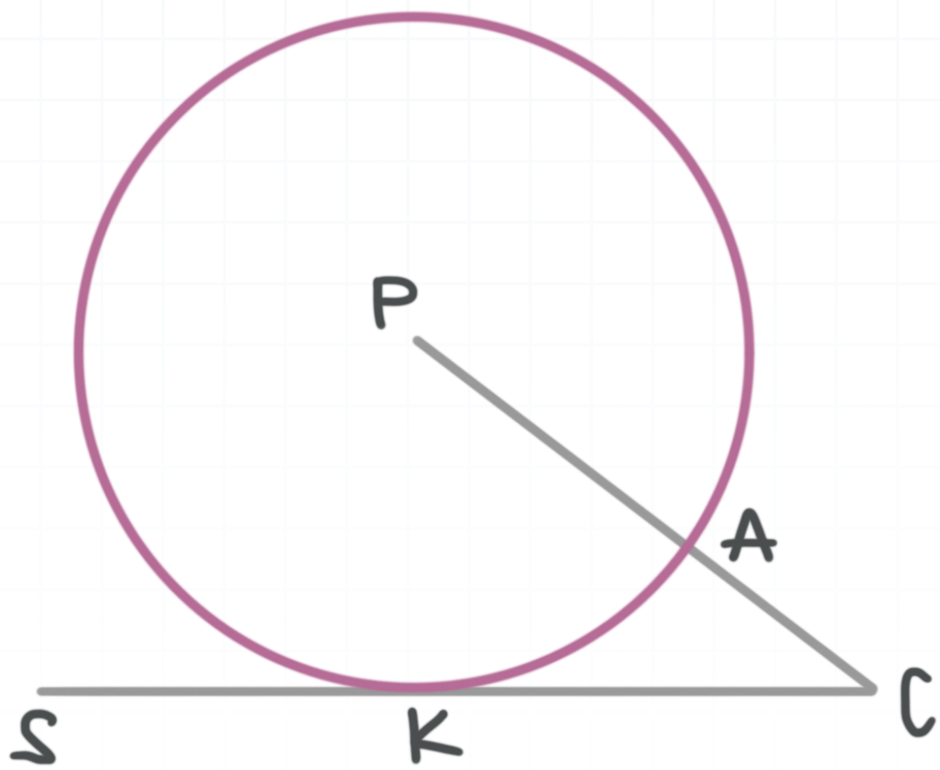
Topic: Tangent lines of circles

Question: In the circle in the figure (with center at P), the radius is 6 and $\overline{CS}$ is tangent to the circle at K. If $\overline{AC} = 4$, how long is $\overline{CK}$?



Answer choices:

