PART ONE

PHYSICAL FUNDAMENTALS OF MECHANICS

1.1. KINEMATICS

• Average vectors of velocity and acceleration of a point:

$$\langle \mathbf{v} \rangle = \frac{\Delta \mathbf{r}}{\Delta t}, \qquad \langle \mathbf{w} \rangle = \frac{\Delta \mathbf{v}}{\Delta t}, \tag{1.1a}$$

where $\Delta\mathbf{r}$ is the displacement vector (an increment of a radius vector).

• Velocity and acceleration of a point:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}, \qquad \mathbf{w} = \frac{d\mathbf{v}}{dt}. \tag{1.1b}$$

• Acceleration of a point expressed in projections on the tangent and the normal to a trajectory:

$$w_\tau = \frac{dv_\tau}{dt}, \qquad w_n = \frac{v^2}{R}, \tag{1.1c}$$

where R is the radius of curvature of the trajectory at the given point.

• Distance covered by a point:

$$s = \int v\, dt, \tag{1.1d}$$

where v is the *modulus* of the velocity vector of a point.

• Angular velocity and angular acceleration of a solid body:

$$\omega = \frac{d\varphi}{dt}, \qquad \beta = \frac{d\omega}{dt}. \tag{1.1e}$$

• Relation between linear and angular quantities for a rotating solid body:

$$\mathbf{v} = [\boldsymbol{\omega}\mathbf{r}], \quad w_n = \omega^2 R, \quad |w_\tau| = \beta R, \tag{1.1.f}$$

where $\mathbf{r}$ is the radius vector of the considered point relative to an arbitrary point on the rotation axis, and R is the distance from the rotation axis.

1.1. A motorboat going downstream overcame a raft at a point A; $\tau = 60$ min later it turned back and after some time passed the raft at a distance $l = 6.0$ km from the point A. Find the flow velocity assuming the duty of the engine to be constant.

1.2. A point traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

1.3. A car starts moving rectilinearly, first with acceleration $w = 5.0\ \text{m/s}^2$ (the initial velocity is equal to zero), then uniformly, and finally, decelerating at the same rate w, comes to a stop. The total time of motion equals $\tau = 25$ s. The average velocity during that time is equal to $\langle v \rangle = 72$ km per hour. How long does the car move uniformly?

1.4. A point moves rectilinearly in one direction. Fig. 1.1 shows

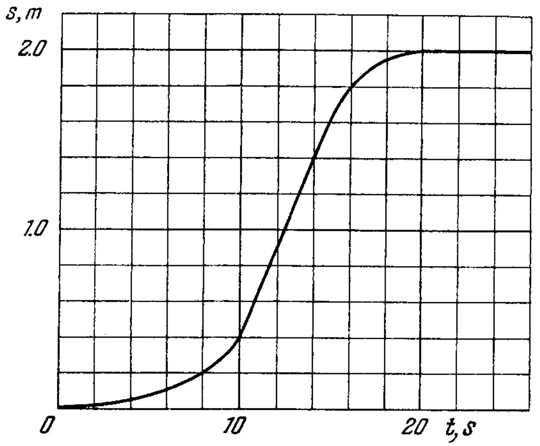

Fig. 1.1.

the distance s traversed by the point as a function of the time t. Using the plot find:

(a) the average velocity of the point during the time of motion;

(b) the maximum velocity;

(c) the time moment t_0 at which the instantaneous velocity is equal to the mean velocity averaged over the first t_0 seconds.

1.5. Two particles, *1* and *2*, move with constant velocities $\mathbf{v}_1$ and $\mathbf{v}_2$. At the initial moment their radius vectors are equal to $\mathbf{r}_1$ and $\mathbf{r}_2$. How must these four vectors be interrelated for the particles to collide?

1.6. A ship moves along the equator to the east with velocity $v_0 = 30$ km/hour. The southeastern wind blows at an angle $\varphi = 60°$ to the equator with velocity $v = 15$ km/hour. Find the wind velocity v' relative to the ship and the angle φ' between the equator and the wind direction in the reference frame fixed to the ship.

1.7. Two swimmers leave point A on one bank of the river to reach point B lying right across on the other bank. One of them crosses the river along the straight line AB while the other swims at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point B. What was the velocity u

of his walking if both swimmers reached the destination simultaneously? The stream velocity $v_0 = 2.0$ km/hour and the velocity v' of each swimmer with respect to water equals 2.5 km per hour.

