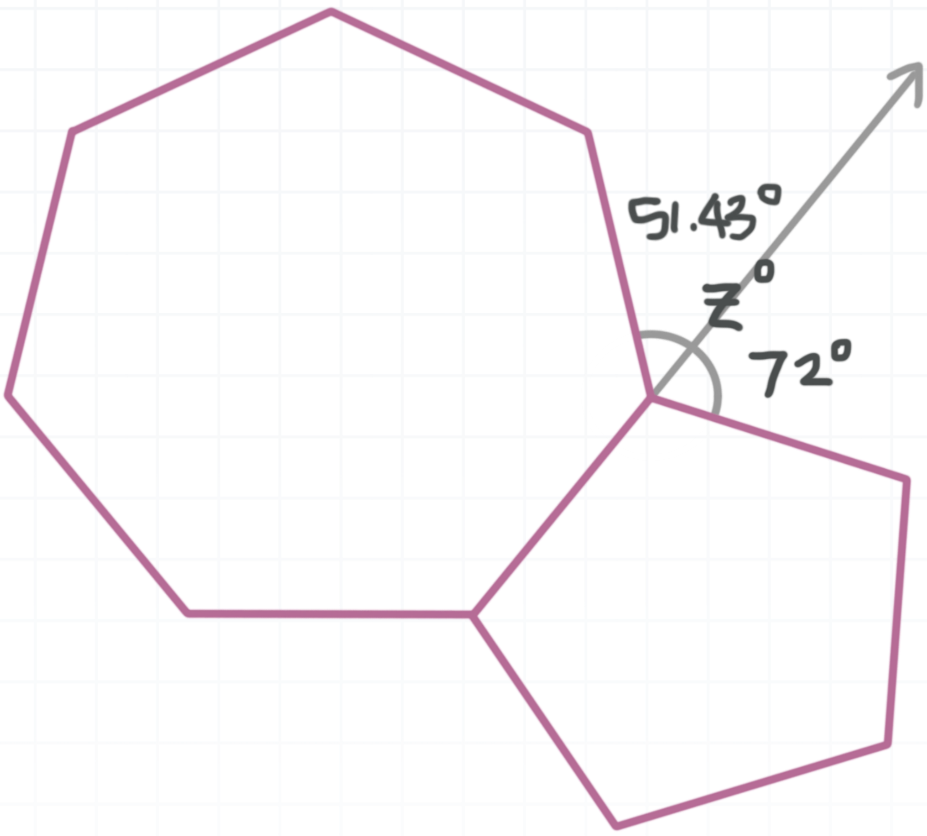
Notice that the angle of measure z° is formed from an exterior angle of the heptagon and an exterior angle of the pentagon, which are a pair of adjacent angles, and that the sum of their measures is z°.

An exterior angle of a regular heptagon has a measure of

$$360^\circ \div 7 \approx 51.43^\circ$$

An exterior angle of a regular pentagon has a measure of

$$360^\circ \div 5 = 72^\circ$$



Therefore,

$$z^\circ \approx 51.43^\circ + 72^\circ$$

$$z^\circ \approx 123.43^\circ$$