1D FDM Package

1D Heat Equation solution using FDM

Lab Exercises

Total Marks: 100

For each exercise

1. *Name the file as exercise-x.py, where x denotes the exercise number*
2. *Save the output in a .dat file. Name the output from FDM as exercise-x-fdm.dat*
3. *Save the output from exact solution also as a .dat file. Name the output from exact solution as exercise-x-exact.dat*
4. *First column of the .dat file should be x or t variable, second column should be corresponding y or u or T variable*

Steps to Solve Exercises 1-3

For each exercise

1. *Identify the domain or interval of the problem*
2. *Discretize the domain, compute the number of points required depending Δt, say N_t*
3. *Declare and Initialize arrays to zero:* ***t[Nt], y[Nt]***
4. *Assign* ***y[0]=y0*** *value and* ***t[0]=0***
5. *Use Δt to compute the element of* ***t*** *array*
6. *Use given formula to compute* ***y[i]***
7. *Use given formula to compute* ***yexact[i]***
8. ***Run the program and save the files as instructions given in previous slides***
9. *Save according to the given instructions in previous slides*
10. *Plot the graph*

Exercise 27

$$y' = -ky, t \in [0,2]$$
$$y(0) = y_0$$

Explicit Euler Method

$$y_{n+1} = y_n - \Delta t k y_n$$

$$y_{n+1} = (1 - \Delta t k) y_n$$

$$y = e^{-kt} y_0$$

Write your Python Code to solve the following problem and then plot the graph from your approximate solution and the real solution. Assume $k = 1, y_0 = 1, \Delta t = 0.1$ **[10 Marks]**

$$y' = -ky, y(0) = y_0, t \in [0,2]$$

Explicit Euler Method

$$\boxed{y_{n+1} = (1 - \Delta t k) y_n}$$

$$y_{n+1} = y_n - \Delta t k y_n$$

$$y_{n+1} = (1 - \Delta t k) y_n$$

$$y = e^{-kt} y_0$$

Exercise 28

$$y' = -ky, t \in [0,2]$$
$$y(0) = y_0$$

Implicit Euler Method

$$y_{n+1} = y_n - \Delta t k y_{n+1}$$

$$(1 + \Delta t k) y_{n+1} = y_n$$

$$y_{n+1} = \frac{y_n}{(1 + \Delta t k)}$$

$$y = y_0 e^{-kt}$$

Write your Python Code to solve the following problem and then plot the graph from your approximate solution and the real solution. Assume $k = 2, y_0 = 2, \Delta t = 0.1$ **[10 Marks]**

$$y_{n+1} = \frac{y_n}{1 + \Delta t k}$$

$$y' = -ky, y(0) = y_0$$

Implicit Euler Method

$$y_{n+1} = y_n - \Delta t k y_{n+1}$$

$$(1 + \Delta t k) y_{n+1} = y_n$$

$$y_{n+1} = \frac{y_n}{(1 + \Delta t k)}$$

$$y = y_0 e^{-kt}$$

Exercise 29

$$y = e^{-kt} y_0$$

$$y' = -ky, t \in [0,1]$$
$$y(0) = y_0$$

$$y_{n+1} = y_n + \Delta t g_{n+\frac{1}{2}}$$
$$g_{n+\frac{1}{2}} = \frac{g_{n+1} + g_n}{2}$$
$$y_{n+1} = y_n - k\Delta t \left(\frac{y_{n+1} + y_n}{2}\right)$$

$$y_{n+1}\left(1 + \frac{k\Delta t}{2}\right) = y_n\left(1 - \frac{k\Delta t}{2}\right)$$

$$y_{n+1} = \frac{y_n\left(1 - \frac{k\Delta t}{2}\right)}{\left(1 + \frac{k\Delta t}{2}\right)}$$

Write your Python Code to solve the following problem and then plot the graph from your approximate solution and the real solution. Assume $\mathrm{k} = 3, \mathrm{y}_0 = -1, \Delta t = 0.1$ **[10 Marks]**

$$y' = -ky, y(0) = y_0$$

$$y_{n+1} = y_n + \Delta t g_{n+\frac{1}{2}}$$

$$\boxed{y_{n+1} = \frac{y_n\left(1 - \frac{k\Delta t}{2}\right)}{\left(1 + \frac{k\Delta t}{2}\right)}}$$

$$y = y_0 e^{-kt}$$