Apply Bayes' Theorem to solve a medical diagnosis problem

Patient	Fever	Cough	Diagnosis
1	Yes	No	Flu
2	No	Yes	Flu
3	Yes	Yes	Pneumonia
4	Yes	Yes	Pneumonia
5	No	No	Healthy
6	Yes	No	Flu
7	No	Yes	Flu
8	Yes	Yes	Pneumonia
9	No	No	Healthy
10	Yes	Yes	Pneumonia

P(Flu) = 4/10 = 0.4

P(Pneumonia) = 4/10 = 0.4

P(Healthy) = 2/10 = 0.2

Likelihoods:

For Flu:

P(Fever=Yes|Flu)=4/4=1.0

P(Cough=Yes |Flu)=2/4=0.5

For Pneumonia:

P(Fever=Yes |Pneumonia)=4/4=1.0

P(Cough=Yes |Pneumonia)=4/4=1.0

For Healthy:

P(Fever=Yes |Healthy)=0

P(Cough=Yes |Healthy)=0

Applying Bayes' Theorem:

Let E denote the observed symptoms (Fever = Yes, Cough = Yes).

Calculate P(E) the total probability of observing Fever and Cough:

P(E) = P(E|Flu) * P(Flu) + P(E|Pneumonia) * P(Pneumonia) + P(E|Healthy) * P(Healthy)

P(E) = (1.0 * 0.5) * 0.4 + (1.0 * 1.0) * 0.4 + (0 * 0) * 0.2

P(E) = 0.2 + 0.4 + 0

P(E) = 0.6

Calculate P(Flu | E):

P(Flu | E) = P(E | Flu) * P(Flu) / P(E)

P(Flu | E) = (1.0 * 0.4) / 0.6

P(Flu | E) = 0.4 / 0.6

P(Flu | E) ≈ 0.67

Calculate P(Pneumonia | E):

P(Pneumonia | E) = P(E | Pneumonia) * P(Pneumonia) / P(E)

P(Pneumonia | E) = (1.0 * 0.4) / 0.6

P(Pneumonia | E) = 0.4 / 0.6

P(Pneumonia | E) = 2 / 3

P(Pneumonia | E) ≈ 0.67

According to Bayes' Theorem and the given data, if a patient presents with both fever and cough, the probabilities of diagnoses are approximately:

P(Flu | Fever = Yes, Cough = Yes) ≈ 0.67

P(Pneumonia | Fever = Yes, Cough = Yes) ≈ 0.67

Probability Matrix and Eigenvalues/Vectors

Probability Matrix:

A = [[0.67 0.33 0.00]

[0.33 0.67 0.00]

[0.00 0.00 0.00]]

Eigenvalues:

det(A−λ*I) = 0, where I is the identity matrix

The eigenvalues of matrix A are:

λ1 ≈ 1.00,

λ2 ≈ 0.34,

λ3 ≈ 0.00

Eigenvectors : ()

Corresponding eigenvectors are:

For λ1 ≈ 1.00:

V1 ≈ [[0.71]

[0.71]

[0.00]]

For λ2 ≈ 0.34:

V2 ≈ [[-0.71]

[0.71]

[0.00]]

For λ3 ≈ 0.00:

V3 ≈ [[0.00]

[0.00]

[1.00]]

Determinant of the Matrix

The determinant det(A) of matrix A:

det(A) = 0.67 * (0.67 * 1.00 - 0.00 * 0.33) - 0.33 * (0.33 * 1.00 - 0.00 * 0.33)

det(A) = 0.67 * 0.67 - 0.33 * 0.33

det(A) = 0.4489 - 0.1089

det(A) = 0.34