Geometry Workbook Solutions

Area and perimeter

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AREA OF A RECTANGLE

■ 1. The base of a rectangle is 8 feet. Find its height if the area of the rectangle is 80 ft^2.

Solution:

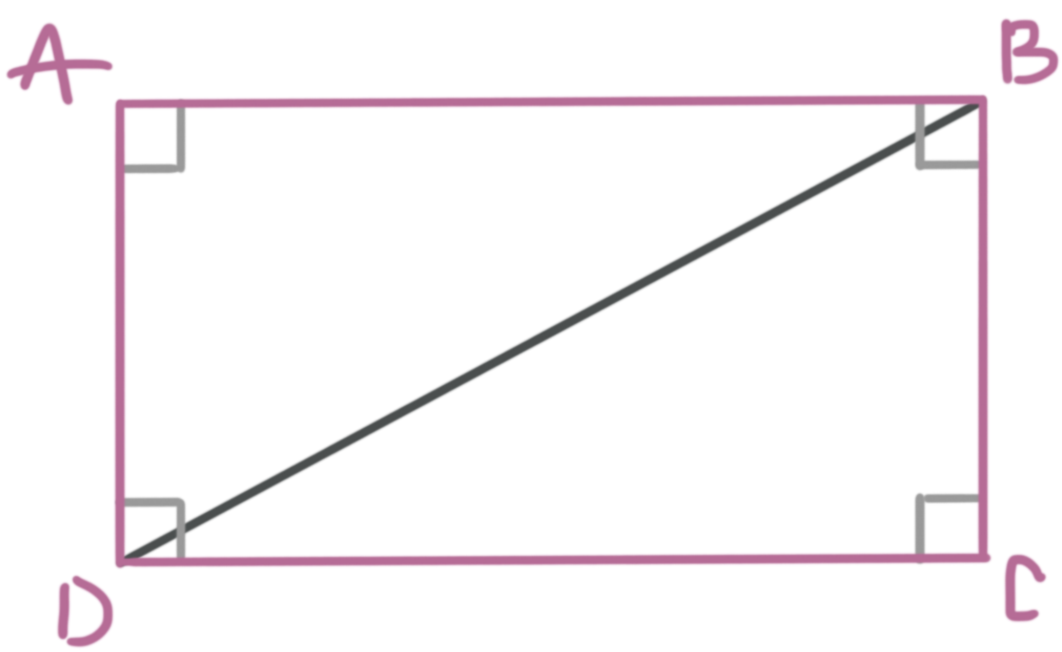
We can use the formula for the area of the rectangle to plug in everything we know, and then solve for the height.

$$A = bh$$

$$80 = 8h$$

$$h = 10 \text{ feet}$$

■ 2. In rectangle $ABCD$, $BD = 13$ and $AB = 12$. Find the area of this rectangle.



Solution:

Use the Pythagorean Theorem to find the length of $\overline{AD}$.

$$AD^2 + AB^2 = BD^2$$

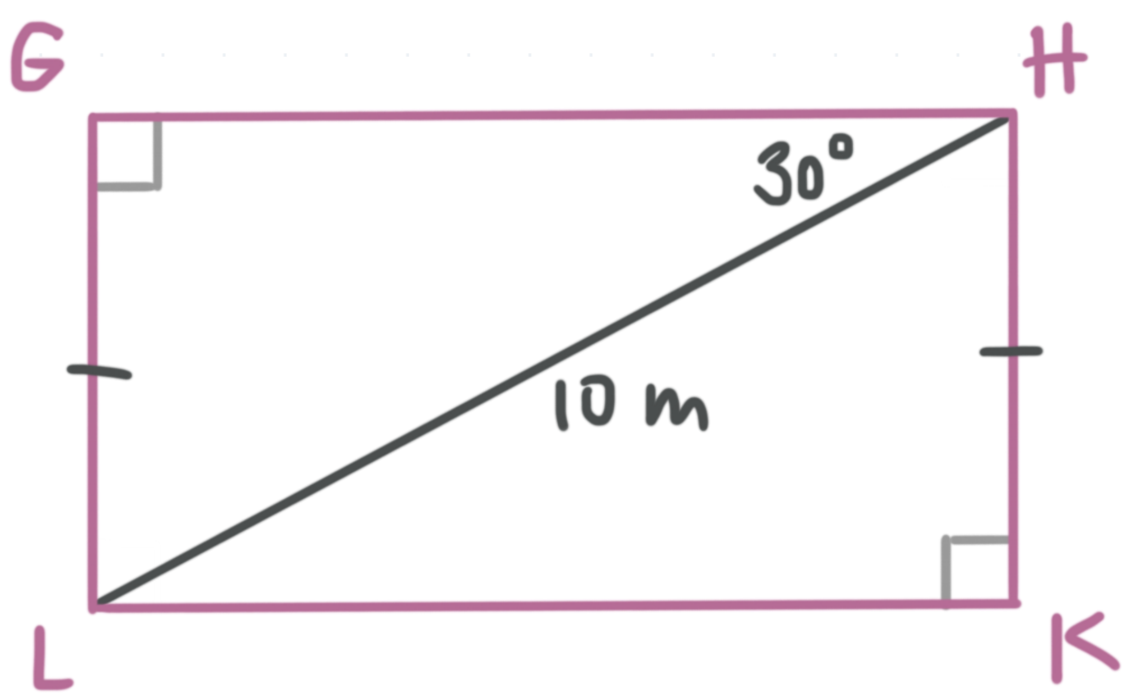
$$AD^2 + 12^2 = 13^2$$

$$AD^2 = 25$$

$$AD = 5$$

Therefore, the area of the rectangle is $A = bh = (5)(12) = 60$.

■ 3. In rectangle $GHKL$, $LH = 10$ and $m\angle GHL = 30$. Find the exact area of the rectangle.



Solution:

$\triangle GHL$ is a special right triangle with degree measures $30-60-90$. The diagonal of the rectangle is 10 and is the hypotenuse of $\triangle GHL$. The shortest leg of the triangle is $\overline{GL}$ and this side is half the length of the hypotenuse. $GL=5$ and $\overline{GH}$ is the product of the shorter leg and $\sqrt{3}$. Therefore, $GH=5\sqrt{3}$. The area of the rectangle is

$$A=bh=(5)\left(5\sqrt{3}\right)=25\sqrt{3}$$

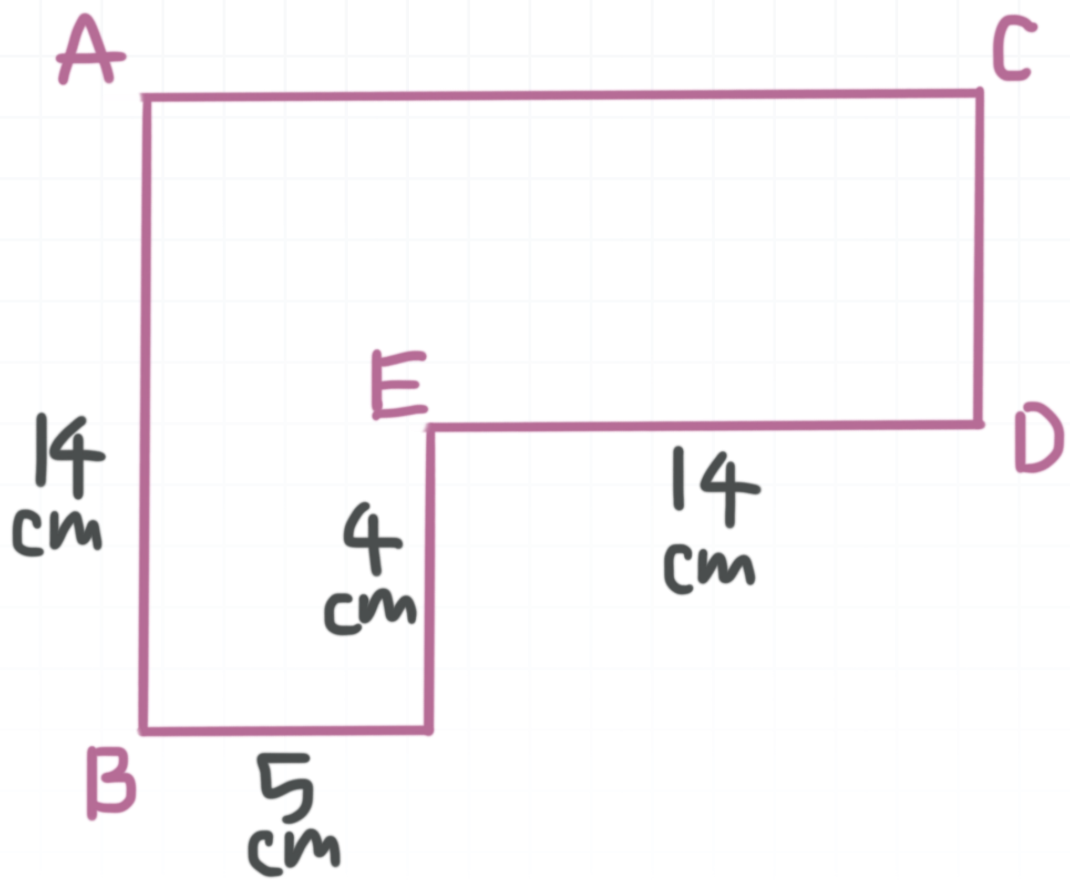
■ 4. The area of a small square flower garden is 49 ft^2. Suppose we wish to make the garden bigger by adding 6 feet to one of the sides. How much more square footage is available in this new rectangular garden?

