Subthreshold poles in electron-positron annihilation. D$\bar{\mathrm{D}}$ final states.

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We show that the ψ(2S) subthreshold pole influences the cross section of the electron-positron annihilation into the D$\bar{\mathrm{D}}$ final states. We make the fits to the BESIII data [1] on $\mathrm{D}^0\bar{\mathrm{D}}^0$ and $\mathrm{D}^+\mathrm{D}^-$ final states, and also two common fits to both processes. The statistical significance of the subthreshold pole is greater than 4σ in all cases.

I. INTRODUCTION

Vector meson dominance (VMD) [2] is a useful phenomenological concept based on the assumption that the interaction of the hadronic system with the electromagnetic field is mediated by truly neutral vector meson resonances of negative C-parity. It is frequently used to interpret the e^+e^- annihilation data to hadrons. Experimentalists use resonances with masses higher than the considered reaction's threshold to fit the excitation curves' salient features (peaks, bumps, dips, steep slopes). VMD has also been used in modeling dilepton production in hadronic [3] and nuclear [4] collisions.

Subthreshold poles are very well-known in hadronic reactions. A typical example is the nucleon pole in the pion-nucleon scattering. Even if it is not accessible in an experiment, it influences the cross-section strongly. Owing to that, its existence can be proven by analyzing the forward scattering amplitude obtained from phase-shift analyses, and its residue proportional to the πNN coupling constant can be determined with reasonable accuracy [5]. Guided by this analogy, we plan to investigate the role of subthreshold poles in the e^+e^- annihilation into hadrons. This paper is a first step.

To our knowledge, nobody has systematically investigated the analytic properties of the amplitudes of the e^+e^- annihilation to various hadronic systems. Here, we will assume that they are similar to those of hadronic amplitudes, discussed, *e.g.*, in [6].

From general principles of causality, locality of interactions, and unitarity, the amplitude is a real analytic function in the complex s-plane with the cut along the positive real axis running from the process threshold to infinity (called the physical cut). The stable states, i.e., those which do not decay into the considered final state, are represented by poles lying in the real axis below the threshold. Resonances are represented by the pairs of complex conjugate poles situated on the higher Riemann sheets, accessible through the physical cut. The contribution of resonance to the amplitude on the upper branch of the physical cut is usually parametrized by some form of the Breit-Wigner formula. They run from the simplest one with constant mass and width to the most sophisticated with running (s-dependent) mass and variable width, which are related through the once-[7] or twice-[8] subtracted dispersion relations.

In this paper, we investigate the influence of the ψ(2S) pole on the electron-positron annihilation cross section into the D$\bar{\mathrm{D}}$ final states. To get a broader view of the problem, we first fit the $\mathrm{D}^+\mathrm{D}^-$ and $\mathrm{D}^0\bar{\mathrm{D}}^0$ processes individually and then perform two common fits, differing in the degree of respecting the distinctiveness of the two processes.

II. MODEL

For the description of the electron-positron annihilation into a D$\bar{\mathrm{D}}$ pair (D represents D^+ or D^0) we use a Vector Meson Dominance (VMD) model based on the Feynman diagram depicted in Fig. 1 and the interaction Lagrangian

$$\mathcal{L}_{V\phi}(x) = \mathrm{i}g_{V\phi}V_\mu(x)\left\{[\partial^\mu\phi^\dagger(x)]\phi(x) - \phi^\dagger(x)\partial^\mu\phi(x)\right\},$$

where $V^\mu(x)$ denotes the (hermitian) vector field and $\phi(x)$ the pseudoscalar D field. The γV junction is parametrized as eM_V^2/g_V in analogy with the $\gamma\rho^0$ junction $eM_{\rho^0}^2/g_\rho$. The formula for the cross section of the e^+e^- annihilation into a D$\bar{\mathrm{D}}$ pair based on the vector-meson-dominance model with n resonances is

$$\sigma(s) = \frac{\pi\alpha^2}{3s}\left(1 - \frac{4m_\mathrm{D}^2}{s}\right)^{3/2}\left|\sum_{k=1}^{n}\frac{\sqrt{Q_k}\,e^{\mathrm{i}\delta_k}}{s - M_k^2 + \mathrm{i}M_k\Gamma_k}\right|^2. \quad (1)$$

In this formula, M_k and Γ_k are the mass and width of the kth resonance, respectively, Q_k is related to the Lagrangians parameters through $Q_k = R_k^2$, $R_k = M_k^2 g_{V_k\phi}/g_{V_k}$, and δ_k is an additional phase shift. We will treat M_ks, Γ_ks, Q_ks, and δ_ks as free parameters [9] (except δ_1, which is kept at 0). We will determine their mean values and dispersions by fitting the formula (1) to experimental cross sections using the standard χ^2 criterion.

