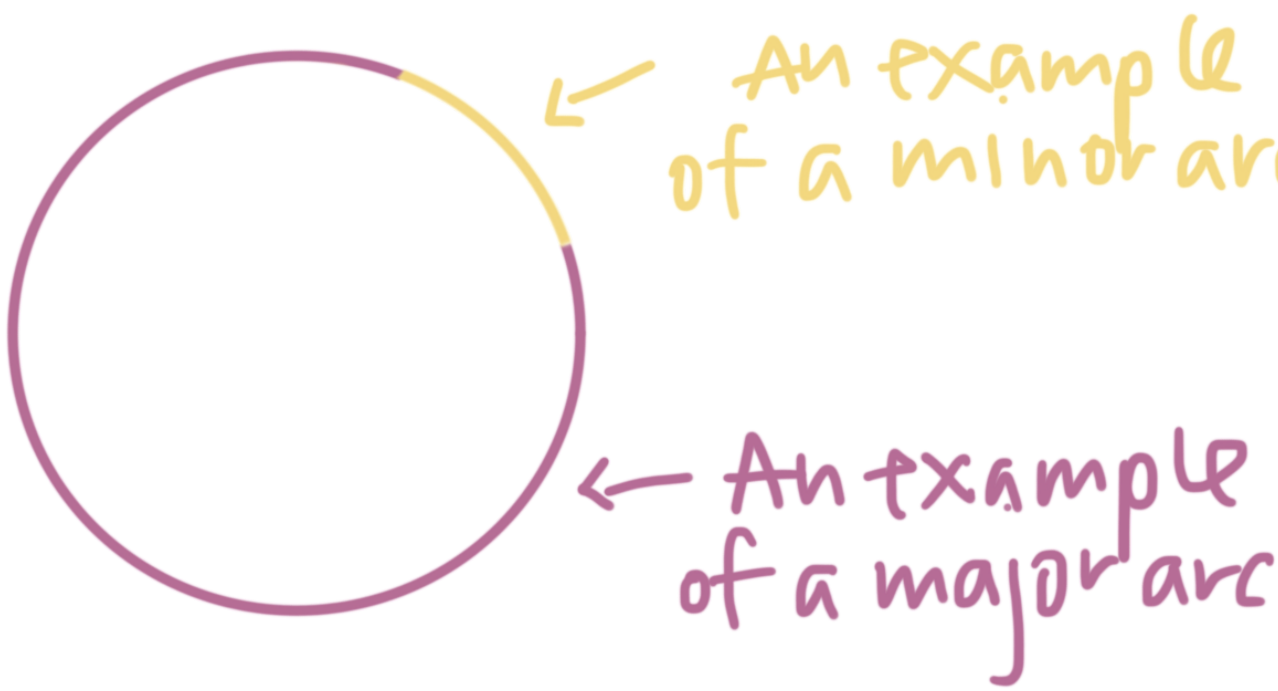
Degree measure of an arc

In this lesson we'll look at arcs of circles and how to find their degree measure. Arcs also have length, but we won't consider that in this lesson, so we'll use "measure" to mean "degree measure."

Arcs

An **arc** is a continuous part of a circle (a part that has no holes in it). There are three types of arcs.

