Differentiation of $\sigma(x)$

$$y = \sigma(x) = \frac{1}{1+e^{-x}} \quad — (1)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1+e^{-x})^{-1}$$

$$\left[\frac{d}{dx}\frac{1}{U(x)} = \frac{d}{dx}\left[U(x)\right]^{-1} = -\frac{\frac{d}{dx}U(x)}{U(x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+e^{-x})^2} \times \frac{d}{dx}(1+e^{-x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+e^{-x})^2} \times \left(\frac{d}{dx}1 + \frac{d}{dx}e^{-x}\right) \qquad \left[\frac{d}{dx}(\text{Const}) = 0\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+e^{-x})^2} \times \left(0 + e^{-x} \times (-1)\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+e^{-x})^2} \times -e^{-x}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}} \quad \text{Ans}$$

Since, we can see $\sigma(x)$ in the derivative, lets try to express it in terms of $\sigma(x)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1+e^{-x})} \times \frac{e^{-x}+1-1}{(1+e^{-x})} = \sigma(x) \times \left[\frac{\cancel{(1+e^{-x})}}{\cancel{(1+e^{-x})}} - \frac{1}{1+e^{-x}}\right]$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \sigma'(x) = \sigma(x)\left(1-\sigma(x)\right)}$$

Derivation of tanh

$$y = \tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)$$

$$\left[\begin{array}{l} y = \frac{u}{v} \\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^{2}} \end{array}\right]$$

$$\frac{dy}{dx} = \frac{(e^{x}+e^{-x})\frac{d}{dx}(e^{x}-e^{-x}) - (e^{x}-e^{-x})\frac{d}{dx}(e^{x}+e^{-x})}{(e^{x}+e^{-x})^{2}}$$

$$\frac{dy}{dx} = \frac{(e^{x}+e^{-x})(e^{x}+e^{-x}) - (e^{x}-e^{-x})(e^{x}-e^{-x})}{(e^{x}+e^{-x})^{2}}$$

$$\frac{dy}{dx} = 1 - \left[\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right]^{2}$$

$$\frac{dy}{dx} = \tanh'(x) = 1 - [\tanh(x)]^{2}$$