Assignment_5_task_5.4

Machine Learning (WiSe 2025/2026)

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Assignment 5 Task 5.4

Given mathematical function:

$f(x) = \frac{1}{4}(x^3 - 3x) + 1$

(a)

For x = -4:

```
f(-4) = 0.25 * (-64 + 12) +1
f(-4) = -12
```

For x = -3:

```
f(-3) = 0.25 * ((-3)^3 - 3(-3)) +1
f(-3) = -3.5
```

For x = -2:

```
f(-2) = 0.25 * ((-2)^3 - 3(-2)) +1
f(-2) = 0.5
```

For x = -1:

```
f(-1) = 0.25 * ((-1)^3 - 3(-1)) +1
f(-1) = 1.5
```

For x = 0:

```
f(-0) = 0.25 * ((-0)^3 - 3(-0)) +1
f(-0) = 1
```

For x = 1:

```
f(1) = 0.25 * ((1)^3 - 3(1)) +1
f(1) = 0.5
```

For x = 2:

```
f(2) = 0.25 * ((2)^3 - 3(2)) +1
```

```
f(2) = 1.5
```

For x = 3:

```
f(3) = 0.25 * ((3)^3 - 3(3)) +1
f(3) = 5.5
```

For x = 4:

```
f(4) = 0.25 * ((4)^3 - 3(4)) +1
f(4) = 14
```

(b)

Given MSE function is:

$\frac{1}{N}\sum_{i}^{N}(y_{avg} - y_i)^2$

Considering our first split to be at -2.5, our tree would split the given values like so:

{-4, -3} {-2, -1, 0, 1, 2, 3, 4}

here y_{avg} would be calculated like so:

$y_{avg} = \frac{y_{-4}+y_{-3}}{2}$

$y_{avg} = \frac{-12+(-3.5)}{2}$

Therefore $y_{avg} = -7.75$

Then we calculate MSE for the entire dataset using y_{avg} for the entire dataset.

So, MSE = 42.17

Now the cost to split a node can be calculated using weighted MSE for every node.

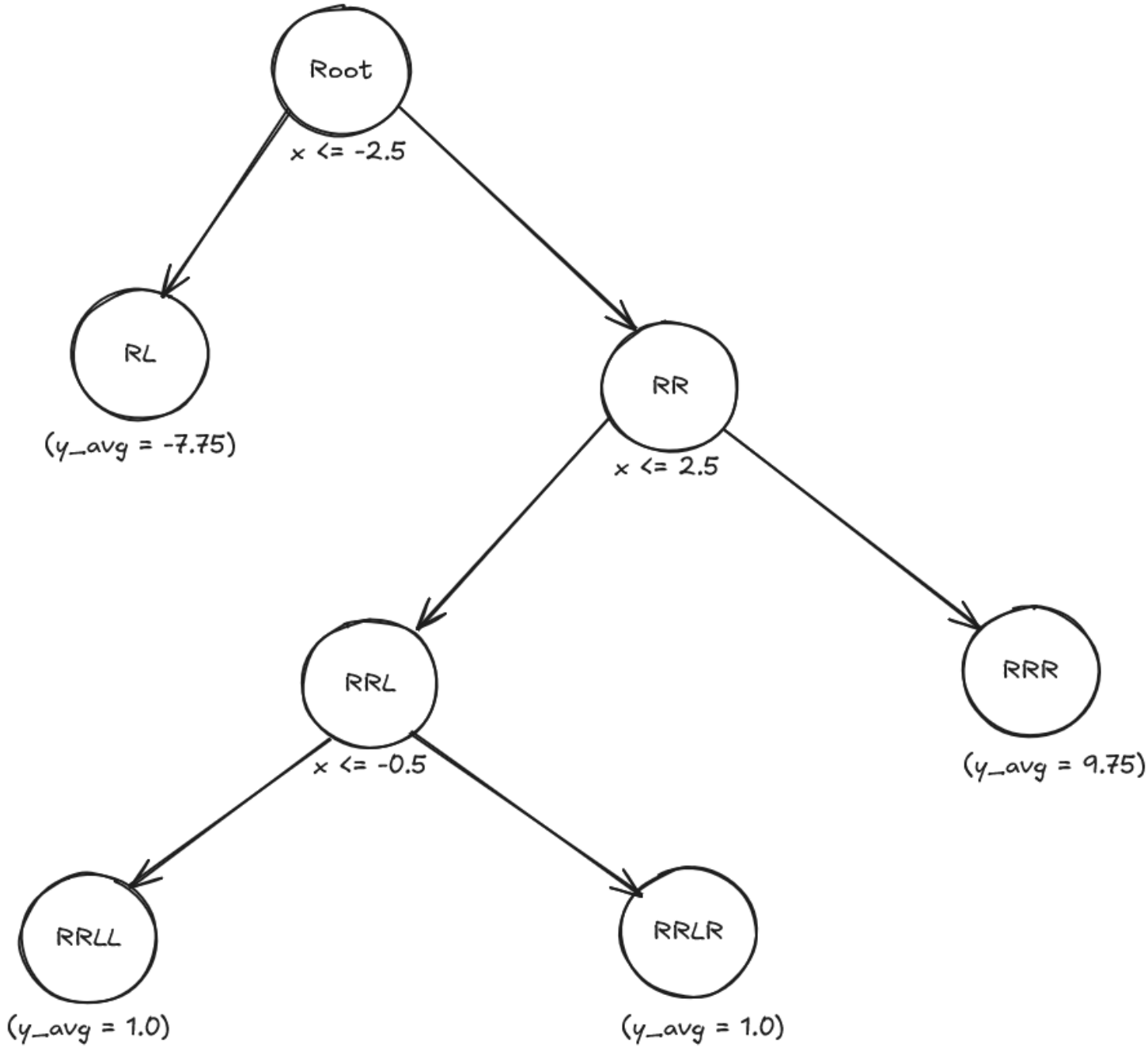
$Cost(s) = \frac{N_L}{N}.MSE(R_L) + \frac{N_R}{N}.MSE(R_R)$

where R_L and R_R are the left and right child nodes, and N_L and N_R are their respective sample sizes.
The constraint is that each child node must contain at least two values.

Thus recursively generating the entire tree yeilds the following:

Leaf Node	Region (x values)	Prediction (y_{avg})	MSE
R_L	x≤-2.5 (i.e., x∈{-4,-3})	-7.75	18.0625
R_{RR}	x>2.5 (i.e., x∈{3,4})	9.75	18.0625
R_{RLL}	-2.5<x≤-0.5 (i.e., x∈{-2,-1})	1.0	0.2500
R_{RLR}	-0.5<x≤2.5 (i.e., x∈{0,1,2})	1.0	0.1667

Here's what the tree would look like:



(c)

The tree is only somewhat good, as there aren't enough data values to actually be certain.