Connections Design

Mini Trash Compactor

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Project Summary:

Municipal solid waste management (MSWM) is an increasingly important issue with growing populations and changes in lifestyles leading to much greater amounts of waste produced per capita. Appropriate waste management has far-reaching impacts, spanning from effects on public health, economies, and climate change. Shortages of landfill space, difficulties in collecting large volumes of trash in developing areas, and greenhouse gas emissions from more frequent waste transportation are all motivations for devising a more accessible method of reducing the volume of solid waste through creation of household trash compactors. Many trash compact compactors are either too large, expansive, or high in energy consumption for household use. As a result, there is a need for a more compact, affordable and manageable device, which can be integrated into small spaces. This small compactor can serve small apartments, dorms, or offices where full sized solutions are not feasible.

After further exploration, our team developed the Mini Trash Compactor, a motorized device designed to compact household waste while being easily affordable and efficient. The device utilizes an electric motor connected to a belt system, driving dual lead screws. These screws control the vertical motion of the ram, creating a crushing force when moving down the frame to the bottom of the waste bin, ultimately crushing any items in the process. Once the compaction process is complete, the user can open the drawer and remove the bin to access the compacted items. This mechanism ensures effective and consistent compaction without relying on bulky hydraulic or pneumatic systems typically found in the preexisting models.

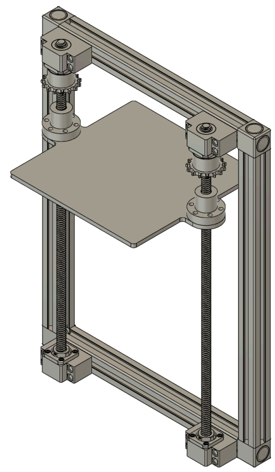


Figure 1 - CAD model of our trash compactor without exterior housing and belt motor system

Design Specifications

Combating the pre existing models, our product must meet design specifications in order to be useful for the consumers. Thus, each design component has their own specifications and the product itself has an overall criteria to meet. The overall device specifications include:

- Dimensions approximately 14” W X 16” D X 20” H
- Total weight of the product should be about up to 25 lb to ensure portability
- Power to be standard 120 AC input from wall socket
- Should last 1000 cycles per year for 5 years, totaling 5000 cycles
- Components must withstand loads required to crush a full bin of aluminum cans and paper
- Ram is constructed of ANSI 330 Stainless Steel
- Lead screws constructed of AISI 1018 Steel

In order to actually build the product, the most important component to meet specifications would be the frame. The frame specifications contains:

- Should be able to withstand a vertical load of 200N applied to the top of the frame
- Protect the mechanism from external forces and possible wear and tear/damage

- Object should remain stationary during cycle
- Never change the orientation of the product
- Constructed of 6061 Aluminum

Drawings, Illustrations and Models

Included below are models of our device with component and dimension call outs. The compactor consists of a t-slot frame connected by t-slot brackets. The lead screws connect to the top and bottom frame members via end supports that have t-slot nuts. A flange nut attaches each side of the ram to the lead screw. On each set screw between the flange nut and the upper end support is a sprocket that will hold the belt that is driven by a motor to rotate the screws and move the ram vertically.

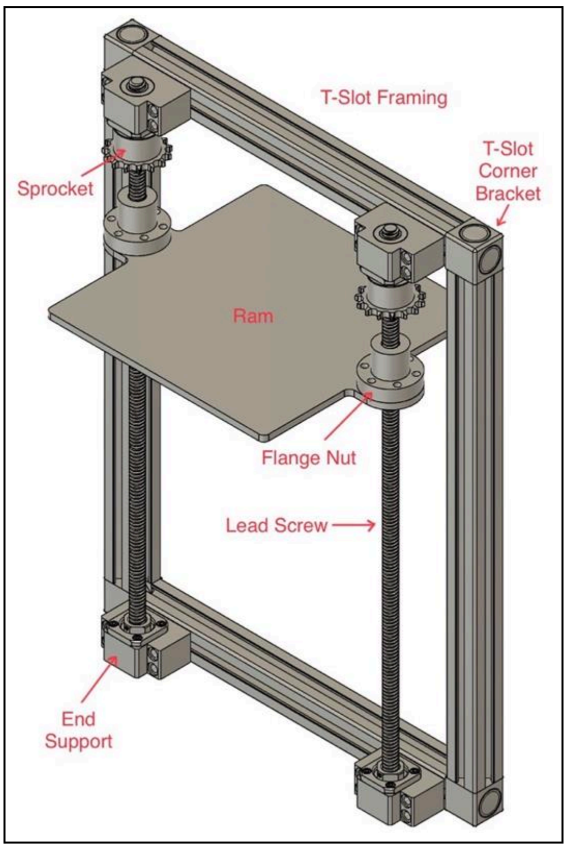


Figure 2 - Labeled Trash Compactor Assembly

Material Properties				
Steel AISI 1018 Lead Screw	330 Stainless Steel Ram	6061 Aluminum T-Slot Framing	304 Stainless Steel #12-24 Socket Cap Screw	Delrin Flange Nut
S_y = 572 MPa S_{ut}' = 696 MPa E = 207 GPa	S_y = 276 MPa S_{ut}' = 568 MPa E = 200 GPA	S_y = 275 MPa S_{ut}' = 310 MPa E = 68.9 GPa	S_y = 30 ksi S_{ut}' = 75 ksi E = 28000 ksi S_p = 120 ksi	S_y = 9.137 ksi S_{ut}' = 5.903 ksi E = 420.609 ksi

Weld Analysis

The trash compactor design does not incorporate any welded joints. All the structural connections, including the ram, frame, and supporting components were achieved using mechanical fasteners and screws. This approach allows a straightforward assembly and part replacement without the need for specialized welding equipment.

Flange Screws Analysis

The flange screws secure the flange nut to the ram of the trash compactor. The force to crush a can under the ram is estimated to be 44.961 lbf. This force will be distributed between the two flanges which have six screws each. We selected #12-24 304 SS screws for this connection. The fastener length was calculated and rounded up to ⅞". Detailed calculations of all subsections can be found in Appendix B. Note that the following analysis was done only for the left flange because the right flange has the same configuration. To analyze these screws for static failure, we calculated the following: external forces on each screw, fraction of the external load carried by the screws, the preload, the safety factor against screw failure under static loading, and the safety factor against loosening, also known as membrane separation. For fatigue analysis, the load amplitude and mean were calculated as well as endurance strength of the screws. These values were used in the Goodman equation to find the safety factor against fatigue.