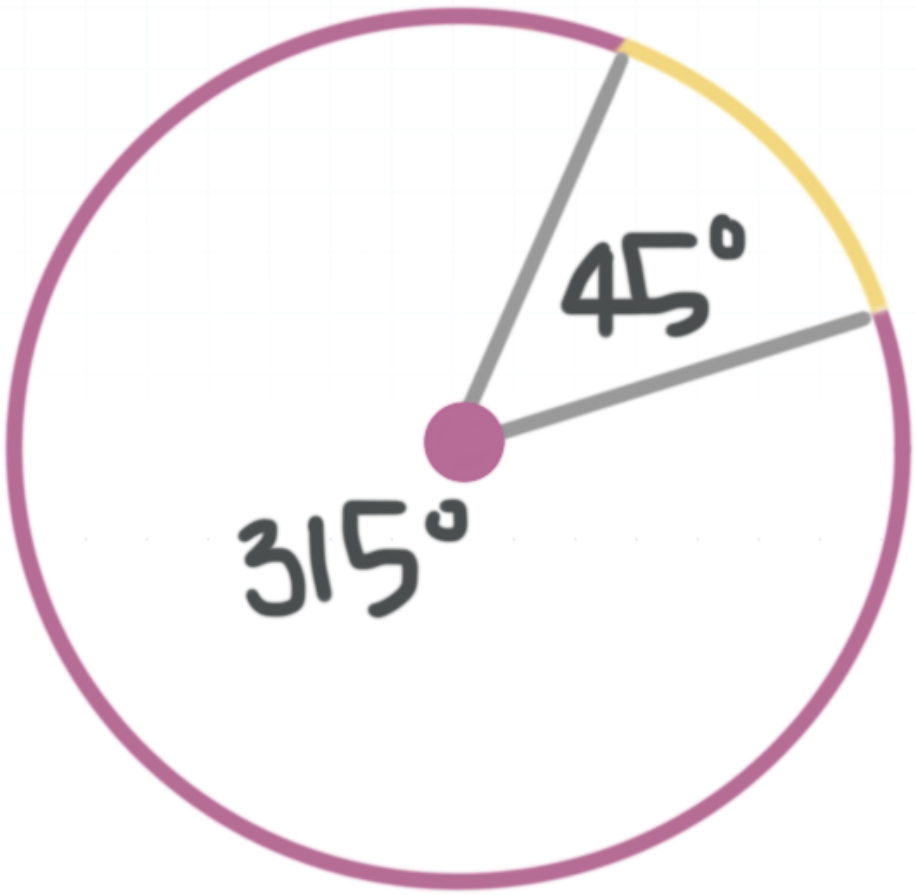
1. **Minor arcs** have measure less than 180°.
2. **Major arcs** have measure more than 180°.
3. **Semicircular arcs** (also called **semicircles**) consist of half a circle and have measure of exactly 180°.



Measures of arcs and central angles

A **central angle** of a circle is an angle whose vertex is at the center of the circle. The central angle and the corresponding arc (the arc whose endpoints are the points of intersection of the sides of the central angle with the circle) have the same measure.

Here the measure of the central angle that corresponds to the minor arc is 45°, so the measure of the minor arc is 45°.



The measure of the central angle that corresponds to the major arc is 315°, so the measure of the major arc is 315°. Notice that the sum of the measures of a pair of central angles of a circle that correspond to a pair of arcs which (together) make up the entire circle (but intersect only at their endpoints) is 360°.

Naming arcs and their measures

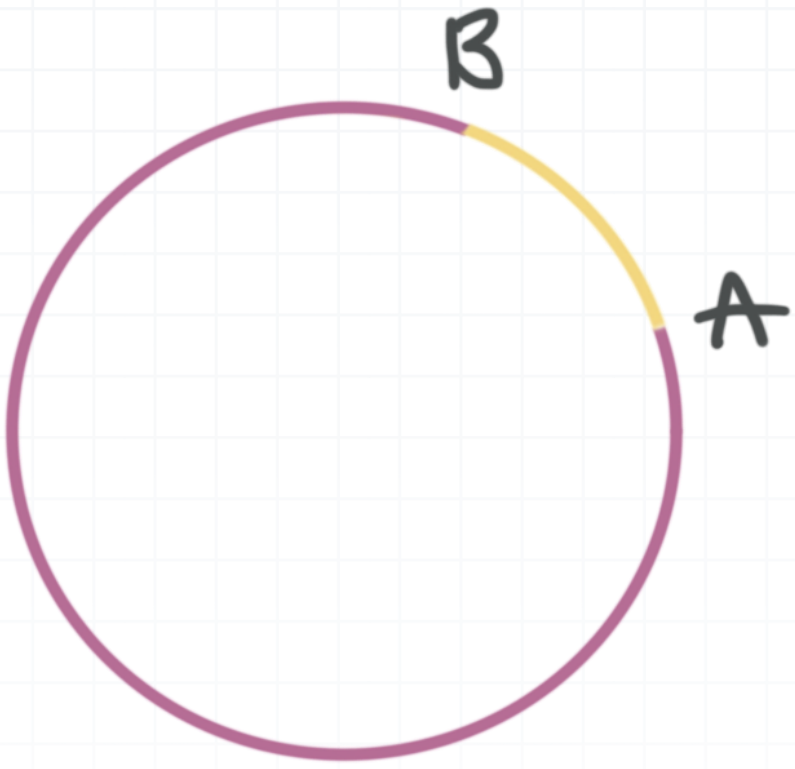
You can name an arc by the letters that name its endpoints.

Notice that you could take a pencil, place it at one of the endpoints of an arc, and "trace out" the arc until you get to the other endpoint. However, you could choose to go around the circle in either direction - clockwise or counterclockwise, when you do the tracing, which would give you two different arcs.

