Geometry Workbook Solutions

Similarity

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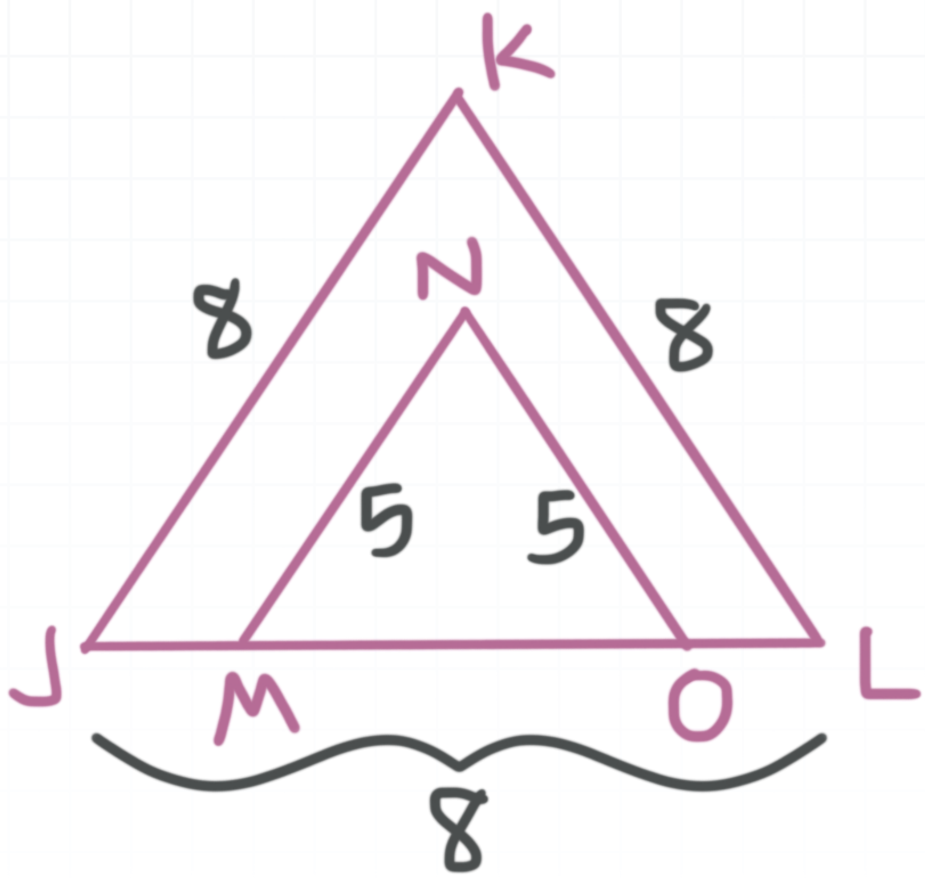
SIMILAR TRIANGLES

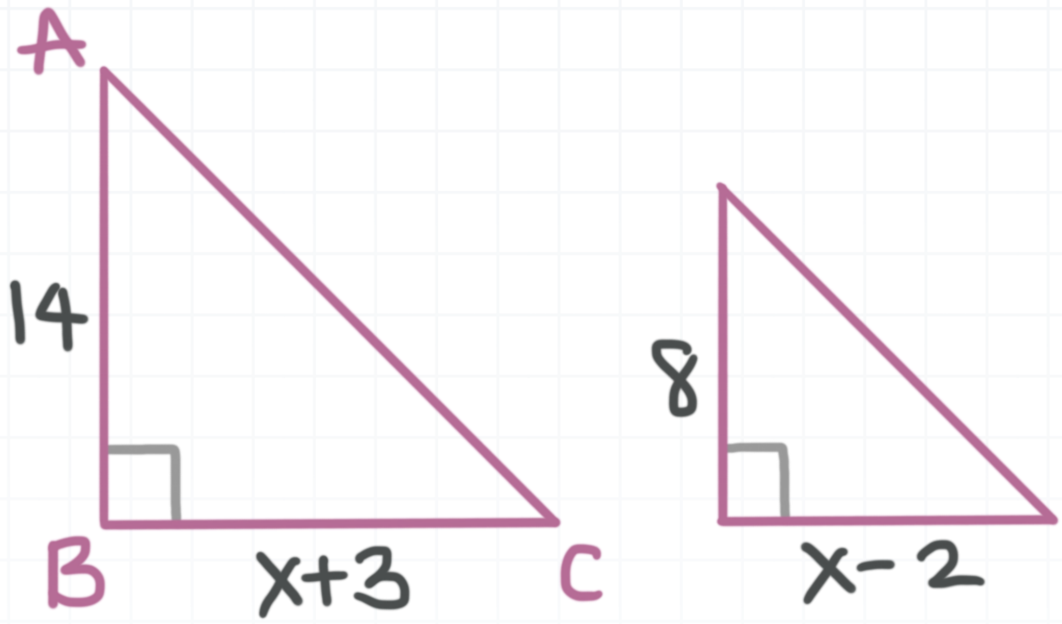
1. $\triangle JKL$ is similar to $\triangle MNO$. Find MO.



Solution:

5. $8/5$ must be the proportion of all corresponding sides of the similar triangles. $\triangle JKL$ is equilateral and $\triangle MNO$ must also be equilateral.

2. $\triangle ABC$ is similar to $\triangle DEF$. Set up a proportion to find the value of x.



Solution:

$x = 26/3$. Set up a proportion, the cross multiply to solve for x.

$$\frac{14}{8} = \frac{x+3}{x-2}$$

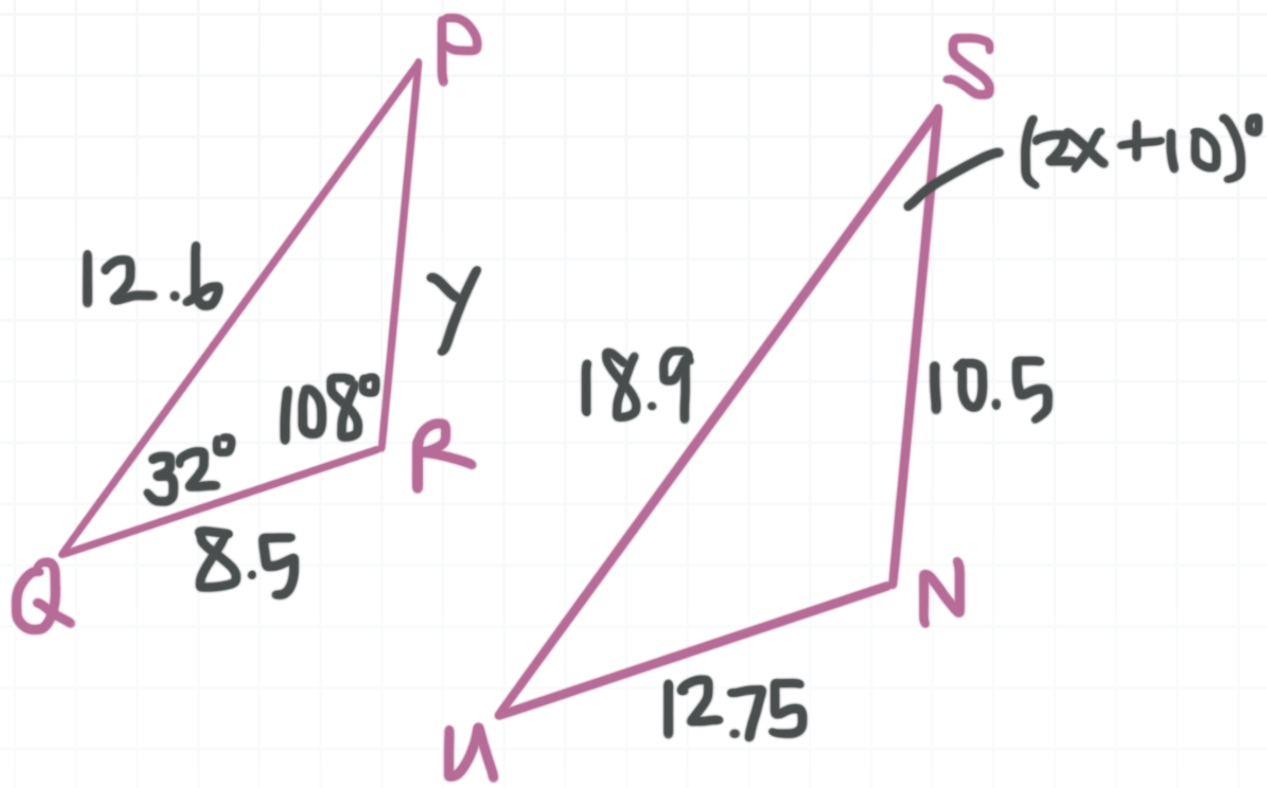
$$14(x-2) = 8(x+3)$$

$$14x - 28 = 8x + 24$$

$$6x = 52$$

$$x = \frac{26}{3}$$

■ 3. $\triangle PQR$ is similar to $\triangle SUN$. Find the values of x and y.



