

PART TWO

THERMODYNAMICS AND MOLECULAR PHYSICS

2.1. EQUATION OF THE GAS STATE. PROCESSES

- Ideal gas law:

$$pV = \frac{m}{M} RT, \quad (2.1a)$$

where M is the molar mass.

- Barometric formula:

$$p = p_0 e^{-Mgh/RT}, \quad (2.1b)$$

where p_0 is the pressure at the height $h = 0$.

- Van der Waals equation of gas state (for a mole):

$$\left(p + \frac{a}{V_M^2}\right)(V_M - b) = RT, \quad (2.1c)$$

where V_M is the molar volume under given p and T .

2.1. A vessel of volume $V = 30$ l contains ideal gas at the temperature 0°C . After a portion of the gas has been let out, the pressure in the vessel decreased by $\Delta p = 0.78$ atm (the temperature remaining constant). Find the mass of the released gas. The gas density under the normal conditions $\rho = 1.3$ g/l.

2.2. Two identical vessels are connected by a tube with a valve letting the gas pass from one vessel into the other if the pressure difference $\Delta p \geq 1.10$ atm. Initially there was a vacuum in one vessel while the other contained ideal gas at a temperature $t_1 = 27^\circ\text{C}$ and pressure $p_1 = 1.00$ atm. Then both vessels were heated to a temperature $t_2 = 107^\circ\text{C}$. Up to what value will the pressure in the first vessel (which had vacuum initially) increase?

2.3. A vessel of volume $V = 20$ l contains a mixture of hydrogen and helium at a temperature $t = 20^\circ\text{C}$ and pressure $p = 2.0$ atm. The mass of the mixture is equal to $m = 5.0$ g. Find the ratio of the mass of hydrogen to that of helium in the given mixture.

2.4. A vessel contains a mixture of nitrogen ($m_1 = 7.0$ g) and carbon dioxide ($m_2 = 11$ g) at a temperature $T = 290$ K and pressure $p_0 = 1.0$ atm. Find the density of this mixture, assuming the gases to be ideal.

2.5. A vessel of volume $V = 7.5$ l contains a mixture of ideal gases at a temperature $T = 300$ K: $\nu_1 = 0.10$ mole of oxygen, $\nu_2 = 0.20$ mole of nitrogen, and $\nu_3 = 0.30$ mole of carbon dioxide. Assuming the gases to be ideal, find:

- (a) the pressure of the mixture;

(b) the mean molar mass M of the given mixture which enters its equation of state $pV = (m/M)RT$, where m is the mass of the mixture.

2.6. A vertical cylinder closed from both ends is equipped with an easily moving piston dividing the volume into two parts, each containing one mole of air. In equilibrium at $T_0 = 300$ K the volume of the upper part is $\eta = 4.0$ times greater than that of the lower part. At what temperature will the ratio of these volumes be equal to $\eta' = 3.0$?

2.7. A vessel of volume V is evacuated by means of a piston air pump. One piston stroke captures the volume ΔV . How many strokes are needed to reduce the pressure in the vessel η times? The process is assumed to be isothermal, and the gas ideal.

2.8. Find the pressure of air in a vessel being evacuated as a function of evacuation time t . The vessel volume is V , the initial pressure is p_0 . The process is assumed to be isothermal, and the evacuation rate equal to C and independent of pressure.

Note. The evacuation rate is the gas volume being evacuated per unit time, with that volume being measured under the gas pressure attained by that moment.

2.9. A chamber of volume $V = 87$ l is evacuated by a pump whose evacuation rate (see Note to the foregoing problem) equals $C = 10$ l/s. How soon will the pressure in the chamber decrease by $\eta = 1000$ times?

2.10. A smooth vertical tube having two different sections is open from both ends and equipped with two pistons of different areas (Fig. 2.1). Each piston slides within a respective tube section. One mole of ideal gas is enclosed between the pistons tied with a non-stretchable thread. The cross-sectional area of the upper piston is $\Delta S = 10$ cm² greater than that of the lower one. The combined mass of the two pistons is equal to $m = 5.0$ kg. The outside air pressure is $p_0 = 1.0$ atm. By how many kelvins must the gas between the pistons be heated to shift the pistons through $l = 5.0$ cm?

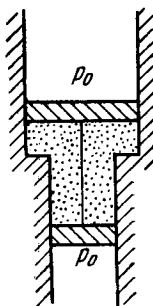


Fig. 2.1.

2.11. Find the maximum attainable temperature of ideal gas in each of the following processes:

(a) $p = p_0 - \alpha V^2$; (b) $p = p_0 e^{-\beta V}$, where p_0 , α and β are positive constants, and V is the volume of one mole of gas.

2.12. Find the minimum attainable pressure of ideal gas in the process $T = T_0 + \alpha V^2$, where T_0 and α are positive constants, and V is the volume of one mole of gas. Draw the approximate p vs V plot of this process.

2.13. A tall cylindrical vessel with gaseous nitrogen is located in a uniform gravitational field in which the free-fall acceleration is equal to g . The temperature of the nitrogen varies along the height

h so that its density is the same throughout the volume. Find the temperature gradient dT/dh .

2.14. Suppose the pressure p and the density ρ of air are related as $p/\rho^n = \text{const}$ regardless of height (n is a constant here). Find the corresponding temperature gradient.

2.15. Let us assume that air is under standard conditions close to the Earth's surface. Presuming that the temperature and the molar mass of air are independent of height, find the air pressure at the height 5.0 km over the surface and in a mine at the depth 5.0 km below the surface.

2.16. Assuming the temperature and the molar mass of air, as well as the free-fall acceleration, to be independent of the height, find the difference in heights at which the air densities at the temperature 0°C differ

(a) e times; (b) by $\eta = 1.0\%$.

2.17. An ideal gas of molar mass M is contained in a tall vertical cylindrical vessel whose base area is S and height h . The temperature of the gas is T , its pressure on the bottom base is p_0 . Assuming the temperature and the free-fall acceleration g to be independent of the height, find the mass of gas in the vessel.

2.18. An ideal gas of molar mass M is contained in a very tall vertical cylindrical vessel in the uniform gravitational field in which the free-fall acceleration equals g . Assuming the gas temperature to be the same and equal to T , find the height at which the centre of gravity of the gas is located.

2.19. An ideal gas of molar mass M is located in the uniform gravitational field in which the free-fall acceleration is equal to g . Find the gas pressure as a function of height h , if $p = p_0$ at $h = 0$, and the temperature varies with height as

(a) $T = T_0(1 - ah)$; (b) $T = T_0(1 + ah)$,
where a is a positive constant.

2.20. A horizontal cylinder closed from one end is rotated with a constant angular velocity ω about a vertical axis passing through the open end of the cylinder. The outside air pressure is equal to p_0 , the temperature to T , and the molar mass of air to M . Find the air pressure as a function of the distance r from the rotation axis. The molar mass is assumed to be independent of r .

2.21. Under what pressure will carbon dioxide have the density $\rho = 500 \text{ g/l}$ at the temperature $T = 300 \text{ K}$? Carry out the calculations both for an ideal and for a Van der Waals gas.

2.22. One mole of nitrogen is contained in a vessel of volume $V = 1.00 \text{ l}$. Find:

(a) the temperature of the nitrogen at which the pressure can be calculated from an ideal gas law with an error $\eta = 10\%$ (as compared with the pressure calculated from the Van der Waals equation of state);

(b) the gas pressure at this temperature.

2.23. One mole of a certain gas is contained in a vessel of volume $V = 0.250 \text{ l}$. At a temperature $T_1 = 300 \text{ K}$ the gas pressure is $p_1 =$

$= 90$ atm, and at a temperature $T_2 = 350$ K the pressure is $p_2 = 110$ atm. Find the Van der Waals parameters for this gas.

2.24. Find the isothermal compressibility κ of a Van der Waals gas as a function of volume V at temperature T .

Note. By definition, $\kappa = -\frac{1}{V} \frac{\partial V}{\partial p}$.

2.25. Making use of the result obtained in the foregoing problem, find at what temperature the isothermal compressibility κ of a Van der Waals gas is greater than that of an ideal gas. Examine the case when the molar volume is much greater than the parameter b .

2.2. THE FIRST LAW OF THERMODYNAMICS. HEAT CAPACITY

- The first law of thermodynamics:

$$Q = \Delta U + A, \quad (2.2a)$$

where ΔU is the increment of the internal energy of the system.

- Work performed by gas:

$$A = \int p \, dV. \quad (2.2b)$$

- Internal energy of an ideal gas:

$$U = \frac{m}{M} C_V T = \frac{m}{M} \frac{RT}{\gamma - 1} = \frac{pV}{\gamma - 1}. \quad (2.2c)$$

- Molar heat capacity in a polytropic process ($pV^n = \text{const}$):

$$C = \frac{R}{\gamma - 1} - \frac{R}{n - 1} = \frac{(n - \gamma) R}{(n - 1)(\gamma - 1)}. \quad (2.2d)$$

- Internal energy of one mole of a Van der Waals gas:

$$U = C_V T - \frac{a}{V_M}. \quad (2.2e)$$

2.26. Demonstrate that the interval energy U of the air in a room is independent of temperature provided the outside pressure p is constant. Calculate U , if p is equal to the normal atmospheric pressure and the room's volume is equal to $V = 40$ m³.

