
📚 Math Notes: Algebra, Calculus, and More!

1️⃣ Algebra: The Language of Mathematics

🔹 Exponents and Logarithms

Laws of Exponents:

1. **Multiplication Rule:** am×an=am+na^m \times a^n = a^{m+n}am×an=am+n
2. **Division Rule:** aman=am−n\frac{a^m}{a^n} = a^{m-n}anam=am−n
3. **Power Rule:** (am)n=am⋅n(a^m)^n = a^{m \cdot n}(am)n=am⋅n
4. **Negative Exponent:** a−n=1ana^{-n} = \frac{1}{a^n}a−n=an1
5. **Zero Exponent:** a0=1a^0 = 1a0=1

Logarithms: The Inverse of Exponents

- If ax=ba^x = bax=b, then loga(b)=x\log_a(b) = xloga(b)=x
- **Log Rules:**
  - log(ab)=loga+logb\log(ab) = \log a + \log blog(ab)=loga+logb
  - log(ab)=loga−logb\log \left(\frac{a}{b}\right) = \log a - \log blog(ba)=loga−logb
  - log(an)=nloga\log(a^n) = n \log alog(an)=nloga
  - loga(a)=1\log_a(a) = 1loga(a)=1, loga(1)=0\log_a(1) = 0loga(1)=0

💡 **Example:**

- log2(8)=3\log_2(8) = 3log2(8)=3 because 23=82^3 = 823=8
- log10(1000)=3\log_{10}(1000) = 3log10(1000)=3 because 103=100010^3 = 1000103=1000

2️⃣ Quadratic Equations

A quadratic equation is in the form:

ax2+bx+c=0ax^2 + bx + c = 0ax2+bx+c=0

where a,b,ca, b, ca,b,c are constants, and a≠0a \neq 0a=0.

🔹 Solving Quadratics

1. **Factoring**
    - Example: x2−5x+6=0x^2 - 5x + 6 = 0x2−5x+6=0
    - Factor: (x−2)(x−3)=0(x - 2)(x - 3) = 0(x−2)(x−3)=0
    - Solutions: x=2,3x = 2, 3x=2,3
2. **Quadratic Formula**
    - Formula:
3. x=−b±b2−4ac2ax = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}x=2a−b±b2−4ac
    - Example: Solve 2x2−3x−5=02x^2 - 3x - 5 = 02x2−3x−5=0
    - a=2,b=−3,c=−5a = 2, b = -3, c = -5a=2,b=−3,c=−5
    - Discriminant: (−3)2−4(2)(−5)=9+40=49(-3)^2 - 4(2)(-5) = 9 + 40 = 49(−3)2−4(2)(−5)=9+40=49
    - Solutions: x=3±74=104,−44x = \frac{3 \pm 7}{4} = \frac{10}{4}, \frac{-4}{4}x=43±7=410,4−4 x=2.5,−1x = 2.5, -1x=2.5,−1

3️⃣ Calculus: The Study of Change

🔹 Limits and Continuity

The limit of a function describes its behavior as xxx approaches a value.

Basic Limit Laws

1. limx→cf(x)+g(x)=limx→cf(x)+limx→cg(x)\lim_{x \to c} f(x) + g(x) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)limx→cf(x)+g(x)=limx→cf(x)+limx→cg(x)
2. limx→c[f(x)g(x)]=limx→cf(x)⋅limx→cg(x)\lim_{x \to c} [f(x)g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)limx→c[f(x)g(x)]=limx→cf(x)⋅limx→cg(x)
3. limx→ckf(x)=klimx→cf(x)\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)limx→ckf(x)=klimx→cf(x)

💡 **Example:**

limx→3(x2+2x)=32+2(3)=9+6=15\lim_{x \to 3} (x^2 + 2x) = 3^2 + 2(3) = 9 + 6 = 15x→3lim(x2+2x)=32+2(3)=9+6=15

🔹 Derivatives: The Rate of Change

The derivative of a function measures how it changes.

- Definition: f′(x)=limh→0f(x+h)−f(x)hf'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}f′(x)=h→0limhf(x+h)−f(x)
- **Power Rule:** ddxxn=nxn−1\frac{d}{dx} x^n = n x^{n-1}dxdxn=nxn−1
- **Product Rule:** (fg)′=f′g+fg′(fg)' = f'g + fg'(fg)′=f′g+fg′

- **Quotient Rule:** (fg)′=f′g−fg′g2\left(\frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}(gf)′=g2f′g−fg′
- **Chain Rule:** ddxf(g(x))=f′(g(x))⋅g′(x)\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)dxdf(g(x))=f′(g(x))⋅g′(x)

💡 **Example:**

f(x)=x3+4x2−2x+7f(x) = x^3 + 4x^2 - 2x + 7f(x)=x3+4x2−2x+7 f′(x)=3x2+8x−2f'(x) = 3x^2 + 8x - 2f′(x)=3x2+8x−2

🔹 Integration: The Reverse of Derivatives

∫xndx=xn+1n+1+C\int x^n dx = \frac{x^{n+1}}{n+1} + C∫xndx=n+1xn+1+C

- **Definite Integral:** ∫abf(x)dx\int_a^b f(x) dx∫abf(x)dx gives the area under the curve from x=ax = ax=a to x=bx = bx=b.

💡 **Example:**

∫(3x2+8x−2)dx=x3+4x2−2x+C\int (3x^2 + 8x - 2) dx = x^3 + 4x^2 - 2x + C∫(3x2+8x−2)dx=x3+4x2−2x+C

4️⃣ Probability and Statistics

🔹 Probability Basics

- Probability of an event: P(A)=Number of favorable outcomesTotal number of outcomesP(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}P(A)=Total number of outcomesNumber of favorable outcomes
- **Addition Rule:** P(A∪B)=P(A)+P(B)−P(A∩B)P(A \cup B) = P(A) + P(B) - P(A \cap B)P(A∪B)=P(A)+P(B)−P(A∩B)
- **Multiplication Rule:** If events are independent, P(A∩B)=P(A)⋅P(B)P(A \cap B) = P(A) \cdot P(B)P(A∩B)=P(A)⋅P(B).

💡 **Example:**

- Rolling a die: P(rolling a 4)=16P(\text{rolling a 4}) = \frac{1}{6}P(rolling a 4)=61.
- Drawing an ace from a deck: P(Ace)=452=113P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}P(Ace)=524=131.

🔹 Normal Distribution (Bell Curve)

- **Mean (μ):** The average value.
- **Standard Deviation (σ):** Measures spread of data.
- **Empirical Rule:**

    - 68% of data falls within **1σ** of the mean.
    - 95% falls within **2σ**.
    - 99.7% falls within **3σ**.

💡 **Example:** If heights of students are normally distributed with μ=170μ = 170μ=170 cm and σ=10σ = 10σ=10 cm:

- 68% of students have heights between **160 cm and 180 cm**.
- 95% are between **150 cm and 190 cm**.

🔥 Fun Math Fact!

The number **π (pi) ≈ 3.1415926535...** is **irrational**, meaning it **never ends** and **never repeats**! 🔥