Therefore, the endpoints of the arc don't uniquely determine the arc. If you ever study trigonometry, you'll learn that an arc with a positive measure is traced out in the counterclockwise direction, while an arc with a negative measure is traced out in the clockwise direction.

In this course, we'll only deal with angles of positive measure, so we'll always name an arc in such a way that the first letter in its name is the endpoint you'd start from if you were to trace the arc by moving counterclockwise around the circle. We'll name the corresponding central angle the same way, where the first letter in the name of the arc is the starting point, and the last letter in the name of the arc is the endpoint point, after tracing out the arc in a counterclockwise direction.

For example, the yellow arc shown below is arc AB. Instead of writing the word "arc," we can use a curved symbol over the letters to indicate the arc: $\overset{\frown}{AB}$.



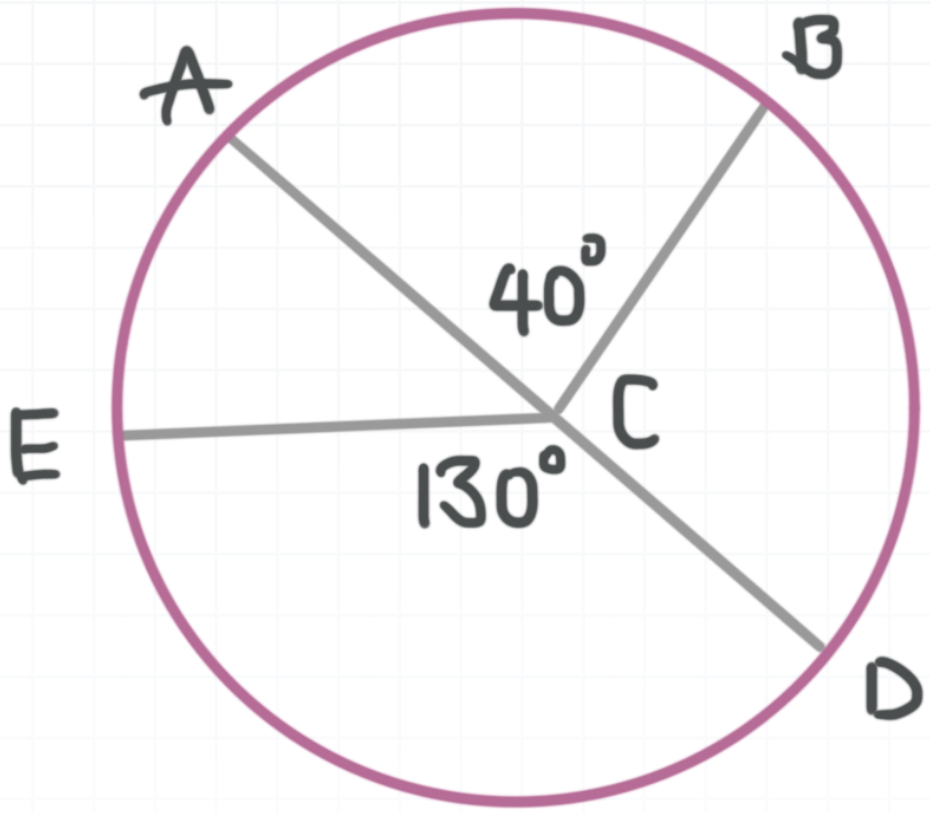
To indicate the measure of an arc, you can use an m or the word "measure." In this case we can say the measure of arc AB is $45°$. If we used symbols instead of words, we could write $m\overset{\frown}{AB} = 45°$.

Let's look at a few examples.

Example

If $\overline{AD}$ is a diameter of the circle (with center at C) in the figure, what is the difference between the measures of $\overset{\frown}{DB}$ and arc $\overset{\frown}{AE}$?

Note: A **diameter** is any line segment that passes through the center of a circle and has both of its endpoints on the circle. Notice that the diameter is equal to twice the radius, and that the diameter splits a circle into two semicircles. In this problem, $\overline{AD}$ is a diameter that splits the circle into semicircles AD and DA.