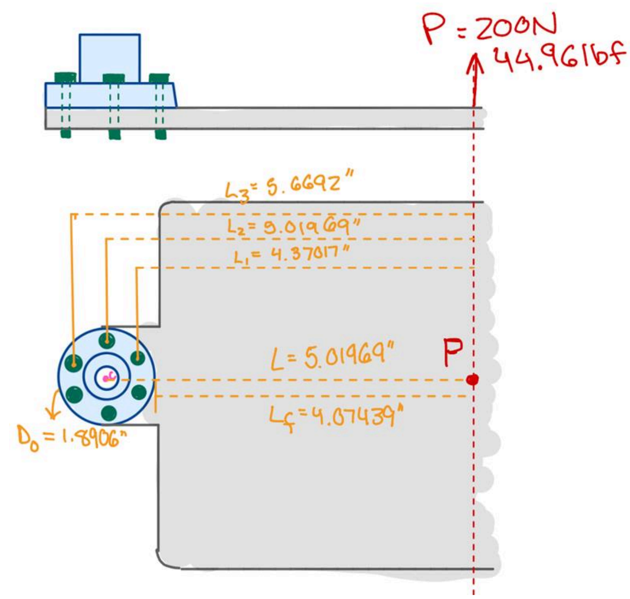


Figure 3 - Side and top views of static loading on left flange screws

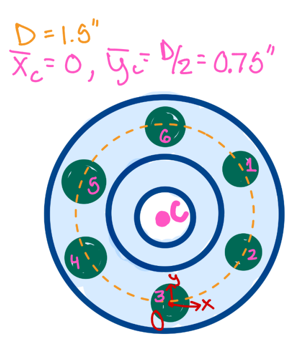


Figure 4 - Screw layout on flange with centroid

External Forces on Screws

The external forces on each screw was calculated based on the distance from the load P to the center of each screw. These dimensions are defined in Figure 4 above. There will be three different external loads dependent on the three centerlines of the flange's screw geometry. Screws 1 and 2 have the same centerline and share distance L1. Screws 3 and 6 share a centerline and distance L2. Screws 4 and 5 have the same centerline sharing distance L3. The external loads were calculated using the following method:

$$Screws\ 1\ and\ 2:\ F_1 = \frac{(P/12)(L_f)}{L_1 - L_f} = 51.61\ lbf \tag{1}$$

$$Screws\ 3\ and\ 6:\ F_2 = \frac{(P/12)(L_f)}{L_2 - L_f} = 16.1491\ lbf \tag{2}$$

$$Screws\ 4\ and\ 5:\ F_3 = \frac{(P/12)(L_f)}{L_3 - L_f} = 9.57213\ lbf \tag{3}$$

Fraction of External Load

To find the fraction of the external load, the stiffness of the fastener and member were calculated. Detailed calculations can be found in Appendix B.

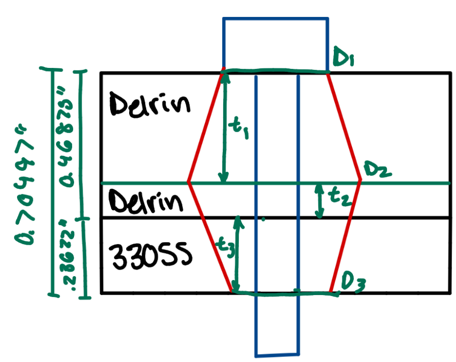


Figure 5 - Frustum used to find member stiffness

$$Fastener\ Stiffness:\ K_b = \frac{A_d A_t E_{screw}}{A_d l_t + A_t l_d} = 1325490\ kip/in \tag{4}$$

$$Member\ Stiffness\text{:}\ K_m = \left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right]^{-1} = 46245.2\ kip/in \tag{5}$$

$$where\ K_n = \frac{0.5774\pi E_n d}{ln\left[\frac{(1.155t_n + D_n - d)(D_n + d)}{(1.155t_n + D_n + d)(D_n - d)}\right]} \tag{6}$$

Thus, the faction of the external load carried by each screw was found using the following:

$$C = \frac{K_b}{K_b + K_m} = 0.996 \tag{7}$$

Preload

These screws are reusable therefore the preload was calculated using the following equation:

$$F_i = 0.75 A_t S_p = 2178\ lbf \tag{8}$$

Screw Failure Safety Factor

Since there are three different external loads, there will be three different safety factors for screw failure under static loading. The screws facing the largest external loads are considered critical points and will have the minimum safety factor needed. Detailed calculations can be found in Appendix B.

$$Screws\ 1\ and\ 2:\ n_{L_1} = \frac{A_t S_p - F_i}{CF_1} = 14.56 \tag{9}$$

$$Screws\ 3\ and\ 6:\ n_{L_3} = \frac{A_t S_p - F_i}{CF_2} = 46.54 \tag{10}$$

$$Screws\ 4\ and\ 5:\ n_{L_3} = \frac{A_t S_p - F_i}{CF_3} = 78.51 \tag{11}$$

The minimum safety factor against static failure is 14.56, which is well above 1, therefore it is unlikely that the #12-25 304 SS screws will fail under static loading.

Loosening Safety Factor

There are three different external loads, thus there will be three different safety factors against membrane separation or loosening. Similar to above, the screws facing the largest external load are considered critical points and will have the minimum safety factor needed.

$$Screws\ 1\ and\ 2:\ n_{O_1} = \frac{F_i}{(1-C)F_1} = 1241.21 \tag{12}$$

$$Screws\ 3\ and\ 6:\ n_{O_3} = \frac{F_i}{(1-C)F_2} = 3966.71 \tag{13}$$

$$Screws\ 4\ and\ 5:\ n_{O_3} = \frac{F_i}{(1-C)F_3} = 6692.22 \tag{14}$$

The minimum safety factor against loosening is 1241.21, which is well above 1, therefore it is unlikely the membrane fastened by these screws will separate.

Fatigue Safety Factor

Screws will not experience compression, therefore during cyclic loading the minimum load will be 0 lbf. This will also occur when the trash compactor is not crushing anything. The maximum load will be 49.961 lbf when a can is being crushed under the ram. Detailed calculations for this safety factor can be found in Appendix B.

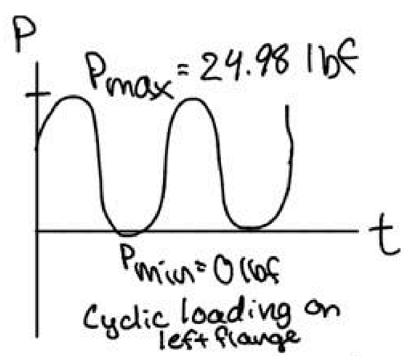


Figure 6 - cyclic loading of left flange nut

The load amplitude and mean were found using the following equations:

$$load\ amplitude\ P_a = \frac{P_{max} - P_{min}}{2} = 12.49\ lbf \quad (15)$$

$$load\ mean\ P_m = \frac{P_{max} + P_{min}}{2} = 12.49\ lbf \quad (16)$$

The endurance strength was found using the method below. The screw threads were assumed to be rolled, the temperature was assumed to be room temperature, and the reliability was 95%.

$$Endurance\ Strength\ S_e = \frac{k_d k_e S_e'}{K_f} = 147955\ psi \quad (17)$$

Therefore the safety factor against fatigue was calculated. The safety factor is extremely high for these screws indicating they will very likely resist fatigue failure.

$$n_f = \frac{S_{ut} A_t - F_i}{C(P_a(\frac{S_{ut}}{S_e}) + P_m)} = 211.55 \quad (18)$$

We acknowledge that these screws are overqualified to support the static loading they experience; However, these screws are industry standard and affordable.

Shear Failure

In addition to static failure, we looked at shear failure in the flange screws due to eccentric loading. For this analysis we calculated the following: moment about the centroid, direct shear forces on the screws, indirect shear forces on the screws, the resultant shear force, resultant shear stress, and safety factor against shear failure. For this section refer to Figures 3 and 4.

$$Moment\ about\ C:\ M_c = PL = 225.69\ lbf \cdot in \quad (19)$$

$$Direct\ Shear\ Forces: F' = F_1' = F_2' = F_3' = F_4' = F_5' = F_6' = (\frac{P}{12}) = 3.747\ lbf \quad (20)$$

$$Indirect\ Shear\ Forces:\ F'' = F_1'' = F_2'' = F_3'' = F_4'' = F_5'' = F_6'' = \frac{M_c}{6r} = 50.15\ lbf \quad (21)$$

$$Resultant\ Shear\ Force:\ V = \sqrt{F'^2 + F''^2} = 50.29\ lbf \quad (22)$$

$$Resultant\ Shear\ Stress:\ \tau = \frac{V}{A} = 1372.41\ psi \quad (23)$$

$$Safety\ Factor\ Against\ Shear\ Failure\ according\ to\ Tresca\ Theory:\ n = \frac{S_y}{2\tau} = 10.93 \qquad (24)$$

End Slot Connection Analysis

The end slot connections secure the power screw from shifting, connecting them to the end supports. The force on the plate is found to be 100 N, as per calculations from the component design report. This force will be distributed between the four hex nut screws which have 4 screws in symmetry across the plate. We selected an M3 x 0.05mm hex nut screw for this connection. The washer length was calculated and rounded up to 0.5 mm. Detailed calculations of all subsections can be found in Appendix C. Note the following analysis was done only for one of the screws as the entire plate was made in symmetry, resulting in the same calculations for each screw.

