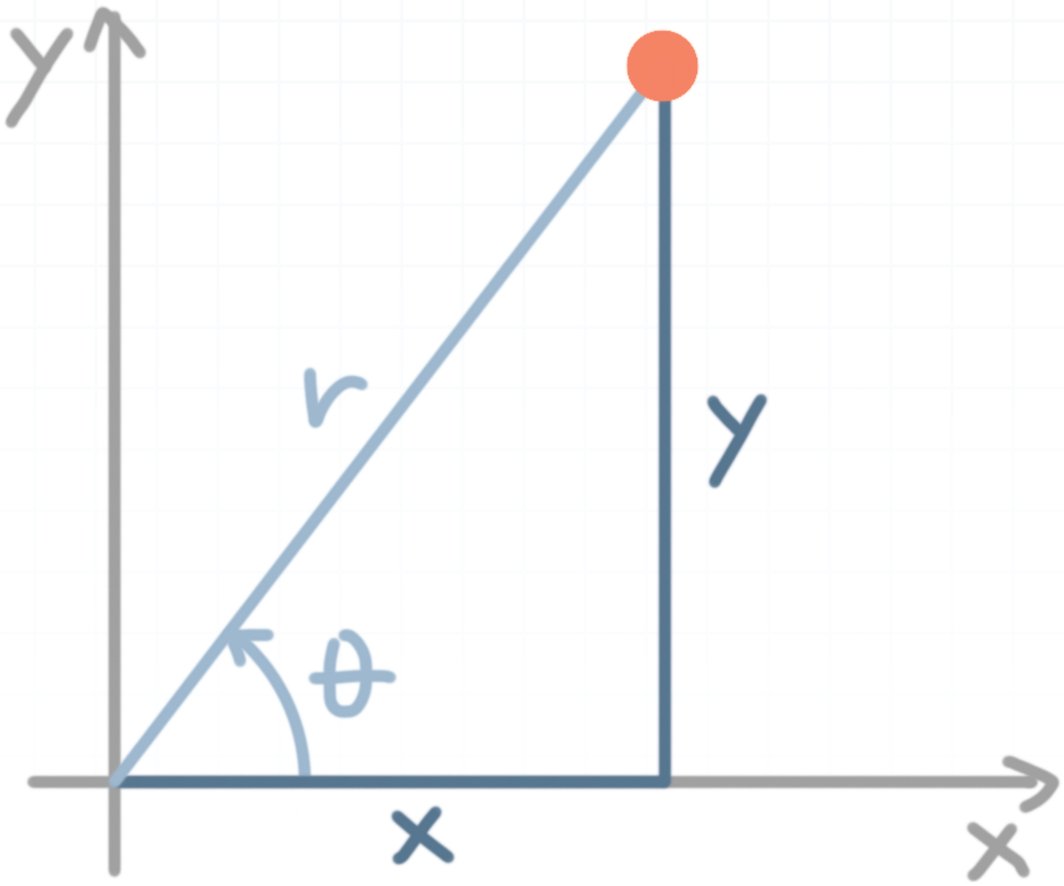
Converting rectangular points to polar

We've used $y = r\sin\theta$ and $x = r\cos\theta$ to convert from polar to rectangular, but there are two other conversion formula worth mentioning, the first of which also comes from the right triangle that we sketched out earlier.



Given a right triangle with legs a and b and hypotenuse c, remember that the Pythagorean Theorem tells us that

$$a^2 + b^2 = c^2$$

If we substitute x, y, and r for a, b, and c, respectively, we get

$$x^2 + y^2 = r^2$$

So to convert between polar and rectangular coordinates, it's also helpful to know

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

The second conversion formula we want to build comes again from

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

We know that tangent is equivalent to sine/cosine, so

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x}$$

If we apply the inverse the inverse tangent $\tan^{-1}$ to both sides of this equation, we get

$$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Let's do an example where we use these two new conversion equations in order to convert a rectangular coordinate point into a polar coordinate point.

Example

Convert $(6,11)$ into polar coordinates.

The value of r for this point will be

$$r = \sqrt{x^2 + y^2} = \sqrt{6^2 + 11^2} = \sqrt{36 + 121} = \sqrt{157} \approx 12.5$$

and the value for θ will be

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{11}{6}\right) \approx \tan^{-1}(1.83) \approx 1.07 \text{ radians}$$