Solution:

The original square garden has dimensions 7 feet by 7 feet. By adding 6 feet onto one of the sides, we get a new rectangle with dimensions 13 feet by 7 feet. The new garden has an area of $(13)(7)=91$ ft^2. To find the area gained, take $91-49=42$ ft^2.

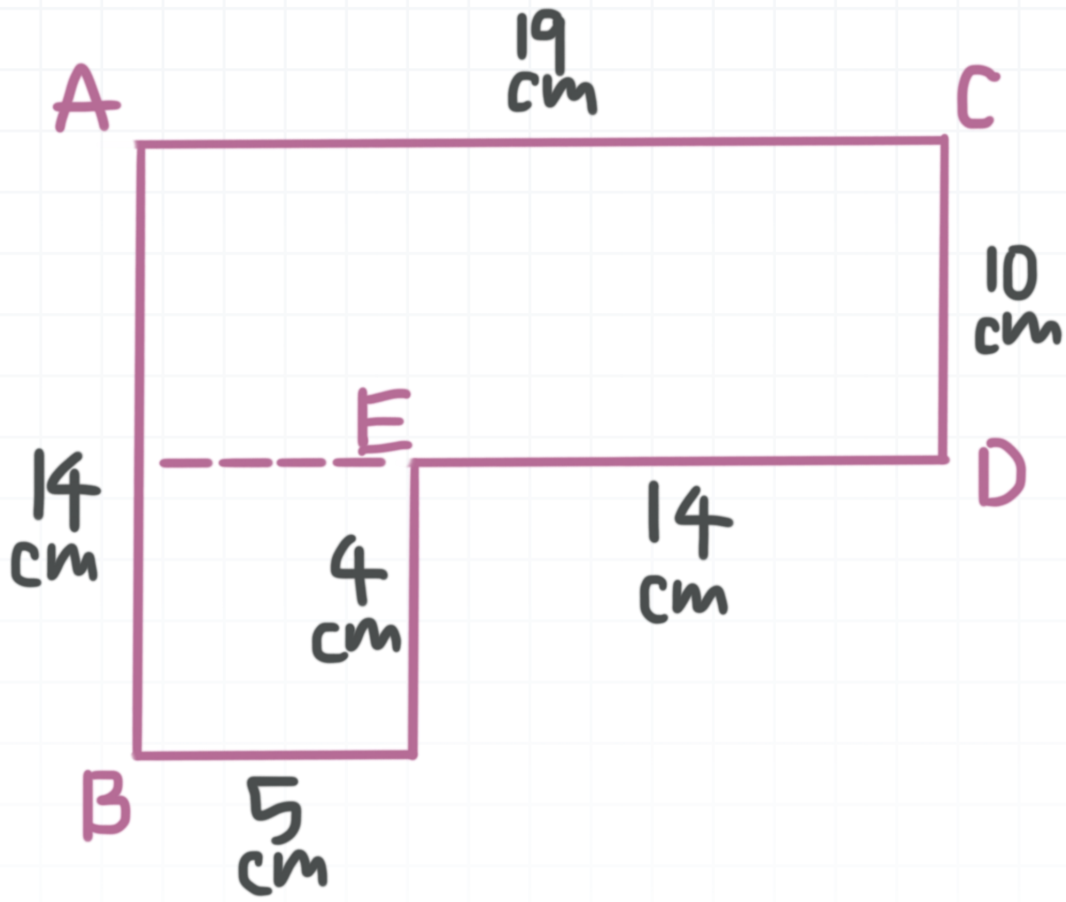
AREA OF A RECTANGLE USING SUMS AND DIFFERENCES

■ 1. Find the area of the figure.



Solution:

Segment the figure into two rectangles, and fill out the rest of the figure.



The area of the larger rectangle is $A_1 = lw = (19)(10) = 190$ cm^2.

The area of the smaller rectangle is $A_2 = lw = (5)(4) = 20$ cm^2.

