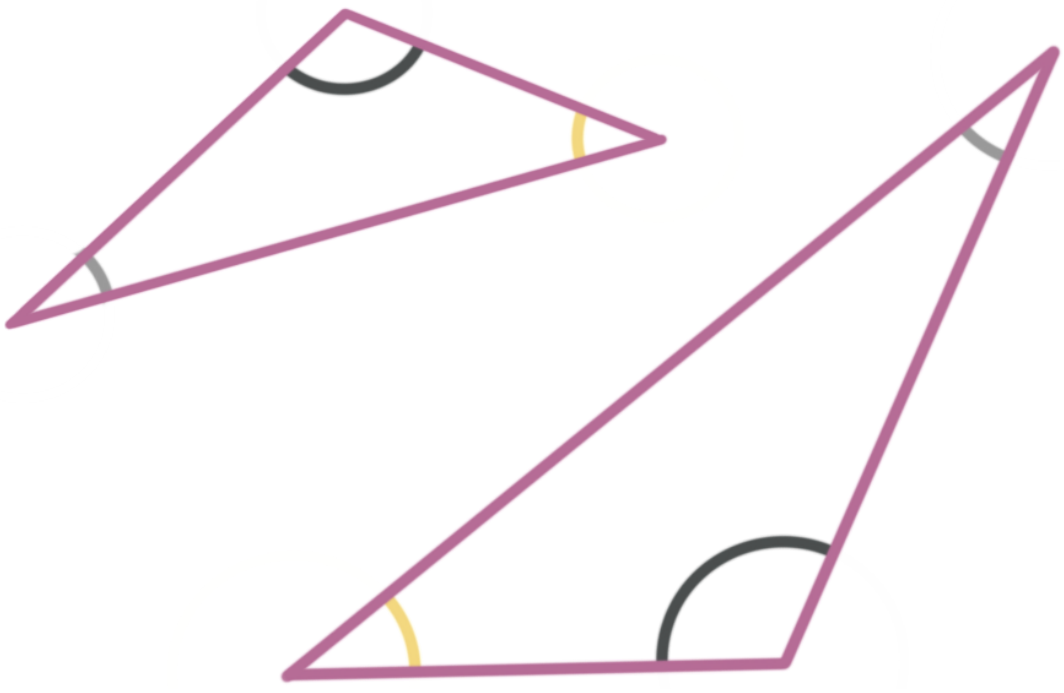
Similar triangles

In this lesson we'll look at similar triangles and how to find an unknown length of a side in one triangle (in a pair of similar triangles) from known lengths of sides in the two triangles.

In a pair of **similar triangles**, all three pairs of angles are congruent, and lengths of corresponding sides are proportional. This means that if you know two triangles are similar, you can use the given information to find an unknown length of a side in one of them from known lengths of sides in the two triangles.

