MAM1044H: Dynamics
A Voyage to Eros Using Hohmann Orbits

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Abstract

In this paper, preparations for a mission to Eros were done. The asteroid was last observed on the 30th of December 2000 and this paper sets forth plans for a mission (NEAR II) which will set up a stabe orbit around the asteroid. In order to minimise costs and efforts, the entire mission depended on Hohmann's orbits of least energy. A launch widow was calculated for and then the orbits were calculated. The duration of flight was calculated and thus the mission was deemed appropriate. In order to minimise efforts and complexity, approximations were made where appropriate and this should not influence the validity of the mission in a large way.

1 Introduction

433 Eros is an elongated asteroid that was discovered in 1898 by G. Witt and is one of the most extensivly studied Near-Earth-Objects in history. Eros belongs to the Amor group of asteroids and is the second-largest Near-Earth-Object, with a diameter of approximately 16.8 km. In 1996, NEAR Shoemaker, a robotic space probe, was sent on a mission to study and land on Eros. The main goal of the mission was to link Eros to meteorites recovered on Earth. However, this was not successful, and no conclusive outcome was ever published.

Hohmann Transfer orbits are eliptical orbits between two circular orbits, that use the least amout of energy to traverse from one orbit to the other. An advantage of such an orbit is that fuel is typically conserved and thus less-weighing payloads need to be deployed. A disadvantage is however, that such orbits take a longer period of time to reach their destinations. Non-Hohmann orbits are used to reduce the length of missions at the expense of greater costs. Due to the reversibility of Hohmann orbits, they can be used for return missions, where the spacecraft's engine is fired in the opposite direction toits current path, slowing the spacecraft and causing it to drop into the lower-energy eliptical transfer orbit.

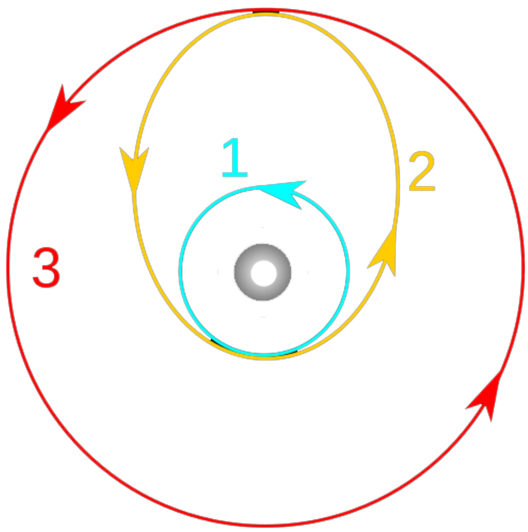

Figure 1: An illustation showing two circular orbits 1 and 3, with 2 being the Hohmann transfer orbit between the two orbits

The typical equation for such orbits is obtained from:

$$E = \frac{mv^2}{2} - \frac{GMm}{r} = \frac{-GMm}{2a} \tag{1}$$

where, r is the distance of the orbiting body from the primary focus, and a is the semi major axis of the object's orbit.

2 Theory and Calculation

Eros has a perihelion and aphelion distance of 1.13 AU and 1.78 AU respectively. It also has dimensions of 33 km x 13km x 13km. From this, the stages of the mission were planned.

2.1 Stage 1

Given that on the 30th of December 2000, the angle between the Earth and Eros was 97°, the orbital period of the Earth is 365.25 days and that of Eros is 643 days;

$$T = \frac{2\pi}{\Omega}$$

Where Ω is the difference in angular velocity between the Earth and Eros. The value was found to be **2 years and 115 days**. This means that the time taken for the same position to be re-established betweeen the two bodies is 2 years and 115 days. The first available launch date was found to be the 17th of January 2003.

