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What is the Hierarchy Problem?

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ABSTRACT

Is there a Hierarchy Problem? If so, what, exactly, is the problem? Almost every theorist has a personal answer to these questions. In this article, I give my answers. I will explain that the Hierarchy Problem is not a formal problem but rather our ignorance of a crucial physics explanation – the explanation of the nature of the Higgs boson. Without the solution to this problem, we cannot make progress on the major questions of our field.

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Contents

1 Introduction

Today, the "Hierarchy Problem" is the question in particle physics about which there is the least consensus. There are many conceptions of this problem and the related question of the "naturalness" of the parameters of the Standard Model. I doubt that — absent any illuminating experimental discovery — a new theoretical paper could meet the stated goal of this special issue of "clarifying common misconceptions". Still, I am glad to have this opportunity to state my personal opinions on what the Hierarchy Problem really asks and offer my own route to a solution. As to whether I am dissolving confusion and simply adding to it, I leave to the reader to judge.

The approach to the Hierarchy Problem in this paper is intentionally narrow. In particular, I will only discuss "the" Hierarchy Problem, the problem of the value of the Higgs mass term μ^2 in the Standard Model. For those who seek a comprehensive review of the Hierarchy Problem, including discussions of the Cosmological Constant Problem, I strongly recommend the paper written by Nathaniel Craig for the Snowmass 2021 study, which attempts a complete survey of the literature [1].

2 Is there a Hierarchy Problem?

The Hierarchy Problem in its simplest formulation concerns the mechanism of the spontaneous breaking of electroweak $SU(2) \times U(1)$ gauge symmetry in the Standard Model (SM). I define the SM as a renormalizable theory with exactly one fundamental Higgs scalar doublet Φ. Under this set of assumptions, there is a unique expression for the potential energy of the Higgs field,

$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4 \ . \tag{1}$$

The parameters μ^2 and λ are parameters of a renormalizable quantum field theory and, as such, must be adjusted by hand to values that fit the experimental data. The observed spontaneous breaking of electroweak symmetry (EWSB) requires $\mu^2 < 0$.

The SM with a single Higgs field has important virtues that must be taken seriously. In particular, it implies that lepton flavor conservation and flavor universality is absolute, up to truly tiny effects due to the neutrino masses, and it implies that flavor mixing and CP violation in the hadronic weak decays is completely described by the four Cabibbo-Kobayashi-Maskawa (CKM) parameters. It implies that there is no flavor violation in Higgs boson couplings and decays. So far, these restrictions hold up very well against experiment. The SM allows arbitrarily large mass hierarchies among the quarks and leptons, and these hierarchies are stable under higher loop corrections.

However, for those who would like to understand the reason for EWSB, the SM is extremely frustrating. In the SM, the parameter μ^2 is inserted by hand with no explanation. In principle, the origin, the value, and, especially, the sign of μ^2 ought to be found from an argument based on physics. There is a long history of models giving mechanisms of EWSB, but every model requires new interactions in addition to those in the SM. The SM alone just throws up its hands. The is the version of the problem that I feel is the most important. So, yes, there is definitely a hierarchy problem.

Most discussions of the hierarchy problem are given in terms of the instability of the value or sign of μ^2 under loop corrections. In the rest of this section, I will describe this approach and comment on it.

Even at the 1-loop order, the radiative corrections to the μ^2 parameter have a troubling form. One finds

$$\mu^2 = \mu^2|_{\text{bare}} - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \frac{3\lambda}{8\pi^2}\Lambda^2 + \frac{9\alpha_w + 3\alpha'}{16\pi}\Lambda^2 + \cdots \ . \tag{2}$$

I have quoted this expression with the Λ parameters being ultraviolet (UV) cutoffs on the Feynman integrals. These can have other interpretations, as I will discuss in the next paragraphs. In all but the most naive models, these parameters take large dimensionful values corresponding to higher mass scales in nature. The three Λ parameters need not be identical. Looking at the problem in this way, it seems that there are large positive and negative contributions to μ^2 that must be arranged to cancel to give the observed value $\mu^2 \approx$ -(100 GeV)^2. This is why a problem that might otherwise be called "the μ^2 ignorance problem" is called the Hierarchy Problem.