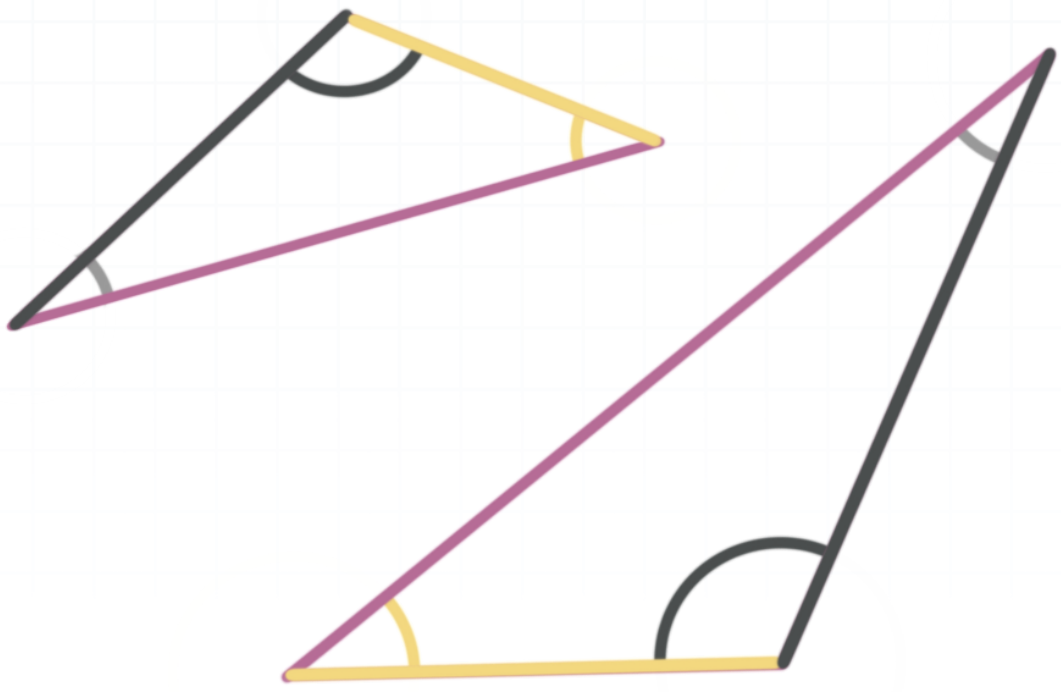
Corresponding angles and sides

In a pair of similar triangles corresponding angles have the same measure. In the pair of similar triangles shown in the figure below, each pair of corresponding angles is in a different color.



And in a pair of similar triangles, the lengths of corresponding sides are proportional. Corresponding sides are the included sides for the same two pairs of corresponding angles.

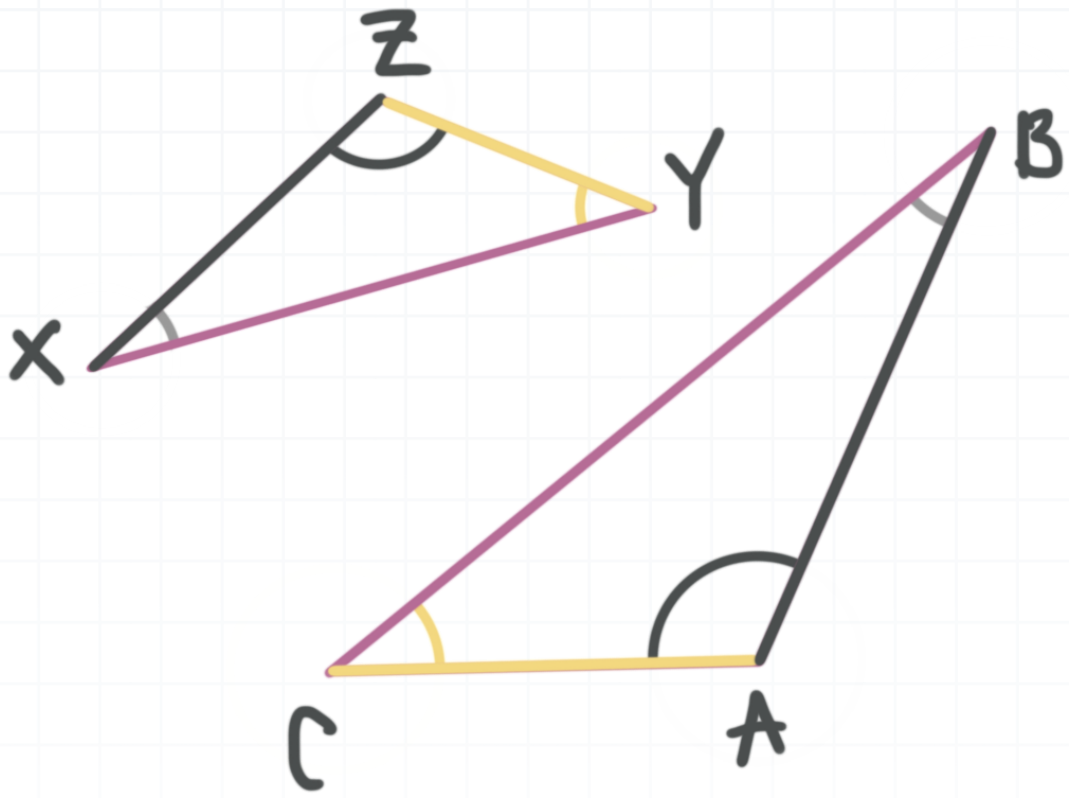
To say that lengths of corresponding sides are proportional means that there is a constant (number) such that the lengths of the sides of one triangle can be found by multiplying the lengths of the sides of the other triangle by that constant.



