Rotating figures in coordinate space

In this lesson we'll look at rotation of a figure in a coordinate plane and how to determine the location and orientation of the figure after the rotation takes place.

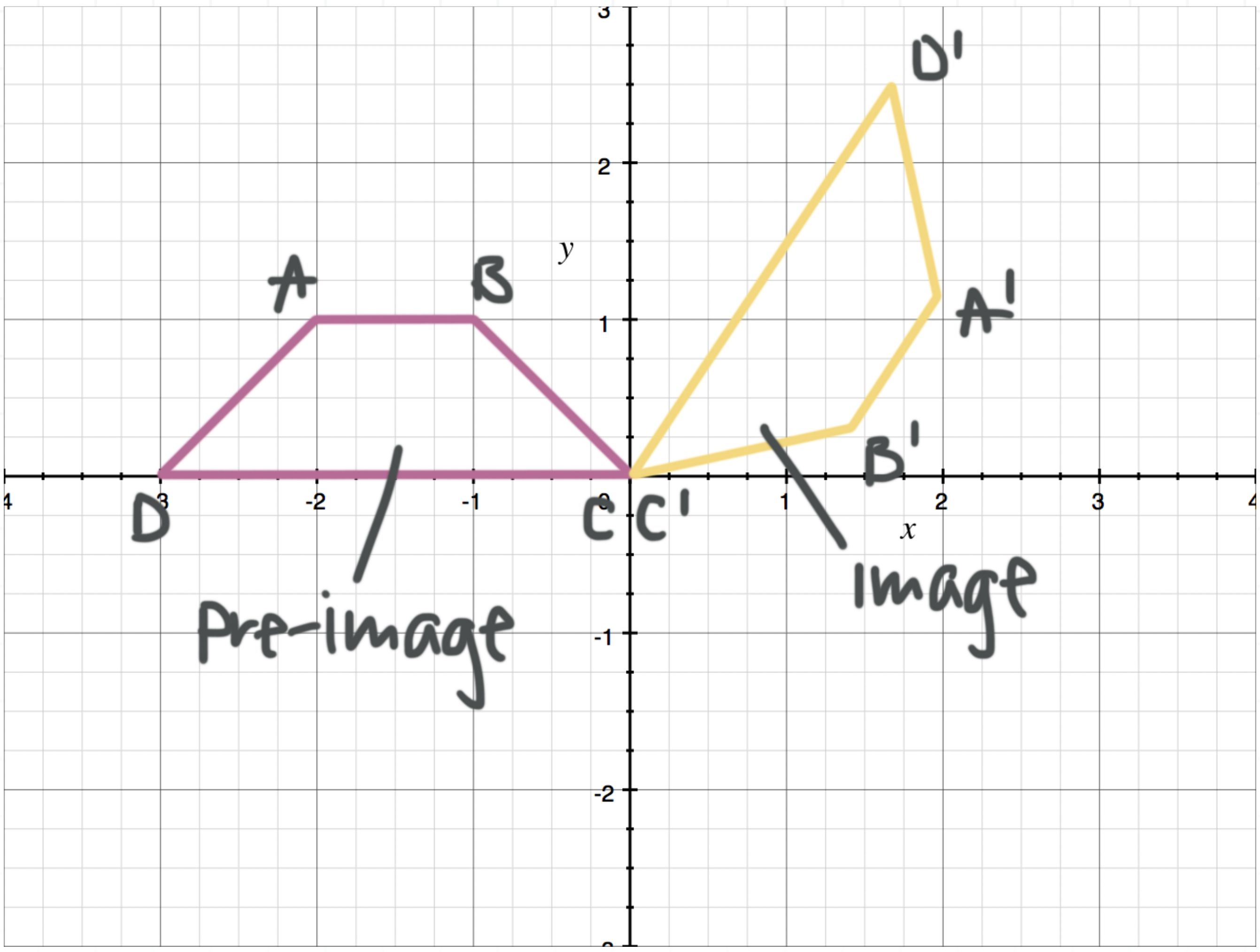
A **rotation** is a type of transformation that turns a figure around a central point, called the **point of rotation**, with no translation of the figure. The point of rotation, which remains fixed (it isn't moved by the rotation), can be inside or outside of the figure.

The pre-image and image

Before a rotation, we have the **pre-image** (the figure in its original location and orientation). Points in the pre-image are usually labeled with capital letters.

After the rotation, we have the **image** (the figure in its final location and orientation). Points in the image are usually labeled with the same capital letters, plus the prime symbol $'$ after each letter. So if figure $ABCD$ is rotated, its image becomes figure $A'B'C'D'$.

In a rotation, the image and pre-image are always congruent, because a rotation never changes the measures of angles or the lengths of line segments and curves in the figure.