When they hear the Hierarchy Problem described in this way, many students are not concerned by this issue. They point out that, in dimensional regularization, the ultraviolet divergences disappear. Isn't that a solution to this problem? I find this explanation unacceptable. Let me first give a technical explanation. Dimensional regularization is a very convenient regulator because it sets integrals with no mass parameters equal to zero. In the case of the above diagrams, dimensional regularization renders them UV finite by subtracting an integral

$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2} = 0 \ . \tag{3}$$

But one should also consider the physical meaning of this subtraction. The massless particles implied by (3) do not exist. If any higher-mass particle of mass M contributes to the loop, it will, in dimensional regularization, give a contribution of the structure of one of the terms in (2) with the Λ^2 replaced by M^2. This includes particles assocated with the completion of the SM to include quantum gravity. Fluctuations of the t, Φ, and W, Z quantum fields at high momenta certainly do exist. Even dimensional regularization tells us that these fluctuations contribute to the renormalization group β function of the λ parameter. In fact, they contribute to the prediction of the SM that its vacuum is unstable [2,3]. So their effect is not so easily dismissed [4].

This technical argument comes back to my earlier displeasure with the SM. Given that any modification of the SM leads to terms of the form of those in (2), the argument from dimensional regularization works only if we dismiss any correction to the SM, at any mass scale. Then we are really giving up on the explanation of μ^2.

The Hierarchy Problem is often presented by quoting (2) and then assuming that the Λ parameters must be proportional to a very large mass scale such as the Planck scale. This poses a striking problem, but it misses the essential physics question. I will expand on this in the following section.

Before going further, though, I would like to call attention to a benefit of taking the fluctuations of the top quark field at large momentum seriously. Consider the vacuum of the top quark quantum field as represented by a Dirac sea. It is obvious that generating a HIggs field vacuum expectation value that gives the top quark a mass opens a mass gap in the spectrum that lowers the energy of the filled Dirac sea, which is the same as lowering the energy of the vacuum. I have found this argument helpful in clarifying the mystery of EWSB for physics department colloquium audiences. Why not take it seriously? The idea that the largeness of the top quark Yukawa coupling relative to other dimensionless couplings of the SM is the driver of EWSB is very enticing. This was first noticed in the context of supersymmetric grand unified theories [5–7], but it applies in a very wide variety of explicit models of EWSB, as I will discuss below.

3 The Wilsonian point of view

Ken Wilson is often credited with the original statement requiring a natural explanation for the Higgs mass parameter, through his comment: “It is interesting to note that there are no weakly coupled scalar particles in nature; scalar particles are the only kind of free particles whose mass term does not break either an internal or a gauge symmetry.” [8]. However, Wilson’s ideas run through this problem at a deeper level than this. At a time when particle theorists believed that hadron were fundamental and rejected equations with pointlike couplings and UV divergences, Wilson held to the belief that quantum field theories were not different from ordinary quantum-mechanical systems. He analyzed them by taking the UV cutoff seriously as the largest energy scale and working downward, rather than considering the UV cutoff scale as an artifact to be removed by appropriate incantations [9]. This led him to consider quantum field theories as related to models of phase transitions in condensed mater and, eventually, to important discoveries about those systems. These discoveries doubled back to particle physics in the concept of Wilsonian effective field theories, which now provide a basic language for our discussions of particle physics.

As a student of Wilson, I am thoroughly infected with that philosophy, for better or worse. Spontaneous breaking of symmetry is found in a great variety of condensed matter systems — magnets, superconductors, liquid crystals, and more. The condensed matter context forces us to think about symmetry-breaking in these systems in a different way. Condensed matter systems are made of atoms, and the laws of atomic physics have been fixed since discovery of quantum mechanics. This means that the explanations for these examples of spontaneous symmetry breaking must be derivable from those laws. This makes the study of phase transitions in condensed matter a fascinating subject, with many surprising mechanisms leading to the observed nontrivial ground states [10]. Thinking in this way get us away from simplifying the hierarchy program to the explanation of a large ratio of mass scales.

Then, *the real goal of our pursuit of the origin of* μ^2 *should be to find a compelling physical mechanism that explains the origin of EWSB.* This mechanism might be one already known in condensed matter physics, or it might be one that is completely new. In any case, that physics is not present in the SM, a theory that is completely weak-coupling at short distances. *This point of view requires that there must be new fundamental interactions working at shorter distances than the ones that we have probed so far.* That gives an important goal for further exploration in high-energy physics.