1.8. Two boats, A and B, move away from a buoy anchored at the middle of a river along the mutually perpendicular straight lines: the boat A along the river, and the boat B across the river. Having moved off an equal distance from the buoy the boats returned. Find the ratio of times of motion of boats τ_A/τ_B if the velocity of each boat with respect to water is $\eta = 1.2$ times greater than the stream velocity.

1.9. A boat moves relative to water with a velocity which is $n = 2.0$ times less than the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting?

1.10. Two bodies were thrown simultaneously from the same point: one, straight up, and the other, at an angle of $\theta = 60°$ to the horizontal. The initial velocity of each body is equal to $v_0 = 25$ m/s. Neglecting the air drag, find the distance between the bodies $t = 1.70$ s later.

1.11. Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point and moved with velocities $v_1 = 3.0$ m/s and $v_2 = 4.0$ m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

1.12. Three points are located at the vertices of an equilateral triangle whose side equals a. They all start moving simultaneously with velocity v constant in modulus, with the first point heading continually for the second, the second for the third, and the third for the first. How soon will the points converge?

1.13. Point A moves uniformly with velocity v so that the vector $\mathbf{v}$ is continually "aimed" at point B which in its turn moves rectilinearly and uniformly with velocity $u < v$. At the initial moment of time $\mathbf{v} \perp \mathbf{u}$ and the points are separated by a distance l. How soon will the points converge?

1.14. A train of length $l = 350$ m starts moving rectilinearly with constant acceleration $w = 3.0 \cdot 10^{-2}$ m/s²; $t = 30$ s after the start the locomotive headlight is switched on (event *1*), and $\tau = 60$ s after that event the tail signal light is switched on (event *2*). Find the distance between these events in the reference frames fixed to the train and to the Earth. How and at what constant velocity V relative to the Earth must a certain reference frame K move for the two events to occur in it at the same point?

1.15. An elevator car whose floor-to-ceiling distance is equal to 2.7 m starts ascending with constant acceleration 1.2 m/s²; 2.0 s after the start a bolt begins falling from the ceiling of the car. Find:

(a) the bolt's free fall time;

(b) the displacement and the distance covered by the bolt during the free fall in the reference frame fixed to the elevator shaft.

1.16. Two particles, *1* and *2*, move with constant velocities v_1 and v_2 along two mutually perpendicular straight lines toward the intersection point O. At the moment $t = 0$ the particles were located at the distances l_1 and l_2 from the point O. How soon will the distance between the particles become the smallest? What is it equal to?

1.17. From point A located on a highway (Fig. 1.2) one has to get by car as soon as possible to point B located in the field at a distance l from the highway. It is known that the car moves in the field η times slower than on the highway. At what distance from point D one must turn off the highway?

1.18. A point travels along the x axis with a velocity whose projection v_x is presented as a function of time by the plot in Fig. 1.3.

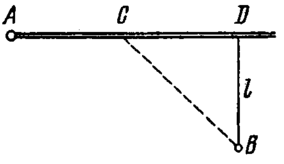

Fig. 1.2.

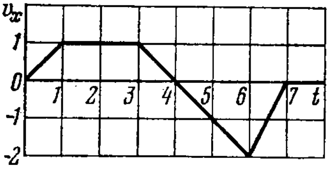

Fig. 1.3.

Assuming the coordinate of the point $x = 0$ at the moment $t = 0$, draw the approximate time dependence plots for the acceleration w_x, the x coordinate, and the distance covered s.

1.19. A point traversed half a circle of radius $R = 160$ cm during time interval $\tau = 10.0$ s. Calculate the following quantities averaged over that time:

(a) the mean velocity $\langle v \rangle$;

(b) the modulus of the mean velocity vector $|\langle \mathbf{v} \rangle|$;

(c) the modulus of the mean vector of the total acceleration $|\langle \mathbf{w} \rangle|$ if the point moved with constant tangent acceleration.

1.20. A radius vector of a particle varies with time t as $\mathbf{r} = \mathbf{a}t(1 - \alpha t)$, where $\mathbf{a}$ is a constant vector and α is a positive factor. Find:

(a) the velocity $\mathbf{v}$ and the acceleration $\mathbf{w}$ of the particle as functions of time;

(b) the time interval Δt taken by the particle to return to the initial points, and the distance s covered during that time.

