PART FOUR

OSCILLATIONS AND WAVES

4.1. MECHANICAL OSCILLATIONS

• Harmonic motion equation and its solution:

$$\ddot{x}+\omega_0^2 x=0, \qquad x=a\cos(\omega_0 t+\alpha), \tag{4.1a}$$

where ω_0 is the natural oscillation frequency.

• Damped oscillation equation and its solution:

$$\ddot{x}+2\beta\dot{x}+\omega_0^2 x=0, \qquad x=a_0 e^{-\beta t}\cos(\omega t+\alpha), \tag{4.1b}$$

where β is the damping coefficient, ω is the frequency of damped oscillations:

$$\omega=\sqrt{\omega_0^2-\beta^2}. \tag{4.1c}$$

• Logarithmic damping decrement λ and quality factor Q:

$$\lambda=\beta T, \quad Q=\pi/\lambda, \tag{4.1d}$$

where $T=2\pi/\omega$.

• Forced oscillation equation and its steady-state solution:

$$\ddot{x}+2\beta\dot{x}+\omega_0^2 x=f_0\cos\omega t, \qquad x=a\cos(\omega t-\varphi), \tag{4.1e}$$

where

$$a=\frac{f_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}}, \qquad \tan\varphi=\frac{2\beta\omega}{\omega_0^2-\omega^2}. \tag{4.1f}$$

• Maximum shift amplitude occurs at

$$\omega_{res}=\sqrt{\omega_0^2-2\beta^2}. \tag{4.1g}$$

4.1. A point oscillates along the x axis according to the law $x = a\cos(\omega t-\pi/4)$. Draw the approximate plots

(a) of displacement x, velocity projection v_x, and acceleration projection w_x as functions of time t;

(b) velocity projection v_x and acceleration projection w_x as functions of the coordinate x.

4.2. A point moves along the x axis according to the law $x = a\sin^2(\omega t-\pi/4)$. Find:

(a) the amplitude and period of oscillations; draw the plot $x(t)$;

(b) the velocity projection v_x as a function of the coordinate x; draw the plot $v_x(x)$.

4.3. A particle performs harmonic oscillations along the x axis about the equilibrium position $x=0$. The oscillation frequency is $\omega = 4.00\ \text{s}^{-1}$. At a certain moment of time the particle has a coordinate $x_0 = 25.0$ cm and its velocity is equal to $v_{x0} = 100$ cm/s.

Find the coordinate x and the velocity v_x of the particle $t = 2.40$ s after that moment.

4.4. Find the angular frequency and the amplitude of harmonic oscillations of a particle if at distances x_1 and x_2 from the equilibrium position its velocity equals v_1 and v_2 respectively.

4.5. A point performs harmonic oscillations along a straight line with a period $T = 0.60$ s and an amplitude $a = 10.0$ cm. Find the mean velocity of the point averaged over the time interval during which it travels a distance $a/2$, starting from

(a) the extreme position;

(b) the equilibrium position.

4.6. At the moment $t = 0$ a point starts oscillating along the x axis according to the law $x = a \sin \omega t$. Find:

(a) the mean value of its velocity vector projection $\langle v_x \rangle$;

(b) the modulus of the mean velocity vector $|\langle \mathbf{v} \rangle|$;

(c) the mean value of the velocity modulus $\langle v \rangle$ averaged over 3/8 of the period after the start.

4.7. A particle moves along the x axis according to the law $x = a \cos \omega t$. Find the distance that the particle covers during the time interval from $t = 0$ to t.

4.8. At the moment $t = 0$ a particle starts moving along the x axis so that its velocity projection varies as $v_x = 35 \cos \pi t$ cm/s, where t is expressed in seconds. Find the distance that this particle covers during $t = 2.80$ s after the start.

4.9. A particle performs harmonic oscillations along the x axis according to the law $x = a \cos \omega t$. Assuming the probability P of the particle to fall within an interval from $-a$ to $+a$ to be equal to unity, find how the probability density dP/dx depends on x. Here dP denotes the probability of the particle falling within an interval from x to $x + dx$. Plot dP/dx as a function of x.

4.10. Using graphical means, find an amplitude a of oscillations resulting from the superposition of the following oscillations of the same direction:

(a) $x_1 = 3.0 \cos (\omega t + \pi/3)$, $x_2 = 8.0 \sin (\omega t + \pi/6)$;

(b) $x_1 = 3.0 \cos \omega t$, $x_2 = 5.0 \cos (\omega t + \pi/4)$, $x_3 = 6.0 \sin \omega t$.

4.11. A point participates simultaneously in two harmonic oscillations of the same direction: $x_1 = a \cos \omega t$ and $x_2 = a \cos 2\omega t$. Find the maximum velocity of the point.

4.12. The superposition of two harmonic oscillations of the same direction results in the oscillation of a point according to the law $x = a \cos 2.1t \cos 50.0t$, where t is expressed in seconds. Find the angular frequencies of the constituent oscillations and the period with which they beat.

4.13. A point A oscillates according to a certain harmonic law in the reference frame K' which in its turn performs harmonic oscillations relative to the reference frame K. Both oscillations occur along the same direction. When the K' frame oscillates at the frequency 20 or 24 Hz, the beat frequency of the point A in the K frame turns out to be equal to ν. At what frequency of oscillation of the frame K' will the beat frequency of the point A become equal to 2ν?

4.14. A point moves in the plane xy according to the law $x = a \sin \omega t$, $y = b \cos \omega t$, where a, b, and ω are positive constants. Find:

(a) the trajectory equation $y(x)$ of the point and the direction of its motion along this trajectory;

(b) the acceleration $\mathbf{w}$ of the point as a function of its radius vector $\mathbf{r}$ relative to the origin of coordinates.

4.15. Find the trajectory equation $y(x)$ of a point if it moves according to the following laws:

(a) $x = a \sin \omega t$, $y = a \sin 2\omega t$;

(b) $x = a \sin \omega t$, $y = a \cos 2\omega t$.

Plot these trajectories.

4.16. A particle of mass m is located in a unidimensional potential field where the potential energy of the particle depends on the coordinate x as $U(x) = U_0(1 - \cos ax)$; U_0 and a are constants. Find the period of small oscillations that the particle performs about the equilibrium position.

4.17. Solve the foregoing problem if the potential energy has the form $U(x) = a/x^2 - b/x$, where a and b are positive constants.

4.18. Find the period of small oscillations in a vertical plane performed by a ball of mass $m = 40$ g fixed at the middle of a horizontally stretched string $l = 1.0$ m in length. The tension of the string is assumed to be constant and equal to $F = 10$ N.

4.19. Determine the period of small oscillations of a mathematical pendulum, that is a ball suspended by a thread $l = 20$ cm in length, if it is located in a liquid whose density is $\eta = 3.0$ times less than that of the ball. The resistance of the liquid is to be neglected.

4.20. A ball is suspended by a thread of length l at the point O on the wall, forming a small angle α with the vertical (Fig. 4.1). Then

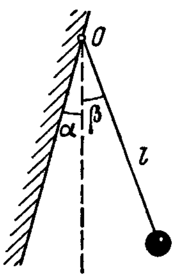

Fig. 4.1.

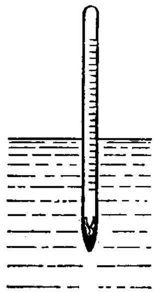

Fig. 4.2.

the thread with the ball was deviated through a small angle β ($\beta > \alpha$) and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

4.21. A pendulum clock is mounted in an elevator car which starts going up with a constant acceleration w, with $w < g$. At a height h the acceleration of the car reverses, its magnitude remaining constant. How soon after the start of the motion will the clock show the right time again?

4.22. Calculate the period of small oscillations of a hydrometer (Fig. 4.2) which was slightly pushed down in the vertical direction. The mass of the hydrometer is $m = 50$ g, the radius of its tube is $r = 3.2$ mm, the density of the liquid is $\rho = 1.00$ g/cm^3. The resistance of the liquid is assumed to be negligible.

4.23. A non-deformed spring whose ends are fixed has a stiffness $\varkappa = 13$ N/m. A small body of mass $m = 25$ g is attached at the point removed from one of the ends by $\eta = 1/3$ of the spring's length. Neglecting the mass of the spring, find the period of small longitudinal oscillations of the body. The force of gravity is assumed to be absent.

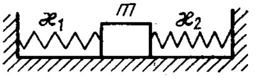

Fig. 4.3.

4.24. Determine the period of small longitudinal oscillations of a body with mass m in the system shown in Fig. 4.3. The stiffness values of the springs are $\varkappa_1$ and $\varkappa_2$. The friction and the masses of the springs are negligible.

4.25. Find the period of small vertical oscillations of a body with mass m in the system illustrated in Fig. 4.4. The stiffness values of the springs are $\varkappa_1$ and $\varkappa_2$, their masses are negligible.

4.26. A small body of mass m is fixed to the middle of a stretched string of length $2l$. In the equilibrium position the string tension is equal to T_0. Find the angular frequency of small oscillations of the body in the transverse direction. The mass of the string is negligible, the gravitational field is absent.

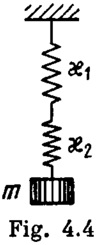

Fig. 4.4.

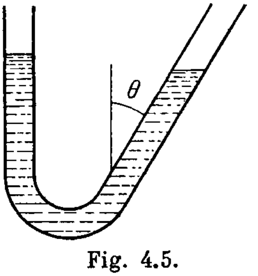

Fig. 4.5.

4.27. Determine the period of oscillations of mercury of mass $m = 200$ g poured into a bent tube (Fig. 4.5) whose right arm forms an angle $\theta = 30°$ with the vertical. The cross-sectional area of the tube is $S = 0.50$ cm^2. The viscosity of mercury is to be neglected.

4.28. A uniform rod is placed on two spinning wheels as shown in Fig. 4.6. The axes of the wheels are separated by a distance $l = 20$ cm, the coefficient of friction between the rod and the wheels is $k = 0.18$. Demonstrate that in this case the rod performs harmonic oscillations. Find the period of these oscillations.

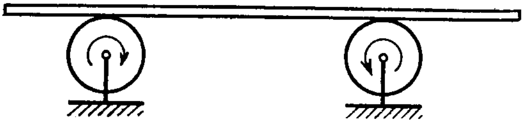

Fig. 4.6.