A $\quad 5$

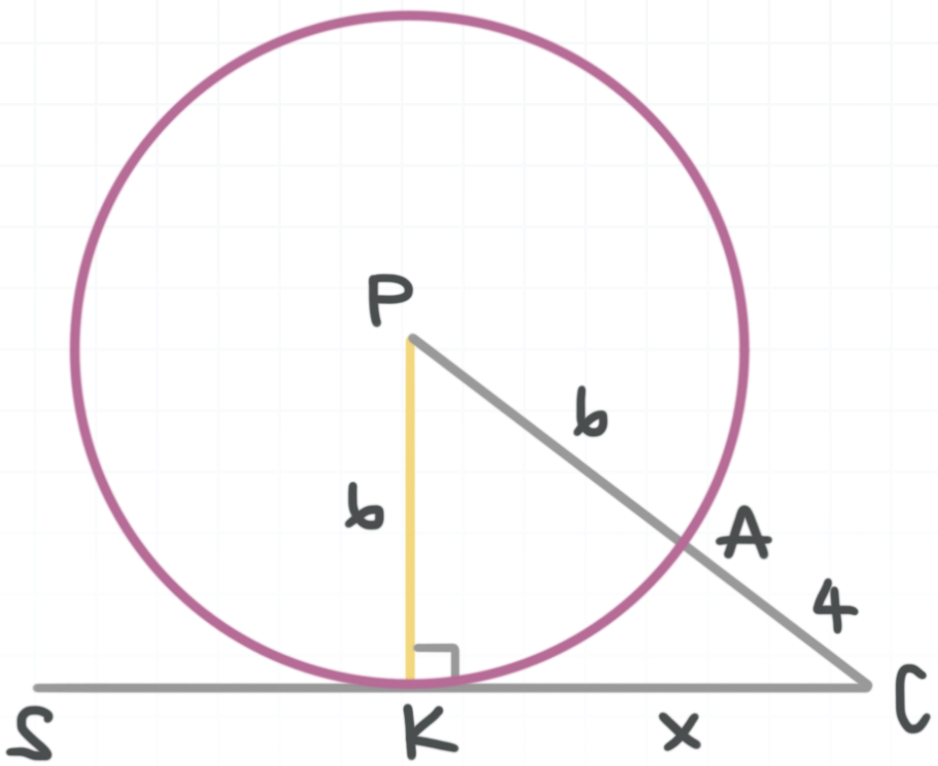
B $\quad 6$

C $\quad 7$

D $\quad 8$

Solution: D

A radius drawn to K will be perpendicular to $\overline{CK}$, making a right triangle. Let $x = \overline{CK}$, and label the segments as shown in the figure.



Use the Pythagorean theorem to find x.

$$(\overline{CK})^2 + (\overline{PK})^2 = (\overline{PC})^2$$

$$(\overline{CK})^2 + (\overline{PK})^2 = (\overline{PA} + \overline{AC})^2$$

$$x^2 + 6^2 = 10^2$$

$$x^2 + 36 = 100$$

$$x^2 = 64$$

$$x = 8$$

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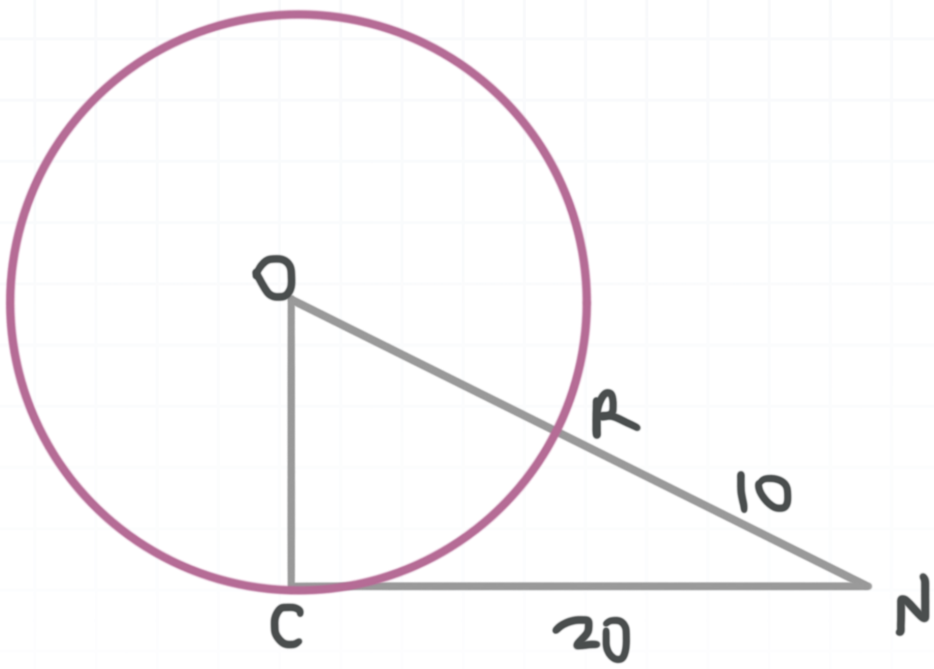
Question: In the circle in the figure (with center at O), $\overline{CN}$ is tangent to the circle at C. If $\overline{CN} = 20$ and $\overline{RN} = 10$, what is the radius of the circle?



