Triangle congruence with AAS, HL

In this lesson we'll look at how to use two more triangle congruence theorems, called angle, angle, side (AAS) and hypotenuse, leg (HL), to show that triangles, or parts of triangles, are congruent.

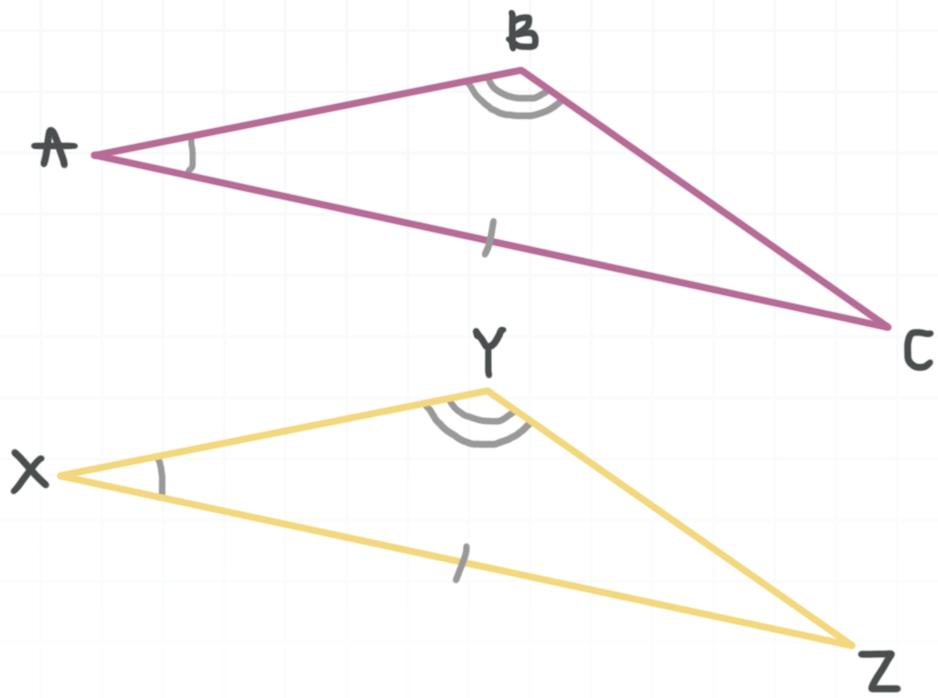
Congruent triangles

A pair of congruent triangles have exactly the same size and shape. That means that we could place one triangle on top of the other in such a way that they're identical, that is, corresponding sides have the same length and corresponding angles have the same measure.

The good news is that, to prove that the two triangles are congruent, we don't have to show that all three pairs of sides and all three pairs of angles match up. There are some triangle theorems that you can use as a short cut to prove that two triangles are congruent.

Angle, angle, side (AAS)