4.29. Imagine a shaft going all the way through the Earth from pole to pole along its rotation axis. Assuming the Earth to be a homogeneous ball and neglecting the air drag, find:

(a) the equation of motion of a body falling down into the shaft;

(b) how long does it take the body to reach the other end of the shaft;

(c) the velocity of the body at the Earth's centre.

4.30. Find the period of small oscillations of a mathematical pendulum of length l if its point of suspension O moves relative to the Earth's surface in an arbitrary direction with a constant acceleration $\mathbf{w}$ (Fig. 4.7). Calculate that period if $l = 21$ cm, $w = g/2$, and the angle between the vectors $\mathbf{w}$ and $\mathbf{g}$ equals $\beta = 120°$.

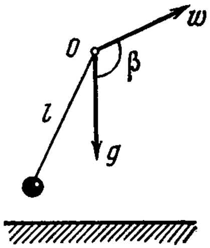

Fig. 4.7.

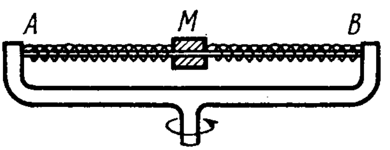

Fig. 4.8.

4.31. In the arrangement shown in Fig. 4.8 the sleeve M of mass $m = 0.20$ kg is fixed between two identical springs whose combined stiffness is equal to $\varkappa = 20$ N/m. The sleeve can slide without friction over a horizontal bar AB. The arrangement rotates with a constant angular velocity $\omega = 4.4$ rad/s about a vertical axis passing through the middle of the bar. Find the period of small oscillations of the sleeve. At what values of ω will there be no oscillations of the sleeve?

4.32. A plank with a bar placed on it performs horizontal harmonic oscillations with amplitude $a = 10$ cm. Find the coefficient of friction between the bar and the plank if the former starts sliding along the plank when the amplitude of oscillation of the plank becomes less than $T = 1.0$ s.

4.33. Find the time dependence of the angle of deviation of a mathematical pendulum 80 cm in length if at the initial moment the pendulum

(a) was deviated through the angle 3.0° and then set free without push;

(b) was in the equilibrium position and its lower end was imparted the horizontal velocity 0.22 m/s;

(c) was deviated through the angle 3.0° and its lower end was imparted the velocity 0.22 m/s directed toward the equilibrium position.

4.34. A body A of mass $m_1 = 1.00$ kg and a body B of mass $m_2 = 4.10$ kg are interconnected by a spring as shown in Fig. 4.9. The body A performs free vertical harmonic oscillations with amplitude $a = 1.6$ cm and frequency $\omega = 25\ \text{s}^{-1}$. Neglecting the mass of the spring, find the maximum and minimum values of force that this system exerts on the bearing surface.

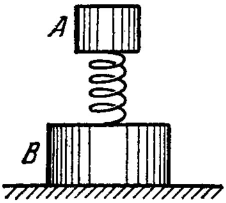

Fig. 4.9.

4.35. A plank with a body of mass m placed on it starts moving straight up according to the law $y = a(1 - \cos \omega t)$, where y is the displacement from the initial position, $\omega = 11\ \text{s}^{-1}$. Find:

(a) the time dependence of the force that the body exerts on the plank if $a = 4.0$ cm; plot this dependence;

(b) the minimum amplitude of oscillation of the plank at which the body starts falling behind the plank;

(c) the amplitude of oscillation of the plank at which the body springs up to a height $h = 50$ cm relative to the initial position (at the moment $t = 0$).

4.36. A body of mass m was suspended by a non-stretched spring, and then set free without push. The stiffness of the spring is $\varkappa$. Neglecting the mass of the spring, find:

(a) the law of motion $y(t)$, where y is the displacement of the body from the equilibrium position;

(b) the maximum and minimum tensions of the spring in the process of motion.

4.37. A particle of mass m moves due to the force $\mathbf{F} = -\alpha m \mathbf{r}$, where α is a positive constant, $\mathbf{r}$ is the radius vector of the particle relative to the origin of coordinates. Find the trajectory of its motion if at the initial moment $\mathbf{r} = r_0 \mathbf{i}$ and the velocity $\mathbf{v} = v_0 \mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes.

4.38. A body of mass m is suspended from a spring fixed to the ceiling of an elevator car. The stiffness of the spring is $\varkappa$. At the moment $t = 0$ the car starts going up with an acceleration w. Neglecting the mass of the spring, find the law of motion $y(t)$ of the body relative to the elevator car if $y(0) = 0$ and $\dot{y}(0) = 0$. Consider the following two cases:

(a) $w = \text{const}$;

(b) $w = \alpha t$, where α is a constant.

4.39. A body of mass $m = 0.50$ kg is suspended from a rubber cord with elasticity coefficient $k = 50$ N/m. Find the maximum distance over which the body can be pulled down for the body's oscillations to remain harmonic. What is the energy of oscillation in this case?

4.40. A body of mass m fell from a height h onto the pan of a spring balance (Fig. 4.10). The masses of the pan and the spring are negligible, the stiffness of the latter is $\varkappa$. Having stuck to the pan, the body starts performing harmonic oscillations in the vertical direction. Find the amplitude and the energy of these oscillations.

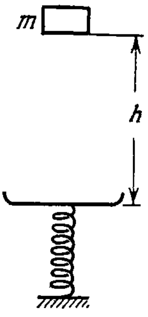

Fig. 4.10.

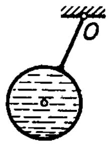

Fig. 4.11.

4.41. Solve the foregoing problem for the case of the pan having a mass M. Find the oscillation amplitude in this case.

4.42. A particle of mass m moves in the plane xy due to the force varying with velocity as $\mathbf{F} = a(\dot{y}\mathbf{i} - \dot{x}\mathbf{j})$, where a is a positive constant, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. At the initial moment $t = 0$ the particle was located at the point $x = y = 0$ and possessed a velocity $\mathbf{v}_0$ directed along the unit vector $\mathbf{j}$. Find the law of motion $x(t)$, $y(t)$ of the particle, and also the equation of its trajectory.

4.43. A pendulum is constructed as a light thin-walled sphere of radius R filled up with water and suspended at the point O from a light rigid rod (Fig. 4.11). The distance between the point O and the centre of the sphere is equal to l. How many times will the small oscillations of such a pendulum change after the water freezes? The viscosity of water and the change of its volume on freezing are to be neglected.

4.44. Find the frequency of small oscillations of a thin uniform vertical rod of mass m and length l hinged at the point O (Fig. 4.12). The combined stiffness of the springs is equal to $\varkappa$. The mass of the springs is negligible.

4.45. A uniform rod of mass $m = 1.5$ kg suspended by two identical threads $l = 90$ cm in length (Fig. 4.13) was turned through a small angle about the vertical axis passing through its middle point C. The threads deviated in the process through an angle $\alpha = 5.0°$. Then the rod was released to start performing small oscillations. Find:

(a) the oscillation period;
(b) the rod's oscillation energy.

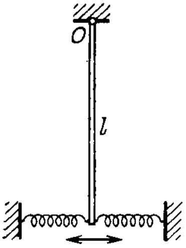

Fig. 4.12.

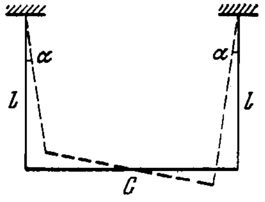

Fig. 4.13.

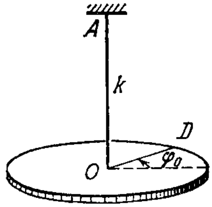

Fig. 4.14.

4.46. An arrangement illustrated in Fig. 4.14 consists of a horizontal uniform disc D of mass m and radius R and a thin rod AO whose torsional coefficient is equal to k. Find the amplitude and the energy of small torsional oscillations if at the initial moment the disc was deviated through an angle φ_0 from the equilibrium position and then imparted an angular velocity $\dot{\varphi}_0$.

4.47. A uniform rod of mass m and length l performs small oscillations about the horizontal axis passing through its upper end. Find the mean kinetic energy of the rod averaged over one oscillation period if at the initial moment it was deflected from the vertical by an angle θ_0 and then imparted an angular velocity $\dot{\theta}_0$.

4.48. A physical pendulum is positioned so that its centre of gravity is above the suspension point. From that position the pendulum started moving toward the stable equilibrium and passed it with an angular velocity ω. Neglecting the friction find the period of small oscillations of the pendulum.

4.49. A physical pendulum performs small oscillations about the horizontal axis with frequency $\omega_1 = 15.0\ \text{s}^{-1}$. When a small body of mass $m = 50$ g is fixed to the pendulum at a distance $l = 20$ cm below the axis, the oscillation frequency becomes equal to $\omega_2 = 10.0\ \text{s}^{-1}$. Find the moment of inertia of the pendulum relative to the oscillation axis.

4.50. Two physical pendulums perform small oscillations about the same horizontal axis with frequencies ω_1 and ω_2. Their moments of inertia relative to the given axis are equal to I_1 and I_2 respectively. In a state of stable equilibrium the pendulums were fastened rigidly together. What will be the frequency of small oscillations of the compound pendulum?

4.51. A uniform rod of length l performs small oscillations about the horizontal axis OO' perpendicular to the rod and passing through one of its points. Find the distance between the centre of inertia of the rod and the axis OO' at which the oscillation period is the shortest. What is it equal to?

4.52. A thin uniform plate shaped as an equilateral triangle with a height h performs small oscillations about the horizontal axis coinciding with one of its sides. Find the oscillation period and the reduced length of the given pendulum.

4.53. A smooth horizontal disc rotates about the vertical axis O (Fig. 4.15) with a constant angular velocity ω. A thin uniform rod AB of length l performs small oscillations about the vertical axis A fixed to the disc at a distance a from the axis of the disc. Find the frequency ω_0 of these oscillations.

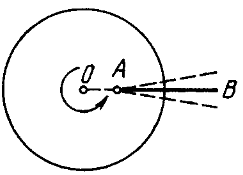

Fig. 4.15.

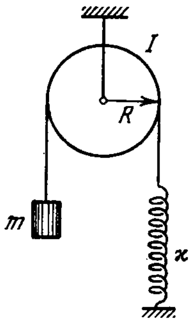

Fig. 4.16.