In the pair of similar triangles in the figure above, each pair of corresponding sides is in a different color.

Naming similar triangles

To state that two triangles are similar, you use the symbol $\sim$. You need to match the letters for the vertices of the first triangle to the letters for the corresponding vertices of the second triangle.

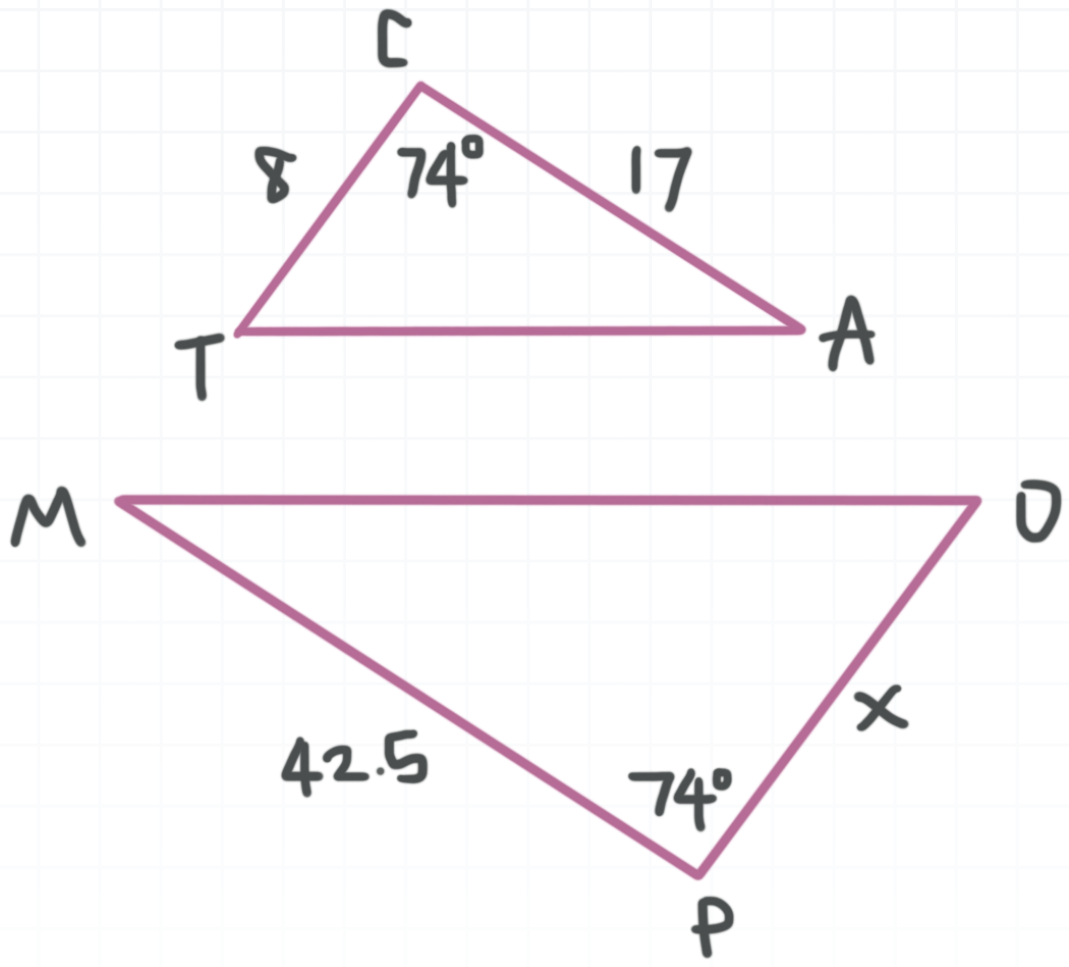


Here, the letters X, Y, and Z for the vertices of the first triangle correspond to the letters B, C, and A, respectively, for the vertices of the second triangle, so we see that $\triangle XYZ \sim \triangle BCA$.

