MATH10232: EXAMPLE SHEET[1] 0

Questions for supervision classes

The first part of the course concerns the solution of ordinary differential equations (ODEs) and this "zeroth" example sheet contains some simple exercises to remind you about basic properties of integral and differential calculus. If you find any of the actual integration or differentiation difficult, then you should start revising your first semester and A-level calculus courses.
Note that from next week, you **must** hand in solutions to selected questions on the example sheets before the supervision. Be sure to check with your supervisor when and where they would like you to hand in the work.

1. **Integration as the inverse of differentiation (The fundamental theorem of calculus)**

   (a) *Differentiation:* For the function

   $$y_1(x) = \sin(x),$$

   what is the derivative, $\mathrm{d}y_1/\mathrm{d}x$? Is the derivative unique?

   (b) *Integration:* If the derivative of the function $y_2(x)$ is

   $$\frac{\mathrm{d}y_2}{\mathrm{d}x} = \cos(x),$$

   what is the function $y_2(x)$? Is your answer unique? If not, how and why do the various solutions differ?
   This is an example of an ordinary differential equation: an equation that contains a derivative of a function of a single variable.

   (c) If the second derivative of the function $y_3(x)$ is

   $$\frac{d^2y_3}{\mathrm{d}x^2} = -\sin(x),$$

   find $y_3(x)$. Is the answer unique? If not, how and why do the various solutions differ?

[1] Any feedback to: *joel.daou@manchester.ac.uk*

2. **Ordinary differential equations (ODEs) that relate a function and its derivatives**

   (a) Determine the function $y_4(x)$ that satisfies the (ordinary differential) equation[2]

   $$\frac{\mathrm{d}y_4}{\mathrm{d}x} = y_4(x).$$

   In other words: "Which function is equal to its first derivative?". Is your answer unique? If not, how do the various solutions differ?

   (b) Determine the function $y_5(x)$ that satisfies[3]

   $$\frac{\mathrm{d}^2y_5}{\mathrm{d}x^2} = -y_5(x).$$

   In other words: "Which function is equal to the negative of its second derivative?". Is the answer to this question unique? If not, how do the various solutions differ?

3. **Computer Exercises** [4]

   (a) Make sure that you are able to run MATLAB.

   (b) Use MATLAB to create plots of the functions y_1, y_2, y_3, y_4 and y_5. Write down (or save) the commands that you used to generate the graphs somewhere safe.

[2] If you have done the C4 module in A-level mathematics, you will have seen the method of separation of variables, which can be used to solve this equation. If not, you should be able to find the answer by trial and error and by using your knowledge of elementary functions.

[3] Feel free to use trial and error to find the answer here, or look at Q.1.(c). A constructive alternative is to multiply by $\mathrm{d}y_5/\mathrm{d}x$ and integrate.

[4] Important advice: Please do not skip a regular weekly practise of MATLAB, staring now, by doing the MATLAB exercises. These exercises provide a gentle progressive introduction and will equip you with a powerful tool for tackling a variety of problems in applied mathematics.