PH3101: Classical Mechanics

Name of student: Suvajit Dey
Roll No.: sd21ms198

Double Pendulum and its Phase Spaces

I have taken a double pendulum system where bobs has same mass m and same length l. Here Angular displacement of first bob is $\theta_1(q_1)$ and that of second bob is $\theta_2(q_2)$.

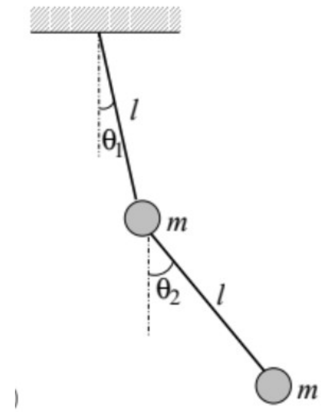


Figure 1: Double Pendulum

Parameters

- Mass of the bobs$(m) = 1$ kg
- Length of the string$(l) = 1$ m
- Acceleration due to gravity$(g) = 9.8 m/s^2$

Initial Conditions:

- Initial angular displacement of first bob$(q_1(0)) = \frac{\pi}{2}$
- Initial angular displacement of second bob$(q_2(0)) = \frac{\pi}{6}$
- Initial angular velocity of first bob$(\dot{q}_1(0)) = 0$
- Initial angular velocity of second bob$(\dot{q}_2(0)) = 0$

Lagrangian of the System is :

$$L = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1 + m_2) g l_1 \cos\theta_1 + m_2 g l_2 \cos\theta_2$$

Generalized Momenta of the System are:

$$p_{\theta_1} = \frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$p_{\theta_2} = \frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2)$$

Hamiltonian of the System is:

$$H = \frac{m_2 l_2^2 p_{\theta_1}^2 + (m_1 + m_2) l_1^2 p_{\theta_2}^2 - 2 m_2 l_1 l_2 p_{\theta_1} p_{\theta_2} \cos(\theta_1 - \theta_2)}{2 m_2 l_1^2 l_2^2 \left[m_1 + m_2 \sin^2(\theta_1 - \theta_2)\right]} - (m_1 + m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$$

First Order Differential Equations are:

$$\dot{\theta}_1 = \frac{\partial H}{\partial p_{\theta_1}} = \frac{l_2 p_{\theta_1} - l_1 p_{\theta_2} \cos(\theta_1 - \theta_2)}{l_1^2 l_2 \left[m_1 + m_2 \sin^2(\theta_1 - \theta_2)\right]}$$

$$\dot{\theta}_2 = \frac{\partial H}{\partial p_{\theta_2}} = \frac{-m_2 l_2 p_{\theta_1} \cos(\theta_1 - \theta_2) + (m_1 + m_2) l_1 p_{\theta_2}}{m_2 l_1 l_2^2 \left[m_1 + m_2 \sin^2(\theta_1 - \theta_2)\right]}$$

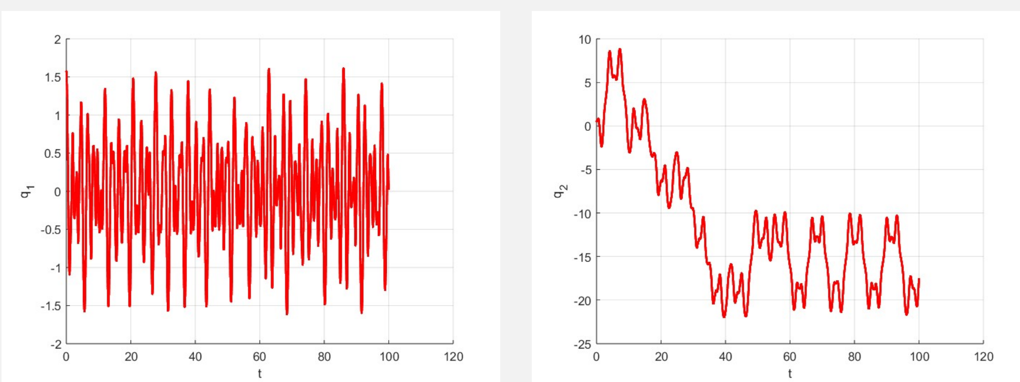
$$\dot{p}_{\theta_1} = -\frac{\partial H}{\partial \theta_1} = -(m_1 + m_2) g l_1 \sin\theta_1 - h_1 + h_2 \sin[2(\theta_1 - \theta_2)]$$