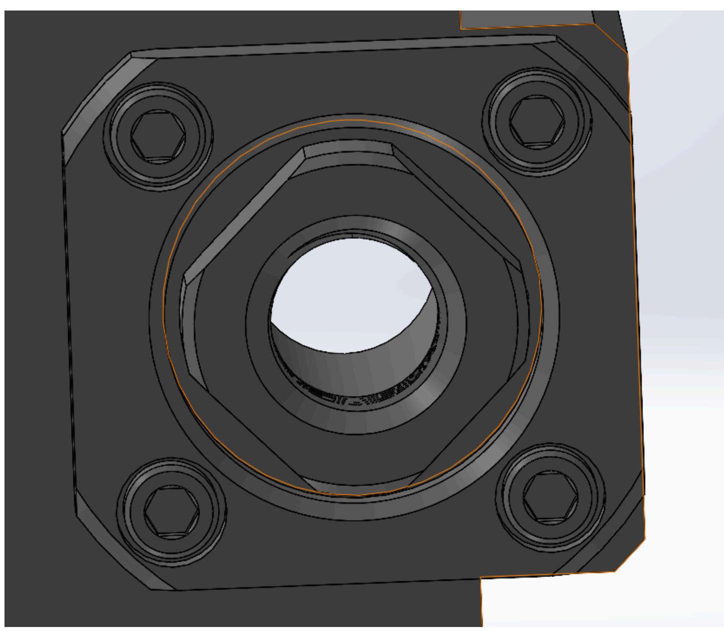


Figure 7: Connecting Screws

External Forces on Screws

The external forces on each screw was calculated based on the distance from the load P to the center of each screw. The force P is directed from the force felt by the power screw, coming from the force felt by the ram; equating to P = 100N. As the entire locations are in perfect symmetry, the force P felt by the screws is all equivalent.

$$F_1 = F_2 = F_3 = F_4 = \frac{P}{4} = 25N \quad (25)$$

Fraction of External Load

To find the fraction of the external load, the stiffness of the fastener and member were calculated. Detailed calculations can be found in Appendix C.

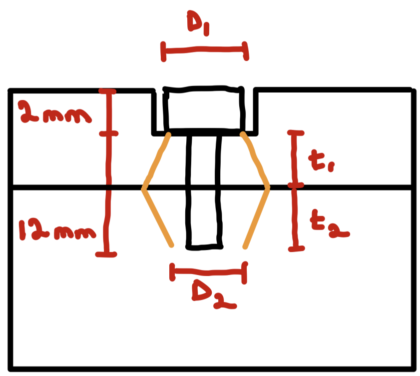


Figure 8: Frustum used to find member stiffness

$$Fastener\ Stiffness:\ K_b = \frac{A_d A_t E_{screw}}{A_d l_t + A_t l_d} = 135\ kN/mm \quad (26)$$

$$Member\ Stiffness:\ K_m = \left[\frac{1}{K_1} + \frac{1}{K_2}\right]^{-1} = 528.5\ kN/mm \quad (27)$$

$$where\ K_2 = \frac{0.5774\pi E_n d}{ln\left[\frac{(1.155t_n + D_n - d)(D_n + d)}{(1.155t_n + D_n + d)(D_n - d)}\right]} \quad (28)$$

Thus, the faction of the external load carried by each screw was found using the following:

$$C = \frac{K_b}{K_b + K_m} = 0.20346 \tag{29}$$

Preload

These screws are reusable therefore the preload was calculated using the following equation:

$$F_i = 0.75 A_t S_p = 1.47\ kN \tag{30}$$

Screw Failure Safety Factor

As all the screws feel the same external load, the safety factor for the screws can be calculated by the following equation. Detailed calculations can be found in Appendix C.

$$n_L = \frac{A_t S_p - F_i}{CF} = 17.4 \tag{31}$$

The minimum safety factor against static failure is 27.557, which is well above 1, therefore it is unlikely that the M3 x 0.5mm screws will fail under static loading.

Loosening Safety Factor

The safety factor against member separation or loosening can be calculated using the following formula.

$$n_O = \frac{F_i}{(1-C)F_1} = 73.9 \tag{32}$$

Fatigue Safety Factor

As the screws will not experience compression, therefore during cyclic loading the minimum load will be 0 N. This will also occur when the trash compactor is not crushing anything. The maximum load will be 50 N, when a can is being crushed under the ram. Detailed calculations for this safety factor can be found in Appendix C.

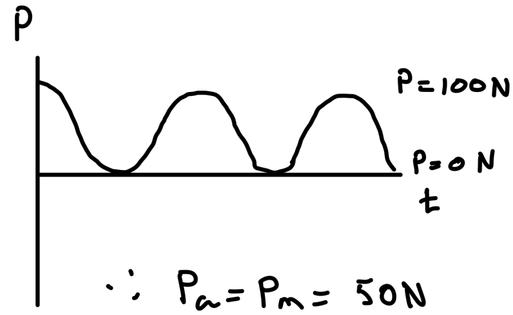


Figure 9: Cyclic Loading Graph

The load amplitude and mean were found using the following equations:

$$P_a = \frac{P_{max} - P_{min}}{2} = 50\ N \tag{33}$$

$$P_m = \frac{P_{max} + P_{min}}{2} = 50\ N \tag{34}$$

The screw threads were assumed to be rolled, the temperature was assumed to be room temperature, and the reliability was 95%. Thus, the endurance strength was found using the following equation.

$$S_e = \frac{k_d k_e S_e'}{K_f} = 95.2\ MPa \tag{35}$$

Therefore the safety factor against fatigue was calculated. The safety factor is extremely high for these screws indicating they will very likely resist fatigue failure.

$$n_f = \frac{S_{ut} A_t - F_i}{C(P_a(\frac{S_{ut}}{S_e}) + P_m)} = 26.6 \tag{36}$$

T-Slot Fastener Analysis

The power screw mechanism is held in place by an end-support as seen in the previous section. This end-support is attached to the T-slot extrusion channels, which will be analyzed in detail within this section. As reported in the component design report, the powerscrews would divide the 200 N force equally into itself, meaning that 100 N of maximum force would be applied at the powerscrew. This force would also be carried over into the end-support, but would contribute to direct shear on the bolts. Additionally an external force of 250 N was applied to bolt fastener system. This calculation can be seen in Appendix D below, and would be used within the static failure analysis section. 2 M5 screws were used to fasten the end support in place.

