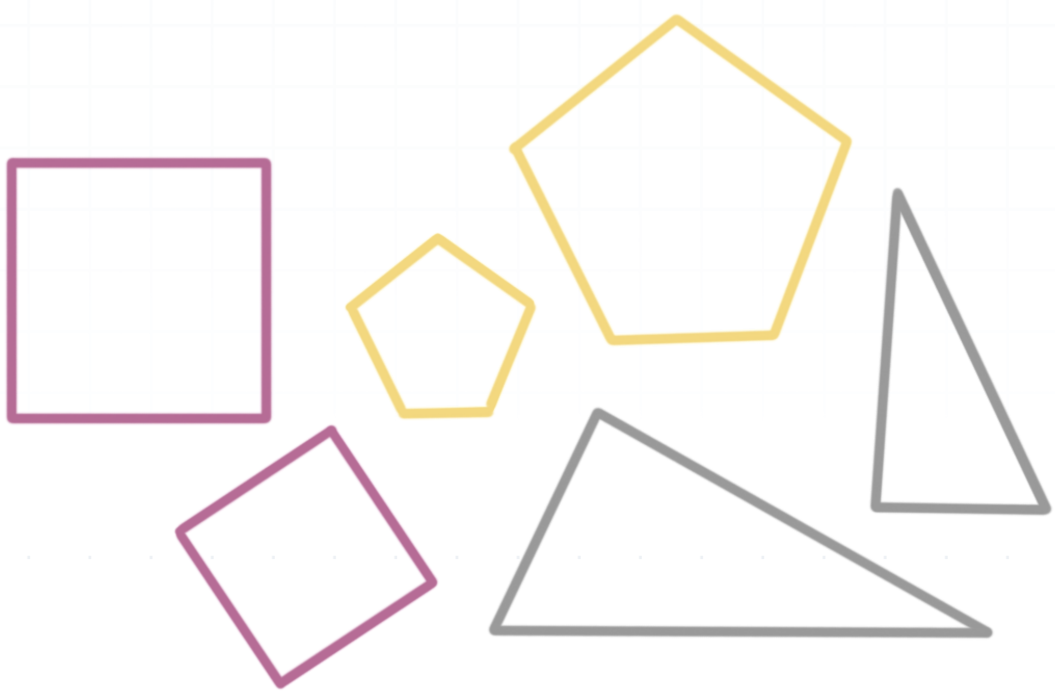
Triangle similarity theorems

In this lesson we'll look at how to prove that a pair of triangles are similar. In math, the word "similarity" has a very specific meaning. Outside of math, when we say two things are similar, we just mean that they're generally like each other.

But in math, to say two figures are similar means that they have exactly the same shape but different sizes. Here are examples of similar squares, similar pentagons, and similar triangles:



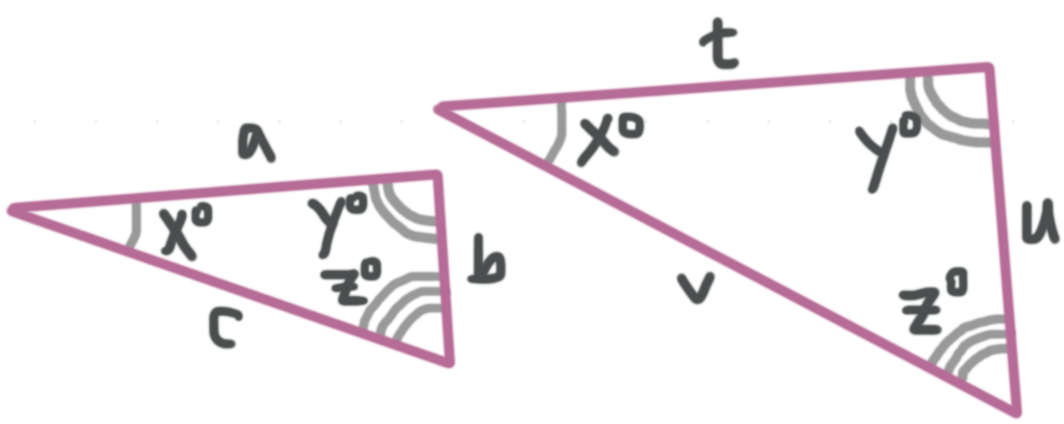
Similar triangles

Similar triangles have the same shape but not the same size. Remember that if two triangles have exactly the same shape and exactly the same size, then we say they're **congruent**. According to one definition of similarity, however, congruence implies similarity; by that definition, congruent triangles are also similar.

