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Paper Code : BCAC201 Discrete Structures
UPID : 200050

Time Allotted : 3 Hours | Full Marks : 70

*The Figures in the margin indicate full marks.*
*Candidate are required to give their answers in their own words as far as practicable*

## Group-A (Very Short Answer Type Question)

1. Answer any ten of the following : [ 1 x 10 = 10 ]

(i) Composition of Mappings is _____ but not _____ in general.

(ii) Which rule of Inference is used in deriving the conclusion: "If it is Sunday, then the Mall will be crowded. It is Sunday. Thus, the Mall is crowded."

(iii) The maximum number of diagonals can be drawn in a hexagon is _____.

(iv) What is the identity element in the group G = {2, 4, 6, 8} under ordinary multiplication modulo 10?

(v) The sum of the out-degrees of all the vertices in a digraph is 20. Then the number of edges in the graph is _____.

(vi) If set A and B have 2 and 5 elements respectively, then the number of subsets of set (A X B) is _____.

(vii) If A = {{Φ}, {Φ, {Φ}}}, then power set of A has _____ elements.

(viii) Let P and Q be propositional symbols such that P is equivalent to Q. Then _____ will be tautology.

(ix) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is _____.

(x) The set of integer modulo *n* forms a field if *n* is _____ number.

(xi) If *F1, F2* and *F3* be three propositions such that both of the following two propositions are Tautologies:
$(F1 \wedge F2) \rightarrow F3 \quad and \quad F1 \wedge (F2 \rightarrow F3)$
Then given the following statements based on the above two propositions, which one is correct?
a: Both F1 and F2 are Tautologies but F3 is Contradiction
b: Both F1 and F2 are Contradictions
c: F1 is Tautology but F2 is Contradiction
d: F1 is Contradiction but F2 is Tautology

(xii) If *T(n)* be the time to *recursively* calculate the factorial of a integer number *n>1*, then *T(n)* must satisfy the recurrence relation _____

## Group-B (Short Answer Type Question)

Answer any three of the following : [ 5 x 3 = 15 ]

2. $Prove\ that\ A - (B \cup C) = (A - B) \cap (A - C)$ [5]

3. Construct the truth table of the following proposition: [5]
$(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q))$

4. *Check the validity of the following argument (H1: 1st Premises, H2: 2nd Premises, C: Conclusion):* [5]
$H1 : P \rightarrow (Q \rightarrow R);\ H2 : P \wedge Q;\ C : R$

5. $If\ (R, +, .)\ is\ a\ Ring\ such\ that\ a^2 = a\ \forall a \in R,\ prove\ that$ [5]
$(i)\ a + a = 0\ \forall a \in R$
$(ii)\ a \cdot b = 0 \Rightarrow a = b$

6. $Solve\ the\ recurrence\ relation :$ [5]
$a_n + 3a_{n-1} + 3a_{n-2} + a_{n-3} = 0;\ a_0 = 1, a_1 = -2, a_2 = -1$

## Group-C (Long Answer Type Question)

Answer any three of the following : [ 15 x 3 = 45 ]

7. (a) $Let\ f : \mathbb{R} \rightarrow \mathbb{R}\ and\ g : \mathbb{R} \rightarrow \mathbb{R}\ be\ defined\ by f(x) = x^2 + 3, g(x) = x + 6.\ Then\ find\ f \circ g\ and\ g \circ f.$ [ 5 ]

(b) Show that the following relation R defined on Z is symmetric, transitive but not reflexive. R = (a, b): a, b ∈ Z and ab>0. [ 5 ]

(c) Show that the following function g is neither surjective nor injective: $g : \mathbb{R} \rightarrow \mathbb{R}\ defined\ by\ g(x) = x^2, x \in \mathbb{R}$ [ 5 ]

8. (a) $Without\ truth\ table,\ prove\ that\ P \wedge (P \vee Q) \equiv P$ [5]

(b) Prove that the following argument is valid: [5]
$P \to Q, P \to R, Q \to R \vdash R$

