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Soln 1 Consider a random sample $(x_1, x_2, x_3, \dots, x_n)$

$\mu = \theta_1$ (mean) and $\sigma^2 = \theta_2$ (variance)

Likelihood function $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{x_i - \theta_1}{2\theta_2}\right)^2$

to max. take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2} \right]$$

(i) Differentiate wrt θ_1 (for θ_1)

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\Rightarrow \theta_1 = \sum_{i=1}^{n} \frac{x_i}{n} \quad [\text{Mean}]$$

(ii) Differentiate wrt θ_2 (for θ_2)

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2 \quad [\text{Variance}]$$

Sol^n 2 Binomial distribution $B(n, \theta)$ $p = \theta$, $q = 1 - \theta$

pmf (Probability mass function)

$$f(x; n, \theta) = {}^{n}C_x \, \theta^x (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i} \, \theta^{x_i} (1-\theta)^{n-x_i}$$

$\Rightarrow$

Taking log on both sides

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln {}^{n}C_{x_i} + x_i \ln\theta + (n - x_i) \ln(1-\theta) \right]$$

differentiate wrt θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n - x_i}{1-\theta} \right] = 0$$

Find θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n - x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{(1-\theta) x_i - \theta (n - x_i)}{\theta (1-\theta)} \right] = 0$$

$$\sum_{i=1}^{n} (1-\theta) x_i - (n - x_i)\theta = 0$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i m$$

$$\Rightarrow \boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}} \rightarrow \text{MLE.}$$