Topic: Chain rule with product rule

Question: Apply product rule and chain rule to find the derivative.

$$y = (4x - 7)^2(2x + 3)$$

Answer choices:

A $y' = (4x - 7)(12x + 5)$

B $y' = 2(4x - 7)(2x + 3)$

C $y' = 2(4x - 7)(12x + 5)$

D $y' = 2(4x - 7)^3(12x + 5)$

Solution: C

Set $f(x) = (4x - 7)^2$ and $g(x) = 2x + 3$. Then

$$f(x) = (4x - 7)^2$$

$$f'(x) = 2(4x - 7)(4)$$

$$f'(x) = 8(4x - 7)$$

and

$$g(x) = 2x + 3$$

$$g'(x) = 2$$

Now we can apply product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = ((4x - 7)^2)(2) + (8(4x - 7))(2x + 3)$$

The two terms $2(4x - 7)^2$ and $8(4x - 7)(2x + 3)$ share a common factor of $2(4x - 7)$, so factor that out.

$$y' = 2(4x - 7)\big[(4x - 7) + 4(2x + 3)\big]$$

$$y' = 2(4x - 7)(4x - 7 + 8x + 12)$$

$$y' = 2(4x - 7)(12x + 5)$$

Topic: Chain rule with product rule

Question: Apply product rule and chain rule to find the derivative.

$$y = 2 \sin x^2 \sec(2x^3 + 3)$$

Answer choices:

A $y' = 12x^2 \sin x^2 \sec(2x^3 + 3)\tan(2x^3 + 3) + 4x \cos x^2 \sec(2x^3 + 3)$

B $y' = 2 \sin x^2 \sec(2x^3 + 3)\tan(2x^3 + 3) + 2x \cos x \sec(2x^3 + 3)$

C $y' = 12x^2 \sin x^2 \sec(2x^3 + 3)\tan(2x^3 + 3) + 4x \cos x \sec(2x^3 + 3)$

D $y' = 12x^2 \sin x^2 \sec(2x^3 + 3) + 4x \sin x^2 \sec(2x^3 + 3)$

Solution: A

Use the product rule with

$$f(x) = 2\sin x^2$$

$$f'(x) = 2\cos x^2(2x)$$

$$f'(x) = 4x\cos x^2$$

and

$$g(x) = \sec(2x^3 + 3)$$

$$g'(x) = \sec(2x^3 + 3)\tan(2x^3 + 3)(6x^2)$$

$$g'(x) = 6x^2\sec(2x^3 + 3)\tan(2x^3 + 3)$$

Then the derivative is

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (2\sin x^2)(6x^2\sec(2x^3 + 3)\tan(2x^3 + 3)) + (4x\cos x^2)(\sec(2x^3 + 3))$$

$$y' = 12x^2\sin x^2\sec(2x^3 + 3)\tan(2x^3 + 3) + 4x\cos x^2\sec(2x^3 + 3)$$

Topic: Chain rule with product rule

Question: Find the derivative of the exponential function.

$$y = 4xe^{5x^2-2}$$

Answer choices:

A $y' = 4e^{5x^2-2}(5x + 1)$

B $y' = 4e^{5x^2-2}(5x^2 + 1)$

C $y' = 4e^{5x^2-2}(10x^2 + 1)$

D $y' = 4e^{5x^2-2}(10x + 1)$

Solution: C

We'll apply product rule with

$$f(x) = 4x$$

$$f'(x) = 4$$

and

$$g(x) = e^{5x^2-2}$$

$$g'(x) = 10xe^{5x^2-2}$$

Then the derivative is

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (4x)(10xe^{5x^2-2}) + (4)(e^{5x^2-2})$$

$$y' = 40x^2e^{5x^2-2} + 4e^{5x^2-2}$$

The terms in the denominator share a common factor of $4e^{5x^2-2}$, so factor that out.

$$y' = 4e^{5x^2-2}(10x^2 + 1)$$