1D FDM Package

1D Heat Equation solution using FDM

Lab Exercises

Total Marks: 100

Exercise 31

Steps to Solve Exercises 31

1. *Construct the matrix **A** using arrays*
2. *If you are not comfortable with A, construct three row vectors, a,b,d as given in last lab exercise.*
3. *Construct the RHS of the linear system*
4. *Use Thomas algorithm to solve the problem*
5. *Use while loop: while(n<Nt)*
   a) *Whenever g_i^n is required in the formula, call the function g(x[i],t+n*dt)*
   b) *For each time step, save the result in a file called exercise-4-fdm-n.dat, where n denotes the time-step number*
   c) *For each time step, calculate the analytical solution Texact[i] and save the result in a file exercise-4-exact-n.dat*

Let us solve the following ODE

$$u'' = k^2u + 2, x \in \left(0, \frac{\pi}{6}\right)$$
$$u(0) = \frac{11}{9}, u\left(\frac{\pi}{6}\right) = \frac{11}{9}$$

From Taylor's approximation,

$$u''(x_0) \approx \frac{u(x_0 + h) - 2u(x_0) + u(x_0 - h)}{h^2}$$

$$u'' = k^2u + 2$$

$$\frac{u(x_0 + h) - 2u(x_0) + u(x_0 - h)}{h^2} = k^2u(x_0) + 2$$

$$u_{i-1} - (2 + k^2h^2)u_i + u_{i+1} = 2h^2$$

i=1

$$(2 + k^2h^2)u_1 + u_2 = 2h^2$$

i=2

$$u_1 + (2 + k^2h^2)u_2 + u_3 = 2h^2$$

i=3

$$u_2 + (2 + k^2h^2)u_3 + u_4 = 2h^2$$

i=N

$$u_{N-1} + (2 + k^2h^2)u_N = 2h^2$$

$$(k^2h^2 - 2)u_1 + u_2 = 2h^2 - c_1$$
$$u_1 + (k^2h^2 - 2)u_2 + u_3 = 2h^2$$
$$u_2 + (k^2h^2 - 2)u_3 + u_4 = 2h^2$$
$$u_{N-1} + (k^2h^2 - 2)u_N = 2h^2 - c_2$$
$$Au = b$$
$$a = (k^2h^2 - 2)$$

$$A = \begin{bmatrix} a & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & a & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & a & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & a \end{bmatrix}$$

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 2h^2 - c_1 \\ 2h^2 \\ \vdots \\ 2h^2 \\ 2h^2 - c_2 \end{bmatrix}$$

$$u_{i-1} - (2 + k^2h^2)u_i + u_{i+1} = 2h^2$$

$$A = \begin{bmatrix} a & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & a & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & a & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & a \end{bmatrix}$$

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 2h^2 - c_1 \\ 2h^2 \\ \vdots \\ 2h^2 \\ 2h^2 - c_2 \end{bmatrix}$$

$$a = k^2h^2 - 2$$

Write your Pythib Code to construct the following A matrix, u and b vectors. Solve the system Au=b using Thomas algorithm. Assume k=9 **[30 Marks]**

$$\boxed{u_{i-1} - (2 + k^2h^2)u_i + u_{i+1} = 2h^2}$$

$$A = \begin{bmatrix} a & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & a & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & a & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & a \end{bmatrix}$$

$$u = \sin(3x) + \cos(3x) + \frac{2}{9}$$

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 2h^2 - c_1 \\ 2h^2 \\ \vdots \\ 2h^2 \\ 2h^2 - c_2 \end{bmatrix}$$

$$a = k^2h^2 - 2$$