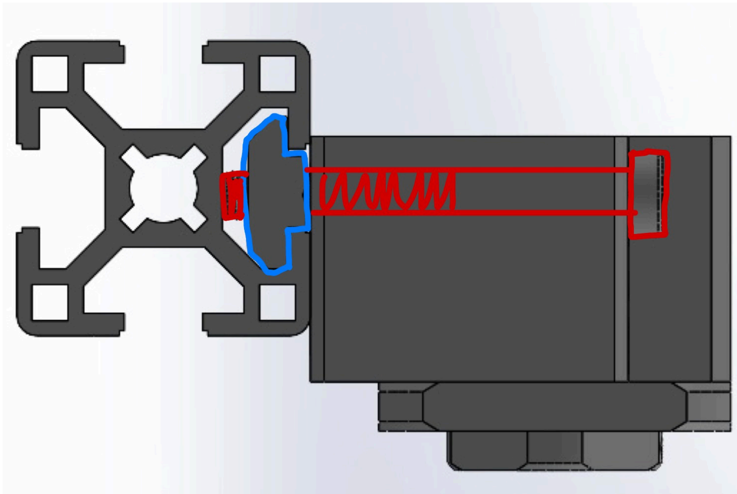


Figure 10: T-Slot fastener

External Forces on Screws

Since an external load of 250 N was applied on the setup, this means that each screw received an external force of 125 N or 0.125 kN.

$$F_1 = F_2 = \frac{P}{2} = 125\ N \tag{37}$$

Fraction of External Load

To find the fraction of the external load, the stiffness of the fastener and member were calculated. Detailed calculations can be found in Appendix D. An image below showcases the diagram used to calculate each of the values. The lengths of each of the screw were determined based on the members involved and is more documented in Appendix D. The end result was that a 40 mm M5 bolt was required to fasten this setup

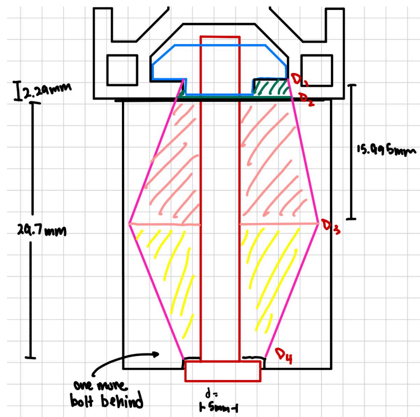


Figure 11: Frustum used to find member stiffness

Due to the combination of 6061 Aluminum and AISI 1018 Steel, 3 frustums were drawn and the stiffness constants were calculated.

$$Fastener\ Stiffness:\ K_b = \frac{A_d A_t E_{screw}}{A_d l_t + A_t l_d} = 101.475\ kN/mm \tag{38}$$

$$Member\ Stiffness:\ K_m = \left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{3}\right]^{-1} = 276.85\ kN/mm \tag{39}$$

$$where\ K_n = \frac{0.5774\pi E_n d}{ln\left[\frac{(1.155t_n + D_n - d)(D_n + d)}{(1.155t_n + D_n + d)(D_n - d)}\right]} \tag{40}$$

Only frustum 1 included the Young's Modulus of Aluminum 6061 whereas the following frustums were composed of stainless steel. Thus, the faction of the external load carried by each screw was found using the following:

$$C = \frac{K_b}{K_b + K_m} = 0.268 \tag{41}$$

Preload

These screws are reusable therefore the preload was calculated using the following equation:

$$F_i = 0.75 A_t S_p = 3.3\ kN \tag{42}$$

Screw Failure Safety Factor

As stated previously, the force applied externally on each bolt was 0.125 kN. This value was used to calculate the factor of safety due to an external load as seen below. The values of each of the variables in this equation is stated in Appendix D.

$$n_L = \frac{A_t S_p - F_i}{CF} = 131304 \tag{43}$$

Since the safety factor is above 1, the screw is unlikely to fail due to the external load applied to the fixture. This high safety factor is contributed to the strong material strength of 304 SS. This material was chosen because it was an industry standard, but due to the high factor of safety, in further iterations, a lower quality material may be used for efficient cost measures.

Loosening Safety Factor

The safety factor against member separation or loosening can be calculated using the following formula.

$$n_0 = \frac{F_i}{(1-C)F_1} = 36.07 \tag{44}$$

The safety factor for loosening is also above 1, therefore it is unlikely to loosen.

Fatigue Safety Factor (Shear)

2 M5 screws hold each of the end supports in place as seen below. The 100 N force by the power screw results in direct shear of the bolts used.

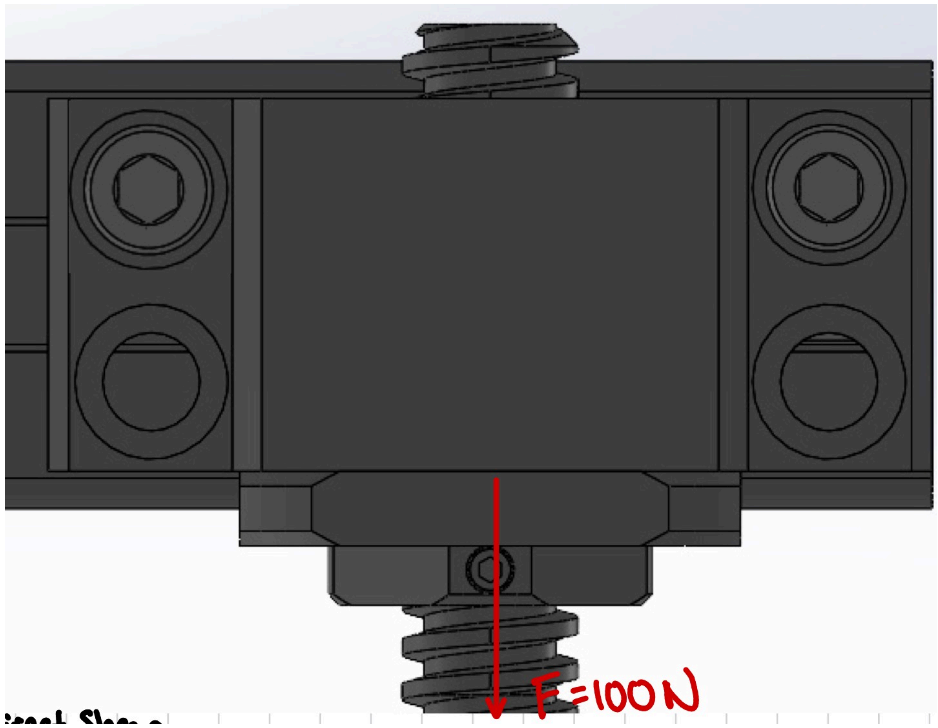


Figure 12: T-Slot fastener

However, this shear force is cyclic due to the operation of the trash compactor, where it supplies 100 N at its peak action and 0 N at rest. This mean direct shear would be a maximum of 50 N or 0.5 kN on each screw. There was no indirect shear acting on the screws because the center of mass of the screws was in line with the force applied, therefore no moment would be acting on the bolts.

$$F'_1 = F'_2 = \frac{100\ N}{2} = 50\ N\ =\ 0.05\ kN \tag{45}$$

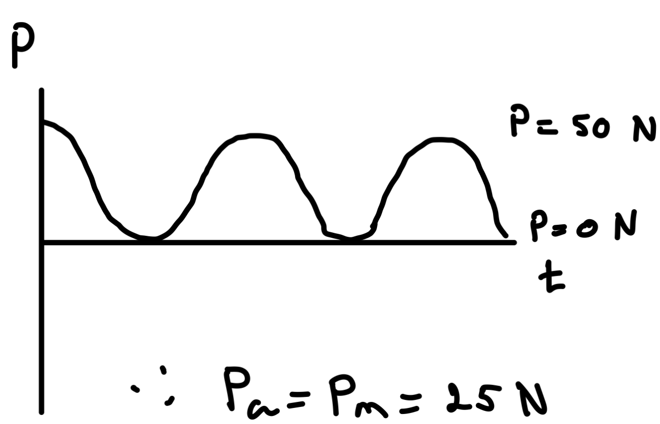


Figure 13: Cyclic Loading Graph

Since the minimum shear is 0 N, the amplitude and mean shear force is 25 N.

$$\tau_a = \tau_m = \frac{V}{A} = \frac{V}{\frac{\pi}{4}d^2} = \frac{0.05}{\frac{\pi}{4}5^2} = 1.273\ MPa \tag{46}$$

This represents the cyclic shear force on each of the bolts. The endurance limit for shear was found using the Equation below

$$S_{se} = (0.59)(0.5S_{ut}) = 141.6\ MPa \tag{47}$$

The ultimate strength in pure shear was also calculated using Equation 6-58, and resulted in the value as seen below

$$S_{su} = (0.67)(S_{ut}) = 321.6\ MPa \tag{48}$$

To ensure infinite life, the safety factor against fatigue failure was calculated and resulted in a value of 77.23.

$$n_f = [\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}]^{-1} = 77.23 \tag{49}$$

This safety factor is above 1, therefore it is unlikely to fail due to shear fatigue.