1.21. At the moment $t = 0$ a particle leaves the origin and moves in the positive direction of the x axis. Its velocity varies with time as $\mathbf{v} = \mathbf{v}_0(1 - t/\tau)$, where $\mathbf{v}_0$ is the initial velocity vector whose modulus equals $v_0 = 10.0$ cm/s; $\tau = 5.0$ s. Find:

(a) the x coordinate of the particle at the moments of time 6.0, 10, and 20 s;

(b) the moments of time when the particle is at the distance 10.0 cm from the origin;

(c) the distance s covered by the particle during the first 4.0 and 8.0 s; draw the approximate plot $s(t)$.

1.22. The velocity of a particle moving in the positive direction of the x axis varies as $v = \alpha\sqrt{x}$, where α is a positive constant. Assuming that at the moment $t = 0$ the particle was located at the point $x = 0$, find:

(a) the time dependence of the velocity and the acceleration of the particle;

(b) the mean velocity of the particle averaged over the time that the particle takes to cover the first s metres of the path.

1.23. A point moves rectilinearly with deceleration whose modulus depends on the velocity v of the particle as $w = a\sqrt{v}$, where a is a positive constant. At the initial moment the velocity of the point is equal to v_0. What distance will it traverse before it stops? What time will it take to cover that distance?

1.24. A radius vector of a point A relative to the origin varies with time t as $\mathbf{r} = at\mathbf{i} - bt^2\mathbf{j}$, where a and b are positive constants, and $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find:

(a) the equation of the point's trajectory $y(x)$; plot this function;

(b) the time dependence of the velocity $\mathbf{v}$ and acceleration $\mathbf{w}$ vectors, as well as of the moduli of these quantities;

(c) the time dependence of the angle α between the vectors $\mathbf{w}$ and $\mathbf{v}$;

(d) the mean velocity vector averaged over the first t seconds of motion, and the modulus of this vector.

1.25. A point moves in the plane xy according to the law $x = at$, $y = at(1 - \alpha t)$, where a and α are positive constants, and t is time. Find:

(a) the equation of the point's trajectory $y(x)$; plot this function;

(b) the velocity v and the acceleration w of the point as functions of time;

(c) the moment t_0 at which the velocity vector forms an angle $\pi/4$ with the acceleration vector.

1.26. A point moves in the plane xy according to the law $x = a \sin \omega t$, $y = a(1 - \cos \omega t)$, where a and ω are positive constants. Find:

(a) the distance s traversed by the point during the time τ;

(b) the angle between the point's velocity and acceleration vectors.

1.27. A particle moves in the plane xy with constant acceleration $\mathbf{w}$ directed along the negative y axis. The equation of motion of the particle has the form $y = ax - bx^2$, where a and b are positive constants. Find the velocity of the particle at the origin of coordinates.

1.28. A small body is thrown at an angle to the horizontal with the initial velocity $\mathbf{v}_0$. Neglecting the air drag, find:

(a) the displacement of the body as a function of time $\mathbf{r}(t)$;

(b) the mean velocity vector $\langle \mathbf{v} \rangle$ averaged over the first t seconds and over the total time of motion.

1.29. A body is thrown from the surface of the Earth at an angle α

to the horizontal with the initial velocity v_0. Assuming the air drag to be negligible, find:

(a) the time of motion;

(b) the maximum height of ascent and the horizontal range; at what value of the angle α they will be equal to each other;

(c) the equation of trajectory $y(x)$, where y and x are displacements of the body along the vertical and the horizontal respectively;

(d) the curvature radii of trajectory at its initial point and at its peak.

1.30. Using the conditions of the foregoing problem, draw the approximate time dependence of moduli of the normal w_n and tangent w_τ acceleration vectors, as well as of the projection of the total acceleration vector w_v on the velocity vector direction.

1.31. A ball starts falling with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time?

1.32. A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial velocity 240 m/s reach the target in the absence of air drag?

1.33. A cannon fires successively two shells with velocity $v_0 = 250$ m/s; the first at the angle $\theta_1 = 60°$ and the second at the angle $\theta_2 = 45°$ to the horizontal, the azimuth being the same. Neglecting the air drag, find the time interval between firings leading to the collision of the shells.

1.34. A balloon starts rising from the surface of the Earth. The ascension rate is constant and equal to v_0. Due to the wind the balloon gathers the horizontal velocity component $v_x = ay$, where a is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent:

(a) the horizontal drift of the balloon $x(y)$;

(b) the total, tangential, and normal accelerations of the balloon.

