PART FIVE

OPTICS

5.1. PHOTOMETRY AND GEOMETRICAL OPTICS

- Spectral response of an eye $V(\lambda)$ is shown in Fig. 5.1.

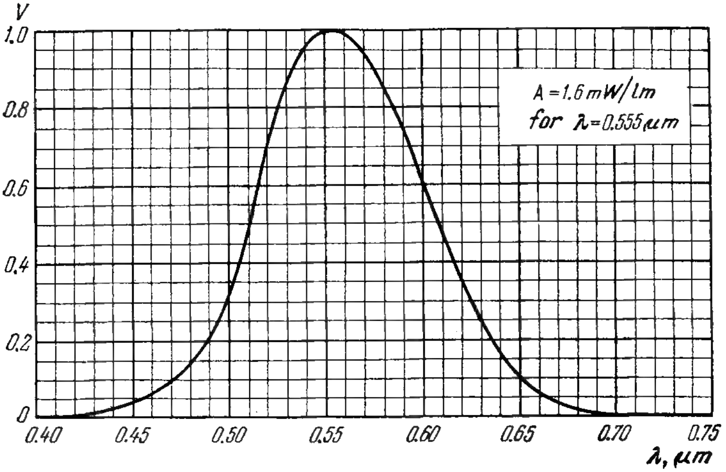

Fig. 5.1.

- Luminous intensity I and illuminance E:

$$I = \frac{d\Phi}{d\Omega}, \quad E = \frac{d\Phi_{inc}}{dS}. \tag{5.1a}$$

- Illuminance produced by a point isotropic source:

$$E = \frac{I \cos \alpha}{r^2}, \tag{5.1b}$$

where α is the angle between the normal to the surface and the direction to the source.

- Luminosity M and luminance L:

$$M = \frac{d\Phi_{emit}}{dS}, \quad L = \frac{d\Phi}{d\Omega\, \Delta S \cos \theta}. \tag{5.1c}$$

- For a Lambert source $L = \text{const}$ and luminosity

$$M = \pi L. \tag{5.1d}$$

• Relation between refractive angle θ of a prism and least deviation angle α:

$$\sin\frac{\alpha+\theta}{2}=n\sin\frac{\theta}{2}, \tag{5.1e}$$

where n is the refractive index of the prism.

• Equation of spherical mirror:

$$\frac{1}{s'}+\frac{1}{s}=\frac{2}{R}, \tag{5.1f}$$

where R is the curvature radius of the mirror.

• Equations for aligned optical system (Fig. 5.2):

$$\frac{n'}{s'}-\frac{n}{s}=\Phi,\quad \frac{f'}{s'}+\frac{f}{s}=1,\quad xx'=ff'. \tag{5.1g}$$

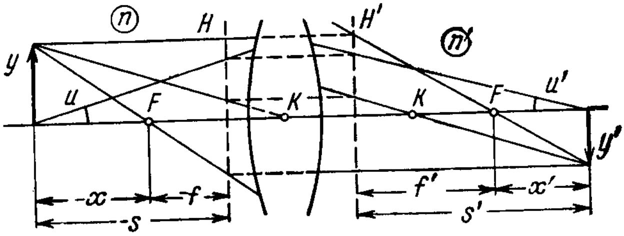

Fig. 5.2.

• Relations between focal lengths and optical power:

$$f'=\frac{n'}{\Phi},\quad f=-\frac{n}{\Phi},\quad \frac{f'}{f}=-\frac{n'}{n}. \tag{5.1h}$$

• Optical power of a spherical refractive surface:

$$\Phi=\frac{n'-n}{R}. \tag{5.1i}$$

• Optical power of a thin lens in a medium with refractive index n_0:

$$\Phi=(n-n_0)\left(\frac{1}{R_1}-\frac{1}{R_2}\right), \tag{5.1j}$$

where n is the refractive index of the lens.

• Optical power of a thick lens:

$$\Phi=\Phi_1+\Phi_2-\frac{d}{n}\Phi_1\Phi_2, \tag{5.1k}$$

where d is the thickness of the lens. This equation is also valid for a system of two thin lenses separated by a medium with refractive index n.

• Principal planes H and H' are removed from the crest points O and O' of surfaces of a thick lens (Fig. 5.3) by the following distances:

$$X = \frac{d}{n}\frac{\Phi_2}{\Phi}, \quad X' = -\frac{d}{n}\frac{\Phi_1}{\Phi}. \qquad (5.1l)$$

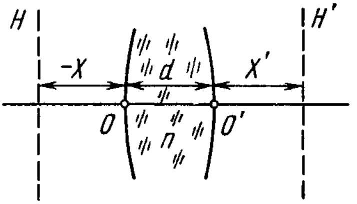

Fig. 5.3.

• Lagrange-Helmholtz invariant:

$$nyu = \text{const.} \qquad (5.1m)$$

• Magnifying power of an optical device:

$$\Gamma = \frac{\tan \psi'}{\tan \psi}, \qquad (5.1n)$$

where ψ' and ψ are the angles subtended at the eye by an image formed by the optical device and by the corresponding object at a distance for convenient viewing (in the case of a microscope or magnifying glass that distance is equal to $l_0 = 25$ cm).

5.1. Making use of the spectral response curve for an eye (see Fig. 5.1), find:

(a) the energy flux corresponding to the luminous flux of 1.0 lm at the wavelengths 0.51 and 0.64 μm;

(b) the luminous flux corresponding to the wavelength interval from 0.58 to 0.63 μm if the respective energy flux, equal to $\Phi_e = 4.5$ mW, is uniformly distributed over all wavelengths of the interval. The function $V(\lambda)$ is assumed to be linear in the given spectral interval.

5.2. A point isotropic source emits a luminous flux $\Phi = 10$ lm with wavelength $\lambda = 0.59$ μm. Find the peak strength values of electric and magnetic fields in the luminous flux at a distance $r = 1.0$ m from the source. Make use of the curve illustrated in Fig. 5.1.

5.3. Find the mean illuminance of the irradiated part of an opaque sphere receiving

(a) a parallel luminous flux resulting in illuminance E_0 at the point of normal incidence;

(b) light from a point isotropic source located at a distance $l = 100$ cm from the centre of the sphere; the radius of the sphere is $R = 60$ cm and the luminous intensity is $I = 36$ cd.

5.4. Determine the luminosity of a surface whose luminance depends on direction as $L = L_0 \cos\theta$, where θ is the angle between the radiation direction and the normal to the surface.

5.5. A certain luminous surface obeys Lambert's law. Its luminance is equal to L. Find:

(a) the luminous flux emitted by an element ΔS of this surface into a cone whose axis is normal to the given element and whose aperture angle is equal to θ;

(b) the luminosity of such a source.

5.6. An illuminant shaped as a plane horizontal disc $S = 100$ cm^2 in area is suspended over the centre of a round table of radius $R = 1.0$ m. Its luminance does not depend on direction and is equal to $L = 1.6 \cdot 10^4$ cd/m^2. At what height over the table should the illuminant be suspended to provide maximum illuminance at the circumference of the table? How great will that illuminance be? The illuminant is assumed to be a point source.

5.7. A point source is suspended at a height $h = 1.0$ m over the centre of a round table of radius $R = 1.0$ m. The luminous intensity I of the source depends on direction so that illuminance at all points of the table is the same. Find the function $I(\theta)$, where θ is the angle between the radiation direction and the vertical, as well as the luminous flux reaching the table if $I(0) = I_0 = 100$ cd.

5.8. A vertical shaft of light from a projector forms a light spot $S = 100$ cm^2 in area on the ceiling of a round room of radius $R = 2.0$ m. The illuminance of the spot is equal to $E = 1000$ lx. The reflection coefficient of the ceiling is equal to $\rho = 0.80$. Find the maximum illuminance of the wall produced by the light reflected from the ceiling. The reflection is assumed to obey Lambert's law.

5.9. A luminous dome shaped as a hemisphere rests on a horizontal plane. Its luminosity is uniform. Determine the illuminance at the centre of that plane if its luminance equals L and is independent of direction.

5.10. A Lambert source has the form of an infinite plane. Its luminance is equal to L. Find the illuminance of an area element oriented parallel to the given source.

5.11. An illuminant shaped as a plane horizontal disc of radius $R = 25$ cm is suspended over a table at a height $h = 75$ cm. The illuminance of the table below the centre of the illuminant is equal to $E_0 = 70$ lx. Assuming the source to obey Lambert's law, find its luminosity.

5.12. A small lamp having the form of a uniformly luminous sphere of radius $R = 6.0$ cm is suspended at a height $h = 3.0$ m above the floor. The luminance of the lamp is equal to $L = 2.0 \cdot 10^4$ cd/m^2 and is independent of direction. Find the illuminance of the floor directly below the lamp.

5.13. Write the law of reflection of a light beam from a mirror in vector form, using the directing unit vectors $\mathbf{e}$ and $\mathbf{e}'$ of the incident and reflected beams and the unit vector **n** of the outside normal to the mirror surface.

5.14. Demonstrate that a light beam reflected from three mutually perpendicular plane mirrors in succession reverses its direction.

5.15. At what value of the angle of incident θ_1 is a shaft of light reflected from the surface of water perpendicular to the refracted shaft?

5.16. Two optical media have a plane boundary between them. Suppose θ_{1cr} is the critical angle of incidence of a beam and θ_1 is the angle of incidence at which the refracted beam is perpendicular to the reflected one (the beam is assumed to come from an optically denser medium). Find the relative refractive index of these media if $\sin\theta_{1cr}/\sin\theta_1 = \eta = 1.28$.

5.17. A light beam falls upon a plane-parallel glass plate $d=6.0$ cm in thickness. The angle of incidence is $\theta = 60°$. Find the value of deflection of the beam which passed through that plate.

5.18. A man standing on the edge of a swimming pool looks at a stone lying on the bottom. The depth of the swimming pool is equal to h. At what distance from the surface of water is the image of the stone formed if the line of vision makes an angle θ with the normal to the surface?

5.19. Demonstrate that in a prism with small refracting angle θ the shaft of light deviates through the angle $\alpha \simeq (n-1)\,\theta$ regardless of the angle of incidence, provided that the latter is also small.

5.20. A shaft of light passes through a prism with refracting angle θ and refractive index n. Let α be the diffraction angle of the shaft. Demonstrate that if the shaft of light passes through the prism symmetrically,

(a) the angle α is the least;

(b) the relationship between the angles α and θ is defined by Eq. (5.1e).

5.21. The least deflection angle of a certain glass prism is equal to its refracting angle. Find the latter.

5.22. Find the minimum and maximum deflection angles for a light ray passing through a glass prism with refracting angle $\theta = 60°$.

5.23. A trihedral prism with refracting angle 60° provides the least deflection angle 37° in air. Find the least deflection angle of that prism in water.

5.24. A light ray composed of two monochromatic components passes through a trihedral prism with refracting angle $\theta = 60°$. Find the angle $\Delta\alpha$ between the components of the ray after its passage through the prism if their respective indices of refraction are equal to 1.515 and 1.520. The prism is oriented to provide the least deflection angle.

5.25. Using Fermat's principle derive the laws of deflection and refraction of light on the plane interface between two media.

5.26. By means of plotting find:

(a) the path of a light ray after reflection from a concave and convex spherical mirrors (see Fig. 5.4, where F is the focal point, OO' is the optical axis);

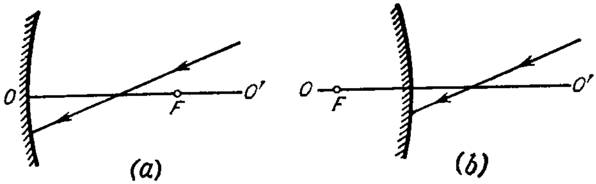

Fig. 5.4.

(b) the positions of the mirror and its focal point in the cases illustrated in Fig. 5.5, where P and P' are the conjugate points.

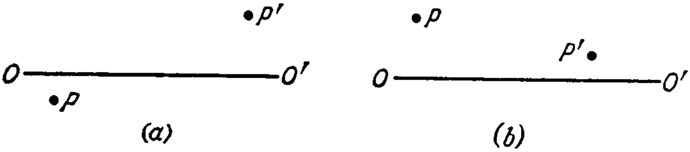

Fig. 5.5.

5.27. Determine the focal length of a concave mirror if:

(a) with the distance between an object and its image being equal to $l = 15$ cm, the transverse magnification $\beta = -2.0$;

(b) in a certain position of the object the transverse magnification is $\beta_1 = -0.50$ and in another position displaced with respect to the former by a distance $l = 5.0$ cm the transverse magnification $\beta_2 = -0.25$.

5.28. A point source with luminous intensity $I_0 = 100$ cd is positioned at a distance $s = 20.0$ cm from the crest of a concave mirror with focal length $f = 25.0$ cm. Find the luminous intensity of the reflected ray if the reflection coefficient of the mirror is $\rho = 0.80$.

5.29. Proceeding from Fermat's principle derive the refraction formula for paraxial rays on a spherical boundary surface of radius R between media with refractive indices n and n'.

5.30. A parallel beam of light falls from vacuum on a surface enclosing a medium with refractive index n (Fig. 5.6). Find the shape of that surface, $x(r)$, if the beam is brought into focus at the point F at a distance f from the crest O. What is the maximum radius of a beam that can still be focussed?

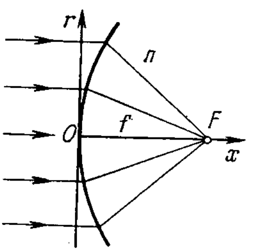

Fig. 5.6.

5.31. A point source is located at a distance of 20 cm from the front surface of a symmetrical glass biconvex lens. The lens is 5.0 cm thick and the curvature radius of its surfaces is 5.0 cm. How far beyond the rear surface of this lens is the image of the source formed?

5.32. An object is placed in front of convex surface of a glass plano-convex lens of thickness $d = 9.0$ cm. The image of that object is formed on the plane surface of the lens serving as a screen. Find:

(a) the transverse magnification if the curvature radius of the lens's convex surface is $R = 2.5$ cm;

(b) the image illuminance if the luminance of the object is $L = 7700$ cd/m^2 and the entrance aperture diameter of the lens is $D = 5.0$ mm; losses of light are negligible.

5.33. Find the optical power and the focal lengths

(a) of a thin glass lens in liquid with refractive index $n_0 = 1.7$ if its optical power in air is $\Phi_0 = -5.0$ D;

(b) of a thin symmetrical biconvex glass lens, with air on one side and water on the other side, if the optical power of that lens in air is $\Phi_0 = +10$ D.

