Redshift

Topics: Doppler shift, cosmological redshift

sources: Chromey Ch. 10.8, 11.9

$$1+z = \frac{\lambda_{obs}}{\lambda_{emit}}$$

Doppler Shift

Change in wavelength because source has a velocity through space.

$$1+z = \sqrt{\frac{1+\beta}{1-\beta}}, \quad \beta = \frac{v_r}{c}$$ Relativistic Doppler Shift

$$z \approx \frac{v_r}{c}$$ Non-relativistic limit

Cosmological Redshift

Change in wavelength because space is expanding as light travels through it.

$$1+z = \frac{a(t_0)}{a(t)} = \frac{1}{a(t)}$$

These effects stack:

$$1+z_{cosmo} = \frac{\lambda_{stretch}}{\lambda_{emit}}, \quad 1+z_{Doppler} = \frac{\lambda_{obs}}{\lambda_{stretch}}$$

$$\Longrightarrow \quad 1+z = (1+z_{cosmo})(1+z_{Doppler})$$

Flux and magnitude

Topics: Flux, magnitude, units

Source: Chromey Ch. 1, 10

Fluxes:

Energy flux, or just flux, is a basic physical description of the apparent brightness of a source.

This is expressed by a few different quantities (that are often confused):

I_ν, I : the specific intensity / intensity in $W\ m^{-2}\ Hz^{-1}\ sr^{-1}$ and $W\ m^{-2}\ sr^{-1}$. aka surface brightness.

F_ν, F : the flux density / flux in $W\ m^{-2}\ Hz^{-1}$ and $W\ m^{-2}$.

L_ν, L : the luminosity density / luminosity in $W\ Hz^{-1}$ and W. Also called the energy flux as above.

In fact, these specific quantities can be expressed as per frequency or wavelength, i.e. f_ν or f_λ.

To convert between f_ν and f_λ you need to realize that 1 Angstrom worth of spectrum contains a different number of Hz depending on what part of the EM spectrum you're in:

$$c = \nu\lambda \Rightarrow \nu = \frac{c}{\lambda} \Rightarrow \frac{\delta\nu}{\delta\lambda} = -\frac{c}{\lambda^2}$$

E.g. A source has a flux density of 3×10^{-14} erg s^{-1} cm^{-2} $Å^{-1}$ at 5000 Å. How many erg s^{-1} cm^{-2} Hz^{-1} is this?

$$\frac{\delta\nu}{\delta\lambda} = -\frac{c}{\lambda^2} = \frac{3\times10^{18}\ Å/s}{(5000\ Å)^2} = -1.2\times10^{11}\ Hz\ Å^{-1}$$

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Luminosity:

The energy output of a source. Related to flux by:

$$L = 4\pi d_L^2 F$$

Flux:

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So then:

$$f_\nu = f_\lambda \cdot \frac{\delta\lambda}{\delta\nu} = \frac{3\times10^{-14}\ erg\ s^{-1}\ cm^{-2}\ \AA^{-1}}{1.2\times10^{11}\ Hz\ \AA^{-1}} = 2.5\times10^{-25}\ erg\ s^{-1}\ cm^{-2}\ Hz^{-1}$$

How many W m^{-2} Hz^{-1} is this?

$$1\ W = 10^7\ erg\ s^{-1}$$

$$1 m^2 = (100\ cm)^2 = 10^4\ cm^2$$

$$f_\nu = 2.5\times10^{-25}\ erg\ s^{-1}\ cm^{-2}\ Hz^{-1} \cdot \frac{1 W}{10^7\ erg\ s^{-1}} \frac{10^4\ cm^2}{1 m^2} = 2.5\times10^{-28}\ W\ m^{-2}\ Hz^{-1}$$

How many Jy is this?