2.27. A thermally insulated vessel containing a gas whose molar mass is equal to M and the ratio of specific heats $C_p/C_V = \gamma$ moves with a velocity v . Find the gas temperature increment resulting from the sudden stoppage of the vessel.

2.28. Two thermally insulated vessels 1 and 2 are filled with air and connected by a short tube equipped with a valve. The volumes of the vessels, the pressures and temperatures of air in them are known (V_1 , p_1 , T_1 and V_2 , p_2 , T_2). Find the air temperature and pressure established after the opening of the valve.

2.29. Gaseous hydrogen contained initially under standard conditions in a sealed vessel of volume $V = 5.0$ l was cooled by $\Delta T =$

= 55 K. Find how much the internal energy of the gas will change and what amount of heat will be lost by the gas.

2.30. What amount of heat is to be transferred to nitrogen in the isobaric heating process for that gas to perform the work $A = 2.0$ J?

2.31. As a result of the isobaric heating by $\Delta T = 72$ K one mole of a certain ideal gas obtains an amount of heat $Q = 1.60$ kJ. Find the work performed by the gas, the increment of its internal energy, and the value of $\gamma = C_p/C_V$.

2.32. Two moles of a certain ideal gas at a temperature $T_0 = 300$ K were cooled isochorically so that the gas pressure reduced $n = 2.0$ times. Then, as a result of the isobaric process, the gas expanded till its temperature got back to the initial value. Find the total amount of heat absorbed by the gas in this process.

2.33. Calculate the value of $\gamma = C_p/C_V$ for a gaseous mixture consisting of $\nu_1 = 2.0$ moles of oxygen and $\nu_2 = 3.0$ moles of carbon dioxide. The gases are assumed to be ideal.

2.34. Find the specific heat capacities c_V and c_p for a gaseous mixture consisting of 7.0 g of nitrogen and 20 g of argon. The gases are assumed to be ideal.

2.35. One mole of a certain ideal gas is contained under a weightless piston of a vertical cylinder at a temperature T . The space over the piston opens into the atmosphere. What work has to be performed in order to increase isothermally the gas volume under the piston n times by slowly raising the piston? The friction of the piston against the cylinder walls is negligibly small.

2.36. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0 , in which an ideal gas is contained under the same pressure p_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly moving the piston?

2.37. Three moles of an ideal gas being initially at a temperature $T_0 = 273$ K were isothermally expanded $n = 5.0$ times its initial volume and then isochorically heated so that the pressure in the final state became equal to that in the initial state. The total amount of heat transferred to the gas during the process equals $Q = 80$ kJ. Find the ratio $\gamma = C_p/C_V$ for this gas.

2.38. Draw the approximate plots of isochoric, isobaric, isothermal, and adiabatic processes for the case of an ideal gas, using the following variables:

(a) p , T ; (b) V , T .

2.39. One mole of oxygen being initially at a temperature $T_0 = 290$ K is adiabatically compressed to increase its pressure $\eta = 10.0$ times. Find:

(a) the gas temperature after the compression;

(b) the work that has been performed on the gas.

2.40. A certain mass of nitrogen was compressed $\eta = 5.0$ times

(in terms of volume), first adiabatically, and then isothermally. In both cases the initial state of the gas was the same. Find the ratio of the respective works expended in each compression.

2.41. A heat-conducting piston can freely move inside a closed thermally insulated cylinder with an ideal gas. In equilibrium the piston divides the cylinder into two equal parts, the gas temperature being equal to T_0 . The piston is slowly displaced. Find the gas temperature as a function of the ratio η of the volumes of the greater and smaller sections. The adiabatic exponent of the gas is equal to γ .

2.42. Find the rate v with which helium flows out of a thermally insulated vessel into vacuum through a small hole. The flow rate of the gas inside the vessel is assumed to be negligible under these conditions. The temperature of helium in the vessel is $T = 1,000$ K.

2.43. The volume of one mole of an ideal gas with the adiabatic exponent γ is varied according to the law $V = a/T$, where a is a constant. Find the amount of heat obtained by the gas in this process if the gas temperature increased by ΔT .

2.44. Demonstrate that the process in which the work performed by an ideal gas is proportional to the corresponding increment of its internal energy is described by the equation $pV^n = \text{const}$, where n is a constant.

2.45. Find the molar heat capacity of an ideal gas in a polytropic process $pV^n = \text{const}$ if the adiabatic exponent of the gas is equal to γ . At what values of the polytropic constant n will the heat capacity of the gas be negative?

2.46. In a certain polytropic process the volume of argon was increased $\alpha = 4.0$ times. Simultaneously, the pressure decreased $\beta = 8.0$ times. Find the molar heat capacity of argon in this process, assuming the gas to be ideal.

2.47. One mole of argon is expanded polytropically, the polytropic constant being $n = 1.50$. In the process, the gas temperature changes by $\Delta T = -26$ K. Find:

- (a) the amount of heat obtained by the gas;
- (b) the work performed by the gas.

2.48. An ideal gas whose adiabatic exponent equals γ is expanded according to the law $p = \alpha V$, where α is a constant. The initial volume of the gas is equal to V_0 . As a result of expansion the volume increases η times. Find:

- (a) the increment of the internal energy of the gas;
- (b) the work performed by the gas;
- (c) the molar heat capacity of the gas in the process.

2.49. An ideal gas whose adiabatic exponent equals γ is expanded so that the amount of heat transferred to the gas is equal to the decrease of its internal energy. Find:

- (a) the molar heat capacity of the gas in this process;
- (b) the equation of the process in the variables T, V ;
- (c) the work performed by one mole of the gas when its volume increases η times if the initial temperature of the gas is T_0 .

2.50. One mole of an ideal gas whose adiabatic exponent equals γ undergoes a process in which the gas pressure relates to the temperature as $p = aT^\alpha$, where a and α are constants. Find:

(a) the work performed by the gas if its temperature gets an increment ΔT ;

(b) the molar heat capacity of the gas in this process; at what value of α will the heat capacity be negative?

2.51. An ideal gas with the adiabatic exponent γ undergoes a process in which its internal energy relates to the volume as $U = aV^\alpha$, where a and α are constants. Find:

(a) the work performed by the gas and the amount of heat to be transferred to this gas to increase its internal energy by ΔU ;

(b) the molar heat capacity of the gas in this process.

2.52. An ideal gas has a molar heat capacity C_V at constant volume. Find the molar heat capacity of this gas as a function of its volume V , if the gas undergoes the following process:

(a) $T = T_0 e^{\alpha V}$; (b) $p = p_0 e^{\alpha V}$,
where T_0 , p_0 , and α are constants.

2.53. One mole of an ideal gas whose adiabatic exponent equals γ undergoes a process $p = p_0 + \alpha/V$, where p_0 and α are positive constants. Find:

(a) heat capacity of the gas as a function of its volume;

(b) the internal energy increment of the gas, the work performed by it, and the amount of heat transferred to the gas, if its volume increased from V_1 to V_2 .

2.54. One mole of an ideal gas with heat capacity at constant pressure C_p undergoes the process $T = T_0 + \alpha V$, where T_0 and α are constants. Find:

(a) heat capacity of the gas as a function of its volume;

(b) the amount of heat transferred to the gas, if its volume increased from V_1 to V_2 .

2.55. For the case of an ideal gas find the equation of the process (in the variables T , V) in which the molar heat capacity varies as:

(a) $C = C_V + \alpha T$; (b) $C = C_V + \beta V$; (c) $C = C_V + ap$,
where α , β , and a are constants.

2.56. An ideal gas has an adiabatic exponent γ . In some process its molar heat capacity varies as $C = \alpha/T$, where α is a constant. Find:

(a) the work performed by one mole of the gas during its heating from the temperature T_0 to the temperature η times higher;

(b) the equation of the process in the variables p , V .

2.57. Find the work performed by one mole of a Van der Waals gas during its isothermal expansion from the volume V_1 to V_2 at a temperature T .

2.58. One mole of oxygen is expanded from a volume $V_1 = 1.00$ l to $V_2 = 5.0$ l at a constant temperature $T = 280$ K. Calculate:

(a) the increment of the internal energy of the gas:

(b) the amount of the absorbed heat.
The gas is assumed to be a Van der Waals gas.

2.59. For a Van der Waals gas find:

- (a) the equation of the adiabatic curve in the variables T , V ;
- (b) the difference of the molar heat capacities $C_p - C_v$ as a function of T and V .

2.60. Two thermally insulated vessels are interconnected by a tube equipped with a valve. One vessel of volume $V_1 = 10$ l contains $\nu = 2.5$ moles of carbon dioxide. The other vessel of volume $V_2 = 100$ l is evacuated. The valve having been opened, the gas adiabatically expanded. Assuming the gas to obey the Van der Waals equation, find its temperature change accompanying the expansion.

2.61. What amount of heat has to be transferred to $\nu = 3.0$ moles of carbon dioxide to keep its temperature constant while it expands into vacuum from the volume $V_1 = 5.0$ l to $V_2 = 10$ l? The gas is assumed to be a Van der Waals gas.

2.3. KINETIC THEORY OF GASES.

BOLTZMANN'S LAW AND MAXWELL'S DISTRIBUTION

- Number of collisions exercised by gas molecules on a unit area of the wall surface per unit time:

$$\nu = \frac{1}{4} n \langle v \rangle, \quad (2.3a)$$

where n is the concentration of molecules, and $\langle v \rangle$ is their mean velocity.