5.34. By means of plotting find:

(a) the path of a ray of light beyond thin converging and diverging lenses (Fig. 5.7, where OO' is the optical axis, F and F' are the front and rear focal points);

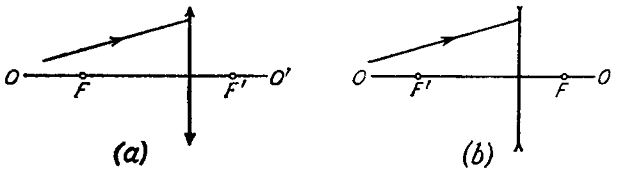

Fig. 5.7.

(b) the position of a thin lens and its focal points if the position of the optical axis OO' and the positions of the cojugate points P, P' (see Fig. 5.5) are known; the media on both sides of the lenses are identical;

(c) the path of ray *2* beyond the converging and diverging lenses (Fig. 5.8) if the path of ray *1* and the positions of the lens and of its

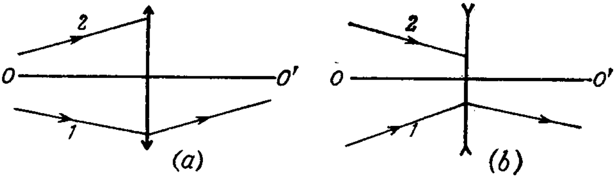

Fig. 5.8.

optical axis OO' are all known; the media on both sides of the lenses are identical.

5.35. A thin converging lens with focal length $f = 25$ cm projects the image of an object on a screen removed from the lens by a distance $l = 5.0$ m. Then the screen was drawn closer to the lens by a distance $\Delta l = 18$ cm. By what distance should the object be shifted for its image to become sharp again?

5.36. A source of light is located at a distance $l = 90$ cm from a screen. A thin converging lens provides the sharp image of the source when placed between the source of light and the screen at two positions. Determine the focal length of the lens if

(a) the distance between the two positions of the lens is $\Delta l = 30$ cm;

(b) the transverse dimensions of the image at one position of the lens are $\eta = 4.0$ greater than those at the other position.

5.37. A thin converging lens is placed between an object and a screen whose positions are fixed. There are two positions of the lens at which the sharp image of the object is formed on the screen. Find the transverse dimension of the object if at one position of the lens the image dimension equals $h' = 2.0$ mm and at the other, $h'' = 4.5$ mm.

5.38. A thin converging lens with aperture ratio $D : f = 1 : 3.5$ (D is the lens diameter, f is its focal length) provides the image of a sufficiently distant object on a photographic plate. The object luminance is $L = 260$ cd/m^2. The losses of light in the lens amount to $\alpha = 0.10$. Find the illuminance of the image.

5.39. How does the luminance of a real image depend on diameter D of a thin converging lens if that image is observed

(a) directly;

(b) on a white screen backscattering according to Lambert's law?

5.40. There are two thin symmetrical lenses: one is converging, with refractive index $n_1 = 1.70$, and the other is diverging with refractive index $n_2 = 1.51$. Both lenses have the same curvature radius of their surfaces equal to $R = 10$ cm. The lenses were put close together and submerged into water. What is the focal length of this system in water?

5.41. Determine the focal length of a concave spherical mirror which is manufactured in the form of a thin symmetric biconvex glass lens one of whose surfaces is silvered. The curvature radius of the lens surface is $R = 40$ cm.

5.42. Figure 5.9 illustrates an aligned system consisting of three thin lenses. The system is located in air. Determine:

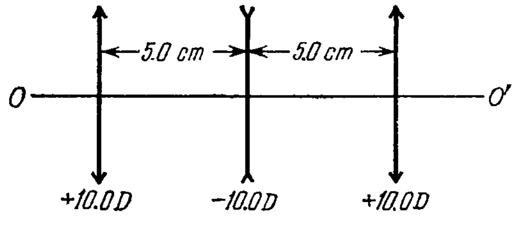

Fig. 5.9.

(a) the position of the point of convergence of a parallel ray incoming from the left after passing through the system;

(b) the distance between the first lens and a point lying on the axis to the left of the system, at which that point and its image are located symmetrically with respect to the lens system.

5.43. A Galilean telescope of 10-fold magnification has the length of 45 cm when adjusted to infinity. Determine:

(a) the focal lengths of the telescope's objective and ocular;

(b) by what distance the ocular should be displaced to adjust the telescope to the distance of 50 m.

5.44. Find the magnification of a Keplerian telescope adjusted to infinity if the mounting of the objective has a diameter D and the image of that mounting formed by the telescope's ocular has a diameter d.

5.45. On passing through a telescope a flux of light increases its intensity $\eta = 4.0 \cdot 10^4$ times. Find the angular dimension of a distant object if its image formed by that telescope has an angular dimension $\psi' = 2.0°$.

5.46. A Keplerian telescope with magnification $\Gamma = 15$ was submerged into water which filled up the inside of the telescope. To make the system work as a telescope again within the former dimensions, the objective was replaced. What has the magnification of the telescope become equal to? The refractive index of the glass of which the ocular is made is equal to $n = 1.50$.

5.47. At what magnification Γ of a telescope with a diameter of the objective $D = 6.0$ cm is the illuminance of the image of an object on the retina not less than without the telescope? The pupil diameter is assumed to be equal to $d_0 = 3.0$ mm. The losses of light in the telescope are negligible.

5.48. The optical powers of the objective and the ocular of a microscope are equal to 100 and 20 D respectively. The microscope magnification is equal to 50. What will the magnification of the microscope be when the distance between the objective and the ocular is increased by 2.0 cm?

5.49. A microscope has a numerical aperture $\sin \alpha = 0.12$, where α is the aperture angle subtended by the entrance pupil of the microscope. Assuming the diameter of an eye's pupil to be equal to $d_0 = 4.0$ mm, determine the microscope magnification at which

(a) the diameter of the beam of light coming from the microscope is equal to the diameter of the eye's pupil;

(b) the illuminance of the image on the retina is independent of magnification (consider the case when the beam of light passing through the system "microscope-eye" is bounded by the mounting of the objective).

5.50. Find the positions of the principal planes, the focal and nodal points of a thin biconvex symmetric glass lens with curvature radius of its surfaces equal to $R = 7.50$ cm. There is air on one side of the lens and water on the other.

5.51. By means of plotting find the positions of focal points and principal planes of aligned optical systems illustrated in Fig. 5.10:
(a) a telephoto lens, that is a combination of a converging and a diverging thin lenses ($f_1 = 1.5\ a$, $f_2 = -1.5\ a$);

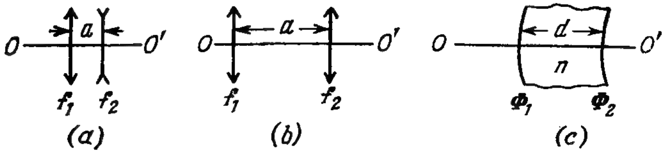

Fig. 5.10.

(b) a system of two thin converging lenses ($f_1 = 1.5\ a$, $f_2 = 0.5\ a$);
(c) a thick convex-concave lens ($d = 4$ cm, $n = 1.5$, $\Phi_1 = +50$ D, $\Phi_2 = -50$ D).

5.52. An optical system is located in air. Let OO' be its optical axis, F and F' are the front and rear focal points, H and H' are the front and rear principal planes, P and P' are the conjugate points. By means of plotting find:
(a) the positions F' and H' (Fig. 5.11a);
(b) the position of the point S' conjugate to the point S (Fig. 5.11b);

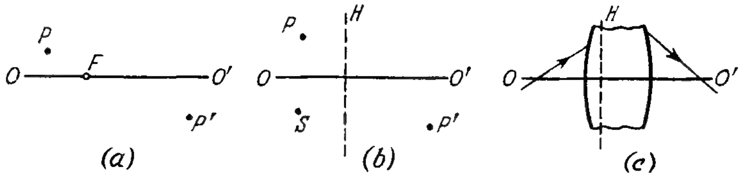

Fig. 5.11.

(c) the positions F, F', and H' (Fig. 5.11c, where the path of the ray of light is shown before and after passing through the system).

5.53. Suppose F and F' are the front and rear focal points of an optical system, and H and H' are its front and rear principal points. By means of plotting find the position of the image S' of the point S for the following relative positions of the points S, F, F', H, and H':
(a) $FSHH'F'$; (b) $HSF'FH'$; (c) $H'SF'FH$; (d) $F'H'SHF$.

5.54. A telephoto lens consists of two thin lenses, the front converging lens and the rear diverging lens with optical powers $\Phi_1 = +10$ D and $\Phi_2 = -10$ D. Find:
(a) the focal length and the positions of principal axes of that system if the lenses are separated by a distance $d = 4.0$ cm;
(b) the distance d between the lenses at which the ratio of a focal length f of the system to a distance l between the converging lens and the rear principal focal point is the highest. What is this ratio equal to?

5.55. Calculate the positions of the principal planes and focal points of a thick convex-concave glass lens if the curvature radius of the convex surface is equal to $R_1 = 10.0$ cm and of the concave surface to $R_2 = 5.0$ cm and the lens thickness is $d = 3.0$ cm.

5.56. An aligned optical system consists of two thin lenses with focal lengths f_1 and f_2, the distance between the lenses being equal to d. The given system has to be replaced by one thin lens which, at any position of an object, would provide the same transverse magnification as the system. What must the focal length of this lens be equal to and in what position must it be placed with respect to the two-lens system?

5.57. A system consists of a thin symmetrical converging glass lens with the curvature radius of its surfaces $R = 38$ cm and a plane mirror oriented at right angles to the optical axis of the lens. The distance between the lens and the mirror is $l = 12$ cm. What is the optical power of this system when the space between the lens and the mirror is filled up with water?

5.58. At what thickness will a thick convex-concave glass lens in air

(a) serve as a telescope provided the curvature radius of its convex surface is $\Delta R = 1.5$ cm greater than that of its concave surface?

(b) have the optical power equal to -1.0 D if the curvature radii of its convex and concave surfaces are equal to 10.0 and 7.5 cm respectively?

5.59. Find the positions of the principal planes, the focal length and the sign of the optical power of a thick convex-concave glass lens

(a) whose thickness is equal to d and curvature radii of the surfaces are the same and equal to R;

(b) whose refractive surfaces are concentric and have the curvature radii R_1 and R_2 ($R_2 > R_1$).

5.60. A telescope system consists of two glass balls with radii $R_1 = 5.0$ cm and $R_2 = 1.0$ cm. What are the distance between the centres of the balls and the magnification of the system if the bigger ball serves as an objective?

5.61. Two identical thick symmetrical biconvex lenses are put close together. The thickness of each lens equals the curvature radius of its surfaces, i.e. $d = R = 3.0$ cm. Find the optical power of this system in air.

5.62. A ray of light propagating in an isotropic medium with refractive index n varying gradually from point to point has a curvature radius ρ determined by the formula

$$\frac{1}{\rho} = \frac{\partial}{\partial N} (\ln n),$$

where the derivative is taken with respect to the principal normal to the ray. Derive this formula, assuming that in such a medium the law of refraction $n \sin \theta = \text{const}$ holds. Here θ is the angle between the ray and the direction of the vector ∇n at a given point.

5.63. Find the curvature radius of a ray of light propagating in a horizontal direction close to the Earth's surface where the gradient of the refractive index in air is equal to approximately $3 \cdot 10^{-8}\ \mathrm{m}^{-1}$. At what value of that gradient would the ray of light propagate all the way round the Earth?

5.2. INTERFERENCE OF LIGHT

• Width of a fringe:

$$\Delta x = \frac{l}{d} \lambda, \tag{5.2a}$$

where l is the distance from the sources to the screen, d is the distance between the sources.

• Temporal and spatial coherences. Coherence length and coherence radius:

$$l_{coh} \approx \frac{\lambda^2}{\Delta\lambda}, \quad \rho_{coh} \approx \frac{\lambda}{\psi}, \tag{5.2b}$$

where ψ is the angular dimension of the source.

• Condition for interference maxima in the case of light reflected from a thin plate of thickness b:

$$2b\sqrt{n^2 - \sin^2 \theta_1} = (k + 1/2)\,\lambda, \tag{5.2c}$$

where k is an integer.

• Newton's rings produced on reflection of light from the surfaces of an air interlayer formed between a lens of radius R and a glass plate with which the convex surface of the lens is in contact. The radii of the rings:

$$r = \sqrt{\lambda R k/2}, \tag{5.2d}$$

with the rings being bright if $k = 1, 3, 5, \ldots$, and dark if $k = 2, 4, 6, \ldots$ The value $k = 0$ corresponds to the middle of the central dark spot.

5.64. Demonstrate that when two harmonic oscillations are added, the time-averaged energy of the resultant oscillation is equal to the sum of the energies of the constituent oscillations, if both of them

(a) have the same direction and are incoherent, and all the values of the phase difference between the oscillations are equally probable;

(b) are mutually perpendicular, have the same frequency and an arbitrary phase difference.

5.65. By means of plotting find the amplitude of the oscillation resulting from the addition of the following three oscillations of the same direction:

$$\xi_1 = a \cos \omega t, \quad \xi_2 = 2a \sin \omega t, \quad \xi_3 = 1.5a \cos(\omega t + \pi/3).$$

5.66. A certain oscillation results from the addition of coherent oscillations of the same direction $\xi_k = a \cos[\omega t + (k - 1)\varphi]$, where k is the number of the oscillation ($k = 1, 2, \ldots, N$), φ is the phase difference between the kth and $(k - 1)$th oscillations. Find the amplitude of the resultant oscillation.

5.67. A system illustrated in Fig. 5.12 consists of two coherent point sources *1* and *2* located in a certain plane so that their dipole moments are oriented at right angles to that plane. The sources are separated by a distance d, the radiation wavelength is equal to λ. Taking into account that the oscillations of source *2* lag in phase behind the oscillations of source *1* by φ ($\varphi < \pi$), find:

(a) the angles θ at which the radiation intensity is maximum;

(b) the conditions under which the radiation intensity in the direction $\theta = \pi$ is maximum and in the opposite direction, minimum.

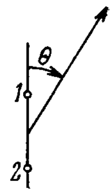

Fig. 5.12.

