

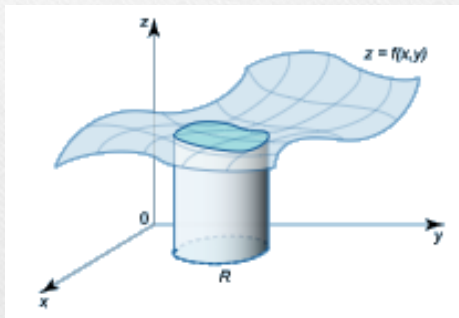
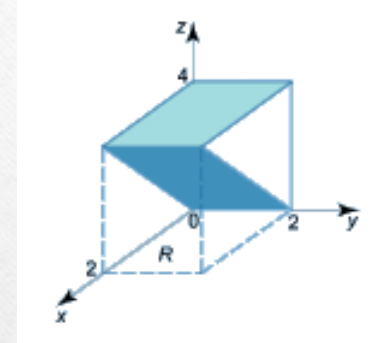


INSTITUTE OF ENGINEERING & MANAGEMENT, KOLKATA

Course Name : Mathematics & Statistics-III (BSC-M301)



Multivariate Calculus (Integration)



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What is Calculus?

- **Calculus** is a branch of mathematics that explores **variables** and how they change by looking at them in infinitely small pieces called **infinitesimals**.
- Calculus, as it is practiced today, was invented in the **17th century** by British scientist **Isaac Newton** (1642-1726) and German scientist **Gottfried Leibnitz** (1646-1716), who independently developed the principles of calculus.
- It provides a framework for modeling systems in which there is change, and a way to deduce the predictions of such models.

Branches of Calculus

Differential Calculus

- **Differentiation** of functions of **one** or **more** variables
- Concerned with instantaneous rates of change and slope of curves

Integral Calculus

- **Integration** of function of **one** or **more** variables
- Concerned with accumulation of quantities and area under or between curves and many more

Functions of one or more variables

Single Variable Calculus

- Involves functions of single variable of the form
- $y = f(x)$

Multivariable Calculus

- Involves functions of more than one variable i.e. multivariable functions of the form
- $z = f(x, y)$
- or $w = f(x, y, z)$
- or $w = f(x_1, x_2, \dots, x_n)$

Two view points of Integration

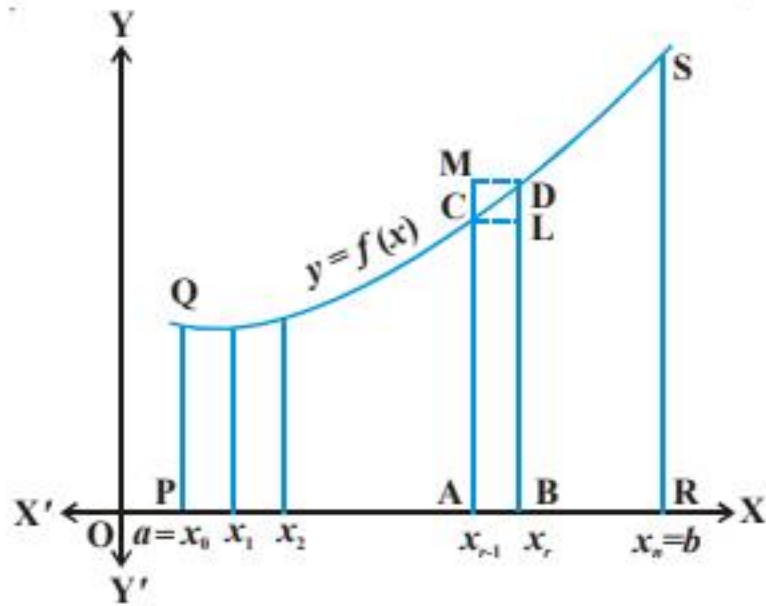
Fundamental
Theorem of
Integral Calculus

As a certain
Summation –
Limit of a sum

Inverse process
of
Differentiation

Geometrical Interpretation of $\int_a^b f(x)dx$

Form the sum $h(f(x_0) + f(x_1) + \cdots + f(x_{n-1}))$



$$\int_a^b f(x)dx = \lim_{h \rightarrow 0} h \sum_{r=0}^{n-1} f(x_r)$$

$$\text{or } \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{r=0}^{n-1} f(x_r)$$

$\int_a^b f(x)dx$ geometrically represents the **area** of the space enclosed by the curve $y = f(x)$, the ordinates $x = a, x = b$ and the x -axis.

Multivariate Integration

Double Integrals

- Concerned with functions of two variables as $z = f(x, y)$ and are integrals of the type
- $\iint f(x, y) dx dy$ or $\iint_R f(x, y) dx dy$

Triple Integrals

- Concerned with functions of three variables as $w = f(x, y, z)$ and are integrals of the type
- $\iiint f(x, y, z) dx dy dz$ or $\iiint_R f(x, y, z) dx dy dz$

Course Contents

➤ Double Integrals

- ❖ Applications in finding Area, Volume, Centre of Mass & Gravity

➤ Triple Integrals with applications

➤ Vector Integration

- ❖ Line Integrals
- ❖ Surface Integrals
- ❖ Theorems of Green, Gauss & Stokes

Learning Outcome

Learn the methods for evaluating multiple integrals and their applications to different geometrical and physical problems.

Text / Reference Books

➤ Text Books:

- ✓ Higher Engineering Mathematics, *B. S. Grewal*, Khanna Publishers, 43rd Edition.
- ✓ Engineering Mathematics-2B, *B. Basu Mallik & Krishanu Deyasi*, Cengage Learning.

➤ Reference Books:

- ✓ Advanced Engineering Mathematics, *Michael Greenberg*, Pearson
- ✓ Advanced Engineering Mathematics, *Jain & Iyengar*, Narosa.
- ✓ Advanced Engineering Mathematics, *H. K. Dass*, SultanChand.

Thank You