4.54. Find the frequency of small oscillations of the arrangement illustrated in Fig. 4.16. The radius of the pulley is R, its moment of inertia relative to the rotation axis is I, the mass of the body is m, and the spring stiffness is $\varkappa$. The mass of the thread and the spring is negligible, the thread does not slide over the pulley, there is no friction in the axis of the pulley.

4.55. A uniform cylindrical pulley of mass M and radius R can freely rotate about the horizontal axis O (Fig. 4.17). The free end of

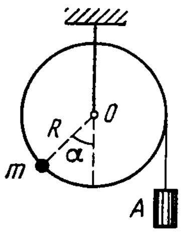

Fig. 4.17.

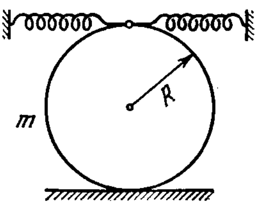

Fig. 4.18.

a thread tightly wound on the pulley carries a deadweight A. At a certain angle α it counterbalances a point mass m fixed at the rim of the pulley. Find the frequency of small oscillations of the arrangement.

4.56. A solid uniform cylinder of radius r rolls without sliding along the inside surface of a cylinder of radius R, performing small oscillations. Find their period.

4.57. A solid uniform cylinder of mass m performs small oscillations due to the action of two springs whose combined stiffness is equal to $\varkappa$ (Fig. 4.18). Find the period of these oscillations in the absence of sliding.

4.58. Two cubes with masses m_1 and m_2 were interconnected by a weightless spring of stiffness $\varkappa$ and placed on a smooth horizontal surface. Then the cubes were drawn closer to each other and released simultaneously. Find the natural oscillation frequency of the system.

4.59. Two balls with masses $m_1 = 1.0$ kg and $m_2 = 2.0$ kg are slipped on a thin smooth horizontal rod (Fig. 4.19). The balls are

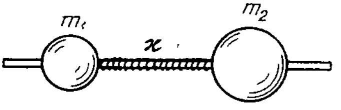

Fig. 4.19.

interconnected by a light spring of stiffness $\varkappa = 24$ N/m. The left-hand ball is imparted the initial velocity $v_1 = 12$ cm/s. Find:

(a) the oscillation frequency of the system in the process of motion;

(b) the energy and the amplitude of oscillations.

4.60. Find the period of small torsional oscillations of a system consisting of two discs slipped on a thin rod with torsional coefficient k. The moments of inertia of the discs relative to the rod's axis are equal to I_1 and I_2.

4.61. A mock-up of a CO_2 molecule consists of three balls interconnected by identical light springs and placed along a straight line in the state of equilibrium. Such a system can freely perform oscillations of two types, as shown by the arrows in Fig. 4.20. Knowing the masses of the atoms, find the ratio of frequencies of these oscillations.

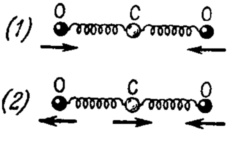

Fig. 4.20.

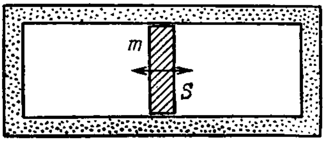

Fig. 4.21.

4.62. In a cylinder filled up with ideal gas and closed from both ends there is a piston of mass m and cross-sectional area S (Fig. 4.21).

In equilibrium the piston divides the cylinder into two equal parts, each with volume V_0. The gas pressure is p_0. The piston was slighlty displaced from the equilibrium position and released. Find its oscillation frequency, assuming the processes in the gas to be adiabatic and the friction negligible.

4.63. A small ball of mass $m = 21$ g suspended by an insulating thread at a height $h = 12$ cm from a large horizontal conducting plane performs small oscillations (Fig. 4.22). After a charge q had been imparted to the ball, the oscillation period changed $\eta = 2.0$ times. Find q.

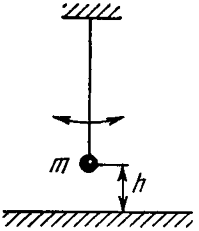

Fig. 4.22.

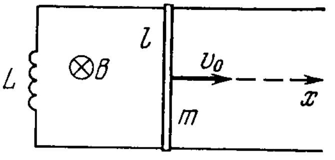

Fig. 4.23.

4.64. A small magnetic needle performs small oscillations about an axis perpendicular to the magnetic induction vector. On changing the magnetic induction the needle's oscillation period decreased $\eta = 5.0$ times. How much and in what way was the magnetic induction changed? The oscillation damping is assumed to be negligible.

4.65. A loop (Fig. 4.23) is formed by two parallel conductors connected by a solenoid with inductance L and a conducting rod of mass m which can freely (without friction) slide over the conductors. The conductors are located in a horizontal plane in a uniform vertical magnetic field with induction B. The distance between the conductors is equal to l. At the moment $t = 0$ the rod is imparted an initial velocity v_0 directed to the right. Find the law of its motion $x(t)$ if the electric resistance of the loop is negligible.

4.66. A coil of inductance L connects the upper ends of two vertical copper bars separated by a distance l. A horizontal conducting connector of mass m starts falling with zero initial velocity along the bars without losing contact with them. The whole system is located in a uniform magnetic field with induction B perpendicular to the plane of the bars. Find the law of motion $x(t)$ of the connector.

4.67. A point performs damped oscillations according to the law $x = a_0 e^{-\beta t} \sin \omega t$. Find:

(a) the oscillation amplitude and the velocity of the point at the moment $t = 0$;

(b) the moments of time at which the point reaches the extreme positions.

4.68. A body performs torsional oscillations according to the law $\varphi = \varphi_0 e^{-\beta t} \cos \omega t$. Find:

(a) the angular velocity $\dot{\varphi}$ and the angular acceleration $\ddot{\varphi}$ of the body at the moment $t = 0$;

(b) the moments of time at which the angular velocity becomes maximum.

4.69. A point performs damped oscillations with frequency ω and damping coefficient β according to the law (4.1b). Find the initial amplitude a_0 and the initial phase α if at the moment $t = 0$ the displacement of the point and its velocity projection are equal to

(a) $x(0) = 0$ and $v_x(0) = \dot{x}_0$;
(b) $x(0) = x_0$ and $v_x(0) = 0$.

4.70. A point performs damped oscillations with frequency $\omega = 25\ \mathrm{s}^{-1}$. Find the damping coefficient β if at the initial moment the velocity of the point is equal to zero and its displacement from the equilibrium position is $\eta = 1.020$ times less than the amplitude at that moment.

4.71. A point performs damped oscillations with frequency ω and damping coefficient β. Find the velocity amplitude of the point as a function of time t if at the moment $t = 0$

(a) its displacement amplitude is equal to a_0;
(b) the displacement of the point $x(0) = 0$ and its velocity projection $v_x(0) = \dot{x}_0$.

4.72. There are two damped oscillations with the following periods T and damping coefficients β: $T_1 = 0.10$ ms, $\beta_1 = 100\ \mathrm{s}^{-1}$ and $T_2 = 10$ ms, $\beta_2 = 10\ \mathrm{s}^{-1}$. Which of them decays faster?

4.73. A mathematical pendulum oscillates in a medium for which the logarithmic damping decrement is equal to $\lambda_0 = 1.50$. What will be the logarithmic damping decrement if the resistance of the medium increases $n = 2.00$ times? How many times has the resistance of the medium to be increased for the oscillations to become impossible?

4.74. A deadweight suspended from a weightless spring extends it by $\Delta x = 9.8$ cm. What will be the oscillation period of the deadweight when it is pushed slightly in the vertical direction? The logarithmic damping decrement is equal to $\lambda = 3.1$.

4.75. Find the quality factor of the oscillator whose displacement amplitude decreases $\eta = 2.0$ times every $n = 110$ oscillations.

4.76. A particle was displaced from the equilibrium position by a distance $l = 1.0$ cm and then left alone. What is the distance that the particle covers in the process of oscillations till the complete stop, if the logarithmic damping decrement is equal to $\lambda = 0.020$?

4.77. Find the quality factor of a mathematical pendulum $l = 50$ cm long if during the time interval $\tau = 5.2$ min its total mechanical energy decreases $\eta = 4.0 \cdot 10^4$ times.

4.78. A uniform disc of radius $R = 13$ cm can rotate about a horizontal axis perpendicular to its plane and passing through the edge of the disc. Find the period of small oscillations of that disc if the logarithmic damping decrement is equal to $\lambda = 1.00$.

4.79. A thin uniform disc of mass m and radius R suspended by an elastic thread in the horizontal plane performs torsional oscillations in a liquid. The moment of elastic forces emerging in the thread is equal to $N = \alpha\varphi$, where α is a constant and φ is the angle of rotation from the equilibrium position. The resistance force acting on a unit area of the disc is equal to $F_1 = \eta v$, where η is a constant and v is the velocity of the given element of the disc relative to the liquid. Find the frequency of small oscillation.

4.80. A disc A of radius R suspended by an elastic thread between two stationary planes (Fig. 4.24) performs torsional oscillations about its axis OO'. The moment of inertia of the disc relative to that axis is equal to I, the clearance between the disc and each of the planes is equal to h, with $h \ll R$. Find the viscosity of the gas surrounding the disc A if the oscillation period of the disc equals T and the logarithmic damping decrement, λ.

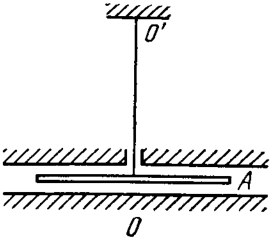

Fig. 4.24.

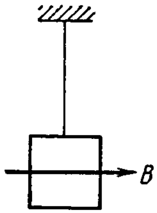

Fig. 4.25.

4.81. A conductor in the shape of a square frame with side a suspended by an elastic thread is located in a uniform horizontal magnetic field with induction B. In equilibrium the plane of the frame is parallel to the vector $\mathbf{B}$ (Fig. 4.25). Having been displaced from the equilibrium position, the frame performs small oscillations about a vertical axis passing through its centre. The moment of inertia of the frame relative to that axis is equal to I, its electric resistance is R. Neglecting the inductance of the frame, find the time interval after which the amplitude of the frame's deviation angle decreases e-fold.

