INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

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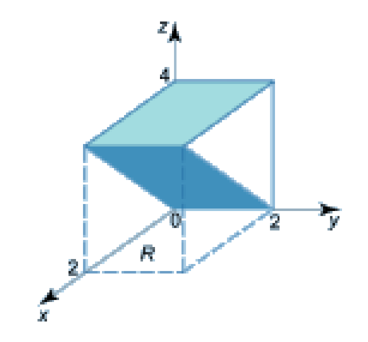
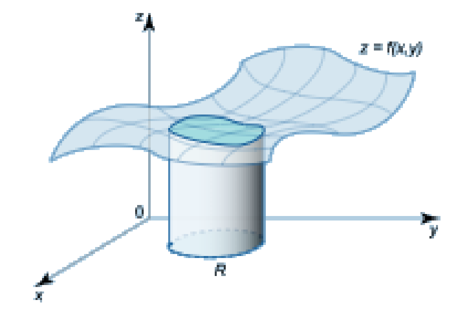
Course Name : Mathematics - II (BSC-203)

Multivariate Calculus (Integration)





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Multivariate Integration

Double Integrals

- Concerned with functions of two variables as $z = f(x, y)$ and are integrals of the type
- $\iint f(x, y)dxdy$ or $\iint_R f(x, y)dxdy$

Triple Integrals

- Concerned with functions of three variables as $w = f(x, y, z)$ and are integrals of the type
- $\iiint f(x, y, z)dxdydz$ or $\iiint_R f(x, y, z)dxdydz$

Understanding Double Integrals

Double Integrals

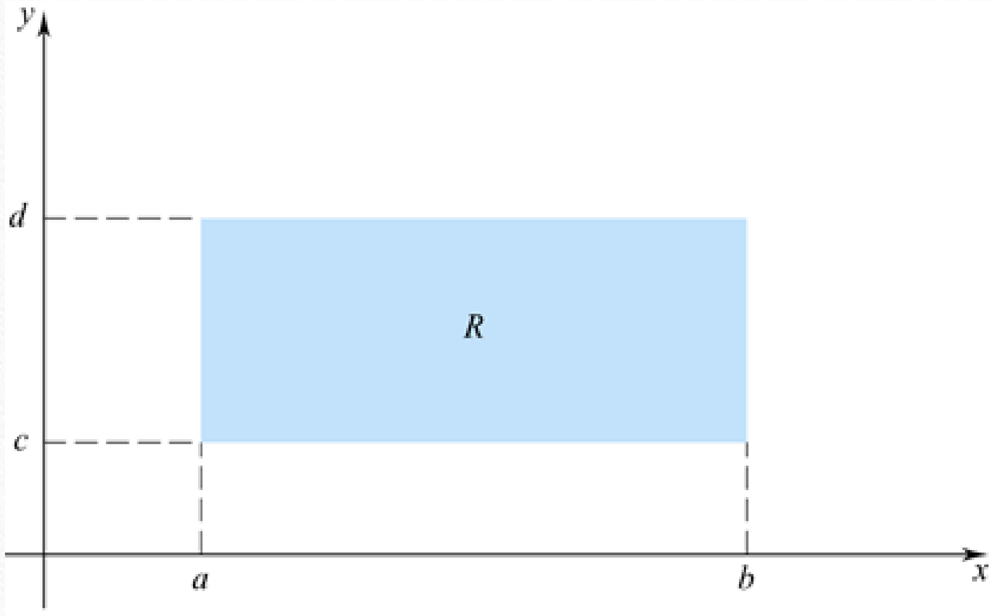
The Double Integral over a Rectangle

We start with a function $f(x, y)$ continuous on a rectangle

$$R : a \leq x \leq b,\ c \leq y \leq d\,.$$

We want to define the double integral of f over R:

