Calculus and Vectors - MATH10131

Problem Sheet for Week 1 — Real and Complex Numbers

***Note*:** This problem sheet introduces some basic concepts in calculus and also serves to demonstrate how problem sheets are to be used in the module. The problem sheet for any week is made available via internet. You should print it out and work through it. Sample answers will be made available on the web later. You will have a small-group supervision class each week and you must work through the problem sheet for that week and hand in your answers (or attempted answers) to the starred questions before your calculus supervision. In the first week there is no time to hand in work based on the problem sheet but make sure that you have worked through the sheet. You must also arrange with your supervisor where, when and how to hand in work for the second and all subsequent weeks. Your participation in these supervision classes will be marked by your supervisor and will count towards 10 % of your final mark for the module. The objective is not to hand in 'the right answer' to each question. It is to attempt each question, identify where you have difficulties and to make sure that your difficulties are discussed in the supervision class. You must make sure that the supervisor is fully aware of your difficulties by handing in your work and, if necessary, by telling the supervisor at or before the start of the class.

***The problem sheets are structured as follows*:**

- The first part consists of easy questions and basic revision material which can be used as warm-up exercises; you should ensure that you can confidently answer these questions. It is *essential* that you raise any difficulties with these questions in your Examples Class and discuss them with your supervisor.
- The second part consists of standard questions that test your understanding; these are for use in Examples Class and for revision at later times. There are also many suitable problems in 'Stewart' with answers to all odd-numbered exercises provided in Appendix H at the back.
- The third part consists of deeper and more difficult (usually interesting) questions that can, optionally, be used to advance your knowledge of calculus even further.

Easy Questions

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1. In each case, for $x \in \mathbb{R}$, find all of the intervals where x satisfies the inequality.
   (a) $|x+3| < 1$ (b) $|3x-2| \leq 4$ (c) $|x+1| \geq 5$ (d) $1/x \geq 2$
2. Sketch the graphs of the following relations between real values of x and y (that is the set of all points satisfying the equation or inequality). Explain very briefly how you got the graphs.
   (a) $2x + y = 1$ (b) $2x + y \leq 1$ (c) $y^2 + x = 4$ (d) $x^2 + 4y^2 < 4$
3. Are the following true or false? (assume that $z \in \mathbb{C}$)
   (a) $i^2 + (-i)^4 = 0$ (b) If $z \neq 0$ then $\mathrm{Im}(z) \notin \mathbb{R}$ (c) $\mathrm{Im}(z) = \mathrm{Im}(\bar{z})$
4. Give the values of the following exactly, without using a calculator. For example $\sin(\frac{\pi}{4}) = 1/\sqrt{2}$.
   (a) $\cos(\frac{\pi}{4})$ (b) $\sin(\frac{\pi}{3})$ (c) $\tan(\frac{\pi}{6})$ (d) $\cos(\frac{2\pi}{3})$ (e) $\tan(\frac{7\pi}{6})$

Standard Questions

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5. (a) Find the set of real numbers x satisfying $|x^2 - 2| < 1$
   (b) Find the set of real numbers u satisfying $|u^2 - 2| < 3$
   (c) Find the set of real numbers y satisfying $1/y \geq -2$

6. Convert each of the following complex numbers into standard form ('`real part`' + i '`imaginary part`')

   (a) $(5+i)(2+3i)$ (b) $(3+4i)(3-4i)$ (c) $(-2+i)^3$ (d) $(2-i)^4$

   (e) $\dfrac{13}{3-2i}$ (f) $\dfrac{i(7+3i)}{3-7i}$ (g) $\dfrac{5-5i}{i(1+i)(2+i)}$ (h) $\dfrac{4+5i}{2-3i}$

7. (a) Sketch, in the complex plane, where $|z-i|<1$ for $z\in\mathbb{C}$

   (b) Sketch, in the complex plane, where $1<|w-2|\leq 3$ for $w\in\mathbb{C}$

Harder Questions

8. Use a calculator or suitable computer package (if you have access to one) to work out values of the formula

$$\left(1+\tfrac{1}{n}\right)^n$$

   for $n=1,\ 2,\ 4,\ 8,\ \cdots,\ 2^k,\ \cdots$ and any other suitably large values of n you wish.

   How large does n need to be before the absolute value of the difference $e-\left(1+\frac{1}{n}\right)^n$ is less than 0.01?

   Note that e is the base of the natural logarithm given by $e\approx 2.718281828$
