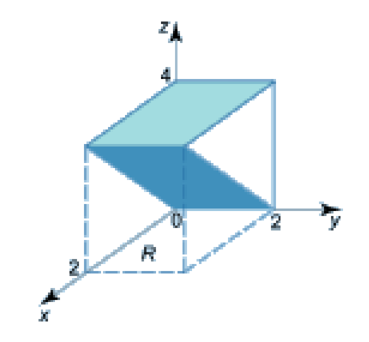
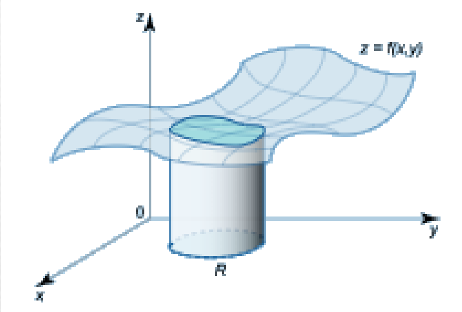
INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

Course Name : Mathematics & Statistics-III (BSC-M301)

INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

Multivariate Calculus (Integration)





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Double Integrals : Change of Order of Integration

Evaluate $I = \int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y}\, dy\, dx$ by changing the order of integration.

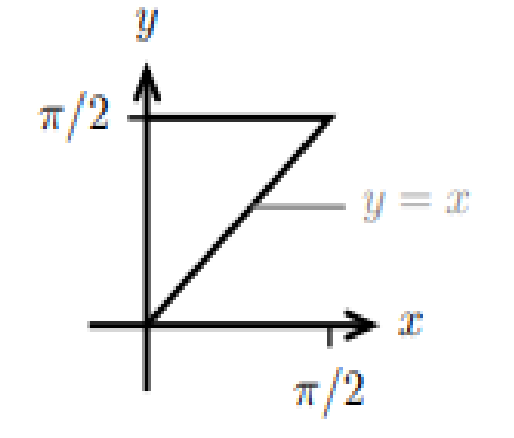
Solution:

The given limits are: (Inner) y from x to $\frac{\pi}{2}$; (Outer) x from 0 to $\frac{\pi}{2}$.

As we reverse the order of integration, the new limits are:

(Inner) x from 0 to y; (Outer) y from 0 to $\frac{\pi}{2}$. Then,

$$I = \int_0^{\frac{\pi}{2}} \int_0^{y} \frac{\sin y}{y}\, dx dy .$$

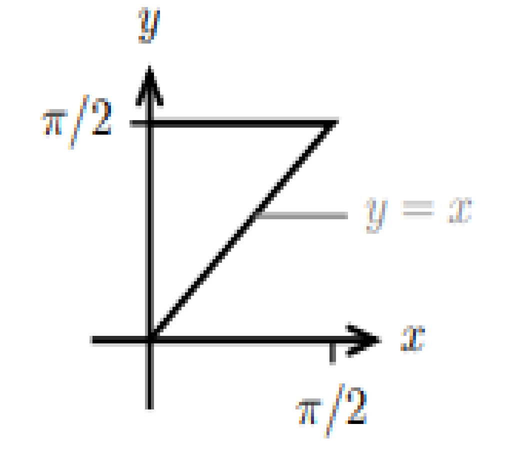


Double Integrals : Change of Order of Integration

Now,

$$I = \int_0^{\frac{\pi}{2}} \int_0^{y} \frac{siny}{y} dxdy = \int_0^{\frac{\pi}{2}} \frac{siny}{y} \left\{ \int_0^{y} dx \right\} dy = \int_0^{\frac{\pi}{2}} \frac{siny}{y} \{x\}_0^y dy$$

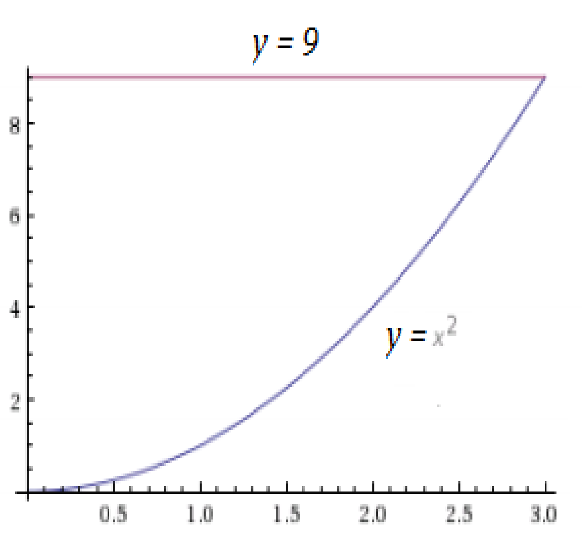
$$= \int_0^{\frac{\pi}{2}} sinydy = \{-cosy\}_0^{\frac{\pi}{2}} = 1$$



Double Integrals : Change of Order of Integration

Evaluate the integral by first reversing the order of integration,

$$\int_{x=0}^{x=3}\int_{y=x^2}^{y=9} x^3 e^{y^3}\, dy\, dx.$$



Solution:

The given limits are: (Inner) y from x^2 to 9; (Outer) x from 0 to 3. As we reverse the order of integration, the new limits are: (Inner) x from 0 to $\sqrt{y}$; (Outer) y from 0 to 9 . Then, the given integral becomes

$$\int_{y=0}^{y=9}\int_{x=0}^{x=\sqrt{y}} x^3 e^{y^3} dxdy$$

Double Integrals : Change of Order of Integration

Now,

$$\int_{y=0}^{y=9}\int_{x=0}^{x=\sqrt{y}} x^3 e^{y^3}\,dx\,dy = \int_{y=0}^{y=9} e^{y^3}\left\{\int_{x=0}^{x=\sqrt{y}} x^3\,dx\right\}dy = \int_{y=0}^{y=9} e^{y^3}\left\{\frac{x^4}{4}\right\}_0^{\sqrt{y}} dy$$

$$= \frac{1}{4}\int_{y=0}^{y=9} e^{y^3} y^2\,dy = \frac{1}{12}\int_{z=0}^{z=9^3} e^z\,dz = \frac{1}{12}\left(e^{729}-1\right)$$

where $y^3 = z$.

Thank You