- Equation of an ideal gas state:

$$p = nkT. \quad (2.3b)$$

- Mean energy of molecules:

$$\langle e \rangle = \frac{i}{2} kT, \quad (2.3c)$$

where i is the sum of translational, rotational, and the double number of vibrational degrees of freedom.

- Maxwellian distribution:

$$dN(v_x) = N \left(\frac{m}{2\pi kT} \right)^{1/2} e^{-mv_x^2/2kT} dv_x, \quad (2.3d)$$

$$dN(v) = N \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv. \quad (2.3e)$$

- Maxwellian distribution in a reduced form:

$$dN(u) = N \frac{4}{\sqrt{\pi}} e^{-u^2} u^2 du, \quad (2.3f)$$

where $u = v/v_p$, v_p is the most probable velocity.

- The most probable, the mean, and the root mean square velocities of molecules:

$$v_p = \sqrt{2 \frac{kT}{m}} \quad \langle v \rangle = \sqrt{\frac{8}{\pi} \frac{kT}{m}}, \quad v_{sq} = \sqrt{3 \frac{kT}{m}}. \quad (2.3g)$$

- Boltzmann's formula:

$$n = n_0 e^{-(U-U_0)/kT}, \quad (2.3h)$$

where U is the potential energy of a molecule.

2.62. Modern vacuum pumps permit the pressures down to $p = 4 \cdot 10^{-15}$ atm to be reached at room temperatures. Assuming that the gas exhausted is nitrogen, find the number of its molecules per 1 cm^3 and the mean distance between them at this pressure.

2.63. A vessel of volume $V = 5.0 \text{ l}$ contains $m = 1.4 \text{ g}$ of nitrogen at a temperature $T = 1800 \text{ K}$. Find the gas pressure, taking into account that $\eta = 30\%$ of molecules are disassociated into atoms at this temperature.

2.64. Under standard conditions the density of the helium and nitrogen mixture equals $\rho = 0.60 \text{ g/l}$. Find the concentration of helium atoms in the given mixture.

2.65. A parallel beam of nitrogen molecules moving with velocity $v = 400 \text{ m/s}$ impinges on a wall at an angle $\theta = 30^\circ$ to its normal. The concentration of molecules in the beam $n = 0.9 \cdot 10^{19} \text{ cm}^{-3}$. Find the pressure exerted by the beam on the wall assuming the molecules to scatter in accordance with the perfectly elastic collision law.

2.66. How many degrees of freedom have the gas molecules, if under standard conditions the gas density is $\rho = 1.3 \text{ mg/cm}^3$ and the velocity of sound propagation in it is $v = 330 \text{ m/s}$.

2.67. Determine the ratio of the sonic velocity v in a gas to the root mean square velocity of molecules of this gas, if the molecules are

- (a) monatomic; (b) rigid diatomic.

2.68. A gas consisting of N -atomic molecules has the temperature T at which all degrees of freedom (translational, rotational, and vibrational) are excited. Find the mean energy of molecules in such a gas. What fraction of this energy corresponds to that of translational motion?

2.69. Suppose a gas is heated up to a temperature at which all degrees of freedom (translational, rotational, and vibrational) of its molecules are excited. Find the molar heat capacity of such a gas in the isochoric process, as well as the adiabatic exponent γ , if the gas consists of

- (a) diatomic;
- (b) linear N -atomic;
- (c) network N -atomic

molecules.

2.70. An ideal gas consisting of N -atomic molecules is expanded isobarically. Assuming that all degrees of freedom (translational, rotational, and vibrational) of the molecules are excited, find what fraction of heat transferred to the gas in this process is spent to perform the work of expansion. How high is this fraction in the case of a monatomic gas?

2.71. Find the molar mass and the number of degrees of freedom of molecules in a gas if its heat capacities are known: $c_V = 0.65 \text{ J/(g}\cdot\text{K)}$ and $c_p = 0.91 \text{ J/(g}\cdot\text{K)}$.

2.72. Find the number of degrees of freedom of molecules in a gas whose molar heat capacity

(a) at constant pressure is equal to $C_p = 29 \text{ J/(mol}\cdot\text{K)}$;

(b) is equal to $C = 29 \text{ J/(mol}\cdot\text{K)}$ in the process $pT = \text{const.}$

2.73. Find the adiabatic exponent γ for a mixture consisting of ν_1 moles of a monatomic gas and ν_2 moles of gas of rigid diatomic molecules.

2.74. A thermally insulated vessel with gaseous nitrogen at a temperature $t = 27^\circ\text{C}$ moves with velocity $v = 100 \text{ m/s}$. How much (in per cent) and in what way will the gas pressure change on a sudden stoppage of the vessel?

2.75. Calculate at the temperature $t = 17^\circ\text{C}$:

(a) the root mean square velocity and the mean kinetic energy of an oxygen molecule in the process of translational motion;

(b) the root mean square velocity of a water droplet of diameter $d = 0.10 \text{ }\mu\text{m}$ suspended in the air.

2.76. A gas consisting of rigid diatomic molecules is expanded adiabatically. How many times has the gas to be expanded to reduce the root mean square velocity of the molecules $\eta = 1.50$ times?

2.77. The mass $m = 15 \text{ g}$ of nitrogen is enclosed in a vessel at a temperature $T = 300 \text{ K}$. What amount of heat has to be transferred to the gas to increase the root mean square velocity of its molecules $\eta = 2.0$ times?

2.78. The temperature of a gas consisting of rigid diatomic molecules is $T = 300 \text{ K}$. Calculate the angular root mean square velocity of a rotating molecule if its moment of inertia is equal to $I = 2.1 \cdot 10^{-39} \text{ g}\cdot\text{cm}^2$.

2.79. A gas consisting of rigid diatomic molecules was initially under standard conditions. Then the gas was compressed adiabatically $\eta = 5.0$ times. Find the mean kinetic energy of a rotating molecule in the final state.

2.80. How will the rate of collisions of rigid diatomic molecules against the vessel's wall change, if the gas is expanded adiabatically η times?

2.81. The volume of gas consisting of rigid diatomic molecules was increased $\eta = 2.0$ times in a polytropic process with the molar heat capacity $C = R$. How many times will the rate of collisions of molecules against a vessel's wall be reduced as a result of this process?

2.82. A gas consisting of rigid diatomic molecules was expanded in a polytropic process so that the rate of collisions of the molecules against the vessel's wall did not change. Find the molar heat capacity of the gas in this process.

2.83. Calculate the most probable, the mean, and the root mean

square velocities of a molecule of a gas whose density under standard atmospheric pressure is equal to $\rho = 1.00 \text{ g/l}$.

2.84. Find the fraction of gas molecules whose velocities differ by less than $\delta\eta = 1.00\%$ from the value of

- (a) the most probable velocity;
- (b) the root mean square velocity.

2.85. Determine the gas temperature at which

(a) the root mean square velocity of hydrogen molecules exceeds their most probable velocity by $\Delta v = 400 \text{ m/s}$;

(b) the velocity distribution function $F(v)$ for the oxygen molecules will have the maximum value at the velocity $v = 420 \text{ m/s}$.

2.86. In the case of gaseous nitrogen find:

(a) the temperature at which the velocities of the molecules $v_1 = 300 \text{ m/s}$ and $v_2 = 600 \text{ m/s}$ are associated with equal values of the Maxwell distribution function $F(v)$;

(b) the velocity of the molecules v at which the value of the Maxwell distribution function $F(v)$ for the temperature T_0 will be the same as that for the temperature η times higher.

2.87. At what temperature of a nitrogen and oxygen mixture do the most probable velocities of nitrogen and oxygen molecules differ by $\Delta v = 30 \text{ m/s}$?

2.88. The temperature of a hydrogen and helium mixture is $T = 300 \text{ K}$. At what value of the molecular velocity v will the Maxwell distribution function $F(v)$ yield the same magnitude for both gases?

2.89. At what temperature of a gas will the number of molecules, whose velocities fall within the given interval from v to $v + dv$, be the greatest? The mass of each molecule is equal to m .

2.90. Find the fraction of molecules whose velocity projections on the x axis fall within the interval from v_x to $v_x + dv_x$, while the moduli of perpendicular velocity components fall within the interval from v_\perp to $v_\perp + dv_\perp$. The mass of each molecule is m , and the temperature is T .

2.91. Using the Maxwell distribution function, calculate the mean velocity projection $\langle v_x \rangle$ and the mean value of the modulus of this projection $\langle |v_x| \rangle$ if the mass of each molecule is equal to m and the gas temperature is T .

2.92. From the Maxwell distribution function find $\langle v_x^2 \rangle$, the mean value of the squared v_x projection of the molecular velocity in a gas at a temperature T . The mass of each molecule is equal to m .

2.93. Making use of the Maxwell distribution function, calculate the number ν of gas molecules reaching a unit area of a wall per unit time, if the concentration of molecules is equal to n , the temperature to T , and the mass of each molecule is m .

2.94. Using the Maxwell distribution function, determine the pressure exerted by gas on a wall, if the gas temperature is T and the concentration of molecules is n .