4.82. A bar of mass $m = 0.50$ kg lying on a horizontal plane with a friction coefficient $k = 0.10$ is attached to the wall by means of a horizontal non-deformed spring. The stiffness of the spring is equal to $\varkappa = 2.45$ N/cm, its mass is negligible. The bar was displaced so that the spring was stretched by $x_0 = 3.0$ cm, and then released. Find:

(a) the period of oscillation of the bar;

(b) the total number of oscillations that the bar performs until it stops completely.

4.83. A ball of mass m can perform undamped harmonic oscillations about the point $x = 0$ with natural frequency ω_0. At the moment $t = 0$, when the ball was in equilibrium, a force $F_x = F_0 \cos \omega t$ coinciding with the x axis was applied to it. Find the law of forced oscillation $x(t)$ for that ball.

4.84. A particle of mass m can perform undamped harmonic oscillations due to an electric force with coefficient k. When the particle was in equilibrium, a permanent force F was applied to it for τ seconds. Find the oscillation amplitude that the particle acquired after the action of the force ceased. Draw the approximate plot $x(t)$ of oscillations. Investigate possible cases.

4.85. A ball of mass m when suspended by a spring stretches the latter by Δl. Due to external vertical force varying according to a harmonic law with amplitude F_0 the ball performs forced oscillations. The logarithmic damping decrement is equal to λ. Neglecting the mass of the spring, find the angular frequency of the external force at which the displacement amplitude of the ball is maximum. What is the magnitude of that amplitude?

4.86. The forced harmonic oscillations have equal displacement amplitudes at frequencies $\omega_1 = 400\ \text{s}^{-1}$ and $\omega_2 = 600\ \text{s}^{-1}$. Find the resonance frequency at which the displacement amplitude is maximum.

4.87. The velocity amplitude of a particle is equal to half the maximum value at the frequencies ω_1 and ω_2 of external harmonic force. Find:

(a) the frequency corresponding to the velocity resonance;

(b) the damping coefficient β and the damped oscillation frequency ω of the particle.

4.88. A certain resonance curve describes a mechanical oscillating system with logarithmic damping decrement $\lambda = 1.60$. For this curve find the ratio of the maximum displacement amplitude to the displacement amplitude at a very low frequency.

4.89. Due to the external vertical force $F_x = F_0 \cos \omega t$ a body suspended by a spring performs forced steady-state oscillations according to the law $x = a \cos(\omega t - \varphi)$. Find the work performed by the force F during one oscillation period.

4.90. A ball of mass $m = 50$ g is suspended by a weightless spring with stiffness $\varkappa = 20.0$ N/m. Due to external vertical harmonic force with frequency $\omega = 25.0\ \text{s}^{-1}$ the ball performs steady-state oscillations with amplitude $a = 1.3$ cm. In this case the displacement of the ball lags in phase behind the external force by $\varphi = \frac{3}{4}\pi$. Find:

(a) the quality factor of the given oscillator;

(b) the work performed by the external force during one oscillation period.

4.91. A ball of mass m suspended by a weightless spring can perform vertical oscillations with damping coefficient β. The natural oscillation frequency is equal to ω_0. Due to the external vertical force varying as $F = F_0 \cos \omega t$ the ball performs steady-state harmonic oscillations. Find:

(a) the mean power $\langle P \rangle$, developed by the force F, averaged over one oscillation period;

(b) the frequency ω of the force F at which $\langle P\rangle$ is maximum; what is $\langle P\rangle_{max}$ equal to?

4.92. An external harmonic force F whose frequency can be varied, with amplitude maintained constant, acts in a vertical direction on a ball suspended by a weightless spring. The damping coefficient is η times less than the natural oscillation frequency ω_0 of the ball. How much, in per cent, does the mean power $\langle P\rangle$ developed by the force F at the frequency of displacement resonance differ from the maximum mean power $\langle P\rangle_{max}$? Averaging is performed over one oscillation period.

4.93. A uniform horizontal disc fixed at its centre to an elastic vertical rod performs forced torsional oscillations due to the moment of forces $N_z = N_m \cos \omega t$. The oscillations obey the law $\varphi = \varphi_m \cos(\omega t - \alpha)$. Find:

(a) the work performed by friction forces acting on the disc during one oscillation period;

(b) the quality factor of the given oscillator if the moment of inertia of the disc relative to the axis is equal to I.

4.2. ELECTRIC OSCILLATIONS

• Damped oscillation in a circuit

$$q = q_m e^{-\beta t} \cos(\omega t + \alpha),$$

where

$$\omega = \sqrt{\omega_0^2 - \beta^2}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad \beta = \frac{R}{2L}. \tag{4.2a}$$

• Logarithmic damping decrement λ and quality factor Q of a circuit are defined by Eqs. (4.1d). When damping is low:

$$\lambda = \pi R \sqrt{\frac{C}{L}}, \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}. \tag{4.2b}$$

• Steady-state forced oscillation in a circuit with a voltage $V = V_m \cos \omega t$ connected in series:

$$I = I_m \cos(\omega t - \varphi), \tag{4.2c}$$

where

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}, \qquad \tan \varphi = \frac{\omega L - \frac{1}{\omega C}}{R}. \tag{4.2d}$$

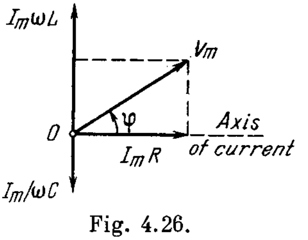

Fig. 4.26.

The corresponding vector diagram for voltages is shown in Fig. 4.26.

• Power generated in an ac circuit:

$$P = VI \cos \varphi, \tag{4.2e}$$

where V and I are the effective values of voltage and current:

$$V = V_m/\sqrt{2}, \quad I = I_m/\sqrt{2}. \tag{4.2f}$$

4.94. Due to a certain cause the free electrons in a plane copper plate shifted over a small distance x at right angles to its surface. As a result, a surface charge and a corresponding restoring force emerged, giving rise to so-called plasma oscillations. Find the angular frequency of these oscillations if the free electron concentration in copper is $n = 0.85 \cdot 10^{29}\ \mathrm{m^{-1}}$.

4.95. An oscillating circuit consisting of a capacitor with capacitance C and a coil of inductance L maintains free undamped oscillations with voltage amplitude across the capacitor equal to V_m. For an arbitrary moment of time find the relation between the current I in the circuit and the voltage V across the capacitor. Solve this problem using Ohm's law and then the energy conservation law.

4.96. An oscillating circuit consists of a capacitor with capacitance C, a coil of inductance L with negligible resistance, and a switch. With the switch disconnected, the capacitor was charged to a voltage V_m and then at the moment $t = 0$ the switch was closed. Find:

(a) the current $I(t)$ in the circuit as a function of time;

(b) the emf of self-inductance in the coil at the moments when the electric energy of the capacitor is equal to that of the current in the coil.

4.97. In an oscillating circuit consisting of a parallel-plate capacitor and an inductance coil with negligible active resistance the oscillations with energy W are sustained. The capacitor plates were slowly drawn apart to increase the oscillation frequency η-fold. What work was done in the process?

4.98. In an oscillating circuit shown in Fig. 4.27 the coil inductance is equal to $L = 2.5$ mH and the capacitor have capacitances $C_1 = 2.0\ \mu\mathrm{F}$ and $C_2 = 3.0\ \mu\mathrm{F}$. The capacitors were charged to a voltage $V = 180$ V, and then the switch Sw was closed. Find:

(a) the natural oscillation frequency;

(b) the peak value of the current flowing through the coil.

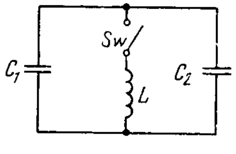

Fig. 4.27.

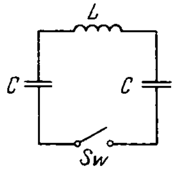

Fig. 4.28.

4.99. An electric circuit shown in Fig. 4.28 has a negligibly small active resistance. The left-hand capacitor was charged to a voltage V_0 and then at the moment $t = 0$ the switch Sw was closed. Find the time dependence of the voltages in left and right capacitors.

4.100. An oscillating circuit consists of an inductance coil L and a capacitor with capacitance C. The resistance of the coil and the lead wires is negligible. The coil is placed in a permanent magnetic field so that the total flux passing through all the turns of the coil is equal to Φ. At the moment $t = 0$ the magnetic field was switched off. Assuming the switching off time to be negligible compared to the natural oscillation period of the circuit, find the circuit current as a function of time t.

4.101. The free damped oscillations are maintained in a circuit, such that the voltage across the capacitor varies as $V = V_m e^{-\beta t} \cos \omega t$. Find the moments of time when the modulus of the voltage across the capacitor reaches

(a) peak values;

(b) maximum (extremum) values.

4.102. A certain oscillating circuit consists of a capacitor with capacitance C, a coil with inductance L and active resistance R, and a switch. When the switch was disconnected, the capacitor was charged; then the switch was closed and oscillations set in. Find the ratio of the voltage across the capacitor to its peak value at the moment immediately after closing the switch.

4.103. A circuit with capacitance C and inductance L generates free damped oscillations with current varying with time as $I = I_m e^{-\beta t} \sin \omega t$. Find the voltage across the capacitor as a function of time, and in particular, at the moment $t = 0$.

4.104. An oscillating circuit consists of a capacitor with capacitance $C = 4.0\ \mu\text{F}$ and a coil with inductance $L = 2.0$ mH and active resistance $R = 10\ \Omega$. Find the ratio of the energy of the coil's magnetic field to that of the capacitor's electric field at the moment when the current has the maximum value.

4.105. An oscillating circuit consists of two coils connected in series whose inductances are L_1 and L_2, active resistances are R_1 and R_2, and mutual inductance is negligible. These coils are to be replaced by one, keeping the frequency and the quality factor of the circuit constant. Find the inductance and the active resistance of such a coil.

4.106. How soon does the current amplitude in an oscillating circuit with quality factor $Q = 5000$ decrease $\eta = 2.0$ times if the oscillation frequency is $\nu = 2.2$ MHz?

4.107. An oscillating circuit consists of capacitance $C = 10\ \mu\text{F}$, inductance $L = 25$ mH, and active resistance $R = 1.0\ \Omega$. How many oscillation periods does it take for the current amplitude to decrease e-fold?

4.108. How much (in per cent) does the free oscillation frequency ω of a circuit with quality factor $Q = 5.0$ differ from the natural oscillation frequency ω_0 of that circuit?

