

age	p _i	n _i	I(p _i , n _i)
<=30	2	8	0.971
31...40	4	0	0
>40	3	2	0.971

1.

$$\text{Info}(D) = I(8,4) = -\frac{8}{12} \log_2 \left(\frac{8}{12} \right) - \frac{4}{12} \log_2 \left(\frac{4}{12} \right) = 0.9183$$

age	income	student	credit rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
<=30	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

$$\text{Info}_{\text{age}}(D) = \frac{4}{12} I(2,2) + \frac{3}{12} I(3,0) + \frac{5}{12} I(3,2)$$

$$I(2,2) = -\frac{2}{4} \log_2 \left(\frac{2}{4} \right) - \frac{2}{4} \log_2 \left(\frac{2}{4} \right) = 1$$

$$= \frac{4}{12} (1) + \frac{3}{12} (0) + \frac{5}{12} (0.971)$$

$$I(3,0) = -\frac{3}{3} \log_2 \left(\frac{3}{3} \right) - \frac{0}{3} \log_2 \left(\frac{0}{3} \right) = 0$$

$$= 0.5761$$

$$I(3,2) = -\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{2}{5} \log_2 \left(\frac{2}{5} \right) = 0.971$$

$$\text{Gain}(\text{age}) = \text{Info}(D) - \text{Info}_{\text{age}}(D) = 0.9183 - 0.5761 = 0.3422$$

income	p _i	n _i
high	2	2
medium	4	1
low	2	1

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \frac{4}{12} I(2,2) + \frac{5}{12} I(4,1) + \frac{3}{12} I(2,1) \\ &= \frac{4}{12} (1) + \frac{5}{12} (0.7219) + \frac{3}{12} (0.9183) \\ &= 0.8837 \end{aligned}$$

$$I(4,1) = -\frac{4}{5} \log_2 \left(\frac{4}{5} \right) - \frac{1}{5} \log_2 \left(\frac{1}{5} \right) = 0.7219$$

$$I(2,1) = -\frac{2}{3} \log_2 \left(\frac{2}{3} \right) - \frac{1}{3} \log_2 \left(\frac{1}{3} \right) = 0.9183$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.9183 - 0.8837 = 0.0346$$

student	p _i	n _i
Yes	5	1
No	3	3

$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \frac{6}{12} I(5,1) + \frac{6}{12} I(3,3) \\ &= \frac{6}{12} (0.65) + \frac{6}{12} (1) \\ &= 0.825 \end{aligned}$$

$$I(5,1) = -\frac{5}{6} \log_2 \left(\frac{5}{6} \right) - \frac{1}{6} \log_2 \left(\frac{1}{6} \right) = 0.6500$$

$$I(3,3) = -\frac{3}{6} \log_2 \left(\frac{3}{6} \right) - \frac{3}{6} \log_2 \left(\frac{3}{6} \right) = 1$$

$$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{student}}(D) = 0.9183 - 0.825 = 0.0933$$

Credit_rating	p _i	n _i
fair	6	1
excellent	2	3

$$\begin{aligned} \text{Info}_{\text{credit-rating}}(D) &= \frac{7}{12} I(6,1) + \frac{5}{12} I(2,3) \\ &= \frac{7}{12} (0.5917) + \frac{5}{12} (0.9710) \\ &= 0.7497 \end{aligned}$$

$$\text{Gain}(\text{Credit_rating}) = \text{Info}(D) - \text{Info}_{\text{credit-rating}}(D) = 0.9183 - 0.7497 = 0.1686$$