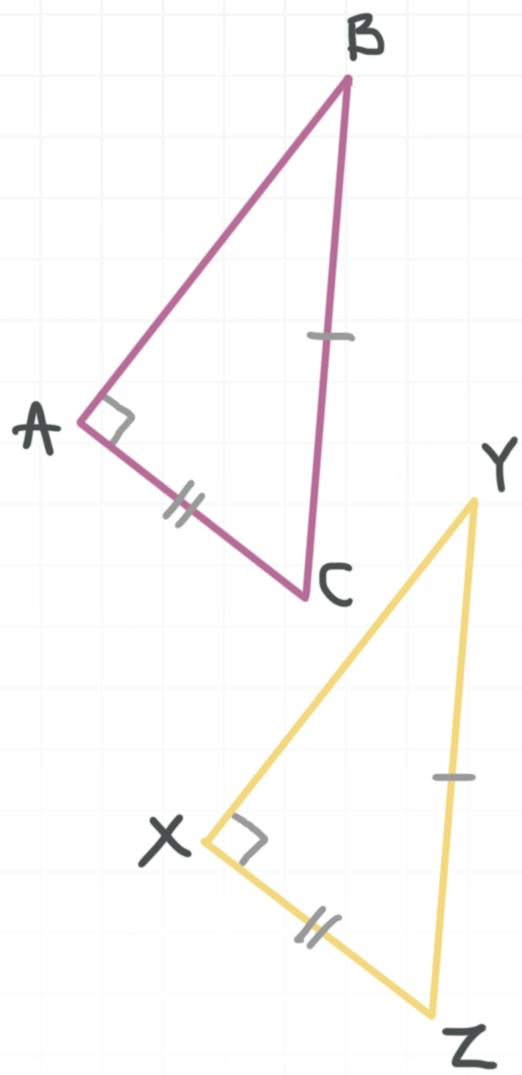
If you can show that two pairs of angles of two triangles are congruent, and that the side opposite an angle in one of those two pairs of congruent angles is congruent to the side opposite the other angle in that pair, then you've proven that the triangles are congruent by "angle, angle, side" (AAS), without needing to check the third pair of angles or the other two pairs of sides. In the figure below, $\triangle ABC \cong \triangle XYZ$ by angle, angle, side: In $\triangle ABC$, $\overline{AC}$ is the side opposite $\angle ABC$ (an angle in one of the pairs of congruent angles, namely the pair $\angle ABC$ and $\angle XYZ$), and in $\triangle XYZ$, $\overline{XZ}$ is the side opposite $\angle XYZ$ (the other angle in that congruent pair).



Hypotenuse, leg (HL)

This theorem can be used only with right triangles, so in order to use "hypotenuse, leg" to prove that a pair of triangles are congruent, we need to know before we even begin that both triangles are right triangles. Then we need congruent hypotenuses and a pair of congruent legs.

For instance, in the figure below, $\triangle ABC \cong \triangle XYZ$ by hypotenuse, leg: The hypotenuse of $\triangle ABC$ (side $\overline{BC}$) is congruent to the hypotenuse of $\triangle XYZ$ (side $\overline{YZ}$), and one of the legs of $\triangle ABC$ (side $\overline{AC}$) is congruent to one of the legs of $\triangle XYZ$ (side $\overline{XZ}$).



Be careful

Whenever you state that two triangles are congruent, you must match the letters for corresponding vertices when you name the triangles. Even if the letters for the vertices of one of the triangles are in alphabetical order, the letters for the corresponding vertices of the other triangle will not necessarily be in alphabetical order.

Write the names so that the letters for the vertices are in the same places. Then the letters for the endpoints of pairs of congruent sides will also be in the same places.

If you have a pair of congruent triangles, $\triangle ABC$ and $\triangle DEF$, then the triangle congruency statement $\triangle ABC \cong \triangle DEF$ means that all of the following are true:

The letters A, B, and C for the vertices of $\triangle ABC$ correspond to the letters D, E, and F, respectively, for the vertices of $\triangle DEF$.

Side $\overline{AB}$ in $\triangle ABC$ is congruent to side $\overline{DE}$ in $\triangle DEF$.

Side $\overline{BC}$ in $\triangle ABC$ is congruent to side $\overline{EF}$ in $\triangle DEF$.

Side $\overline{AC}$ in $\triangle ABC$ is congruent to side $\overline{DF}$ in $\triangle DEF$.

For instance, in the figure above, it would be correct to say that $\triangle ABC \cong \triangle XYZ$. But it would be incorrect to say $\triangle ABC \cong \triangle YZX$, since this statement doesn't list the letters for the vertices of the lower triangle (in the figure) in the same order as the letters for the corresponding vertices of the upper triangle. In other words, we could correctly write this congruence statement as any of the following:

$\triangle ABC \cong \triangle XYZ$

$\triangle BCA \cong \triangle YZX$

$\triangle CAB \cong \triangle ZXY$

$\triangle ACB \cong \triangle XZY$

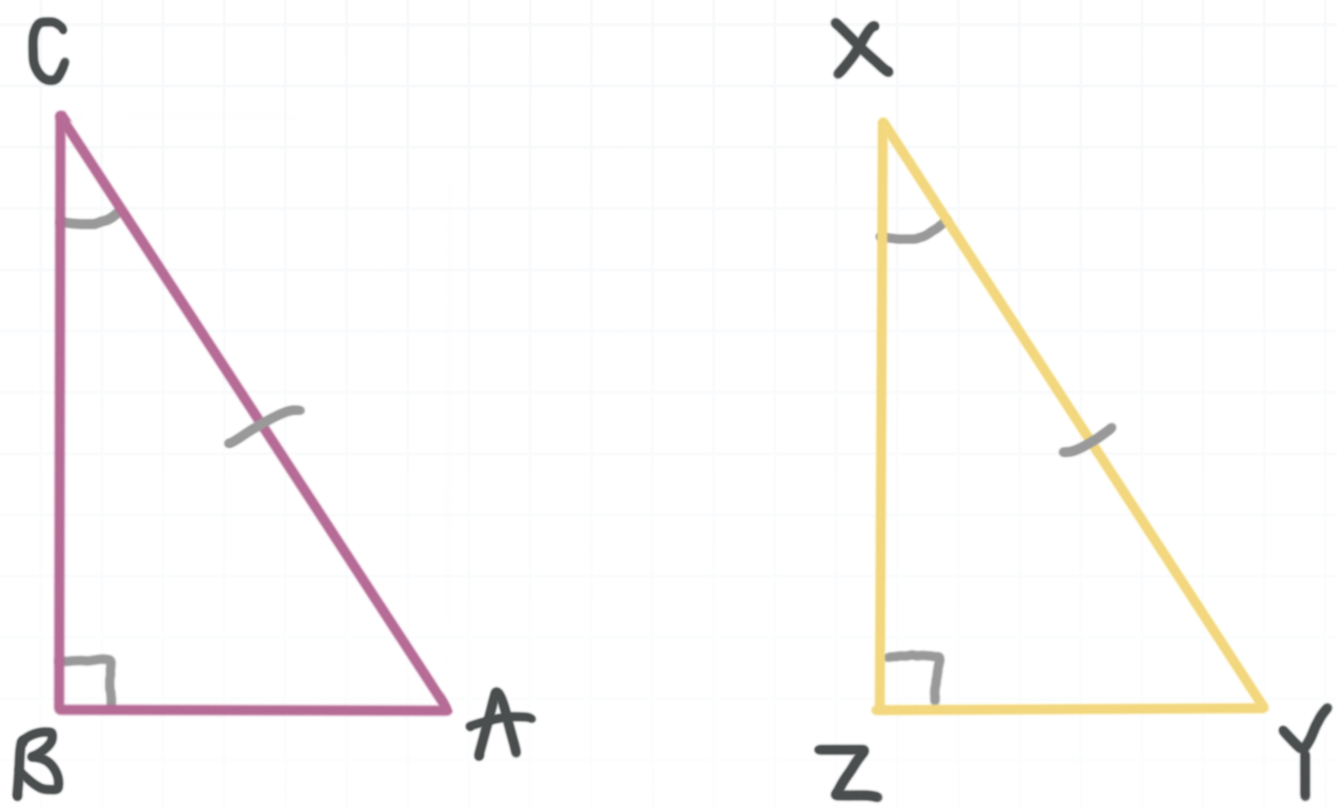
$\triangle BAC \cong \triangle YXZ$

$\triangle CBA \cong \triangle ZYX$

Let's do an example.

Example

Can you prove that the two triangles are congruent? If so, how?