5.68. A stationary radiating system consists of a linear chain of parallel oscillators separated by a distance d, with the oscillation phase varying linearly along the chain. Find the time dependence of the phase difference $\Delta\varphi$ between the neighbouring oscillators at which the principal radiation maximum of the system will be "scanning" the surroundings with the constant angular velocity ω.

5.69. In Lloyd's mirror experiment (Fig. 5.13) a light wave emitted directly by the source S (narrow slit) interferes with the wave reflected from a mirror M. As a result, an interference fringe pattern is

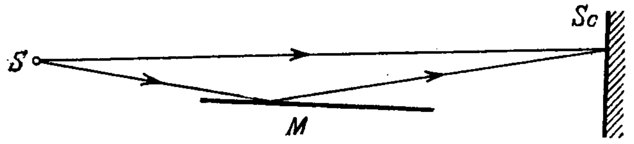

Fig. 5.13.

formed on the screen Sc. The source and the mirror are separated by a distance $l = 100$ cm. At a certain position of the source the fringe width on the screen was equal to $\Delta x = 0.25$ mm, and after the source was moved away from the mirror plane by $\Delta h = 0.60$ mm, the fringe width decreased $\eta = 1.5$ times. Find the wavelength of light.

5.70. Two coherent plane light waves propagating with a divergence angle $\psi \ll 1$ fall almost normally on a screen. The amplitudes of the waves are equal. Demonstrate that the distance between the neighbouring maxima on the screen is equal to $\Delta x = \lambda/\psi$, where λ is the wavelength.

5.71. Figure 5.14 illustrates the interference experiment with Fresnel mirrors. The angle between the mirrors is $\alpha = 12'$, the distances from the mirrors' intersection line to the narrow slit S and the screen Sc are equal to $r = 10.0$ cm and $b = 130$ cm respectively. The wavelength of light is $\lambda = 0.55$ μm. Find:

(a) the width of a fringe on the screen and the number of possible maxima;

(b) the shift of the interference pattern on the screen when the slit is displaced by $\delta l = 1.0$ mm along the arc of radius r with centre at the point O;

(c) at what maximum width δ_{max} of the slit the interference fringes on the screen are still observed sufficiently sharp.

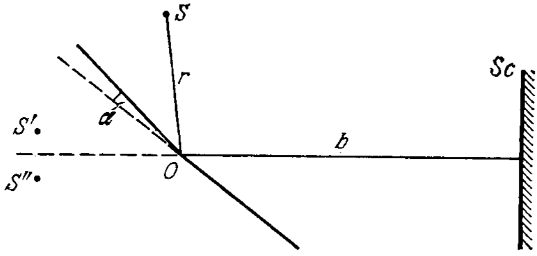

Fig. 5.14.

5.72. A plane light wave falls on Fresnel mirrors with an angle $\alpha = 2.0'$ between them. Determine the wavelength of light if the width of the fringe on the screen $\Delta x = 0.55$ mm.

5.73. A lens of diameter 5.0 cm and focal length $f = 25.0$ cm was cut along the diameter into two identical halves. In the process, the layer of the lens $a = 1.00$ mm in thickness was lost. Then the halves were put together to form a composite lens. In this focal plane a narrow slit was placed, emitting monochromatic light with wavelength $\lambda = 0.60$ μm. Behind the lens a screen was located at a distance $b = 50$ cm from it. Find:

(a) the width of a fringe on the screen and the number of possible maxima;

(b) the maximum width of the slit δ_{max} at which the fringes on the screen will be still observed sufficiently sharp.

5.74. The distances from a Fresnel biprism to a narrow slit and a screen are equal to $a = 25$ cm and $b = 100$ cm respectively. The refracting angle of the glass biprism is equal to $\theta = 20'$. Find the wavelength of light if the width of the fringe on the screen is $\Delta x = 0.55$ mm.

5.75. A plane light wave with wavelength $\lambda = 0.70$ μm falls normally on the base of a biprism made of glass ($n = 1.520$) with refracting angle $\theta = 5.0°$. Behind the biprism (Fig. 5.15) there is a plane-parallel plate, with the space between them filled up with benzene ($n' = 1.500$). Find the width of a fringe on the screen Sc placed behind this system.

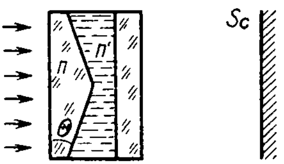

Fig. 5.15.

5.76. A plane monochromatic light wave falls normally on a diaphragm with two narrow slits separated by a distance $d = 2.5$ mm.

A fringe pattern is formed on a screen placed at a distance $l = 100$ cm behind the diaphragm. By what distance and in which direction will these fringes be displaced when one of the slits is covered by a glass plate of thickness $h = 10$ µm?

5.77. Figure 5.16 illustrates an interferometer used in measurements of refractive indices of transparent substances. Here S is

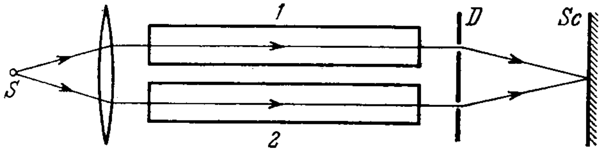

Fig. 5.16.

a narrow slit illuminated by monochromatic light with wavelength $\lambda = 589$ nm, *1* and *2* are identical tubes with air of length $l = 10.0$ cm each, D is a diaphragm with two slits. After the air in tube *1* was replaced with ammonia gas, the interference pattern on the screen Sc was displaced upward by $N = 17$ fringes. The refractive index of air is equal to $n = 1.000277$. Determine the refractive index of ammonia gas.

5.78. An electromagnetic wave falls normally on the boundary between two isotropic dielectrics with refractive indices n_1 and n_2. Making use of the continuity condition for the tangential components, **E** and **H** across the boundary, demonstrate that at the interface the electric field vector **E**

(a) of the transmitted wave experiences no phase jump;

(b) of the reflected wave is subjected to the phase jump equal to π if it is reflected from a medium of higher optical density.

5.79. A parallel beam of white light falls on a thin film whose refractive index is equal to $n = 1.33$. The angle of indices is $\theta_1 = 52°$. What must the film thickness be equal to for the reflected light to be coloured yellow ($\lambda = 0.60$ µm) most intensively?

5.80. Find the minimum thickness of a film with refractive index 1.33 at which light with wavelength 0.64 µm experiences maximum reflection while light with wavelength 0.40 µm is not reflected at all. The incidence angle of light is equal to 30°.

5.81. To decrease light losses due to reflection from the glass surface the latter is coated with a thin layer of substance whose refractive index $n' = \sqrt{n}$, where n is the refractive index of the glass. In this case the amplitudes of electromagnetic oscillations reflected from both coated surfaces are equal. At what thickness of that coating is the glass reflectivity in the direction of the normal equal to zero for light with wavelength λ?

5.82. Diffused monochromatic light with wavelength $\lambda = 0.60$ µm falls on a thin film with refractive index $n = 1.5$. Determine the film thickness if the angular separation of neighbouring maxima observed in reflected light at the angles close to $\theta = 45°$ to the normal is equal to $\delta\theta = 3.0°$.

5.83. Monochromatic light passes through an orifice in a screen Sc (Fig. 5.17) and being reflected from a thin transparent plate P produces fringes of equal inclination on the screen. The thickness of the plate is equal to d, the distance between the plate and the screen is l, the radii of the ith and kth dark rings are r_i and r_k. Find the wavelength of light taking into account that $r_{i,k} \ll l$.

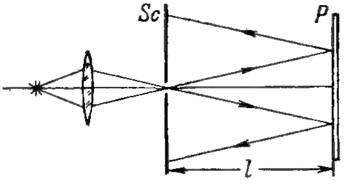

Fig. 5.17.

5.84. A plane monochromatic light wave with wavelength λ falls on the surface of a glass wedge whose faces form an angle $\alpha \ll 1$. The plane of incidence is perpendicular to the edge, the angle of incidence is θ_1. Find the distance between the neighbouring fringe maxima on the screen placed at right angles to reflected light.

5.85. Light with wavelength $\lambda = 0.55$ µm from a distant point source falls normally on the surface of a glass wedge. A fringe pattern whose neighbouring maxima on the surface of the wedge are separated by a distance $\Delta x = 0.21$ mm is observed in reflected light. Find:

(a) the angle between the wedge faces;

(b) the degree of light monochromatism ($\Delta\lambda/\lambda$) if the fringes disappear at a distance $l \simeq 1.5$ cm from the wedge's edge.

5.86. The convex surface of a plano-convex glass lens comes into contact with a glass plate. The curvature radius of the lens's convex surface is R, the wavelength of light is equal to λ. Find the width Δr of a Newton ring as a function of its radius r in the region where $\Delta r \ll r$.

5.87. The convex surface of a plano-convex glass lens with curvature radius $R = 40$ cm comes into contact with a glass plate. A certain ring observed in reflected light has a radius $r = 2.5$ mm. Watching the given ring, the lens was gradually removed from the plate by a distance $\Delta h = 5.0$ µm. What has the radius of that ring become equal to?

5.88. At the crest of a spherical surface of a plano-convex lens there is a ground-off plane spot of radius $r_0 = 3.0$ mm through which the lens comes into contact with a glass plate. The curvature radius of the lens's convex surface is equal to $R = 150$ cm. Find the radius of the sixth bright ring when observed in reflected light with wavelength $\lambda = 655$ nm.

5.89. A plano-convex glass lens with curvature radius of spherical surface $R = 12.5$ cm is *pressed* against a glass plate. The diameters of the tenth and fifteenth dark Newton's rings in reflected light are equal to $d_1 = 1.00$ mm and $d_2 = 1.50$ mm. Find the wavelength of light.

5.90. Two plano-convex thin glass lenses are brought into contact with their spherical surfaces. Find the optical power of such a system if in reflected light with wavelength $\lambda = 0.60$ μm the diameter of the fifth bright ring is $d = 1.50$ mm.

5.91. Two thin symmetric glass lenses, one biconvex and the other biconcave, are brought into contact to make a system with optical power $\Phi = 0.50$ D. Newton's rings are observed in reflected light with wavelength $\lambda = 0.61$ μm. Determine:

(a) the radius of the tenth dark ring;

(b) how the radius of that ring will change when the space between the lenses is filled up with water.

5.92. The spherical surface of a plano-convex lens comes into contact with a glass plate. The space between the lens and the plate is filled up with carbon dioxide. The refractive indices of the lens, carbon dioxide, and the plate are equal to $n_1 = 1.50$, $n_2 = 1.63$, and $n_3 = 1.70$ respectively. The curvature radius of the spherical surface of the lens is equal to $R = 100$ cm. Determine the radius of the fifth dark Newton's ring in reflected light with wavelength $\lambda = 0.50$ μm.

5.93. In a two-beam interferometer the orange mercury line composed of two wavelengths $\lambda_1 = 576.97$ nm and $\lambda_2 = 579.03$ nm is employed. What is the least order of interference at which the sharpness of the fringe pattern is the worst?

5.94. In Michelson's interferometer the yellow sodium line composed of two wavelengths $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm was used. In the process of translational displacement of one of the mirrors the interference pattern vanished periodically (why?). Find the displacement of the mirror between two successive appearances of the sharpest pattern.

5.95. When a Fabry-Perot étalon is illuminated by monochromatic light with wavelength λ an interference pattern, the system of con-

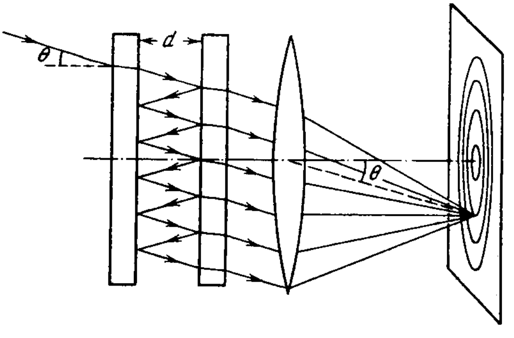

Fig. 5.18.

centric rings, appears in the focal plane of a lens (Fig. 5.18). The thickness of the étalon is equal to d. Determine how

(a) the position of rings;
(b) the angular width of fringes
depends on the order of interference.

5.96. For the Fabry-Perot étalon of thickness $d = 2.5$ cm find:
(a) the highest order of interference of light with wavelength $\lambda = 0.50$ µm;
(b) the dispersion region $\Delta\lambda$, i.e. the spectral interval of wavelengths, within which there is still no overlap with other orders of interference if the observation is carried out approximately at wavelength $\lambda = 0.50$ µm.

5.3. DIFFRACTION OF LIGHT

• Radius of the periphery of the kth Fresnel zone:

$$r_k = \sqrt{k\lambda \frac{ab}{a+b}}, \quad k = 1, 2, 3, \ldots \tag{5.3a}$$

• Cornu's spiral (Fig. 5.19). The numbers along that spiral correspond to the values of parameter v. In the case of a plane wave $v = x\sqrt{2/b\lambda}$, where x

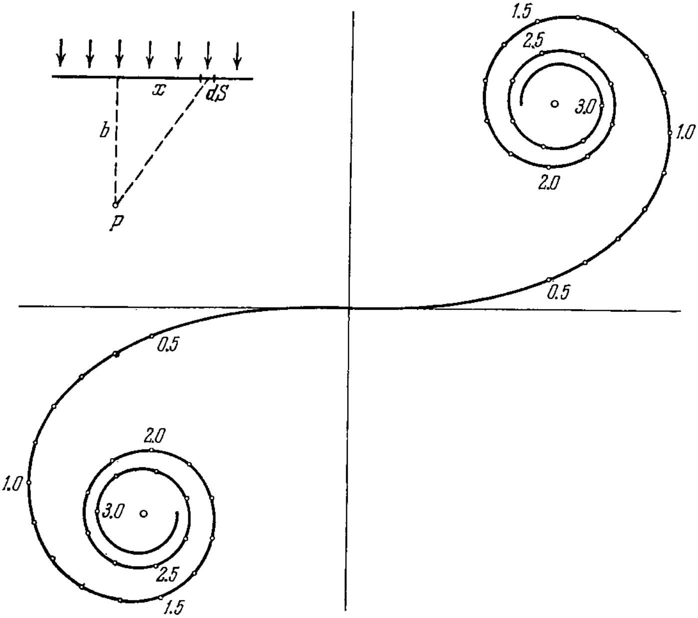

Fig. 5.19.

and b are the distances defining the position of the element dS of a wavefront relative to the observation point P as shown in the upper left corner of the figure.