$\overline{AD}$ is a diameter of the circle, which means that the sum of the measures of arc DB and arc BA is $180°$.

$$m\overset{\frown}{DB} + m\overset{\frown}{BA} = 180°$$

$$m\overset{\frown}{DB} + 40° = 180°$$

$$m\overset{\frown}{DB} = 140°$$

Likewise, the sum of the measures of $\overset{\frown}{AE}$ and $\overset{\frown}{ED}$ is $180°$.

$$m\overset{\frown}{AE} + m\overset{\frown}{ED} = 180°$$

$$m\overset{\frown}{AE} + 130° = 180°$$

$$m\overset{\frown}{AE} = 50°$$

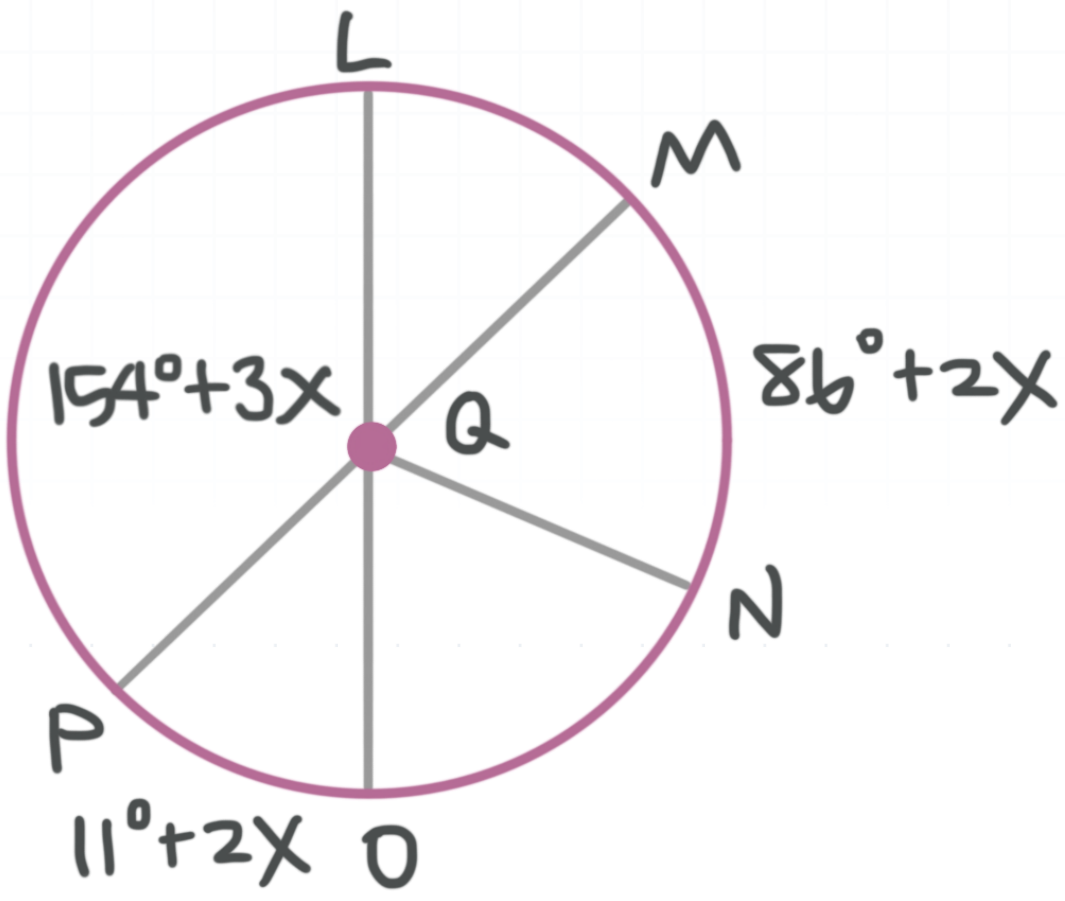
The difference between the measures of arcs $\overset{\frown}{DB}$ and $\overset{\frown}{AE}$ is

$$140^\circ - 50^\circ = 90^\circ$$

Let's look at one more example.

Example

What is the measure of $\overset{\frown}{NM}$, given that $\overline{LO}$ is a diameter of the circle (with center at Q) in the figure?



We need to solve for x and then plug it back into the expression for the measure of $\overset{\frown}{NM}$. We know that $\overline{LO}$ is a diameter of the circle, so the sum of the measures of arc LP and arc PO is 180°.

We can use this information to find the value of x.

$$m\widehat{LP} + m\widehat{PO} = 180^\circ$$

Central angle LQP has measure $154^\circ + 3x$, which means that $\widehat{LP}$ also has measure $154^\circ + 3x$. Therefore,

$$(154^\circ + 3x) + (11^\circ + 2x) = 180^\circ$$

$$165^\circ + 5x = 180^\circ$$

$$5x = 15^\circ$$

$$x = 3^\circ$$

Now we can find the measure of $\widehat{NM}$.

$$m\widehat{NM} = 86^\circ + 2x$$

$$m\widehat{NM} = 86^\circ + 2(3^\circ)$$

$$m\widehat{NM} = 92^\circ$$