Let’s start by working through an example.

Example

The triangles in the figure are similar. Solve for the value of x.



In a pair of similar triangles, lengths of corresponding sides are proportional. The sides of length x and 42.5 in the bottom triangle correspond to the sides of length 8 and 17, respectively, in the top triangle. So we have the following proportion:

$$\frac{x}{8} = \frac{42.5}{17}$$

Cross multiply.

$$17x = 8(42.5)$$

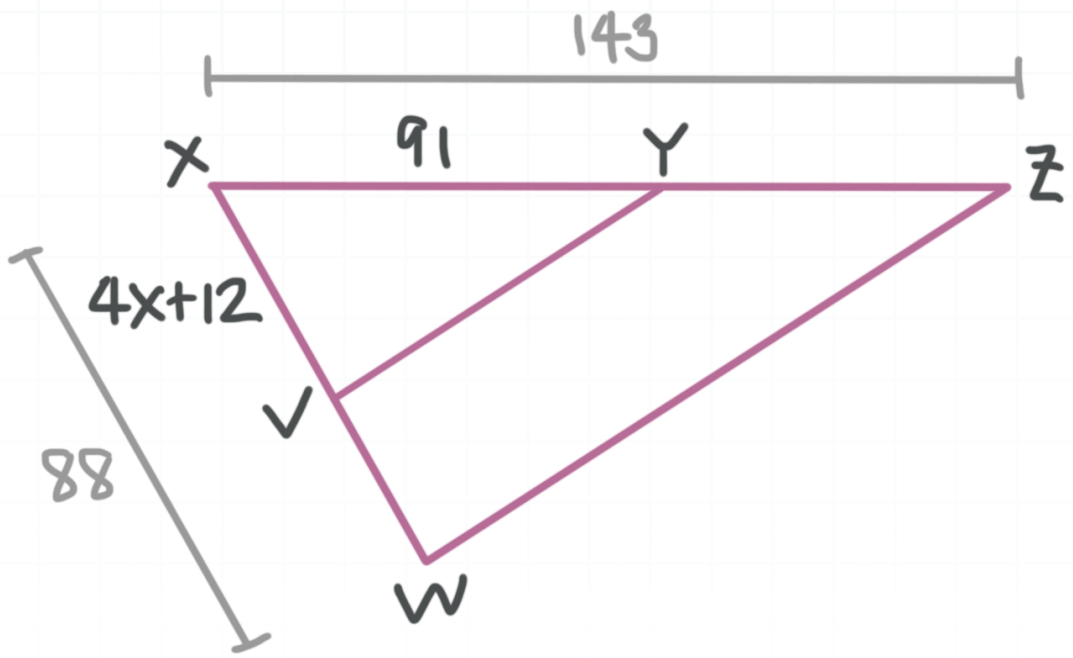
$$17x = 340$$

$$x = 20$$

Let's try one more.

Example

In the figure, $\triangle XVY \sim \triangle XWZ$. Find the value of x.



In a pair of similar triangles, lengths of corresponding sides are proportional. The sides of length $4x + 12$ and 91 in $\triangle XVY$ correspond to the sides of length 88 and 143, respectively, in $\triangle XWZ$. So we have the following proportion:

$$\frac{4x + 12}{88} = \frac{91}{143}$$

Cross multiply.

$$(4x + 12)143 = 91(88)$$

$$(4x + 12)143 = 8{,}008$$

$$\frac{(4x + 12)143}{143} = \frac{8{,}008}{143}$$

$$4x + 12 = 56$$

$$4x = 44$$

$$x = 11$$