$1\ Jy = 10^{-26}\ W\ m^{-2}\ Hz^{-1}$

$f_\nu = 2.5\times10^{-28}\ W\ m^{-2}\ Hz^{-1} \cdot \frac{1\ Jy}{10^{-26}\ W m^{-2} Hz^{-1}} = 2.5\times10^{-2}\ Jy = 25\ mJy$

How many $ph\ s^{-1}\ cm^{-2}\ Å^{-1}$ is this?

$h = 6.624\times10^{-27}\ erg\ s$

$1\ ph = E = h\nu = h\,c/\lambda = 6.624\times10^{-27}\ erg\ s \cdot \frac{3\times10^{18}\ Å/s}{5000\ Å} = 3.97\times10^{-12}\ erg$

$f_\lambda = \frac{3\times10^{-14}\ erg\ s^{-1}\ cm^{-2}\ Å^{-1}}{3.97\times10^{-12}\ erg/ph} = 7.5\times10^{-3}\ ph\ s^{-1}\ cm^{-2}\ Å^{-1}$

Magnitudes

A (backwards) way to describe brightness. It is actually a way to compare the brightness of two sources:

$$m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$$

$$\frac{F_2}{F_1} = 10^{0.4(m_2 - m_1)}$$

A common refrence system is based on Vega, which has a magnitude of 0 (by design) in the Johnson-Cousins filters (i.e. UBVRI). Vega is bright, so it is now more common to refer to a group of Landolt standard stars. But in Vega magnitudes:

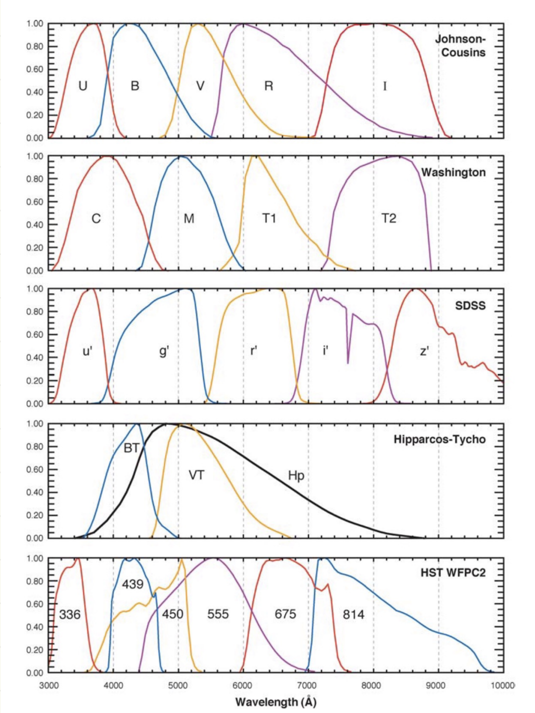
$$f_\nu\,(Jy) = F_{vega,0}\, 10^{-0.4m}$$

Band	λ_{eff} (μm)	f_ν(Jy)
U	0.36	1880
B	0.44	4440
V	0.56	3540
R	0.70	2870
I	0.90	2250

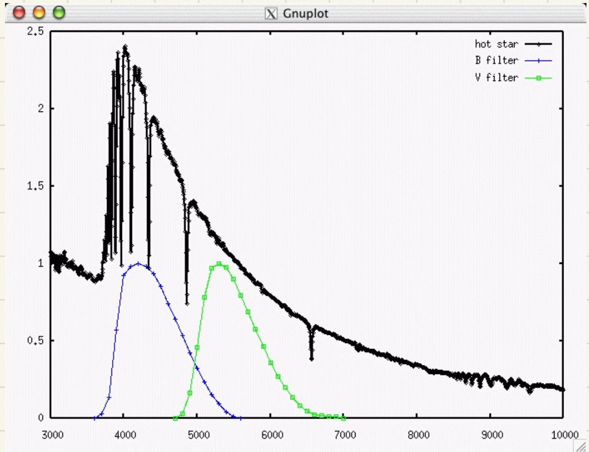
Another system attempts to address the fact that Vega's magnitude is not actually 0 in all these filters:

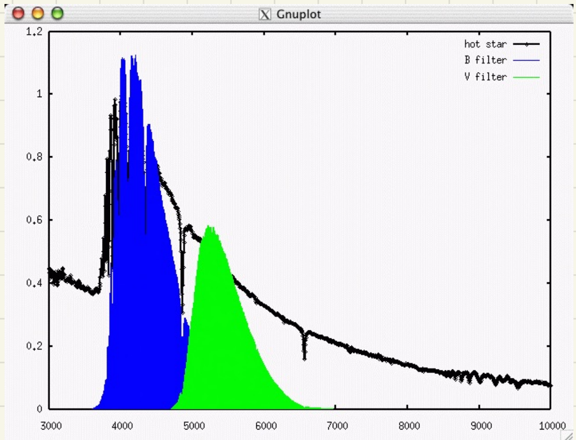
$$m_{AB} = -2.5 \log\left[f_\nu\,(Jy)\right] + 8.9$$

$$f_\nu(Jy) = 3631\,Jy\; 10^{-0.4 m_{AB}}$$



Bessell (2005)





spiff.rit.edu/classes/phys440/lectures/color/color.html

$$F_x = \int_{\lambda_{x,-}}^{\lambda_{x,+}} R_x(\lambda)\, F_\lambda \, d\lambda$$ flux in a band

- can convert fluxes in band to a magnitude
- A "color" is the difference between magnitudes in two bands.

Absolute Magnitude:

$$F = \frac{L}{4\pi d^2}$$

The absolute magnitude is the magnitude a source would have at 10pc.

$$m - M = 5\log\left(\frac{d}{10pc}\right)$$

$$\to \boxed{m - M = 5\log d - 5}$$

distance modulus

d: dist. in pc

$$(m-M)_\lambda = (m-M)_0 + A_\lambda$$

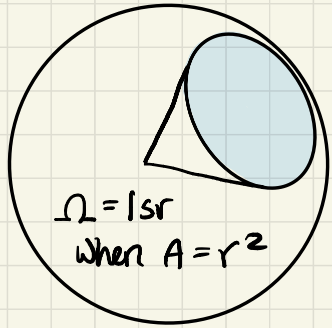
A_λ: extinction in magnitudes

Solid angle (Ω)

Solid angle is a three dimensional analog of angle, it measures the part of your field of view that an object covers.

$$\Omega = \frac{A}{r^2}\ \mathrm{sr}$$

1 steradian is the unit of solid angle that a unit area subtends on the surface of a unit sphere.



There are 4π steradians.

Square degrees are another unit of solid angle.

There are $4\pi\,\mathrm{sr} \cdot \left(\frac{180}{\pi}\right)^2 \frac{\deg^2}{\mathrm{sr}} = 41{,}252.96\ \deg^2$

2.B. You observe an extended nebula with surface brightness (or specific intensity) of 1 MJy/sr. What is the flux you would measure through a circular aperture with a radius of 10 arcsec?

Aperture area is:

$$A = \pi r^2 = \pi \cdot (10\,\mathrm{arcsec})^2 = 314\,\mathrm{arcsec}^2$$

$$f_\nu = I_\nu \cdot A = 1\,\frac{\mathrm{MJy}}{\mathrm{sr}} \cdot 314\,\mathrm{arcsec}^2 \cdot \frac{1\,\mathrm{sr}}{\left(60''/' \cdot 60'/^\circ \cdot 360^\circ/2\pi\,\mathrm{rad}\right)^2}$$

$$f_\nu = 1\,\frac{\mathrm{MJy}}{\mathrm{sr}} \cdot 314\,\mathrm{arcsec}^2 \cdot \frac{1\,\mathrm{sr}}{4.25\times10^{10}\,\mathrm{arcsec}^2} = 7.38\,\mathrm{mJy}$$

But! If the surface brightness is measured in mag arcsec^{-2} you can't just multiply by an area. Similarly, surface brightness is not additive when expressed as magnitudes per area.

$$\mu = m + 2.5\log\Theta$$