• Fraunhofer diffraction produced by light falling normally from a slit. Condition of intensity minima:

$$b \sin \theta = \pm k\lambda, \quad k = 1, 2, 3, \ldots, \tag{5.3b}$$

where b is the width of the slit, θ is the diffraction angle.

• Diffraction grating, with light falling normally. The main Fraunhofer maxima appear under the condition

$$d \sin \theta = \pm k\lambda, \quad k = 0, 1, 2, \ldots \tag{5.3c}$$

the condition of additional minima:

$$d \sin \theta = \pm \frac{k'}{N} \lambda, \tag{5.3d}$$

where $k' = 1, 2, \ldots$, except for $0, N, 2N, \ldots$.

• Angular dispersion of a diffraction grating:

$$D = \frac{\delta\theta}{\delta\lambda} = \frac{k}{d \cos \theta}. \tag{5.3e}$$

• Resolving power of a diffraction grating:

$$R = \frac{\lambda}{\delta\lambda} = kN, \tag{5.3f}$$

where N is the number of lines of the grating.

• Resolving power of an objective

$$R = \frac{1}{\delta\psi} = \frac{D}{1.22\,\lambda}, \tag{5.3g}$$

where $\delta\psi$ is the least angular separation resolved by the objective, D is the diameter of the objective.

• Bragg's equation. The condition of diffraction maxima:

$$2d \sin \alpha = \pm k\lambda, \tag{5.3h}$$

where d is the interplanar distance, α is the glancing angle, $k = 1, 2, 3, \ldots$.

5.97. A plane light wave falls normally on a diaphragm with round aperture opening the first N Fresnel zones for a point P on a screen located at a distance b from the diaphragm. The wavelength of light is equal to λ. Find the intensity of light I_0 in front of the diaphragm if the distribution of intensity of light $I(r)$ on the screen is known. Here r is the distance from the point P.

5.98. A point source of light with wavelength $\lambda = 0.50$ μm is located at a distance $a = 100$ cm in front of a diaphragm with round aperture of radius $r = 1.0$ mm. Find the distance b between the diaphragm and the observation point for which the number of Fresnel zones in the aperture equals $k = 3$.

5.99. A diaphragm with round aperture, whose radius r can be varied during the experiment, is placed between a point source of light and a screen. The distances from the diaphragm to the source and the screen are equal to $a = 100$ cm and $b = 125$ cm. Determine the wavelength of light if the intensity maximum at the centre of the diffraction pattern of the screen is observed at $r_1 = 1.00$ mm and the next maximum at $r_2 = 1.29$ mm.

5.100. A plane monochromatic light wave with intensity I_0 falls normally on an opaque screen with a round aperture. What is the intensity of light I behind the screen at the point for which the aperture

(a) is equal to the first Fresnel zone; to the internal half of the first zone;

(b) was made equal to the first Fresnel zone and then half of it was closed (along the diameter)?

5.101. A plane monochromatic light wave with intensity I_0 falls normally on an opaque disc closing the first Fresnel zone for the observation point P. What did the intensity of light I at the point P become equal to after

(a) half of the disc (along the diameter) was removed;

(b) half of the external half of the first Fresnel zone was removed (along the diameter)?

5.102. A plane monochromatic light wave with intensity I_0 falls normally on the surfaces of the opaque screens shown in Fig. 5.20. Find the intensity of light I at a point P

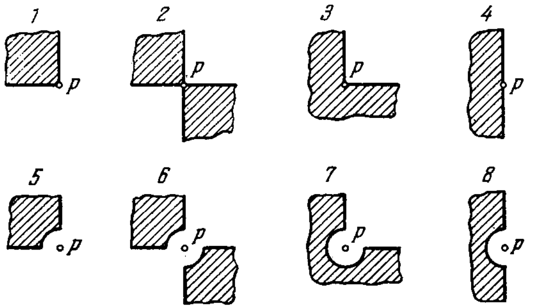

Fig. 5.20.

(a) located behind the corner points of screens *1-3* and behind the edge of half-plane *4*;

(b) for which the rounded-off edge of screens *5-8* coincides with the boundary of the first Fresnel zone.

Derive the general formula describing the results obtained for screens *1-4*; the same, for screens *5-8*.

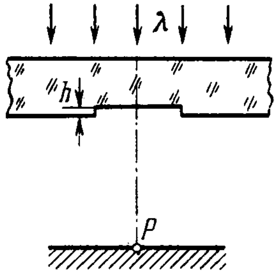

Fig. 5.21.

5.103. A plane light wave with wavelength $\lambda = 0.60$ μm falls normally on a sufficiently large glass plate having a round recess on the opposite side (Fig. 5.21). For the observation point P that recess corresponds to the first one and a half Fresnel zones. Find the depth h of the recess at which the intensity of light at the point P is

(a) maximum;

(b) minimum;

(c) equal to the intensity of incident light.

5.104. A plane light wave with wavelength λ and intensity I_0 falls normally on a large glass plate whose opposite side serves as an opaque screen with a round aperture equal to the first Fresnel zone for the observation point P. In the middle of the aperture there is a round recess equal to half the Fresnel zone. What must the depth h of that recess be for the intensity of light at the point P to be the highest? What is this intensity equal to?

5.105. A plane light wave with wavelength $\lambda = 0.57$ µm falls normally on a surface of a glass ($n = 1.60$) disc which shuts one and a half Fresnel zones for the observation point P. What must the minimum thickness of that disc be for the intensity of light at the point P to be the highest? Take into account the interference of light on its passing through the disc.

5.106. A plane light wave with wavelength $\lambda = 0.54$ µm goes through a thin converging lens with focal length $f = 50$ cm and an aperture stop fixed immediately after the lens, and reaches a screen placed at a distance $b = 75$ cm from the aperture stop. At what aperture radii has the centre of the diffraction pattern on the screen the maximum illuminance?

5.107. A plane monochromatic light wave falls normally on a round aperture. At a distance $b = 9.0$ m from it there is a screen showing a certain diffraction pattern. The aperture diameter was decreased $\eta = 3.0$ times. Find the new distance b' at which the screen should be positioned to obtain the diffraction pattern similar to the previous one but diminished η times.

5.108. An opaque ball of diameter $D = 40$ mm is placed between a source of light with wavelength $\lambda = 0.55$ µm and a photographic plate. The distance between the source and the ball is equal to $a = 12$ m and that between the ball and the photographic plate is equal to $b = 18$ m. Find:

(a) the image dimension y' on the plate if the transverse dimension of the source is $y = 6.0$ mm;

(b) the minimum height of irregularities, covering the surface of the ball at random, at which the ball obstructs light.

Note. As calculations and experience show, that happens when the height of irregularities is comparable with the width of the Fresnel zone along which the edge of an opaque screen passes.

5.109. A point source of monochromatic light is positioned in front of a zone plate at a distance $a = 1.5$ m from it. The image of the source is formed at a distance $b = 1.0$ m from the plate. Find the focal length of the zone plate.

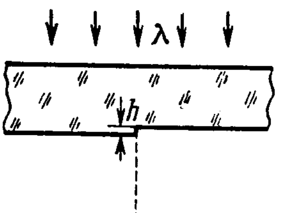

Fig. 5.22.

5.110. A plane light wave with wavelength $\lambda = 0.60$ µm and intensity I_0 falls normally on a large glass plate whose side view is shown in

Fig. 5.22. At what height h of the ledge will the intensity of light at points located directly below be

(a) minimum;

(b) twice as low as I_0 (the losses due to reflection are to be neglected).

5.111. A plane monochromatic light wave falls normally on an opaque half-plane. A screen is located at a distance $b = 100$ cm behind the half-plane. Making use of the Cornu spiral (Fig. 5.19), find:

(a) the ratio of intensities of the first maximum and the neighbouring minimum;

(b) the wavelength of light if the first two maxima are separated by a distance $\Delta x = 0.63$ mm.

5.112. A plane light wave with wavelength 0.60 μm falls normally on a long opaque strip 0.70 mm wide. Behind it a screen is placed at a distance 100 cm. Using Fig. 5.19, find the ratio of intensities of light in the middle of the diffraction pattern and at the edge of the geometrical shadow.

5.113. A plane monochromatic light wave falls normally on a long rectangular slit behind which a screen is positioned at a distance $b = 60$ cm. First the width of the slit was adjusted so that in the middle of the diffraction pattern the lowest minimum was observed. After widening the slit by $\Delta h = 0.70$ mm, the next minimum was obtained in the centre of the pattern. Find the wavelength of light.

5.114. A plane light wave with wavelength $\lambda = 0.65$ μm falls normally on a large glass plate whose opposite side has a long rectangular recess 0.60 mm wide. Using Fig. 5.19, find the depth h of the recess at which the diffraction pattern on the screen 77 cm away from the plate has the maximum illuminance at its centre.

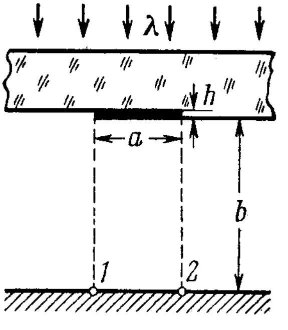

Fig. 5.23.

5.115. A plane light wave with wavelength $\lambda = 0.65$ μm falls normally on a large glass plate whose opposite side has a ledge and an opaque strip of width $a = 0.30$ mm (Fig. 5.23). A screen is placed at a distance $b = 110$ cm from the plate. The height h of the ledge is such that the intensity of light at point *2* of the screen is the highest possible. Making use of Fig. 5.19, find the ratio of intensities at points *1* and *2*.

5.116. A plane monochromatic light wave of intensity I_0 falls normally on an opaque screen with a long slit having a semicircular

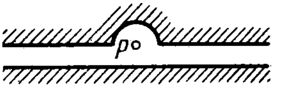

Fig. 5.24.

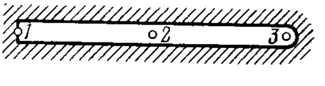

Fig. 5.25.

cut on one side (Fig. 5.24). The edge of the cut coincides with the boundary line of the first Fresnel zone for the observation point P. The width of the slit measures 0.90 of the radius of the cut. Using Fig. 5.19, find the intensity of light at the point P.

5.117. A plane monochromatic light wave falls normally on an opaque screen with a long slit whose shape is shown in Fig. 5.25. Making use of Fig. 5.19, find the ratio of intensities of light at points *1*, *2*, and *3* located behind the screen at equal distances from it. For point *3* the rounded-off edge of the slit coincides with the boundary line of the first Fresnel zone.

5.118. A plane monochromatic light wave falls normally on an opaque screen shaped as a long strip with a round hole in the middle. For the observation point P the hole corresponds to half the Fresnel zone, with the hole diameter being $\eta = 1.07$ times less than the width of the strip. Using Fig. 5.19, find the intensity of light at the point P provided that the intensity of the incident light is equal to I_0.

5.119. Light with wavelength λ falls normally on a long rectangular slit of width b. Find the angular distribution of the intensity of light in the case of Fraunhofer diffraction, as well as the angular position of minima.

5.120. Making use of the result obtained in the foregoing problem, find the conditions defining the angular position of maxima of the first, the second, and the third order.

5.121. Light with wavelength $\lambda = 0.50$ μm falls on a slit of width $b = 10$ μm at an angle $\theta_0 = 30°$ to its normal. Find the angular position of the first minima located on both sides of the central Fraunhofer maximum.

5.122. A plane light wave with wavelength $\lambda = 0.60$ μm falls normally on the face of a glass wedge with refracting angle $\Theta = 15°$. The opposite face of the wedge is opaque and has a slit of width $b = 10$ μm parallel to the edge. Find:

(a) the angle $\Delta\theta$ between the direction to the Fraunhofer maximum of zeroth order and that of incident light;

(b) the angular width of the Fraunhofer maximum of the zeroth order.

5.123. A monochromatic beam falls on a reflection grating with period $d = 1.0$ mm at a glancing angle $\alpha_0 = 1.0°$. When it is diffracted at a glancing angle $\alpha = 3.0°$ a Fraunhofer maximum of second order occurs. Find the wavelength of light.

5.124. Draw the approximate diffraction pattern originating in the case of the Fraunhofer diffraction from a grating consisting of three identical slits if the ratio of the grating period to the slit width is equal to

(a) two;

(b) three.

5.125. With light falling normally on a diffraction grating, the angle of diffraction of second order is equal to 45° for a wavelength

$\lambda_1 = 0.65$ μm. Find the angle of diffraction of third order for a wave length $\lambda_2 = 0.50$ μm.

5.126. Light with wavelength 535 nm falls normally on a diffraction grating. Find its period if the diffraction angle 35° corresponds to one of the Fraunhofer maxima and the highest order of spectrum is equal to five.

5.127. Find the wavelength of monochromatic light falling normally on a diffraction grating with period $d = 2.2$ μm if the angle between the directions to the Fraunhofer maxima of the first and the second order is equal to $\Delta\theta = 15°$.

5.128. Light with wavelength 530 nm falls on a transparent diffraction grating with period 1.50 μm. Find the angle, relative to the grating normal, at which the Fraunhofer maximum of highest order is observed provided the light falls on the grating

(a) at right angles;

(b) at the angle 60° to the normal.

5.129. Light with wavelength $\lambda = 0.60$ μm falls normally on a diffraction grating inscribed on a plane surface of a plano-convex cylindrical glass lens with curvature radius $R = 20$ cm. The period of the grating is equal to $d = 6.0$ μm. Find the distance between the principal maxima of first order located symmetrically in the focal plane of that lens.

5.130. A plane light wave with wavelength $\lambda = 0.50$ μm falls normally on the face of a glass wedge with an angle $\Theta = 30°$. On the opposite face of the wedge a transparent diffraction grating with period $d = 2.00$ μm is inscribed, whose lines are parallel to the wedge's edge. Find the angles that the direction of incident light forms with the directions to the principal Fraunhofer maxima of the zero and the first order. What is the highest order of the spectrum? At what angle to the direction of incident light is it observed?

5.131. A plane light wave with wavelength λ falls normally on a phase diffraction grating whose side view is shown in Fig. 5.26. The grating is cut on a glass plate with refractive index n. Find the depth h of the lines at which the intensity of the central Fraunhofer maximum is equal to zero. What is in this case the diffraction angle corresponding to the first maximum?