2.95. Making use of the Maxwell distribution function, find $\langle 1/v \rangle$, the mean value of the reciprocal of the velocity of molecules

in an ideal gas at a temperature T , if the mass of each molecule is equal to m . Compare the value obtained with the reciprocal of the mean velocity.

2.96. A gas consists of molecules of mass m and is at a temperature T . Making use of the Maxwell velocity distribution function, find the corresponding distribution of the molecules over the kinetic energies ε . Determine the most probable value of the kinetic energy ε_p . Does ε_p correspond to the most probable velocity?

2.97. What fraction of monatomic molecules of a gas in a thermal equilibrium possesses kinetic energies differing from the mean value by $\delta\eta = 1.0\%$ and less?

2.98. What fraction of molecules in a gas at a temperature T has the kinetic energy of translational motion exceeding ε_0 if $\varepsilon_0 \gg kT$?

2.99. The velocity distribution of molecules in a beam coming out of a hole in a vessel is described by the function $F(v) = Av^3 e^{-mv^2/2kT}$, where T is the temperature of the gas in the vessel. Find the most probable values of

(a) the velocity of the molecules in the beam; compare the result obtained with the most probable velocity of the molecules in the vessel;

(b) the kinetic energy of the molecules in the beam.

2.100. An ideal gas consisting of molecules of mass m with concentration n has a temperature T . Using the Maxwell distribution function, find the number of molecules reaching a unit area of a wall at the angles between θ and $\theta + d\theta$ to its normal per unit time.

2.101. From the conditions of the foregoing problem find the number of molecules reaching a unit area of a wall with the velocities in the interval from v to $v + dv$ per unit time.

2.102. Find the force exerted on a particle by a uniform field if the concentrations of these particles at two levels separated by the distance $\Delta h = 3.0$ cm (along the field) differ by $\eta = 2.0$ times. The temperature of the system is equal to $T = 280$ K.

2.103. When examining the suspended gamboge droplets under a microscope, their average numbers in the layers separated by the distance $h = 40$ μm were found to differ by $\eta = 2.0$ times. The environmental temperature is equal to $T = 290$ K. The diameter of the droplets is $d = 0.40$ μm , and their density exceeds that of the surrounding fluid by $\Delta\rho = 0.20$ g/cm^3 . Find Avogadro's number from these data.

2.104. Suppose that η_0 is the ratio of the molecular concentration of hydrogen to that of nitrogen at the Earth's surface, while η is the corresponding ratio at the height $h = 3000$ m. Find the ratio η/η_0 at the temperature $T = 280$ K, assuming that the temperature and the free fall acceleration are independent of the height.

2.105. A tall vertical vessel contains a gas composed of two kinds of molecules of masses m_1 and m_2 , with $m_2 > m_1$. The concentrations of these molecules at the bottom of the vessel are equal to n_1 and n_2

respectively, with $n_2 > n_1$. Assuming the temperature T and the free-fall acceleration g to be independent of the height, find the height at which the concentrations of these kinds of molecules are equal.

2.106. A very tall vertical cylinder contains carbon dioxide at a certain temperature T . Assuming the gravitational field to be uniform, find how the gas pressure on the bottom of the vessel will change when the gas temperature increases η times.

2.107. A very tall vertical cylinder contains a gas at a temperature T . Assuming the gravitational field to be uniform, find the mean value of the potential energy of the gas molecules. Does this value depend on whether the gas consists of one kind of molecules or of several kinds?

2.108. A horizontal tube of length $l = 100$ cm closed from both ends is displaced lengthwise with a constant acceleration w . The tube contains argon at a temperature $T = 330$ K. At what value of w will the argon concentrations at the tube's ends differ by $\eta = 1.0\%$?

2.109. Find the mass of a mole of colloid particles if during their centrifuging with an angular velocity ω about a vertical axis the concentration of the particles at the distance r_2 from the rotation axis is η times greater than that at the distance r_1 (in the same horizontal plane). The densities of the particles and the solvent are equal to ρ and to ρ_0 respectively.

2.110. A horizontal tube with closed ends is rotated with a constant angular velocity ω about a vertical axis passing through one of its ends. The tube contains carbon dioxide at a temperature $T = 300$ K. The length of the tube is $l = 100$ cm. Find the value ω at which the ratio of molecular concentrations at the opposite ends of the tube is equal to $\eta = 2.0$.

2.111. The potential energy of gas molecules in a certain central field depends on the distance r from the field's centre as $U(r) = ar^2$, where a is a positive constant. The gas temperature is T , the concentration of molecules at the centre of the field is n_0 . Find:

(a) the number of molecules located at the distances between r and $r + dr$ from the centre of the field;

(b) the most probable distance separating the molecules from the centre of the field;

(c) the fraction of molecules located in the spherical layer between r and $r + dr$;

(d) how many times the concentration of molecules in the centre of the field will change if the temperature decreases η times.

2.112. From the conditions of the foregoing problem find:

(a) the number of molecules whose potential energy lies within the interval from U to $U + dU$;

(b) the most probable value of the potential energy of a molecule; compare this value with the potential energy of a molecule located at its most probable distance from the centre of the field.

2.4. THE SECOND LAW OF THERMODYNAMICS.

ENTROPY

- Heat engine efficiency:

$$\eta = \frac{A}{Q_1} = 1 - \frac{Q'_2}{Q_1}, \quad (2.4a)$$

where Q_1 is the heat obtained by the working substance, Q'_2 is the heat released by the working substance.

- Efficiency of a Carnot cycle:

$$\eta = \frac{T_1 - T_2}{T_1}, \quad (2.4b)$$

where T_1 and T_2 are the temperatures of the hot and cold bodies respectively.

- Clausius inequality:

$$\oint \frac{\delta Q}{T} \leq 0, \quad (2.4c)$$

where δQ is the elementary amount of heat transferred to the system (δQ is an algebraic quantity).

- Entropy increment of a system:

$$\Delta S \geq \int \frac{\delta Q}{T}. \quad (2.4d)$$

- Fundamental relation of thermodynamics:

$$T dS \geq dU + p dV. \quad (2.4e)$$

- Relation between the entropy and the statistical weight Ω (the thermodynamic probability):

$$S = k \ln \Omega. \quad (2.4f)$$

where k is the Boltzmann constant.

2.113. In which case will the efficiency of a Carnot cycle be higher: when the hot body temperature is increased by ΔT , or when the cold body temperature is decreased by the same magnitude?

2.114. Hydrogen is used in a Carnot cycle as a working substance. Find the efficiency of the cycle, if as a result of an adiabatic expansion

(a) the gas volume increases $n = 2.0$ times;

(b) the pressure decreases $n = 2.0$ times.

2.115. A heat engine employing a Carnot cycle with an efficiency of $\eta = 10\%$ is used as a refrigerating machine, the thermal reservoirs being the same. Find its refrigerating efficiency ε .

2.116. An ideal gas goes through a cycle consisting of alternate isothermal and adiabatic curves (Fig. 2.2). The isothermal processes proceed at the temperatures T_1 , T_2 , and T_3 . Find the efficiency of such a cycle, if in each isothermal expansion the gas volume increases in the same proportion.

2.117. Find the efficiency of a cycle consisting of two isochoric and two adiabatic lines, if the volume of the ideal gas changes $n = 10$ times within the cycle. The working substance is nitrogen.

2.118. Find the efficiency of a cycle consisting of two isobaric and two adiabatic lines, if the pressure changes n times within the cycle. The working substance is an ideal gas whose adiabatic exponent is equal to γ .

2.119. An ideal gas whose adiabatic exponent equals γ goes through a cycle consisting of two isochoric and two isobaric lines. Find the efficiency of such a cycle, if the absolute temperature of the gas rises n times both in the isochoric heating and in the isobaric expansion.

2.120. An ideal gas goes through a cycle consisting of

(a) isochoric, adiabatic, and isothermal lines;

(b) isobaric, adiabatic, and isothermal lines,

with the isothermal process proceeding at the *minimum* temperature of the whole cycle.

Find the efficiency of each cycle if the absolute temperature varies n -fold within the cycle.

2.121. The conditions are the same as in the foregoing problem with the exception that the isothermal process proceeds at the *maximum* temperature of the whole cycle.

2.122. An ideal gas goes through a cycle consisting of isothermal, polytropic, and adiabatic lines, with the isothermal process proceeding at the *maximum* temperature of the whole cycle. Find the efficiency of such a cycle if the absolute temperature varies n -fold within the cycle.

2.123. An ideal gas with the adiabatic exponent γ goes through a direct (clockwise) cycle consisting of adiabatic, isobaric, and isochoric lines. Find the efficiency of the cycle if in the adiabatic process the volume of the ideal gas

(a) increases n -fold; (b) decreases n -fold.

2.124. Calculate the efficiency of a cycle consisting of isothermal, isobaric, and isochoric lines, if in the isothermal process the volume of the ideal gas with the adiabatic exponent γ

(a) increases n -fold; (b) decreases n -fold.

2.125. Find the efficiency of a cycle consisting of two isochoric and two isothermal lines if the volume varies v -fold and the absolute temperature τ -fold within the cycle. The working substance is an ideal gas with the adiabatic exponent γ .

2.126. Find the efficiency of a cycle consisting of two isobaric and two isothermal lines if the pressure varies n -fold and the absolute temperature τ -fold within the cycle. The working substance is an ideal gas with the adiabatic exponent γ .

