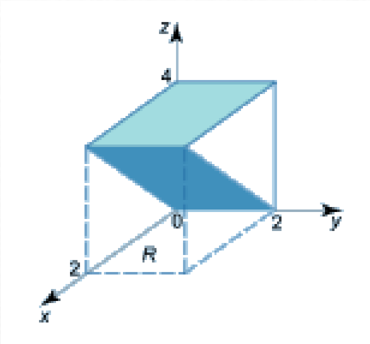
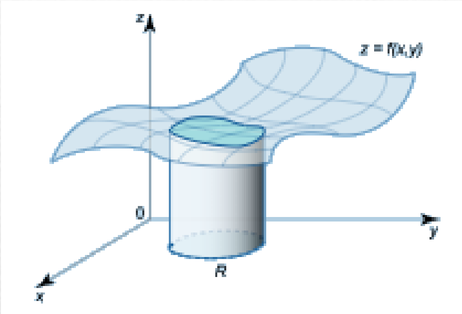
INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

Course Name : Mathematics & Statistics-III (BSC-M301)

Multivariate Calculus (Integration)





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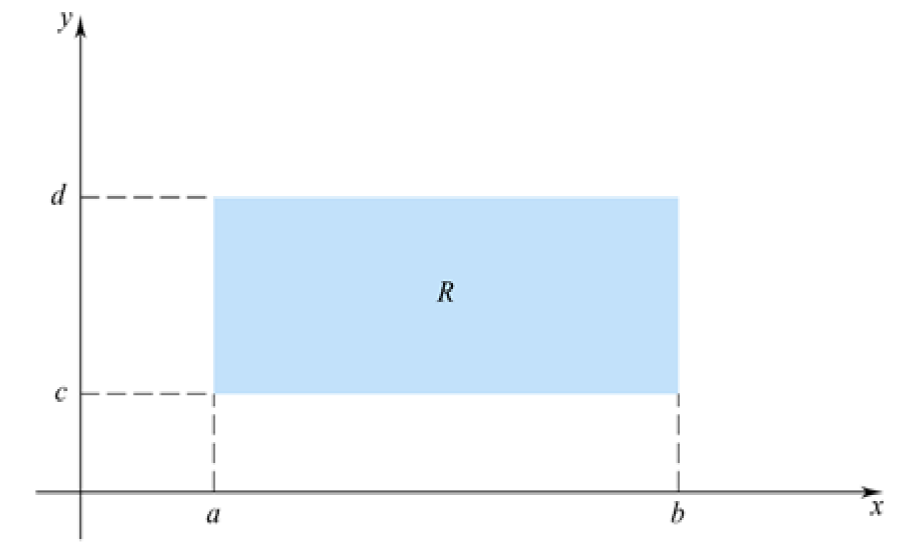
Evaluation of Double Integrals by Repeated Single Integrals

Rectangular Region : $R = \{(x, y) : a \le x \le b, c \le y \le d\}$

Fubini's Theorem

$$\iint_R f(x,y)\,dx\,dy = \int_c^d \left\{ \int_a^b f(x,y)\,dx \right\} dy$$

$$\text{or } = \int_a^b \left\{ \int_c^d f(x,y)\,dy \right\} dx$$

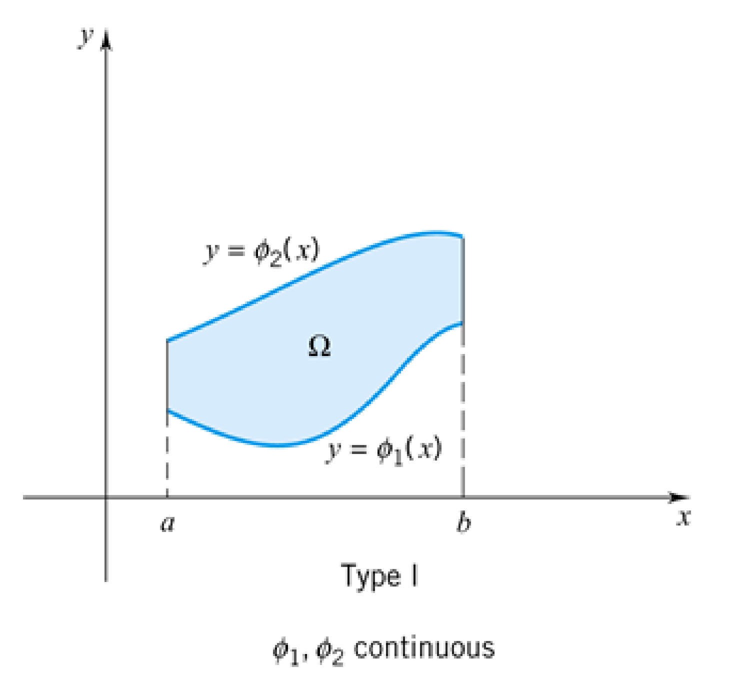


Evaluation of Double Integrals by Repeated Single Integrals

Type I Region :