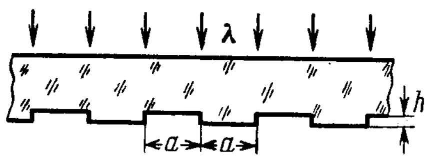

Fig. 5.26.

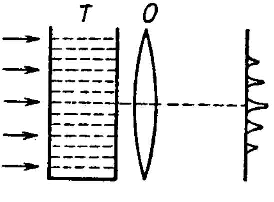

Fig. 5.27.

5.132. Figure 5.27 illustrates an arrangement employed in observations of diffraction of light by ultrasound. A plane light wave with wavelength $\lambda = 0.55$ μm passes through the water-filled tank T

in which a standing ultrasonic wave is sustained at a frequency $\nu = 4.7$ MHz. As a result of diffraction of light by the optically inhomogeneous periodic structure a diffraction spectrum can be observed in the focal plane of the objective O with focal length $f = 35$ cm. The separation between neighbouring maxima is $\Delta x = 0.60$ mm. Find the propagation velocity of ultrasonic oscillations in water.

5.133. To measure the angular distance ψ between the components of a double star by Michelson's method, in front of a telescope's lens a diaphragm was placed, which had two narrow parallel slits separated by an adjustable distance d. While diminishing d, the first smearing of the pattern was observed in the focal plane of the objective at $d = 95$ cm. Find ψ, assuming the wavelength of light to be equal to $\lambda = 0.55$ μm.

5.134. A transparent diffraction grating has a period $d = 1.50$ μm. Find the angular dispersion D (in angular minutes per nanometres) corresponding to the maximum of highest order for a spectral line of wavelength $\lambda = 530$ nm of light falling on the grating

(a) at right angles;

(b) at the angle $\theta_0 = 45°$ to the normal.

5.135. Light with wavelength λ falls on a diffraction grating at right angles. Find the angular dispersion of the grating as a function of diffraction angle θ.

5.136. Light with wavelength $\lambda = 589.0$ nm falls normally on a diffraction grating with period $d = 2.5$ μm, comprising $N = 10\,000$ lines. Find the angular width of the diffraction maximum of second order.

5.137. Demonstrate that when light falls on a diffraction grating at right angles, the maximum resolving power of the grating cannot exceed the value l/λ, where l is the width of the grating and λ is the wavelength of light.

5.138. Using a diffraction grating as an example, demonstrate that the frequency difference of two maxima resolved according to Rayleigh's criterion is equal to the reciprocal of the difference of propagation times of the extreme interfering oscillations, i.e. $\delta\nu = 1/\delta t$.

5.139. Light composed of two spectral lines with wavelengths 600.000 and 600.050 nm falls normally on a diffraction grating 10.0 mm wide. At a certain diffraction angle θ these lines are close to being resolved (according to Rayleigh's criterion). Find θ.

5.140. Light falls normally on a transparent diffraction grating of width $l = 6.5$ cm with 200 lines per millimetre. The spectrum under investigation includes a spectral line with $\lambda = 670.8$ nm consisting of two components differing by $\delta\lambda = 0.015$ nm. Find:

(a) in what order of the spectrum these components will be resolved;

(b) the least difference of wavelengths that can be resolved by this grating in a wavelength region $\lambda \approx 670$ nm.

5.141. With light falling normally on a transparent diffraction grating 10 mm wide, it was found that the components of the yellow line of sodium (589.0 and 589.6 nm) are resolved beginning with the fifth order of the spectrum. Evaluate:

(a) the period of this grating;

(b) what must be the width of the grating with the same period for a doublet $\lambda = 460.0$ nm whose components differ by 0.13 nm to be resolved in the third order of the spectrum.

5.142. A transparent diffraction grating of a quartz spectrograph is 25 mm wide and has 250 lines per millimetre. The focal length of an objective in whose focal plane a photographic plate is located is equal to 80 cm. Light falls on the grating at right angles. The spectrum under investigation includes a doublet with components of wavelengths 310.154 and 310.184 nm. Determine:

(a) the distances on the photographic plate between the components of this doublet in the spectra of the first and the second order;

(b) whether these components will be resolved in these orders of the spectrum.

5.143. The ultimate resolving power $\lambda/\delta\lambda$ of the spectrograph's trihedral prism is determined by diffraction of light at the prism edges (as in the case of a slit). When the prism is oriented to the least deviation angle in accordance with Rayleigh's criterion,

$$\lambda/\delta\lambda = b \mid dn/d\lambda \mid,$$

where b is the width of the prism's base (Fig. 5.28), and $dn/d\lambda$ is the dispersion of its material. Derive this formula.

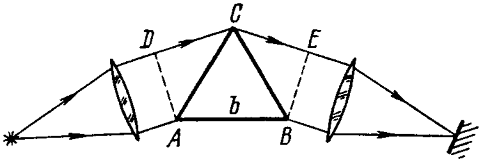

Fig. 5.28.

5.144. A spectrograph's trihedral prism is manufactured from glass whose refractive index varies with wavelength as $n = A + B/\lambda^2$, where A and B are constants, with B being equal to 0.010 μm^2. Making use of the formula from the foregoing problem, find:

(a) how the resolving power of the prism depends on λ; calculate the value of $\lambda/\delta\lambda$ in the vicinity of $\lambda_1 = 434$ nm and $\lambda_2 = 656$ nm if the width of the prism's base is $b = 5.0$ cm;

(b) the width of the prism's base capable of resolving the yellow doublet of sodium (589.0 and 589.6 nm).

5.145. How wide is the base of a trihedral prism which has the same resolving power as a diffraction grating with 10 000 lines in the second order of the spectrum if $|dn/d\lambda| = 0.10\ \mu m^{-1}$?

5.146. There is a telescope whose objective has a diameter $D = 5.0$ cm. Find the resolving power of the objective and the minimum separation between two points at a distance $l = 3.0$ km from the telescope, which it can resolve (assume $\lambda = 0.55$ μm).

5.147. Calculate the minimum separation between two points on the Moon which can be resolved by a reflecting telescope with mirror diameter 5 m. The wavelength of light is assumed to be equal to $\lambda = 0.55$ μm.

5.148. Determine the minimum multiplication of a telescope with diameter of objective $D = 5.0$ cm with which the resolving power of the objective is totally employed if the diameter of the eye's pupil is $d_0 = 4.0$ mm.

5.149. There is a microscope whose objective's numerical aperture is $\sin \alpha = 0.24$, where α is the half-angle subtended by the objective's rim. Find the minimum separation resolved by this microscope when an object is illuminated by light with wavelength $\lambda = 0.55$ μm.

5.150. Find the minimum magnification of a microscope, whose objective's numerical aperture is $\sin \alpha = 0.24$, at which the resolving power of the objective is totally employed if the diameter of the eye's pupil is $d_0 = 4.0$ mm.

5.151. A beam of X-rays with wavelength λ falls at a glancing angle 60.0° on a linear chain of scattering centres with period a. Find the angles of incidence corresponding to all diffraction maxima if $\lambda = 2a/5$.

5.152. A beam of X-rays with wavelength $\lambda = 40$ pm falls normally on a plane rectangular array of scattering centres and produces a system of diffraction maxima (Fig. 5.29) on a plane screen removed from the array by a distance $l = 10$ cm. Find the array periods a and b along the x and y axes if the distances between symmetrically located maxima of second order are equal to $\Delta x = 60$ mm (along the x axis) and $\Delta y = 40$ mm (along the y axis).

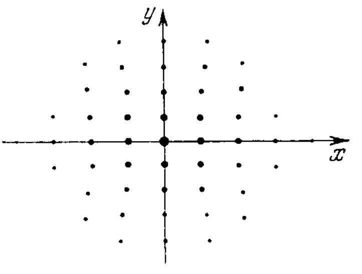

Fig. 5.29.

5.153. A beam of X-rays impinges on a three-dimensional rectangular array whose periods are a, b, and c. The direction of the incident beam coincides with the direction along which the array period is equal to a. Find the directions to the diffraction maxima and the wavelengths at which these maxima will be observed.

5.154. A narrow beam of X-rays impinges on the natural facet of a NaCl single crystal, whose density is $\rho = 2.16$ g/cm^3 at a glancing angle $\alpha = 60.0°$. The mirror reflection from this facet produces a maximum of second order. Find the wavelength of radiation.

5.155. A beam of X-rays with wavelength $\lambda = 174$ pm falls on the surface of a single crystal rotating about its axis which is parallel to its surface and perpendicular to the direction of the incident beam. In this case the directions to the maxima of second and third order from the system of planes parallel to the surface of the single crystal form an angle $\alpha = 60°$ between them. Find the corresponding interplanar distance.

5.156. On transmitting a beam of X-rays with wavelength $\lambda = 17.8$ pm through a polycrystalline specimen a system of diffraction rings is produced on a screen located at a distance $l = 15$ cm from the specimen. Determine the radius of the bright ring corresponding to second order of reflection from the system of planes with interplanar distance $d = 155$ pm.

5.4. POLARIZATION OF LIGHT

- Degree of polarization of light:

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}. \tag{5.4a}$$

- Malus's law:

$$I = I_0 \cos^2 \varphi. \tag{5.4b}$$

- Brewster's law:

$$\tan \theta_B = n_2/n_1. \tag{5.4c}$$

- Fresnel equations for intensity of light reflected at the boundary between two dielectrics:

$$I'_\perp = I_\perp \frac{\sin^2(\theta_1 - \theta_2)}{\sin^2(\theta_1 + \theta_2)}, \quad I'_\parallel = I_\parallel \frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)}, \tag{5.4d}$$

where $I_\perp$ and $I_\parallel$ are the intensities of incident light whose electric vector oscillations are respectively perpendicular and parallel to the plane of incidence.

- A crystalline plate between two polarizers P and P'. If the angle between the plane of polarizer P and the optical axis OO' of the plate is equal to 45°, the intensity I' of light which passes through the polarizer P' turns out to be either maximum or minimum under the following conditions:

Polarizers P and P'	$\delta = 2\pi k$	$\delta = (2k+1)\pi$
parallel	$I'_\parallel = \max$	$I'_\parallel = \min$
crossed	$I'_\perp = \min$	$I'_\perp = \max$

(5.4e)

Here $\delta = 2\pi(n_0 - n_e)d/\lambda$ is the phase difference between the ordinary and extraordinary rays, $k = 0, 1, 2, \ldots$

- Natural and magnetic rotation of the plane of polarization:

$$\varphi_{nat} = \alpha l, \quad \varphi_{magn} = VlH, \tag{5.4f}$$

where α is the rotation constant, V is Verdet's constant.

5.157. A plane monochromatic wave of natural light with intensity I_0 falls normally on a screen composed of two touching Polaroid half-planes. The principal direction of one Polaroid is parallel, and of the other perpendicular, to the boundary between them. What kind of diffraction pattern is formed behind the screen? What is the intensity of light behind the screen at the points of the plane perpendicular to the screen and passing through the boundary between the Polaroids?

5.158. A plane monochromatic wave of natural light with intensity I_0 falls normally on an opaque screen with round hole corresponding to the first Fresnel zone for the observation point P. Find the intensity of light at the point P after the hole was covered with two identical Polaroids whose principal directions are mutually perpendicular and the boundary between them passes

(a) along the diameter of the hole;

(b) along the circumference of the circle limiting the first half of the Fresnel zone.

5.159. A beam of plane-polarized light falls on a polarizer which rotates about the axis of the ray with angular velocity $\omega = 21$ rad/s. Find the energy of light passing through the polarizer per one revolution if the flux of energy of the incident ray is equal to $\Phi_0 = 4.0$ mW.

5.160. A beam of natural light falls on a system of $N = 6$ Nicol prisms whose transmission planes are turned each through an angle $\varphi = 30°$ with respect to that of the foregoing prism. What fraction of luminous flux passes through this system?

5.161. Natural light falls on a system of three identical in-line Polaroids, the principal direction of the middle Polaroid forming an angle $\varphi = 60°$ with those of two other Polaroids. The maximum transmission coefficient of each Polaroid is equal to $\tau = 0.81$ when plane-polarized light falls on them. How many times will the intensity of the light decrease after its passing through the system?

5.162. The degree of polarization of partially polarized light is $P = 0.25$. Find the ratio of intensities of the polarized component of this light and the natural component.

5.163. A Nicol prism is placed in the way of partially polarized beam of light. When the prism is turned from the position of maximum transmission through an angle $\varphi = 60°$, the intensity of transmitted light decreased by a factor of $\eta = 3.0$. Find the degree of polarization of incident light.

5.164. Two identical imperfect polarizers are placed in the way of a natural beam of light. When the polarizers' planes are parallel, the system transmits $\eta = 10.0$ times more light than in the case of crossed planes. Find the degree of polarization of light produced

(a) by each polarizer separately;

(b) by the whole system when the planes of the polarizers are parallel.

5.165. Two parallel plane-polarized beams of light of equal intensity whose oscillation planes N_1 and N_2 form a small angle φ between them (Fig. 5.30) fall on a Nicol prism. To equalize the intensities of the beams emerging behind the prism, its principal direction N must be aligned along the bisecting line A or B. Find the value of the angle φ at which the rotation of the Nicol prism through a small angle $\delta\varphi \ll \varphi$ from the position A results in the fractional change of intensities of the beams $\Delta I/I$ by the value $\eta = 100$ times exceeding that resulting due to rotation through the same angle from the position B.

5.166. Resorting to the Fresnel equations, demonstrate that light reflected from the surface of dielectric will be totally polarized if the angle of incidence θ_1 satisfies the condition $\tan\theta_1 = n$, where n is the refractive index of the dielectric. What is in this case the angle between the reflected and refracted rays?

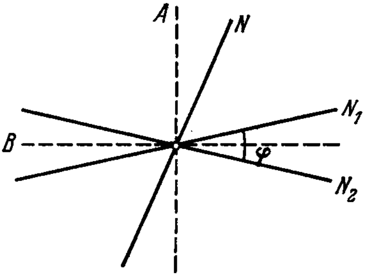

Fig. 5.30.

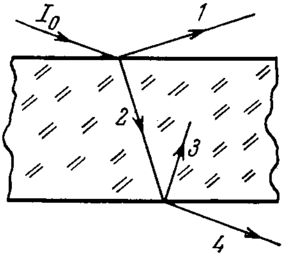

Fig. 5.31.

5.167. Natural light falls at the Brewster angle on the surface of glass. Using the Fresnel equations, find

(a) the reflection coefficient;

(b) the degree of polarization of refracted light.