2.127. An ideal gas with the adiabatic exponent γ goes through a cycle (Fig. 2.3) within which the absolute temperature varies τ -fold. Find the efficiency of this cycle.

2.128. Making use of the Clausius inequality, demonstrate that

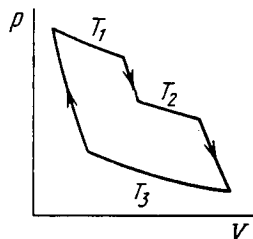


Fig. 2.2.

all cycles having the same maximum temperature T_{max} and the same minimum temperature T_{min} are less efficient compared to the Carnot cycle with the same T_{max} and T_{min} .

2.129. Making use of the Carnot theorem, show that in the case of a physically uniform substance whose state is defined by the parameters T and V

$$(\partial U / \partial V)_T = T (\partial p / \partial T)_V - p,$$

where $U(T, V)$ is the internal energy of the substance.

Instruction. Consider the infinitesimal Carnot cycle in the variables p, V .

2.130. Find the entropy increment of one mole of carbon dioxide when its absolute temperature increases $n = 2.0$ times if the process of heating is

(a) isochoric; (b) isobaric.

The gas is to be regarded as ideal.

2.131. The entropy of $\nu = 4.0$ moles of an ideal gas increases by $\Delta S = 23 \text{ J/K}$ due to the isothermal expansion. How many times should the volume $\nu = 4.0$ moles of the gas be increased?

2.132. Two moles of an ideal gas are cooled isochorically and then expanded isobarically to lower the gas temperature back to the initial value. Find the entropy increment of the gas if in this process the gas pressure changed $n = 3.3$ times.

2.133. Helium of mass $m = 1.7 \text{ g}$ is expanded adiabatically $n = 3.0$ times and then compressed isobarically down to the initial volume. Find the entropy increment of the gas in this process.

2.134. Find the entropy increment of $\nu = 2.0$ moles of an ideal gas whose adiabatic exponent $\gamma = 1.30$ if, as a result of a certain process, the gas volume increased $\alpha = 2.0$ times while the pressure dropped $\beta = 3.0$ times.

2.135. Vessels 1 and 2 contain $\nu = 1.2$ moles of gaseous helium. The ratio of the vessels' volumes $V_2/V_1 = \alpha = 2.0$, and the ratio of the absolute temperatures of helium in them $T_1/T_2 = \beta = 1.5$. Assuming the gas to be ideal, find the difference of gas entropies in these vessels, $S_2 - S_1$.

2.136. One mole of an ideal gas with the adiabatic exponent γ goes through a polytropic process as a result of which the absolute temperature of the gas increases τ -fold. The polytropic constant equals n . Find the entropy increment of the gas in this process.

2.137. The expansion process of $\nu = 2.0$ moles of argon proceeds so that the gas pressure increases in direct proportion to its volume.

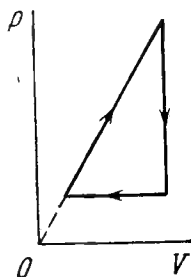


Fig. 2.3.

Find the entropy increment of the gas in this process provided its volume increases $\alpha = 2.0$ times.

2.138. An ideal gas with the adiabatic exponent γ goes through a process $p = p_0 - \alpha V$, where p_0 and α are positive constants, and V is the volume. At what volume will the gas entropy have the maximum value?

2.139. One mole of an ideal gas goes through a process in which the entropy of the gas changes with temperature T as $S = aT + C_V \ln T$, where a is a positive constant, C_V is the molar heat capacity of this gas at constant volume. Find the volume dependence of the gas temperature in this process if $T = T_0$ at $V = V_0$.

2.140. Find the entropy increment of one mole of a Van der Waals gas due to the isothermal variation of volume from V_1 to V_2 . The Van der Waals corrections are assumed to be known.

2.141. One mole of a Van der Waals gas which had initially the volume V_1 and the temperature T_1 was transferred to the state with the volume V_2 and the temperature T_2 . Find the corresponding entropy increment of the gas, assuming its molar heat capacity C_V to be known.

2.142. At very low temperatures the heat capacity of crystals is equal to $C = aT^3$, where a is a constant. Find the entropy of a crystal as a function of temperature in this temperature interval.

2.143. Find the entropy increment of an aluminum bar of mass $m = 3.0$ kg on its heating from the temperature $T_1 = 300$ K up to $T_2 = 600$ K if in this temperature interval the specific heat capacity of aluminum varies as $c = a + bT$, where $a = 0.77$ J/(g·K), $b = 0.46$ mJ/(g·K²).

2.144. In some process the temperature of a substance depends on its entropy S as $T = aS^n$, where a and n are constants. Find the corresponding heat capacity C of the substance as a function of S . At what condition is $C < 0$?

2.145. Find the temperature T as a function of the entropy S of a substance for a polytropic process in which the heat capacity of the substance equals C . The entropy of the substance is known to be equal to S_0 at the temperature T_0 . Draw the approximate plots $T(S)$ for $C > 0$ and $C < 0$.

2.146. One mole of an ideal gas with heat capacity C_V goes through a process in which its entropy S depends on T as $S = \alpha/T$, where α is a constant. The gas temperature varies from T_1 to T_2 . Find:

(a) the molar heat capacity of the gas as a function of its temperature;

(b) the amount of heat transferred to the gas;

(c) the work performed by the gas.

2.147. A working substance goes through a cycle within which the absolute temperature varies n -fold, and the shape of the cycle is shown in (a) Fig. 2.4a; (b) Fig. 2.4b, where T is the absolute temperature, and S the entropy. Find the efficiency of each cycle.

2.148. One of the two thermally insulated vessels interconnected by a tube with a valve contains $\nu = 2.2$ moles of an ideal gas. The other vessel is evacuated. The valve having been opened, the gas increased its volume $n = 3.0$ times. Find the entropy increment of the gas.

2.149. A weightless piston divides a thermally insulated cylinder into two equal parts. One part contains one mole of an ideal gas with adiabatic exponent γ , the other is evacuated. The initial gas temperature is T_0 . The piston is released and the gas fills the whole

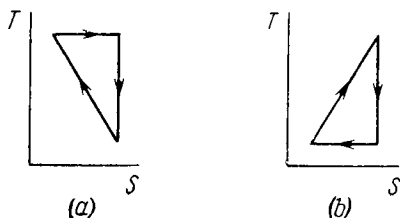


Fig. 2.4.

volume of the cylinder. Then the piston is slowly displaced back to the initial position. Find the increment of the internal energy and the entropy of the gas resulting from these two processes.

2.150. An ideal gas was expanded from the initial state to the volume V without any heat exchange with the surrounding bodies. Will the final gas pressure be the same in the case of (a) a fast and in the case of (b) a very slow expansion process?

2.151. A thermally insulated vessel is partitioned into two parts so that the volume of one part is $n = 2.0$ times greater than that of the other. The smaller part contains $\nu_1 = 0.30$ mole of nitrogen, and the greater one $\nu_2 = 0.70$ mole of oxygen. The temperature of the gases is the same. A hole is punctured in the partition and the gases are mixed. Find the corresponding increment of the system's entropy, assuming the gases to be ideal.

2.152. A piece of copper of mass $m_1 = 300$ g with initial temperature $t_1 = 97^\circ\text{C}$ is placed into a calorimeter in which the water of mass $m_2 = 100$ g is at a temperature $t_2 = 7^\circ\text{C}$. Find the entropy increment of the system by the moment the temperatures equalize. The heat capacity of the calorimeter itself is negligibly small.

2.153. Two identical thermally insulated vessels interconnected by a tube with a valve contain one mole of the same ideal gas each. The gas temperature in one vessel is equal to T_1 and in the other, T_2 . The molar heat capacity of the gas of constant volume equals C_V . The valve having been opened, the gas comes to a new equilibrium state. Find the entropy increment ΔS of the gas. Demonstrate that $\Delta S > 0$.

2.154. N atoms of gaseous helium are enclosed in a cubic vessel of volume 1.0 cm^3 at room temperature. Find:

(a) the probability of atoms gathering in one half of the vessel;
 (b) the approximate numerical value of N ensuring the occurrence of this event within the time interval $t \approx 10^{10}$ years (the age of the Universe).

2.155. Find the statistical weight of the most probable distribution of $N = 10$ identical molecules over two halves of the cylinder's volume. Find also the probability of such a distribution.

2.156. A vessel contains N molecules of an ideal gas. Dividing mentally the vessel into two halves A and B , find the probability that the half A contains n molecules. Consider the cases when $N = 5$ and $n = 0, 1, 2, 3, 4, 5$.

2.157. A vessel of volume V_0 contains N molecules of an ideal gas. Find the probability of n molecules getting into a certain separated part of the vessel of volume V . Examine, in particular, the case $V = V_0/2$.

2.158. An ideal gas is under standard conditions. Find the diameter of the sphere within whose volume the relative fluctuation of the number of molecules is equal to $\eta = 1.0 \cdot 10^{-3}$. What is the average number of molecules inside such a sphere?

2.159. One mole of an ideal gas consisting of monatomic molecules is enclosed in a vessel at a temperature $T_0 = 300$ K. How many times and in what way will the statistical weight of this system (gas) vary if it is heated isochorically by $\Delta T = 1.0$ K?