$\Omega = \{(x, y): a \le x \le b, \phi_1(x) \le y \le \phi_2(x)\}$

where $\phi_1(x)$ and $\phi_2(x)$ are continuous on $[a,b]$



Fubini's Theorem

$$\iint_{\Omega} f(x, y)\, dx\, dy = \int_a^b \left(\int_{\phi_1(x)}^{\phi_2(x)} f(x, y)\, dy \right) dx.$$

Evaluation of Double Integrals by Repeated Single Integrals

Calculate the integral $\iint\limits_R (x+y)\,dxdy$. The region of integration R is bounded by the lines $x=0,\ y=0,\ x+y=2.$

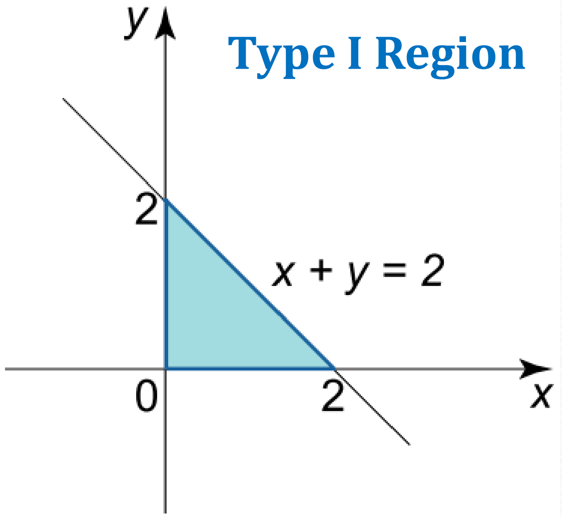
Solution.

We can represent the region R as the set $R=\{(x,y)\mid 0\le x\le 2,\quad 0\le y\le 2-x\}$

$$\iint\limits_R (x+y)\,dxdy=\int_0^2\int_0^{2-x}(x+y)\,dydx=\int_0^2\left[\int_0^{2-x}(x+y)\,dy\right]dx$$

$$=\int_0^2\left[\left(xy+\frac{y^2}{2}\right)\Big|_{y=0}^{2-x}\right]dx=\int_0^2\left[x(2-x)+\frac{(2-x)^2}{2}\right]dx=\int_0^2\left(2-\frac{x^2}{2}\right)dx$$

$$=\left(2x-\frac{x^3}{6}\right)\Big|_0^2=\frac{8}{3}.$$



Evaluation of Double Integrals by Repeated Single Integrals

Calculate the double integral $\iint\limits_R (x-y)\,dxdy$. The region of integration R is bounded by $x=0$, $x=1$, $y=x$, $y=2-x^2$.

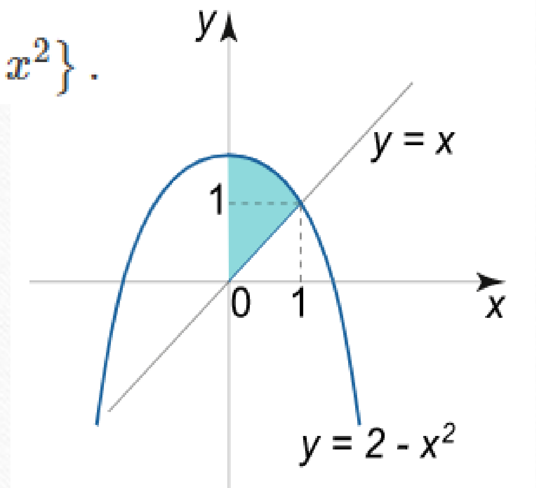
Solution.

We can represent the region R in the form $R=\{(x,y)\,|\,0\le x\le 1,\ \ x\le y\le 2-x^2\}$.

So R is the region of type I

$$\iint\limits_R (x-y)\,dxdy=\int\limits_0^1\int\limits_x^{2-x^2}(x-y)\,dydx=\int\limits_0^1\left[\int\limits_x^{2-x^2}(x-y)\,dy\right]dx.$$

Type I Region



Evaluation of Double Integrals by Repeated Single Integrals

Calculate first the inner integral:

$$\int_{x}^{2-x^2} (x-y)\,dy = \left(xy - \frac{y^2}{2}\right)\Bigg|_{y=x}^{2-x^2} = [x\left(2-x^2\right) - \frac{\left(2-x^2\right)^2}{2}] - \left[x^2 - \frac{x^2}{2}\right]$$

$$= -\frac{x^4}{2} - x^3 + \frac{3x^2}{2} + 2x - 2.$$

Now we can compute the outer integral:

$$\int_{0}^{1} \left(-\frac{x^4}{2} - x^3 + \frac{3x^2}{2} + 2x - 2\right) dx = \left(-\frac{x^5}{10} - \frac{x^4}{4} + \frac{x^3}{2} + x^2 - 2x\right)\Bigg|_{0}^{1} = -\frac{17}{20}.$$

Thank You