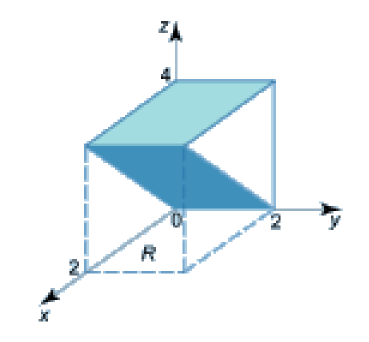
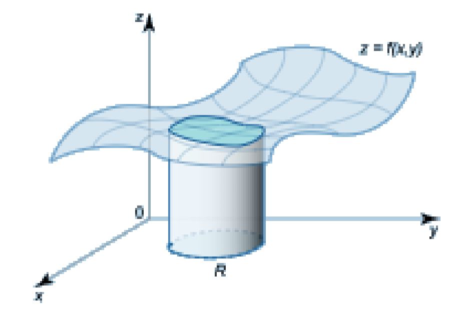
# INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

**Course Name :** Mathematics & Statistics-III (BSC-M301)

INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

# Multivariate Calculus (Integration)

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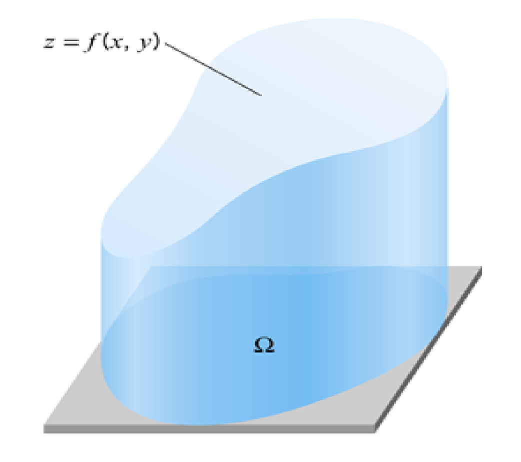
# Application of Double Integrals : Area

**Try to remember:**

If $f(x, y) = 1$, then the double integral $\iint_{\Omega} f(x,y)\,dxdy,$ reduces to

$$\iint_{\Omega} dxdy$$

which represents the **area of the region Ω .**



# Application of Double Integrals : Area

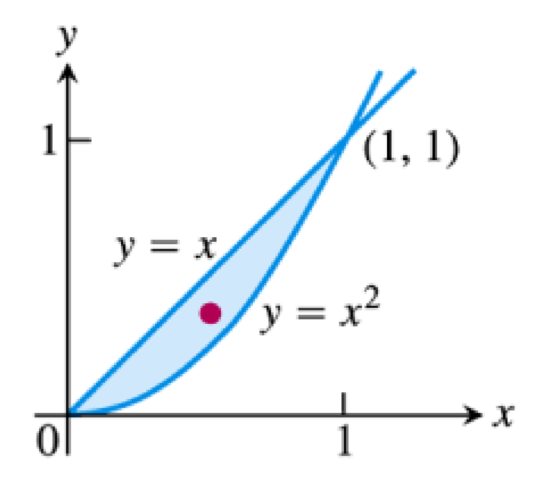
Find the area of the region $R$ bounded by $y = x$ and $y = x^2$ in the first quadrant.

*Solution* :

The required area is given by

$$A = \int_0^1 \int_{x^2}^{x} dy\, dx = \int_0^1 \Big[ y \Big]_{x^2}^{x} dx$$

$$= \int_0^1 (x - x^2)\, dx = \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{6}.$$



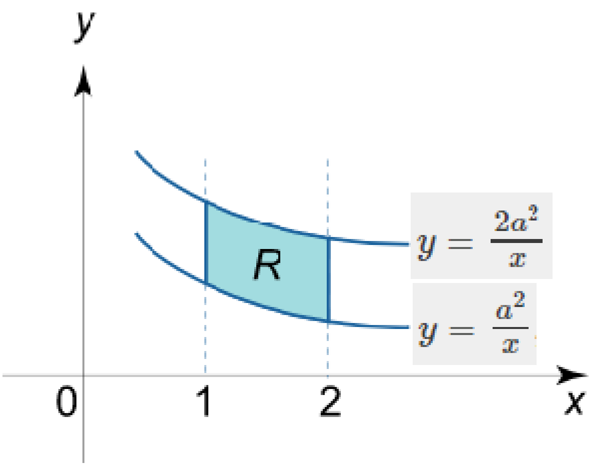
# Application of Double Integrals : Area

Find the area of the region $R$ bounded by the hyperbolas $y = \frac{a^2}{x}$, $y = \frac{2a^2}{x}$ $(a > 0)$ and the vertical lines $x = 1$, $x = 2$.

