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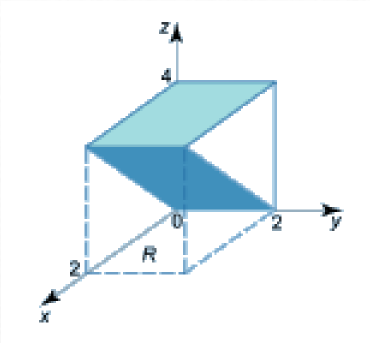
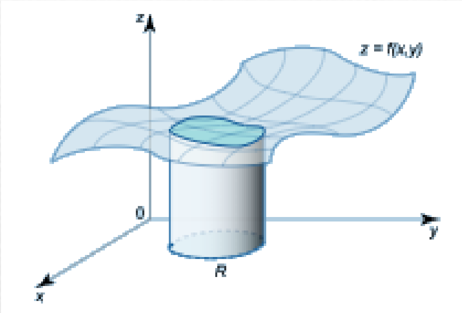
Course Name : Mathematics & Statistics-III (BSC-M301)

Multivariate Calculus (Integration)





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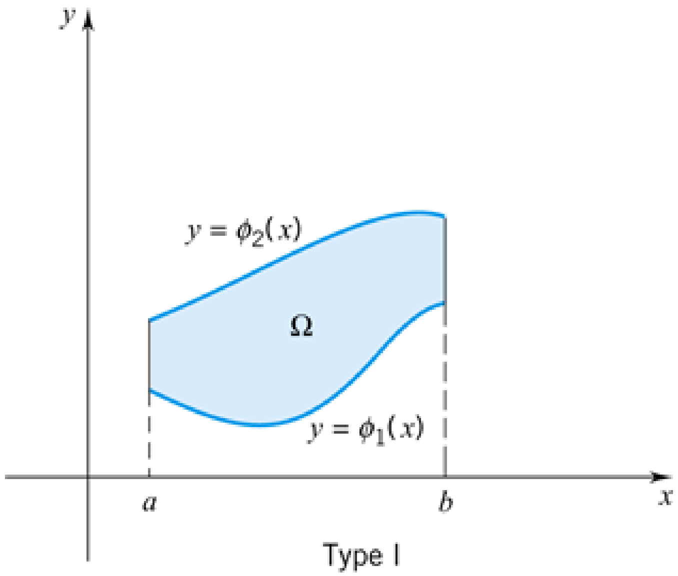
Evaluation of Double Integrals by Repeated Single Integrals

Type I Region :

$\Omega = \{(x, y): a \leq x \leq b, \phi_1(x) \leq y \leq \phi_2(x)\}$

where $\phi_1(x)$ and $\phi_2(x)$ are continuous on $[a,b]$



Fubini's Theorem

$$\iint_\Omega f(x, y)\,dx\,dy = \int_a^b \left(\int_{\phi_1(x)}^{\phi_2(x)} f(x, y)\,dy \right) dx.$$

Evaluation of Double Integrals by Repeated Single Integrals

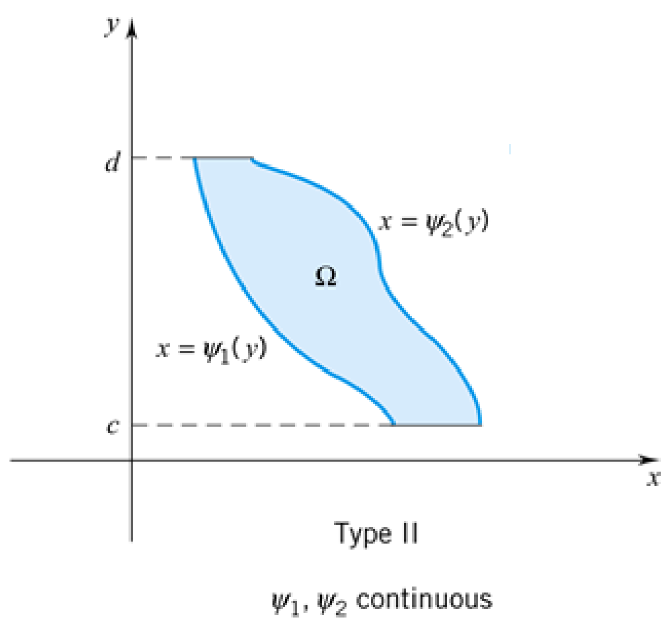
Type II Region :

$\Omega = \{(x, y): c \leq y \leq d, \psi_1(y) \leq x \leq \psi_2(y)\}$

where $\psi_1(y)$ and $\psi_2(y)$ are continuous on $[c,d]$

Fubini's Theorem

$$\iint_{\Omega} f(x, y)\, dx\, dy = \int_c^d \left(\int_{\psi_1(y)}^{\psi_2(y)} f(x, y)\, dx \right) dy.$$



Evaluation of Double Integrals by Repeated Single Integrals

Find the double integral $\iint_R (x+y)\,dxdy$, where the region R is a parallelogram with the sides $y=x$, $y=x+a$, $y=a$, $y=2a$, a is a parameter.

Solution.

