

If you are using a jupyter notebook (recommended), then keep all your programs in a single notebook. A good programming style is to define a function for one task with clearly defined input (arguments) and output. For plots you may use matplotlib (if you are using python) or gnuplot (if you are using c or fortran) or LsqFit module if you are using Julia.

If you are planning to submit separate programs, then please follow the guideline below:

- Keep all files of a worksheet in a single folder.
- Follow a systematic naming convention. You may name the program files as Q1.py or Q1a.py, Q1b.py for question 1 (if you have created multiple files for a single question). The data file should be named as Q1-data-a.dat and so on.
- Finally compress the entire folder as a single .zip or .tgz (using `tar cvfz archive.tgz folder-name/`, and submit the file in WeLearn.

1. In this problem, we shall generate a set of random numbers whose distribution (not normalized) is given as

$$w(x) = \frac{1}{1 + (x - 1)^2} \quad \forall x \in \mathbb{R}.$$

To achieve this we shall use the Metropolis algorithm in the following way:

- Plot the distribution as a function of x to get an idea about its behavior.
- Start a random walk of steps $1e6$ steps from $x_0 \sim 0.5$ (*why?*). We are choosing a starting point which is close to the maximum of this distribution.
 - Take a step δx by choosing a floating point random number between -1 and 1 (corresponds to a maximum step size 1).
 - Find

$$a = \min \left[1, \frac{w(x_i + \delta x)}{w(x_i)} \right].$$
 - Accept the step with a probability a .
 - Store the result and take the next step.
- Generate the histogram data and normalize it.
- Plot the normalized distribution. On the same plot, draw normalized $w(x)$.

2. Calculate the integral

$$\int_0^1 w(x) dx, = \int_0^1 \frac{x^3}{e^x - 1}$$

using *importance sampling* method. You may proceed as follows:

- Choose a trial function of the form

$$\frac{1}{b + (x - x_o)^2}$$

. Choose b and x_{circ} (intuitively).

- Calculate the integral using the importance sampling method.