task_10.3

Machine Learning (WiSe 2025/2026)

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Assignment 10 Task 3

Part A

Iteration 1

Centroids: $C_1(1,0)$ and $C_2(5,6)$
Points: $A(1,2), B(3,2), C(5,2), D(1,1) and E(5,1)$

Distance of C_1 to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1-1)^2 + (2-0)^2} = 2.0000$$

$$D(C_1, B) = \sqrt{(3-1)^2 + (2-0)^2} = 2.8284$$

$$D(C_1, C) = \sqrt{(5-1)^2 + (2-0)^2} = 4.4721$$

$$D(C_1, D) = \sqrt{(1-1)^2 + (1-0)^2} = 1.0000$$

$$D(C_1, E) = \sqrt{(5-1)^2 + (1-0)^2} = 4.1231$$

Distance of C_2 to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1-5)^2 + (2-6)^2} = 5.6569$$

$$D(C_1, B) = \sqrt{(3-5)^2 + (2-6)^2} = 4.4721$$

$$D(C_1, C) = \sqrt{(5-5)^2 + (2-6)^2} = 4.0000$$

$$D(C_1, D) = \sqrt{(1-5)^2 + (1-6)^2} = 6.4031$$

$$D(C_1, E) = \sqrt{(5-5)^2 + (1-6)^2} = 5.0000$$

Now we assign the points to their clusters:

- Cluster 1: A, B, D, E
- Cluster 2: C

Now we create new centroids:

$C_1'(x,y) = \frac{(1,2)+(3,2)+(1,1)+(5,1)}{4} = \frac{(9,6)}{4} = (2.25, 1.5)$
$C_2'(x,y) = \frac{(5,2)}{1} = (5,2)$

Iteration 2

Distance of C_1' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1-2.25)^2 + (2-1.5)^2} = 1.3463$$

$$D(C_1, B) = \sqrt{(3 - 2.25)^2 + (2 - 1.5)^2} = 0.9014$$

$$D(C_1, C) = \sqrt{(5 - 2.25)^2 + (2 - 1.5)^2} = 2.7951$$

$$D(C_1, D) = \sqrt{(1 - 2.25)^2 + (1 - 1.5)^2} = 1.3463$$

$$D(C_1, E) = \sqrt{(5 - 2.25)^2 + (1 - 1.5)^2} = 2.7951$$

Distance of C_2' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1 - 5)^2 + (2 - 2)^2} = 4.0000$$

$$D(C_1, B) = \sqrt{(3 - 5)^2 + (2 - 2)^2} = 2.0000$$

$$D(C_1, C) = \sqrt{(5 - 5)^2 + (2 - 2)^2} = 0.0000$$

$$D(C_1, D) = \sqrt{(1 - 5)^2 + (1 - 2)^2} = 4.1231$$

$$D(C_1, E) = \sqrt{(5 - 5)^2 + (1 - 2)^2} = 1.0000$$

Assigning points to clusters:

- Cluster 1: A, B, D
- Cluster 2: C, E

Now we create new centroids:

$C_1''(x, y) = \frac{(1,2)+(3,2)+(1,1)}{3} = \frac{(4,5)}{3} = (1.3333, 1.6666)$
$C_2''(x, y) = \frac{(5,2)+(5,1)}{2} = \frac{(10,3)}{2} = (5, 1.5)$

Iteration 3

Distance of C_1'' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1 - 1.3333)^2 + (2 - 1.6666)^2} = 0.4714$$

$$D(C_1, B) = \sqrt{(3 - 1.3333)^2 + (2 - 1.6666)^2} = 1.6997$$

$$D(C_1, C) = \sqrt{(5 - 1.3333)^2 + (2 - 1.6666)^2} = 3.6818$$

$$D(C_1, D) = \sqrt{(1 - 1.3333)^2 + (1 - 1.6666)^2} = 0.7453$$

$$D(C_1, E) = \sqrt{(5 - 1.3333)^2 + (1 - 1.6666)^2} = 3.7268$$

Distance of C_2'' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1 - 5)^2 + (2 - 1.5)^2} = 4.0311$$

$$D(C_1, B) = \sqrt{(3 - 5)^2 + (2 - 1.5)^2} = 2.0616$$

$$D(C_1, C) = \sqrt{(5 - 5)^2 + (2 - 1.5)^2} = 0.5000$$

$$D(C_1, D) = \sqrt{(1 - 5)^2 + (1 - 1.5)^2} = 4.0311$$

$$D(C_1, E) = \sqrt{(5 - 5)^2 + (1 - 1.5)^2} = 0.5000$$

Assigning points to clusters:

- Cluster 1: A, B, D

- Cluster 2: C, E

Now we create new centroids:

$$C_1'''(x,y) = \frac{(1,2)+(3,2)+(1,1)}{3} = \frac{(4,5)}{3} = (1.3333, 1.6666)$$
$$C_2'''(x,y) = \frac{(5,2)+(5,1)}{2} = \frac{(10,3)}{2} = (5, 1.5)$$

At this point we see that the points making up the clusters didn't move and neither did the centroids.

This means that the clusters are final and the points are correctly assigned.