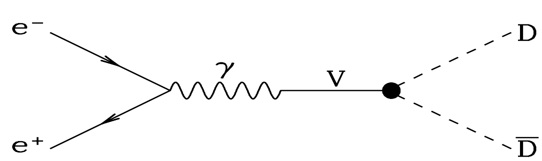

FIG. 1. Feynman diagram defining our model

III. THE EXPLORED DATA

The experimentalists working at the spectrometer BESIII situated at the electron-positron collider BEPCII based at the IHEP laboratory in Beijing, China, have recently published [1] precise data on e^+e^- annihilation to the $\mathrm{D\bar{D}}$ final states. In the Supplemental Material [10], they provide the Born cross sections of the $e^+e^- \to \mathrm{D^0\bar{D}^0}$ (Table 1) and $e^+e^- \to \mathrm{D^+D^-}$ (Table 2) processes at 150 cms energies between 3.80 and 4.95 GeV. Also, correction factors are tabulated, enabling conversion of the Born cross sections to the dressed ones. The dressed cross sections were a subject of the fit presented in the Supplemental Material [10] with results $\chi^2/\mathrm{DOF} = 346/276$, which implies the confidence level (CL) of 0.27% [11].

We follow the experimentalists and use the dressed cross sections when fitting our model to the BESIII data. We determine the free parameters by minimizing the usual χ^2. However, we do not fix the masses and widths of resonances at individual PDG values as the experimentalists did, but consider them free parameters. Our motivation is that the experimental resolution and acceptance influence the final shape of the data. As a result, the original narrow resonances are blurred out, and their positions may shift somehow. We are unable to include these effects in our fitting formula, so we have to fit the final product without trying to find the parameters of the genuine resonances.

IV. $\mathrm{D^+D^-}$ FINAL STATES

To help the minimization code cope with many parameters, we started the fitting with two resonances and gradually added further using the already found parameters as the starting ones for the next run. In this way, we reached the six-resonance fit with $\chi^2/\mathrm{DOF} = 152.4/127$ and CL=6%. To get a better fit, we added a new resonance with unconstrained parameters. The quality of the fit increased to $\chi^2/\mathrm{DOF} = 107.2/123$, CL=84%. But the parameters $M = (3722\pm50)$ MeV and $\Gamma = (0\pm310)$ MeV suggest that to improve the goodness of the fit, a subthreshold pole is needed lying on the real axis of the s-plane, not a resonance. There are two obvious candidates with the $J^{PC} = 1^{--}$ quantum numbers and the quark composition ensuring the strong coupling to the $D\bar{D}$ system: $J/\psi(1\mathrm{S})$ with mass about 3097 MeV and $\psi(2\mathrm{S})$ with mass 3686.097(11) MeV [12]. The mass of the subthreshold pole we have found leads us to choose the $\psi(2\mathrm{S})$ for further processing.

The results of the fit assuming the $\psi(2\mathrm{S})$ subthreshold pole and six resonances are shown in Table I. The fit quality characterized by $\chi^2/\mathrm{DOF} = 107.6/125$ and CL=87% is somewhat better than that of the "seven-resonance" one, thanks to fewer free parameters. We fixed the mass of the subthreshold pole and zeroed its width. The presence of the subthreshold pole is indicated by a statistical significance of 4.1σ. Further adding a resonance led to a slight decrease of the χ^2, which, due to the increased number of parameters, did not bring a better confidence level.

V. $\mathrm{D^0\bar{D}^0}$ FINAL STATES

When processing the BESIII [1] cross-section data on the $e^+e^- \to \mathrm{D^0\bar{D}^0}$ process, we reached a good fit already with the six-resonance Anzatz: $\chi^2/\mathrm{DOF} = 118.2/127$, CL=70%. Anyhow, we tried to improve the fit further by adding another resonance. Its mass, which came out as (3629 ± 55) MeV, and vanishing width of (0 ± 110) MeV again points to a subthreshold pole. Moreover, a simple average of this mass and that found in the "seven-resonance" $\mathrm{D^+D^-}$ fit is (3680 ± 37) MeV, which again hints to the $\psi(2\mathrm{S})$ resonance as an agent acting in both processes.