1.35. A particle moves in the plane xy with velocity $\mathbf{v} = a\mathbf{i} + bx\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes, and a and b are constants. At the initial moment of time the particle was located at the point $x = y = 0$. Find:

(a) the equation of the particle's trajectory $y(x)$;

(b) the curvature radius of trajectory as a function of x.

1.36. A particle A moves in one direction along a given trajectory with a tangential acceleration $w_\tau = \mathbf{a\tau}$, where $\mathbf{a}$ is a constant vector coinciding in direction with the x axis (Fig. 1.4), and $\boldsymbol{\tau}$ is a unit vector coinciding in direction with the velocity vector at a given point. Find how the velocity of the particle depends on x provided that its velocity is negligible at the point $x = 0$.

1.37. A point moves along a circle with a velocity $v = at$, where $a = 0.50$ m/s^2. Find the total acceleration of the point at the moment when it covered the n-th ($n = 0.10$) fraction of the circle after the beginning of motion.

1.38. A point moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations

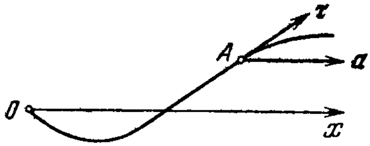

Fig. 1.4.

are equal in moduli. At the initial moment $t = 0$ the velocity of the point equals v_0. Find:

(a) the velocity of the point as a function of time and as a function of the distance covered s;

(b) the total acceleration of the point as a function of velocity and the distance covered.

1.39. A point moves along an arc of a circle of radius R. Its velocity depends on the distance covered s as $v = a\sqrt{s}$, where a is a constant. Find the angle α between the vector of the total acceleration and the vector of velocity as a function of s.

1.40. A particle moves along an arc of a circle of radius R according to the law $l = a \sin \omega t$, where l is the displacement from the initial position measured along the arc, and a and ω are constants. Assuming $R = 1.00$ m, $a = 0.80$ m, and $\omega = 2.00$ rad/s, find:

(a) the magnitude of the total acceleration of the particle at the points $l = 0$ and $l = \pm a$;

(b) the minimum value of the total acceleration w_{min} and the corresponding displacement l_m.

1.41. A point moves in the plane so that its tangential acceleration $w_\tau = a$, and its normal acceleration $w_n = bt^4$, where a and b are positive constants, and t is time. At the moment $t = 0$ the point was at rest. Find how the curvature radius R of the point's trajectory and the total acceleration w depend on the distance covered s.

1.42. A particle moves along the plane trajectory $y(x)$ with velocity v whose modulus is constant. Find the acceleration of the particle at the point $x = 0$ and the curvature radius of the trajectory at that point if the trajectory has the form

(a) of a parabola $y = ax^2$;

(b) of an ellipse $(x/a)^2 + (y/b)^2 = 1$; a and b are constants here.

1.43. A particle A moves along a circle of radius $R = 50$ cm so that its radius vector $\mathbf{r}$ relative to the point O (Fig. 1.5) rotates with the constant angular velocity $\omega = 0.40$ rad/s. Find the modulus of the velocity of the particle, and the modulus and direction of its total acceleration.

1.44. A wheel rotates around a stationary axis so that the rotation angle φ varies with time as $\varphi = at^2$, where $a = 0.20$ rad/s². Find the total acceleration w of the point A at the rim at the moment $t = 2.5$ s if the linear velocity of the point A at this moment $v = 0.65$ m/s.

1.45. A shell acquires the initial velocity $v = 320$ m/s, having made $n = 2.0$ turns inside the barrel whose length is equal to $l = 2.0$ m. Assuming that the shell moves inside the barrel with a uniform acceleration, find the angular velocity of its axial rotation at the moment when the shell escapes the barrel.

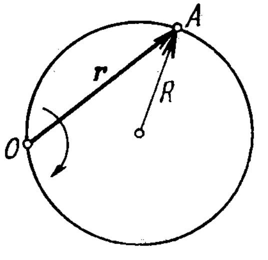

Fig. 1.5.

1.46. A solid body rotates about a stationary axis according to the law $\varphi = at - bt^3$, where $a = 6.0$ rad/s and $b = 2.0$ rad/s³. Find:

(a) the mean values of the angular velocity and angular acceleration averaged over the time interval between $t = 0$ and the complete stop;

(b) the angular acceleration at the moment when the body stops.