$$\dot{p}_{\theta_2} = -\frac{\partial H}{\partial \theta_2} = -m_2 g l_2 \sin\theta_2 + h_1 - h_2 \sin[2(\theta_1 - \theta_2)]$$

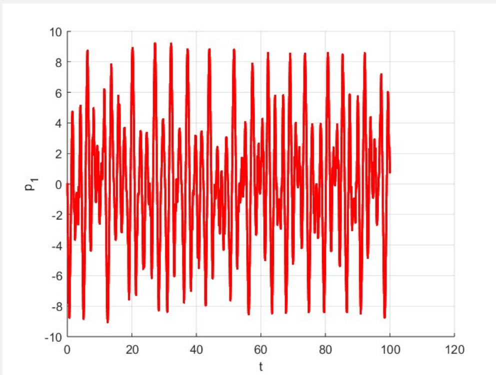
where:

$$h_1 = \frac{p_{\theta_1} p_{\theta_2} \sin(\theta_1 - \theta_2)}{l_1 l_2 \left[m_1 + m_2 \sin^2(\theta_1 - \theta_2)\right]}$$

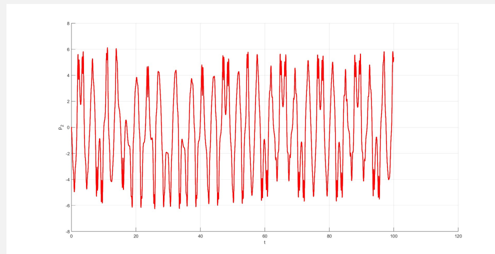
$$h_2 = \frac{m_2 l_2^2 p_{\theta_1}^2 + (m_1 + m_2) l_1^2 p_{\theta_2}^2 - 2 m_2 l_1 l_2 p_{\theta_1} p_{\theta_2} \cos(\theta_1 - \theta_2)}{2 l_1^2 l_2^2 \left[m_1 + m_2 \sin^2(\theta_1 - \theta_2)\right]^2}$$