2.5. LIQUIDS. CAPILLARY EFFECTS

• Additional (capillary) pressure in a liquid under an arbitrary surface (Laplace's formula):

$$\Delta p = \alpha \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (2.5a)$$

where α is the surface tension of a given liquid.

• Free energy increment of the surface layer of a liquid:

$$dF = \alpha dS, \quad (2.5b)$$

where dS is the area increment of the surface layer.

• Amount of heat required to form a unit area of the liquid surface layer during the isothermal increase of its surface:

$$q = -T \frac{d\alpha}{dT}. \quad (2.5c)$$

2.160. Find the capillary pressure

(a) in mercury droplets of diameter $d = 1.5 \mu\text{m}$;

(b) inside a soap bubble of diameter $d = 3.0$ mm if the surface tension of the soap water solution is $\alpha = 45$ mN/m.

2.161. In the bottom of a vessel with mercury there is a round hole of diameter $d = 70 \mu\text{m}$. At what maximum thickness of the mercury layer will the liquid still not flow out through this hole?

2.162. A vessel filled with air under pressure p_0 contains a soap bubble of diameter d . The air pressure having been reduced isothermally n -fold, the bubble diameter increased η -fold. Find the surface tension of the soap water solution.

2.163. Find the pressure in an air bubble of diameter $d = 4.0 \mu\text{m}$, located in water at a depth $h = 5.0 \text{ m}$. The atmospheric pressure has the standard value p_0 .

2.164. The diameter of a gas bubble formed at the bottom of a pond is $d = 4.0 \mu\text{m}$. When the bubble rises to the surface its diameter increases $n = 1.1$ times. Find how deep is the pond at that spot. The atmospheric pressure is standard, the gas expansion is assumed to be isothermal.

2.165. Find the difference in height of mercury columns in two communicating vertical capillaries whose diameters are $d_1 = 0.50 \text{ mm}$ and $d_2 = 1.00 \text{ mm}$, if the contact angle $\theta = 138^\circ$.

2.166. A vertical capillary with inside diameter 0.50 mm is submerged into water so that the length of its part protruding over the water surface is equal to $h = 25 \text{ mm}$. Find the curvature radius of the meniscus.

2.167. A glass capillary of length $l = 110 \text{ mm}$ and inside diameter $d = 20 \mu\text{m}$ is submerged vertically into water. The upper end of the capillary is sealed. The outside pressure is standard. To what length x has the capillary to be submerged to make the water levels inside and outside the capillary coincide?

2.168. When a vertical capillary of length l with the sealed upper end was brought in contact with the surface of a liquid, the level of this liquid rose to the height h . The liquid density is ρ , the inside diameter of the capillary is d , the contact angle is θ , the atmospheric pressure is p_0 . Find the surface tension of the liquid.

2.169. A glass rod of diameter $d_1 = 1.5 \text{ mm}$ is inserted symmetrically into a glass capillary with inside diameter $d_2 = 2.0 \text{ mm}$. Then the whole arrangement is vertically oriented and brought in contact with the surface of water. To what height will the water rise in the capillary?

2.170. Two vertical plates submerged partially in a wetting liquid form a wedge with a very small angle $\delta\varphi$. The edge of this wedge is vertical. The density of the liquid is ρ , its surface tension is α , the contact angle is θ . Find the height h , to which the liquid rises, as a function of the distance x from the edge.

2.171. A vertical water jet flows out of a round hole. One of the horizontal sections of the jet has the diameter $d = 2.0 \text{ mm}$ while the other section located $l = 20 \text{ mm}$ lower has the diameter which is $n = 1.5$ times less. Find the volume of the water flowing from the hole each second.

2.172. A water drop falls in air with a uniform velocity. Find the difference between the curvature radii of the drop's surface at the upper and lower points of the drop separated by the distance $h = 2.3 \text{ mm}$.

2.173. A mercury drop shaped as a round tablet of radius R and thickness h is located between two horizontal glass plates. Assuming that $h \ll R$, find the mass m of a weight which has to be placed on the upper plate to diminish the distance between the plates n -times. The contact angle equals θ . Calculate m if $R = 2.0$ cm, $h = 0.38$ mm, $n = 2.0$, and $\theta = 135^\circ$.

2.174. Find the attraction force between two parallel glass plates, separated by a distance $h = 0.10$ mm, after a water drop of mass $m = 70$ mg was introduced between them. The wetting is assumed to be complete.

2.175. Two glass discs of radius $R = 5.0$ cm were wetted with water and put together so that the thickness of the water layer between them was $h = 1.9$ μm . Assuming the wetting to be complete, find the force that has to be applied at right angles to the plates in order to pull them apart.

2.176. Two vertical parallel glass plates are partially submerged in water. The distance between the plates is $d = 0.10$ mm, and their width is $l = 12$ cm. Assuming that the water between the plates does not reach the upper edges of the plates and that the wetting is complete, find the force of their mutual attraction.

2.177. Find the lifetime of a soap bubble of radius R connected with the atmosphere through a capillary of length l and inside radius r . The surface tension is α , the viscosity coefficient of the gas is η .

2.178. A vertical capillary is brought in contact with the water surface. What amount of heat is liberated while the water rises along the capillary? The wetting is assumed to be complete, the surface tension equals α .

2.179. Find the free energy of the surface layer of

(a) a mercury droplet of diameter $d = 1.4$ mm;

(b) a soap bubble of diameter $d = 6.0$ mm if the surface tension of the soap water solution is equal to $\alpha = 45$ mN/m.

2.180. Find the increment of the free energy of the surface layer when two identical mercury droplets, each of diameter $d = 1.5$ mm, merge isothermally.

2.181. Find the work to be performed in order to blow a soap bubble of radius R if the outside air pressure is equal to p_0 and the surface tension of the soap water solution is equal to α .

2.182. A soap bubble of radius r is inflated with an ideal gas. The atmospheric pressure is p_0 , the surface tension of the soap water solution is α . Find the difference between the molar heat capacity of the gas during its heating inside the bubble and the molar heat capacity of the gas under constant pressure, $C - C_p$.

2.183. Considering the Carnot cycle as applied to a liquid film, show that in an isothermal process the amount of heat required for the formation of a unit area of the surface layer is equal to $q = -T \cdot d\alpha/dT$, where $d\alpha/dT$ is the temperature derivative of the surface tension.

2.184. The surface of a soap film was increased isothermally by $\Delta\sigma$ at a temperature T . Knowing the surface tension of the soap water solution α and the temperature coefficient $d\alpha/dT$, find the increment

- (a) of the entropy of the film's surface layer;
- (b) of the internal energy of the surface layer.

2.6. PHASE TRANSFORMATIONS

- Relations between Van der Waals constants and the parameters of the critical state of a substance:

$$V_{M\ cr} = 3b, \quad p_{cr} = \frac{a}{27b^2}, \quad T_{cr} = \frac{8a}{27Rb}. \quad (2.6a)$$

- Relation between the critical parameters for a mole of substance:

$$p_{cr}V_{M\ cr} = (3/8)RT_{cr}. \quad (2.6b)$$

- Clausius-Clapeyron equation:

$$\frac{dp}{dT} = \frac{q_{12}}{T(V'_2 - V'_1)}, \quad (2.6c)$$

where q_{12} is the specific heat absorbed in the transformation $1 \rightarrow 2$, V'_1 and V'_2 are the specific volumes of phases 1 and 2.

2.185. A saturated water vapour is contained in a cylindrical vessel under a weightless piston at a temperature $t = 100^\circ\text{C}$. As a result of a slow introduction of the piston a small fraction of the vapour $\Delta m = 0.70\text{ g}$ gets condensed. What amount of work was performed over the gas? The vapour is assumed to be ideal, the volume of the liquid is to be neglected.

2.186. A vessel of volume $V = 6.0\text{ l}$ contains water together with its saturated vapour under a pressure of 40 atm and at a temperature of 250°C . The specific volume of the vapour is equal to $V'_v = 50\text{ l/kg}$ under these conditions. The total mass of the system water-vapour equals $m = 5.0\text{ kg}$. Find the mass and the volume of the vapour.

2.187. The saturated water vapour is enclosed in a cylinder under a piston and occupies a volume $V_0 = 5.0\text{ l}$ at the temperature $t = 100^\circ\text{C}$. Find the mass of the liquid phase formed after the volume under the piston decreased isothermally to $V = 1.6\text{ l}$. The saturated vapour is assumed to be ideal.

2.188. A volume occupied by a saturated vapour is reduced isothermally n -fold. Find what fraction η of the final volume is occupied by the liquid phase if the specific volumes of the saturated vapour and the liquid phase differ by N times ($N > n$). Solve the same problem under the condition that the final volume of the substance corresponds to the midpoint of a horizontal portion of the isothermal line in the diagram p, V .

2.189. An amount of water of mass $m = 1.00\text{ kg}$, boiling at standard atmospheric pressure, turns completely into saturated vapour.

Assuming the saturated vapour to be an ideal gas find the increment of entropy and internal energy of the system.

2.190. Water of mass $m = 20$ g is enclosed in a thermally insulated cylinder at the temperature of 0°C under a weightless piston whose area is $S = 410$ cm². The outside pressure is equal to standard atmospheric pressure. To what height will the piston rise when the water absorbs $Q = 20.0$ kJ of heat?