Solution:

$x = 15$ and $y = 7$. Set up a proportion to find y.

$$\frac{8.5}{12.75} = \frac{y}{10.5}$$

$$(8.5)(10.5) = 12.75y$$

$$12.75y = 89.25$$

$$y = 7$$

Then we can find $m\angle P$ as $m\angle P = 180^\circ - 32^\circ - 108^\circ = 40^\circ$. And since $m\angle P = m\angle S$, we can find x be setting up an equation.

$$40 = 2x + 10$$

$$30 = 2x$$

$$x = 15$$

■ 4. A 14-foot tree casts a 6-foot long shadow. A 3.5-foot tall child would have a shadow length of how many feet?

Solution:

0.75 feet. Set up the proportion.

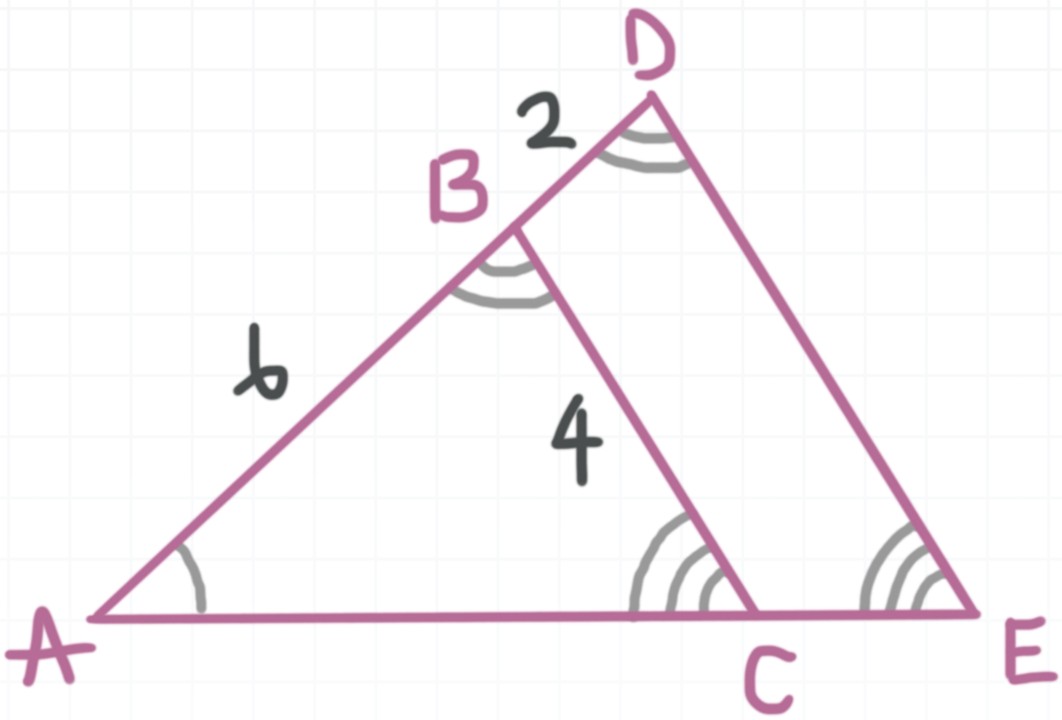
$$\frac{14}{3} = \frac{3.5}{x}$$

$$14x = (3)(3.5)$$

$$14x = 10.5$$

$$x = 0.75$$

■ 5. Find DE.



Solution:

$DE=16/3$. $\triangle ABC$ is similar to $\triangle ADE$, so

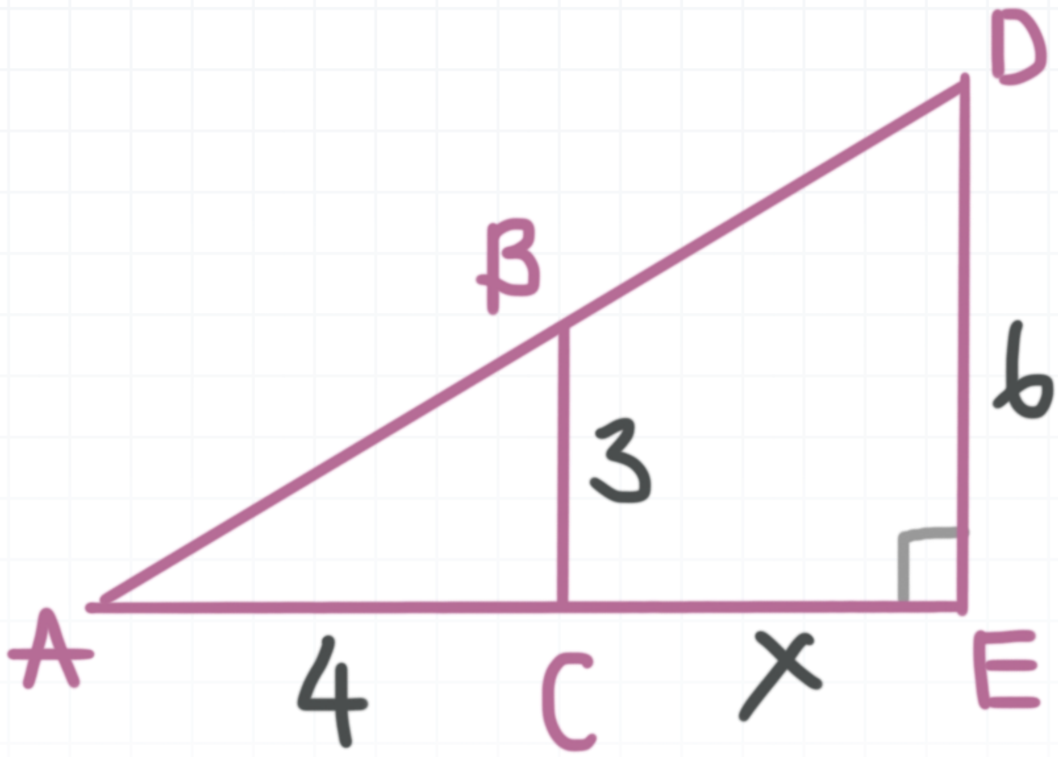
$$\frac{AB}{AD}=\frac{BC}{DE}$$

$$\frac{6}{4}=\frac{8}{DE}$$

$$6DE=32$$

$$DE=\frac{16}{3}$$

■ 6. Find CE.



Solution:

$CE = 4$. $\triangle ABC$ is similar to $\triangle ADE$, so

$$\frac{AC}{AE} = \frac{BC}{DE}$$

$$\frac{4}{4+x} = \frac{3}{6}$$

$$24 = 3(4+x)$$

$$24 = 12 + 3x$$

$$12 = 3x$$

$$x = 4$$

45-45-90 TRIANGLES

■ 1. $\triangle PDX$ is an isosceles right triangle with vertex $\angle D$, and $PD = 4$. Find DX and XP.