In a pair of **similar triangles**, all three pairs of corresponding angles are congruent and the lengths of all three pairs of corresponding sides are proportional. In fact, if all three pairs of corresponding angles are congruent, then the lengths of all three pairs of corresponding sides are automatically proportional, and vice versa. The symbol for similarity is $\sim$, so if we want to say that triangles A and B are similar, we can write that as $A \sim B$.

The triangles below are similar because the corresponding interior angles are congruent, so the lengths of corresponding sides are proportional like this:

$$\frac{a}{t} = \frac{b}{u} = \frac{c}{v}$$



We're going to look at three theorems that allow you to prove that triangles are similar.

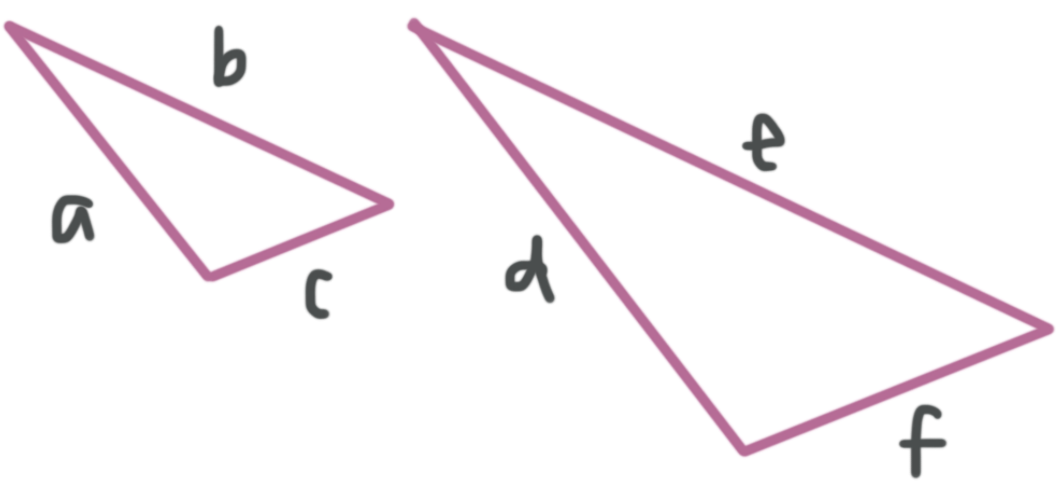
Angle, angle (AA)

If a pair of triangles have two pairs of congruent angles, then the triangles are similar. The reason is that if two pairs of angles are congruent, then the third pair of angles have to be congruent as well because the measures of the interior angles of a triangle always sum to 180°.

Side, side, side (SSS)

If the lengths of all three pairs of sides of a pair of triangles are proportional, then the triangles are similar. The reason is that, if the lengths of all three pairs of sides are proportional, then that forces all three pairs of their interior angles to be congruent.

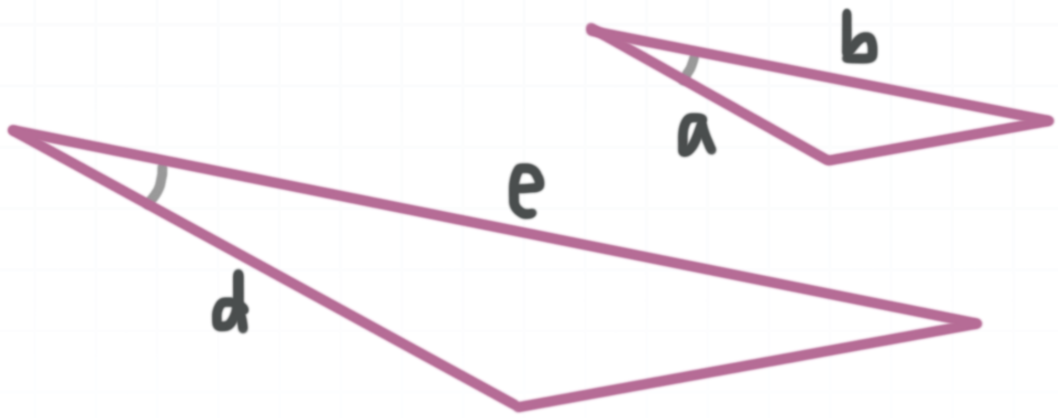
$$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$



