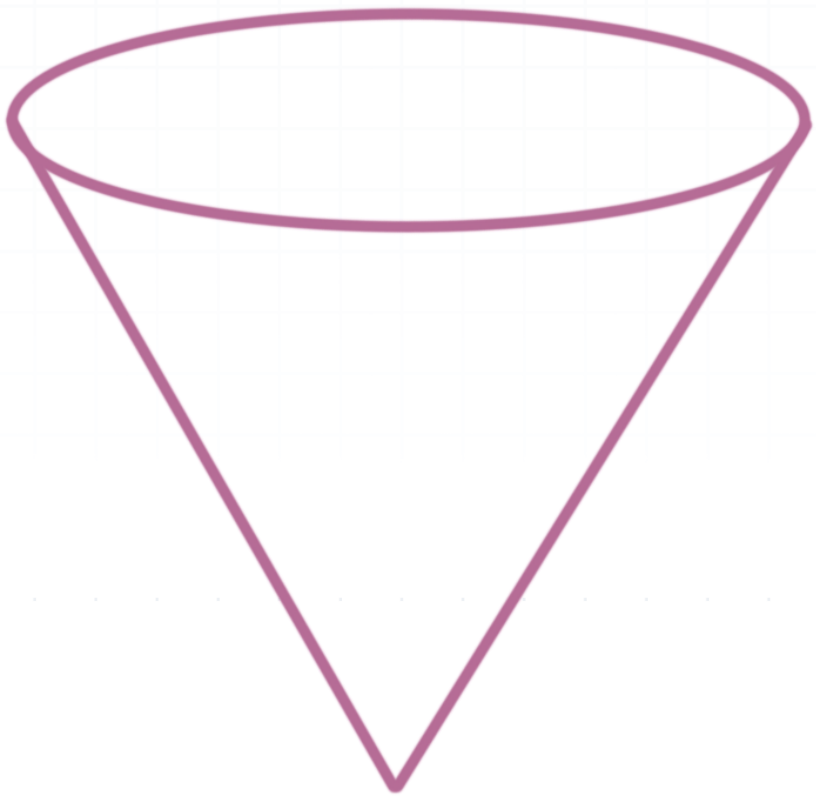
Nets/volume/surface area of cones

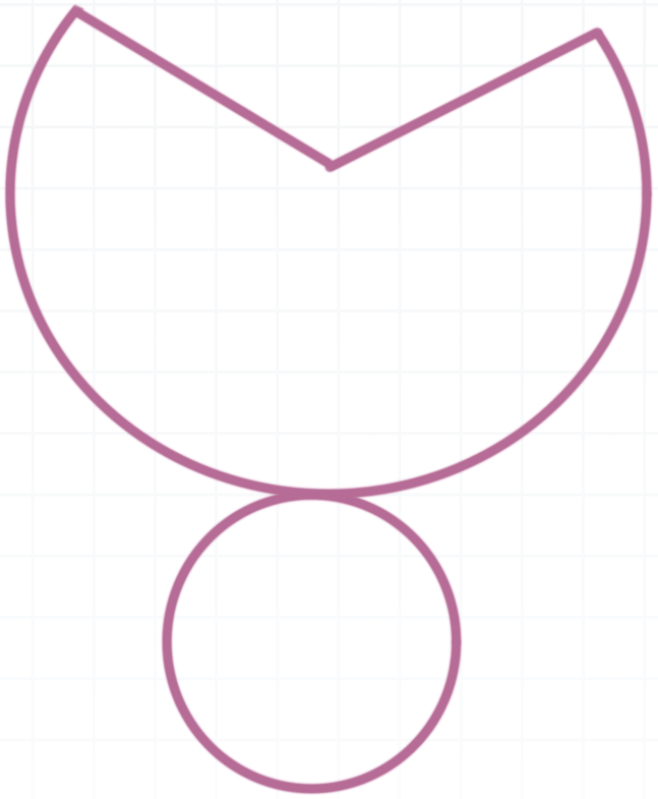
In this lesson we'll look at the nets, volume, and surface area of cones.