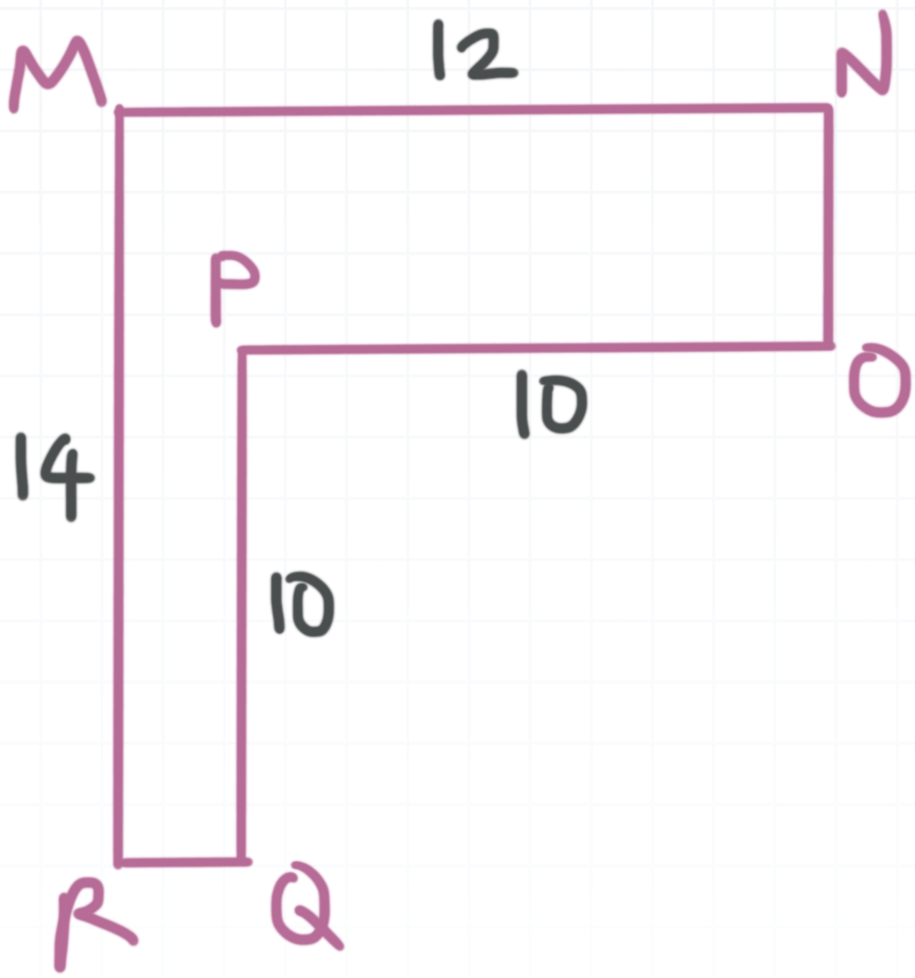
Then the area of the whole figure is

$$A = A_1 + A_2$$

$$A = 190 \text{ cm}^2 + 20 \text{ cm}^2$$

$$A = 210 \text{ cm}^2$$

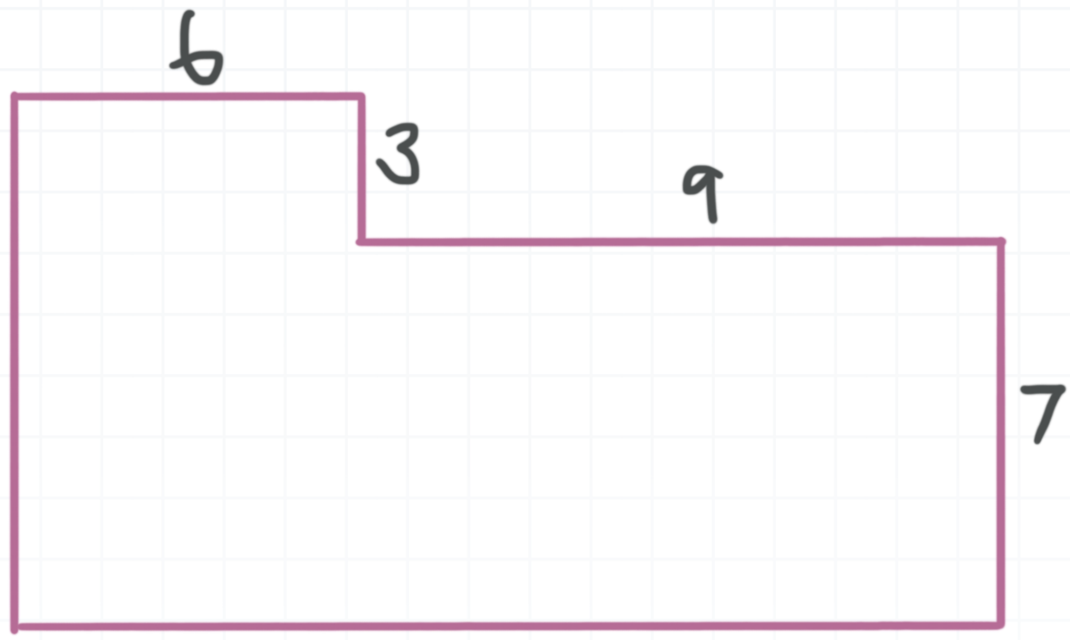
■ 2. Find the area of the figure.



Solution:

The area of larger rectangle with three vertices at M, N, and R is $A=(14)(12)=168$. The area of the smaller rectangle with three vertices at P, O, and Q is $A=(10)(10)=100$. Using the difference method, the area of the figure is $168-100=68$.

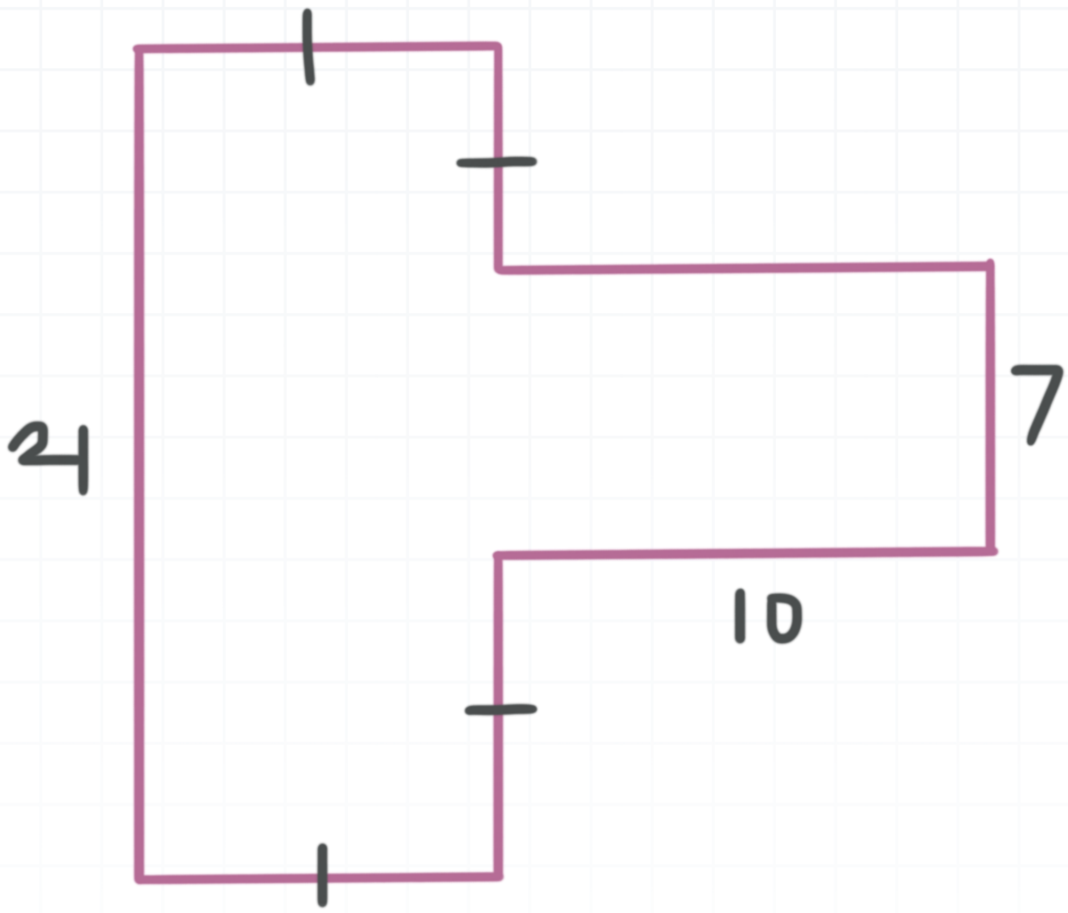
■ 3. Find the area of the figure.



Solution:

The area of the little rectangle in the upper left is $A = (6)(3) = 18$. The area of the larger rectangle at the bottom is $A = (6 + 9)(7) = 105$. Using the sum method, the area of the figure is $18 + 105 = 123$.

■ 4. Find the area of the figure.



Solution:

The area of the rectangle on the left is $A = (21)(7) = 147$. The area of the rectangle on the right is $A = 10(7) = 70$. Using the sum method, the area of the figure is $A = 147 + 70 = 217$.

PERIMETER OF A RECTANGLE

■ 1. A rectangle has a base of 10 meters. The height is 4 meters greater than the base. Find the perimeter of this rectangle.

Solution:

By the formula for the perimeter of a rectangle, we get

$$P = 2b + 2h$$

$$P = 2(10) + 2(10 + 4)$$

$$P = 20 + 18$$

$$P = 48$$

■ 2. The area of a rectangle is 40 ft^2. Find the perimeter of this rectangle if the length of the rectangle is 3 feet longer than the width.

Solution:

First, we'll write the equation for the area and plug in what we know.

$$A = bh$$

$$40 = b(b+3)$$

$$40 = b^2 + 3b$$

$$0 = b^2 + 3b - 40$$

$$0 = (b+8)(b-5)$$

$$b = -8, 5$$

The base of the rectangle can't be defined by a negative number, so the base must be 5 feet long. The height is therefore $h = 5 + 3 = 8$ feet, and the perimeter is

$$p = 2b + 2h$$

$$p = 2(5) + 2(8)$$

$$p = 10 + 16$$

$$p = 26$$

■ 3. Find the perimeter of a rectangle with vertices at $A(-3,0)$, $B(0,4)$, $C(4,1)$, and $D(1,-3)$.

Solution:

We need to use the distance formula to calculate the distance between adjacent points, which will give us the length of each side of the rectangle.

$$d_{AB} = \sqrt{(4-0)^2 + (0-(-3))^2} = 5$$

$$d_{BC} = \sqrt{(1-4)^2 + (4-0))^2} = 5$$

$$d_{CD} = \sqrt{((-3)-1)^2 + (1-4)^2} = 5$$

$$d_{AD} = \sqrt{(-3)-0)^2 + (1-(-3))^2} = 5$$

Therefore, the perimeter is

$$p = AB + BC + CD + AD$$

$$p = 5 + 5 + 5 + 5$$

$$p = 20$$

■ 4. Find the value of x if the base of the rectangle has length $x + 4$, the height of the rectangle is x, and the perimeter of a rectangle is 20 units.