An example of a physical mechanism leading to spontaneous symmetry breaking in particle physics is the explanation of chiral symmetry breaking in the strong interactions. From the development of the theory of weak interactions in the 1950's, it was understood that the strong interactions should be invariant under chiral symmetries, even though QCD, which makes this statement obvious, was many years in the future. In 1961, Nambu and Jona-Lasinio proposed that the chiral symmetry of the strong interactions could be spontaneously broken by same mechanism that causes superconductivity – fermion pair condensation [11]. Throughout the 1960's, this idea and the related idea of the quark model were considered to be hopelessly naive. Progress was made through the language of "current algebra" [12] in papers that are very difficult to read today. Finally, these physically transparent ideas found their place within the theory of QCD with asymptotic freedom.

My goal for the hierarchy problem is similar. Can we find a cogent explanation for EWSB based on an explicit physical model? This does not need to be an ultimate explanation or one correct all the way to the Planck scale. We can build our model of fundamental physics step by step.

This is the step that we need to take now. Different models of EWSB lead to different models of the Higgs boson, for example, as a member of a supersymmetry multiplet, as member of a global symmetry multiplet, as a composite particle. Each hypothesis leads to distinct theories of the most important issues in elementary particle physics – the origin of the hierarchy of fermion masses and mixings, the origin of the baryon asymmetry in nature, the origin of neutrino masses. Many hypotheses for the problem of the origin of dark matter also depend of this choice. Without the solution to the Hierarchy Problem, as I have framed it here, it will be difficult to find the correct path to address any of these questions, much less to find the correct model.

4 The Hierarchy Problem before the LHC Results

I saw the Hierarchy Problem in this way also before the start of the LHC experimental program. At that time, I was optimistic that we would obtain clues to this problem from the discovery of new particles at the LHC.

For many members of our community, the idea that drove the search for new particles was the beauty of supersymmetry. Supersymmetry is, after all, a unique extension of Poincaré invariance. It is a central element of string theory and of most successful theories of grand unification. Supersymmetry also provides a natural mechanism for cutting off the quadratic divergences in (2), since the quadratic divergences in scalar masses parameters must cancel in a completely supersymmetric theory. And, if the Higgs boson should be found at a mass below 1 TeV, its superpartners should also be found there.

For me, though, supersymmetry had a different attraction. Supersymmetry provides a quantitative physical mechanism by which the top quark Yukawa coupling can provide a physical mechanism for EWSB. In supersymmetric theories, the logarithmic renormalization of soft supersymmetry breaking scalar mass terms by Yukawa couplings is negative. So, if supersymmetry is spontaneously broken, as it must be to provide a realistic theory, then the renormalization of the symmetry-breaking mass terms will lead to a vacuum instability for one or more scalar fields. There are many, many new scalar fields in a supersymmetric extension of the SM, leading to many possibilities for symmetry breaking. But, due to the large size of the top quark Yukawa coupling and some convenient group theory factors, it is the Higgs doublet scalars for which this effect is largest, leading to the exactly the pattern of EWSB found in the SM [5–7]. This mechanism is highlighted in my 2006 TASI lectures on supersymmetry [13].

If we give up the idea that the solution to EWSB must also provide a fundamental theory up to very high energies, then there are many models that use the largeness of the top quark Yukawa coupling to drive other mechanisms for EWSB. In 1984, Kaplan and Georgi introduced the idea that the Higgs boson could be a composite particle bound by a new set of strong interactions [14]. The Higgs boson could be light compared to the symmetry-breaking scale of the strong interaction theory if it were a

Goldstone boson of a dynamically broken global symmetry of that theory. The Higgs boson mass term would be supplied by radiative corrections induced by terms in the theory that did not respect that global symmetry. Later, this mechanism would be explicitly realized (and some difficulties of the Kaplan-Georgi model solved) in Little Higgs theories [15,16]. In these models, a vector-like fermion top quark partner would cancel the quadratic divergences of the top quark loop correction to μ^2, leaving over a naturally negative contribution.

A similar mechanism was found to work in models with an extra space dimension. In these models, the Higgs boson doublet is identified with the 5th component of a set of gauge fields in the higher dimension. This identification leads to the cancellation of quadratic divergences. The contribution to the Higgs mass term from the top quark is again negative, but the contributions from the Kaluza-Klein resonances of the 5-dimensional top quark field are positive and cut off the ultraviolet divergence. Here also, radiative corrections due to the top quark Yukawa coupling lead to a naturally negative value of μ^2 [17].

