# 1D FDM Package

## 1D Heat Equation solution using FDM

Lab Exercises

Total Marks: 100

# Exercise 30

# Steps to Solve Exercises 30

1. *Identify the domain or interval of the problem*
2. *Discretize the time domain and space domain, compute the number of points required depending $\Delta t, \Delta x$ say $N_t$ and $N_x$*
3. *Declare and Initialize arrays to zero: x**[Nx], T[Nx],Tnew[Nx], Texact[Nx]***
4. *Assign **T[i]=0** for all $0 \leq i < N_x$ and **n=0***
5. *Use given formula to compute $T_i^{n+1}$ from given formula, assume LHS of the formula as **Tnew[i]** and values at RHS as **T[i]***
6. *Write a function with two arguments: float g(float x, float t)*
7. *Use while loop: while(n<Nt)*
   a) *Whenever $g_i^n$ is required in the formula, call the function g(x[i],t+n*dt)*
   b) *For each time step, save the Tnew[i] and x[i] for all $0 \leq i < N_x$ in a file called exercise-4-fdm-n.dat, where n denotes the time-step number*
   c) *For each time step, calculate the analytical solution Texact[i] and save the result in a file exercise-4-exact-n.dat*
   d) *After saving the data, assign T[i]=Tnew[i], for all $0 \leq i < N_x$*

# 1D Heat Equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + g \qquad \begin{array}{l} 0 < x < l, \\ 0 < t < \infty \end{array}$$

$$T(0,t) = 0, 0 < t < \infty$$

$$T(l,t) = 0, 0 < t < \infty$$

$$T(x,0) = 0,0 \leq x \leq l$$

$$g(x,t) = 10\alpha t + 5x(l-x)$$

Analytical Solution: $T(x,t) = 5xt(l-x)$

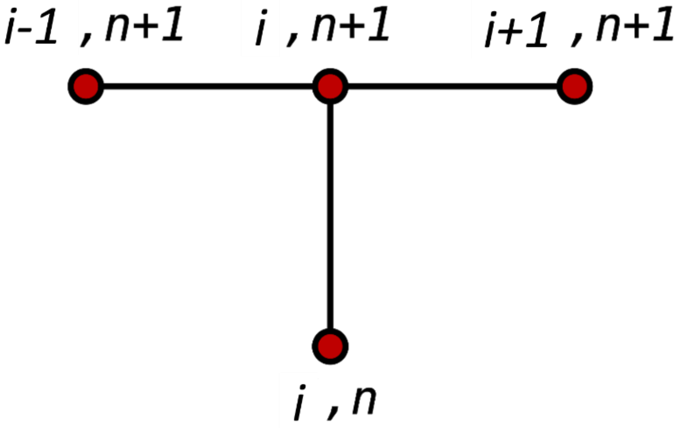
$T(x,0) = s(x)$

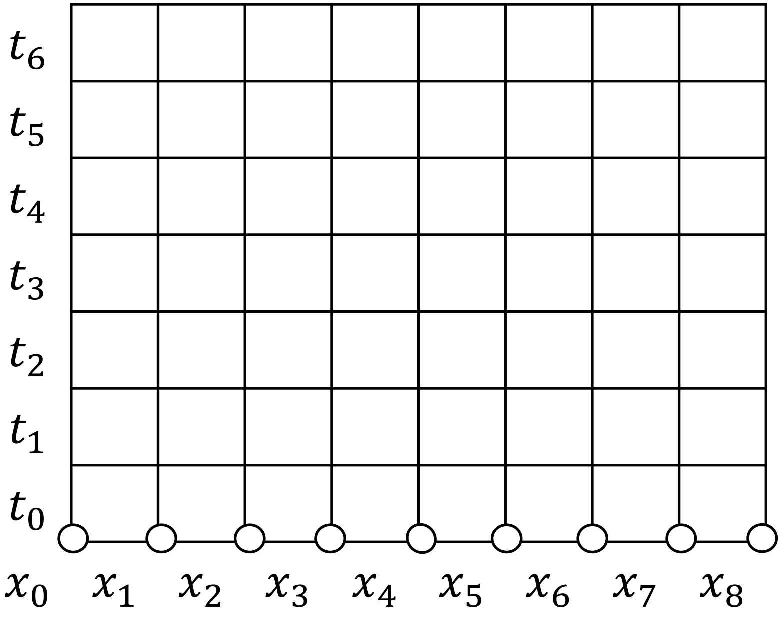
$T(0,t) = 0$

$T(l,t) = 0$

# Forward Euler Scheme

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + g$$




$$T(x_i, t_n) = T_i^n$$

$$T_i^{n+1} = T_i^n + F(T_{i+1}^n - 2T_i^n + T_{i-1}^n) + g_i^n \Delta t$$

$$F = \alpha \frac{\Delta t}{\Delta x^2}$$

$F$ is the dimensionless number that lumps the key physical parameter in the problem, $\alpha$, and the discretization parameters $\Delta x$ and $\Delta t$ into a single parameter. Depending on $F$, the numerical method schemes are chosen.

**Write your Python Code to solve the following problem and then plot the graph from your approximate solution and the real solution. Assume $\alpha = 1, l = 5, \Delta x = 0.1, \Delta t = 0.1\,, T_{fin} = 10$. Compare your approximate solution against the analytical solution**

**[20 Marks]**

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + g(x,t)$$

$$T_i^{n+1} = T_i^n + F(T_{i+1}^n - 2T_i^n + T_{i-1}^n) + g_i^n \Delta t \qquad F = \alpha \frac{\Delta t}{\Delta x^2}$$

$$g(x,t) = 10\alpha t + 5x(l - x)$$

Analytical Solution: $T(x,t) = 5xt(l - x)$