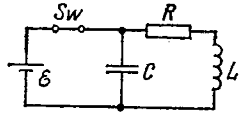

Fig. 4.29.

4.109. In a circuit shown in Fig. 4.29 the battery emf is equal to $\mathscr{E} = 2.0$ V, its internal resistance is $r = 9.0\ \Omega$, the capacitance of the capacitor is $C = 10\ \mu\text{F}$, the coil inductance is $L = 100$ mH, and the resistance is $R = 1.0\ \Omega$. At a certain moment the switch Sw was disconnected. Find the energy of oscillations in the circuit

(a) immediately after the switch was disconnected;

(b) $t = 0.30$ s after the switch was disconnected.

4.110. Damped oscillations are induced in a circuit whose quality factor is $Q = 50$ and natural oscillation frequency is $\nu_0 = 5.5$ kHz. How soon will the energy stored in the circuit decrease $\eta = 2.0$ times?

4.111. An oscillating circuit incorporates a leaking capacitor. Its capacitance is equal to C and active resistance to R. The coil inductance is L. The resistance of the coil and the wires is negligible. Find:

(a) the damped oscillation frequency of such a circuit;

(b) its quality factor.

4.112. Find the quality factor of a circuit with capacitance $C = 2.0\ \mu F$ and inductance $L = 5.0$ mH if the maintenance of undamped oscillations in the circuit with the voltage amplitude across the capacitor being equal to $V_m = 1.0$ V requires a power $\langle P \rangle = 0.10$ mW. The damping of oscillations is sufficiently low.

4.113. What mean power should be fed to an oscillating circuit with active resistance $R = 0.45\ \Omega$ to maintain undamped harmonic oscillations with current amplitude $I_m = 30$ mA?

4.114. An oscillating circuit consists of a capacitor with capacitance $C = 1.2$ nF and a coil with inductance $L = 6.0\ \mu H$ and active resistance $R = 0.50\ \Omega$. What mean power should be fed to the circuit to maintain undamped harmonic oscillations with voltage amplitude across the capacitor being equal to $V_m = 10$ V?

4.115. Find the damped oscillation frequency of the circuit shown in Fig. 4.30. The capacitance C, inductance L, and active resistance R are supposed to be known. Find how must C, L, and R be interrelated to make oscillations possible.

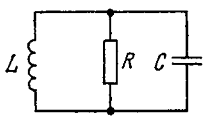

Fig. 4.30.

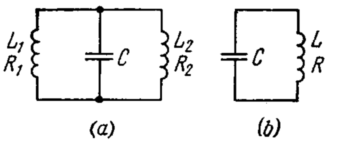

Fig. 4.31.

4.116. There are two oscillating circuits (Fig. 4.31) with capacitors of equal capacitances. How must inductances and active resistances of the coils be interrelated for the frequencies and damping of free oscillations in both circuits to be equal? The mutual inductance of coils in the left circuit is negligible.

4.117. A circuit consists of a capacitor with capacitance C and a coil of inductance L connected in series, as well as a switch and a resistance equal to the critical value for this circuit. With the switch disconnected, the capacitor was charged to a voltage V_0, and at the moment $t = 0$ the switch was closed. Find the current I in the circuit as a function of time t. What is I_{max} equal to?

4.118. A coil with active resistance R and inductance L was connected at the moment $t = 0$ to a source of voltage $V = V_m \cos \omega t$. Find the current in the coil as a function of time t.

4.119. A circuit consisting of a capacitor with capacitance C and a resistance R connected in series was connected at the moment $t = 0$ to a source of ac voltage $V = V_m \cos \omega t$. Find the current in the circuit as a function of time t.

4.120. A long one-layer solenoid tightly wound of wire with resistivity ρ has n turns per unit length. The thickness of the wire insulation is negligible. The cross-sectional radius of the solenoid is equal to a. Find the phase difference between current and alternating voltage fed to the solenoid with frequency ν.

4.121. A circuit consisting of a capacitor and an active resistance $R = 110\ \Omega$ connected in series is fed an alternating voltage with amplitude $V_m = 110$ V. In this case the amplitude of steady-state current is equal to $I_m = 0.50$ A. Find the phase difference between the current and the voltage fed.

4.122. Fig. 4.32 illustrates the simplest ripple filter. A voltage $V = V_0 (1 + \cos \omega t)$ is fed to the left input. Find:

(a) the output voltage $V'(t)$;

(b) the magnitude of the product RC at which the output amplitude of alternating voltage component is $\eta = 7.0$ times less than the direct voltage component, if $\omega = 314\ \text{s}^{-1}$.

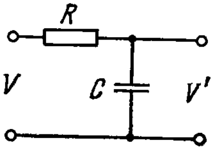

Fig. 4.32.

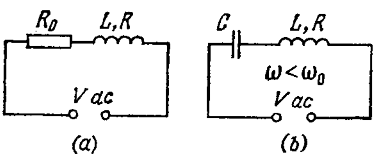

Fig. 4.33.

4.123. Draw the approximate voltage vector diagrams in the electric circuits shown in Fig. 4.33 *a*, *b*. The external voltage V is assumed to be alternating harmonically with frequency ω.

4.124. A series circuit consisting of a capacitor with capacitance $C = 22\ \mu$F and a coil with active resistance $R = 20\ \Omega$ and inductance $L = 0.35$ H is connected to a source of alternating voltage with amplitude $V_m = 180$ V and frequency $\omega = 314\ \text{s}^{-1}$. Find:

(a) the current amplitude in the circuit;

(b) the phase difference between the current and the external voltage;

(c) the amplitudes of voltage across the capacitor and the coil.

4.125. A series circuit consisting of a capacitor with capacitance C, a resistance R, and a coil with inductance L and negligible active resistance is connected to an oscillator whose frequency can be varied without changing the voltage amplitude. Find the frequency at which the voltage amplitude is maximum

(a) across the capacitor;

(b) across the coil.

4.126. An alternating voltage with frequency $\omega = 314\ \text{s}^{-1}$ and amplitude $V_m = 180$ V is fed to a series circuit consisting of a capacitor and a coil with active resistance $R = 40\ \Omega$ and inductance $L = 0.36$ H. At what value of the capacitor's capacitance will the voltage amplitude across the coil be maximum? What is this amplitude equal to? What is the corresponding voltage amplitude across the condenser?

4.127. A capacitor with capacitance C whose interelectrode space is filled up with poorly conducting medium with active resistance R is connected to a source of alternating voltage $V = V_m \cos \omega t$. Find the time dependence of the steady-state current flowing in lead wires. The resistance of the wires is to be neglected.

4.128. An oscillating circuit consists of a capacitor of capacitance C and a solenoid with inductance L_1. The solenoid is inductively connected with a short-circuited coil having an inductance L_2 and a negligible active resistance. Their mutual inductance coefficient is equal to L_{12}. Find the natural frequency of the given oscillating circuit.

4.129. Find the quality factor of an oscillating circuit connected in series to a source of alternating emf if at resonance the voltage across the capacitor is n times that of the source.

4.130. An oscillating circuit consisting of a coil and a capacitor connected in series is fed an alternating emf, with coil inductance being chosen to provide the maximum current in the circuit. Find the quality factor of the system, provided an n-fold increase of inductance results in an η-fold decrease of the current in the circuit.

4.131. A series circuit consisting of a capacitor and a coil with active resistance is connected to a source of harmonic voltage whose frequency can be varied, keeping the voltage amplitude constant. At frequencies ω_1 and ω_2 the current amplitudes are n times less than the resonance amplitude. Find:

(a) the resonance frequency;

(b) the quality factor of the circuit.

4.132. Demonstrate that at low damping the quality factor Q of a circuit maintaining forced oscillations is approximately equal to $\omega_0/\Delta\omega$, where ω_0 is the natural oscillation frequency, $\Delta\omega$ is the width of the resonance curve $I(\omega)$ at the "height" which is $\sqrt{2}$ times less than the resonance current amplitude.

4.133. A circuit consisting of a capacitor and a coil connected in series is fed two alternating voltages of equal amplitudes but different frequencies. The frequency of one voltage is equal to the natural oscillation frequency (ω_0) of the circuit, the frequency of the other voltage is η times higher. Find the ratio of the current amplitudes

(I_0/I) generated by the two voltages if the quality factor of the system is equal to Q. Calculate this ratio for $Q = 10$ and 100, if $\eta = 1.10$.

4.134. It takes t_0 hours for a direct current I_0 to charge a storage battery. How long will it take to charge such a battery from the mains using a half-wave rectifier, if the effective current value is also equal to I_0?

4.135. Find the effective value of current if its mean value is I_0 and its time dependence is

(a) shown in Fig. 4.34;

(b) $I \sim |\sin \omega t|$.

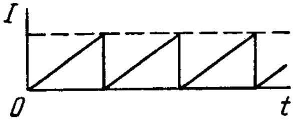

Fig. 4.34.

4.136. A solenoid with inductance $L = 7$ mH and active resistance $R = 44\ \Omega$ is first connected to a source of direct voltage V_0 and then to a source of sinusoidal voltage with effective value $V = V_0$. At what frequency of the oscillator will the power consumed by the solenoid be $\eta = 5.0$ times less than in the former case?

4.137. A coil with inductive resistance $X_L = 30\ \Omega$ and impedance $Z = 50\ \Omega$ is connected to the mains with effective voltage value $V = 100$ V. Find the phase difference between the current and the voltage, as well as the heat power generated in the coil.

4.138. A coil with inductance $L = 0.70$ H and active resistance $r = 20\ \Omega$ is connected in series with an inductance-free resistance R. An alternating voltage with effective value $V = 220$ V and frequency $\omega = 314\ \text{s}^{-1}$ is applied across the terminals of this circuit. At what value of the resistance R will the maximum heat power be generated in the circuit? What is it equal to?

4.139. A circuit consisting of a capacitor and a coil in series is connected to the mains. Varying the capacitance of the capacitor, the heat power generated in the coil was increased $n = 1.7$ times. How much (in per cent) was the value of $\cos \varphi$ changed in the process?

4.140. A source of sinusoidal emf with constant voltage is connected in series with an oscillating circuit with quality factor $Q = 100$. At a certain frequency of the external voltage the heat power generated in the circuit reaches the maximum value. How much (in per cent) should this frequency be shifted to decrease the power generated $n = 2.0$ times?