Solution:

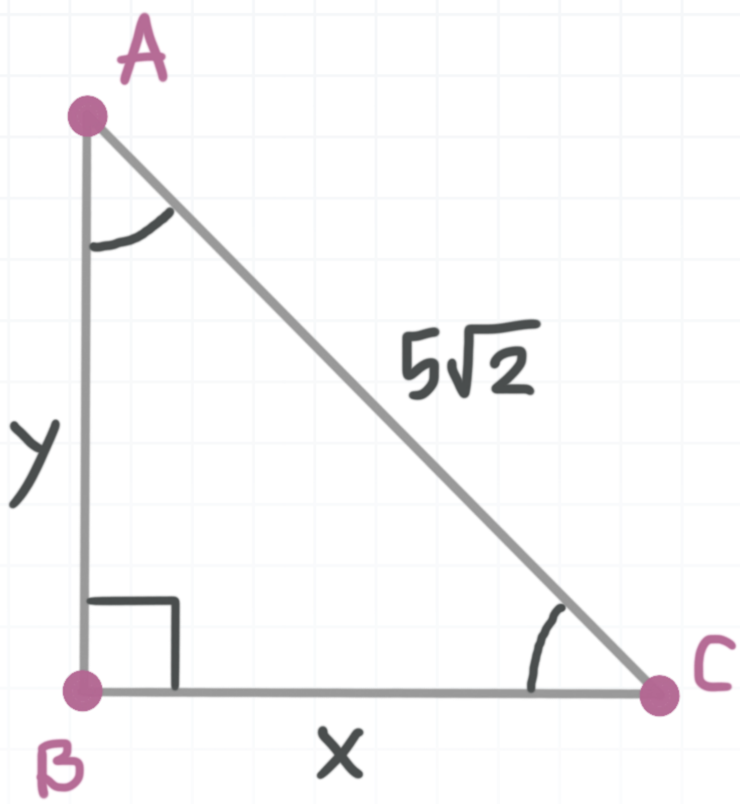
$DX = 4$ and $XP = 4\sqrt{2}$. By the $45-45-90$ rule of right triangles, legs of the triangle are congruent and the hypotenuse has a measure of $\sqrt{2}$. The legs are PD and DX and both have measures of 4. The hypotenuse is XP and has measure $4\sqrt{2}$.

■ 2. A square has a perimeter of 40 meters. Find the length of the diagonal of the square.

Solution:

$10\sqrt{2}$. Since the perimeter is 40, we know the length of each side is 10. Using the $45-45-90$ rule of right triangles, we get the length of the diagonal to be $10\sqrt{2}$.

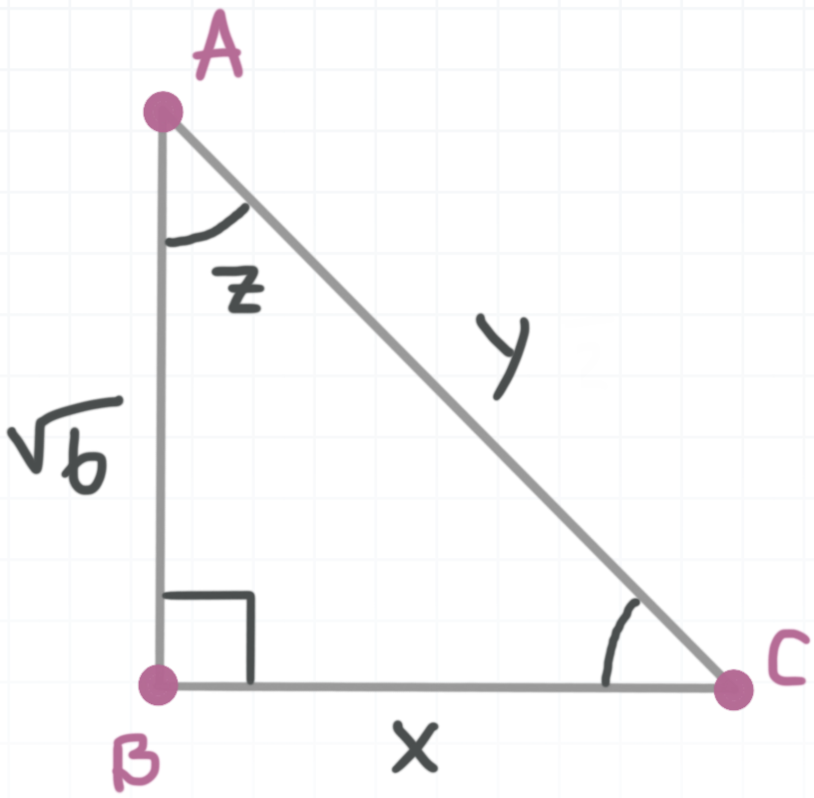
■ 3. Find the values of x and y.



Solution:

$x = 5$ and $y = 5$. By the $45 - 45 - 90$ rule of right triangles, legs of the triangle are congruent and the hypotenuse has a measure of $\sqrt{2}$. The value of each leg must equal 5.

4. Find the values of x, y, and z.



Solution:

$x = \sqrt{6}$, $y = 2\sqrt{3}$, and $z = 45$. In our $45 - 45 - 90$ special right triangle, the legs are congruent. $\overline{AB} \cong \overline{CB}$, which means they both have a measure of $\sqrt{6}$. The length of the hypotenuse can be found by taking the measure of the leg and multiplying it by $\sqrt{2}$.

$$y = \sqrt{6}\sqrt{2}$$

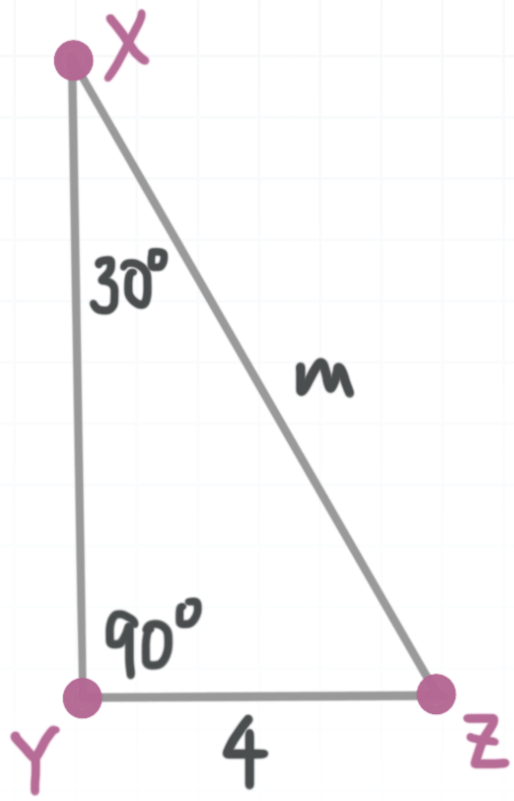
$$y = \sqrt{12}$$

$$y = \sqrt{4}\sqrt{3}$$

$$y = 2\sqrt{3}$$

30-60-90 TRIANGLES

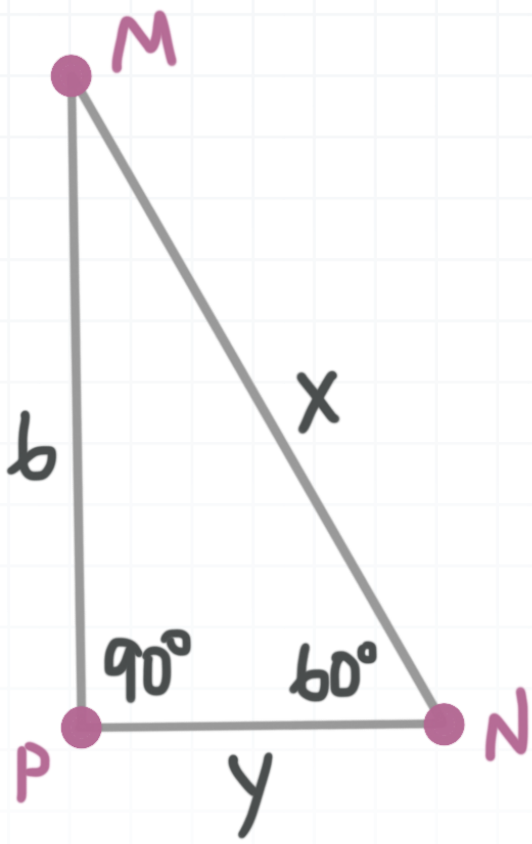
■ 1. Find the value of m in the given triangle.



Solution:

$m = 8$. m represents the length of the hypotenuse of this $30-60-90$ triangle. The hypotenuse is always twice as long as the shortest leg, so $m = 2(4) = 8$.

■ 2. Find the values of x and y in the given triangle.



