Perimeter of a triangle

To find the perimeter of a triangle, we can use the Pythagorean theorem, regardless of whether or not the triangle is a right triangle.

Right triangles

When the triangle *is* right, we only need to know two of the three sides of the triangle. Remember that the Pythagorean theorem tells us that the sum of the squares of the legs, a and b, is equal to the square of the hypotenuse, c.

$$a^2 + b^2 = c^2$$

Which means, given two of the values a, b, and c, we'll be able to use the Pythagorean theorem to solve for the third value that we don't know. Then once we know all three of the values a, b, and c, we can add them to find the perimeter of the triangle.

Example

One leg of a right triangle is 4 meters long, and the triangle's hypotenuse is 5 meters long. Find the perimeter of the triangle.

Let's call the known leg $a = 4$, the unknown leg b, and the hypotenuse $c = 5$. Then substitute into the Pythagorean theorem to find the length of the leg b.

$$a^2 + b^2 = c^2$$

$$4^2 + b^2 = 5^2$$

$$16 + b^2 = 25$$

$$b^2 = 9$$

$$b = 3$$

Now that we know the lengths of all three sides, we add them together to find the triangle's perimeter.

$$P = a + b + c$$

$$P = 4 + 3 + 5$$

$$P = 12$$

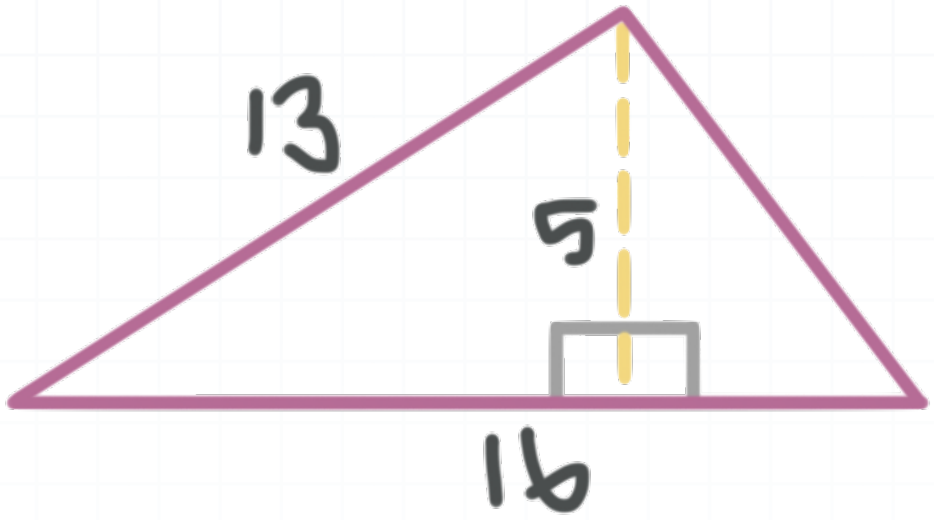
Oblique triangles

Any triangle that isn't a right triangle is an oblique triangle. When a triangle is oblique and we want to find its perimeter, we start by separating the oblique triangle into two right triangles. From there, we find

the lengths of sides of the right triangles, and use those lengths to find the perimeter of the oblique triangle.

Example

Find the perimeter of the oblique triangle.



The height of the oblique triangle is 5. We can consider that height as a leg of the right triangle on the left. The hypotenuse of the triangle on the left is 13, so we can use these two side lengths in the Pythagorean theorem to find the length of the unknown leg.

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 13^2$$

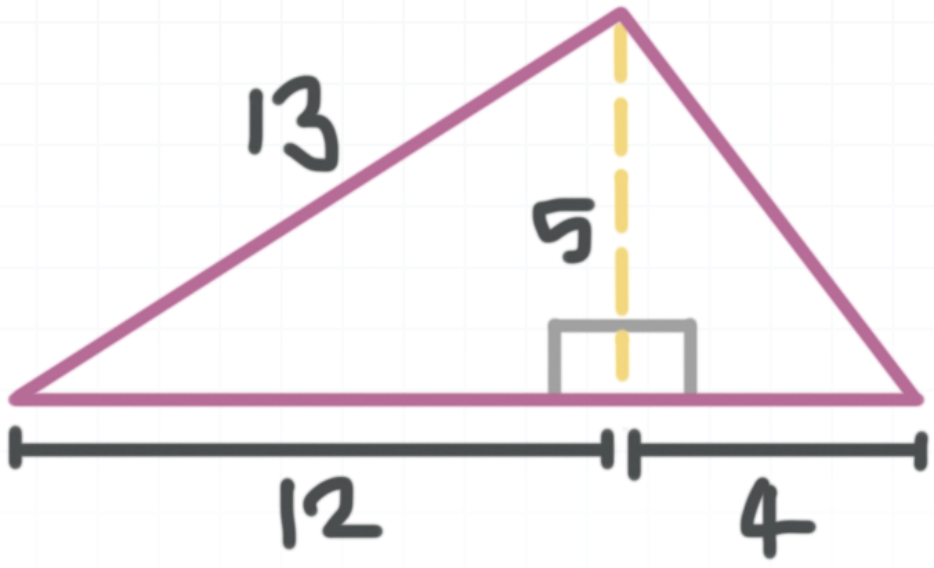
$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = \sqrt{144}$$

$$b = 12$$

From the figure, the base length of the oblique triangle is 16, but we just found $b = 12$, which means the length of the base leg of the right triangle on the right is $16 - 12 = 4$.



For the right triangle on the right side, the vertical leg is the height $a = 5$, and we just found that the length of the base leg is $b = 4$, so from the Pythagorean theorem the hypotenuse of the triangle on the right is

$$a^2 + b^2 = c^2$$

$$5^2 + 4^2 = c^2$$

$$25 + 16 = c^2$$

$$41 = c^2$$

$$c = \sqrt{41}$$

Therefore, the three side lengths of the oblique triangle are 13, 16, and $\sqrt{41}$, which means its perimeter is

$$P = 13 + 16 + \sqrt{41}$$

$$P = 29 + \sqrt{41}$$