2.2 Stage 2:The Transfer Orbit

In order to get into orbit, the craft must be launced from Earth into a geosynchronous orbit. In order to find the the escape velocity, the equation:

$$E = \frac{mv^2}{2} - \frac{GMm}{r} = \frac{-GMm}{2a} \tag{2}$$

was used. From this, a new equation

$$v = \sqrt{Gm}(\frac{2}{r} * \frac{1}{a})^{1/2} \tag{3}$$

was derived and the velocity calculated from the appropriate values. The velocity needed to get into a geosynchronous orbit was fround to be $10.761 * 10^3$ **meters per second** $\approx$ **11 kilometers per second**.

The energy needed using a Hohmann orbit to arrive at Eros is given by

$$E = \frac{-GM_{sun}m_r}{r_{earth} + r_{eros}} \tag{4}$$

The radii of the two bodies was averaged from their aphelion and perihelion distances. This was found to be $r_{earth} = 14.96 * 10^{10}$ and $r_{eros} = 2.18 * 10^{11}$. Using the equation:

$$\frac{1}{2}m_r v^2 - \frac{GM_{sun}m_r}{r_{earth}} = \frac{-GM_{sun}m_r}{r_{earth} + r_{eros}} \tag{5}$$

From simplifying the equation, we get;

$$v^2 = 2GM_{sun}(\frac{r_{eros}}{r_{earth}(r_{earth} + r_{eros})}) \tag{6}$$

And the value was found to be $3.2 * 10^4$ **meters per second** $\approx$ **32 kilometers per second**. The velocity of the earth around the sun is given by this equation;

$$\frac{1}{2}m_{earth}v^2 - \frac{GM_{sun}m_{earth}}{r_{earth}} = \frac{-GM_{sun}m_{earth}}{2r_{earth}} \tag{7}$$

And from this, the velocity was found to be $2.97 * 10^4$ **meters per second = 29.7 kilometers per second**. Therefore, the spacecraft only needs to produce enough thrust to travel at $32 - 29.7 =$ **2.3 kilometers per second**.

2.3 Stage 3: Times of Flight and Arrival

The time spent in flight is calculated from the equation;

$$T = \frac{1}{2} \frac{\pi (r_{earth} + r_{eros})^{3/2}}{(2GM_{sun})^{1/2}} \tag{8}$$

The final result was found to be **248.69 days ≈ 8.2 months**. From the 17th of January, the date of arrival was found to be the 25th of October 2003.

2.4 Stage 4: Required Velocities for Entry Into an Orbit

The velocity at which the spacecraft would arrive at Eros would be given by the equation;

$$v^2 = 2GM_{sun}\left(\frac{1}{r_{eros}} - \frac{1}{r_{earth} + r_{eros}}\right) \tag{9}$$

From this, the value obtained is **22 261.49 meters per second ≈ 22.26 kilometers per second**. The velocity at which Eros is travelling is given by

$$v^2 = \frac{GM_{sun}}{r_{eros}} \tag{10}$$

which is **24 675.23 meters per second ≈ 24.68 kilometers per second**. Therefore, the spacecraft would have to produce thrust what will result in an extra **2.42 kilometers per second** to get into an Eros orbit.

2.5 Stage 5: Launch Dates from Eros

The next available launch date is the 26th of March 2005. This was found by cycling though all dates at wich the Earth and Eros were advanced by an angle of . This was done by using the equation;

$$T = \frac{2\pi}{\Omega} \tag{11}$$

. As seen in Stage 1, the two bodies are at the required angle every 2 years and 155 days, and so, the next available launch is at some date, the 26th of March 2005, that is a multiple of the cycle.

2.6 Stage 6: The return of NEAR II

The duration of the return trip of NEAR II was found to be ≈ 1 year and 5 months from the date of departure from Eros.

3 Conclusion

Another mission to Eros is possible. It can be acomplished in a reasonable short period of time while preserving energy and thus requiring less payload mass, making it more economical. The success of the mission is beneficial to humanity because caeful observation and in-depth studies in Near-Earth Objects allows us to predict and have a more general understanding in how these objects are created, how the move, and possibly how they can be catastrophic to life on Earth.