Solution:

$x = 4\sqrt{3}$ and $y = 2\sqrt{3}$. In a $30 - 60 - 90$ triangle, the length of the longer leg is always the product of the length of the shorter leg and $\sqrt{3}$.

$$6 = y\sqrt{3}$$

$$y = \frac{6\sqrt{3}}{3}$$

$$y = 2\sqrt{3}$$

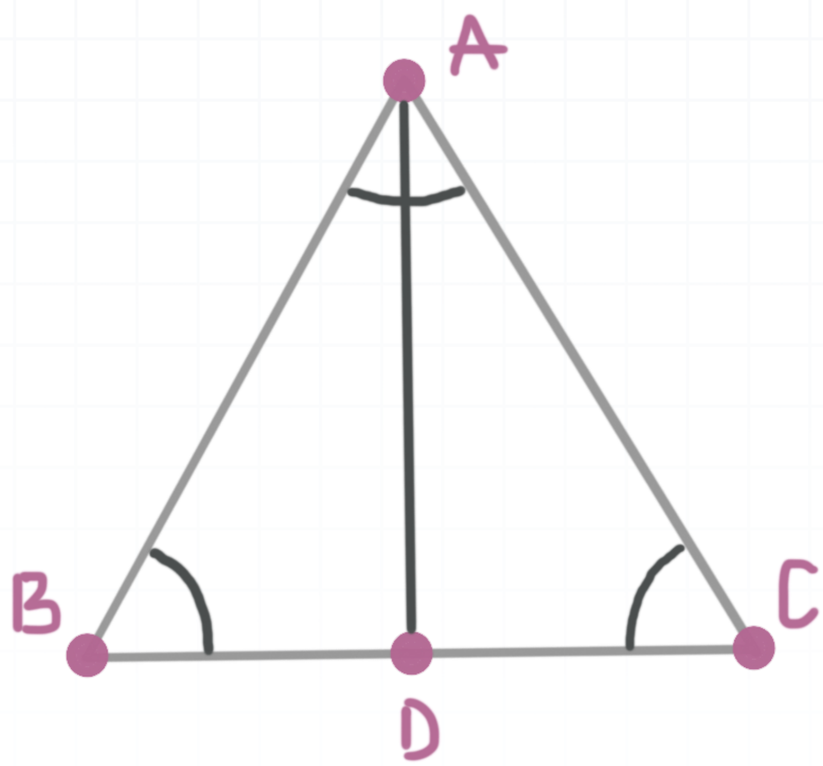
The length of the hypotenuse is twice the length of the shortest leg.

$$x = 2(y)$$

$$x = 2\left(2\sqrt{3}\right)$$

$$x = 4\sqrt{3}$$

■ 3. $\triangle BAC$ is an equilateral triangle. The perimeter is 42 cm and $m\angle ADC = 90$. Find AD.



Solution:

$AD = 7\sqrt{3}$. The perimeter is 42, therefore the length of each side of the triangle is 14. $\overline{AD}$ is an altitude of the triangle, so two $30-60-90$ triangles are formed. $BD = CD = 7$, which is the shortest leg of each of your special right triangles. The longer leg of a $30-60-90$ triangles is the product of the shortest leg and $\sqrt{3}$. So $AD = 7\sqrt{3}$.

4. $\triangle XYZ$ is an equilateral triangle. $\overline{XM}$ is an altitude, median, and angle bisector of the triangle. If $XM = 9$, find the perimeter of the triangle.

Solution:

The perimeter is $18\sqrt{3}$. Draw an equilateral triangle and label $\overline{XM}$. Find the length of YM.

$$XM = YM\sqrt{3}$$

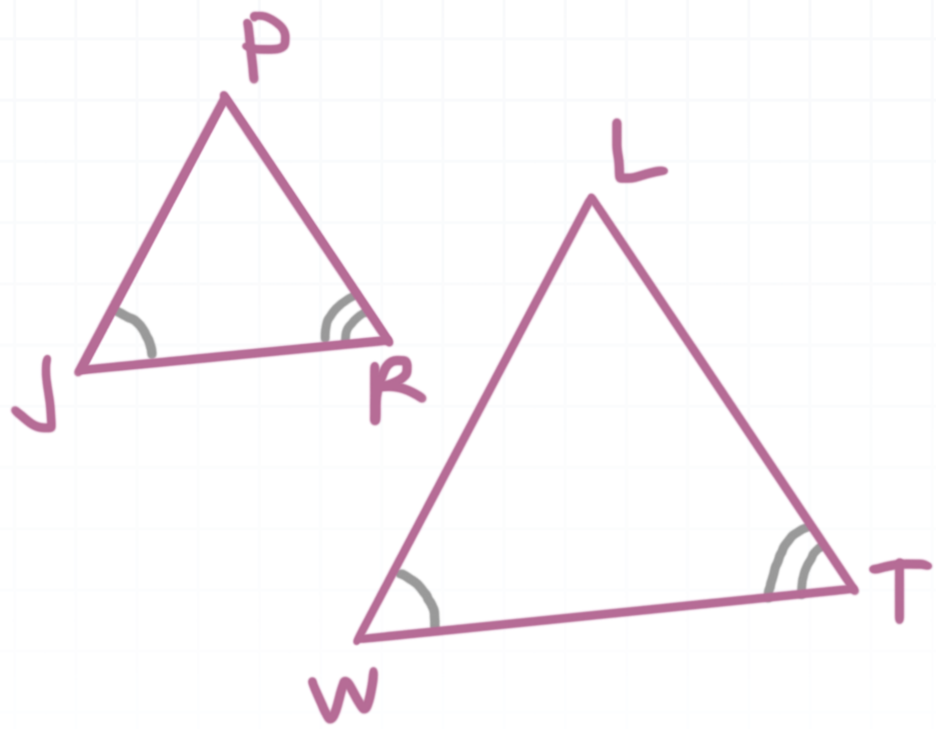
$$YM = \frac{XM}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

Now find the length of YZ. $YZ = 6\sqrt{3}$. So the perimeter is

$$3\left(6\sqrt{3}\right) = 18\sqrt{3}$$

TRIANGLE SIMILARITY STATEMENTS

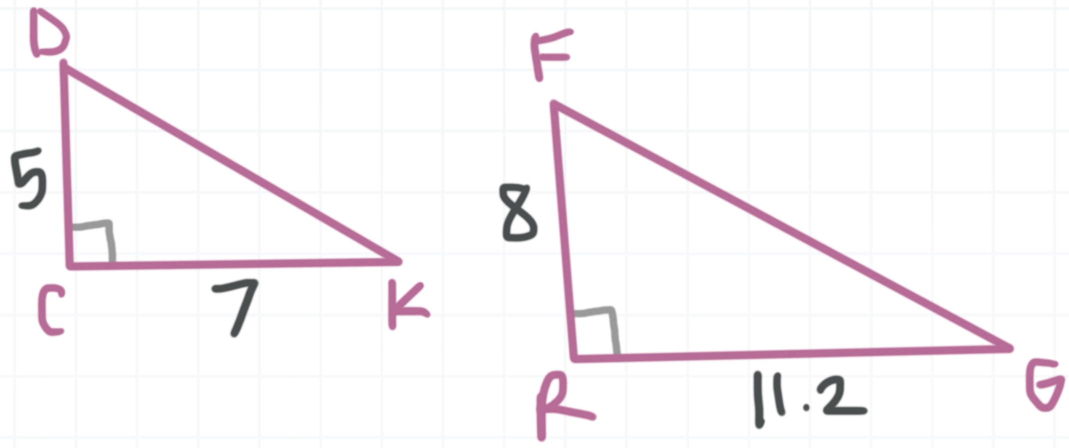
■ 1. Write a similarity statement for the triangles and provide the theorem that proves they're similar.



Solution:

$\triangle JPR \sim \triangle WLT$ by AA. If two angles of a triangle are congruent to two angles of another triangle, then the triangles must be similar.