Solution:

Plug what you know into the formula for the perimeter of a rectangle.

$$P = 2b + 2h$$

$$20 = 2(x + 4) + 2(x)$$

$$20 = 2x + 8 + 2x$$

$$20 = 4x + 8$$

$$12 = 4x$$

$$x = 3$$

AREA OF A PARALLELOGRAM

■ 1. Find the area of a parallelogram with $b=14$ yards and $h=10$ yards.

Solution:

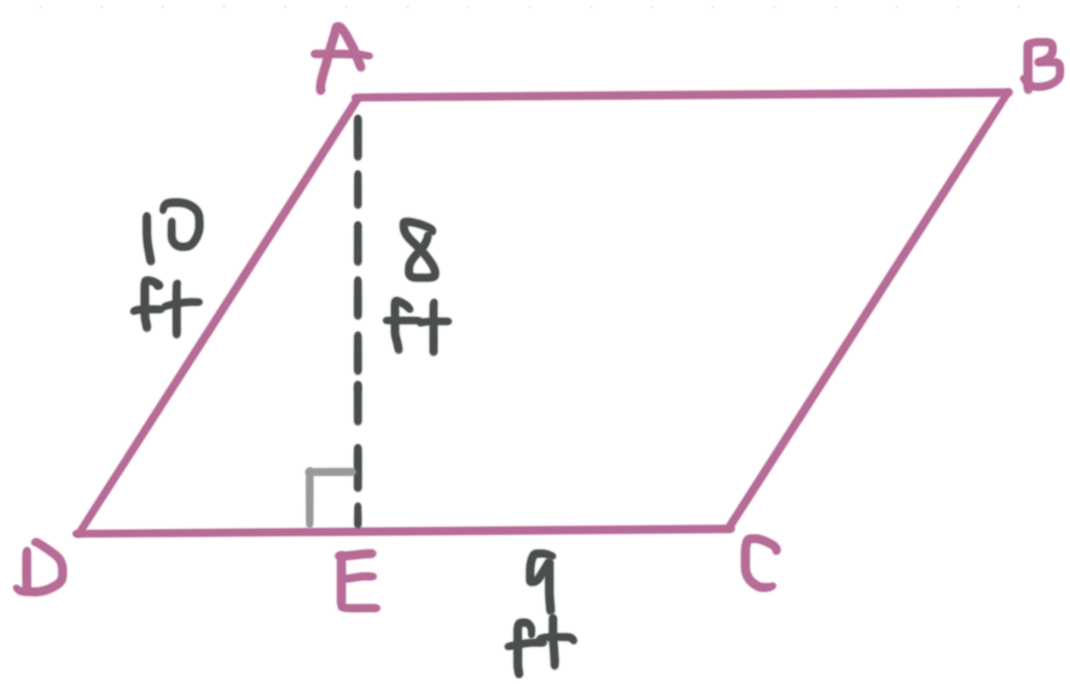
The area of a parallelogram is given by the product of its base and height.

$$A=bh$$

$$A=(14)(10)$$

$$A=140\text{ yd}^2$$

■ 2. Find the area of the parallelogram.



Solution:

Find the missing side, ED, of the right triangle using Pythagorean Theorem.

$$ED^2 + 8^2 = 10^2$$

$$ED = 6$$

Find the length of the base of the parallelogram, $\overline{DC}$.

$$DC = 6 + 9$$

Then the area is

$$A = bh = (15)(8) = 120$$

■ 3. Find the area of parallelogram $JKLM$, if $J(0,0)$, $K(1,3)$, $L(-5,3)$, and $M(-6,0)$.

Solution:

Graph the parallelogram and find the base and height. The base is $b = 6$ and the height is $h = 3$. Then the area is

$$A = bh = (6)(3) = 18$$

■ 4. A parallelogram has a base that is 3 feet longer than it is tall. The area of the parallelogram is 88 square feet. Find the height of the parallelogram.

Solution:

Using the equation for area, we can find the height.

$$A = bh$$

$$88 = (h + 3)(h)$$

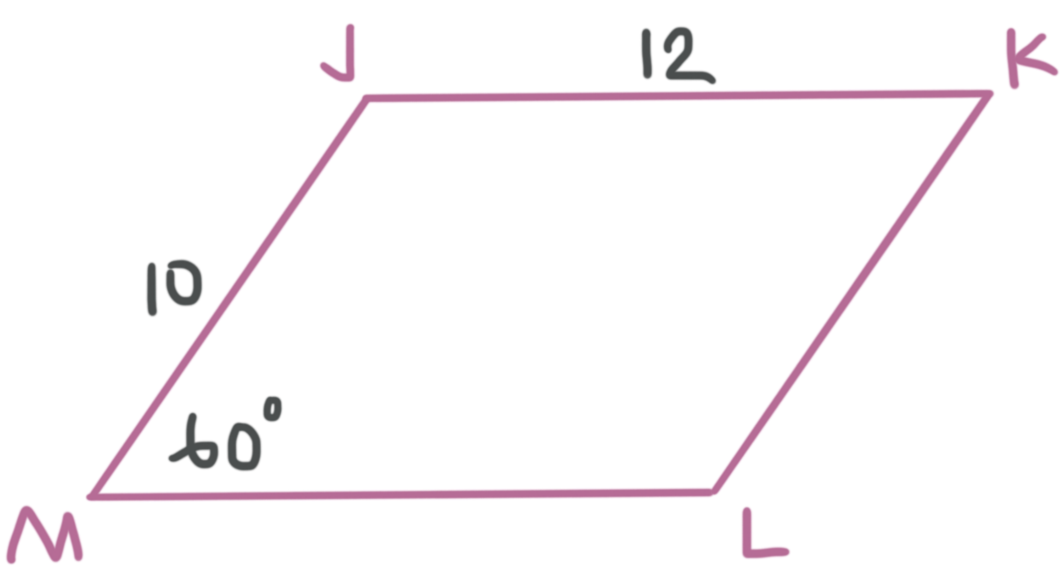
$$88 = h^2 + 3h$$

$$0 = h^2 + 3h - 88$$

$$0 = (h + 11)(h - 8)$$

$$h = 8$$

■ 5. Find the exact area of the parallelogram.



Solution:

The height forms a right angle with the base. A $30-60-90$ triangle is formed with 10 as its hypotenuse. The height can be found by applying $30-60-90$ rules to get $h=5\sqrt{3}$. Then the area is the parallelogram is

$$A=bh=12(5\sqrt{3})=60\sqrt{3}$$

AREA OF A TRAPEZOID

■ 1. Find the area of a trapezoid with base lengths 16 and 18, and height 10.

Solution:

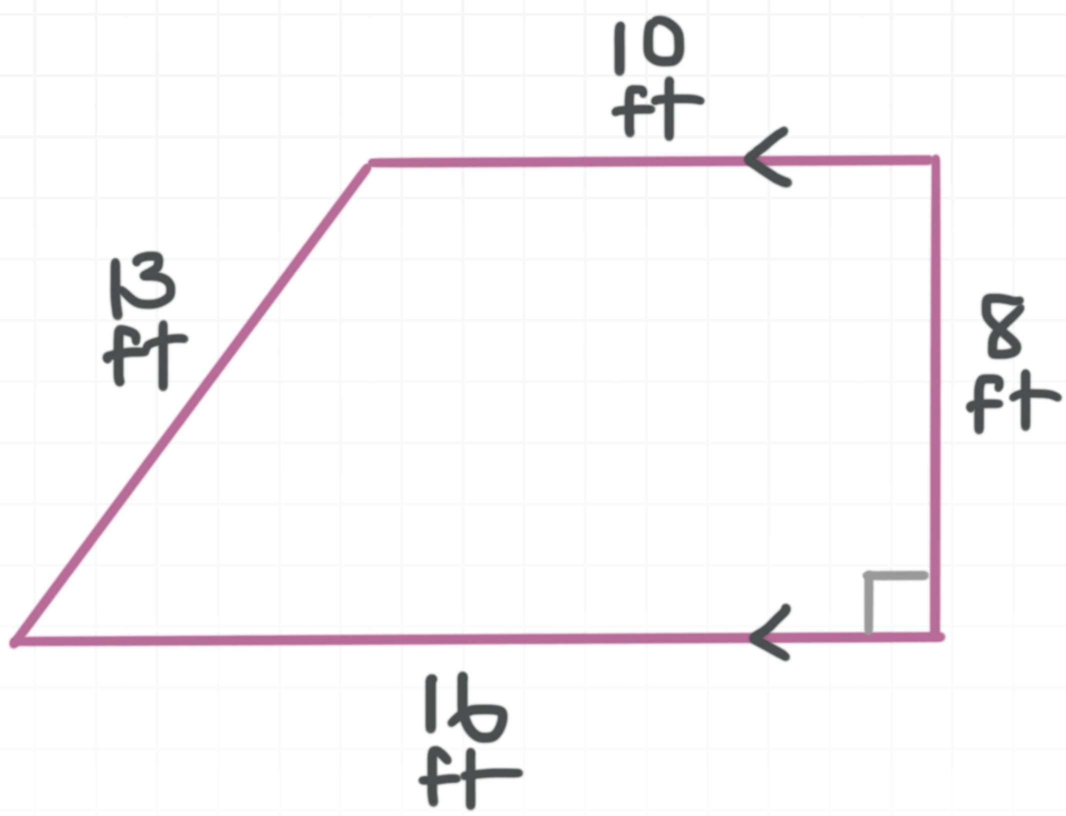
If we plug what we've been given into the formula for the area of a trapezoid, we get