5.168. A plane beam of natural light with intensity I_0 falls on the surface of water at the Brewster angle. A fraction $\rho = 0.039$ of luminous flux is reflected. Find the intensity of the refracted beam.

5.169. A beam of plane-polarized light falls on the surface of water at the Brewster angle. The polarization plane of the electric vector of the electromagnetic wave makes an angle $\varphi = 45°$ with the incidence plane. Find the reflection coefficient.

5.170. A narrow beam of natural light falls on the surface of a thick transparent plane-parallel plate at the Brewster angle. As a result, a fraction $\rho = 0.080$ of luminous flux is reflected from its top surface. Find the degree of polarization of beams *1-4* (Fig. 5.31)

5.171. A narrow beam of light of intensity I_0 falls on a plane-parallel glass plate (Fig. 5.31) at the Brewster angle. Using the Fresnel equations, find:

(a) the intensity of the transmitted beam I_4 if the oscillation plane of the incident plane-polarized light is perpendicular to the incidence plane;

(b) the degree of polarization of the transmitted light if the light falling on the plate is natural.

5.172. A narrow beam of natural light falls on a set of N thick plane-parallel glass plates at the Brewster angle. Find:

(a) the degree P of polarization of the transmitted beam;

(b) what P is equal to when $N = 1$, 2, 5, and 10.

5.173. Using the Fresnel equations, find:

(a) the reflection coefficient of natural light falling normally on the surface of glass;

(b) the relative loss of luminous flux due to reflections of a paraxial ray of natural light passing through an aligned optical system comprising five glass lenses (secondary reflections of light are to be neglected).

5.174. A light wave falls normally on the surface of glass coated with a layer of transparent substance. Neglecting secondary reflections, demonstrate that the amplitudes of light waves reflected from the two surfaces of such a layer will be equal under the condition $n' = \sqrt{n}$, where n' and n are the refractive indices of the layer and the glass respectively.

5.175. A beam of natural light falls on the surface of glass at an angle of 45°. Using the Fresnel equations, find the degree of polarization of

(a) reflected light;

(b) refracted light.

5.176. Using Huygens's principle, construct the wavefronts and the propagation directions of the ordinary and extraordinary rays in a positive uniaxial crystal whose optical axis

(a) is perpendicular to the incidence plane and parallel to the surface of the crystal;

(b) lies in the incidence plane and is parallel to the surface of the crystal;

(c) lies in the incidence plane at an angle of 45° to the surface of the crystal, and light falls at right angles to the optical axis.

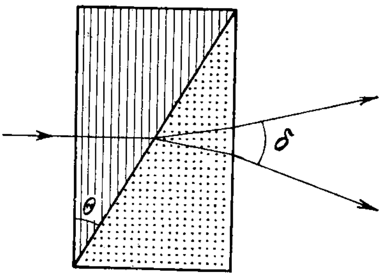

Fig. 5.32.

5.177. A narrow beam of natural light with wavelength $\lambda = 589$ nm falls normally on the surface of a Wollaston polarizing prism made of Iceland spar as shown in Fig. 5.32. The optical axes of the two parts of the prism are mutually perpendicular. Find the angle δ between the directions of the beams behind the prism if the angle θ is equal to 30°.

5.178. What kind of polarization has a plane electromagnetic wave if the projections of the vector **E** on the x and y axes are perpendicular to the propagation direction and are defined by the following equations:

(a) $E_x = E \cos(\omega t - kz)$, $E_y = E \sin(\omega t - kz)$;
(b) $E_x = E \cos(\omega t - kz)$, $E_y = E \cos(\omega t - kz + \pi/4)$;
(c) $E_x = E \cos(\omega t - kz)$, $E_y = E \cos(\omega t - kz + \pi)$?

5.179. One has to manufacture a quartz plate cut parallel to its optical axis and not exceeding 0.50 mm in thickness. Find the maximum thickness of the plate allowing plane-polarized light with wavelength $\lambda = 589$ nm
(a) to experience only rotation of polarization plane;
(b) to acquire circular polarization
after passing through that plate.

5.180. A quartz plate cut parallel to the optical axis is placed between two crossed Nicol prisms. The angle between the principal directions of the Nicol prisms and the plate is equal to 45°. The thickness of the plate is $d = 0.50$ mm. At what wavelengths in the interval from 0.50 to 0.60 μm is the intensity of light which passed through that system independent of rotation of the rear prism? The difference of refractive indices for ordinary and extraordinary rays in that wavelength interval is assumed to be $\Delta n = 0.0090$.

5.181. White natural light falls on a system of two crossed Nicol prisms having between them a quartz plate 1.50 mm thick, cut parallel to the optical axis. The axis of the plate forms an angle of 45° with the principal directions of the Nicol prisms. The light transmitted through that system was split into the spectrum. How many dark fringes will be observed in the wavelength interval from 0.55 to 0.66 μm? The difference of refractive indices for ordinary and extraordinary rays in that wavelength interval is assumed to be equal to 0.0090.

5.182. A crystalline plate cut parallel to its optical axis is 0.25 mm thick and serves as a quarter-wave plate for a wavelength $\lambda = 530$ nm. At what other wavelengths of visible spectrum will it also serve as a quarter-wave plate? The difference of refractive indices for extraordinary and ordinary rays is assumed to be constant and equal to $n_e - n_o = 0.0090$ at all wavelengths of the visible spectrum.

5.183. A quartz plate cut parallel to its optical axis is placed between two crossed Nicol prisms so that their principle directions form an angle of 45° with the optical axis of the plate. What is the minimum thickness of that plate transmitting light of wavelength $\lambda_1 = 643$ nm with maximum intensity while greatly reducing the intensity of transmitting light of wavelength $\lambda_2 = 564$ nm? The difference of refractive indices for extraordinary and ordinary rays is assumed to be equal to $n_e - n_o = 0.0090$ for both wavelengths.

5.184. A quartz wedge with refracting angle $\Theta = 3.5°$ is inserted between two crossed Polaroids. The optical axis of the wedge is parallel to its edge and forms an angle of 45° with the principal directions of the Polaroids. On transmission of light with wavelength $\lambda = 550$ nm through this system, an interference fringe pattern is formed. The width of each fringe is $\Delta x = 1.0$ mm. Find the difference of refractive indices of quartz for ordinary and extraordinary rays at the wavelength indicated above.

5.185. Natural monochromatic light of intensity I_0 falls on a system of two Polaroids between which a crystalline plate is inserted, cut parallel to its optical axis. The plate introduces a phase difference δ between the ordinary and extraordinary rays. Demonstrate that the intensity of light transmitted through that system is equal to

$$I = \frac{1}{2} I_0 [\cos^2(\varphi - \varphi') - \sin 2\varphi \cdot \sin 2\varphi' \ \sin^2(\delta/2)],$$

where φ and φ' are the angles between the optical axis of the crystal and the principal directions of the Polaroids. In particular, consider the cases of crossed and parallel Polaroids.

5.186. Monochromatic light with circular polarization falls normally on a crystalline plate cut parallel to the optical axis. Behind the plate there is a Nicol prism whose principal direction forms an angle φ with the optical axis of the plate. Demonstrate that the intensity of light transmitted through that system is equal to

$$I = I_0 (1 + \sin 2\varphi \cdot \sin \delta),$$

where δ is the phase difference between the ordinary and extraordinary rays which is introduced by the plate.

5.187. Explain how, using a Polaroid and a quarter-wave plate made of positive uniaxial crystal ($n_e > n_o$), to distinguish

(a) light with left-hand circular polarization from that with right-hand polarization;

(b) natural light from light with circular polarization and from the composition of natural light and that with circular polarization.

5.188. Light with wavelength λ falls on a system of crossed polarizer P and analyzer A between which a Babinet compensator C is inserted (Fig. 5.33). The compensator consists of two quartz wedges with the optical axis of one of them being parallel to the edge, and of the other, perpendicular to it. The principal directions of the polarizer and the analyser form an angle of 45° with the optical axes of the compensator. The refracting angle of the wedges is equal to Θ ($\Theta \ll 1$) and the difference of refractive indices of quartz is $n_e - n_o$. The insertion of investigated birefringent sample S, with the optical axis oriented as shown in the figure, results in displacement of the fringes upward by δx mm. Find:

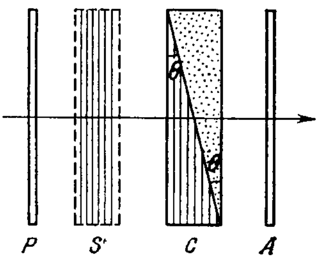

Fig. 5.33.

(a) the width of the fringe Δx;

(b) the magnitude and the sign of the optical path difference of ordinary and extraordinary rays, which appears due to the sample S.

5.189. Using the tables of the Appendix, calculate the difference of refractive indices of quartz for light of wavelength $\lambda = 589.5$ nm with right-hand and left-hand circular polarizations.

5.190. Plane-polarized light of wavelength 0.59 μm falls on a trihedral quartz prism P (Fig. 5.34) with refracting angle $\Theta = 30°$. Inside the prism light propagates along the optical axis whose direction is shown by hatching. Behind the Polaroid Pol an interference pattern of bright and dark fringes of width $\Delta x = 15.0$ mm is observed. Find the specific rotation constant of quartz and the distribution of intensity of light behind the Polaroid.

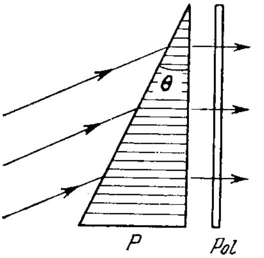

Fig. 5.34.

5.191. Natural monochromatic light falls on a system of two crossed Nicol prisms between which a quartz plate cut at right angles to its optical axis is inserted. Find the minimum thickness of the plate at which this system will transmit $\eta = 0.30$ of luminous flux if the specific rotation constant of quartz is equal to $\alpha = 17$ ang.deg/mm.

5.192. Light passes through a system of two crossed Nicol prisms between which a quartz plate cut at right angles to its optical axis is placed. Determine the minimum thickness of the plate which allows light of wavelength 436 nm to be completely cut off by the system and transmits half the light of wavelength 497 nm. The specific rotation constant of quartz for these wavelengths is equal to 41.5 and 31.1 angular degrees per mm respectively.

5.193. Plane-polarized light of wavelength 589 nm propagates along the axis of a cylindrical glass vessel filled with slightly turbid sugar solution of concentration 500 g/l. Viewing from the side, one can see a system of helical fringes, with 50 cm between neighbouring dark fringes along the axis. Explain the emergence of the fringes and determine the specific rotation constant of the solution.

5.194. A Kerr cell is positioned between two crossed Nicol prisms so that the direction of electric field **E** in the capacitor forms an angle of 45° with the principal directions of the prisms. The capacitor has the length $l = 10$ cm and is filled up with nitrobenzene. Light of wavelength $\lambda = 0.50$ μm passes through the system. Taking into account that in this case the Kerr constant is equal to $B = 2.2 \cdot 10^{-10}$ cm/V^2, find:

(a) the minimum strength of electric field E in the capacitor at which the intensity of light that passes through this system is independent of rotation of the rear prism;

(b) how many times per second light will be interrupted when a sinusoidal voltage of frequency $\nu = 10$ MHz and strength amplitude $E_m = 50$ kV/cm is applied to the capacitor.

Note. The Kerr constant is the coefficient B in the equation $n_e - n_o = B\lambda E^2$.

5.195. Monochromatic plane-polarized light with angular frequency ω passes through a certain substance along a uniform magnetic field H. Find the difference of refractive indices for right-hand and left-hand components of light beam with circular polarization if the Verdet constant is equal to V.

5.196. A certain substance is placed in a longitudinal magnetic field of a solenoid located between two Polaroids. The length of the tube with substance is equal to $l = 30$ cm. Find the Verdet constant if at a field strength $H = 56.5$ kA/m the angle of rotation of polarization plane is equal to $\varphi_1 = +5°10'$ for one direction of the field and to $\varphi_2 = -3°20'$, for the opposite direction.

5.197. A narrow beam of plane-polarized light passes through dextrorotatory positive compound placed into a longitudinal magnetic field as shown in Fig. 5.35. Find the angle through which the

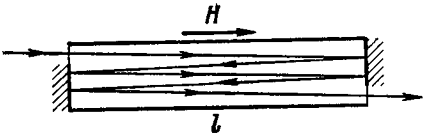

Fig. 5.35.

polarization plane of the transmitted beam will turn if the length of the tube with the compound is equal to l, the specific rotation constant of the compound is equal to α, the Verdet constant is V, and the magnetic field strength is H.

5.198. A tube of length $l = 26$ cm is filled with benzene and placed in a longitudinal magnetic field of a solenoid positioned between two Polaroids. The angle between the principle directions of the Polaroids is equal to 45°. Find the minimum strength of the magnetic field at which light of the wavelength 589 nm propagates through that system only in one direction (optical valve). What happens if the direction of the given magnetic field is changed to the opposite one?

5.199. Experience shows that a body irradiated with light with circular polarization acquires a torque. This happens because such a light possesses an angular momentum whose flow density in vacuum is equal to $M = I/\omega$, where I is the intensity of light, ω is the angular oscillation frequency. Suppose light with circular polarization and wavelength $\lambda = 0.70$ μm falls normally on a uniform black disc of mass $m = 10$ mg which can freely rotate about its axis. How soon will its angular velocity become equal to $\omega_0 = 1.0$ rad/s provided $I = 10$ W/cm²?

5.5. DISPERSION AND ABSORPTION OF LIGHT

- Permittivity of substance according to elementary theory of dispersion:

$$\varepsilon = 1 + \sum_k \frac{n_k e^2/m\varepsilon_0}{\omega_{0k}^2 - \omega^2}, \tag{5.5a}$$

where n_k is the concentration of electrons of natural frequency ω_{0k}.

- Relation between refractive index and permittivity of substance:

$$n = \sqrt{\varepsilon}. \tag{5.5b}$$

- Phase velocity v and group velocity u:

$$v = \omega/k, \quad u = d\omega/dk. \tag{5.5c}$$

- Rayleigh's formula:

$$u = v - \lambda \frac{dv}{d\lambda}. \tag{5.5d}$$

- Attenuation of a narrow beam of electromagnetic radiation:

$$I = I_0 e^{-\mu d}, \tag{5.5e}$$

where $\mu = \varkappa + \varkappa'$, μ, $\varkappa$, $\varkappa'$ are the coefficients of linear attenuation, absorption, and scattering.