4.141. A series circuit consisting of an inductance-free resistance $R = 0.16$ kΩ and a coil with active resistance is connected to the mains with effective voltage $V = 220$ V. Find the heat power generated in the coil if the effective voltage values across the resistance R and the coil are equal to $V_1 = 80$ V and $V_2 = 180$ V respectively.

4.142. A coil and an inductance-free resistance $R = 25\ \Omega$ are connected in parallel to the ac mains. Find the heat power generated in the coil provided a current $I = 0.90$ A is drawn from the mains. The coil and the resistance R carry currents $I_1 = 0.50$ A and $I_2 = 0.60$ A respectively.

4.143. An alternating current of frequency $\omega = 314\ \text{s}^{-1}$ is fed to a circuit consisting of a capacitor of capacitance $C = 73\ \mu\text{F}$ and an active resistance $R = 100\ \Omega$ connected in parallel. Find the impedance of the circuit.

4.144. Draw the approximate vector diagrams of currents in the circuits shown in Fig. 4.35. The voltage applied across the points A and B is assumed to be sinusoidal; the parameters of each circuit are so chosen that the total current I_0 lags in phase behind the external voltage by an angle φ.

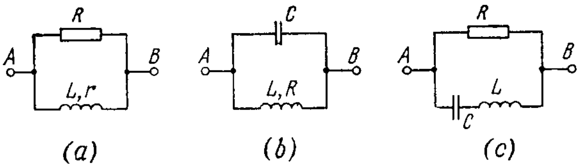

Fig. 4.35.

4.145. A capacitor with capacitance $C = 1.0\ \mu\text{F}$ and a coil with active resistance $R = 0.10\ \Omega$ and inductance $L = 1.0$ mH are connected in parallel to a source of sinusoidal voltage $V = 31$ V. Find:

(a) the frequency ω at which the resonance sets in;

(b) the effective value of the fed current in resonance, as well as the corresponding currents flowing through the coil and through the capacitor.

4.146. A capacitor with capacitance C and a coil with active resistance R and inductance L are connected in parallel to a source of sinusoidal voltage of frequency ω. Find the phase difference between the current fed to the circuit and the source voltage.

4.147. A circuit consists of a capacitor with capacitance C and a coil with active resistance R and inductance L connected in parallel. Find the impedance of the circuit at frequency ω of alternating voltage.

4.148. A ring of thin wire with active resistance R and inductance L rotates with constant angular velocity ω in the external uniform magnetic field perpendicular to the rotation axis. In the process, the flux of magnetic induction of external field across the ring varies with time as $\Phi = \Phi_0 \cos \omega t$. Demonstrate that

(a) the inductive current in the ring varies with time as $I = I_m \sin(\omega t - \varphi)$, where $I_m = \omega\Phi_0/\sqrt{R^2 + \omega^2 L^2}$ with $\tan\varphi = \omega L/R$;

(b) the mean mechanical power developed by external forces to maintain rotation is defined by the formula $P = {}^1/_2\,\omega^2\Phi_0^2 R/(R^2 + \omega^2 L^2)$.

4.149. A wooden core (Fig. 4.36) supports two coils: coil *1* with inductance L_1 and short-circuited coil *2* with active resistance R and inductance L_2. The mutual inductance of the coils depends on

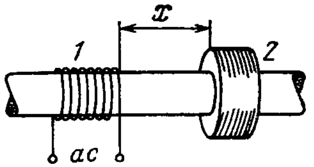

Fig. 4.36.

the distance x between them according to the law $L_{12}(x)$. Find the mean (averaged over time) value of the interaction force between the coils when coil *1* carries an alternating current $I_1 = I_0 \cos \omega t$.

4.3. ELASTIC WAVES. ACOUSTICS

• Equations of plane and spherical waves:

$$\xi = a\cos(\omega t - kx), \quad \xi = \frac{a_0}{r}\cos(\omega t - kr). \tag{4.3a}$$

In the case of a homogeneous absorbing medium the factors $e^{-\gamma x}$ and $e^{-\gamma r}$ respectively appear in the formulas, where γ is the wave damping coefficient.

• Wave equation:

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \xi}{\partial t^2}. \tag{4.3b}$$

• Phase velocity of longitudinal waves in an elastic medium ($v_{\parallel}$) and transverse waves in a string ($v_{\perp}$):

$$v_{\parallel} = \sqrt{E/\rho}, \quad v_{\perp} = \sqrt{T/\rho_1}, \tag{4.3c}$$

where E is Young's modulus, ρ is the density of the medium, T is the tension of the string, ρ_1 is its linear density.

• Volume density of energy of an elastic wave:

$$w = \rho a^2 \omega^2 \sin^2(\omega t - kx), \quad \langle w \rangle = {}^1/_2 \rho a^2 \omega^2. \tag{4.3d}$$

• Energy flow density, or the Umov vector for a travelling wave:

$$\mathbf{j} = w\mathbf{v}, \quad \langle \mathbf{j} \rangle = {}^1/_2 \rho a^2 \omega^2 \mathbf{v}. \tag{4.3e}$$

• Standing wave equation:

$$\xi = a\cos kx \cdot \cos \omega t. \tag{4.3f}$$

• Acoustical Doppler effect:

$$\nu = \nu_0 \frac{v + v_{ob}}{v - v_s}. \tag{4.3g}$$

• Loudness level (in bels):

$$L = \log(I/I_0). \tag{4.3h}$$

• Relation between the intensity I of a sound wave and the pressure oscillation amplitude $(\Delta p)_m$:

$$I = (\Delta p)_m^2 / 2\rho v. \tag{4.3i}$$

4.150. How long will it take sound waves to travel the distance l between the points A and B if the air temperature between them varies linearly from T_1 to T_2? The velocity of sound propagation in air is equal to $v = \alpha\sqrt{T}$, where α is a constant.

4.151. A plane harmonic wave with frequency ω propagates at a velocity v in a direction forming angles α, β, γ with the x, y, z axes. Find the phase difference between the oscillations at the points of medium with coordinates x_1, y_1, z_1 and x_2, y_2, z_2.

4.152. A plane wave of frequency ω propagates so that a certain phase of oscillation moves along the x, y, z axes with velocities v_1, v_2, v_3 respectively. Find the wave vector $\mathbf{k}$, assuming the unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ of the coordinate axes to be assigned.

4.153. A plane elastic wave $\xi = a \cos(\omega t - kx)$ propagates in a medium K. Find the equation of this wave in a reference frame K' moving in the positive direction of the x axis with a constant velocity V relative to the medium K. Investigate the expression obtained.

4.154. Demonstrate that any differentiable function $f(t + \alpha x)$, where α is a constant, provides a solution of wave equation. What is the physical meaning of the constant α?

4.155. The equation of a travelling plane sound wave has the form $\xi = 60 \cos(1800t - 5.3x)$, where ξ is expressed in micrometres, t in seconds, and x in metres. Find:

(a) the ratio of the displacement amplitude, with which the particles of medium oscillate, to the wavelength;

(b) the velocity oscillation amplitude of particles of the medium and its ratio to the wave propagation velocity;

(c) the oscillation amplitude of relative deformation of the medium and its relation to the velocity oscillation amplitude of particles of the medium.

4.156. A plane wave $\xi = a \cos(\omega t - kx)$ propagates in a homogeneous elastic medium. For the moment $t = 0$ draw

(a) the plots of ξ, $\partial\xi/\partial t$, and $\partial\xi/\partial x$ vs x;

(b) the velocity direction of the particles of the medium at the points where $\xi = 0$, for the cases of longitudinal and transverse waves;

(c) the approximate plot of density distribution $\rho(x)$ of the medium for the case of longitudinal waves.

4.157. A plane elastic wave $\xi = ae^{-\gamma x} \cos(\omega t - kx)$, where a, γ, ω, and k are constants, propagates in a homogeneous medium. Find the phase difference between the oscillations at the points where the particles' displacement amplitudes differ by $\eta = 1.0\%$, if $\gamma = 0.42\ \text{m}^{-1}$ and the wavelength is $\lambda = 50$ cm.

4.158. Find the radius vector defining the position of a point source of spherical waves if that source is known to be located on the straight line between the points with radius vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ at which the oscillation amplitudes of particles of the medium are equal to a_1 and a_2. The damping of the wave is negligible, the medium is homogeneous.

4.159. A point isotropic source generates sound oscillations with frequency $\nu = 1.45$ kHz. At a distance $r_0 = 5.0$ m from the source the displacement amplitude of particles of the medium is equal to $a_0 = 50$ μm, and at the point A located at a distance $r = 10.0$ m from the source the displacement amplitude is $\eta = 3.0$ times less than a_0. Find:

(a) the damping coefficient γ of the wave;

(b) the velocity oscillation amplitude of particles of the medium at the point A.

4.160. Two plane waves propagate in a homogeneous elastic medium, one along the x axis and the other along the y axis: $\xi_1 = a \cos (\omega t - kx)$, $\xi_2 = a \cos (\omega t - ky)$. Find the wave motion pattern of particles in the plane xy if both waves

(a) are transverse and their oscillation directions coincide;

(b) are longitudinal.

4.161. A plane undamped harmonic wave propagates in a medium. Find the mean space density of the total oscillation energy $\langle w \rangle$, if at any point of the medium the space density of energy becomes equal to w_0 one-sixth of an oscillation period after passing the displacement maximum.

4.162. A point isotropic sound source is located on the perpendicular to the plane of a ring drawn through the centre O of the ring. The distance between the point O and the source is $l = 1.00$ m, the radius of the ring is $R = 0.50$ m. Find the mean energy flow across the area enclosed by the ring if at the point O the intensity of sound is equal to $I_0 = 30$ μW/m². The damping of the waves is negligible.

4.163. A point isotropic source with sonic power $P = 0.10$ W is located at the centre of a round hollow cylinder with radius $R = 1.0$ m and height $h = 2.0$ m. Assuming the sound to be completely absorbed by the walls of the cylinder, find the mean energy flow reaching the lateral surface of the cylinder.