Let's write down what we know.

$$\angle ZXY \cong \angle BCA$$

$$\overline{XY} \cong \overline{CA}$$

$$m\angle YZX = 90^\circ$$

$$m\angle ABC = 90^\circ$$

Because $m\angle YZX = 90^\circ$ and $m\angle ABC = 90^\circ$, we know that

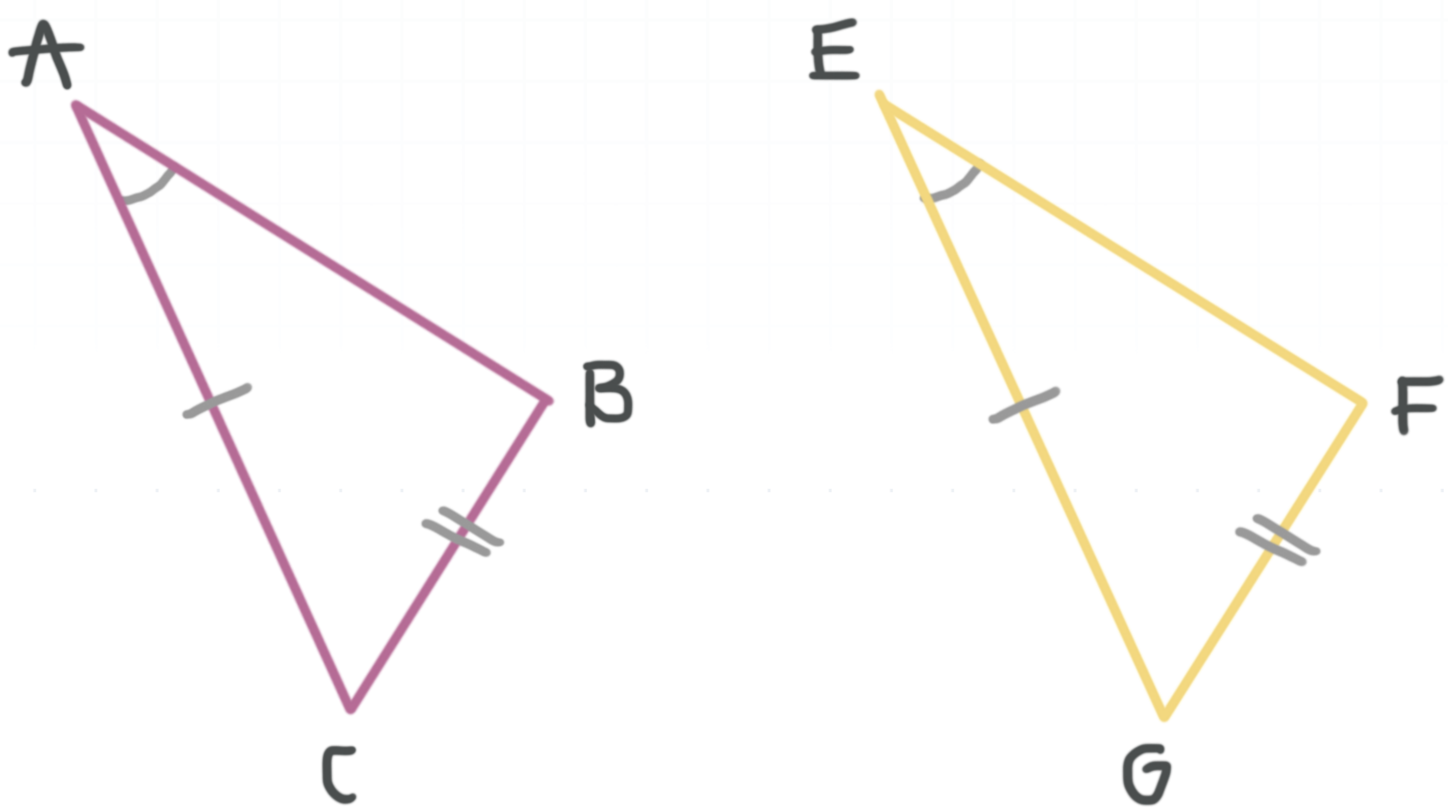
$$\angle YZX \cong \angle ABC$$

Therefore, the triangles are congruent by angle, angle, side: In $\triangle ABC$, $\overline{CA}$ is the side opposite $\angle ABC$ (an angle in one of the two pairs of congruent angles, namely the pair $\angle ABC$ and $\angle YZX$); and in $\triangle XYZ$, $\overline{XY}$ is the side opposite $\angle YZX$ (the other angle in that congruent pair).