Answer choices:

A 15

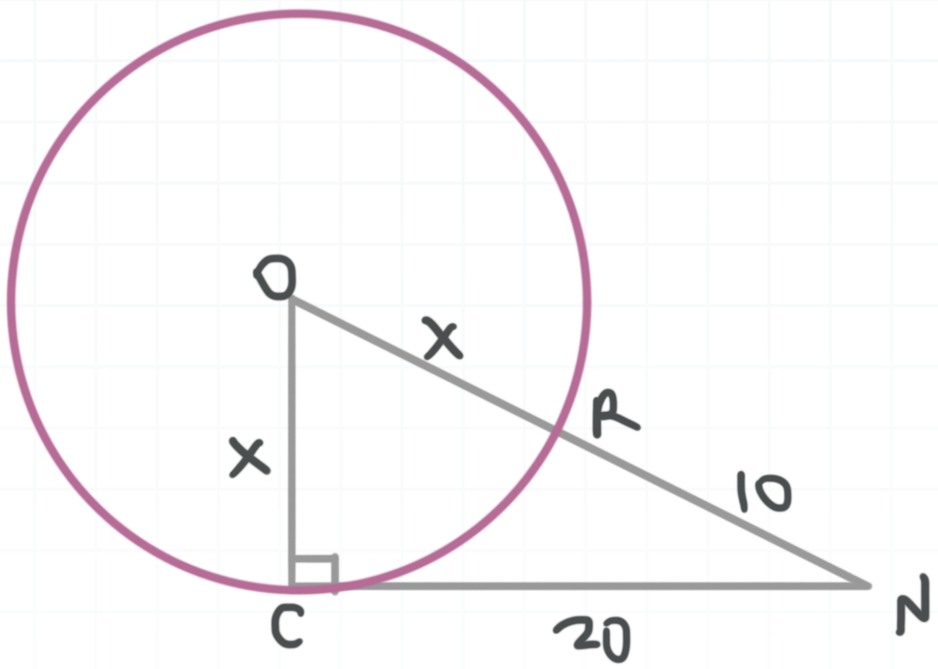
B 12

C 9

D 6

Solution: A

A radius drawn to C will be perpendicular to $\overline{CN}$, forming a right triangle.



Let $x = \overline{OC}$ (and therefore that $\overline{OR} = x$ as well) and use the Pythagorean theorem.

$$x^2 + 20^2 = (x + 10)^2$$

$$x^2 + 400 = x^2 + 20x + 100$$

Subtract x^2 and 100 from each side.