1.47. A solid body starts rotating about a stationary axis with an angular acceleration $\beta = at$, where $a = 2.0 \cdot 10^{-2}$ rad/s³. How soon after the beginning of rotation will the total acceleration vector of an arbitrary point of the body form an angle $\alpha = 60°$ with its velocity vector?

1.48. A solid body rotates with deceleration about a stationary axis with an angular deceleration $\beta \propto \sqrt{\omega}$, where ω is its angular velocity. Find the mean angular velocity of the body averaged over the whole time of rotation if at the initial moment of time its angular velocity was equal to ω_0.

1.49. A solid body rotates about a stationary axis so that its angular velocity depends on the rotation angle φ as $\omega = \omega_0 - a\varphi$, where ω_0 and a are positive constants. At the moment $t = 0$ the angle $\varphi = 0$. Find the time dependence of

(a) the rotation angle;

(b) the angular velocity.

1.50. A solid body starts rotating about a stationary axis with an angular acceleration $\boldsymbol{\beta} = \boldsymbol{\beta}_0 \cos\varphi$, where $\boldsymbol{\beta}_0$ is a constant vector and φ is an angle of rotation from the initial position. Find the angular velocity of the body as a function of the angle φ. Draw the plot of this dependence.

1.51. A rotating disc (Fig. 1.6) moves in the positive direction of the x axis. Find the equation $y(x)$ describing the position of the instantaneous axis of rotation, if at the initial moment the axis C of the disc was located at the point O after which it moved

(a) with a constant velocity v, while the disc started rotating counterclockwise with a constant angular acceleration β (the initial angular velocity is equal to zero);

(b) with a constant acceleration w (and the zero initial velocity), while the disc rotates counterclockwise with a constant angular velocity ω.

1.52. A point A is located on the rim of a wheel of radius $R = 0.50$ m which rolls without slipping along a horizontal surface with velocity $v = 1.00$ m/s. Find:

(a) the modulus and the direction of the acceleration vector of the point A;

(b) the total distance s traversed by the point A between the two successive moments at which it touches the surface.

1.53. A ball of radius $R = 10.0$ cm rolls without slipping down an inclined plane so that its centre moves with constant acceleration

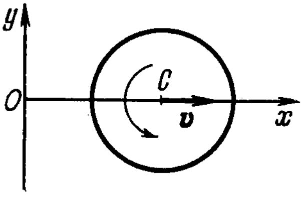

Fig. 1.6.

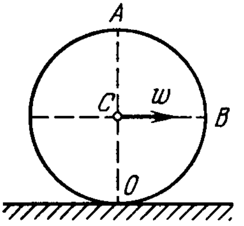

Fig. 1.7.

$w = 2.50$ cm/s^2; $t = 2.00$ s after the beginning of motion its position corresponds to that shown in Fig. 1.7. Find:

(a) the velocities of the points A, B, and O;

(b) the accelerations of these points.

1.54. A cylinder rolls without slipping over a horizontal plane. The radius of the cylinder is equal to r. Find the curvature radii of trajectories traced out by the points A and B (see Fig. 1.7).

1.55. Two solid bodies rotate about stationary mutually perpendicular intersecting axes with constant angular velocities $\omega_1 = 3.0$ rad/s and $\omega_2 = 4.0$ rad/s. Find the angular velocity and angular acceleration of one body relative to the other.

1.56. A solid body rotates with angular velocity $\boldsymbol{\omega} = at\mathbf{i} + bt^2\mathbf{j}$, where $a = 0.50$ rad/s^2, $b = 0.060$ rad/s^3, and $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes. Find:

(a) the moduli of the angular velocity and the angular acceleration at the moment $t = 10.0$ s;

(b) the angle between the vectors of the angular velocity and the angular acceleration at that moment.

1.57. A round cone with half-angle $\alpha = 30°$ and the radius of the base $R = 5.0$ cm rolls uniformly and without slipping over a horizontal plane as shown in Fig. 1.8. The cone apex is hinged at the point O which is on the same level with the point C, the cone base centre. The velocity of point C is $v = 10.0$ cm/s. Find the moduli of

(a) the vector of the angular velocity of the cone and the angle it forms with the vertical;

(b) the vector of the angular acceleration of the cone.

1.58. A solid body rotates with a constant angular velocity $\omega_0 = 0.50$ rad/s about a horizontal axis AB. At the moment $t = 0$

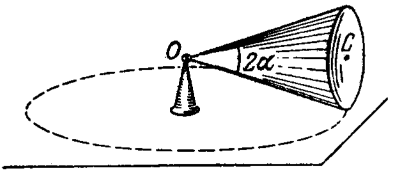

Fig. 1.8.

the axis AB starts turning about the vertical with a constant angular acceleration $\beta_0 = 0.10$ rad/s^2. Find the angular velocity and angular acceleration of the body after $t = 3.5$ s.

