9月22日课后作业

1 第一题

解:

(1) 由信道转移矩阵得:

$p(y_1|x_1) = 0.6 \quad p(y_2|x_1) = 0.4$

$p(y_1|x_2) = 0.4 \quad p(y_2|x_2) = 0.6$

又由$p(x_1) = 0.75 \quad p(x_2) = 0.25$得:

$p(x_1y_1) = p(y_1|x_1)p(x_1) = 0.6 * 0.75 = 0.45$

$p(x_1y_2) = p(y_2|x_1)p(x_1) = 0.4 * 0.75 = 0.30$

$p(x_2y_1) = p(y_1|x_2)p(x_2) = 0.4 * 0.25 = 0.10$

$p(x_2y_2) = p(y_2|x_2)p(x_2) = 0.6 * 0.25 = 0.15$

由$p(y_i) = \sum_j^n p(y_i|x_j)p(x_j)$得:

$p(y_1) = p(y_1|x_1)p(x_1) + p(y_1|x_2)p(x_2) = 0.55$

$p(y_2) = p(y_2|x_1)p(x_1) + p(y_2|x_2)p(x_2) = 0.45$

$\therefore p(x_1|y_1) = p(x_1y_1)/p(y_1) = 0.45/0.55 = 0.818$

$p(x_2|y_1) = p(x_2y_1)/p(y_1) = 0.10/0.55 = 0.181$

$p(x_1|y_2) = p(x_1y_2)/p(y_2) = 0.30/0.45 = 0.666$

$p(x_2|y_2) = p(x_2y_2)/p(y_2) = 0.15/0.45 = 0.333$

因此

$$H(X) = -\sum_{i=1}^{2} p(x_i)logp(x_i) = 0.811(bits/symbol)$$

$$H(X|Y) = -\sum_{i=1}^{2}\sum_{j=1}^{2} p(x_iy_j)logp(x_i|y_j) = 0.788(bits/symbol)$$

$$I(X;Y) = H(X) - H(X|Y) = 0.023(bits/symbol)$$

(2) 信道容量为:

$C = 1 - H(q) = 1 - 0.97 = 0.03$

(3) 当信源分布满足$p(x_i) = 1/2$时，信道容量达到最大值。最大值为0.03

2 第二题

3 第三题

解:

记4个不同的状态分别为$S_1 = 00$，$S_2 = 01$，$S_3 = 10$和$S_4 = 11$

因此状态转移概率如下:

$p(S_1|S_1) = 0.20 \quad p(S_2|S_1) = 0.80 \quad p(S_3|S_1) = 0.00 \quad p(S_4|S_1) = 0.00$

$p(S_1|S_2) = 0.00 \quad p(S_2|S_2) = 0.00 \quad p(S_3|S_2) = 0.75 \quad p(S_4|S_2) = 0.25$

$p(S_1|S_3) = 0.75 \quad p(S_2|S_3) = 0.25 \quad p(S_3|S_3) = 0.00 \quad p(S_4|S_3) = 0.00$

$p(S_1|S_4) = 0.00 \quad p(S_2|S_4) = 0.00 \quad p(S_3|S_4) = 0.80 \quad p(S_4|S_4) = 0.20$

由$p(S_j)=\sum_{i=1}^{4}p(S_i)p(S_j|S_i)$得:

$$\begin{cases}p(S_1)=p(S_1)p(S_1|S_1)+p(S_3)p(S_1|S_3)=0.20p(S_1)+0.75p(S_3)\\p(S_2)=p(S_1)p(S_2|S_1)+p(S_3)p(S_2|S_3)=0.80p(S_1)+0.25p(S_3)\\p(S_3)=p(S_2)p(S_3|S_2)+p(S_4)p(S_3|S_4)=0.75p(S_2)+0.80p(S_4)\\p(S_2)=p(S_2)p(S_4|S_2)+p(S_4)p(S_4|S_4)=0.25p(S_2)+0.20p(S_4)\\p(S_1)+p(S_2)+p(S_3)+p(S_4)=1\end{cases}$$

解得:

$$\begin{cases}p(S_1)=5/21\\p(S_2)=p(S_3)=p(S_4)=16/63\end{cases}$$

因此极限熵:

$$\begin{aligned}H_\infty&=H_{2+1}\\&=-\sum_{i=1}^{4}\sum_{j=1}^{4}p(S_i)p(S_j|S_i)logp(S_j|S_i)\\&=0.766(bits/symbol)\end{aligned}$$