Let's try two more.

Example

Can you prove that the two triangles are congruent? If so, how?



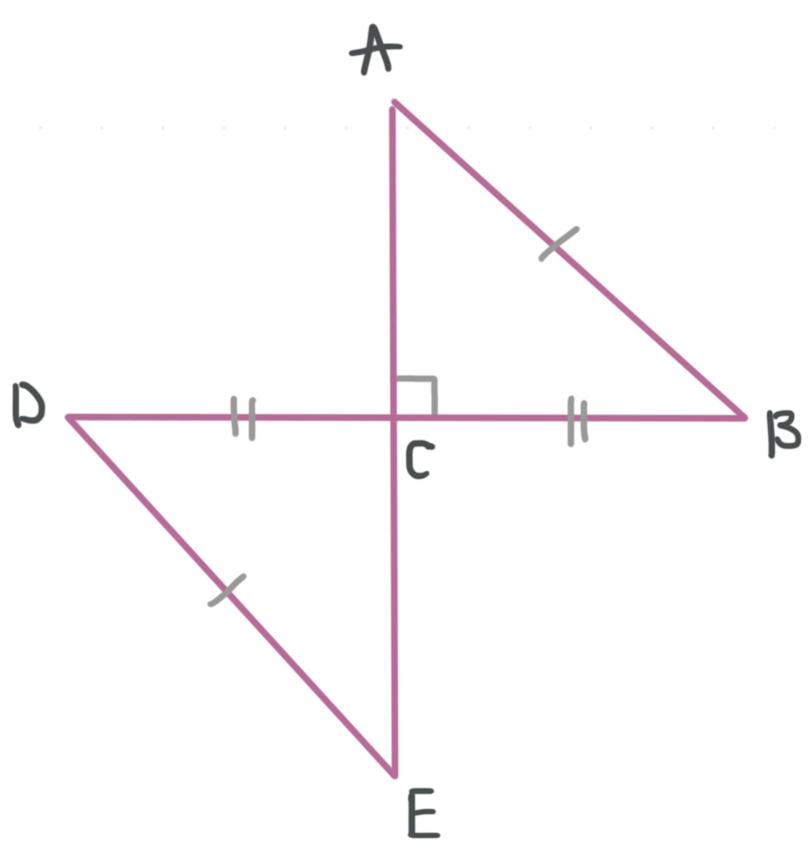
Here's a case where it might be tempting to use hypotenuse, leg because we have two pairs of congruent sides. Unfortunately, we weren't told whether the angles that *look* like right angles actually *are* right angles, so we don't know for sure that these are right triangles, even though they look as if they could be.

We have side, side, angle (where the angles in the pair of congruent angles, $\angle CAB$ and $\angle GEF$, are opposite sides $\overline{BC}$ in $\triangle ABC$ and $\overline{GF}$ in $\triangle EFG$, respectively), but that's not a triangle congruence theorem, so we don't have a way to prove that these triangles are congruent without being given more information.

Let's do the last example.

Example

Can you prove that the two triangles are congruent? If so, state how and write a triangle congruency statement for them.



We know that $\angle BCA \cong \angle DCE$ because they are a pair of vertical angles. Which means that because $\angle BCA$ is a right angle, $\angle DCE$ is a right angle as well. This means that both triangles are right triangles.

$\overline{ED}$ is the hypotenuse of $\triangle EDC$, and $\overline{AB}$ is the hypotenuse of $\triangle ABC$. According to the diagram, $\overline{ED} \cong \overline{AB}$, so the hypotenuses are congruent. We also have a pair of congruent legs because $\overline{DC} \cong \overline{BC}$, so $\triangle ABC \cong \triangle EDC$ by hypotenuse, leg.