$$300 = 20x$$

$$x = 15$$

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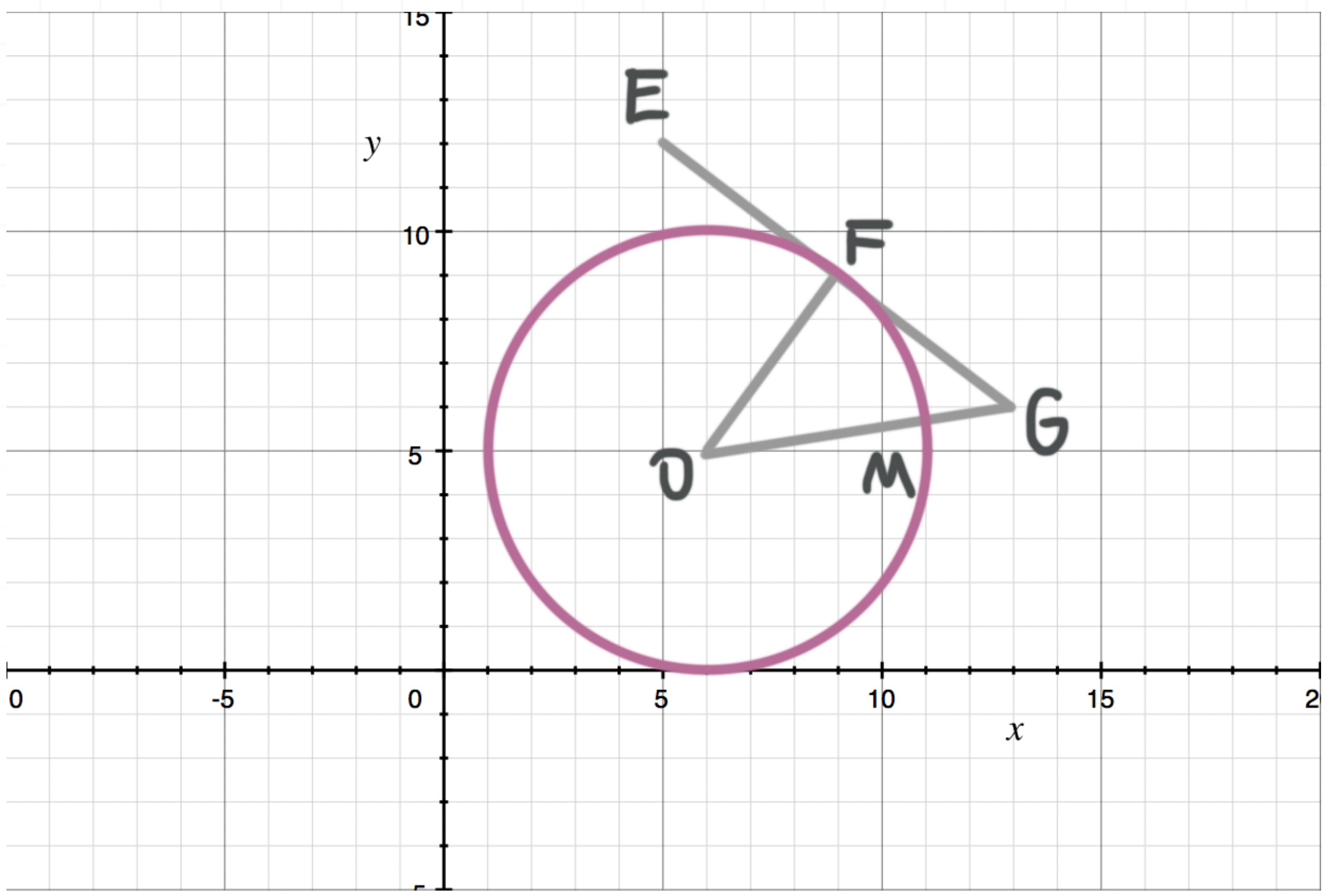
Question: In the circle in the figure, the center (point O) is at $(6,5)$, F is at $(9,9)$, and G is at $(13,6)$. $\overline{EG}$ is tangent to the circle at F. How long is $\overline{MG}$?



Answer choices:

