MATH10212 Linear Algebra B • Homework 1

Students are **strongly** advised to acquire a copy of the Textbook:

D. C. Lay Linear Algebra and its Applications. Pearson, 2006. ISBN 0-521-28713-4.

Normally, homework assignments will consist of some odd numbered exercises from the sections of the Textbook covered in the lectures on the previous week. The Textbook contains answers to most odd numbered exercises.

Introductory problems for Week 1

These problems are only for discussion at supervisions in Week 1 (no submissions by students).

1. [**Lay 1.1.1**] Solve system of equations:

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}$$

2.[1] Solve system of equations:

$$\begin{aligned} x + 2y &= -1 \\ 2x + y &= 1 \end{aligned}$$

3. (a) One equation

$$x + 2y = -1$$

defines a straight line on the (x, y)-plane — this line consists of all points (x, y) satisfying this equation. Similarly, consider

$$2x + y = 1.$$

Which points of these lines simultaneously satisfy both equations?

(b) Draw the lines in part (a) and explain how the picture corresponds to Question 2.

4. (a) How is the line

$$x + 2y = a$$

related to the line

$$x + 2y = -1$$

where a is an arbitrary real number?

(b) How is the line

$$2x + y = b$$

related to the line

$$2x + y = 1$$

where b is an arbitrary real number?

(c) For arbitrary a and b how do the lines

$$x + 2y = a$$

and

$$2x + y = b$$

intersect?

5. Assume that the system of equations

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

has exactly one solution. Show that the three lines

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \\ (a + c)x + (b + d)y &= e + f \end{aligned}$$

are *concurrent*, that is, pass through the same point.

[1]Exercises 2–4 are from the textbook D. Poole, *Linear Algebra: A Modern Introduction.* Thompson, 2006. ISBN 0-534-40596-7.

Homework 1: Solutions

1. The solution is $(x_1, x_2) = (-8, 3)$.

2. The solution is $(x, y) = (1, -1)$.

In Questions 1 and 2, any *ad hoc* method will do: for example, in Question 2 first equation is equivalent to $x = -1 - 2y$; substitute it into the 2nd: $2(-1-2y) + y = 1$; $-3y = 3$; $y = -1$; $x = -1 - 2.(-1) = 1$. Note that this solution clearly implies uniqueness. But if somebody just spots a solution $1, -1$, it is not clear if it is unique.

A systematic row elimination procedure is of course preferred.

3. Points simultaneously satisfying both equations are clearly on both lines, that is, the intersection (one point in this case).

The picture clearly gives one intersection point corresponding to the unique solution in Question 2.

4. (a) The line $x + 2y = a$ is parallel to the line $x + 2y = -1$ and different if $a \neq -1$, because clearly in this case the system (simultaneous equations)

$$\begin{aligned} x + 2y &= a \\ x + 2y &= -1 \end{aligned}$$

has no solutions, so no intersection points (which means parallel).

(b) Solution for the line $2x + y = b$ is similar to that of (a).

(c) The lines $x + 2y = a$ and $2x + y = b$ intersect in exactly one point for any a, b, because they are parallel, correspondingly, to the lines $x + 2y = -1$ and $2x + y = 1$ which intersect in one point by the argument above.

5. The equation of the third line is the sum of the equations for the first two lines, hence any common solution of the first two equations is a solution of the third one. Hence the third line passes through the point of intersection of the first two lines.