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(10)(16 + 18)$$

$$A = 170$$

■ 2. Find the area of the trapezoid.



Solution:

If we plug what we've been given into the formula for the area of a trapezoid, we get

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(8)(16 + 10)$$

$$A = 104$$

■ 3. Find the exact area of the trapezoid that has congruent 2-meter bases and a height of 4 meters.

Solution:

The area of the trapezoid is

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(4)(2 + 2)$$

$$A = \frac{1}{2}(16)$$

$$A = 8$$

■ 4. The area of a trapezoid is 60 m^2. One of the bases has a measure of 7 m and the height of the trapezoid is 10 m. Find the length of the other base.

Solution:

We can plug what we know into the formula for the area of a trapezoid, and then solve for the length of the second base.

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$60 = \frac{1}{2}(10)(7 + \text{base}_2)$$

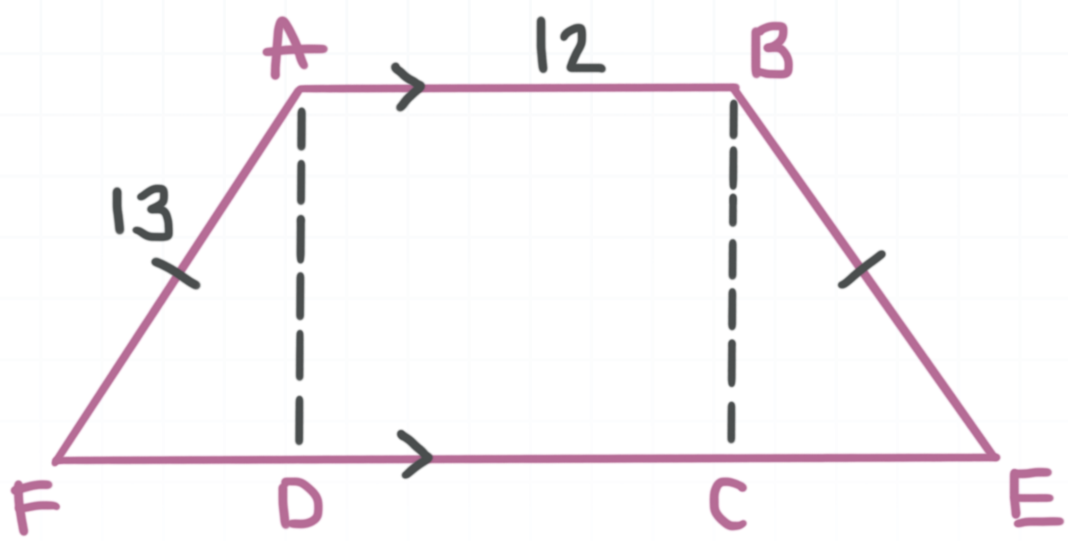
$$120 = 10(7 + \text{base}_2)$$

$$120 = 70 + 10\text{base}_2$$

$$50 = 10\text{base}_2$$

$$\text{base}_2 = 5$$

■ 5. Find the area of trapezoid $ABEF$, if $ABCD$ is a square.



Solution:

We know that $AB = AD$ because $ABCD$ is a square. The height of the trapezoid is 12. Use the Pythagorean Theorem to find FD and CE.

$$FD^2 + AD^2 = AF^2$$

$$FD^2 + 12^2 = 13^2$$

$$FD = 5 = CE$$

Then $\overline{FE} = 5 + 12 + 5 = 22$. Therefore, the area of the trapezoid is

$$A = \frac{1}{2}(\text{height})(\text{base}_1 + \text{base}_2)$$

$$A = \frac{1}{2}(12)(22 + 12)$$

$$A = 204$$

AREA OF A TRIANGLE

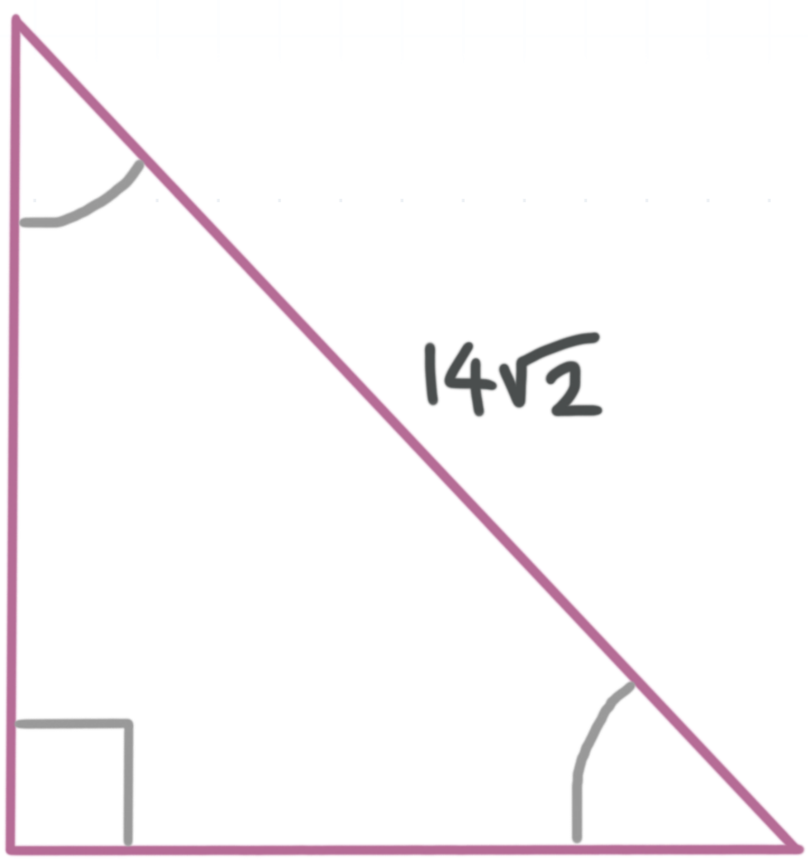
■ 1. Find the area of a triangle that has base length 16 and height 14.

Solution:

The area of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(16)(14) = 112$$

■ 2. Find the area of the triangle.