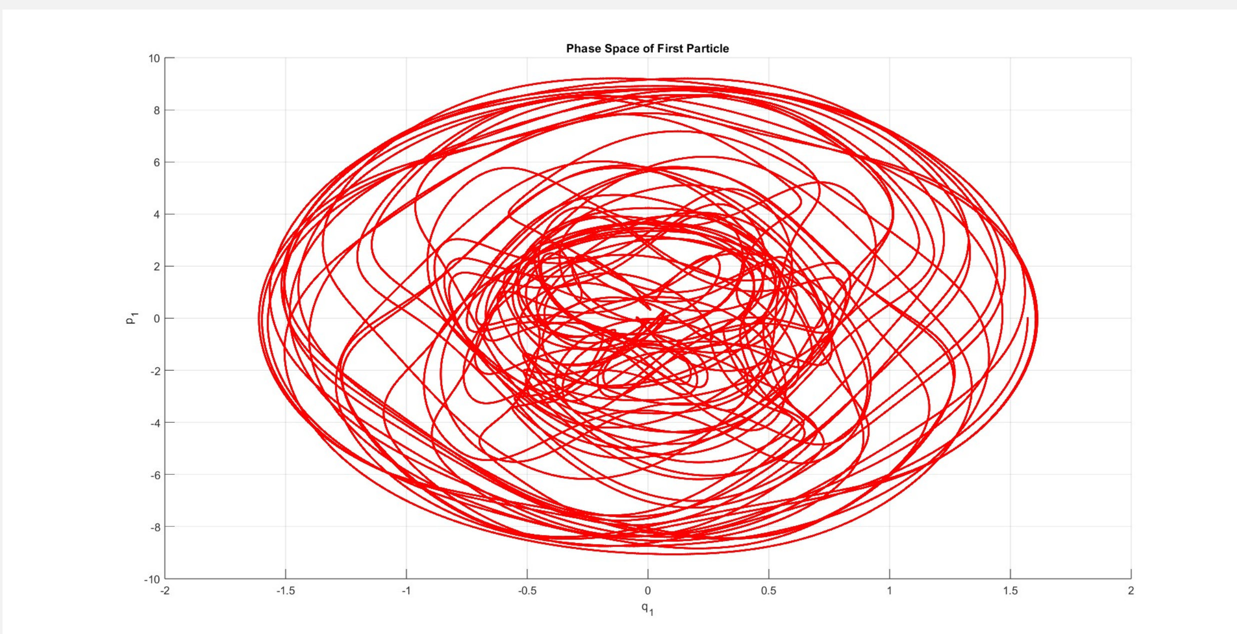
(a) q_1 versus time (b) q_2 versus time



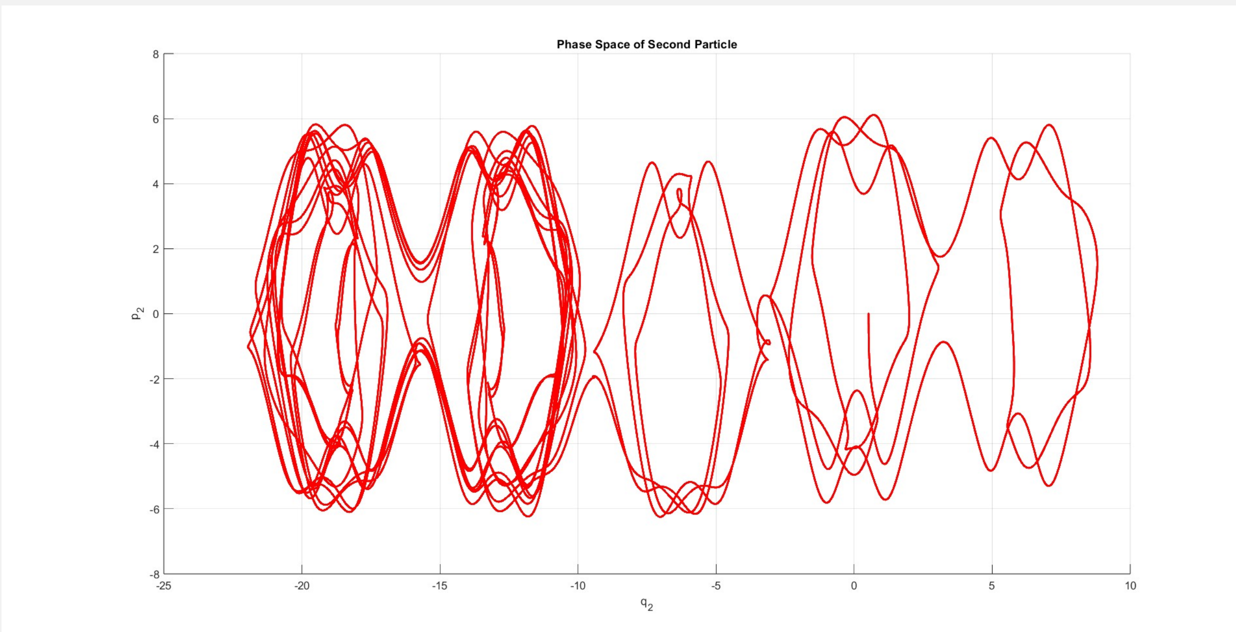
(a) p_1 versus time



(b) p_2 versus time



Phase Plot of First Particle



Phase plot of Second Particle