2.191. One gram of saturated water vapour is enclosed in a thermally insulated cylinder under a weightless piston. The outside pressure being standard, $m = 1.0$ g of water is introduced into the cylinder at a temperature $t_0 = 22^\circ\text{C}$. Neglecting the heat capacity of the cylinder and the friction of the piston against the cylinder's walls, find the work performed by the force of the atmospheric pressure during the lowering of the piston.

2.192. If an additional pressure Δp of a saturated vapour over a convex spherical surface of a liquid is considerably less than the vapour pressure over a plane surface, then $\Delta p = (\rho_v/\rho_l) 2\alpha/r$, where ρ_v and ρ_l are the densities of the vapour and the liquid, α is the surface tension, and r is the radius of curvature of the surface. Using this formula, find the diameter of water droplets at which the saturated vapour pressure exceeds the vapour pressure over the plane surface by $\eta = 1.0\%$ at a temperature $t = 27^\circ\text{C}$. The vapour is assumed to be an ideal gas.

2.193. Find the mass of all molecules leaving one square centimetre of water surface per second into a saturated water vapour above it at a temperature $t = 100^\circ\text{C}$. It is assumed that $\eta = 3.6\%$ of all water vapour molecules falling on the water surface are retained in the liquid phase.

2.194. Find the pressure of saturated tungsten vapour at a temperature $T = 2000$ K if a tungsten filament is known to lose a mass $\mu = 1.2 \cdot 10^{-13}$ g/(s·cm²) from a unit area per unit time when evaporating into high vacuum at this temperature.

2.195. By what magnitude would the pressure exerted by water on the walls of the vessel have increased if the intermolecular attraction forces had vanished?

2.196. Find the internal pressure p_i of a liquid if its density ρ and specific latent heat of vaporization q are known. The heat q is assumed to be equal to the work performed against the forces of the internal pressure, and the liquid obeys the Van der Waals equation. Calculate p_i in water.

2.197. Demonstrate that Eqs. (2.6a) and (2.6b) are valid for a substance, obeying the Van der Waals equation, in critical state.

Instruction. Make use of the fact that the critical state corresponds to the point of inflection in the isothermal curve $p(V)$.

2.198. Calculate the Van der Waals constants for carbon dioxide if its critical temperature $T_{cr} = 304$ K and critical pressure $p_{cr} = 73$ atm.

2.199. Find the specific volume of benzene (C_6H_6) in critical state if its critical temperature $T_{cr} = 562$ K and critical pressure $p_{cr} = 47$ atm.

2.200. Write the Van der Waals equation via the reduced parameters π , v , and τ , having taken the corresponding critical values for the units of pressure, volume, and temperature. Using the equation obtained, find how many times the gas temperature exceeds its critical temperature if the gas pressure is 12 times as high as critical pressure, and the volume of gas is equal to half the critical volume.

2.201. Knowing the Van der Waals constants, find:

(a) the maximum volume which water of mass $m = 1.00$ kg can occupy in liquid state;

(b) the maximum pressure of the saturated water vapour.

2.202. Calculate the temperature and density of carbon dioxide in critical state, assuming the gas to be a Van der Waals one.

2.203. What fraction of the volume of a vessel must liquid ether occupy at room temperature in order to pass into critical state when critical temperature is reached? Ether has $T_{cr} = 467$ K, $p_{cr} = 35.5$ atm, $\rho M = 74$ g/mol.

2.204. Demonstrate that the straight line 1-5 corresponding to the isothermal-isobaric phase transition cuts the Van der Waals isotherm so that areas I and II are equal (Fig. 2.5).

2.205. What fraction of water supercooled down to the temperature $t = -20^\circ\text{C}$ under standard pressure turns into ice when the system passes into the equilibrium state? At what temperature of the supercooled water does it turn into ice completely?

2.206. Find the increment of the ice melting temperature in the vicinity of 0°C when the pressure is increased by $\Delta p = 1.00$ atm. The specific volume of ice exceeds that of water by $\Delta V' = 0.091$ cm³/g.

2.207. Find the specific volume of saturated water vapour under standard pressure if a decrease of pressure by $\Delta p = 3.2$ kPa is known to decrease the water boiling temperature by $\Delta T = 0.9$ K.

2.208. Assuming the saturated water vapour to be ideal, find its pressure at the temperature 101.1°C .

2.209. A small amount of water and its saturated vapour are enclosed in a vessel at a temperature $t = 100^\circ\text{C}$. How much (in per cent) will the mass of the saturated vapour increase if the temperature of the system goes up by $\Delta T = 1.5$ K? Assume that the vapour is an ideal gas and the specific volume of water is negligible as compared to that of vapour.

2.210. Find the pressure of saturated vapour as a function of temperature $p(T)$ if at a temperature T_0 its pressure equals p_0 .

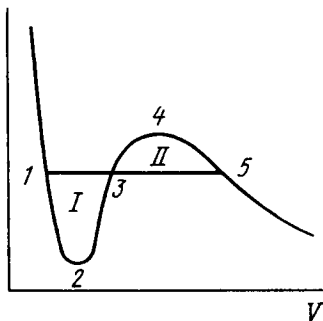


Fig. 2.5.

Assume that: the specific latent heat of vaporization q is independent of T , the specific volume of liquid is negligible as compared to that of vapour, saturated vapour obeys the equation of state for an ideal gas. Investigate under what conditions these assumptions are permissible.

2.211. An ice which was initially under standard conditions was compressed up to the pressure $p = 640$ atm. Assuming the lowering of the ice melting temperature to be a linear function of pressure under the given conditions, find what fraction of the ice melted. The specific volume of water is less than that of ice by $\Delta V' = 0.09$ cm³/g.

2.212. In the vicinity of the triple point the saturated vapour pressure p of carbon dioxide depends on temperature T as $\log p = a - b/T$, where a and b are constants. If p is expressed in atmospheres, then for the sublimation process $a = 9.05$ and $b = 1.80$ kK, and for the vaporization process $a = 6.78$ and $b = 1.31$ kK. Find:

(a) temperature and pressure at the triple point;

(b) the values of the specific latent heats of sublimation, vaporization, and melting in the vicinity of the triple point.

2.213. Water of mass $m = 1.00$ kg is heated from the temperature $t_1 = 10^\circ\text{C}$ up to $t_2 = 100^\circ\text{C}$ at which it evaporates completely. Find the entropy increment of the system.

2.214. The ice with the initial temperature $t_1 = 0^\circ\text{C}$ was first melted, then heated to the temperature $t_2 = 100^\circ\text{C}$ and evaporated. Find the increment of the system's specific entropy.

2.215. A piece of copper of mass $m = 90$ g at a temperature $t_1 = 90^\circ\text{C}$ was placed in a calorimeter in which ice of mass 50 g was at a temperature -3°C . Find the entropy increment of the piece of copper by the moment the thermal equilibrium is reached.

2.216. A chunk of ice of mass $m_1 = 100$ g at a temperature $t_1 = 0^\circ\text{C}$ was placed in a calorimeter in which water of mass $m_2 = 100$ g was at a temperature t_2 . Assuming the heat capacity of the calorimeter to be negligible, find the entropy increment of the system by the moment the thermal equilibrium is reached. Consider two cases: (a) $t_2 = 60^\circ\text{C}$; (b) $t_2 = 94^\circ\text{C}$.

2.217. Molten lead of mass $m = 5.0$ g at a temperature $t_2 = 327^\circ\text{C}$ (the melting temperature of lead) was poured into a calorimeter packed with a large amount of ice at a temperature $t_1 = 0^\circ\text{C}$. Find the entropy increment of the system lead-ice by the moment the thermal equilibrium is reached. The specific latent heat of melting of lead is equal to $q = 22.5$ J/g and its specific heat capacity is equal to $c = 0.125$ J/(g·K).

2.218. A water vapour filling the space under the piston of a cylinder, is compressed (or expanded) so that it remains saturated all the time, being just on the verge of condensation. Find the molar heat capacity C of the vapour in this process as a function of temperature T , assuming the vapour to be an ideal gas and neglecting the specific volume of water in comparison with that of vapour. Calculate C at a temperature $t = 100^\circ\text{C}$.

2.219. One mole of water being in equilibrium with a negligible amount of its saturated vapour at a temperature T_1 was completely converted into saturated vapour at a temperature T_2 . Find the entropy increment of the system. The vapour is assumed to be an ideal gas, the specific volume of the liquid is negligible in comparison with that of the vapour.

2.7. TRANSPORT PHENOMENA

● Relative number of gas molecules traversing the distance s without collisions:

$$N/N_0 = e^{-s/\lambda} \quad (2.7a)$$

where λ is the mean free path.

● Mean free path of a gas molecule:

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}, \quad (2.7b)$$

where d is the effective diameter of a molecule, and n is the number of molecules per unit volume.

● Coefficients of diffusion D , viscosity η , and heat conductivity κ of gases:

$$D = \frac{1}{3} \langle v \rangle \lambda, \quad \eta = \frac{1}{3} \langle v \rangle \lambda \rho, \quad \kappa = \frac{1}{3} \langle v \rangle \lambda \rho c_V, \quad (2.7c)$$

where ρ is the gas density, and c_V is its specific heat capacity at constant volume.

● Friction force acting on a unit area of plates during their motion parallel to each other in a highly rarefied gas:

$$F = \frac{1}{6} \langle v \rangle \rho |u_1 - u_2|, \quad (2.7d)$$

where u_1 and u_2 are the velocities of the plates.