Rotating figures

To rotate a figure you need three things:

1. a direction (clockwise or counterclockwise),

2. an angle (the number of degrees through which you're rotating the figure), and

3. the point of rotation

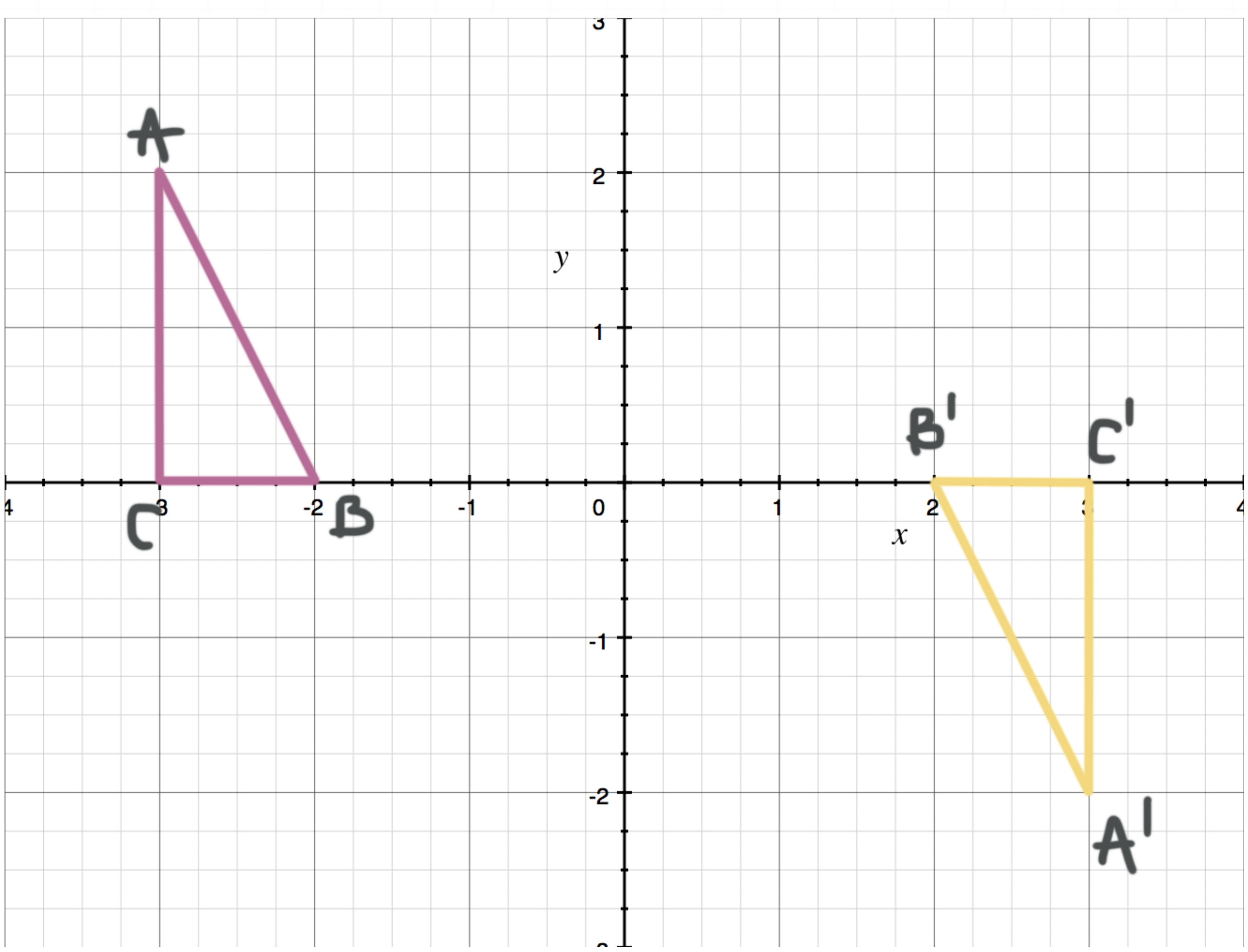
There are some common rotations about the origin, $(0,0)$, that we can easily use rules for:

Angle	Direction	Rule
90 degrees	Clockwise	(x, y) --> (y, -x)
90 degrees	Counterclockwise	(x, y) --> (-y, x)
180 degrees	Clockwise	(x, y) --> (-x, -y)
180 degrees	Counterclockwise	(x, y) --> (-x, -y)
270 degrees	Clockwise	(x, y) --> (-y, x)
270 degrees	Counterclockwise	(x, y) --> (y, -x)

Let's look at some examples.

Example

Write a rule to describe the rotation shown in the graph.