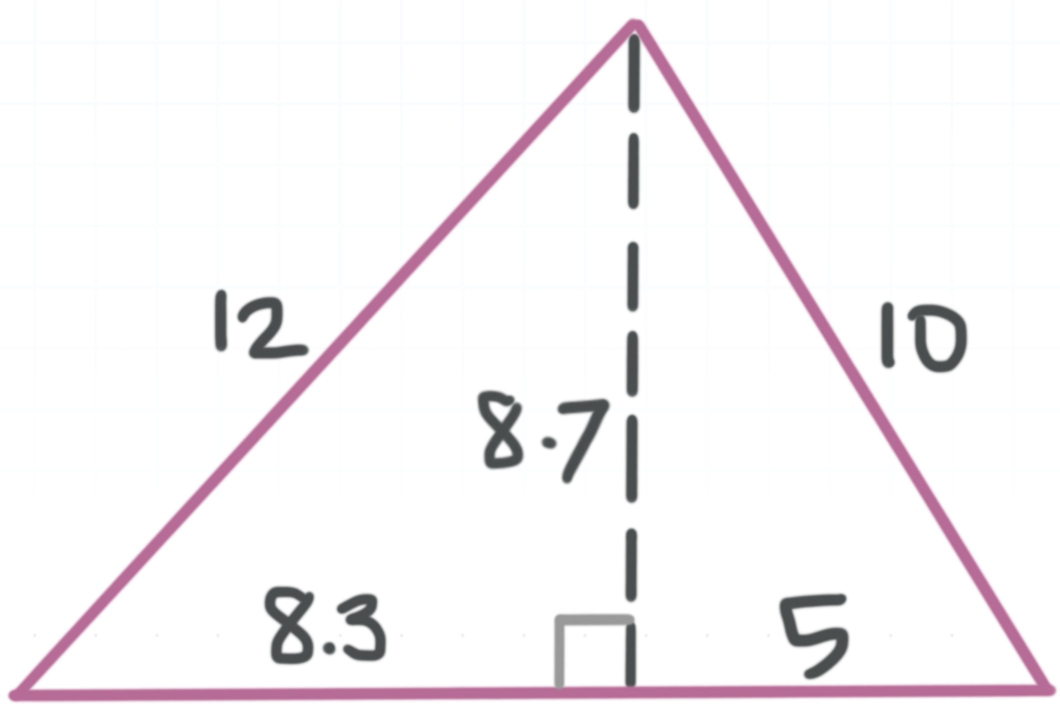


Solution:

Using $45-45-90$ rules, we find that the base of the triangle has length $b=14$ and height $h=14$. Then the area of the triangle is

$$A=\frac{1}{2}bh=\frac{1}{2}(14)(14)=98$$

■ 3. Find the area of the triangle.

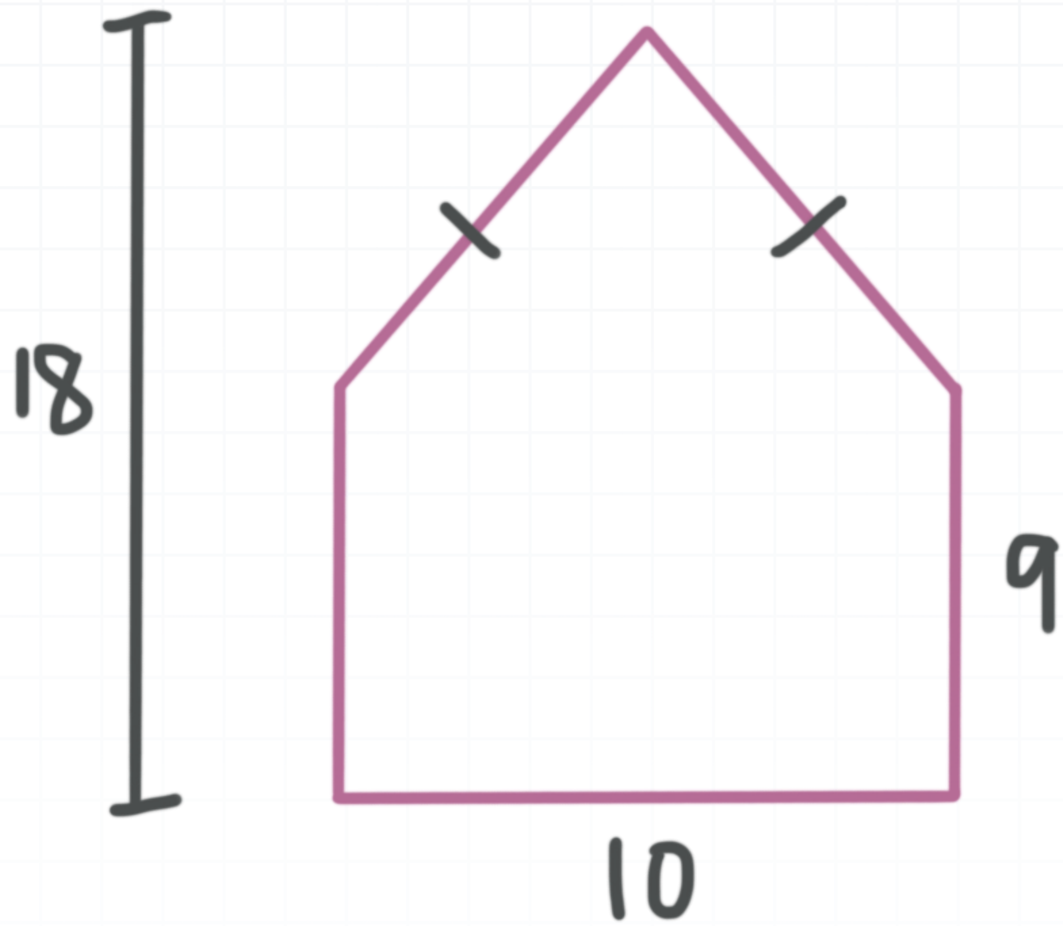


Solution:

The area of the triangle is

$$A=\frac{1}{2}bh=\frac{1}{2}(13.3)(8.7)=57.855$$

4. Find the area of the figure below.



Solution:

The area of the rectangle is

$$A_R = bh = (10)(9) = 90$$

The area of the triangle is

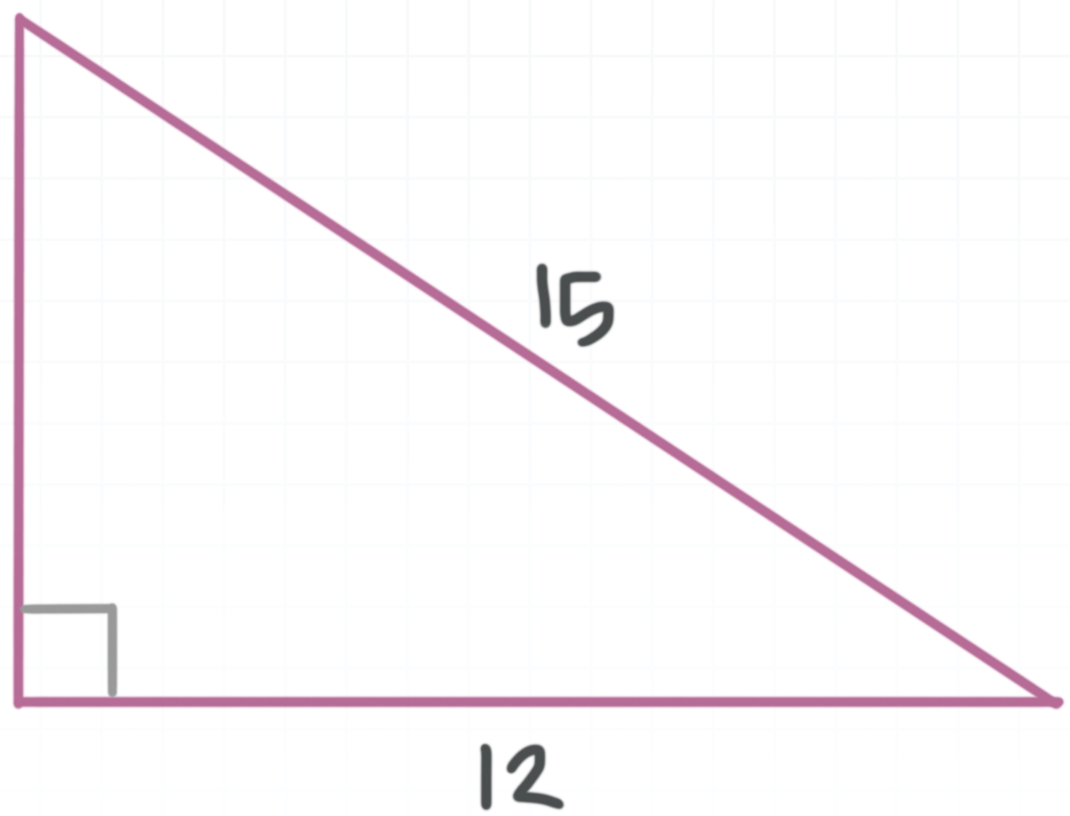
$$A_T = \frac{1}{2}(bh) = \frac{1}{2}(10)(9) = 45$$

Therefore, the area of the entire region is

$$A = A_R + A_T = 90 + 45 = 135$$

PERIMETER OF A TRIANGLE

■ 1. Find the perimeter of the triangle.