● Density of a thermal flux transferred between two walls by highly rarefied gas:

$$q = \frac{1}{6} \langle v \rangle \rho c_V |T_1 - T_2|, \quad (2.7e)$$

where T_1 and T_2 are the temperatures of the walls.

2.220. Calculate what fraction of gas molecules

(a) traverses without collisions the distances exceeding the mean free path λ ;

(b) has the free path values lying within the interval from λ to 2λ .

2.221. A narrow molecular beam makes its way into a vessel filled with gas under low pressure. Find the mean free path of molecules if the beam intensity decreases η -fold over the distance Δl .

2.222. Let αdt be the probability of a gas molecule experiencing a collision during the time interval dt ; α is a constant. Find:

(a) the probability of a molecule experiencing no collisions during the time interval t ;

(b) the mean time interval between successive collisions.

2.223. Find the mean free path and the mean time interval between successive collisions of gaseous nitrogen molecules

(a) under standard conditions;

(b) at temperature $t = 0^\circ\text{C}$ and pressure $p = 1.0\text{ nPa}$ (such a pressure can be reached by means of contemporary vacuum pumps).

2.224. How many times does the mean free path of nitrogen molecules exceed the mean distance between the molecules under standard conditions?

2.225. Find the mean free path of gas molecules under standard conditions if the Van der Waals constant of this gas is equal to $b = 40\text{ ml/mol}$.

2.226. An acoustic wave propagates through nitrogen under standard conditions. At what frequency will the wavelength be equal to the mean free path of the gas molecules?

2.227. Oxygen is enclosed at the temperature 0°C in a vessel with the characteristic dimension $l = 10\text{ mm}$ (this is the linear dimension determining the character of a physical process in question). Find:

(a) the gas pressure below which the mean free path of the molecules $\lambda > l$;

(b) the corresponding molecular concentration and the mean distance between the molecules.

2.228. For the case of nitrogen under standard conditions find:

(a) the mean number of collisions experienced by each molecule per second;

(b) the total number of collisions occurring between the molecules within 1 cm^3 of nitrogen per second.

2.229. How does the mean free path λ and the number of collisions of each molecule per unit time ν depend on the absolute temperature of an ideal gas undergoing

(a) an isochoric process;

(b) an isobaric process?

2.230. As a result of some process the pressure of an ideal gas increases n -fold. How many times have the mean free path λ and the number of collisions of each molecule per unit time ν changed and how, if the process is

(a) isochoric; (b) isothermal?

2.231. An ideal gas consisting of rigid diatomic molecules goes through an adiabatic process. How do the mean free path λ and the number of collisions of each molecule per second ν depend in this process on

(a) the volume V ; (b) the pressure p ; (c) the temperature T ?

2.232. An ideal gas goes through a polytropic process with exponent n . Find the mean free path λ and the number of collisions of each molecule per second ν as a function of

(a) the volume V ; (b) the pressure p ; (c) the temperature T .

2.233. Determine the molar heat capacity of a polytropic process through which an ideal gas consisting of rigid diatomic molecules goes and in which the number of collisions between the molecules remains constant

(a) in a unit volume; (b) in the total volume of the gas.

2.234. An ideal gas of molar mass M is enclosed in a vessel of volume V whose thin walls are kept at a constant temperature T . At a moment $t = 0$ a small hole of area S is opened, and the gas starts escaping into vacuum. Find the gas concentration n as a function of time t if at the initial moment $n(0) = n_0$.

2.235. A vessel filled with gas is divided into two equal parts 1 and 2 by a thin heat-insulating partition with two holes. One hole has a small diameter, and the other has a very large diameter (in comparison with the mean free path of molecules). In part 2 the gas is kept at a temperature η times higher than that of part 1. How will the concentration of molecules in part 2 change and how many times after the large hole is closed?

2.236. As a result of a certain process the viscosity coefficient of an ideal gas increases $\alpha = 2.0$ times and its diffusion coefficient $\beta = 4.0$ times. How does the gas pressure change and how many times?

2.237. How will a diffusion coefficient D and the viscosity coefficient η of an ideal gas change if its volume increases n times:

(a) isothermally; (b) isobarically?

2.238. An ideal gas consists of rigid diatomic molecules. How will a diffusion coefficient D and viscosity coefficient η change and how many times if the gas volume is decreased adiabatically $n = 10$ times?

2.239. An ideal gas goes through a polytropic process. Find the polytropic exponent n if in this process the coefficient

(a) of diffusion; (b) of viscosity; (c) of heat conductivity remains constant.

2.240. Knowing the viscosity coefficient of helium under standard conditions, calculate the effective diameter of the helium atom.

2.241. The heat conductivity of helium is 8.7 times that of argon (under standard conditions). Find the ratio of effective diameters of argon and helium atoms.

2.242. Under standard conditions helium fills up the space between two long coaxial cylinders. The mean radius of the cylinders is equal to R , the gap between them is equal to ΔR , with $\Delta R \ll R$. The outer cylinder rotates with a fairly low angular velocity ω about the stationary inner cylinder. Find the moment of friction forces acting on a unit length of the inner cylinder. Down to what magnitude should the helium pressure be lowered (keeping the temperature constant) to decrease the sought moment of friction forces $n = 10$ times if $\Delta R = 6$ mm?

2.243. A gas fills up the space between two long coaxial cylinders of radii R_1 and R_2 , with $R_1 < R_2$. The outer cylinder rotates with a fairly low angular velocity ω about the stationary inner cylinder. The moment of friction forces acting on a unit length of the inner cylinder is equal to N_1 . Find the viscosity coefficient η of the gas taking into account that the friction force acting on a unit area of the cylindrical surface of radius r is determined by the formula $\sigma = \eta r (\partial \omega / \partial r)$.

2.244. Two identical parallel discs have a common axis and are located at a distance h from each other. The radius of each disc is equal to a , with $a \gg h$. One disc is rotated with a low angular velocity ω relative to the other, stationary, disc. Find the moment of friction forces acting on the stationary disc if the viscosity coefficient of the gas between the discs is equal to η .

2.245. Solve the foregoing problem, assuming that the discs are located in an ultra-rarefied gas of molar mass M , at temperature T and under pressure p .

2.246. Making use of Poiseuille's equation (1.7d), find the mass μ of gas flowing per unit time through the pipe of length l and radius a if constant pressures p_1 and p_2 are maintained at its ends.

2.247. One end of a rod, enclosed in a thermally insulating sheath, is kept at a temperature T_1 while the other, at T_2 . The rod is composed of two sections whose lengths are l_1 and l_2 and heat conductivity coefficients κ_1 and κ_2 . Find the temperature of the interface.

2.248. Two rods whose lengths are l_1 and l_2 and heat conductivity coefficients κ_1 and κ_2 are placed end to end. Find the heat conductivity coefficient of a uniform rod of length $l_1 + l_2$ whose conductivity is the same as that of the system of these two rods. The lateral surfaces of the rods are assumed to be thermally insulated.

2.249. A rod of length l with thermally insulated lateral surface consists of material whose heat conductivity coefficient varies with temperature as $\kappa = \alpha/T$, where α is a constant. The ends of the rod are kept at temperatures T_1 and T_2 . Find the function $T(x)$, where x is the distance from the end whose temperature is T_1 , and the heat flow density.

2.250. Two chunks of metal with heat capacities C_1 and C_2 are interconnected by a rod of length l and cross-sectional area S and fairly low heat conductivity κ . The whole system is thermally insulated from the environment. At a moment $t = 0$ the temperature difference between the two chunks of metal equals $(\Delta T)_0$. Assuming the heat capacity of the rod to be negligible, find the temperature difference between the chunks as a function of time.

2.251. Find the temperature distribution in a substance placed between two parallel plates kept at temperatures T_1 and T_2 . The plate separation is equal to l , the heat conductivity coefficient of the substance $\kappa \propto \sqrt{T}$.

2.252. The space between two large horizontal plates is filled with helium. The plate separation equals $l = 50$ mm. The lower plate is kept at a temperature $T_1 = 290$ K, the upper, at $T_2 = 330$ K. Find the heat flow density if the gas pressure is close to standard.

2.253. The space between two large parallel plates separated by a distance $l = 5.0$ mm is filled with helium under a pressure $p = 1.0$ Pa. One plate is kept at a temperature $t_1 = 17^\circ\text{C}$ and the other, at a temperature $t_2 = 37^\circ\text{C}$. Find the mean free path of helium atoms and the heat flow density.

2.254. Find the temperature distribution in the space between two coaxial cylinders of radii R_1 and R_2 filled with a uniform heat conducting substance if the temperatures of the cylinders are constant and are equal to T_1 and T_2 respectively.

2.255. Solve the foregoing problem for the case of two concentric spheres of radii R_1 and R_2 and temperatures T_1 and T_2 .

2.256. A constant electric current flows along a uniform wire with cross-sectional radius R and heat conductivity coefficient κ . A unit volume of the wire generates a thermal power w . Find the temperature distribution across the wire provided the steady-state temperature at the wire surface is equal to T_0 .

2.257. The thermal power of density w is generated uniformly inside a uniform sphere of radius R and heat conductivity coefficient κ . Find the temperature distribution in the sphere provided the steady-state temperature at its surface is equal to T_0 .