Corner Bracket Analysis

The top beam and bottom beams are tied to the vertical ones through the cubic corner brackets. The crushing load of 200N splits across the two lead screws, allowing for a reactant force of 100N for each corner. The bracket uses two M8x1.25 stainless steel flat head screws.

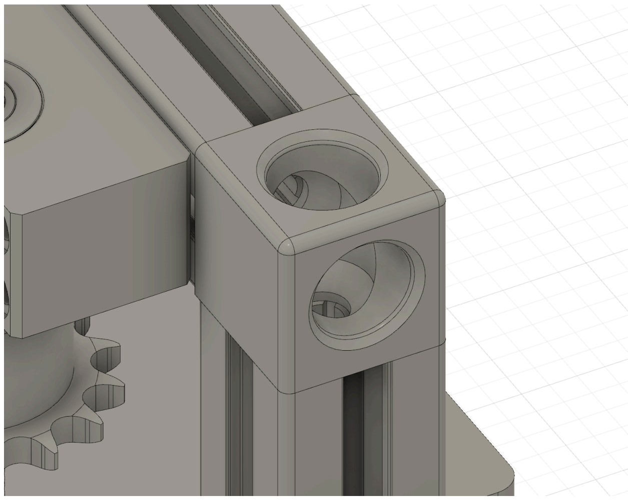
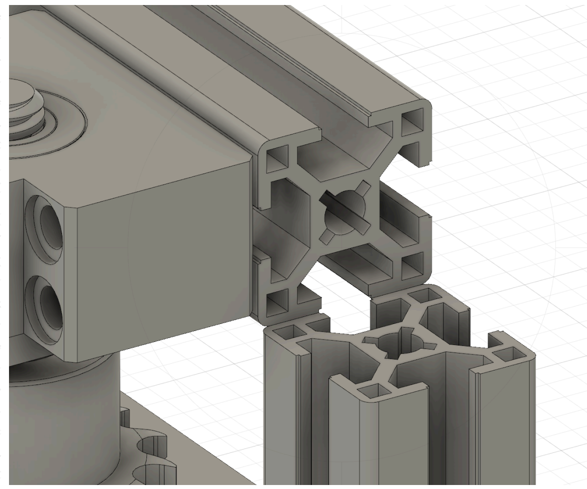
 

Figure 14 and 15. Corner brackets on T-slot rail frame

External Forces on Screws

With the two screws sharing the corner reaction, there is a direct split of 50N per screw. This makes the resultant per screw, 70.7N.

$$F_1 = \frac{P}{2} = 50\ N \tag{50}$$

Fraction of External Load

The fastener and member stiffness were necessary in order to find the fraction of external load.

$$Fastener\ Stiffness:\ K_b = \frac{A_d A_t E_{screw}}{A_d l_t + A_t l_d} = 276\ kN/mm \tag{51}$$

$$Member\ Stiffness:\ K_m = \left[\frac{1}{K_1} + \frac{1}{K_2}\right]^{-1} = 932\ kN/mm \tag{52}$$

$$where\ K_n = \frac{0.5774\pi E_n d}{ln\left[\frac{(1.155t_n + D_n - d)(D_n + d)}{(1.155t_n + D_n + d)(D_n - d)}\right]} \tag{53}$$

Through these values, a fraction of external load was able to be found.

$$C = \frac{K_b}{K_b + K_m} = 0.228 \tag{54}$$

Preload

The preload for the reusable screws was then calculated using the below equation.

$$F_i = 0.75 A_t S_p = 12.4\ kN \tag{55}$$

Screw Failure Safety Factor

Using the values from the preload above, we are able to calculate the failure safety factor using the below equation.

$$n_L = \frac{A_t S_p - F_i}{CF} = 255 \tag{56}$$

As seen from the large safety factor calculated, it is unlikely that the screw fails from the reaction forces of the external load applied.

Loosening Safety Factor

Through the following formula, we are also able to calculate the safety factor against the member separation.

$$n_O = \frac{F_i}{(1-C)F_1} = 226 \tag{57}$$

This makes sense due to the components' strong material strength.

Fatigue Safety Factor

With a minimum load of 0N, we are able to use the relation below to calculate the safety factor for fatigue.

$$n_f = \frac{S_{ut} A_t - F_i}{C(P_a(\frac{S_{ut}}{S_e}) + P_m)} = 318 \tag{58}$$

In this calculation, an ultimate strength of 700MPa and modified endurance limit of 168Mpa was used for the material. With the large safety factor, it is clear that the material is unlikely to fail.

Appendices

Appendix A: Team Contribution

Below is a table with the individual contributions of team members to the Connections Design Report. All members contributed to report writing.

Member	**Responsibilities**
Alvin Reji	Static failure, fatigue, and shear analysis of T slot fastener and its respective report section and appendix;
Marlena Eichelroth	Static failure, fatigue, and shear analysis of flange screws and its respective report section and appendix;
Nathaniel Boutin	Free Body Diagrams, corner bracket Analysis; CAD modelling;
Salim Hamzaoui	Static failure, fatigue, and shear analysis of Corner Bracket and its respective report section and appendix;
Sandra Chai	flange screw, T-slot analysis; Report Review, Appendix Rework
Suyash Gundecha	Static failure, and fatigue failure analysis of End slot connection and its respective report section and appendix;

Appendix B: Flange Screw Analysis Detailed Explanation

Screw Characteristics:

Grip Length: $t_2 \geq d \therefore l = h + d/2 = 0.46875 + \frac{0.2160}{2} = 0.57675''$

Fastener length: $L > h + 1.5d = 0.46875 + 1.5(0.2160) = 0.79275$
$\therefore$ round to 7/8" (0.875") screw

Threaded length: $L \leq 6\text{ in} \therefore L_T = 2d + \frac{1}{4} = 2(0.2160) + .25 = 0.682''$

Unthreaded length in grip: $l_d = L - L_T = 0.875 - 0.682 = 0.193''$

Threaded length in grip: $l_t = l - l_d = 0.57675 - 0.193 = 0.38375''$

Unthreaded area: $A_d = \pi d^2/4 = \pi(0.2160)^2/4 = 0.036644\text{ in}^2$

External load carried by screws:

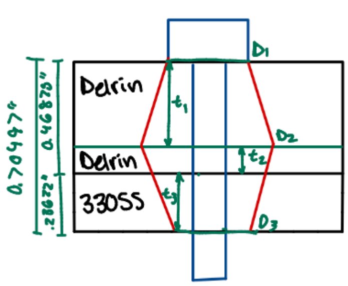
$$S_1 \text{ and } S_2: F_1 = \frac{\left(\frac{P}{12}\right)(L_f)}{(L_1 - L_f)} = \frac{\left(\frac{44.961}{12}\right)(4.07439)}{(4.37017 - 4.07439)} = 51.61\text{ lbf}$$

$$S_3 \text{ and } S_6: F_2 = \frac{\left(\frac{P}{12}\right)(L_f)}{(L_2 - L_f)} = \frac{\left(\frac{44.961}{12}\right)(4.07439)}{(5.01969 - 4.07439)} = 16.1491\text{ lbf}$$

$$S_4 \text{ and } S_5: F_3 = \frac{\left(\frac{P}{12}\right)(L_f)}{(L_3 - L_f)} = \frac{\left(\frac{44.961}{12}\right)(4.07439)}{(5.6692 - 4.07439)} = 9.57213\text{ lbf}$$

Fraction of external load carried by screws:

$$\text{Fastener stiffness: } K_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(0.036644)(0.0242)(28000\text{ ksi})}{(0.036644)(0.38375) + (0.0242)(0.193)} = 1325990\text{ kip/in}$$



$t_1 = 0.352485''$ $E_1 = 420609\text{ psi}$

$t_2 = t_1 - t_3 = 0.116265''$ $E_2 = 420609\text{ psi}$

$t_3 = 0.23622''$ $E_3 = 149037\text{ psi}$

$D_1 = 1.5d = 1.5(.2160) = 0.324''$

$D_2 = D_1 + 2t_1 \tan 30° = 0.324 + 2(.352485)\tan 30° = 0.731015''$

$D_3 = 1.5d = 1.5(.2160) = 0.324''$

$$K_1 = \frac{0.5774\pi E_1 d}{\ln\left[\frac{(1.155t_1+D_1-d)(D_1+d)}{(1.155t_1+D_1+d)(D_1-d)}\right]} = \frac{0.5774\pi(420609)(0.2160)}{\ln\left[\frac{(1.155\cdot .352485+0.324-.2160)(.324+.2160)}{(1.155\cdot .352485+0.324+.2160)(.324-.2160)}\right]} = 164733 \text{ kip/in}$$

$$K_2 = \frac{0.5774\pi E_2 d}{\ln\left[\frac{(1.155t_2+D_2-d)(D_2+d)}{(1.155t_2+D_2+d)(D_2-d)}\right]} = \frac{0.5774\pi(420609)(0.2160)}{\ln\left[\frac{(1.155\cdot .116265+.731015-.2160)(.731015+.2160)}{(1.155\cdot .116265+.731015+.2160)(.731015-.2160)}\right]} = 1663060 \text{ kip/in}$$

$$K_3 = \frac{0.5774\pi E_3 d}{\ln\left[\frac{(1.155t_3+D_3-d)(D_3+d)}{(1.155t_3+D_3+d)(D_3-d)}\right]} = \frac{0.5774\pi(145307)(0.2160)}{\ln\left[\frac{(1.155\cdot .23622+.324-.2160)(.324+.2160)}{(1.155\cdot .23622+.324+.2160)(.324-.2160)}\right]} = 66880.1 \text{ kip/in}$$

$$K_m = \left(\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}\right)^{-1} = 46245.2 \text{ kip/in}$$

$$C = \frac{K_b}{K_b+K_m} = \frac{1325490}{1325490+46245.2} = 0.966$$

Preload for reusable screw:

$$F_i = 0.75A_t S_p = 0.75(0.0242)(120000) = 2178 \text{ lbf}$$

Safety for screw failure under static loading:

$$S_1 \text{ and } S_2: n_{L_1} = \frac{A_t S_p - F_i}{CF_1} = \frac{(0.0242)(120000)-2178}{(0.966)(51.61)} = 14.56$$

$$S_3 \text{ and } S_6: n_{L_2} = \frac{A_t S_p - F_i}{CF_2} = \frac{(0.0242)(120000)-2178}{(0.966)(16.1491)} = 46.54$$

$$S_4 \text{ and } S_5: n_{L_3} = \frac{A_t S_p - F_i}{CF_3} = \frac{(0.0242)(120000)-2178}{(0.966)(9.57213)} = 78.51$$

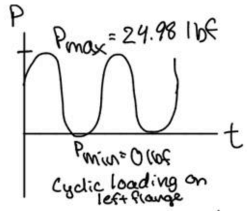
Safety factor against membrane separation:

$$S_1 \text{ and } S_2: n_{0_1} = \frac{F_i}{(1-C)F_1} = \frac{2178}{(1-.966)(51.61)} = 1241.21$$

$$S_3 \text{ and } S_6: n_{0_2} = \frac{F_i}{(1-C)F_2} = \frac{2178}{(1-.966)(16.1491)} = 3966.71$$

$$S_4 \text{ and } S_5: n_{0_3} = \frac{F_i}{(1-C)F_3} = \frac{2178}{(1-.966)(9.57213)} = 6692.2$$

Fatigue failure:



- Load amplitude + mean →

$$P_a = \frac{P_{max} - P_{min}}{2} = \left(\frac{24.98 - 0}{2}\right) = 12.49 \text{ lbf}$$

$$P_m = \frac{P_{max} + P_{min}}{2} = \left(\frac{24.98 + 0}{2}\right) = 12.49 \text{ lbf}$$

- Endurance strength

$$S_e = \frac{K_d K_e S_e'}{K_f} = \frac{(1)(0.868)(375000)}{2.2} = 147955 \text{ psi}$$

$S_e' = 0.5 S_{ut} = 0.5(750000) = 375000$ psi

$K_d = 1$ room temp.

$K_e = 0.868$ for 95% reliability

$K_f = 2.2$ for rolled threads

- safety factor against fatigue

$$n_f = \frac{S_{ut}A_t - F_i}{(C)\left(P_a\left(\frac{S_{ut}}{S_e}\right) + P_m\right)} = \frac{(750000)(0.0242) - 2178}{0.996\left(12.49\left(\frac{750000}{147955}\right) + 12.49\right)} = 211.55$$

Shear due to eccentric loading:

- Moment about centroid

$M_c = P(L) = (49.96)(5.01969) = 225.69$ lbf·in

- Direct Shear Forces

$$F_1' = F_2' = F_3' = F_4' = F_5' = F_6' = (P/6)(\underbrace{1/2}_{\text{two flanges}}) = 3.747 \text{ lbf}$$

- Indirect Shear Forces

Distance from centroid is the same for all

$$F_1'' = F_2'' = F_3'' = F_4'' = F_5'' = F_6'' = \frac{M}{6(r)} = \frac{225.69}{6(0.75)} = 50.15 \text{ lbf}$$

- Resultant Shear Force

$$V = \sqrt{(F')^2 + (F'')^2} = \sqrt{(3.747)^2 + (50.15)^2} = 50.29 \text{ lbf}$$

- Resultant Shear stress

$$\tau = \frac{V}{A} = \frac{V}{\pi d^2/4} = \frac{50.29}{(\pi \cdot .2160^2)/4} = 1372.41 \text{ psi}$$

- Safety factor against Shear failure

Tresca: $n = \frac{S_y}{2\tau} = \frac{30000}{2(1372.41)} = 10.93$

Appendix C: End Slot connection Analysis Detailed Explanation

Screw Characteristics:

$t_2 \geq d$

Grip Length: $l = h + \frac{d}{2} = 5 + \frac{5.5}{2} = 7.75 \text{ mm}$

$L = 12\text{mm}$

Threaded Length: $2d + 6\text{mm} = 10 \text{ mm}$

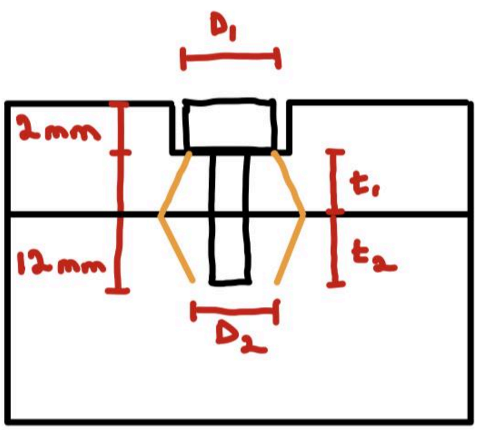
Unthreaded Length: $l_d = L - l_t = 2\text{mm}$

Threaded Length in grip: $l_t = l - l_d = 5.75 \text{ mm}$

Unthreaded area: $A_d = \frac{\pi d^2}{4} = \frac{\pi \cdot 3^2}{4} = 7.0686 \text{ mm}^2$

Threaded area: Table 8-1 $A_t = 5.03 \text{ mm}$

Frustum calculations:



$t_1 = 5\text{mm}$

$t_2 = 7 \text{ mm}$

$D_1 = 1.5d = 4.5$

$D_2 = 1.5d = 4.5$

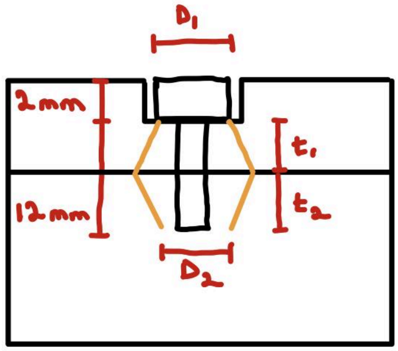
External load carried by screws:

All screws:

$$F_1 = F_2 = F_3 = F_4 = \frac{P}{4} = 25 \text{ N}$$

Fraction of external load carried by screws:

Fastener Stiffness: $K_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d} = 135335 \ \frac{N}{mm} = 135 \ \frac{kN}{mm}$



$t_1 = 5mm$

$t_2 = 7mm$

$D_1 = 1.5d = 4.5$

$D_2 = 1.5d = 4.5$

$$k_1 = \frac{0.5774 \pi E d}{\ln\left[\dfrac{(1.155t_1 + D_1 - d)(D_1 + d)}{(1.155t_1 + D_1 + d)(D_1 - d)}\right]} = \frac{0.5774 \pi \cdot 3 \cdot 207000}{\ln\left[\dfrac{(1.155 \cdot 7 + 4.5 - 3)(4.5 + 3)}{(1.155 \cdot 7 + 4.5 + 3)(4.5 - 3)}\right]} = 1117527.15$$

$$k_2 = \frac{0.5774 \pi E d}{\ln\left[\dfrac{(1.155t_2 + D_2 - d)(D_2 + d)}{(1.155t_2 + D_2 + d)(D_2 - d)}\right]} = \frac{0.5774 \pi \cdot 3 \cdot 207000}{\ln\left[\dfrac{(1.155 \cdot 7 + 4.5 - 3)(4.5 + 3)}{(1.155 \cdot 7 + 4.5 + 3)(4.5 - 3)}\right]} = 1002793.529$$

$$k_m = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1} = 528528.0691 = 528.5 \ \frac{kN}{mm}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{135}{135 + 528.5} = 0.2034578585$$

Preload for reusable screw:

$S_p = 310 \ MPa$

$$F_i = 0.75 A_t S_p = 0.75 \cdot 5.03 \cdot 310 = 1471 \ N = 1.47 \ kN$$

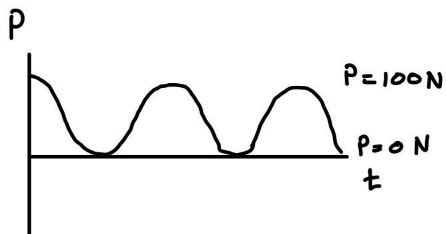
Safety for screw failure under static loading:

$$n_L = \frac{A_t S_p - F_i}{C \cdot F_i} = \frac{5.03 \cdot 310 - 1471}{0.20346 \cdot 25} = 17.3598603 = 17.4$$

Safety factor against membrane separation:

$$n_0 = \frac{F_i}{(1-C)F_1} = \frac{1471}{(1-0.20346)\cdot 25} = 73.8692 = 73.9$$

Fatigue failure:



$$P_a = \frac{P_{max} - P_{min}}{2} = \frac{100-0}{2} = 50\text{ N}$$

$$P_m = \frac{P_{max} + P_{min}}{2} = \frac{100+0}{2} = 50\text{ N}$$

$\therefore P_a = P_m = 50\text{N}$

$S_{ut} = 482.6$ MPa

k_d at room temp $= 1$

k_e at 95% reliability $= 0.868$

$k_f = 2.2$ rolled threads

$S_e' = 0.5\, S_{ut} = 0.5 \cdot 482.6 = 241.3$ MPa

$$S_e = \frac{k_d\, k_e\, S_e'}{k_f} = \frac{1 \cdot 0.868 \cdot 241.3}{2.2} = 95.2 \text{ MPa}$$

$$n_f = \frac{S_{ut} A_t - F_i}{C \cdot \left[P_a \cdot \frac{S_{ut}}{S_e} + P_m\right]} = \frac{482.6 \cdot 5.03 - 1471}{0.20346 \cdot \left[50 \cdot \frac{482.6}{95.2} + 50\right]} = 26.6$$

Appendix D: T-Slot Fastener Analysis Detailed Explanation

Screw Characteristics:

18-8SS / AISI304SS

M5×0.88 , 40mm - McMaster

$S_{ut,304} = 70000 psi$ (McMaster) $= 480$ MPa

$E_{304} = 193 GPa = 28000 ksi$

$l = 29.7 + 2.29 = 31.99 mm$

$H = 4.1 mm$ (McMaster)

$L > l + H = 36.09 \rightarrow 40mm$

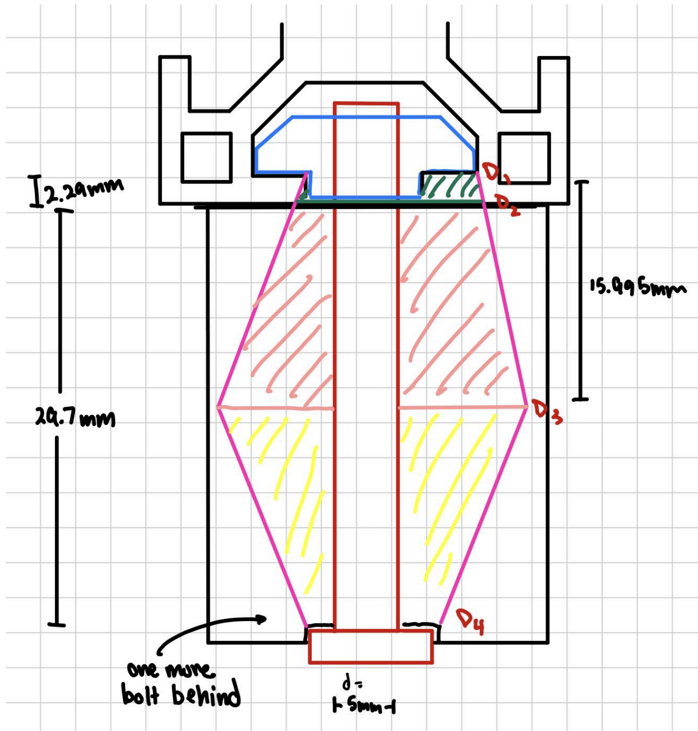
$l_d = L - L_T = 40 - 22 = 18mm$

$l_t = l - l_d = (29.7 + 2.29) - 18 = 13.99mm$

$A_d = \frac{\pi (5)^2}{4} = 19.635 mm^2$

$A_t = 14.2 mm^2$ (Table 8-1)