You can visually see that the triangle has been rotated 180° about the origin, but you could also look at the rules to see if this follows any of them. Let's compare the coordinates of the points in the pre-image to the coordinates of the corresponding points in the image.

The points in the pre-image are

$$A = (-3,2)$$

$$B = (-2,0)$$

$$C = (-3,0)$$

The corresponding points in the image are

$$A' = (3, -2)$$

$$B' = (2,0)$$

$$C' = (3,0)$$

Now we compare the points:

$$A = (-3,2) \quad \to \quad A' = (3, -2)$$

$$B = (-2,0) \quad \to \quad B' = (2,0)$$

$$C = (-3,0) \quad \to \quad C' = (3,0)$$

When you compare the coordinates, you can see they're following the rule for a 180° rotation clockwise around the origin.

Angle	Direction	Rule
180 degrees	Clockwise	(x, y) --> (-x, -y)

So the relationship between the coordinates of any point in the pre-image and the coordinates of the corresponding point in the image is

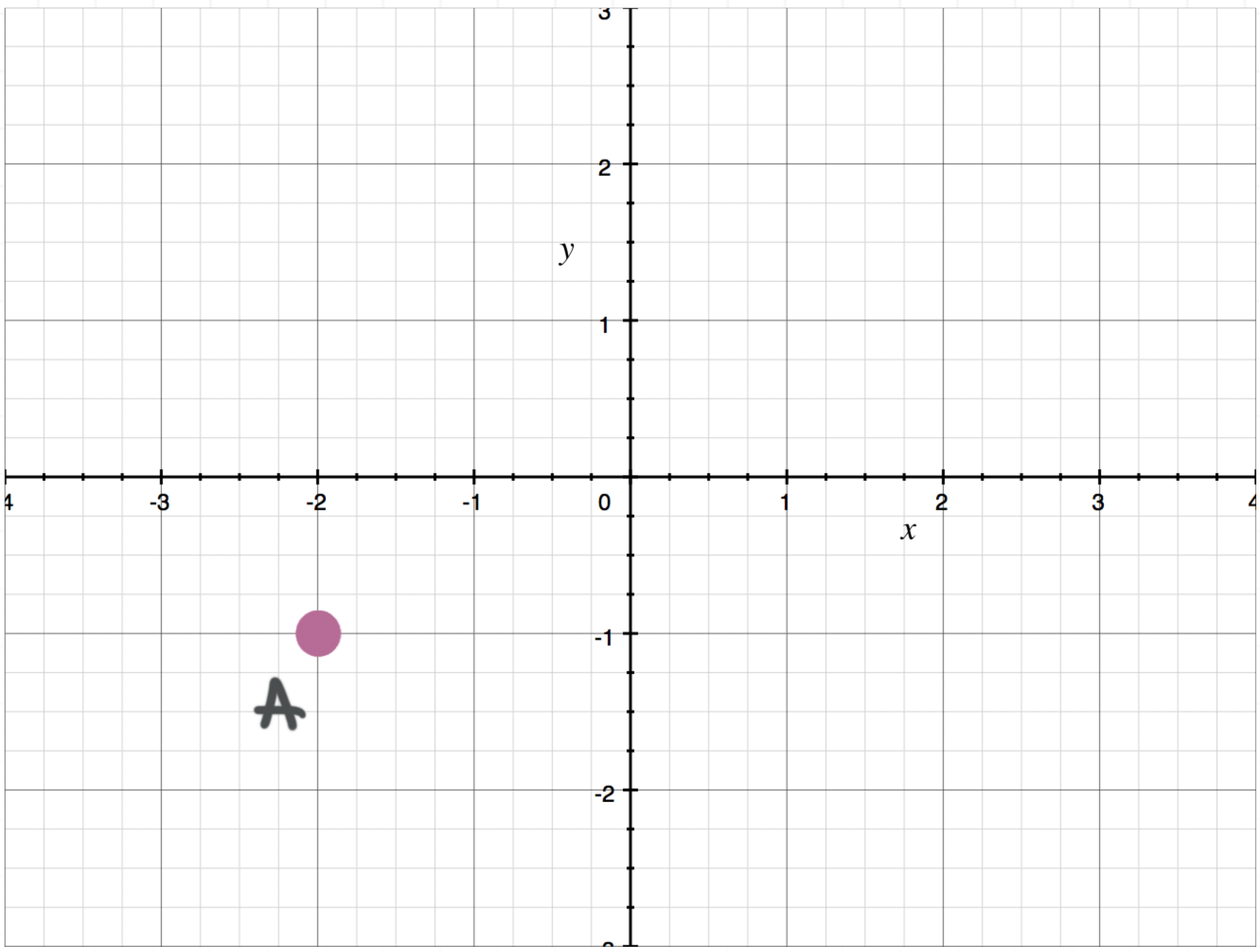
$$(x, y) \to (-x, -y)$$

Since the rule for a counterclockwise rotation of 180° about the origin is the same as the rule for a clockwise rotation of 180° about the origin, you could say either that the figure in this example has been rotated 180° clockwise about the origin or that it's been rotated 180° counterclockwise about the origin.