5.200. A free electron is located in the field of a monochromatic light wave. The intensity of light is $I = 150$ W/m², its frequency is $\omega = 3.4 \cdot 10^{15}$ s^{-1}. Find:

(a) the electron's oscillation amplitude and its velocity amplitude;

(b) the ratio F_m/F_e, where F_m and F_e are the amplitudes of forces with which the magnetic and electric components of the light wave field act on the electron; demonstrate that that ratio is equal to $\frac{1}{2} v/c$, where v is the electron's velocity amplitude and c is the velocity of light.

Instruction. The action of the magnetic field component can be disregarded in the equation of motion of the electron since the calculations show it to be negligible.

5.201. An electromagnetic wave of frequency ω propagates in dilute plasma. The free electron concentration in plasma is equal to n_0. Neglecting the interaction of the wave and plasma ions, find:

(a) the frequency dependence of plasma permittivity;

(b) how the phase velocity of the electromagnetic wave depends on its wavelength λ in plasma.

5.202. Find the free electron concentration in ionosphere if its refractive index is equal to $n = 0.90$ for radiowaves of frequency $\nu = 100$ MHz.

5.203. Assuming electrons of substance to be free when subjected to hard X-rays, determine by what magnitude the refractive index of graphite differs from unity in the case of X-rays whose wavelength in vacuum is equal to $\lambda = 50$ pm.

5.204. An electron experiences a quasi-elastic force kx and a "friction force" $\gamma\dot{x}$ in the field of electromagnetic radiation. The E-component of the field varies as $E = E_0 \cos \omega t$. Neglecting the action of the magnetic component of the field, find:

(a) the motion equation of the electron;

(b) the mean power absorbed by the electron; the frequency at which that power is maximum and the expression for the maximum mean power.

5.205. In some cases permittivity of substance turns out to be a complex or a negative quantity, and refractive index, respectively, a complex ($n' = n + i\varkappa$) or an imaginary ($n' = i\varkappa$) quantity. Write the equation of a plane wave for both of these cases and find out the physical meaning of such refractive indices.

5.206. A sounding of dilute plasma by radiowaves of various frequencies reveals that radiowaves with wavelengths exceeding $\lambda_0 = 0.75$ m experience total internal reflection. Find the free electron concentration in that plasma.

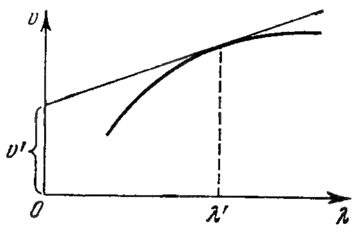

Fig. 5.36.

5.207. Using the definition of the group velocity u, derive Rayleigh's formula (5.5d). Demonstrate that in the vicinity of $\lambda = \lambda'$ the velocity u is equal to the segment v' cut by the tangent of the curve $v(\lambda)$ at the point λ' (Fig. 5.36).

5.208. Find the relation between the group velocity u and phase velocity v for the following dispersion laws:

(a) $v \propto 1/\sqrt{\lambda}$;

(b) $v \propto k$;

(c) $v \propto 1/\omega^2$.

Here λ, k, and ω are the wavelength, wave number, and angular frequency.

5.209. In a certain medium the relationship between the group and phase velocities of an electromagnetic wave has the form $uv = c^2$, where c is the velocity of light in vacuum. Find the dependence of permittivity of that medium on wave frequency, $\varepsilon(\omega)$.

5.210. The refractive index of carbon dioxide at the wavelengths 509, 534, and 589 nm is equal to 1.647, 1.640, and 1.630 respectively. Calculate the phase and group velocities of light in the vicinity of $\lambda = 534$ nm.

5.211. A train of plane light waves propagates in the medium where the phase velocity v is a linear function of wavelength: $v = a + b\lambda$, where a and b are some positive constants. Demonstrate that in such a medium the shape of an arbitrary train of light waves is restored after the time interval $\tau = 1/b$.

5.212. A beam of natural light of intensity I_0 falls on a system of two crossed Nicol prisms between which a tube filled with certain solution is placed in a longitudinal magnetic field of strength H. The length of the tube is l, the coefficient of linear absorption of solution is $\varkappa$, and the Verdet constant is V. Find the intensity of light transmitted through that system.

5.213. A plane monochromatic light wave of intensity I_0 falls normally on a plane-parallel plate both of whose surfaces have a reflection coefficient ρ. Taking into account multiple reflections, find the intensity of the transmitted light if

(a) the plate is perfectly transparent, i.e. the absorption is absent;

(b) the coefficient of linear absorption is equal to $\varkappa$, and the plate thickness is d.

5.214. Two plates, one of thickness $d_1 = 3.8$ mm and the other of thickness $d_2 = 9.0$ mm, are manufactured from a certain substance. When placed alternately in the way of monochromatic light, the first transmits $\tau_1 = 0.84$ fraction of luminous flux and the second, $\tau_2 = 0.70$. Find the coefficient of linear absorption of that substance. Light falls at right angles to the plates. The secondary reflections are to be neglected.

5.215. A beam of monochromatic light passes through a pile of $N = 5$ identical plane-parallel glass plates each of thickness $l = 0.50$ cm. The coefficient of reflection at each surface of the plates is $\rho = 0.050$. The ratio of the intensity of light transmitted through the pile of plates to the intensity of incident light is $\tau = 0.55$. Neglecting the secondary reflections of light, find the absorption coefficient of the given glass.

5.216. A beam of monochromatic light falls normally on the surface of a plane-parallel plate of thickness l. The absorption coefficient of the substance the plate is made of varies linearly along the normal to its surface from $\varkappa_1$ to $\varkappa_2$. The coefficient of reflection at each surface of the plate is equal to ρ. Neglecting the secondary reflections, find the transmission coefficient of such a plate.

5.217. A beam of light of intensity I_0 falls normally on a transparent plane-parallel plate of thickness l. The beam contains all the wavelengths in the interval from λ_1 to λ_2 of equal spectral intensity. Find the intensity of the transmitted beam if in this wavelength interval the absorption coefficient is a linear function of λ, with extreme values $\varkappa_1$ and $\varkappa_2$. The coefficient of reflection at each surface is equal to ρ. The secondary reflections are to be neglected.

5.218. A light filter is a plate of thickness d whose absorption coefficient depends on wavelength λ as

$$\varkappa(\lambda) = \alpha(1 - \lambda/\lambda_0)^2 \text{ cm}^{-1},$$

where α and λ_0 are constants. Find the passband $\Delta\lambda$ of this light filter, that is the band at whose edges the attenuation of light is η times that at the wavelength λ_0. The coefficient of reflection from the surfaces of the light filter is assumed to be the same at all wavelengths.

5.219. A point source of monochromatic light emitting a luminous flux Φ is positioned at the centre of a spherical layer of substance. The inside radius of the layer is a, the outside one is b. The coefficient of linear absorption of the substance is equal to $\varkappa$, the reflection coefficient of the surfaces is equal to ρ. Neglecting the secondary reflections, find the intensity of light that passes through that layer.

5.220. How many times will the intensity of a narrow X-ray beam of wavelength 20 pm decrease after passing through a lead plate of thickness $d = 1.0$ mm if the mass absorption coefficient for the given radiation wavelength is equal to $\mu/\rho = 3.6$ cm^2/g?

5.221. A narrow beam of X-ray radiation of wavelength 62 pm penetrates an aluminium screen 2.6 cm thick. How thick must a lead screen be to attenuate the beam just as much? The mass absorption coefficients of aluminium and lead for this radiation are equal to 3.48 and 72.0 cm^2/g respectively.

5.222. Find the thickness of aluminium layer which reduces by half the intensity of a narrow monochromatic X-ray beam if the corresponding mass absorption coefficient is $\mu/\rho = 0.32$ cm^2/g.

5.223. How many 50%-absorption layers are there in the plate reducing the intensity of a narrow X-ray beam $\eta = 50$ times?

5.6. OPTICS OF MOVING SOURCES

- Doppler effect for $\ll c$:

$$\frac{\Delta\omega}{\omega} = \frac{v}{c}\cos\theta \tag{5.6a}$$

where v is the velocity of a source, θ is the angle between the source's motion direction and the observation line.

- Doppler effect in the general case:

$$\omega = \omega_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}, \tag{5.6b}$$

where $\beta = v/c$.

- If $\theta = 0$, the Doppler effect is called radial, and if $\theta = \pi/2$, transverse.
- Vavilov-Cherenkov effect:

$$\cos\theta = \frac{c}{nv} \tag{5.6c}$$

where θ is the angle between the radiation propagation direction and the velocity vector v of a particle.

5.224. In the Fizeau experiment on measurement of the velocity of light the distance between the gear wheel and the mirror is $l = 7.0$ km, the number of teeth is $z = 720$. Two successive disappearances of light are observed at the following rotation velocities of the wheel: $n_1 = 283$ rps and $n_2 = 313$ rps. Find the velocity of light.

5.225. A source of light moves with velocity v relative to a receiver. Demonstrate that for $v \ll c$ the fractional variation of frequency of light is defined by Eq. (5.6a).

5.226. One of the spectral lines emitted by excited He^+ ions has a wavelength $\lambda = 410$ nm. Find the Doppler shift $\Delta\lambda$ of that line when observed at an angle $\theta = 30°$ to the beam of moving ions possessing kinetic energy $T = 10$ MeV.

5.227. When a spectral line of wavelength $\lambda = 0.59$ μm is observed in the directions to the opposite edges of the solar disc along its equator, there is a difference in wavelengths equal to $\delta\lambda = 8.0$ pm. Find the period of the Sun's revolution about its own axis.

5.228. The Doppler effect has made it possible to discover the double stars which are so distant that their resolution by means of a telescope is impossible. The spectral lines of such stars periodically become doublets indicating that the radiation does come from two stars revolving about their centre of mass. Assuming the masses of the two stars to be equal, find the distance between them and their masses if the maximum splitting of the spectral lines is equal to $(\Delta\lambda/\lambda)_m = 1.2 \cdot 10^{-4}$ and occurs every $\tau = 30$ days.

5.229. A plane electromagnetic wave of frequency ω_0 falls normally on the surface of a mirror approaching with a relativistic velocity V. Making use of the Doppler formula, find the frequency of the reflected wave. Simplify the obtained expression for the case $V \ll c$.

5.230. A radar operates at a wavelength $\lambda = 50.0$ cm. Find the velocity of an approaching aircraft if the beat frequency between the transmitted signal and the signal reflected from the aircraft is equal to $\Delta\nu = 1.00$ kHz at the radar location.

5.231. Taking into account that the wave phase $\omega t - kx$ is an invariant, i.e. it retains its value on transition from one inertial frame to another, determine how the frequency ω and the wave number k entering the expression for the wave phase are transformed. Examine the unidimensional case.

5.232. How fast does a certain nebula recede if the hydrogen line $\lambda = 434$ nm in its spectrum is displaced by 130 nm toward longer wavelengths?

5.233. How fast should a car move for the driver to perceive a red traffic light ($\lambda \approx 0.70$ μm) as a green one ($\lambda' \approx 0.55$ μm)?

5.234. An observer moves with velocity $v_1 = \frac{1}{2}c$ along a straight line. In front of him a source of monochromatic light moves with velocity $v_2 = \frac{3}{4}c$ in the same direction and along the same straight line. The proper frequency of light is equal to ω_0. Find the frequency of light registered by the observer.

5.235. One of the spectral lines of atomic hydrogen has the wavelength $\lambda = 656.3$ nm. Find the Doppler shift $\Delta\lambda$ of that line when observed at right angles to the beam of hydrogen atoms with kinetic energy $T = 1.0$ MeV (the transverse Doppler effect).

5.236. A source emitting electromagnetic signals with proper frequency $\omega_0 = 3.0 \cdot 10^{10}$ s^{-1} moves at a constant velocity $v = 0.80\,c$ along a straight line separated from a stationary observer P by a distance l (Fig. 5.37). Find the frequency of the signals perceived by the observer at the moment when

(a) the source is at the point O;

(b) the observer sees it at the point O.

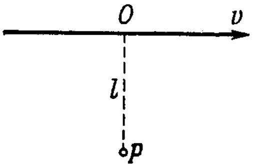

Fig. 5.37.

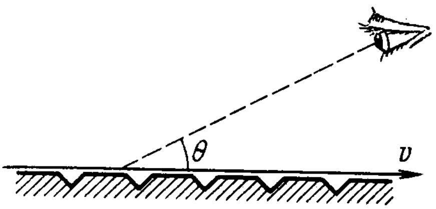

Fig. 5.38.

5.237. A narrow beam of electrons passes immediately over the surface of a metallic mirror with a diffraction grating with period $d = 2.0$ µm inscribed on it. The electrons move with velocity v, comparable to c, at right angles to the lines of the grating. The trajectory of the electrons can be seen in the form of a strip, whose colouring depends on the observation angle θ (Fig. 5.38). Interpret this phenomenon. Find the wavelength of the radiation observed at an angle $\theta = 45°$.

5.238. A gas consists of atoms of mass m being in thermodynamic equilibrium at temperature T. Suppose ω_0 is the natural frequency of light emitted by the atoms.

(a) Demonstrate that the spectral distribution of the emitted light is defined by the formula

$$I_\omega = I_0 e^{-a(1-\omega/\omega_0)^2},$$

(I_0 is the spectral intensity corresponding to the frequency ω_0, $a = mc^2/2kT$).

(b) Find the relative width $\Delta\omega/\omega_0$ of a given spectral line, i.e. the width of the line between the frequencies at which $I_\omega = I_0/2$.

5.239. A plane electromagnetic wave propagates in a medium moving with constant velocity $V \ll c$ relative to an inertial frame K. Find the velocity of that wave in the frame K if the refractive index of the medium is equal to n and the propagation direction of the wave coincides with that of the medium.

5.240. Aberration of light is the apparent displacement of stars attributable to the effect of the orbital motion of the Earth. The direction to a star in the ecliptic plane varies periodically, and the star performs apparent oscillations within an angle $\delta\theta = 41''$. Find the orbital velocity of the Earth.

5.241. Demonstrate that the angle θ between the propagation direction of light and the x axis transforms on transition from the reference frame K to K' according to the formula

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta},$$

where $\beta = V/c$ and V is the velocity of the frame K' with respect to the frame K. The x and x' axes of the reference frames coincide.

