CV Numerical Exercises

1- Edge Detection: Sobel Filter

- Show that Sobel mask filtering is equivalent to applying a 1-D differentiation mask [-1 0 1] followed by a smoothing mask [1 2 1]’

- Sobel Filter: image window

-1	**0**	**1**
-2	**0**	**2**
-1	**0**	**1**

a1	**a2**	**a3**
a4	**a5**	**a6**
a7	**a8**	**a9**

2- Discrete Approximation of the Second Derivative

- Kernel [1 -2 1] is used as the discrete approximation of the second derivative $\frac{\partial^2 I}{\partial x^2}$.
- How is the 3x3 kernel, used to approximate the 2D Laplacian derived from the given kernel? Apply the 2D Laplacian kernel to the center pixel of the following image (show all convolution steps).

3	2	1
6	5	4
9	8	7

3- Laplacian of Gaussian Derivation

- Consider the Laplacian of a Gaussian function to design a filter, compute:

$$h(x, y) = \frac{\partial^2 G(x,y)}{\partial x^2} + \frac{\partial^2 G(x,y)}{\partial y^2},$$

where *G(x,y)* is the Gaussian function:

$$G(x, y) = e^{-\frac{1}{2\sigma^2}(x^2+y^2)}$$

4- LoG Discrete Example Derivation

- Use the *h(x, y)* obtained in **(3)** to design the 3×3 mask with point coordinates (-1,-1), (-1,0), (-1, 1), (0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1).
- Assume that $\sigma = \sqrt{0.5}$ and calculate the coefficients of the mask. Note that they do not need to be integers.

- Does the mask you designed in (4) correspond to a low-pass or a high-pass filter? Explain?

5) Morphological Processing Application

- Suppose a satellite image of a region can be thresholded so that the water pixels are 1’s. However, bridges across rivers produce thin lines of 0’s.

- a) Describe how to restore the bridge pixels to the water region.
- b) Describe how to detect the thin bridges as separate objects.

6) Image Enhancement: Mean Filter

- Assume that we have many noisy versions $g_i(x, y)$of the same image $f(x, y)$ i.e.

$$g_i(x, y) = f(x, y) + \eta_i(x, y)$$

- where the noise η_i is zero-mean and all point-pairs (x, y) are uncorrelated. Then, we can reduce noise by taking the mean of all the noisy images

$$\bar{g}(x, y) = \frac{1}{M}\sum_{i=1}^{M} g_i(x, y)$$

- Prove that

$$E\{\bar{g}(x, y)\} = f(x, y)$$

7) Similarity Transformation

- Suppose we have the template of a square with known corner coordinates:
  - $x_1=(-1,-1)$, $x_2=(1,-1)$, $x_3=(1,1)$, $x_4=(-1,1)$.
- We are looking for the same shape in a database of images where the best candidate is a similar looking shape with corner coordinates:
  - $x'_1=(-0.6,0.6)$, $x'_2=(-0.6,1.4)$, $x'_3=(-1.4,1.4)$, $x'_4=(-1.4,0.6)$.

- Which is the relative orientation, scale and translation (similarity transform) between the two shapes?

8) Hough Transform

- Coarsely estimate the line defined by the edge points at locations (1.1, 9), (3, 15.3), (5.1, 20,9) using Hough transform

- Line equation: $y = mx + b$

- In Hough space, each point of an edge, forms a line, following the given equation:
  - $b = x_{(i)}m - y_{(i)}$

9) Co-occurrence Matrix

- Construct the co-occurrence matrices $C_{(1,2)}$, $C_{(2,2)}$ and $C_{(2,3)}$ for the image below:

1	**1**	**0**	**0**
1	**1**	**0**	**0**
0	**0**	**2**	**2**
0	**0**	**2**	**2**