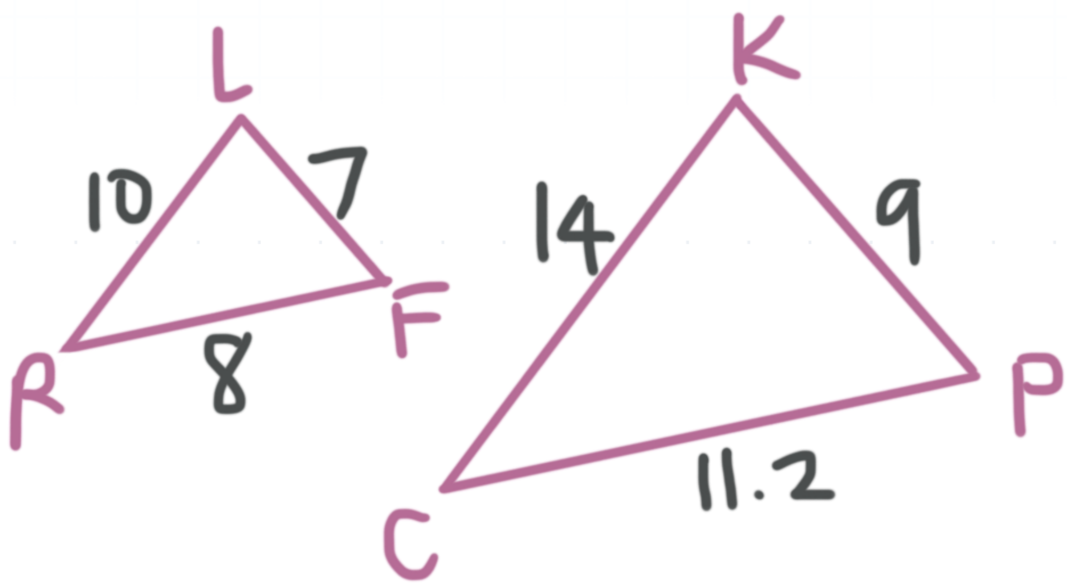
■ 2. Write a similarity statement for the triangles and provide the theorem that proves they're similar.



Solution:

$\triangle DCK \sim \triangle FRG$ by SAS. $\angle C \cong \angle R$ and $5/8 = 7/11.2$.

■ 3. Is $\triangle RLF \sim \triangle CKP$? Explain.



Solution:

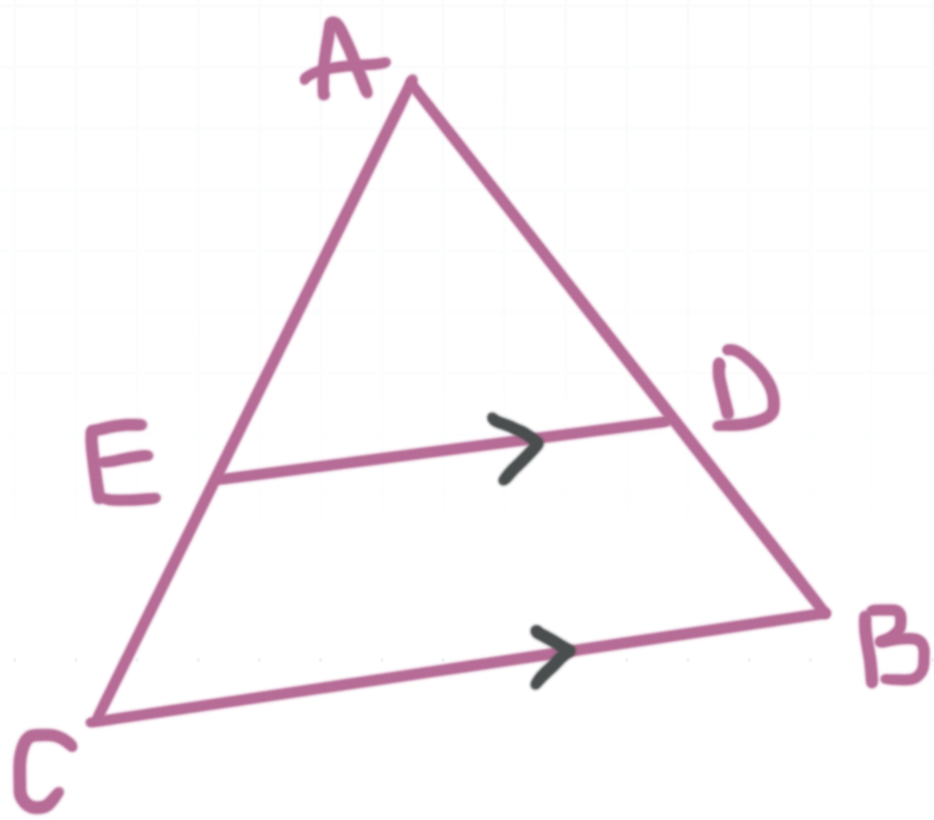
No, these triangles are not similar. Setting up proportions, it's true that

$$\frac{10}{14} = \frac{8}{11}$$

However, the triangles cannot be similar because

$$\frac{10}{14} \neq \frac{7}{9}$$

■ 4. Prove $\triangle AED \sim \triangle ACB$.

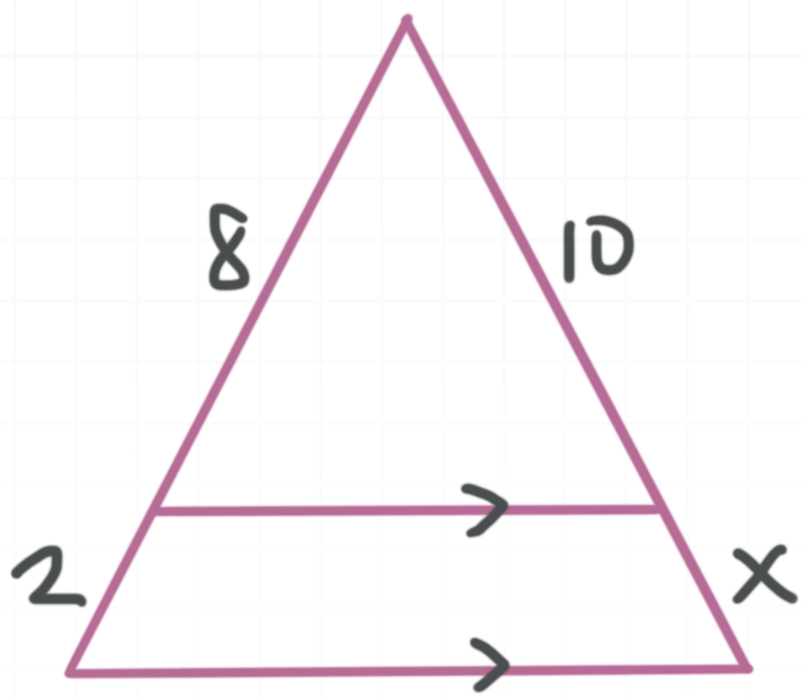


Solution:

$\triangle AED \sim \triangle ACB$ by AA. $\angle A \cong \angle A$ by the Reflexive Property of congruence, and $\angle EDA \cong \angle CBA$ because they are corresponding angles.

TRIANGLE SIDE-SPLITTING THEOREM

■ 1. Solve for x.



Solution:

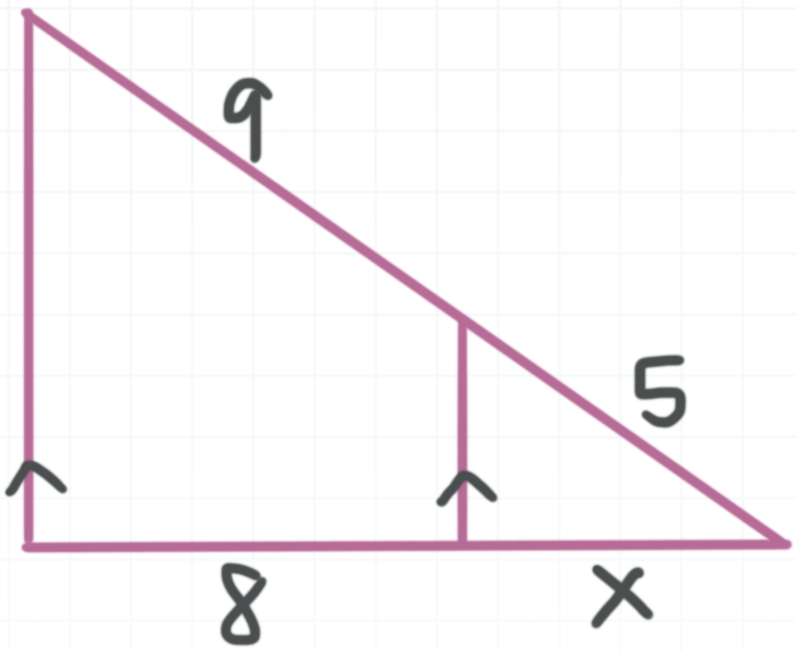
2.5. Set up a proportion.

$$\frac{8}{10}=\frac{2}{x}$$

$$8x=20$$

$$x=2.5$$

■ 2. Solve for x.