A $5\sqrt{2}$

B $5\sqrt{2}+5$

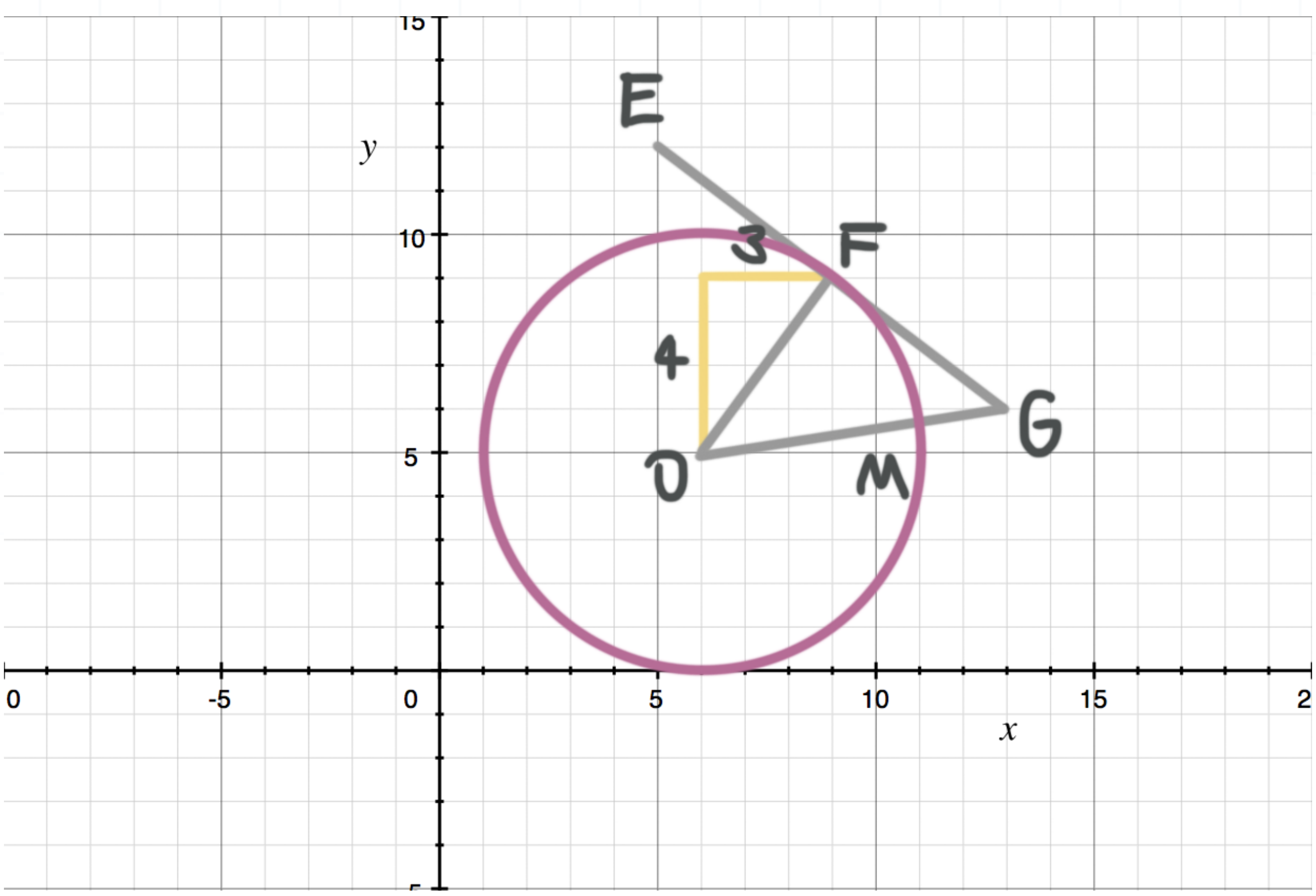
C $2\sqrt{5}$

D $5\sqrt{2}-5$

Solution: D

Notice that $\overline{OF}$ is the hypotenuse of a right triangle with legs of length 4 and 3.

The leg with length 4 is the vertical line segment from O, which is at $(6,5)$, to the point at $(6,9)$; those two points are 4 units apart. The leg with length 3 is the horizontal line segment from the point at $(6,9)$ to F, which is at $(9,9)$; those two points are 3 units apart.



We can find $\overline{OF}$ by applying the Pythagorean theorem to that right triangle.