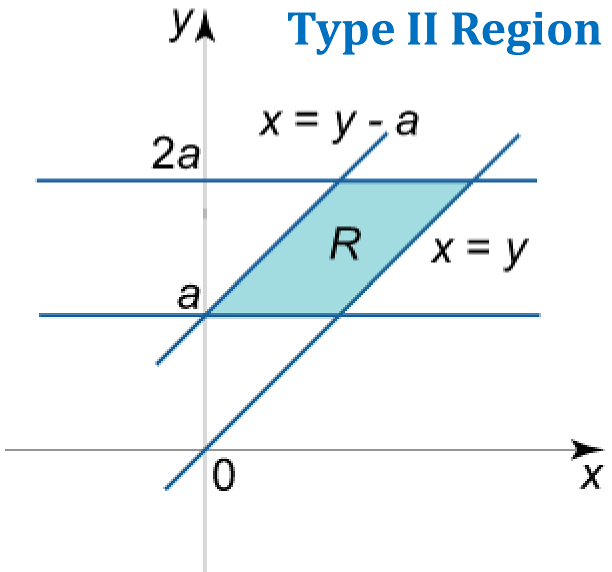
We will consider R as a region of type II

The values of y are those between a and $2a$,

while, given y, the relevant values of x are those

between $x=y-a$ and $x=y$. So the double integral is

Type II Region



Evaluation of Double Integrals by Repeated Single Integrals

$$\iint_R (x+y)\,dxdy = \int_a^{2a}\left[\int_{y-a}^{y}(x+y)\,dx\right]dy = \int_a^{2a}\left[\left(\frac{x^2}{2}+yx\right)\Big|_{x=y-a}^{y}\right]dy$$

$$= \int_a^{2a}\left[\left(\frac{y^2}{2}+y^2\right)-\left(\frac{(y-a)^2}{2}+y(y-a)\right)\right]dy$$

$$= \int_a^{2a}\left(\frac{3y^2}{2}-\frac{y^2-2ay+a^2}{2}-y^2+ay\right)dy = \int_a^{2a}\left(2ay-\frac{a^2}{2}\right)dy$$

$$= \left(\frac{2ay^2}{2}-\frac{a^2y}{2}\right)\Big|_a^{2a} = \left(ay^2-\frac{a^2y}{2}\right)\Big|_a^{2a} = \left(a\cdot(2a)^2-\frac{a^2}{2}\cdot 2a\right)$$

$$-\left(a\cdot a^2-\frac{a^2}{2}\cdot a\right) = 4a^3-a^3-a^3+\frac{a^3}{2} = \frac{5a^3}{2}.$$

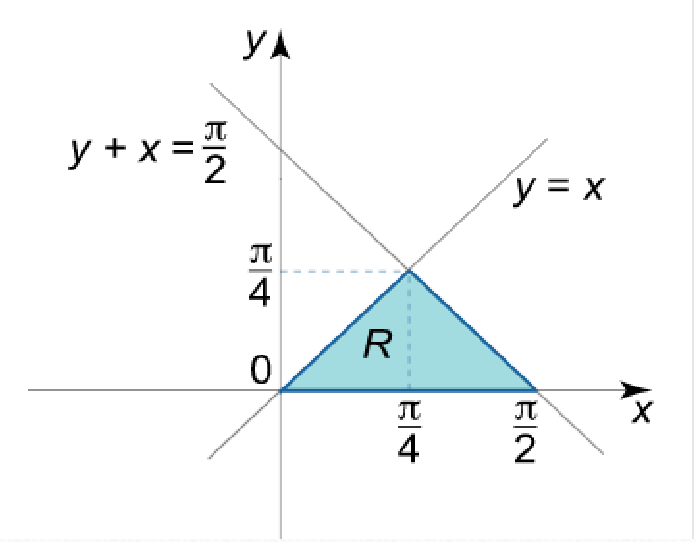
Evaluation of Double Integrals by Repeated Single Integrals

Find the double integral $\iint_R \sin(x+y)\,dx\,dy$, defined in the region R bounded by the lines $y = x, x + y = \frac{\pi}{2}, y = 0.$

Solution.

Considering the region of integration R as a type II region

x varies from $x = y$ to $x = \frac{\pi}{2} - y$ and y varies from 0 to $\frac{\pi}{4}$.

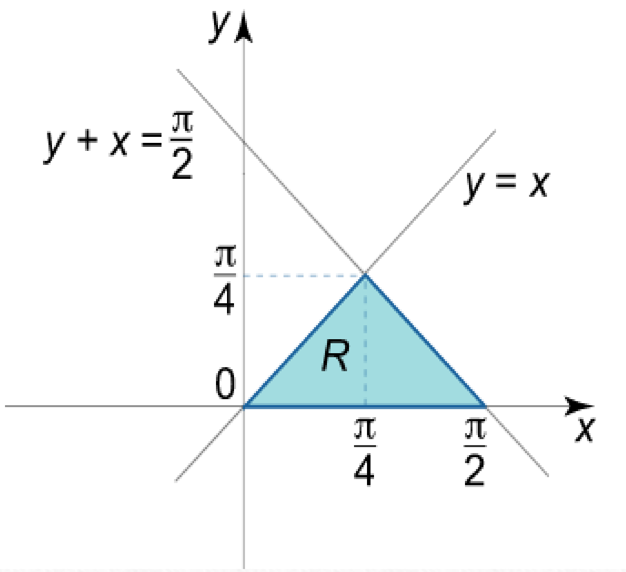


Evaluation of Double Integrals by Repeated Single Integrals

$$\iint\limits_R \sin(x+y)dxdy = \int_0^{\frac{\pi}{4}} \left[\int_y^{\frac{\pi}{2}-y} \sin(x+y)dx \right] dy = \int_0^{\frac{\pi}{4}} \left[\left(-\cos(x+y)\right)\Big|_{x=y}^{\frac{\pi}{2}-y} \right] dy$$

$$= -\int_0^{\frac{\pi}{4}} \left[\cos\left(\frac{\pi}{2} - y + y\right) - \cos 2y \right] dy = \int_0^{\frac{\pi}{4}} \cos 2y dy = \left(\frac{\sin 2y}{2}\right)\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right) = \frac{1}{2}.$$



Thank You