Side, angle, side (SAS)

If the lengths of two pairs of sides of a pair of triangles are proportional and the corresponding pair of included angles are congruent, then the triangles are similar. Remember that the included angle of two sides of a triangle is the angle whose vertex is the point of intersection of those two sides.

$$\frac{a}{d} = \frac{b}{e}$$

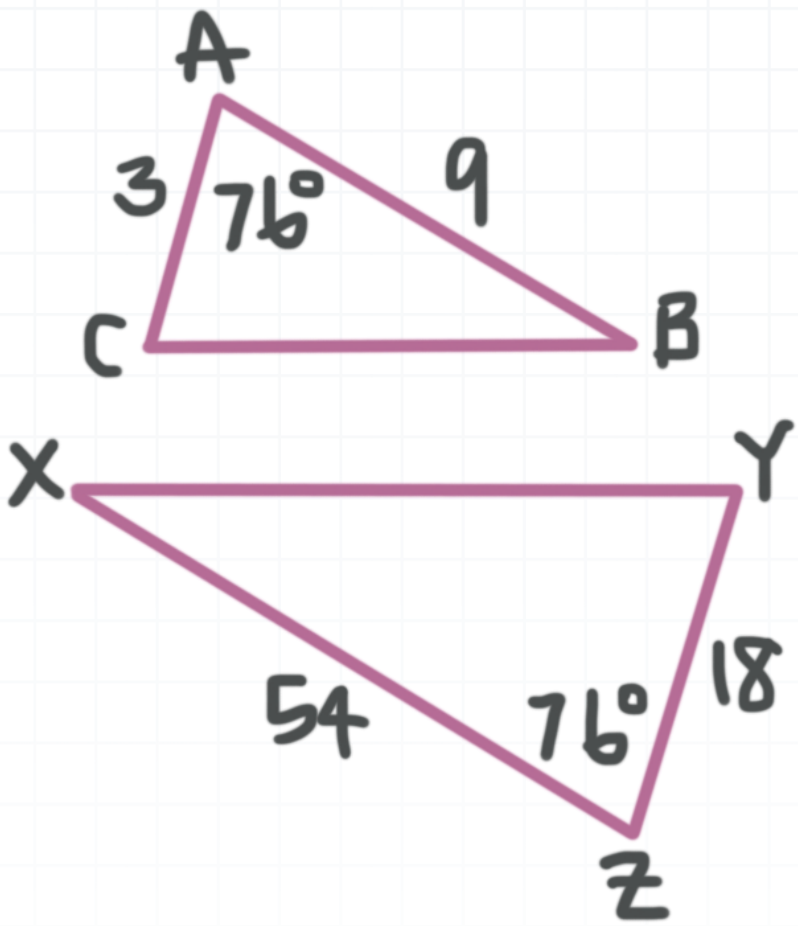


Let's do some practice problems where we use these three theorems to prove similarity of pairs of triangles.

Example

Are the triangles similar? If so, determine which theorem proves that they're similar and complete the similarity statement.

$$\triangle ABC \sim \triangle \_\_\_\_\_$$



We know from the figure that $\angle A \cong \angle Z$, because both of those angles have measure 76°. So we have a pair of congruent angles, and we need to see if the lengths of the pair of sides of $\triangle XYZ$ whose included angle has measure 76° (sides $\overline{ZY}$ and $\overline{ZX}$) are proportional to the lengths of the pair of sides of $\triangle ABC$ whose included angle has measure 76° (sides $\overline{AC}$ and $\overline{AB}$, respectively).

$$\frac{\overline{ZY}}{\overline{AC}} = \frac{18}{3} = 6 \qquad\qquad \frac{\overline{ZX}}{\overline{AB}} = \frac{54}{9} = 6$$

We have the same ratio for the lengths of those two pairs of sides.