The results of the fit with $\psi(2\mathrm{S})$ and six resonances to the BESIII $\mathrm{D\bar{D}}$ cross section data are presented in Table II. Quality of the fit is characterized by $\chi^2/\mathrm{DOF} = 102.0/125$ and a confidence level of 93%. The statistical significance of the subthreshold pole is determined at 4.8σ.

VI. BOTH PROCESSES COMBINED

A. Same parameters for $\mathrm{D^+D^-}$ and $\mathrm{D^0\bar{D}^0}$ final states

We follow the BESIII Collaboration and perform a common fit to their $\mathrm{D^+D^-}$ and $\mathrm{D^0\bar{D}^0}$ cross-section data, assuming all parameters in the cross-section formula (1) are the same for both processes. Assuming six resonances, the fit yielded the results $\chi^2/\mathrm{DOF} = 304.3/277$, CL=12%. Adding the subthreshold pole with the mass of $\psi(2\mathrm{S})$ improved the quality of the fit to $\chi^2/\mathrm{DOF} = 246.8/275$, CL=89%. The parameters of the fit are shown in Table III.

B. Accounting for differences between $\mathrm{D^+D^-}$ and $\mathrm{D^0\bar{D}^0}$ processes

Here, we take into account the differences between the two processes. Their origins are physical (different masses of $\mathrm{D^+}$ and $\mathrm{D^0}$, possibly different couplings to intermediate vector mesons) and experimental (different acceptances rooted in different methods of identifying particles in final states).

We split each R_k and δ_k parameter into two. Those pertinent to the $\mathrm{D^+D^-}$ process will be marked by subscript a, those to the $\mathrm{D^0\bar{D}^0}$ process by b. Already, six resonances provided quite a good fit to the data: $\chi^2/\mathrm{DOF} = 284.8/266$, CL=20%. After including the $\psi(2\mathrm{S})$ subthreshold pole, the quality jumped to

$\chi^2/\mathrm{DOF} = 223.8/262$, CL=96%. The statistical significance of the subthreshold pole reached 4.0σ in the $\mathrm{D}^0\bar{\mathrm{D}}^0$ subset and 4.6σ in the $\mathrm{D}^+\mathrm{D}^-$ subset. The fit details are presented in Table IV.

The $e^+e^- \to \mathrm{D}^+\mathrm{D}^-$ cross section calculated from Eq. (1) using the common-fit parameters from Table IV is compared to data in Fig. 2. Also, the analogous curve describing the pure six-resonance fit is displayed. The inconsistency with data of the fit with (without) the $\psi(2\mathrm{S})$ is characterized by $\chi^2 = 111.6$ ($\chi^2 = 156.0$). Obviously, including the $\psi(2\mathrm{S})$ subthreshold pole leads to a much better description of data. Comparison with the χ^2s of the pure $\mathrm{D}^+\mathrm{D}^-$ fits (107.6 and 152.4, respectively) in Section IV shows that a penalty is paid for sharing the resonance parameters with the $\mathrm{D}^0\bar{\mathrm{D}}^0$ process. In Fig. 2, it is not easy to see the difference between the two fit curves. The exception is the region close to the threshold, where it is visible.

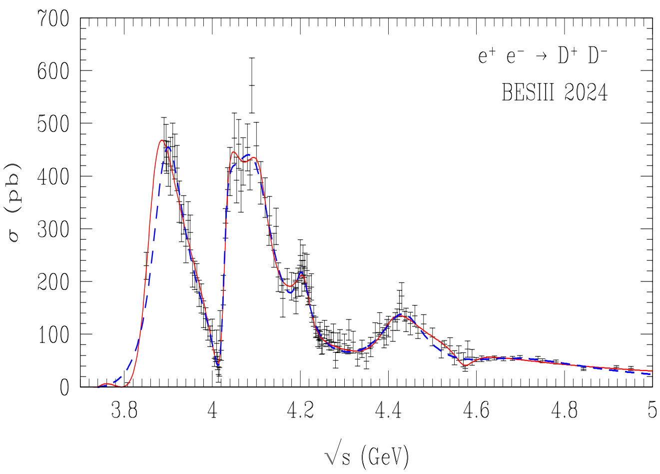

FIG. 2. The $e^+e^- \to \mathrm{D}^+\mathrm{D}^-$ cross section calculated from the common-fit parameters in Table IV (full) compared to data and the analogous six-resonance fit (dashed).