In all of these cases, new particles are needed to build a complete model of EWSB — the supersymmetric partners of the top quark in the case of supersymmetry, vectorlike fermion top quark partners in the cases of nonsupersymmetric models. These particles can be heavier than the Higgs boson, but fine-tuning is needed to push their masses above 1 TeV. In the case of supersymmetry, where the entire theory is described by weak-coupling interactions, a rather sophisticated literature on fine-tuning developed, suggesting quite strong upper limits on the masses of these particles [18, 19]. In the non-supersymmetric cases, the limits are weaker but still in the region of 1-3 TeV.

So far, none of these new particles has been discovered at the LHC. Especially because of the stringent expectations for supersymmetry, most members of our community have come to the opinion that these particles do not exist. Still, especially for the vectorlike fermions, there is opportunity to extend the searches and discover these particles at the HL-LHC. I hope that LHC experimenters will take this opportunity seriously.

Many theorists now are trying to explain that they never actually predicted the discovery of new particles at the LHC, or that they gave reasons why these potential discoveries were already excluded. I am not one of them. I feel that the models that I have reviewed in this section remain compelling ideas for the mechanism of EWSB. Even if nature does not choose models of these types, we ought to own the reasoning that led to them. The true explanation for EWSB might be very close, if only some further new idea can be added to the mix.

5 The "Post-Naturalness Era"

The failure of the LHC experiments to discover new particles has been discouraging for many theorists. This has led to a search for a solution to the Hierarchy Problem outside of the realm of particle physics model-building. Suggestions include the use of the Anthropic Principle, mechanisms in the physics of the early universe, and possible UV/IR connections in the quantum theory of gravity. Many of these models are explained in Anson Hook's review paper [20], which specifically concerns nontrivial mechanisms for relieving the fine-tuning of μ^2 in models of fundamental scalar fields.

Excuse me that I am very cool to these ideas. Particle theorists often view scalars as part of the general equipment of nature, to be added at will to any theory. As I have already stated above, I believe that scalar fields in nature should have a purpose. If they obtain vacuum expectation values, those expectation values should be tied to the masses of particles or to another dimensionful reference point. It is possible that that the vacuum expectation value of the Higgs field could be chosen randomly. But it would be much more pleasing to explain this value in an underly physical picture.

In a 2017 paper, Gian Giudice presented this direction of research in a very optimistic way [21]. In an article titled "The Dawn of the Post-Naturalness Era", he describes the exclusion by the LHC of the models described in the previous section as a true crisis in theoretical particle physics, a crisis that would, in his view, be resolved only by the invention of revolutionary new ideas. Particle theorists are eager for revolutions, and also for "final theories" that fill the gap between currently explored energies and the Planck scale. It is bold to try to imagine these new ideas, but I feel it is not yet necessary. Perhaps conventional quantum field theory has still not shown us all of its tricks.

6 Three Hierarchy Problems

In this more pragmatic way of thinking, it is useful to divide the traditional Hierarchly Problem into three problems.

The first is the traditional "Hierarchy Problem": Why does the physics of electroweak symmetry breaking occur at energies so much lower than the Planck scale? There are many possible answers to this question. Supersymmetry with a low scale of its spontaneous breaking is one. Asymptotic freedom, for which a small coupling constant at an original scale leads to a dynamical generation of a new scale that is exponentially smaller, is another. Actually, we know that this is the explanation for the hierarchy problem of QCD, explaining why the mass of the proton is so much smaller than the Planck scale.

To formulate a solution to this problem, we first need to know what are the basic particles and fields that cause EWSB. Because new particles have not been discovered at the LHC, the nature of these fundamental particles is still hidden from us.

The second is what one might call the "Problem of Scalars". This is the flip side of the idea that one should add scalars to a model to give it more interesting dynamics. Scalars have dimension 1 and can couple to other fields almost without restriction. Even extending the Higgs sector of the SM to two Higgs doublets extends the parameter set to 4 mass parameters and 10 quartic parameters, counting possibly complex coefficients as two real parameters.