Frustum calculations:



$E_{6061} = 68.9\,GPa$; $E_{ss} = 207\,GPa$

$D_1 = 1.5d = 7.5mm \ (E_{6061})$

$D_2 = D_1 + 2(2.29)\tan(30) = 10.14mm$

$D_3 = D_1 + 2(15.995)\tan(30) = 25.97mm \ (E_{ss})$

$D_4 = D_1 = 7.5mm \ (E_{ss})$

$t_1 = 2.29mm$

$t_2 = 13.705mm$

$t_3 = 15.995mm$

External load carried by screws:

Direct Shear

$$F_1' = F_2' = \frac{0.1\,kN}{2} = 0.05\,kN$$

$$V = 0.05\,kN$$

Fraction of external load carried by screws:

$$k_1 = \frac{0.5774\,\pi\,E_{6061}\,d}{\ln\left[\frac{(1.155t_1 + D_1 - d)(D_1 + d)}{(1.155t_1 + D_1 + d)(D_1 - d)}\right]} = 984.24\ \frac{kN}{mm}$$

$$k_2 = \frac{0.5774\,\pi\,E_{ss}\,d}{\ln\left[\frac{(1.155t_2 + D_2 - d)(D_2 + d)}{(1.155t_2 + D_2 + d)(D_2 - d)}\right]} = 2471.57\ \frac{kN}{mm}$$

$$k_3 = \frac{0.5774\,\pi\,E_{ss}\,d}{\ln\left[\frac{(1.155t_3 + D_4 - d)(D_4 + d)}{(1.155t_3 + D_4 + d)(D_4 - d)}\right]} = 1446.37\ \frac{kN}{mm}$$

$$k_m = \left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]^{-1} = 276.85\ \frac{kN}{mm}$$

$$C = \frac{k_b}{k_b + k_m} = 0.268$$

Preload for reusable screw:

$$S_p = 310\,MPa$$

$$F_i = 0.75 A_t S_p = 0.75(14.2)(310) = 3301.5\,N$$
$$= 3.3\,kN$$

$$T = KF_i d = (0.2)(3.3kN)(5mm) = 16.5\,kNmm/Nm$$

Safety for screw failure under static loading:

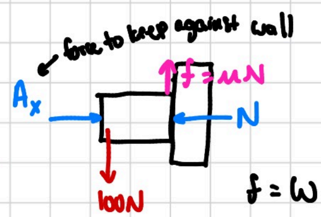
Force on each bolt

$$P = \frac{250}{2} = 125 = 0.125\,kN$$

Failure from static loading:

$$n_L = \frac{A_t S_p - F_i}{C \cdot P} = \frac{(14.2)\left(310\frac{kN}{mm^2}\right) - (3.3kN)}{(0.268)(0.125kN)}$$

$$= 131304$$



$$A_x = N$$

$$f = W$$

$$\mu N = 100 \qquad \mu \approx 0.4$$

$$N = \frac{100}{\mu}$$

$$= 250\,N$$

Safety factor against membrane separation:

Joint sep:

$$n_b = \frac{F_i}{(1-C)P} = \frac{3.3}{(1-0.268)(0.125)} = 36.07$$

Fatigue failure due to shear forces:

$$\tau = \frac{V}{A} = \frac{0.05\,kN}{\frac{\pi}{4} d^2} = \frac{0.05}{\frac{\pi}{4}(5)^2} = 2.546\ MPa$$

Cyclic: $0 \rightarrow 0.05\,kN$

$$\tau_a = \frac{1}{2}\tau = 1.273\ MPa = \tau_m$$

k_c $\qquad$ $S_{ut} = 480\ MPa$

$$S_{se} = \underbrace{(0.59)}_{k_c}\left[0.5\, \underbrace{S_{ut}}_{S_{ut}=480\,MPa}\right] = 141.6\ MPa$$

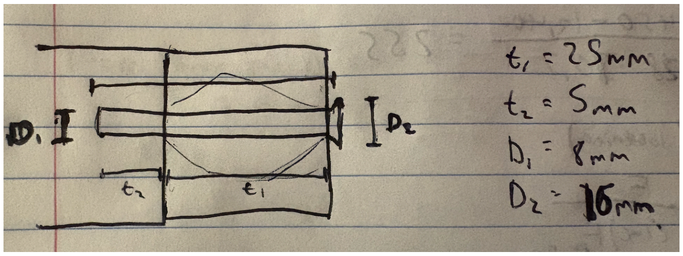
$$S_{su} = 0.67\, S_{ut} = 321.6\ MPa$$

$$n_f = \left[\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right]^{-1} = 77.23$$

↳ FOS against fatigue failure

Appendix E: T-Slot Extrusion Channel Corner Bracket Detailed Explanation

Frustum calculations:



Direct shear per screw:

$$F_i' = \frac{P}{2} = \frac{100N}{2} = 50N$$

$$F_i'' = \frac{Mr}{\Sigma r^2} \quad \Big| \quad \text{Cube length} = 30mm$$

$$M = 100N \cdot 15mm = 1500 N/mm$$

$$F_i'' = \frac{1500 \cdot 15}{2 \cdot 15^2} = 50N$$

Resultant: $V_i = \sqrt{(F_i')^2 + (F_i'')^2} = \sqrt{50^2 + 50^2}$

$$V_i = 70.7N$$

Preload F_i:

$$F_i = 0.75 A_t \cdot S_p \quad \Big| \quad A_t = 36.6 mm^2$$

$$F_i = 0.75 \cdot 36.6 \cdot 450 = 12.4kN \quad \Big| \quad S_p = 450 Mpa$$

Saftey factor: (screw failure)

$$n_L = \frac{A_t S_p - F_i}{CP} = \frac{36.6 \cdot 450 - 12400}{0.9 \cdot 70.7}$$

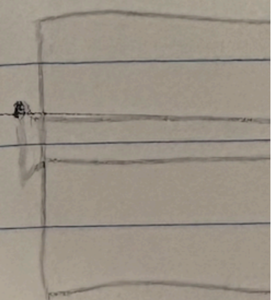
$$k_b = \frac{A_d A_t E_{screw}}{A_d L_t + A_t l_d} = 276 \frac{kN}{mm}$$

$$k_m = \frac{1}{1.39 \cdot 10^6} + \frac{1}{2.83 \cdot 10^6} = 9.32 \cdot 10^5 N/mm$$

fraction of external load

$$C = \frac{k_b}{k_b + k_m} = \frac{2.76 \cdot 10^5}{2.76 \cdot 10^5 + 9.32 \cdot 10^5} = 0.228$$

$$n_L = \frac{36.6 \cdot 450 - 12{,}400}{0.228 \cdot 30.7} = 255$$



(loosening)

saftey factor

$$n_0 = \frac{F}{(1-C)F_{per\ screw}}$$

$$n_0 = \frac{12400}{0.776 \cdot 70} = 226$$

fatigue safety factor:

$$P_a = P_m = \frac{F_{max}}{2} = 35.35\,N$$

$$n_f = \frac{A_t S_{ut} - F_i}{C\left(P_a \frac{S_{ut}}{S_e} + P_m\right)}$$

$$n_f = \frac{36.6 \cdot 700 - 12400}{0.228\left(35.35 \frac{700}{168} + 35.35\right)}$$

$$n_f = 318$$