Solution:

Let the missing side be x.

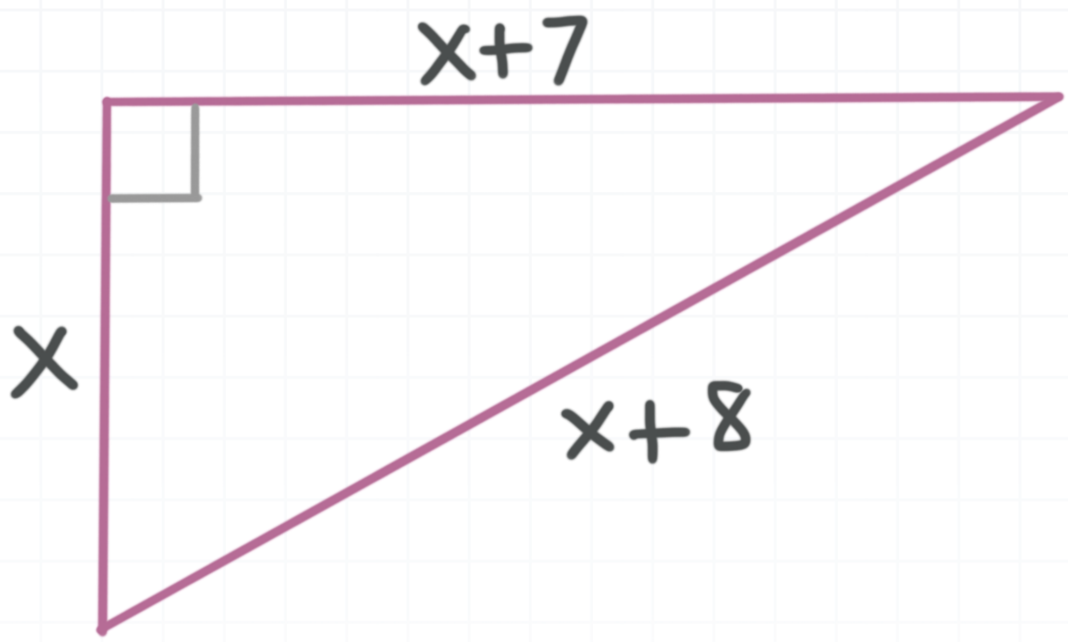
$$x^2 + 12^2 = 15^2$$

$$x^2 = 15^2 - 12^2 = 81$$

$$x = \sqrt{81} = 9$$

The perimeter of the triangle is $9 + 12 + 15 = 36$.

■ 2. Find the perimeter of the triangle.



Solution:

The perimeter can be found by plugging the side lengths into the Pythagorean Theorem.

$$x^2 + (x+7)^2 = (x+8)^2$$

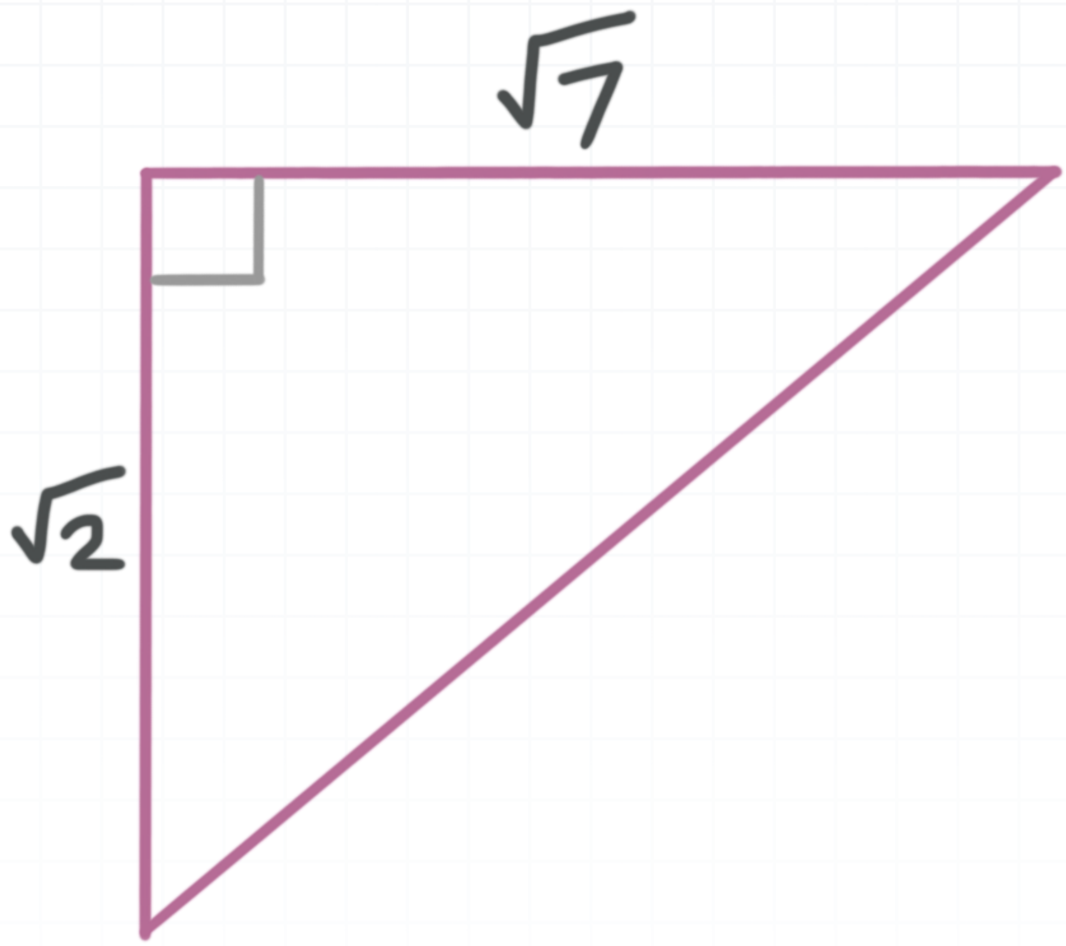
$$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

So $x = 5$ or $x = -3$. But one of the legs of the triangle is x, which means x cannot have a negative value, because that would mean we'd have a negative side length. Therefore, $x = 5$ and the side lengths must be 5, 12, and 13. Which means the perimeter of the triangle is $5 + 12 + 13 = 30$.

3. Find the exact perimeter of the triangle.



Solution:

Plug the side lengths into the Pythagorean Theorem to find the length of the hypotenuse.

$$(\sqrt{2})^2 + (\sqrt{7})^2 = c^2$$

$$2 + 7 = c^2$$

$$c^2 = 9$$

$$c = \sqrt{9} = 3$$

Therefore, the perimeter of the triangle is $\sqrt{2} + \sqrt{7} + 3$.

■ 4. Find the perimeter of a right, isosceles triangle, to the nearest hundredth, in which one of the legs measures 5 inches.

Solution:

Draw a right, isosceles triangle and note that both legs must be 5 inches long. Find the hypotenuse using the Pythagorean Theorem.

$$c^2 = 5^2 + 5^2$$

$$c^2 = 50$$

$$c = \sqrt{50} = 5\sqrt{2}$$

Then the perimeter is

$$P = 5 + 5 + 5\sqrt{2} = 10 + 5\sqrt{2} = 10 + 7.07 \approx 17.07 \text{ inches}$$

AREA OF A CIRCLE

■ 1. Find the area of a circle to the nearest hundredth with a diameter of 44 inches.

Solution:

If the diameter is 44 inches, then the radius is half that: 22 inches. Plug the radius into the formula for the area of a circle.

$$A = \pi r^2$$

$$A = \pi(22)^2$$

$$A = 1{,}520.53$$

■ 2. The area of a circle is 300 cm^2. Find the length of the radius to the nearest tenth of a centimeter.

Solution:

Plug the area into the formula for the area of a circle, and then solve for the radius, r.

$$A = \pi r^2$$

$$300 = \pi r^2$$

$$r^2 = \frac{300}{\pi} = 95.5$$

$$r = \sqrt{95.5} = \approx 9.8$$

■ 3. Find the exact area of a circle with a circumference of 18π.