Solution:

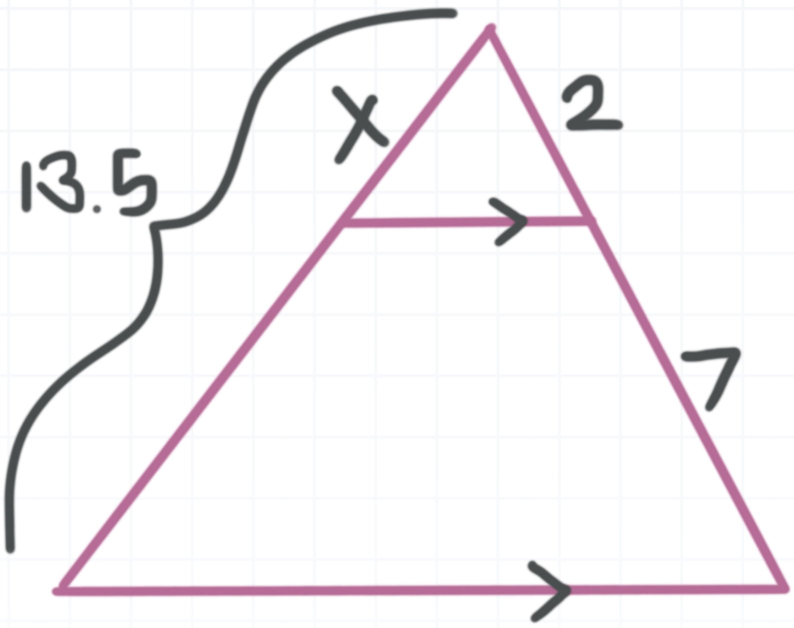
$40/9$. Set up a proportion.

$$\frac{9}{8} = \frac{5}{x}$$

$$9x = 40$$

$$x = \frac{40}{9}$$

■ 3. Solve for x.



Solution:

3. Set up a proportion.

$$\frac{2}{x}=\frac{7}{13.5-x}$$

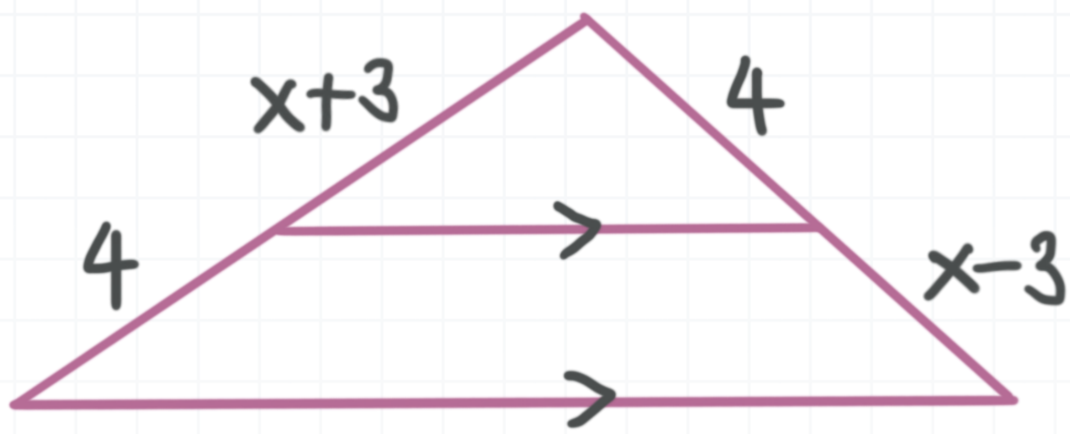
$$2(13.5-x)=7x$$

$$27-2x=7x$$

$$27=9x$$

$$x=3$$

■ 4. Solve for x.



Solution:

5. Set up a proportion.

$$\frac{x+3}{4} = \frac{4}{x-3}$$

$$(x+3)(x-3) = (4)(4)$$

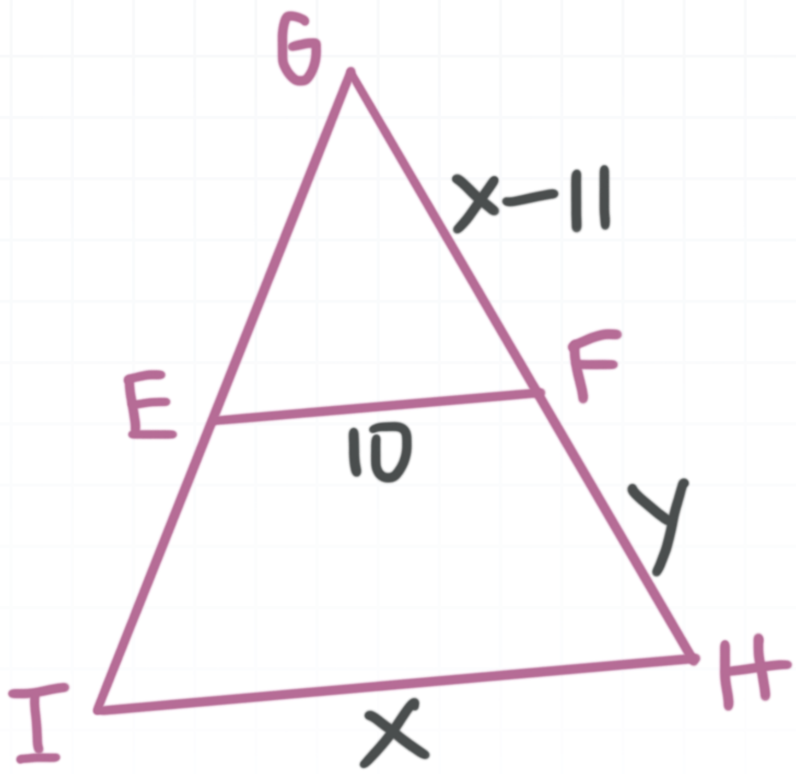
$$x^2 - 9 = 16$$

$$x^2 = 25$$

$$x = 5$$

MIDSEGMENTS OF TRIANGLES

1. $\overline{EF}$ is a midsegment of $\triangle IGH$. Find x and y.



Solution:

$x = 20$ and $y = 9$. Because EF is a midsegment of $\triangle IGH$,

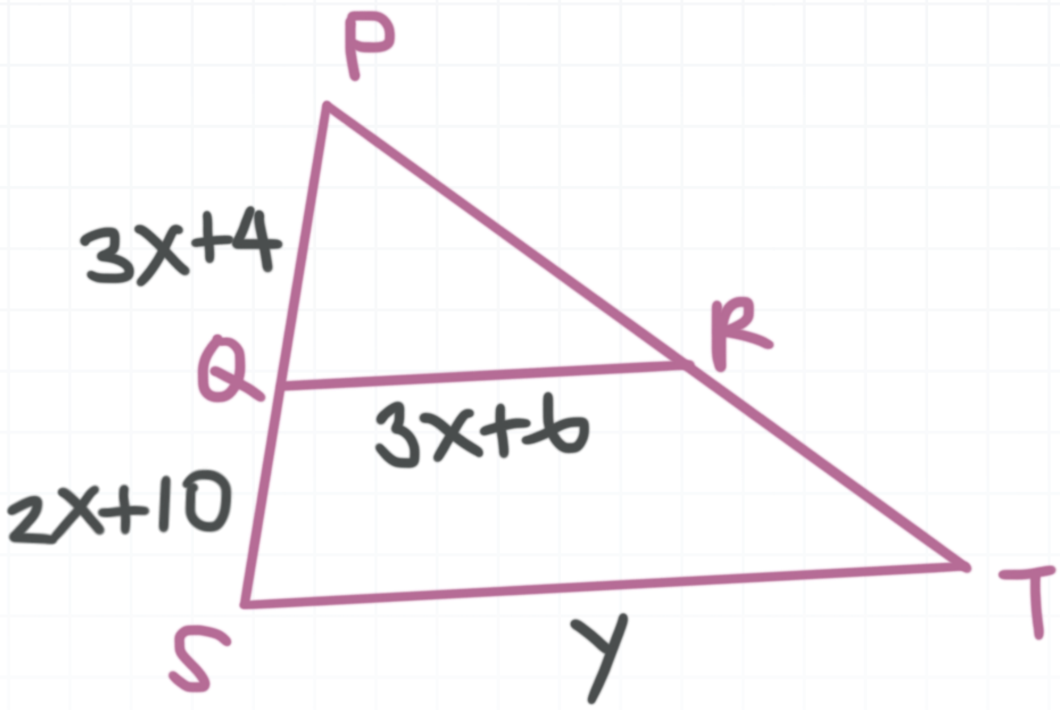
$$IH = 2(EF)$$

$$x = 2(10)$$

$$x = 20$$

Then $GF = 20 - 11 = 9$, we know that $y = 9$.

