Noise: (Howell Ch 4.4)

- for N uncorrelated incoming photons, the uncertainty is $\sqrt{N}$

let

N_S – total number of photons detected from source

n_{pix} – number of pixels over which light is spread

N_B – total number of photons/pixel from background light

detector noise $\begin{cases} N_D & \text{– total number of photons/pixel from dark current} \\ N_R^2 & \text{– total number of photons (electrons)/pixel of read noise} \end{cases}$

- dark current – electrons have thermal energy and sometimes it causes them to "jump out of their buckets" and be read as a detection, detectors are cooled to reduce this. You have to correct for dark current and account for it in noise estimates.
- read noise – the noise associated with reading out the array, direct trade off between read time and noise
  - common to use subarray to reduce time

signal-to-noise:

The signal, S, is just N_S. The noise is the sum, added in quadrature, of all noise sources. Using Poisson statistics, these are:

$\sigma_S = \sqrt{N_S}$ uncertainty on counts from star (source noise)

$\sigma_B = \sqrt{n_{pix} N_B}$ uncertainty due to counts in background (background noise)

$\sigma_D = \sqrt{n_{pix} N_D}$ uncertainty due to dark current (detector noise)

$\sigma_R = \sqrt{n_{pix} N_R^2}$ uncertainty due to read noise (detector noise)

→ not Poisson noise b/c doesn't depend on source, goes as N

defined as N_R^2 electrons per pix from readout

To compute the S/N as a function of integration time, you need to know the rate of detection of photons from the source, R_S, the rate of detection of photons from the background per pixel, R_B, and the rate of dark current per pixel, R_D. (Note that the definition of rate assumes that quantum efficiencies have already been accounted for.) Then,

$$S/N = \frac{R_S t}{\sqrt{R_S t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

<u>Object Limited Case:</u>

Ideally, you'd like to be source noise limited, but it's not always possible for faint sources or bright background.

$$S/N = \frac{R_S t}{\sqrt{R_S t}} = \sqrt{R_S t}$$

How does S/N scale with important properties?

Integration time (t):	$S \propto t$	$N \propto \sqrt{t}$	$\Rightarrow$	$S/N \propto \sqrt{t}$
Telescope Aperture (A):	$S \propto A^2$	$N \propto \sqrt{A^2}$	$\Rightarrow$	$S/N \propto A$
Source flux (R_S):	$S \propto R_S$	$N \propto \sqrt{R_S}$	$\Rightarrow$	$S/N \propto \sqrt{R_S}$
Quantum Efficiency (Q):	$S \propto Q$	$N \propto \sqrt{Q}$	$\Rightarrow$	$S/N \propto \sqrt{Q}$
Source Distance (D):	$S \propto D^{-2}$	$N \propto \sqrt{D^{-2}}$	$\Rightarrow$	$S/N \propto 1/D$
Bandwidth (B):	$S \propto B$	$N \propto \sqrt{B}$	$\Rightarrow$	$S/N \propto \sqrt{B}$
Extraction Aperture (W):	$S \not\propto W$	$N \not\propto W$	$\Rightarrow$	unaffected (point)
Extraction Aperture (W):	$S \propto W^2$	$N \propto \sqrt{W^2}$	$\Rightarrow$	$S/N \propto W$ (extended)

Detector Noise limited case:

$$S/N = \frac{R_s t}{\sqrt{n_{pix}(R_D t + N_e^2)}}$$

Int. time, read noise limited (t):	$S \propto t$	$N \propto \sqrt{n_{pix} N_e^2}$	$S/N \propto t$
Int. time, dark noise limited (t):	$S \propto t$	$N \propto \sqrt{n_{pix} R_D t}$	$S/N \propto \sqrt{t}$
Telescope Aperture (A):	$S \propto A^2$	$N \not\propto A$	$S/N \propto A^2$
Source flux (R_s):	$S \propto R_s$	$N \not\propto R_s$	$S/N \propto R_s$
Quantum Efficiency (Q):	$S \propto Q$	$N \not\propto Q$	$S/N \propto Q$
Source Distance (D):	$S \propto D^{-2}$	$N \not\propto D$	$S/N \propto D^{-2}$
Bandwidth (B):	$S \propto B$	$N \propto B$	$S/N \propto B$
Extraction Aperture, point (W):	$S \not\propto W$	$N \propto \sqrt{n_{pix}} \propto \sqrt{W^2}$	$S/N \propto W^{-1}$
Extraction Aperture, ext (W):	$S \propto W^2$	$N \propto \sqrt{W^2}$	$S/N \propto W$

Background limited case:

$$S/N = \frac{R_s t}{\sqrt{n_{pix}(R_B t)}}$$

Integration time (t):	$S \propto t$	$N \propto \sqrt{n_{pix} R_B t}$	$S/N \propto \sqrt{t}$
Telescope Aperture (A):	$S \propto A^2$	$N \propto \sqrt{A^2}$	$S/N \propto A$
Source flux (R_s):	$S \propto R_s$	$N \propto \sqrt{n_{pix} R_B t}$	$S/N \propto R_s$
Quantum Efficiency (Q):	$S \propto Q$	$N \propto \sqrt{Q}$	$S/N \propto \sqrt{Q}$
Source Distance (D):	$S \propto D^{-2}$	$N \propto \sqrt{n_{pix} R_B t}$	$S/N \propto \sqrt{D^{-2}}$
Bandwidth (B):	$S \propto B$	$N \propto \sqrt{B}$	$S/N \propto \sqrt{B}$
Ext. Aperture, point (W):	$S \not\propto W$	$N \propto \sqrt{W^2}$	$S/N \propto W^{-1}$
Ext. Aperture, ext. (W):	$S \propto W^2$	$N \propto \sqrt{W^2}$	$S/N \propto W$

The rate of photons is related to the flux by:

$$R = \eta_{total} \cdot \Delta\lambda \cdot \overset{\text{aperture area}}{A} \cdot f_\lambda$$

→ f_λ usually in ph s^{-1} cm^{-2} $Å^{-1}$

↳ $\Delta\lambda$: bandpass, can also be $\Delta\nu$ for f_ν

↳ η_{total}: total efficiency, can do $\eta_{atmo} = (2.5^{x \cdot k})^{-1}$

Z.B. You get 1 Ph/s from a star whose PSF is 2pix FWHM, meaning the light is spread over ~16 pixels. The CCD readnoise is $4e^-$/pix and the sky brightness is 0.5 ph/s/pix and the dark current is 0.1 ph/s/pix. What exposure time is required to get S/N = 50?

$$\frac{S}{N} = \frac{R_S t}{\sqrt{R_S t + n_{pix}\left(R_B t + R_D t + N_R^2\right)}}$$

try: $t = 26{,}600$ gives $S/N = 50.0715$

Z.B. For a 25th magnitude star in the R band, how long would be required to make a 10σ detection with an f/10 2m telescope and a total system efficiency of 50%? Readnoise is $5\,e^-$/pix, dark current is $36\,e^-$/pix per hour, and sky background brightness is 25 mag/arcsec^2 using a CCD of 13 μm pixels and 1'' seeing. The R zero point is 2870 Jy.

Translate the magnitude into flux units:

$$f_\nu = 2870\,\text{Jy}\cdot 10^{-0.4\cdot 25} = 2.87\times10^{-7}\,\text{Jy}\cdot 10^{-26}\,\text{W m}^{-2}\,\text{Hz}^{-1}\,\text{Jy}^{-1}$$

$$= 2.87\times10^{-33}\,\text{W m}^{-2}\,\text{Hz}^{-1} = 2.87\times10^{-30}\,\text{erg cm}^{-2}\,\text{Hz}^{-1}\,\text{s}^{-1}$$