In the SM, the masses of quarks and leptons come from Yukawa interactions of the of these fermions with Higgs field. These couplings are given in terms of 3 complex 3×3 matrices, a total of 54 fundamental parameters. Of these, only 14 – the nine quark and lepton masses, the 4 CKM parameters, and the θ parameter, are potentially observable. The rest can be removed by field redefinitions. This leads to enormous ambiguity in the search for a theory of fermion masses and mixings. The history of theories of flavor reflects this. In 1977, Harold Fritzsch gave a model [22] that successfully predicted the relation

$$\tan\theta_c \approx \sqrt{m_d/m_s}\ . \tag{4}$$

It has been all downhill since then. Despite the efforts of many of the leading theorists of the my generation — including Jogesh Pati, Savas Dimopoulos, Lawrence Hall, Graham Ross, and Yossi Nir — no one would say today, "Give me one more decimal place on the CKM parameters and this problem will be solved." The explanation for the neutrino masses, which involve a 3×3 complex seesaw mass matrix in additional to a Yukawa matrix, is even further away.

In addressing this problem, it would be tremendously illuminating to discover the organizing principle for the scalar couplings. Do they come from compositeness the Higgs boson? from the mixing of the lightest Higgs field with heavier partners? from a hierarchy of couplings generated by perturbation theory? An aspect of quantum field theory that we know little about today is the dynamics and possible composite particles of chiral gauge theories. Maybe this is a place that we will find clues to these questions.

It would be wonderful to learn the correct explanation from experiment. But, again, our ignorance of the fundamental nature of the Higgs field and our lack of clues about what stands behind the SM is an impediment here.

The third problem is the "Little Hierarchy Problem". This is the question of why the masses of new particles needed for a dynamical model of EWSB are so much heavier than the mass of Higgs boson. In the context of a search for the dynamical explanation of EWSB, this is a constraint on the possible answers: They must have some feature that leads to an unanticipated mass gap between the μ^2 parameter and the masses of the lightest new particles.

In my opinion, the Little Hierarchy Problem is the one that offers the best chance for a solution now. Only a small hierarchy is needed. In addition, we need the answer to this question to make progress on either of the previous two hierarchy problems, which cannot even be properly posed without new information about the nature of the new particles to be found at energies above those currently probed by the LHC.

7 Solutions to the Little Hierarchy Problem

There are solutions to the Little Hierarchy Problem in the literature, but no one is very impressed with them. In this section, I will discuss three of these. In all three cases, the next set of fundamental interactions beyond the SM has two levels, one immediately generating the Higgs potential and another, at a higher mass scale, being the more fundamental cause of the symmetry-breaking. I will refer to these as the intermediate and the fundamental mass scales. In their current state, these models seem artificial and overly complex. However, they provide starting points.

The first idea is to include more fields in the region between the intermediate and fundamental scales in the model. There are many examples of models that use this strategy implicitly. The strategy is made more explicit in work of mine with Yoon, where we call it "competing forces" [23]. In the example we discuss, the chiral top quark naturally generates a term in the Higgs potential with a negative μ^2. In the same model, an additional vector-like fermion generates a term in the Higgs potential with a positive μ^2. Playing these two effects against one another give a Higgs potential that requires only a modest level of tuning to move new particles out of the range explored by the LHC. The new competing fermion might have its own reason for existence, for example, to generate the cosmic dark matter.

The second idea, "2-stage symmetry breaking" takes advantage of a property of models in which the fundamental symmetry breaking is due to fermion condensation, with the Higgs boson identified with a Goldstone boson created in this process. In such models, mass generation for the Goldstone boson is forbidden by two different chiral symmetries, one associated with the left-handed fundamental fermions, the other with the right-handed fundamental fermions. Breaking both symmetries can require two insertions of weakly coupled chiral symmetry breaking perturbations, leading to a formula for the induced μ^2 term of the form

$$\mu^2 = -\frac{3\alpha_w y_t^2}{8\pi^2} f^2 \ , \tag{5}$$

with f the mass scale of the fundamental level; often, this is enhanced by a factor $\log f^2/m_t^2$. This 2-stage generation of μ^2 is a property of the Littlest Higgs model [15] and is analyzed further in [24]. The model does require a new vectorlike top partner at the intermediate scale, which perhaps could be generated as a massless composite fermion of the fundamental-level theory.