■ 2. $\overline{QR}$ is a midsegment of $\triangle SPT$. Find x and y.



Solution:

$x = 6$ and $y = 48$. Because $SQ = PQ$, we get

$$2x + 10 = 3x + 4$$

$$x = 6$$

Then we can use $x = 6$ to find the length of the midsegment.

$$QR = 3x + 6$$

$$QR = 3(6) + 6$$

$$QR = 24$$

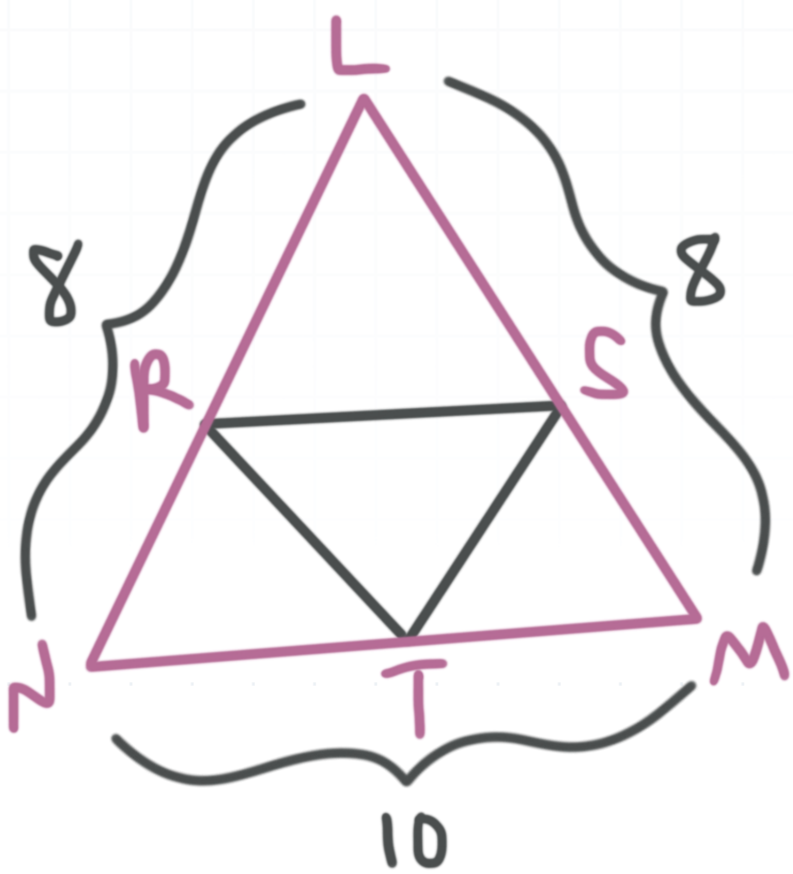
Then the value of y is given by

$$ST = 2QR$$

$$y = 2(24)$$

$$y = 48$$

■ 3. $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$ are midsegments of $\triangle NLM$. Find the perimeter of quadrilateral $RTMS$.



Solution:

18. The four side lengths are given by

$$RT = \frac{1}{2}(8) = 4$$

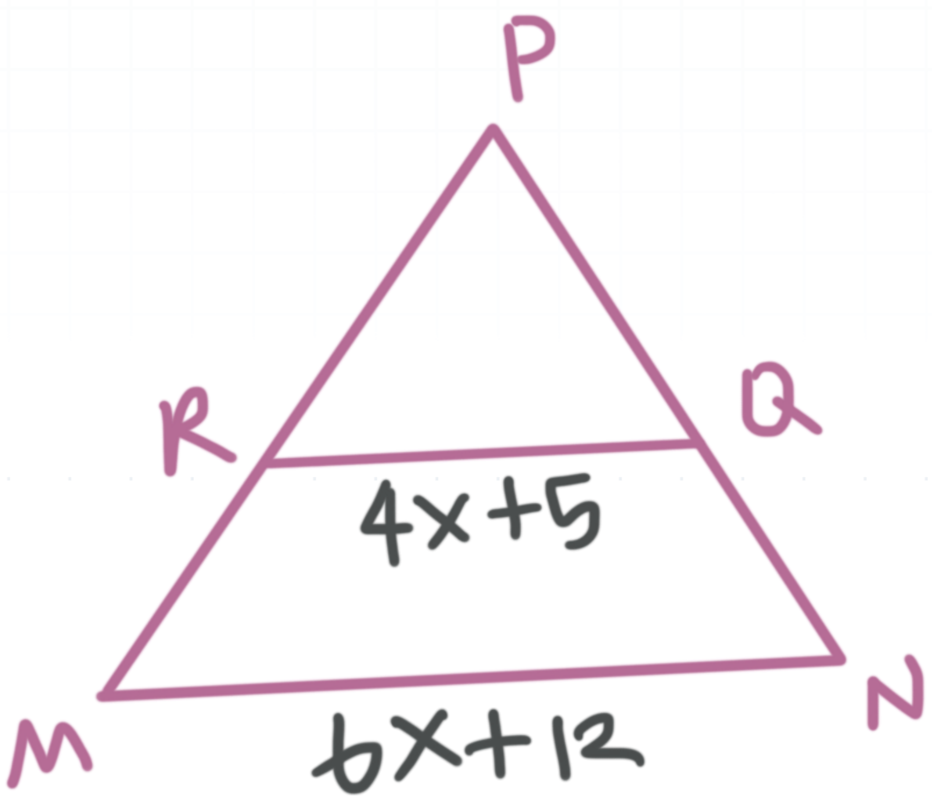
$$SM = \frac{1}{2}(8) = 4$$

$$TM = \frac{1}{2}(10) = 5$$

$$RS = \frac{1}{2}(10) = 5$$

Then the perimeter of $RTMS$ is $4 + 4 + 5 + 5 = 18$.

■ 4. $\overline{RQ}$ is a midsegment of $\triangle MPN$. Find x and MN.



Solution:

$x = 1$ and $MN = 18$. From the formula for the midsegment of a triangle, we get

$$RQ = \frac{1}{2}MN$$

$$4x + 5 = \frac{1}{2}(6x + 12)$$

$$8x + 10 = 6x + 12$$

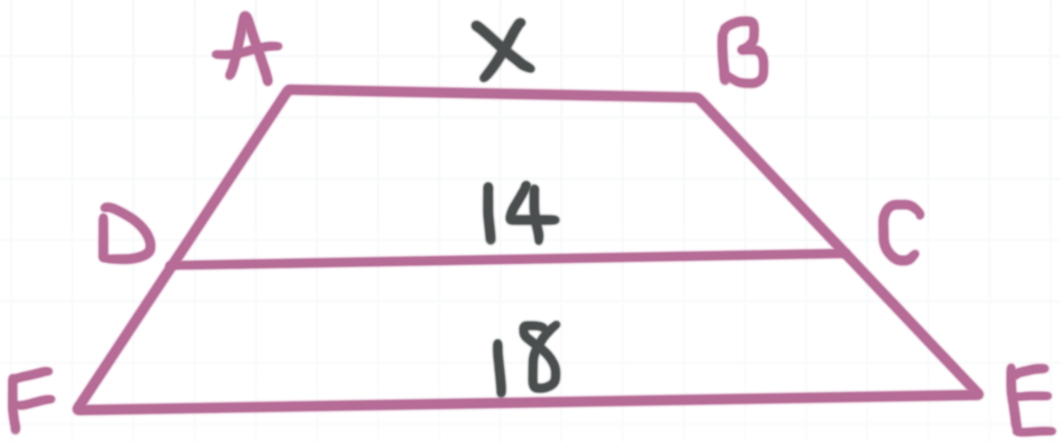
$$2x = 2$$

$$x = 1$$

Then MN is $MN = 6(1) + 12 = 18$.