5.242. Find the aperture angle of a cone in which all the stars located in the semi-sphere for an observer on the Earth will be visible if one moves relative to the Earth with relativistic velocity V differing by 1.0% from the velocity of light. Make use of the formula of the foregoing problem.

5.243. Find the conditions under which a charged particle moving uniformly through a medium with refractive index n emits light (the Vavilov-Cherenkov effect). Find also the direction of that radiation.

Instruction. Consider the interference of oscillations induced by the particle at various moments of time.

5.244. Find the lowest values of the kinetic energy of an electron and a proton causing the emergence of Cherenkov's radiation in a medium with refractive index $n = 1.60$. For what particles is this minimum value of kinetic energy equal to $T_{min} = 29.6$ MeV?

5.245. Find the kinetic energy of electrons emitting light in a medium with refractive index $n = 1.50$ at an angle $\theta = 30°$ to their propagation direction.

5.7. THERMAL RADIATION. QUANTUM NATURE OF LIGHT

- Radiosity

$$M_e = \frac{c}{4} u, \tag{5.7a}$$

where u is the space density of thermal radiation energy.

- Wien's formula and Wien's displacement law:

$$u_\omega = \omega^3 F(\omega/T), \quad T\lambda_m = b, \tag{5.7b}$$

where λ_m is the wavelength corresponding to the maximum of the function u_λ.

- Stefan-Boltzmann law:

$$M_e = \sigma T^4, \tag{5.7c}$$

where σ is the Stefan-Boltzmann constant.

- Planck's formula:

$$u_\omega = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{e^{\hbar\omega/kT} - 1}. \tag{5.7d}$$

- Einstein's photoelectric equation:

$$\hbar\omega = A + \frac{mv_{max}^2}{2}. \tag{5.7e}$$

• Compton effect:

$$\Delta\lambda = 2\pi \lambda\!\!\!^{-}_C (1 - \cos\theta), \tag{5.7f}$$

where $\lambda\!\!\!^{-}_C = \hbar/mc$ is Compton's wavelength.

5.246. Using Wien's formula, demonstrate that
(a) the most probable radiation frequency $\omega_{pr} \propto T$;
(b) the maximum spectral density of thermal radiation $(u_\omega)_{max} \propto T^3$;
(c) the radiosity $M_e \propto T^4$.

5.247. The temperature of one of the two heated black bodies is $T_1 = 2500$ K. Find the temperature of the other body if the wavelength corresponding to its maximum emissive capacity exceeds by $\Delta\lambda = 0.50$ μm the wavelength corresponding to the maximum emissive capacity of the first black body.

5.248. The radiosity of a black body is $M_e = 3.0$ W/cm². Find the wavelength corresponding to the maximum emissive capacity of that body.

5.249. The spectral composition of solar radiation is much the same as that of a black body whose maximum emission corresponds to the wavelength 0.48 μm. Find the mass lost by the Sun every second due to radiation. Evaluate the time interval during which the mass of the Sun diminishes by 1 per cent.

5.250. Find the temperature of totally ionized hydrogen plasma of density $\rho = 0.10$ g/cm³ at which the thermal radiation pressure is equal to the gas kinetic pressure of the particles of plasma. Take into account that the thermal radiation pressure $p = u/3$, where u is the space density of radiation energy, and at high temperatures all substances obey the equation of state of an ideal gas.

5.251. A copper ball of diameter $d = 1.2$ cm was placed in an evacuated vessel whose walls are kept at the absolute zero temperature. The initial temperature of the ball is $T_0 = 300$ K. Assuming the surface of the ball to be absolutely black, find how soon its temperature decreases $\eta = 2.0$ times.

5.252. There are two cavities (Fig. 5.39) with small holes of equal diameters $d = 1.0$ cm and perfectly reflecting outer surfaces. The

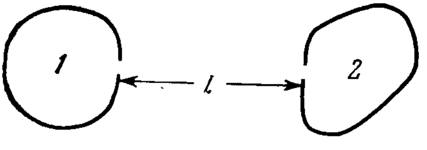

Fig. 5.39.

distance between the holes is $l = 10$ cm. A constant temperature $T_1 = 1700$ K is maintained in cavity *1*. Calculate the steady-state temperature inside cavity *2*.

Instruction. Take into account that a black body radiation obeys the cosine emission law.

5.253. A cavity of volume $V = 1.0$ l is filled with thermal radiation at a temperature $T = 1000$ K. Find:

(a) the heat capacity C_V; (b) the entropy S of that radiation.

5.254. Assuming the spectral distribution of thermal radiation energy to obey Wien's formula $u(\omega, T) = A\omega^3 \exp(-a\omega/T)$, where $a = 7.64$ ps·K, find for a temperature $T = 2000$ K the most probable

(a) radiation frequency; (b) radiation wavelength.

5.255. Using Planck's formula, derive the approximate expressions for the space spectral density u_ω of radiation

(a) in the range where $\hbar\omega \ll kT$ (Rayleigh-Jeans formula);

(b) in the range where $\hbar\omega \gg kT$ (Wien's formula).

5.256. Transform Planck's formula for space spectral density u_ω of radiation from the variable ω to the variables ν (linear frequency) and λ (wavelength).

5.257. Using Planck's formula, find the power radiated by a unit area of a black body within a narrow wavelength interval $\Delta\lambda = 1.0$ nm close to the maximum of spectral radiation density at a temperature $T = 3000$ K of the body.

5.258. Fig. 5.40 shows the plot of the function $y(x)$ representing a fraction of the total power of thermal radiation falling within

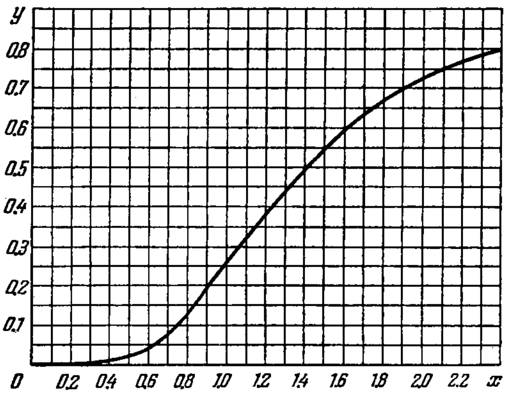

Fig. 5.40.

the spectral interval from 0 to x. Here $x = \lambda/\lambda_m$ (λ_m is the wavelength corresponding to the maximum of spectral radiation density).

Using this plot, find:

(a) the wavelength which divides the radiation spectrum into two equal (in terms of energy) parts at the temperature 3700 K;

(b) the fraction of the total radiation power falling within the visible range of the spectrum (0.40-0.76 μm) at the temperature 5000 K;

(c) how many times the power radiated at wavelengths exceeding 0.76 μm will increase if the temperature rises from 3000 to 5000 K.

5.259. Making use of Planck's formula, derive the expressions determining the number of photons per 1 cm^3 of a cavity at a temperature T in the spectral intervals $(\omega, \omega + d\omega)$ and $(\lambda, \lambda + d\lambda)$.

5.260. An isotropic point source emits light with wavelength $\lambda = 589$ nm. The radiation power of the source is $P = 10$ W. Find:

(a) the mean density of the flow of photons at a distance $r = 2.0$ m from the source;

(b) the distance between the source and the point at which the mean concentration of photons is equal to $n = 100$ cm^{-3}.

5.261. From the standpoint of the corpuscular theory demonstrate that the momentum transferred by a beam of parallel light rays per unit time does not depend on its spectral composition but depends only on the energy flux Φ_e.

5.262. A laser emits a light pulse of duration $\tau = 0.13$ ms and energy $E = 10$ J. Find the mean pressure exerted by such a light pulse when it is focussed into a spot of diameter $d = 10$ µm on a surface perpendicular to the beam and possessing a reflection coefficient $\rho = 0.50$.

5.263. A short light pulse of energy $E = 7.5$ J falls in the form of a narrow and almost parallel beam on a mirror plate whose reflection coefficient is $\rho = 0.60$. The angle of incidence is 30°. In terms of the corpuscular theory find the momentum transferred to the plate.

5.264. A plane light wave of intensity $I = 0.20$ W/cm^2 falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45°. In terms of the corpuscular theory find the magnitude of the normal pressure exerted by light on that surface.

5.265. A plane light wave of intensity $I = 0.70$ W/cm^2 illuminates a sphere with ideal mirror surface. The radius of the sphere is $R = 5.0$ cm. From the standpoint of the corpuscular theory find the force that light exerts on the sphere.

5.266. An isotropic point source of radiation power P is located on the axis of an ideal mirror plate. The distance between the source and the plate exceeds the radius of the plate η-fold. In terms of the corpuscular theory find the force that light exerts on the plate.

5.267. In a reference frame K a photon of frequency ω falls normally on a mirror approaching it with relativistic velocity V. Find the momentum imparted to the mirror during the reflection of the photon

(a) in the reference frame fixed to the mirror;

(b) in the frame K.

5.268. A small ideal mirror of mass $m = 10$ mg is suspended by a weightless thread of length $l = 10$ cm. Find the angle through which the thread will be deflected when a short laser pulse with energy $E = 13$ J is shot in the horizontal direction at right angles to the mirror. Where does the mirror get its kinetic energy?

5.269. A photon of frequency ω_0 is emitted from the surface of a star whose mass is M and radius R. Find the gravitational shift of frequency $\Delta\omega/\omega_0$ of the photon at a very great distance from the star.

5.270. A voltage applied to an X-ray tube being increased $\eta = 1.5$ times, the short-wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 26$ pm. Find the initial voltage applied to the tube.

5.271. A narrow X-ray beam falls on a NaCl single crystal. The least angle of incidence at which the mirror reflection from the system of crystallographic planes is still observed is equal to $\alpha = 4.1°$. The interplanar distance is $d = 0.28$ nm. How high is the voltage applied to the X-ray tube?

5.272. Find the wavelength of the short-wave limit of an X-ray continuous spectrum if electrons approach the anticathode of the tube with velocity $v = 0.85\ c$, where c is the velocity of light.

5.273. Find the photoelectric threshold for zinc and the maximum velocity of photoelectrons liberated from its surface by electromagnetic radiation with wavelength 250 nm.

5.274. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35$ μm and $\lambda_2 = 0.54$ μm, it was found that the corresponding maximum velocities of photoelectrons differ by a factor $\eta = 2.0$. Find the work function of that metal.

5.275. Up to what maximum potential will a copper ball, remote from all other bodies, be charged when irradiated by electromagnetic radiation of wavelength $\lambda = 140$ nm?

5.276. Find the maximum kinetic energy of photoelectrons liberated from the surface of lithium by electromagnetic radiation whose electric component varies with time as $E = a\,(1 + \cos\omega t)\cos\omega_0 t$, where a is a constant, $\omega = 6.0\cdot10^{14}\ \text{s}^{-1}$ and $\omega_0 = 3.60\cdot10^{15}\ \text{s}^{-1}$.

5.277. Electromagnetic radiation of wavelength $\lambda = 0.30$ μm falls on a photocell operating in the saturation mode. The corresponding spectral sensitivity of the photocell is $J = 4.8$ mA/W. Find the yield of photoelectrons, i.e. the number of photoelectrons produced by each incident photon.

5.278. There is a vacuum photocell whose one electrode is made of cesium and the other of copper. Find the maximum velocity of photoelectrons approaching the copper electrode when the cesium electrode is subjected to electromagnetic radiation of wavelength 0.22 μm and the electrodes are shorted outside the cell.

5.279. A photoelectric current emerging in the circuit of a vacuum photocell when its zinc electrode is subjected to electromagnetic radiation of wavelength 262 nm is cancelled if an external decelerating voltage 1.5 V is applied. Find the magnitude and polarity of the outer contact potential difference of the given photocell.

5.280. Compose the expression for a quantity whose dimension is length, using velocity of light c, mass of a particle m, and Planck's constant $\hbar$. What is that quantity?

5.281. Using the conservation laws, demonstrate that a free electron cannot absorb a photon completely.

5.282. Explain the following features of Compton scattering of light by matter:

(a) the increase in wavelength $\Delta\lambda$ is independent of the nature of the scattering substance;

(b) the intensity of the displaced component of scattered light grows with the increasing angle of scattering and with the diminishing atomic number of the substance;

(c) the presence of a non-displaced component in the scattered radiation.

5.283. A narrow monochromatic X-ray beam falls on a scattering substance. The wavelengths of radiation scattered at angles $\theta_1 = 60°$ and $\theta_2 = 120°$ differ by a factor $\eta = 2.0$. Assuming the free electrons to be responsible for the scattering, find the incident radiation wavelength.

5.284. A photon with energy $\hbar\omega = 1.00$ MeV is scattered by a stationary free electron. Find the kinetic energy of a Compton electron if the photon's wavelength changed by $\eta = 25\%$ due to scattering.

5.285. A photon of wavelength $\lambda = 6.0$ pm is scattered at right angles by a stationary free electron. Find:

(a) the frequency of the scattered photon;

(b) the kinetic energy of the Compton electron.

5.286. A photon with energy $\hbar\omega = 250$ keV is scattered at an angle $\theta = 120°$ by a stationary free electron. Find the energy of the scattered photon.

5.287. A photon with momentum $p = 1.02$ MeV/c, where c is the velocity of light, is scattered by a stationary free electron, changing in the process its momentum to the value $p' = 0.255$ MeV/c. At what angle is the photon scattered?

5.288. A photon is scattered at an angle $\theta = 120°$ by a stationary free electron. As a result, the electron acquires a kinetic energy $T = 0.45$ MeV. Find the energy that the photon had prior to scattering.

5.289. Find the wavelength of X-ray radiation if the maximum kinetic energy of Compton electrons is $T_{max} = 0.19$ MeV.

5.290. A photon with energy $\hbar\omega = 0.15$ MeV is scattered by a stationary free electron changing its wavelength by $\Delta\lambda = 3.0$ pm. Find the angle at which the Compton electron moves.

5.291. A photon with energy exceeding $\eta = 2.0$ times the rest energy of an electron experienced a head-on collision with a stationary free electron. Find the curvature radius of the trajectory of the Compton electron in a magnetic field $B = 0.12$ T. The Compton electron is assumed to move at right angles to the direction of the field.

5.292. Having collided with a relativistic electron, a photon is deflected through an angle $\theta = 60°$ while the electron stops. Find the Compton displacement of the wavelength of the scattered photon.