Cones

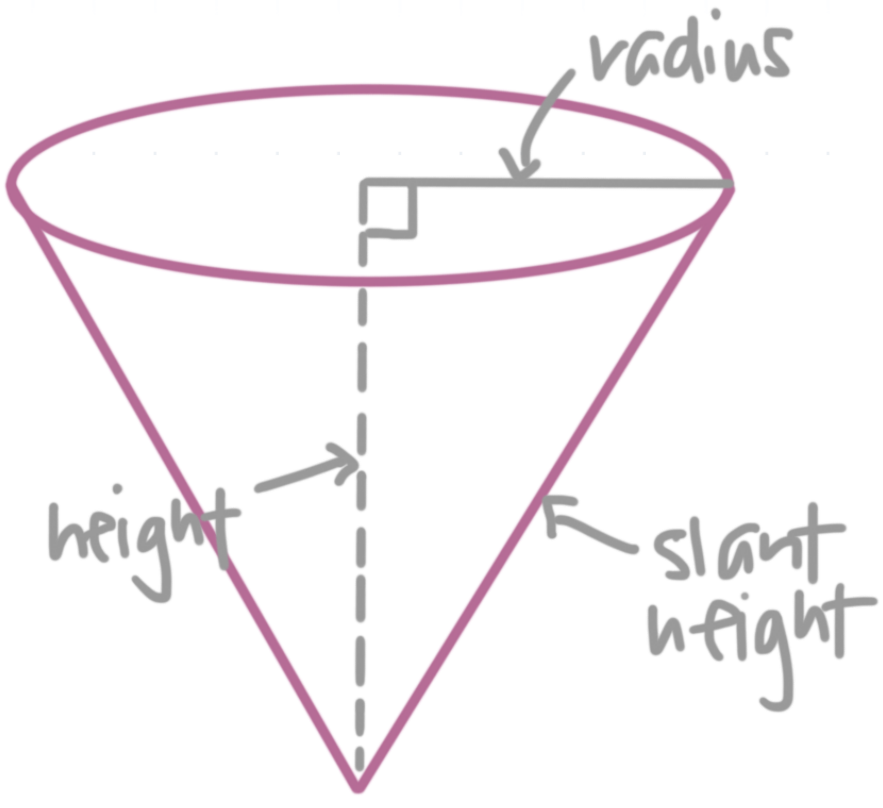
A **right circular cone** (the only kind of cone we're dealing with in this lesson) has a circular base.



The net of a cone consists of a full circle for the base and part of another circle for the wall of the cone.



We'll also need to know the base radius, which is the radius of the base of the cone, the height, which is the distance from the base to the tip of the cone, and the slant height, which is the length along the side of the cone from the base to the tip.



The radius r, height h, and slant height l are all related to one another by the Pythagorean theorem.

$$r^2 + h^2 = l^2$$

Volume and surface area

The volume of a cone is given by

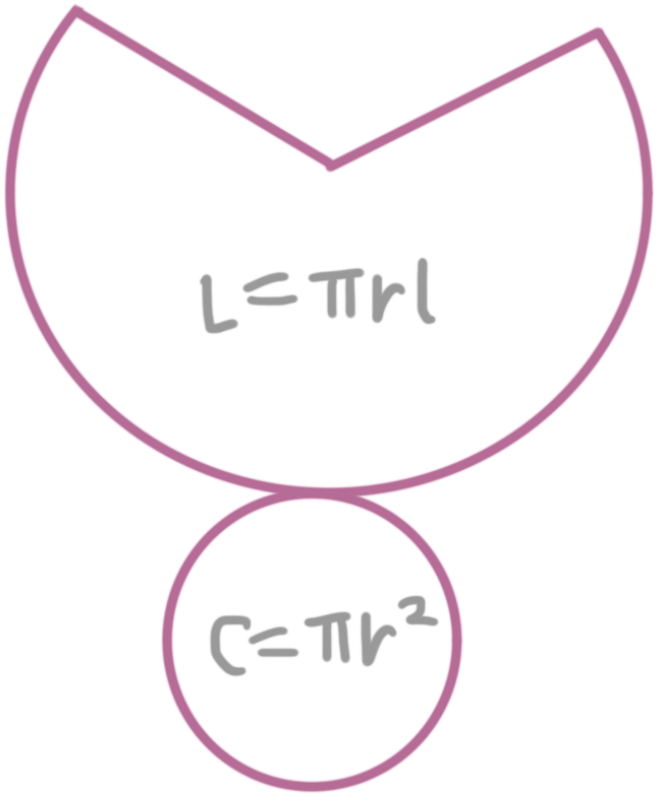
$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius and h is the height of the cone.

The surface area of a cone is given by

$$S = \pi rl + \pi r^2$$

where r is the radius and l is the slant height of the cone.



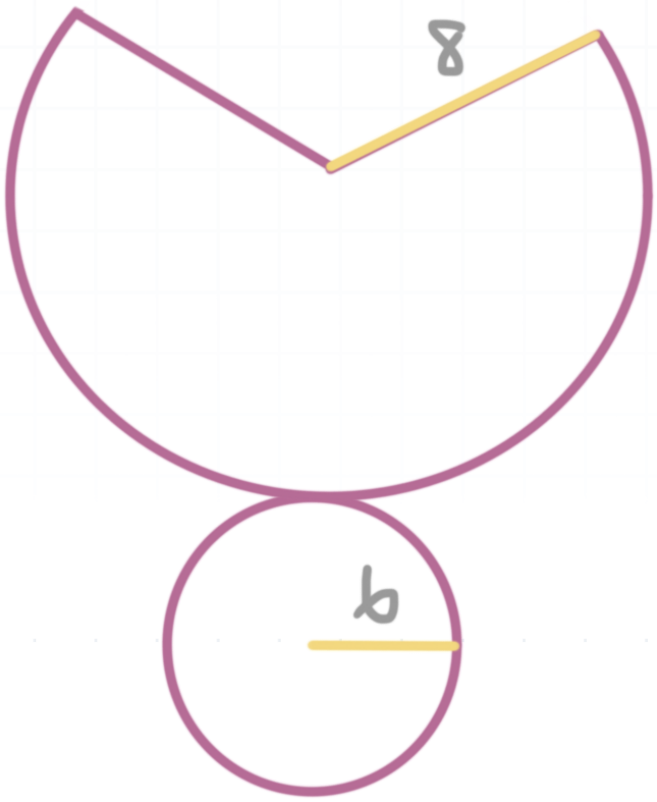
L is called the lateral area, so we can also write the surface area of the cone as

$$S = L + \pi r^2$$

Let's do a few examples.