$$\boxed{\iint\limits_R f(x, y)\,dx\,dy}$$



Double Integrals

The Double Integral over a Rectangle

First we partition the rectangle R as follows.
We begin with a partition

$$P_1 = \{x_0, x_1, \ldots, x_m\} \quad \text{of} \quad [a, b],$$

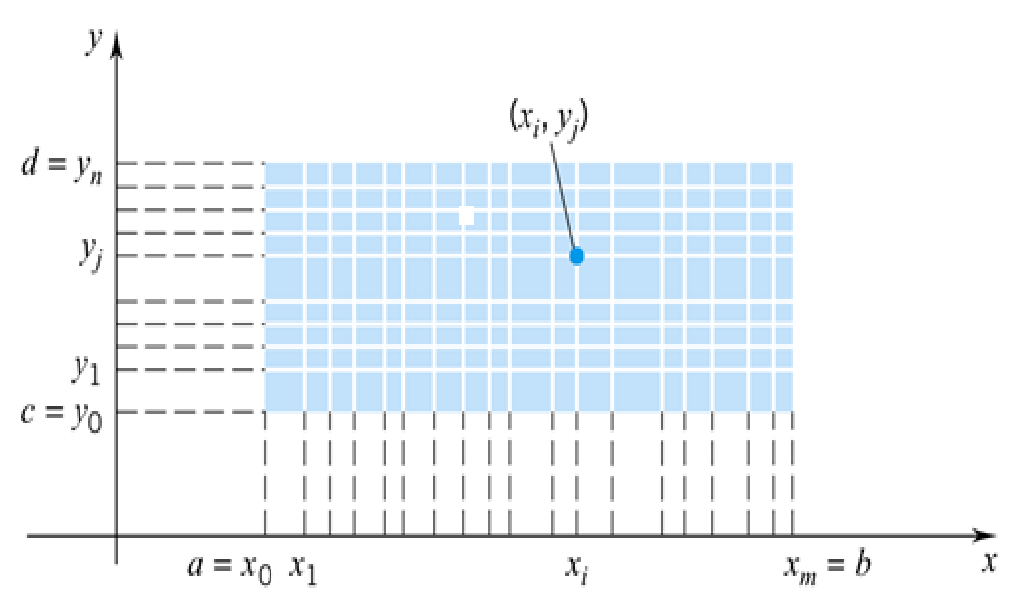
and a partition

$$P_2 = \{y_0, y_1, \ldots, y_n\} \quad \text{of} \quad [c, d].$$

The set

$$P = P_1 \times P_2 = \{(x_i, y_j) : \ x_i \in P_1, \ y_j \in P_2\}$$

is called a *partition of R*. The set P consists of all the grid points (x_i, y_j).

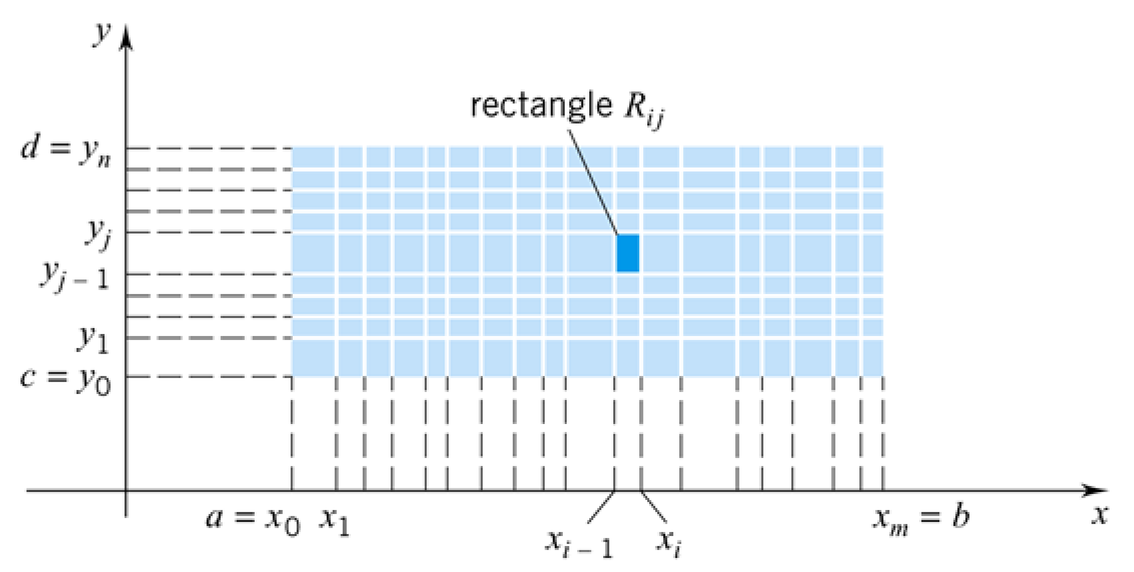


Double Integrals

The Double Integral over a Rectangle

Using the partition P, we break up R into $m \times n = N$, say, non-overlapping rectangles

$$R_{ij}: \quad x_{i-1} \le x \le x_i, \quad y_{j-1} \le y \le y_j, \qquad \text{where} \quad 1 \le i \le m,\ 1 \le j \le n.$$



