1D FDM Package

1D Heat Equation solution using FDM

Lab Exercises

Total Marks: 100

Exercise 32

Steps to Solve Exercises 32

1. *Construct the matrix **A** using arrays*
2. *If you are not comfortable with A, construct three row vectors, a,b,d as given in last lab exercise.*
3. *Construct the RHS of the linear system you can use the function float g(float x, float t) used in the exercise 4*
4. *Use Thomas algorithm to solve the problem*
5. *Use while loop: while(n<Nt)*
   a) *Whenever g_i^n is required in the formula, call the function g(x[i],t+n*dt)*
   b) *For each time step, save the Tnew[i] and x[i] for all $0 \leq i < N_x$ in a file called exercise-4-fdm-n.dat, where n denotes the time-step number*
   c) *For each time step, calculate the analytical solution Texact[i] and save the result in a file exercise-4-exact-n.dat*
   d) *After saving the data, assign T[i]=Tnew[i], for all $0 \leq i < N_x$*

1D Heat Equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + g \qquad \begin{array}{l} 0 < x < l, \\ 0 < t < \infty \end{array}$$

$$T(0, t) = 0, 0 < t < \infty$$

$$T(l, t) = 0, 0 < t < \infty$$

$$T(x, 0) = 0,0 \leq x \leq l$$

$$g(x, t) = 10\alpha t + 5x(l - x)$$

Analytical Solution: $T(x, t) = 5xt(l - x)$

$T(x, 0) = s(x)$

$T(0, t) = 0$  $T(l, t) = 0$

Backward Euler Scheme

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + g \qquad T(x_i, t_n) = T_i^n$$

$$-FT_{i-1}^{n} + (1 + 2F)T_i^n - FT_{i+1}^{n} = T_{i-1}^{n-1} + \Delta t g_i^n$$

$$AT = b$$

$$A = \begin{bmatrix}
a_{00} & a_{01} & 0 & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\
a_{10} & a_{11} & a_{12} & \ddots & \dots & \dots & \dots & \dots & \dots & \vdots \\
0 & a_{21} & a_{22} & a_{23} & \ddots & \dots & \dots & \dots & \dots & \vdots \\
\vdots & 0 & a_{32} & a_{33} & a_{34} & 0 & \dots & \dots & \dots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \dots & \dots & \dots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \dots & \dots & \vdots \\
\vdots & \vdots & \dots & 0 & a_{ii-1} & a_{ii} & a_{ii+1} & \ddots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & a_{N_x-1N_x} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_{N_xN_x-1} & a_{N_xN_x}
\end{bmatrix}$$

$$a_{ii-1} = a_{ii+1} = -F$$

$$a_{ii} = 1 + 2F$$

Write your Python Code to solve 1D Heat Equation and then plot the graph from your approximate solution and the real solution. Assume $\alpha = 1, l = 5, \Delta x = 0.1, \Delta t = 0.1, T_{fin} = 10$. Compare your approximate solution against the analytical solution

[30 Marks]

$$-FT_{i-1}^{n} + (1+2F)T_i^n - FT_{i+1}^n = T_{i-1}^{n-1} + \Delta t g_i^n$$

$$AT^n = b$$

$$A = \begin{bmatrix} a_{00} & a_{01} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ a_{10} & a_{11} & a_{12} & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & a_{21} & a_{22} & a_{23} & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & 0 & a_{32} & a_{33} & a_{34} & 0 & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & 0 & a_{ii-1} & a_{ii} & a_{ii+1} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & a_{N_x-1N_x} \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_{N_xN_x-1} & a_{N_xN_x} \end{bmatrix}$$

$$a_{ii-1} = a_{ii+1} = -F$$

$$a_{ii} = 1 + 2F$$

$$b_0 = b_{N_x} = 0, b_i = T_{i-1}^{n-1} + \Delta t g_i^n$$

$$T^n = \begin{bmatrix} T_0^n \\ T_1^n \\ T_2^n \\ \vdots \\ T_{N_x-1}^n \\ T_{N_x}^n \end{bmatrix} \qquad b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_{N_x-1} \\ b_{N_x} \end{bmatrix}$$