Part B

Iter 1

Centroids: $C_1(2.75, 3)$ and $C_2(3.25, 0)$
Points: $A(1,2), B(3,2), C(5,2), D(1,1) and E(5,1)$

Distance of C_1 to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1-2.75)^2 + (2-3)^2} = 2.0156$$

$$D(C_1, B) = \sqrt{(3-2.75)^2 + (2-3)^2} = 1.0308$$

$$D(C_1, C) = \sqrt{(5-2.75)^2 + (2-3)^2} = 2.4622$$

$$D(C_1, D) = \sqrt{(1-2.75)^2 + (1-3)^2} = 2.6575$$

$$D(C_1, E) = \sqrt{(5-2.75)^2 + (1-3)^2} = 3.0104$$

Distance of C_2 to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1-3.25)^2 + (2-0)^2} = 3.0104$$

$$D(C_1, B) = \sqrt{(3-3.25)^2 + (2-0)^2} = 2.0156$$

$$D(C_1, C) = \sqrt{(5-3.25)^2 + (2-0)^2} = 2.6575$$

$$D(C_1, D) = \sqrt{(1-3.25)^2 + (1-0)^2} = 2.4622$$

$$D(C_1, E) = \sqrt{(5-3.25)^2 + (1-0)^2} = 2.0156$$

Assigning the points to clusters:

- Cluster 1: A, B, C, E
- Cluster 2: D

Updating centroids:

$$C_1'(x,y) = \frac{(1,2)+(3,2)+(5,2)+(5,1)}{4} = \frac{(14,7)}{4} = (3.5, 1.75)$$
$$C_2'(x,y) = \frac{(1,1)}{1} = (1, 1)$$

Iter 2

Distance of C_1' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1-3.5)^2 + (2-1.75)^2} = 2.5125$$

$$D(C_1, B) = \sqrt{(3-3.5)^2 + (2-1.75)^2} = 0.5590$$

$$D(C_1, C) = \sqrt{(5 - 3.5)^2 + (2 - 1.75)^2} = 1.5207$$

$$D(C_1, D) = \sqrt{(1 - 3.5)^2 + (1 - 1.75)^2} = 2.6101$$

$$D(C_1, E) = \sqrt{(5 - 3.5)^2 + (1 - 1.75)^2} = 1.6771$$

Distance of C_2' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1 - 1)^2 + (2 - 1)^2} = 1.0000$$

$$D(C_1, B) = \sqrt{(3 - 1)^2 + (2 - 1)^2} = 2.2361$$

$$D(C_1, C) = \sqrt{(5 - 1)^2 + (2 - 1)^2} = 4.1231$$

$$D(C_1, D) = \sqrt{(1 - 1)^2 + (1 - 1)^2} = 0.0000$$

$$D(C_1, E) = \sqrt{(5 - 1)^2 + (1 - 1)^2} = 4.0000$$

Assigning the points to clusters:

- Cluster 1: B, C, E
- Cluster 2: A, D

Updating centroids:

$C_1''(x, y) = \frac{(3,2)+(5,2)+(5,1)}{3} = \frac{(13,5)}{3} = (4.3333, 1.6666)$
$C_2''(x, y) = \frac{(1,2)+(1,1)}{2} = \frac{(2,3)}{2} = (1, 1.5)$

Iter 3

Distance of C_1'' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1 - 4.3333)^2 + (2 - 1.6666)^2} = 3.3499$$

$$D(C_1, B) = \sqrt{(3 - 4.3333)^2 + (2 - 1.6666)^2} = 1.3744$$

$$D(C_1, C) = \sqrt{(5 - 4.3333)^2 + (2 - 1.6666)^2} = 0.7454$$

$$D(C_1, D) = \sqrt{(1 - 4.3333)^2 + (1 - 1.6666)^2} = 3.3993$$

$$D(C_1, E) = \sqrt{(5 - 4.3333)^2 + (1 - 1.6666)^2} = 0.9428$$

Distance of C_2'' to A, B, C, D & E:

$$D(C_1, A) = \sqrt{(1 - 1)^2 + (2 - 1.5)^2} = 0.5000$$

$$D(C_1, B) = \sqrt{(3 - 1)^2 + (2 - 1.5)^2} = 2.0616$$

$$D(C_1, C) = \sqrt{(5 - 1)^2 + (2 - 1.5)^2} = 4.0311$$

$$D(C_1, D) = \sqrt{(1 - 1)^2 + (1 - 1.5)^2} = 0.5000$$

$$D(C_1, E) = \sqrt{(5 - 1)^2 + (1 - 1.5)^2} = 4.0311$$

Assigning the points to clusters:

- Cluster 1: B, C, E
- Cluster 2: A, D

Updating centroids:

$$C_1'''(x,y) = \frac{(3,2)+(5,2)+(5,1)}{3} = \frac{(13,5)}{3} = (4.3333, 1.6666)$$
$$C_2'''(x,y) = \frac{(1,2)+(1,1)}{2} = \frac{(2,3)}{2} = (1, 1.5)$$

At this point we see that the points making up the clusters didn't move and neither did the centroids.

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