Solution:

Plug the circumference into the formula for the circumference of a circle.

$$C = d\pi$$

$$18\pi = d\pi$$

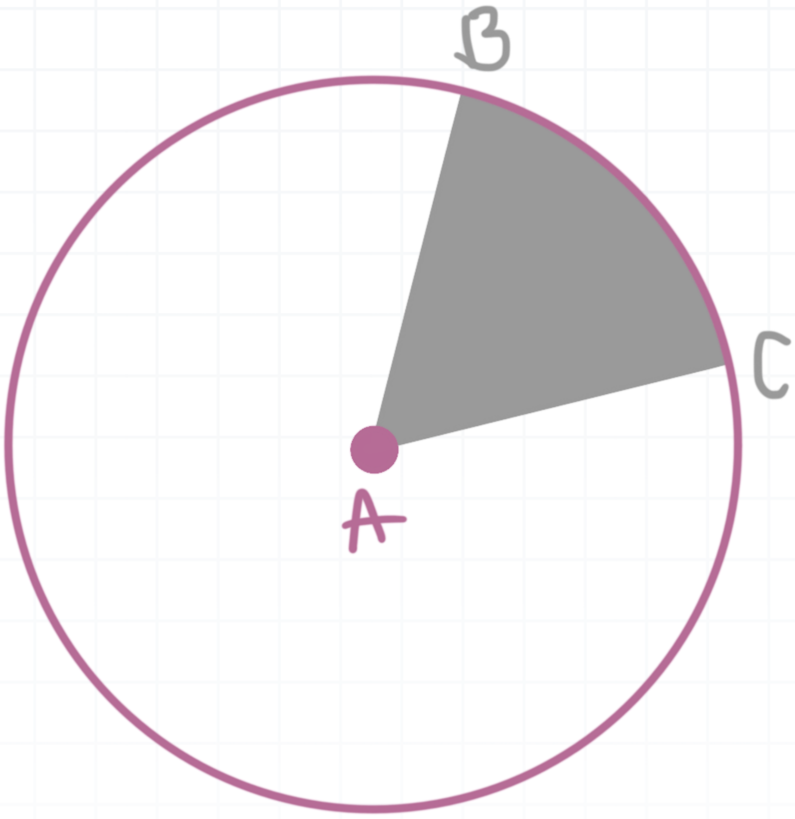
$$d = 18$$

Because the diameter has length 18, the length of the radius is $r = 9$. Therefore, the area of a circle is

$$A = \pi(9)^2$$

$$A = 81\pi$$

■ 4. Find the area of the shaded region to the nearest tenth if $m\angle BAC = 60^\circ$ and $AC = 16$ feet.



Solution:

The shaded area represents $60/360$, or $1/6$ of the total area. The area of the full circle is

$$A = \pi r^2 = \pi (16)^2 \approx 804.2$$

so the area of the shaded region, which is $1/6$th of the circle, is

$$A = \frac{1}{6}(804.2) \approx 134.0$$

CIRCUMFERENCE OF A CIRCLE

■ 1. To the nearest hundredth, find the circumference of a circle that has a radius of 14 feet.

Solution:

The circumference of the circle is

$$C = 2\pi r = 2\pi(14) = 28\pi \approx 87.96$$

■ 2. Find the area of a circle with a circumference of 400 ft.

Solution:

We can use the formula for circumference.

$$C = 2\pi r$$

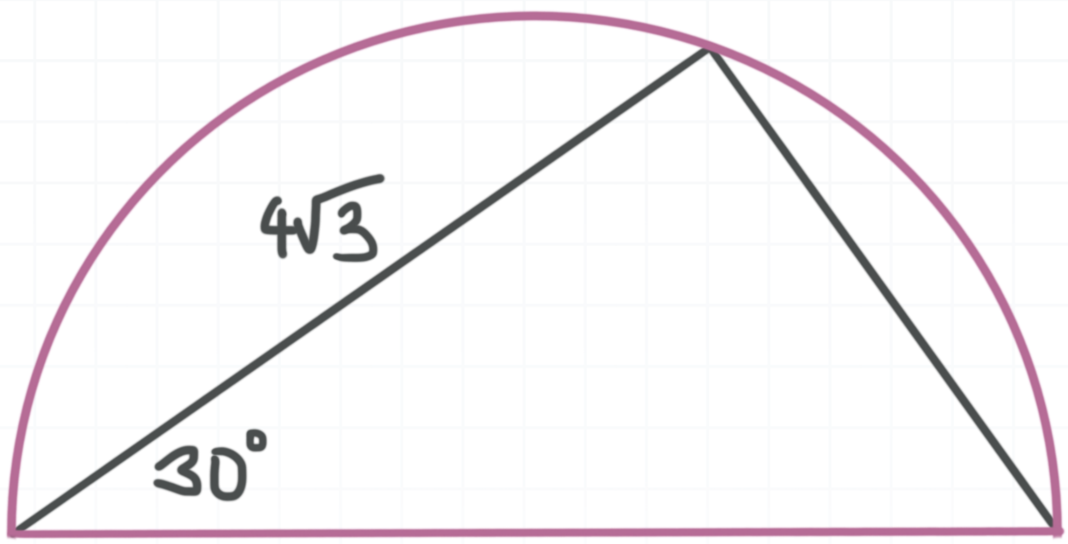
$$400 = 2\pi r$$

$$r = \frac{200}{\pi} \approx 63.66$$

Then the area of the circle is

$$A = \pi(63.66)^2 \approx 12{,}731.61$$

■ 3. Find the exact circumference of the semicircle.



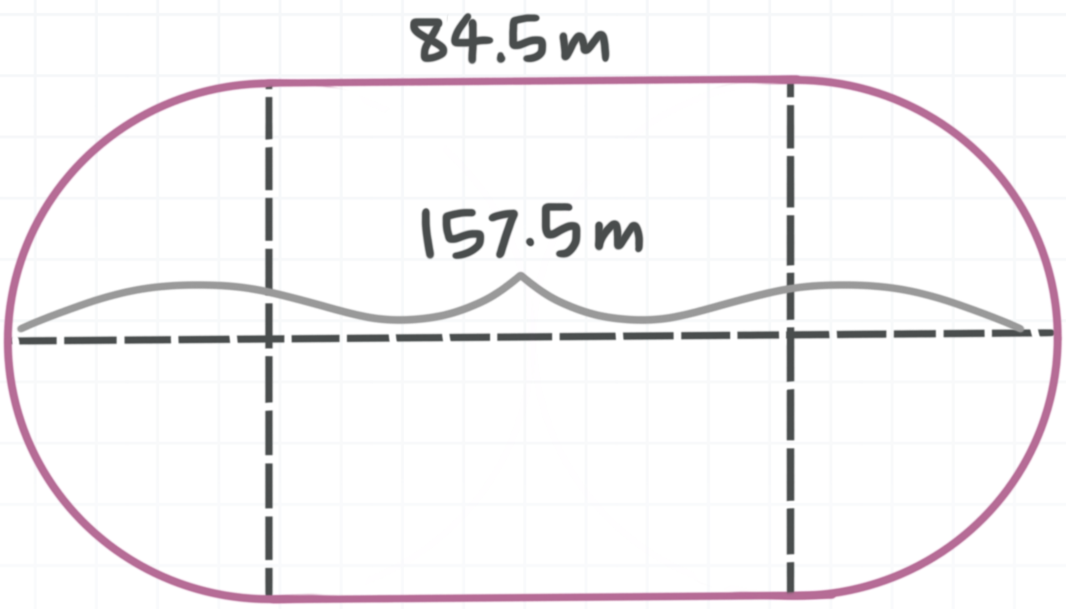
Solution:

Find the diameter of the circle using $30-60-90$ rules to find $d = 8$.

$$C = 2\pi r = d\pi = 8\pi$$

The semicircle has a circumference that is half of the overall circumference of the circle.

■ 4. To the nearest tenth, find the distance around the following track.



Solution:

The length is comprised of two straight stretches and two semicircles. The length of the radius of each semicircle must be

$$\frac{(157.5 - 84.5)}{2} = 36.5$$

The circumference of each semicircle is

$$\frac{1}{2}(2)(36.5)\pi \approx 114.67$$

$84.5 + 84.5 + 114.67 + 114.67 = 398.3$ meters

W W W . K R I S T A K I N G M A T H . C O M