MIDSEGMENTS OF TRAPEZOIDS

■ 1. The trapezoid has midsegment $\overline{DC}$. Find the value of x.



Solution:

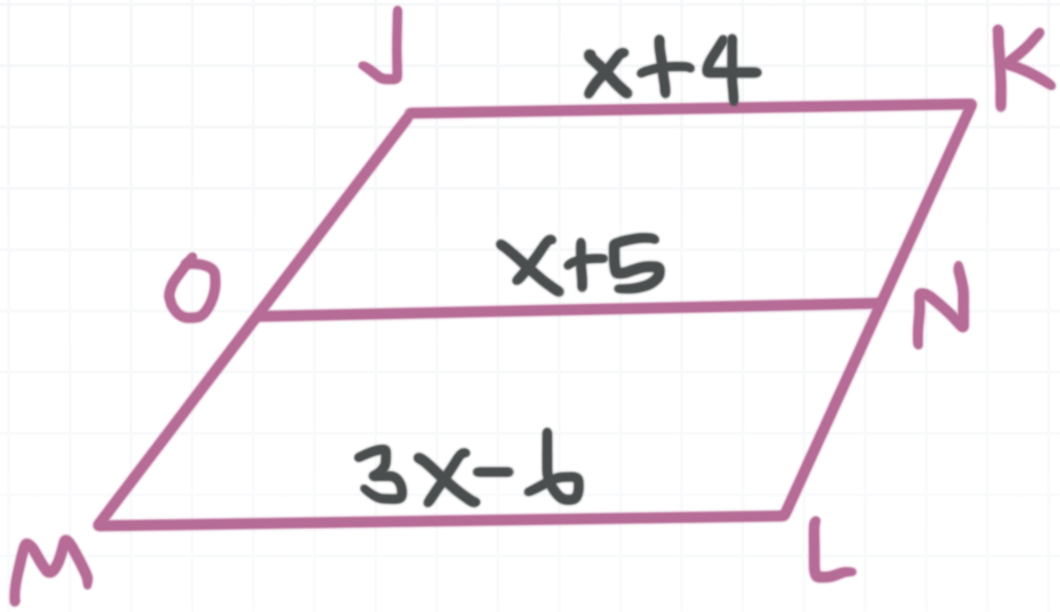
10. The length of the midsegment will be given by

$$DC = \frac{1}{2}(AB + FE)$$

$$14 = \frac{1}{2}(x + 18)$$

$$x = 10$$

■ 2. $\overline{ON}$ is a midsegment of trapezoid $JKLM$. Find JK, ON, and ML.



Solution:

10, 11, 12. The length of ON will be given by

$$ON = \frac{1}{2}(JK + ML)$$

$$x + 5 = \frac{1}{2}(x + 4 + 3x - 6)$$

$$x + 5 = 2x - 1$$

$$x = 6$$

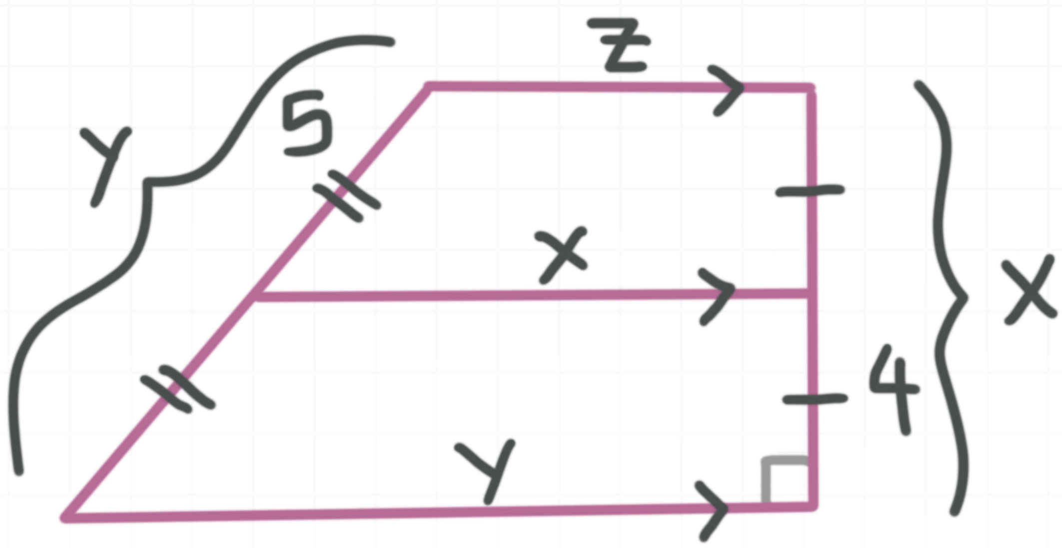
If $x = 6$, then the lengths of the line segments are

$$JK = 6 + 4 = 10$$

$$ON = 6 + 5 = 11$$

$$ML = 3(6) - 6 = 12$$

■ 3. Find x, y, and z.



Solution:

8, 10, 6. We know from the formula for the midsegment of a trapezoid, and the fact that $x = 2(4) = 8$ and $y = 2(5) = 10$, we get

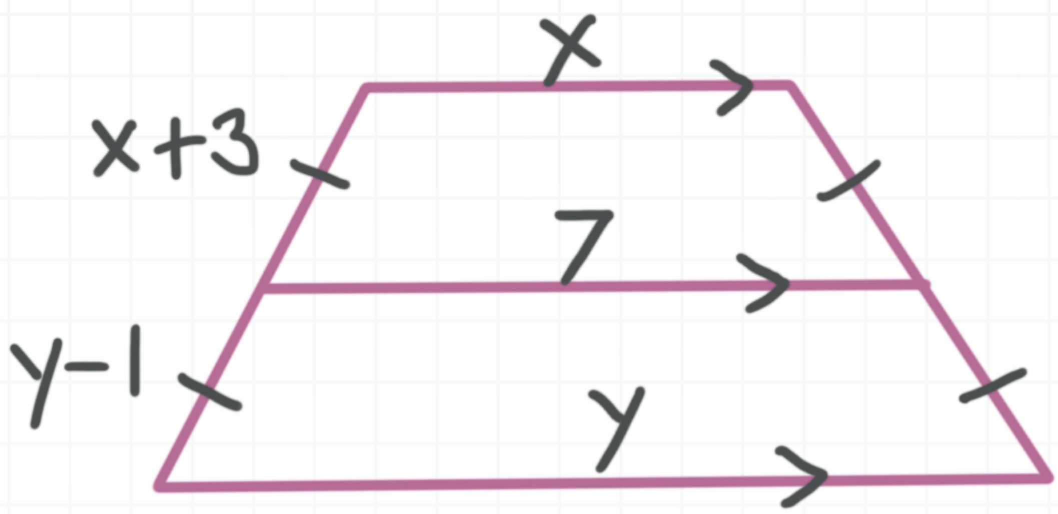
$$x = \frac{1}{2}(y + z)$$

$$8 = \frac{1}{2}(10 + z)$$

$$10 + z = 16$$

$$z = 6$$

■ 4. Find x and y.



Solution:

$x = 5$ and $y = 9$. Given congruent segments on the left side of the trapezoid, we get

$$x + 3 = y - 1$$

$$x - y = -4$$

And from the formula for the midsegment of a trapezoid, we get

$$\frac{1}{2}(x + y) = 7$$

$$x + y = 14$$

Use elimination to solve the system of equations by subtracting $x - y = -4$ from $x + y = 14$.

$$x + y - (x - y) = 14 - (-4)$$

$$x + y - x + y = 14 + 4$$

$$2y = 18$$

$$y = 9$$

Then

$$x + y = 14$$

$$x + 9 = 14$$

$$x = 5$$

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