The third idea is one special to supersymmetry, "Dirac gauginos" [25]. In this strategy, the gaugino sector is $N = 2$ supersymmetric, while the matter sector has $N = 1$ supersymmetry. Supersymmetry can be broken by a D term in the N=2 sector. In the matter sector, this generates supersymmetry-breaking mass terms that are "supersoft", that is, very insensitive to details of the fundamental scale. The μ^2 term is generated both directly, by an electroweak supersoft term, and in two stages, using the top quark Yukawa coupling. Schematically

$$\mu^2 \approx \frac{\alpha_w M_1^2}{\pi} - \frac{3\lambda_t^2 m_{\tilde{q}}^2}{4\pi^2} \log \frac{M_3}{m_{\tilde{q}}} , \tag{6}$$

where M_a are gaugino masses and $m_{\tilde{q}}$ is a squark mass. Effectively, this mechanism automatically uses both of the above strategies to lower μ^2 relative to the fundamental supersymmetry breaking mass scale. Some further development of this idea relevant to the hierarchy problem can be found in [26–28].

There is a ample room to improve these strategies, and perhaps find new ones, in pursuit of an elegant model that solves the Little Hierarchy problem. This direction deserves more attention from the particle theory community.

8 The Hierarchy Problem and future high-energy colliders

Particle theorists often confine their thinking to their own narrow domain. However, our beliefs about the Hierarchy Problem have important implications for experimental particle physics that I feel cannot be ignored.

In the development of the SM, the progress of theory relied strongly on surprises from experiment. The τ-θ puzzle of kaon decays opened the path to an understanding of the weak interactions. The SLAC-MIT deep inelastic scattering experiments and the discovery of the J/ψ shattered the notion that hadrons were fundamental particles and opened the door to the quark model and QCD. Even after the SM was formulated and began to be tested with precision, the heaviness of the top quark was a new discovery that now informs our ideas about beyond-SM physics. We need new experimental surprises to guide further progress.

For the next high energy physics collider, there is a general consensus and a well-developed physics case for an e^+e^- Higgs factory [29]. This will make precision measurements that may point to the next energy scale. But in order to actually reach that scale, we will need a collider at energies substantially higher than those of the

LHC. There is much interest now in planning for a 100 TeV proton collider, or, more generally, for a "10 TeV pCM" (parton CM energy) collider [30].

The cost of any 10 TeV pCM collider will be in the $10 B range. A less expensive machine might be justified by the importance of exploring for new fundamental interactions beyond our current knowledge. For particle physicists, the importance of this goal is obvious. But for our colleagues in other fields of science, and for our government sponsors, this is much less clear. Already, we are hearing from many sources that, with the SM complete, particle physics is finished, and that the funding it requires for a next step is better spent in other areas. Especially at this level of cost, we will be asked what, more precisely, we expect to discover, and what energy is actually needed to achieve that goal. Our physics arguments for a higher energy collider need to be much stronger.

Because the SM can be extrapolated to the Planck scale, there is no guaranteed discovery at such a machine (as there was, for example, for the LHC). Still, we can make a strong argument for a 10 TeV pCM collider if we can argue that this collider gives us an *opportunity* to discover and characterize a new fundamental interaction of nature.

Do you believe that this is so? The mechanistic view of the Hierarchy Problem that I have presented in this paper leads to a very different answer from models that simply motivate a hierarchy of unknown size from the randomness of nature or from the influence of constraints from gravity or cosmology. In those models, higher energy scales in physics may be very far away, or, even, might not be needed at all below the scale of gravity. Mechanistic models of the Hierarchy Problem are different in this respect. They call for new particles at energy scales that are higher than those currently studied but are linked to the weak interaction scale.

I feel strongly the development of new mechanistic models of EWSB is essential if we wish to advocate for higher-energy colliders. Can we claim that there is an opportunity to discover new fundamental interactions if we can obtain another factor of 10 in collision energy? Can we illustrate this with compelling models that address EWSB and other major questions of particle physics? Can we make a quantitative argument that 10 TeV is a important milestone? If we cannot answer these questions for ourselves, we will not be able to persuade others.

Today, 10 TeV pCM colliders are still out of reach. In the next decades, the particle physics community must put serious effort into the development of new tools and technologies to reach those energies. In parallel, theorists must work to define the experimental program that these accelerators will carry out. We cannot do this without confronting the Hierarch Problem and bringing new ideas to its solution. This is a responsibility that the particle theory community must address.

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