Double Integrals

The Double Integral over a Rectangle

Now consider sum of all the products

$f(x_i, y_j)(x_i - x_{i-1})(y_j - y_{j-1}) = f_{ij}\Delta x_i \Delta y_j$ for all $1 \leq i \leq m$, $1 \leq j \leq n$ *i.e.*

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\Delta x_i \Delta y_j$$

Let the number of partitions $N \to \infty$ in such a way that the area $\Delta x_i \Delta y_j$ of each small rectangle $R_{i,j} \to 0$. If the above sum exists under this limit, i.e.

$$\lim_{N\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\Delta x_i \Delta y_j$$

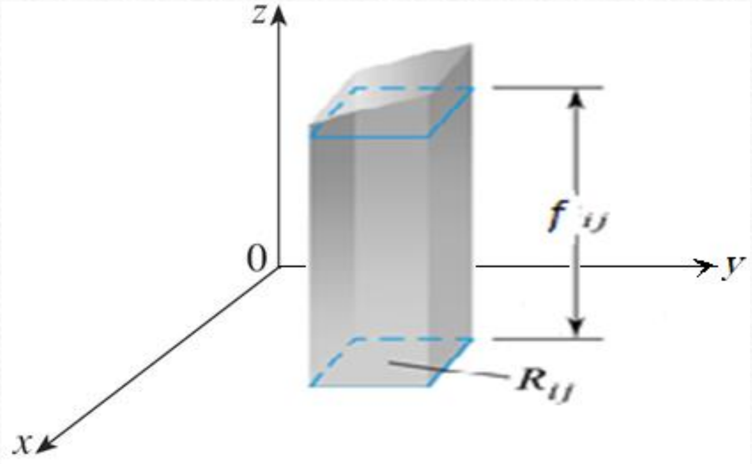
exists, it is called the ***Double Integral*** of the function $\boldsymbol{f(x, y)}$ over the region $\boldsymbol{R}$ and is denoted as

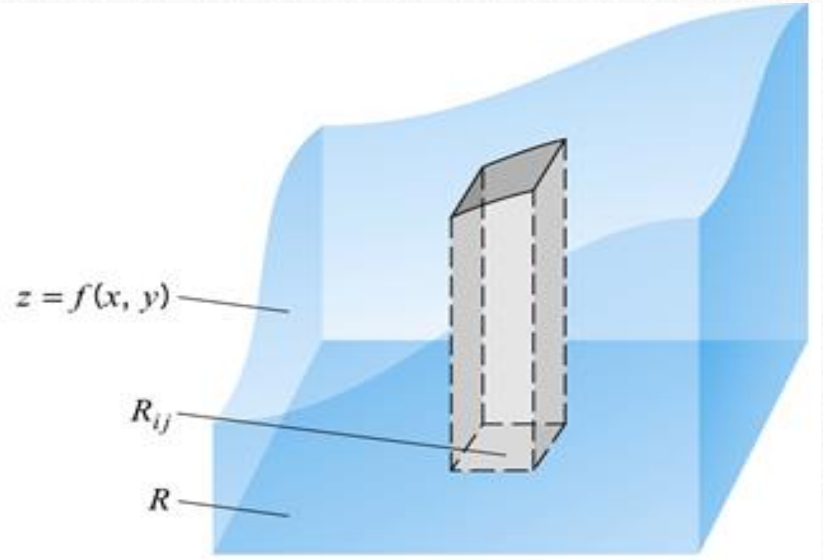
$$\iint_R f(x, y)\,dx\,dy.$$

Double Integrals

The Double Integral as a Volume

If f is continuous and nonnegative on the rectangle R, the equation $z = f(x, y)$ represents a surface that lies above R. In this case, the double integral $\boxed{\iint_R f(x, y)\, dx\, dy}$ gives the **volume of the solid** that is bounded below by R and bounded above by the surface $z = f(x, y)$.





Double Integrals

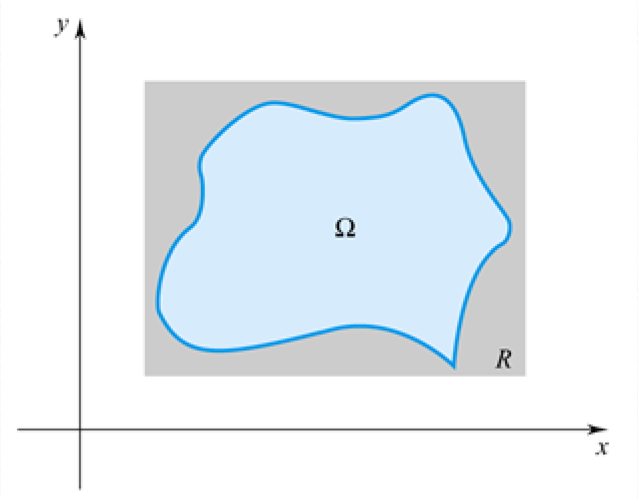
The Double Integral over an arbitrary region Ω

Let **Ω** be a bounded region of more general shape, and let **Ω** be enclosed by a rectangle ***R***.