4.164. The equation of a plane standing wave in a homogeneous elastic medium has the form $\xi = a \cos kx \cdot \cos \omega t$. Plot:

(a) ξ and $\partial\xi/\partial x$ as functions of x at the moments $t = 0$ and $t = T/2$, where T is the oscillation period;

(b) the distribution of density $\rho (x)$ of the medium at the moments $t = 0$ and $t = T/2$ in the case of longitudinal oscillations;

(c) the velocity distribution of particles of the medium at the moment $t = T/4$; indicate the directions of velocities at the antinodes, both for longitudinal and transverse oscillations.

4.165. A longitudinal standing wave $\xi = a \cos kx \cdot \cos \omega t$ is maintained in a homogeneous medium of density ρ. Find the expressions for the space density of

(a) potential energy $w_p (x, t)$;

(b) kinetic energy $w_k (x, t)$.

Plot the space density distribution of the total energy w in the space between the displacement nodes at the moments $t = 0$ and $t = T/4$, where T is the oscillation period.

4.166. A string 120 cm in length sustains a standing wave, with the points of the string at which the displacement amplitude is equal to 3.5 mm being separated by 15.0 cm. Find the maximum displacement amplitude. To which overtone do these oscillations correspond?

4.167. Find the ratio of the fundamental tone frequencies of two identical strings after one of them was stretched by $\eta_1 = 2.0\%$ and the other, by $\eta_2 = 4.0\%$. The tension is assumed to be proportional to the elongation.

4.168. Determine in what way and how many times will the fundamental tone frequency of a stretched wire change if its length is shortened by 35% and the tension increased by 70%.

4.169. To determine the sound propagation velocity in air by acoustic resonance technique one can use a pipe with a piston and a sonic membrane closing one of its ends. Find the velocity of sound if the distance between the adjacent positions of the piston at which resonance is observed at a frequency $\nu = 2000$ Hz is equal to $l = 8.5$ cm.

4.170. Find the number of possible natural oscillations of air column in a pipe whose frequencies lie below $\nu_0 = 1250$ Hz. The length of the pipe is $l = 85$ cm. The velocity of sound is $v = 340$ m/s. Consider the two cases:

(a) the pipe is closed from one end;

(b) the pipe is opened from both ends.

The open ends of the pipe are assumed to be the antinodes of displacement.

4.171. A copper rod of length $l = 50$ cm is clamped at its midpoint. Find the number of natural longitudinal oscillations of the rod in the frequency range from 20 to 50 kHz. What are those frequencies equal to?

4.172. A string of mass m is fixed at both ends. The fundamental tone oscillations are excited with circular frequency ω and maximum displacement amplitude a_{max}. Find:

(a) the maximum kinetic energy of the string;

(b) the mean kinetic energy of the string averaged over one oscillation period.

4.173. A standing wave $\xi = a \sin kx \cdot \cos \omega t$ is maintained in a homogeneous rod with cross-sectional area S and density ρ. Find the total mechanical energy confined between the sections corresponding to the adjacent displacement nodes.

4.174. A source of sonic oscillations with frequency $\nu_0 = 1000$ Hz moves at right angles to the wall with a velocity $u = 0.17$ m/s. Two stationary receivers R_1 and R_2 are located on a straight line, coinciding with the trajectory of the source, in the following succession: R_1-source-R_2-wall. Which receiver registers the beatings and what is the beat frequency? The velocity of sound is equal to $v = 340$ m/s.

4.175. A stationary observer receives sonic oscillations from two tuning forks one of which approaches, and the other recedes with the same velocity. As this takes place, the observer hears the beatings with frequency $\nu = 2.0$ Hz. Find the velocity of each tuning fork if their oscillation frequency is $\nu_0 = 680$ Hz and the velocity of sound in air is $v = 340$ m/s.

4.176. A receiver and a source of sonic oscillations of frequency $\nu_0 = 2000$ Hz are located on the x axis. The source swings harmonically along that axis with a circular frequency ω and an amplitude $a = 50$ cm. At what value of ω will the frequency bandwidth registered by the stationary receiver be equal to $\Delta\nu = 200$ Hz? The velocity of sound is equal to $v = 340$ m/s.

4.177. A source of sonic oscillations with frequency $\nu_0 = 1700$ Hz and a receiver are located at the same point. At the moment $t = 0$ the source starts receding from the receiver with constant acceleration $w = 10.0$ m/s^2. Assuming the velocity of sound to be equal to $v = 340$ m/s, find the oscillation frequency registered by the stationary receiver $t = 10.0$ s after the start of motion.

4.178. A source of sound with natural frequency $\nu_0 = 1.8$ kHz moves uniformly along a straight line separated from a stationary observer by a distance $l = 250$ m. The velocity of the source is equal to $\eta = 0.80$ fraction of the velocity of sound. Find:

(a) the frequency of sound received by the observer at the moment when the source gets closest to him;

(b) the distance between the source and the observer at the moment when the observer receives a frequency $\nu = \nu_0$.

4.179. A stationary source sends forth monochromatic sound. A wall approaches it with velocity $u = 33$ cm/s. The propagation velocity of sound in the medium is $v = 330$ m/s. In what way and how much, in per cent, does the wavelength of sound change on reflection from the wall?

4.180. A source of sonic oscillations with frequency $\nu_0 = 1700$ Hz and a receiver are located on the same normal to a wall. Both the source and the receiver are stationary, and the wall recedes from the source with velocity $u = 6.0$ cm/s. Find the beat frequency registered by the receiver. The velocity of sound is equal to $v = 340$ m/s.

4.181. Find the damping coefficient γ of a sound wave if at distances $r_1 = 10$ m and $r_2 = 20$ m from a point isotropic source of sound the sound wave intensity values differ by a factor $\eta = 4.5$.

4.182. A plane sound wave propagates along the x axis. The damping coefficient of the wave is $\gamma = 0.0230$ m^{-1}. At the point $x = 0$ the loudness level is $L = 60$ dB. Find:

(a) the loudness level at a point with coordinate $x = 50$ m;

(b) the coordinate x of the point at which the sound is not heard any more.

4.183. At a distance $r_0 = 20.0$ m from a point isotropic source of sound the loudness level $L_0 = 30.0$ dB. Neglecting the damping of the sound wave, find:

(a) the loudness level at a distance $r = 10.0$ m from the source;

(b) the distance from the source at which the sound is not heard.

4.184. An observer A located at a distance $r_A = 5.0$ m from a ringing tuning fork notes the sound to fade away $\tau = 19$ s later than an observer B who is located at a distance $r_B = 50$ m from the tuning fork. Find the damping coefficient β of oscillations of the tuning fork. The sound velocity $v = 340$ m/s.

4.185. A plane longitudinal harmonic wave propagates in a medium with density ρ. The velocity of the wave propagation is v. Assuming that the density variations of the medium, induced by the propagating wave, $\Delta\rho \ll \rho$, demonstrate that

(a) the pressure increment in the medium $\Delta p = -\rho v^2 (\partial\xi/\partial x)$, where $\partial\xi/\partial x$ is the relative deformation;

(b) the wave intensity is defined by Eq. (4.3i).

4.186. A ball of radius $R = 50$ cm is located in the way of propagation of a plane sound wave. The sonic wavelength is $\lambda = 20$ cm, the frequency is $\nu = 1700$ Hz, the pressure oscillation amplitude in air is $(\Delta p)_m = 3.5$ Pa. Find the mean energy flow, averaged over an oscillation period, reaching the surface of the ball.

4.187. A point A is located at a distance $r = 1.5$ m from a point isotropic source of sound of frequency $\nu = 600$ Hz. The sonic power of the source is $P = 0.80$ W. Neglecting the damping of the waves and assuming the velocity of sound in air to be equal to $v = 340$ m/s, find at the point A:

(a) the pressure oscillation amplitude $(\Delta p)_m$ and its ratio to the air pressure;

(b) the oscillation amplitude of particles of the medium; compare it with the wavelength of sound.

4.188. At a distance $r = 100$ m from a point isotropic source of sound of frequency 200 Hz the loudness level is equal to $L = 50$ dB. The audibility threshold at this frequency corresponds to the sound intensity $I_0 = 0.10$ nW/m^2. The damping coefficient of the sound wave is $\gamma = 5.0\cdot10^{-4}$ m^{-1}. Find the sonic power of the source.

4.4. ELECTROMAGNETIC WAVES. RADIATION

- Phase velocity of an electromagnetic wave:

$$v = c/\sqrt{\varepsilon\mu}, \quad \text{where } c = 1/\sqrt{\varepsilon_0\mu_0}. \tag{4.4a}$$

- In a travelling electromagnetic wave:

$$E\sqrt{\varepsilon\varepsilon_0} = H\sqrt{\mu\mu_0}. \tag{4.4b}$$

- Space density of the energy of an electromagnetic field:

$$w = \frac{\mathbf{ED}}{2} + \frac{\mathbf{BH}}{2}. \tag{4.4c}$$

- Flow density of electromagnetic energy, the Poynting vector:

$$\mathbf{S} = [\mathbf{EH}]. \tag{4.4d}$$

- Energy flow density of electric dipole radiation in a far field zone:

$$S \sim \frac{1}{r^2}\sin^2\theta, \tag{4.4e}$$

where r is the distance from the dipole, θ is the angle between the radius vector $\mathbf{r}$ and the axis of the dipole.

• Radiation power of an electric dipole with moment $\mathbf{p}(t)$ and of a charge q, moving with acceleration $\mathbf{w}$:

$$P = \frac{1}{4\pi\varepsilon_0} \frac{2\ddot{\mathbf{p}}^2}{3c^3}, \quad P = \frac{1}{4\pi\varepsilon_0} \frac{2q^2\mathbf{w}^2}{3c^3}. \tag{4.4f}$$

4.189. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a non-magnetic medium with permittivity $\varepsilon = 4.0$. Find the increment of its wavelength.

4.190. A plane electromagnetic wave falls at right angles to the surface of a plane-parallel plate of thickness l. The plate is made of non-magnetic substance whose permittivity decreases exponentially from a value ε_1 at the front surface down to a value ε_2 at the rear one. How long does it take a given wave phase to travel across this plate?

4.191. A plane electromagnetic wave of frequency $\nu = 10$ MHz propagates in a poorly conducting medium with conductivity $\sigma = 10$ mS/m and permittivity $\varepsilon = 9$. Find the ratio of amplitudes of conduction and displacement current densities.