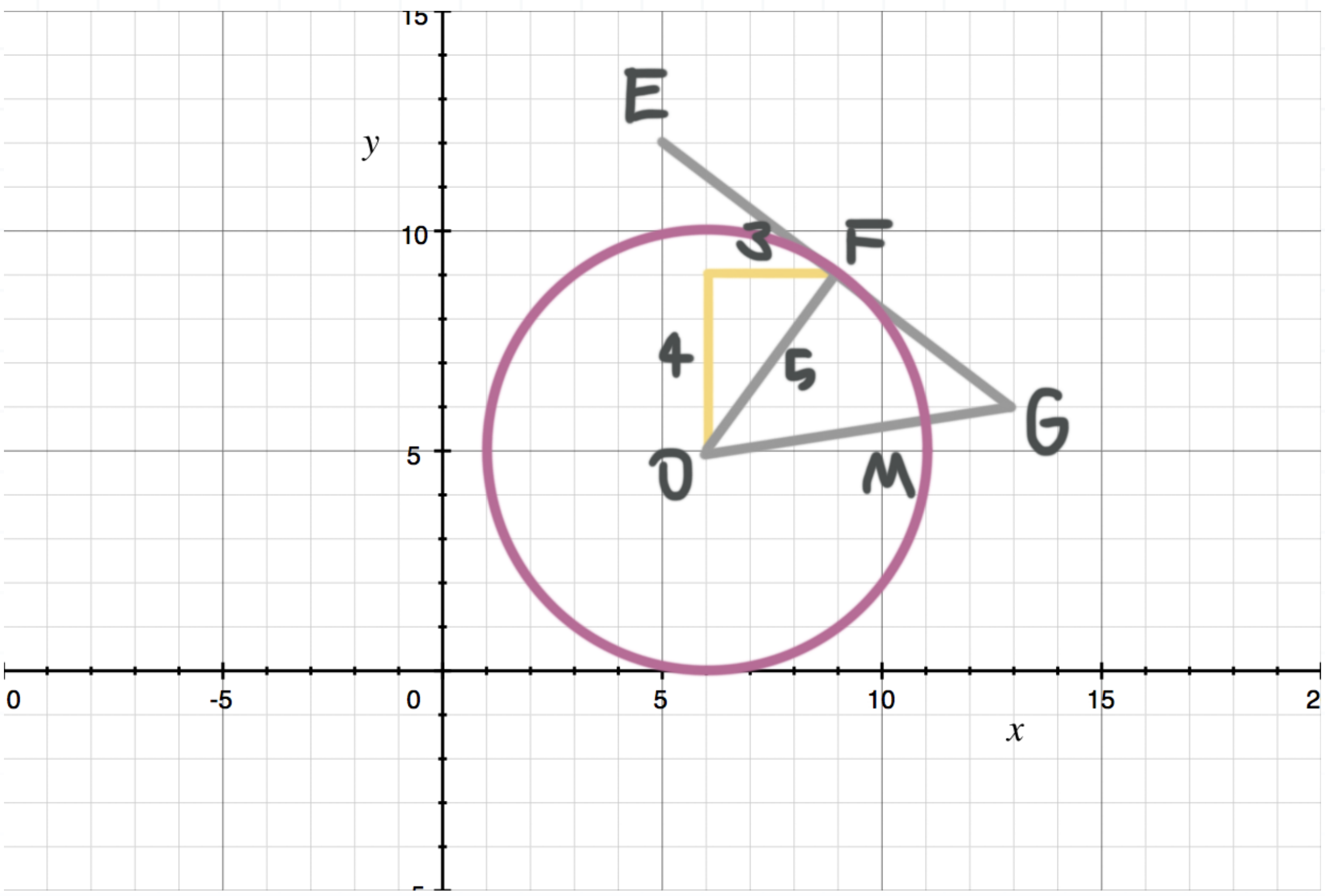
$$4^2 + 3^2 = (\overline{OF})^2$$

$$16 + 9 = (\overline{OF})^2$$

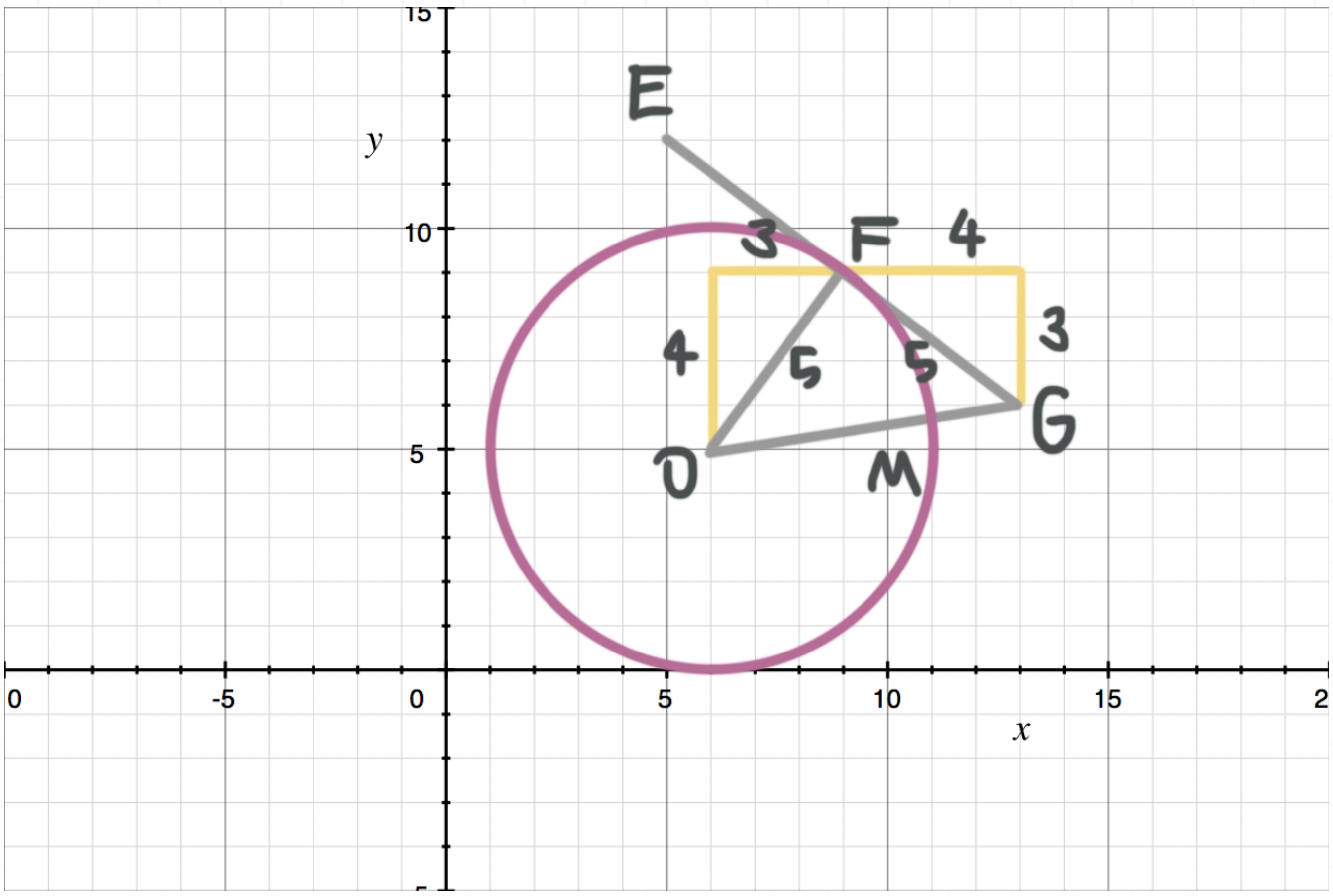
$$25 = (\overline{OF})^2$$

$$5 = \overline{OF}$$

Since $\overline{EG}$ is tangent to the circle at F, that makes $\overline{OF}$ a radius of the circle, so the radius is 5.



Also notice that $\overline{FG}$ is the hypotenuse of a different right triangle with legs of length 4 and 3. That makes $\overline{FG} = 5$ also.



Now focus on $\triangle FOG$, and notice that $\overline{OF} \perp \overline{FG}$ ($\overline{OF}$ is perpendicular to $\overline{FG}$), making $\triangle FOG$ a right triangle with legs $\overline{OF}$ and $\overline{FG}$ and hypotenuse $\overline{OG}$.

Using the Pythagorean theorem, we can find the length of $\overline{OG}$.

$$(\overline{OF})^2 + (\overline{FG})^2 = (\overline{OG})^2$$

$$5^2 + 5^2 = (\overline{OG})^2$$

$$25 + 25 = (\overline{OG})^2$$

$$50 = (\overline{OG})^2$$

$$\sqrt{50} = \overline{OG}$$

$$\sqrt{25 \cdot 2} = \overline{OG}$$

$$5\sqrt{2} = \overline{OG}$$

Notice that $\overline{OG} = \overline{OM} + \overline{MG}$, and that $\overline{OM}$ is a radius of the circle (so $\overline{OM} = 5$). Therefore,

$$5\sqrt{2} = 5 + \overline{MG}$$

$$\overline{MG} = 5\sqrt{2} - 5$$