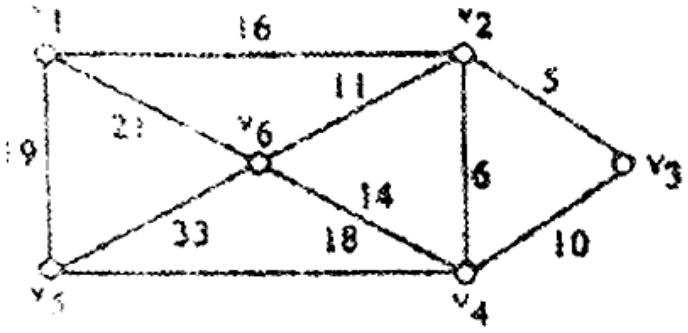
(c) Without using truth table, prove that the following proposition is a Tautology: [5]
$(P \wedge Q) \to (P \to Q)$

9. (a) For any three sets $A, B, C$, show that $A - (B - C) = (A - B) \cup (A \cap C)$ [5]

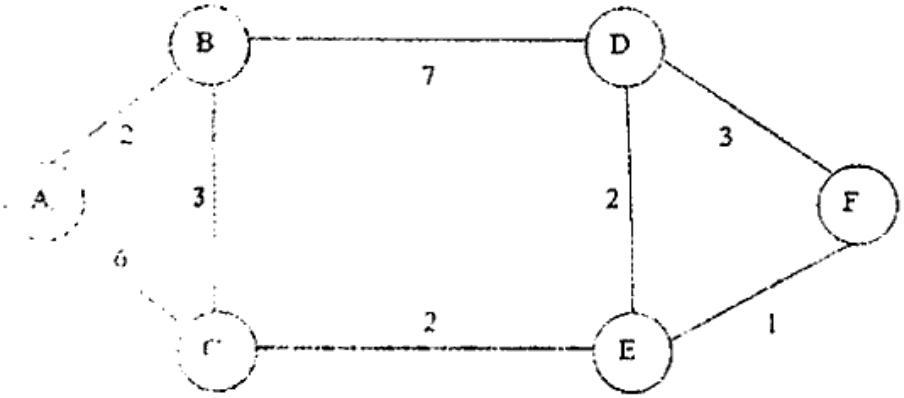
(b) How many numbers must be selected from the set {1, 2, 3, 4} to guarantee that at least one pair of these numbers add up to 7? [5]

(c) If $A, B$ are non-empty sets, then prove that $(A - B)$ and $(A \cap B)$ are pairwise disjoint. [5]

10. (a) Find the minimal spanning tree (MST) of the following graph using Kruskal's Algorithm and also calculate the weight of the MST: [5]




(b) Using Dijkstra Algorithm, find the shortest path between A to F in the following graph and also calculate the length of the shortest path: [5]




(c) Given the following distance matrix of an weighted graph, find the minimal spanning tree (MST) using Prim's Algorithm and also determine the weight of the MST (Distance ∞ means no direct edge between the vertices): [5]



| Distance | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 0 | 12 | ∞ | ∞ | 14 | ∞ | 20 |
| B | 12 | 0 | 12 | 10 | 6 | ∞ | ∞ |
| C | ∞ | 12 | 0 | 4 | ∞ | ∞ | ∞ |
| D | ∞ | 10 | 4 | 0 | ∞ | 6 | ∞ |
| E | 14 | 6 | ∞ | ∞ | 0 | 6 | 8 |
| F | ∞ | ∞ | ∞ | 6 | 6 | 0 | 4 |
| G | 20 | ∞ | ∞ | ∞ | 8 | 4 | 0 |

11. (a) If G is a simple graph with n vertices and k components, prove that G can have atmost $\frac{(n-k)(n-k+1)}{2}$ number of edges. [9]

(b) Prove that the number of internal vertices in a binary tree is one less than the number of pendant vertices. [3]

(c) If $\delta(G)$ and $\Delta(G)$ be the min degree and max degree of an $(p,q)$ graph respectively, prove that $\delta(G) \leq \frac{2q}{p} \leq \Delta(G)$ [3]

*** END OF PAPER ***