Figure 3 depicts the comparison of two $e^+e^- \to \mathrm{D}^0\bar{\mathrm{D}}^0$ excitation curves obtained using parameters from two simultaneous fits to the $\mathrm{D}^+\mathrm{D}^-$ and $\mathrm{D}^0\bar{\mathrm{D}}^0$ data. One fit assumed only six resonances, the other also the $\psi(2\mathrm{S})$ subthreshold pole. The excitation curve resulting from the former is characterized by $\chi^2 = 128.8$, from the latter by $\chi^2 = 112.2$. Similarly to the pure fits of Section V, these numbers differ less than in the $\mathrm{D}^+\mathrm{D}^-$ case. Comparison with the pure $\mathrm{D}^0\bar{\mathrm{D}}^0$ fits ($\chi^2 = 118.2$ and 102.0, respectively) shows that the penalization for using the same resonance masses and widths as in the $\mathrm{D}^+\mathrm{D}^-$ process is more severe than that mentioned in the previous paragraph.

VII. CONCLUSIONS

We recapitulate the main findings of this study in Table V. In all four six-resonance fits, the inclusion of the $\psi(2\mathrm{S})$ subthreshold pole improves the fit quality expressed by CL values. In the case of the $\mathrm{D}^0\bar{\mathrm{D}}^0$ final state only marginally, in the other cases significantly. The statistical significance of the $\psi(2\mathrm{S})$ pole is higher than 4σ in all four cases. We consider it evidence of the subthreshold pole in the amplitudes of electron-positron annihilation to the $\mathrm{D}\bar{\mathrm{D}}$ final states.

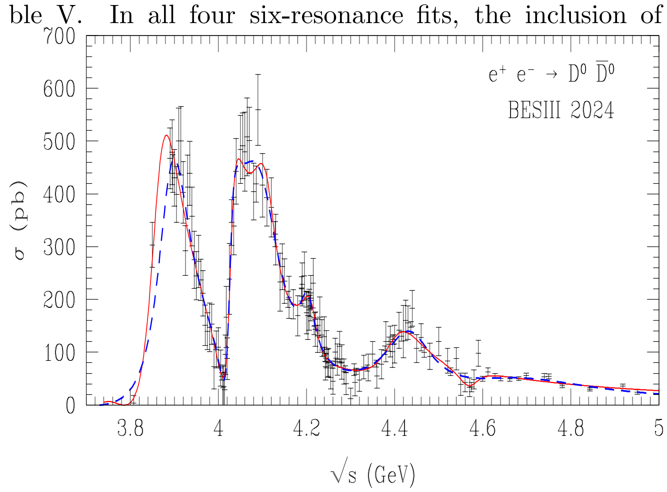

FIG. 3. The $e^+e^- \to \mathrm{D}^0\bar{\mathrm{D}}^0$ cross section calculated from the common-fit parameters in Table IV (full) compared to data and the analogous six-resonance fit (dashed).

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[10] Supplemental material to Ref. [1]: `https://journals.aps.org/prl/supplemental/10.1103/PhysRevLett.133.081901/Supplemental_Material.pdf.`
[11] Erroneously called a p-value in Table 3 of [10].
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TABLE I. Parameters of the fit to the BESIII $e^+e^- \to D^+D^-$ data [1] using the formula (1) with the $\psi(2S)$ subthreshold pole and six resonances. Symbol (f) marks values that we kept fixed.

χ^2/DOF = 107.6/125 CL = 87%							
	$1 \equiv \psi(2S)$	2	3	4	5	6	7
M (MeV)	3686.097(f)	3863.9±3.4	4027.3±1.3	4102.5±5.7	4212.7±2.9	4397.5±7.5	4571.2±8.2
Γ (MeV)	0(f)	83.9±9.0	31.1±2.9	57±10	34.8±4.7	116±15	62±20
Q (GeV^4)	3720±910	2260±330	50±10	34±13	3.3±1.1	610±160	2.6±1.8
δ (rad)	0(f)	-0.473(77)	-0.949(73)	1.47(28)	2.51(17)	-1.634(21)	-2.70(23)
S	4.1σ	6.8σ	5.0σ	2.6σ	3.0σ	3.8σ	1.4σ

TABLE II. Parameters of the fit to the BESIII $e^+e^- \to D^0\bar{D}^0$ data [1] using the formula (1) with the $\psi(2S)$ subthreshold pole and six resonances.

