编码 1027 作业

1 第一题

4,35

(i) 6 unknowns and 3 equations, so it is obviously that the codes are linear dependent. Therefore C is a linear code.

(ii) Generator matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] = \left[\begin{array}{cc} I_3 & Q \end{array}\right]$$

$\Leftrightarrow$ parity-check matrix: $H = [P \quad I_3]$

$$\because G \cdot H^T = 0$$
$$\therefore [I_3 \quad Q][P \quad I_3]^T = 0$$
$$\therefore Q + P^T = 0$$
$$\therefore P = Q^T = \left[\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$$
$$\therefore H = [P \quad I_3] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

2 第二题

4.43

3 第三题

5.15

$$\because n = 2^r - 1 = 15$$
$$\therefore r = 4$$
$$\therefore It\ is\ Ham(4, 2)$$
$$k = n - r = 11$$

parity-check matrix:

$$H = \left[\begin{array}{ccccccccccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right]$$

Coset leader:$\boldsymbol{u}$	Syndrome:$S(\boldsymbol{u}) = \boldsymbol{u}H^T$
000000000000000	0000
100000000000000	0001
010000000000000	0010
001000000000000	0011
000100000000000	0100
000010000000000	0101
000001000000000	0110
000000100000000	0111
000000010000000	1000
000000001000000	1001
000000000100000	1010
000000000010000	1011
000000000001000	1100
000000000000100	1101
000000000000010	1110
000000000000001	1111

(a) 01010 01010 01000

令 $\vec{w_1} = [01010\ 01010\ 01000]$, 则

$\because \vec{w_1} \cdot H^T = 0100$

$\therefore Decode\ Result:$

$\vec{w_1} + \vec{e_1} = [01010\ 01010\ 01000] + [00010\ 00000\ 00000] = [01000\ 01010\ 01000]$

(b) 11100 01110 00111

令 $\vec{w_2} = [11100\ 01110\ 00111]$, 则

$\because \vec{w_2} \cdot H^T = 1010$

$\therefore Decode\ Result:$

$\vec{w_2} + \vec{e_2} = [11100\ 01110\ 00111] + [00000\ 00001\ 00000] = [11100\ 01111\ 00111]$

(c) 11001 11001 11000

令 $\vec{w_3} = [11001\ 11001\ 11000]$, 则

$\because \vec{w_3} \cdot H^T = 1010$

$\therefore Decode\ Result:$

$\vec{w_3} + \vec{e_3} = [11001\ 11001\ 11000] + [00000\ 00001\ 00000] = [11001\ 11000\ 11000]$