*Solution* : The required area is given by

$$A = \iint_R dx\,dy = \int_1^2 \left[ \int_{\frac{a^2}{x}}^{\frac{2a^2}{x}} dy \right] dx = \int_1^2 \left[ y \Big|_{\frac{a^2}{x}}^{\frac{2a^2}{x}} \right] dx = \int_1^2 \left( \frac{2a^2}{x} - \frac{a^2}{x} \right) dx = a^2 \int_1^2 \frac{dx}{x}$$

$$= a^2 (\ln 2 - \ln 1) = a^2 \ln 2.$$

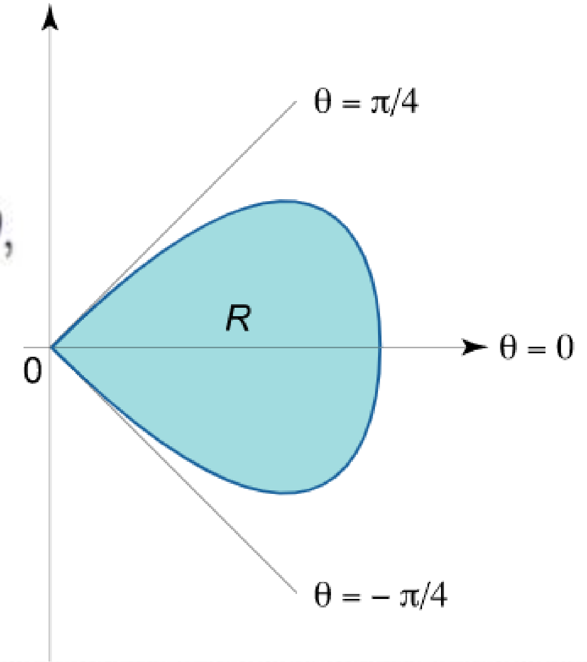


# Application of Double Integrals : Area

Find the area of one loop of the rose defined by the equation $r = \cos 2\theta$.

*Solution* :

The region of integration $R$ can be written in the form $R = \{(r, \theta) \mid 0 \le r \le \cos 2\theta, -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}\}$. Hence, the area of the region in polar coordinates is

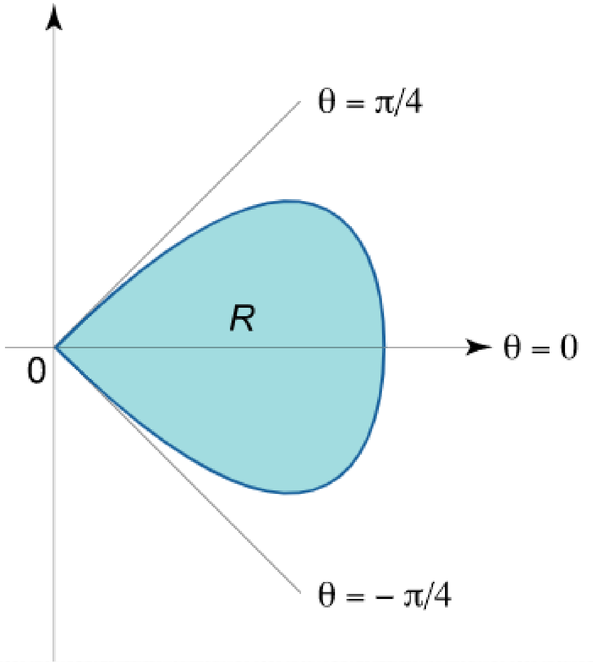


# Application of Double Integrals : Area

$$A = \iint_R r\,dr\,d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\int_0^{\cos 2\theta} r\,dr\right] d\theta = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\left(\frac{r^2}{2}\right)\Big|_0^{\cos 2\theta}\right] d\theta = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2(2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1+\cos 4\theta}{2}\,d\theta = \frac{1}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1+\cos 4\theta)\,d\theta = \frac{1}{4}\left(\theta + \frac{\sin 4\theta}{4}\right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{1}{4}\left[\left(\frac{\pi}{4} + \frac{\sin \pi}{4}\right) - \left(-\frac{\pi}{4} + \frac{\sin(-\pi)}{4}\right)\right] = \frac{\pi}{8}.$$



# Thank You