Let's try another example.

Example

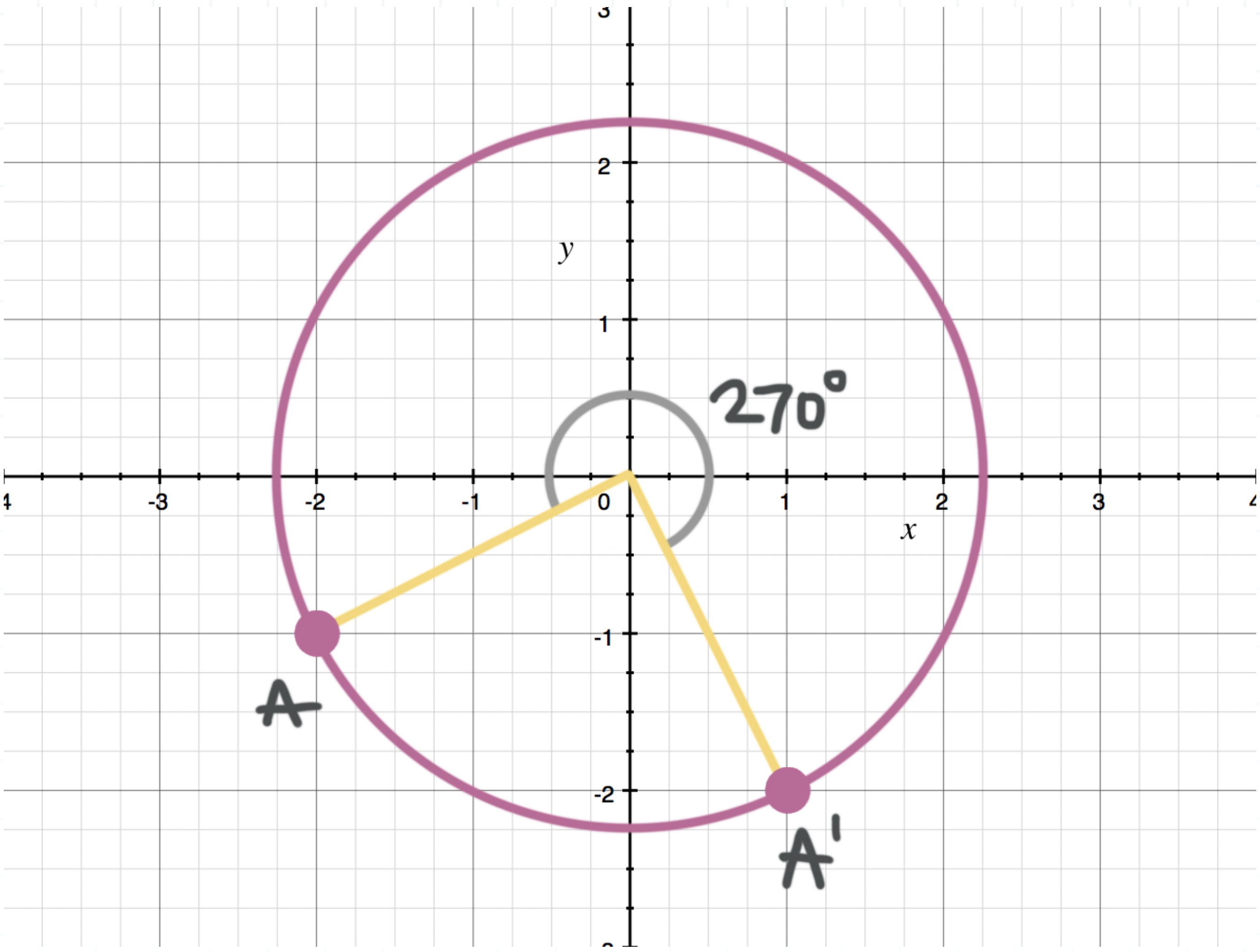
To what point A' will point A be moved in a 270° clockwise rotation around the origin?



Look up the rule for a rotation of 270° clockwise around the origin.

Angle	Direction	Rule
270 degrees	Clockwise	(x, y) --> (-y, x)

Point A is located at $(-2,-1)$, and we use the rule $(x,y) \to (-y,x)$, so $(-2,-1) \to (1,-2)$. The figure shows what happens to point A in a 270° clockwise rotation around the origin: It's moved to point $A' = (1,-2)$.


270°
A
A'
x
y