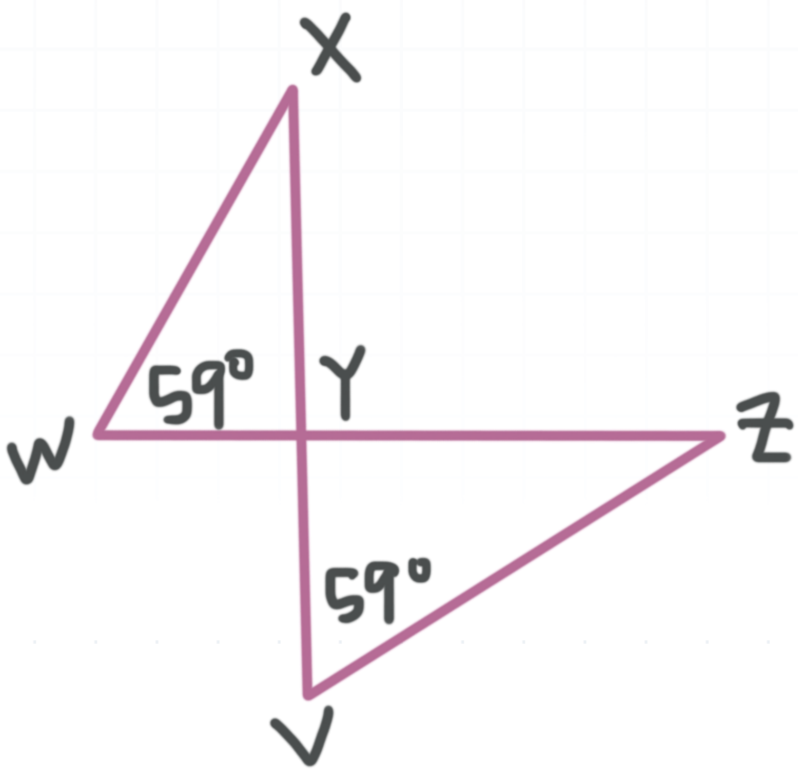
Putting all this together, we know that the triangles are similar by side, angle, side (SAS). When we match up the corresponding parts, we see that the similarity statement is $\triangle ABC \sim \triangle ZXY$.

Let's try another.

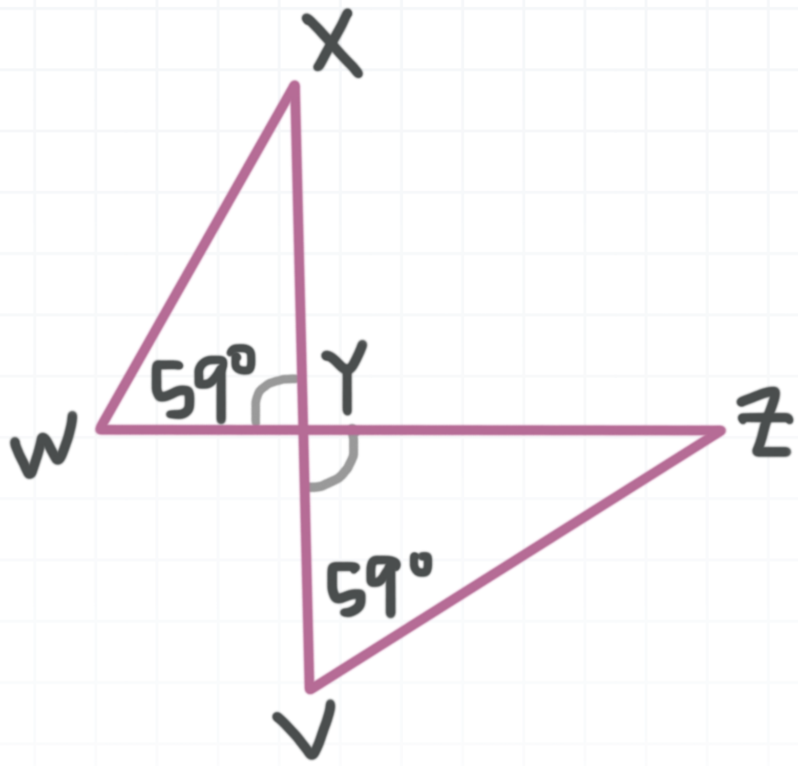
Example

Are the triangles similar? If so, determine which theorem proves that they're similar and complete the similarity statement.

$\triangle WXY \sim \triangle$ ____



We know from the figure that $\angle YWX \cong \angle ZVY$, because both of those angles have measure 59°. We also have a pair of vertical angles at Y, and remember that vertical angles are congruent.



Putting all this together, we know that the triangles are similar by angle, angle (AA). When we match up the corresponding parts, we see that the similarity statement is $\triangle WXY \sim \triangle VZY$.
