

Task_7.2

Machine Learning (WiSe 2025/2026)

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Assignment 7 Task 7.2

threshold expression:

$$w_0 + w_1x_1 + w_2x_2 > 0$$

Initialized weight values:

$$w_0 = 0, \quad w_1 = 0, \quad w_2 = 0$$

Table of data:

x_1	x_2	t (target)
1	0	0
3	-1	1
2	2	0
4	4	0
1	-2	1

I need to apply the perceptron rule in batch with $\eta = 0.1$ in batch mode.

Perceptron rule:

$$w_i \leftarrow w_i + \eta \sum_d (t_d - o_d) x_{i,d}$$

d	x1	x2	t	Net Input ($w_0+w_1x_1+w_2x_2$)	Output (o)	Error ($E=t-o$)	Δw_0 ($E \cdot 1$)	Δw_1 ($E \cdot x_1$)	Δw_2 ($E \cdot x_2$)
1	1	0	0	$0(1)+0(0)+0(0)=0$	0	$0-0=0$	$0 \cdot 1=0$	$0 \cdot 1=0$	$0 \cdot 0=0$
2	3	-1	1	$0(1)+0(3)+0(-1)=0$	0	$1-0=1$	$1 \cdot 1=1$	$1 \cdot 3=3$	$1 \cdot (-1)=-1$
3	2	2	0	$0(1)+0(2)+0(2)=0$	0	$0-0=0$	$0 \cdot 1=0$	$0 \cdot 2=0$	$0 \cdot 2=0$
4	4	4	0	$0(1)+0(4)+0(4)=0$	0	$0-0=0$	$0 \cdot 1=0$	$0 \cdot 4=0$	$0 \cdot 4=0$
5	1	-2	1	$0(1)+0(1)+0(-2)=0$	0	$1-0=1$	$1 \cdot 1=1$	$1 \cdot 1=1$	$1 \cdot (-2)=-2$
SUM							$\sum \Delta w_0=2$	$\sum \Delta w_1=4$	$\sum \Delta w_2=-3$

Update Weights (End of Iteration 1):

Apply the update rule $w_i \leftarrow w_i + \eta \sum \Delta w_i$ with $\eta=0.1$:

$$w_0 = 0 + 0.1 \cdot (2) = 0.2$$
$$w_1 = 0 + 0.1 \cdot (4) = 0.4$$
$$w_2 = 0 + 0.1 \cdot (-3) = -0.3$$

Weights from Iteration 1: $w_0 = 0.2, w_1 = 0.4, w_2 = -0.3$

We repeat the process using these new weights

d	x1	x2	t	Net Input ($0.2+0.4x_1-0.3x_2$)	Output (o)	Error (E=t-o)	Δw_0 (E·1)
1	1	0	0	$0.2+0.4(1)-0.3(0)=0.6$	1	$0-1=-1$	$-1 \cdot 1 =$
2	3	-1	1	$0.2+0.4(3)-0.3(-1)=0.2+1.2+0.3=1.7$	1	$1-1=0$	$0 \cdot 1 = 0$
3	2	2	0	$0.2+0.4(2)-0.3(2)=0.2+0.8-0.6=0.4$	1	$0-1=-1$	$-1 \cdot 1 =$
4	4	4	0	$0.2+0.4(4)-0.3(4)=0.2+1.6-1.2=0.6$	1	$0-1=-1$	$-1 \cdot 1 =$
5	1	-2	1	$0.2+0.4(1)-0.3(-2)=0.2+0.4+0.6=1.2$	1	$1-1=0$	$0 \cdot 1 = 0$
SUM							$\sum \Delta w_0 = -3$

Final Weight Update (End of Iteration 2)

Apply the update rule to the weights from Iteration 1:

$$w_0 = 0.2 + 0.1 \cdot (-3) = 0.2 - 0.3 = -0.1$$
$$w_1 = 0.4 + 0.1 \cdot (-7) = 0.4 - 0.7 = -0.3$$
$$w_2 = -0.3 + 0.1 \cdot (-6) = -0.3 - 0.6 = -0.9$$

Final Perceptron

The final perceptron weights after two updates (two iterations) are:
 $w_0 = -0.1, w_1 = -0.3, w_2 = -0.9$

The final perceptron uses the decision boundary (threshold expression):

$$-0.1 - 0.3x_1 - 0.9x_2 > 0$$