To find the double integral $\boxed{\iint_{\Omega} f(x,y)dxdy,}$ we define a new function $F(x,y)$ with domain ***R*** as follows:

$$F(x,y) = \begin{cases} f(x,y) & \text{if } (x,y) \text{ is in } \Omega \\ 0 & \text{if } (x,y) \text{ is in } R \text{ but not in } \Omega \end{cases}$$

Then, $\boxed{\iint_{\Omega} f(x,y)dxdy = \iint_{R} F(x,y)dxdy}$

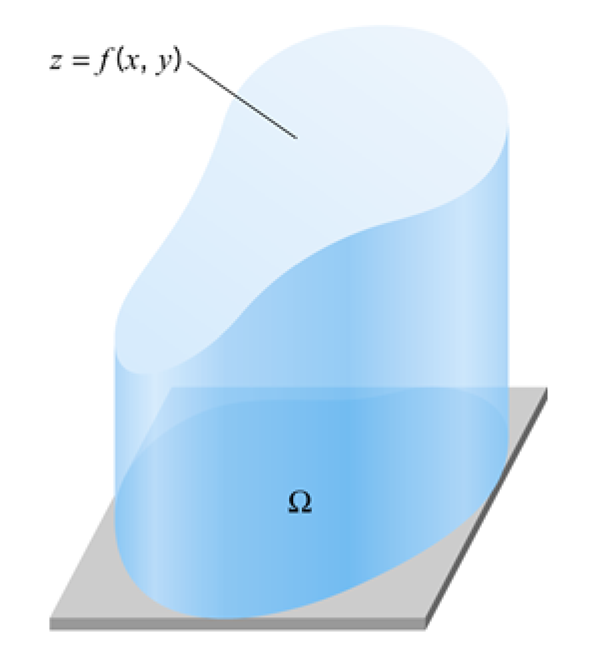


Double Integrals

The Double Integral over an arbitrary region Ω

If f is continuous and nonnegative over Ω, the extended f is nonnegative on all of R. The ***volume of the solid T*** bounded above by $z = f(x, y)$ and bounded below by Ω is given by:

$$\iint_{\Omega} f(x,y)\,dx\,dy$$



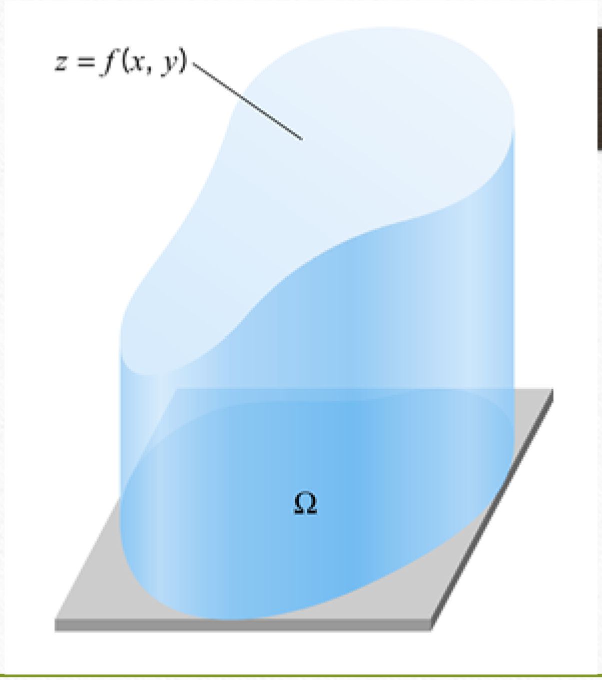
Double Integrals

The Double Integral representing Area

Special Case: If $f(x,y) = 1$, then the double integral $\boxed{\iint_{\Omega} f(x,y)dxdy,}$ reduces to

$$\boxed{\iint_{\Omega} dxdy}$$

which now represents the **area of the region Ω .**



***Question* :**

How to evaluate Double Integrals?

Thank You