编码 1103 作业

November 2022

1 第一题

5.31

Note the generator matrix of C as G. Note the parity-check matrix of C as H, it is obvious that $GH^T = 0$

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & \alpha & \alpha^2 \end{bmatrix} = \begin{bmatrix} I_2 & Q \end{bmatrix}$$

Let$H = [P \quad I_2]$

$$\begin{aligned}
&\because G \cdot H^T = 0 \\
&\therefore [I_2 \quad Q][P \quad I_2]^T = 0 \\
&\therefore Q + P^T = 0 \\
&\therefore P = -Q^T = \begin{bmatrix} -1 & -\alpha \\ -1 & -\alpha^2 \end{bmatrix} \\
&\therefore H = [P \quad I_2] = \begin{bmatrix} -1 & -\alpha & 1 & 0 \\ -1 & -\alpha^2 & 0 & 1 \end{bmatrix}
\end{aligned}$$

(i) Obviously, n=4. Two columns of G are linear independent, so k=2. Two columns of H are linear independent and Three columns of H are linear dependent, so d=3.

We can get d=n-k+1, therefore C is MDS code.

(ii) The generator matrix of $C^\perp$ is the parity-check matrix of C, so,

$$G^\perp = H = \begin{bmatrix} -1 & -\alpha & 1 & 0 \\ -1 & -\alpha^2 & 0 & 1 \end{bmatrix}$$

(iii) It is obvious that: n=4, k=2, d=3, d=n-k+1, therefore $C^\perp$ is MDS code.

2 第二题

6.6

(a) $\{\boldsymbol{c_1}+\boldsymbol{c_2}\}$: $n_1 = n$, $\{\boldsymbol{c_1}-\boldsymbol{c_2}\}$: $n_2 = n$

$\therefore (\boldsymbol{c_1}+\boldsymbol{c_2}, \boldsymbol{c_1}-\boldsymbol{c_2})$: $n = n_1 + n_2 = 2n$

$\{\boldsymbol{c_1}+\boldsymbol{c_2}\}$ or $\{\boldsymbol{c_1}-\boldsymbol{c_2}\}$ makes no influence on linear independent columns of G, so $k = k_1 + k_2$.

$\therefore C_1()C_2$ is a$[2n, k_1 + k_2]$-linear code.

(b) $G = [G_1|G_2]$

(c)