4.192. A plane electromagnetic wave $\mathbf{E} = \mathbf{E}_m \cos(\omega t - \mathbf{kr})$ propagates in vacuum. Assuming the vectors $\mathbf{E}_m$ and $\mathbf{k}$ to be known, find the vector $\mathbf{H}$ as a function of time t at the point with radius vector $\mathbf{r} = 0$.

4.193. A plane electromagnetic wave $\mathbf{E} = \mathbf{E}_m \cos(\omega t - \mathbf{kr})$, where $\mathbf{E}_m = E_m\mathbf{e}_y$, $\mathbf{k} = k\mathbf{e}_x$, $\mathbf{e}_x$, $\mathbf{e}_y$ are the unit vectors of the x, y axes, propagates in vacuum. Find the vector $\mathbf{H}$ at the point with radius vector $\mathbf{r} = x\mathbf{e}_x$ at the moment (a) $t = 0$, (b) $t = t_0$. Consider the case when $E_m = 160$ V/m, $k = 0.51$ m^{-1}, $x = 7.7$ m, and $t_0 = 33$ ns.

4.194. A plane electromagnetic wave $\mathbf{E} = \mathbf{E}_m \cos(\omega t - kx)$ propagating in vacuum induces the emf $\mathscr{E}_{ind}$ in a square frame with side l. The orientation of the frame is shown in Fig. 4.37. Find the amplitude value $\mathscr{E}_{ind}$, if $E_m = 0.50$ mV/m, the frequency $\nu = 5.0$ MHz and $l = 50$ cm.

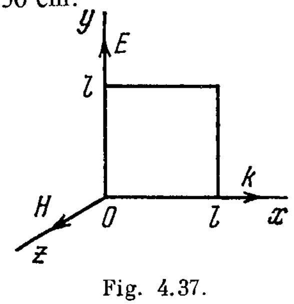

Fig. 4.37.

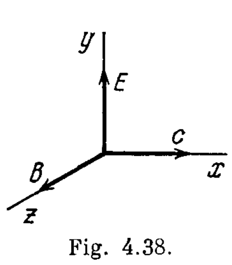

Fig. 4.38.

4.195. Proceeding from Maxwell's equations show that in the case of a plane electromagnetic wave (Fig. 4.38) propagating in vacuum the following relations hold:

$$\frac{\partial E}{\partial t} = -c^2 \frac{\partial B}{\partial x}, \quad \frac{\partial B}{\partial t} = -\frac{\partial E}{\partial x}.$$

4.196. Find the mean Poynting vector $\langle \mathbf{S} \rangle$ of a plane electromagnetic wave $\mathbf{E} = \mathbf{E}_m \cos(\omega t - \mathbf{kr})$ if the wave propagates in vacuum.

4.197. A plane harmonic electromagnetic wave with plane polarization propagates in vacuum. The electric component of the wave has a strength amplitude $E_m = 50$ mV/m, the frequency is $\nu = 100$ MHz. Find:

(a) the efficient value of the displacement current density;

(b) the mean energy flow density averaged over an oscillation period.

4.198. A ball of radius $R = 50$ cm is located in a non-magnetic medium with permittivity $\varepsilon = 4.0$. In that medium a plane electromagnetic wave propagates, the strength amplitude of whose electric component is equal to $E_m = 200$ V/m. What amount of energy reaches the ball during a time interval $t = 1.0$ min?

4.199. A standing electromagnetic wave with electric component $\mathbf{E} = \mathbf{E}_m \cos kx \cdot \cos \omega t$ is sustained along the x axis in vacuum. Find the magnetic component of the wave $\mathbf{B}(x, t)$. Draw the approximate distribution pattern of the wave's electric and magnetic components ($\mathbf{E}$ and $\mathbf{B}$) at the moments $t = 0$ and $t = T/4$, where T is the oscillation period.

4.200. A standing electromagnetic wave $\mathbf{E} = \mathbf{E}_m \cos kx \cdot \cos \omega t$ is sustained along the x axis in vacuum. Find the projection of the Poynting vector on the x axis $S_x(x, t)$ and the mean value of that projection averaged over an oscillation period.

4.201. A parallel-plate air capacitor whose electrodes are shaped as discs of radius $R = 6.0$ cm is connected to a source of an alternating sinusoidal voltage with frequency $\omega = 1000\ \text{s}^{-1}$. Find the ratio of peak values of magnetic and electric energies within the capacitor.

4.202. An alternating sinusoidal current of frequency $\omega = 1000\ \text{s}^{-1}$ flows in the winding of a straight solenoid whose cross-sectional radius is equal to $R = 6.0$ cm. Find the ratio of peak values of electric and magnetic energies within the solenoid.

4.203. A parallel-plate capacity whose electrodes are shaped as round discs is charged slowly. Demonstrate that the flux of the Poynting vector across the capacitor's lateral surface is equal to the increment of the capacitor's energy per unit time. The dissipation of field at the edge is to be neglected in calculations.

4.204. A current I flows along a straight conductor with round cross-section. Find the flux of the Poynting vector across the lateral surface of the conductor's segment with resistance R.

4.205. Non-relativistic protons accelerated by a potential difference U form a round beam with current I. Find the magnitude and direction of the Poynting vector outside the beam at a distance r from its axis.

4.206. A current flowing in the winding of a long straight solenoid is increased at a sufficiently slow rate. Demonstrate that the rate at which the energy of the magnetic field in the solenoid increases is equal to the flux of the Poynting vector across the lateral surface of the solenoid.

4.207. Fig. 4.39 illustrates a segment of a double line carrying direct current whose direction is indicated by the arrows. Taking into account that the potential $\varphi_2 > \varphi_1$, and making use of the Poynting vector, establish on which side (left or right) the source of the current is located.

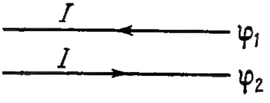

Fig. 4.39.

4.208. The energy is transferred from a source of constant voltage V to a consumer by means of a long straight coaxial cable with negligible active resistance. The consumed current is I. Find the energy flux across the cross-section of the cable. The conductive sheath is supposed to be thin.

4.209. A source of ac voltage $V = V_0 \cos \omega t$ delivers energy to a consumer by means of a long straight coaxial cable with negligible active resistance. The current in the circuit varies as $I = I_0 \cos (\omega t - \varphi)$. Find the time-averaged energy flux through the cross-section of the cable. The sheath is thin.

4.210. Demonstrate that at the boundary between two media the normal components of the Poynting vector are continuous, i.e. $S_{1n} = S_{2n}$.

4.211. Demonstrate that a closed system of charged non-relativistic particles with identical specific charges emits no dipole radiation.

4.212. Find the mean radiation power of an electron performing harmonic oscillations with amplitude $a = 0.10$ nm and frequency $\omega = 6.5 \cdot 10^{14}$ s^{-1}

4.213. Find the radiation power developed by a non-relativistic particle with charge e and mass m, moving along a circular orbit of radius R in the field of a stationary point charge q.

4.214. A particle with charge e and mass m flies with non-relativistic velocity v at a distance b past a stationary particle with charge q. Neglecting the bending of the trajectory of the moving particle, find the energy lost by this particle due to radiation during the total flight time.

4.215. A non-relativistic proton enters a half-space along the normal to the transverse uniform magnetic field whose induction equals $B = 1.0$ T. Find the ratio of the energy lost by the proton due to radiation during its motion in the field to its initial kinetic energy.

4.216. A non-relativistic charged particle moves in a transverse uniform magnetic field with induction B. Find the time dependence of the particle's kinetic energy diminishing due to radiation. How soon will its kinetic energy decrease e-fold? Calculate this time interval for the case (a) of an electron, (b) of a proton.

4.217. A charged particle moves along the y axis according to the law $y = a \cos \omega t$, and the point of observation P is located on the x axis at a distance l from the particle ($l \gg a$). Find the ratio of electromagnetic radiation flow densities S_1/S_2 at the point P at the moments when the coordinate of the particle $y_1 = 0$ and $y_2 = a$. Calculate that ratio if $\omega = 3.3 \cdot 10^6$ s^{-1} and $l = 190$ m.

4.218. A charged particle moves uniformly with velocity v along a circle of radius R in the plane xy (Fig. 4.40). An observer is located

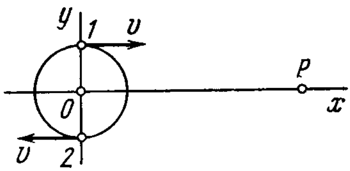

Fig. 4.40.

on the x axis at a point P which is removed from the centre of the circle by a distance much exceeding R. Find:

(a) the relationship between the observed values of the y projection of the particle's acceleration and the y coordinate of the particle;

(b) the ratio of electromagnetic radiation flow densities S_1/S_2 at the point P at the moments of time when the particle moves, from the standpoint of the observer P, toward him and away from him, as shown in the figure.

4.219. An electromagnetic wave emitted by an elementary dipole propagates in vacuum so that in the far field zone the mean value of the energy flow density is equal to S_0 at the point removed from the dipole by a distance r along the perpendicular drawn to the dipole's axis. Find the mean radiation power of the dipole.

4.220. The mean power radiated by an elementary dipole is equal to P_0. Find the mean space density of energy of the electromagnetic field in vacuum in the far field zone at the point removed from the dipole by a distance r along the perpendicular drawn to the dipole's axis.

4.221. An electric dipole whose modulus is constant and whose moment is equal to p rotates with constant angular velocity ω about the axis drawn at right angles to the axis of the dipole and passing through its midpoint. Find the power radiated by such a dipole.

4.222. A free electron is located in the field of a plane electromagnetic wave. Neglecting the magnetic component of the wave disturbing its motion, find the ratio of the mean energy radiated by the oscillating electron per unit time to the mean value of the energy flow density of the incident wave.

4.223. A plane electromagnetic wave with frequency ω falls upon an elastically bonded electron whose natural frequency equals ω_0. Neglecting the damping of oscillations, find the ratio of the mean energy dissipated by the electron per unit time to the mean value of the energy flow density of the incident wave.

4.224. Assuming a particle to have the form of a ball and to absorb all incident light, find the radius of a particle for which its gravitational attraction to the Sun is counterbalanced by the force that light exerts on it. The power of light radiated by the Sun equals $P = 4 \cdot 10^{26}$ W, and the density of the particle is $\rho = 1.0$ g/cm^3.