χ^2/DOF = 102.0/125 CL = 93%							
	$1 \equiv \psi(2S)$	2	3	4	5	6	7
M (MeV)	3686.097(f)	3851.2±4.5	4025.4±2.3	4166.2±3.3	4189.4±2.7	4426.5±6.6	4739±16
Γ (MeV)	0(f)	87±10	50.0±4.7	124.0±9.3	75.1±4.7	91±12	67±22
Q (GeV^4)	6120±1270	2760±500	183±40	2600±300	520±110	29.4±9.1	1.29±0.82
δ (rad)	0(f)	-0.54(13)	-0.74(10)	-2.147(53)	2.204(78)	0.30(22)	1.36(42)
S	4.8σ	5.5σ	4.6σ	8.7σ	4.7σ	3.2σ	1.6σ

TABLE III. Parameters of the common fit to the $e^+e^- \to D^+D^-$ and $e^+e^- \to D^0\bar{D}^0$ data using formula (1) with the $\psi(2S)$ subthreshold pole and six resonances. All parameters are assumed to be equal for both processes.

χ^2/DOF = 246.8/275 CL = 89%							
	$1 \equiv \psi(2S)$	2	3	4	5	6	7
M (MeV)	3686.097(f)	3862.3±3.1	4026.9±1.3	4109.2±6.1	4211.9±3.0	4401.0±5.6	4571.5±7.4
Γ (MeV)	0(f)	86.8±7.8	34.0±2.9	67±11	42.4±5.1	112±11	59±16
Q (GeV^4)	3490±790	2370±290	61±12	42±17	5.3±1.7	550±110	2.1±1.2
δ (rad)	0(f)	-0.481(56)	-0.980(74)	1.80(28)	2.51(18)	-1.642(24)	-2.56(22)
S	4.4σ	8.2σ	5.1σ	2.5σ	3.1σ	5.0σ	1.7σ

TABLE IV. Parameters of the common fit to the BESIII D^+D^- (subscript a) and $D^0\bar{D}^0$ (subscript b) data [1] using Eq. (1) with the $\psi(2S)$ pole and six resonances. Pole masses M and widths Γ are assumed to be the same in both processes. The other parameters in Eq. (1) are different, together with the poles' statistical significances S_a and S_b.

χ^2/DOF = 223.8/262 CL = 96%							
	$1 \equiv \psi(2S)$	2	3	4	5	6	7
M (MeV)	3686.097(f)	3861.8±2.9	4026.8±1.3	4110.2±6.1	4211.8±3.0	4401.1±5.5	4571.3±7.0
Γ (MeV)	0(f)	85.6±7.7	34.8±3.0	68±11	42.4±5.2	111±11	57±14
Q_a (GeV^4)	4080 ± 880	2350±280	66±13	39±16	5.2±1.8	560±110	2.0±1.2
Q_b (GeV^4)	2970 ± 740	2330±290	61±14	49±20	5.5±2.0	532±99	2.3±1.4
δ_a (rad)	0(f)	-0.498(80)	-1.005(68)	1.87(30)	2.47(18)	-1.653(26)	-2.69(23)
δ_b (rad)	0(f)	-0.473(80)	-0.926(98)	1.81(29)	2.55(19)	-1.628(29)	-2.41(24)
S_a	4.6σ	8.4σ	5.1σ	2.9σ	2.9σ	5.1σ	1.7σ
S_b	4.0σ	8.0σ	4.4σ	2.4σ	2.7σ	5.4σ	1.6σ

TABLE V. Comparison of the main parameters of the four six-resonance fits without and with the $\psi(2S)$ subthreshold pole. The first statistical significance in the last column refers to the D^+D^- process, the other to the $D^0\bar{D}^0$ one.

	D^+D^-		$D^0\bar{D}^0$		$D\bar{D}$, same parameters		$D\bar{D}$, different params	
	6 resonances	$\psi(2S)$+6 res.	6 resonances	$\psi(2S)$+6 res.	6 resonances	$\psi(2S)$+6 res.	6 resonances	$\psi(2S)$+6 res.
χ^2/DOF	152.4/127	107.6/125	118.2/127	102.0/125	304.3/277	246.8/275	284.8/266	223.8/262
CL	6%	87%	70%	93%	12%	89%	20%	96%
$S[\psi(2S)]$	/	4.1σ	/	4.8σ	/	4.4σ	/	$4.6\sigma/4.0\sigma$