$$\nu f_\nu = \lambda f_\lambda \Rightarrow f_\lambda = \frac{c}{\lambda^2} f_\nu = \frac{3\times10^{18}\,\text{Å/s}}{(7000\,\text{Å})^2}\cdot 2.87\times10^{-30}\,\text{erg s}^{-1}\,\text{cm}^{-2}\,\text{Hz}^{-1}$$

$$f_\lambda = 1.75\times10^{-19}\,\text{erg s}^{-1}\,\text{cm}^{-2}\,\text{Å}^{-1}$$

$$L_{ph} = \frac{6.626\times10^{-27}\,\text{erg s}\cdot 3\times10^{18}\,\text{Å/s}}{7000\,\text{Å}} = 2.8\times10^{-12}\,\text{erg}$$

$$f_\lambda = \frac{1.75\times10^{-19}\,\text{erg s}^{-1}\,\text{cm}^{-2}\,\text{Å}^{-1}}{2.8\times10^{-12}\,\text{erg ph}^{-1}} = 6.25\times10^{-8}\,\text{ph s}^{-1}\,\text{cm}^{-2}\,\text{Å}^{-1}$$

Multiply by the collecting area, bandpass, and total efficiency:

$$R_S = Q \cdot \Delta\lambda_{bandpass} \cdot A_{aperture} \cdot f_\lambda \, [\text{ph s}^{-1} \text{cm}^{-2} \text{Å}^{-1}]$$

$$= 0.5 \cdot 1000\,\cancel{\text{Å}} \cdot \pi \left(1\cancel{\text{m}} \cdot 100\,\text{cm}\,\cancel{\text{m}}^{-1}\right)^2 \cdot 6.25\times10^{-8}\,\text{ph s}^{-1}\,\cancel{\text{cm}}^{-2}\,\cancel{\text{Å}}^{-1}$$

$$R_S = 0.98 \text{ ph s}^{-1}$$

Doing the same calculation for the sky background gives:

$$R_B = 0.98 \text{ ph s}^{-1} \text{arcsec}^{-2}$$

The platescale is:

$$f = R \cdot D = 10 \cdot 2\text{m} = 20\text{m} \cdot 1000\,\text{mm}\,\text{m}^{-1} = 20{,}000\,\text{mm}$$

$$s = \frac{206265}{f} = \frac{206265}{20{,}000\,\text{mm}} = 10.3 \text{ arcsec mm}^{-1}$$

So one pixel subtends:

$$\text{pix} = 13\,\mu\text{m} \cdot 10^{-3}\,\text{mm}/\mu\text{m} \cdot 10.3 \text{ arcsec mm}^{-1} = 0.13 \text{ arcsec}$$

With 1'' seeing, star images will have a FWHM of ~7 pixels, so most of the light will be distributed over $n_{pix} = 49$ pixels assuming a 7x7 pixel square aperture.

Now calculate the background rate in one pixel. A pixel subtends:

$$(0.13 \text{ arcsec})^2 = 0.0169 \text{ arcsec}^2$$

so

$$R_B = 0.98 \text{ ph s}^{-1} \text{arcsec}^{-2} \cdot 0.0169 \frac{\text{arcsec}^2}{\text{pix}} = 0.02 \text{ ph s}^{-1} \text{pix}^{-1}$$

Convert the dark current:

$$R_D = 36\,e^- \,hr^{-1} \cdot 60\,s\,m^{-1} \cdot 60\,m\,hr^{-1} = 0.01\,e^-\,s^{-1}$$

Then:

$$\frac{S}{N} = \frac{0.98\,ph\,s^{-1}\,t}{\sqrt{0.98\,ph\,s^{-1} \cdot t + 49\,pix\,(0.02\,ph\,s^{-1}\,pix^{-1} \cdot t + 0.01\,e^-\,s^{-1}\,t + 5\,e^-)}}$$

By iteration, $t = 340\,s$ gives $S/N = 10.15$