Example

Find the surface area of the cone that's represented by the net.



The formula for the surface area of a cone is

$$S = \pi rl + \pi r^2$$

In this case, the slant height is $l = 8$ and the radius is $r = 6$. Plugging these into the formula, we get

$$S = \pi(6)(8) + \pi(6)^2$$

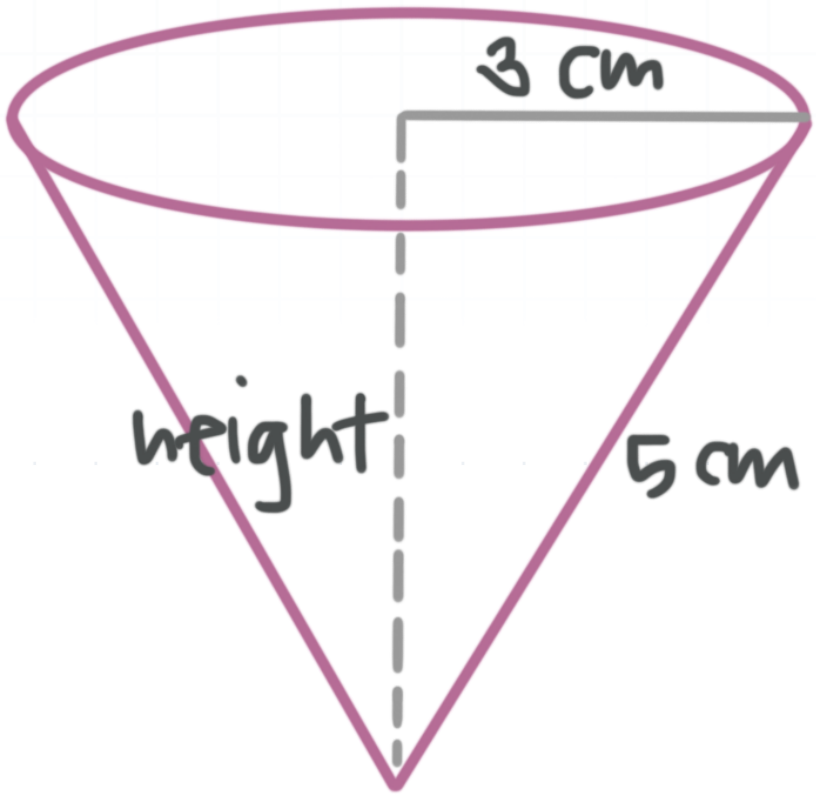
$$S = 48\pi + 36\pi$$

$$S = 84\pi$$

Let's do a few more examples.

Example

What is the volume of the cone?



The formula for volume is

$$V = \frac{1}{3}\pi r^2 h$$

We already know the radius of the cone is 3 cm, and we need to find the height of the cone in order to find the volume. Remember, the radius, height, and slant height are related by the Pythagorean Theorem.

$$r^2 + h^2 = l^2$$

Plugging in, we get

$$(3 \text{ cm})^2 + h^2 = (5 \text{ cm})^2$$

$$9 \text{ cm}^2 + h^2 = 25 \text{ cm}^2$$

$$h^2 = 16 \text{ cm}^2$$

$$h = 4 \text{ cm}$$

Now we can use the volume formula.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(3 \text{ cm})^2(4 \text{ cm})$$

$$V = \frac{1}{3}\pi(9 \text{ cm}^2)(4 \text{ cm})$$

$$V = 12\pi \text{ cm}^3$$

Let’s do one more.

Example

What is the surface area of a cone with a slant height of 16.8 cm and a diameter of 16 cm? Use $\pi = 3.14$.

The formula for the surface area is

$$S = \pi rl + \pi r^2$$

The slant height is $l = 16.8$ cm. We can use the diameter to find the radius.

$$r = \frac{d}{2} = \frac{16 \text{ cm}}{2} = 8 \text{ cm}$$

The radius is $r = 8$. Plug the values of r and l into the formula for surface area.

$$S = \pi rl + \pi r^2$$

$$S = 3.14(8 \text{ cm})(16.8 \text{ cm}) + 3.14(8 \text{ cm})^2$$

$$S = 422.016 \text{ cm}^2 + 200.960 \text{ cm}^2$$

$$S = 622.976 \text{ cm}^2$$