1.2. THE FUNDAMENTAL EQUATION OF DYNAMICS

• The fundamental equation of dynamics of a mass point (Newton's second law):

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F}. \qquad (1.2a)$$

• The same equation expressed in projections on the tangent and the normal of the point's trajectory:

$$m\frac{dv_\tau}{dt} = F_\tau, \qquad m\frac{v^2}{R} = F_n. \qquad (1.2b)$$

• The equation of dynamics of a point in the non-inertial reference frame K' which rotates with a constant angular velocity ω about an axis translating with an acceleration $\mathbf{w}_0$:

$$m\mathbf{w}' = \mathbf{F} - m\mathbf{w}_0 + m\omega^2\mathbf{R} + 2m\,[\mathbf{v}'\boldsymbol{\omega}], \qquad (1.2c)$$

where $\mathbf{R}$ is the radius vector of the point relative to the axis of rotation of the K' frame.

1.59. An aerostat of mass m starts coming down with a constant acceleration w. Determine the ballast mass to be dumped for the aerostat to reach the upward acceleration of the same magnitude. The air drag is to be neglected.

1.60. In the arrangement of Fig. 1.9 the masses m_0, m_1, and m_2 of bodies are equal, the masses of the pulley and the threads are negligible, and there is no friction in the pulley. Find the acceleration $\mathbf{w}$ with which the body m_0 comes down, and the tension of the thread binding together the bodies m_1 and m_2, if the coefficient of friction between these bodies and the horizontal surface is equal to k. Consider possible cases.

1.61. Two touching bars *1* and *2* are placed on an inclined plane forming an angle α with the horizontal (Fig. 1.10). The masses of the bars are equal to m_1 and m_2, and the coefficients of friction be-

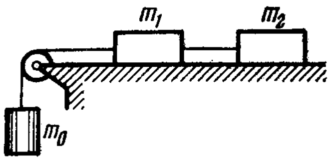

Fig. 1.9.

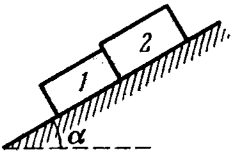

Fig. 1.10.

tween the inclined plane and these bars are equal to k_1 and k_2 respectively, with $k_1 > k_2$. Find:

(a) the force of interaction of the bars in the process of motion;

(b) the minimum value of the angle α at which the bars start sliding down.

1.62. A small body was launched up an inclined plane set at an angle $\alpha = 15°$ against the horizontal. Find the coefficient of friction, if the time of the ascent of the body is $\eta = 2.0$ times less than the time of its descent.

1.63. The following parameters of the arrangement of Fig. 1.11 are available: the angle α which the inclined plane forms with the horizontal, and the coefficient of friction k between the body m_1 and the inclined plane. The masses of the pulley and the threads, as well as the friction in the pulley, are negligible. Assuming both bodies to be motionless at the initial moment, find the mass ratio m_2/m_1 at which the body m_2

(a) starts coming down;

(b) starts going up;

(c) is at rest.

1.64. The inclined plane of Fig. 1.11 forms an angle $\alpha = 30°$ with the horizontal. The mass ratio $m_2/m_1 = \eta = 2/3$. The coefficient of friction between the body m_1 and the inclined plane is equal to $k = 0.10$. The masses of the pulley and the threads are negligible. Find the magnitude and the direction of acceleration of the body m_2 when the formerly stationary system of masses starts moving.

1.65. A plank of mass m_1 with a bar of mass m_2 placed on it lies on a smooth horizontal plane. A horizontal force growing with time t as $F = at$ (a is constant) is applied to the bar. Find how the accelerations of the plank w_1 and of the bar w_2 depend on t, if the coefficient of friction between the plank and the bar is equal to k. Draw the approximate plots of these dependences.

1.66. A small body A starts sliding down from the top of a wedge (Fig. 1.12) whose base is equal to $l = 2.10$ m. The coefficient of friction between the body and the wedge surface is $k = 0.140$. At what value of the angle α will the time of sliding be the least? What will it be equal to?

1.67. A bar of mass m is pulled by means of a thread up an inclined plane forming an angle α with the horizontal (Fig. 1.13). The coef-

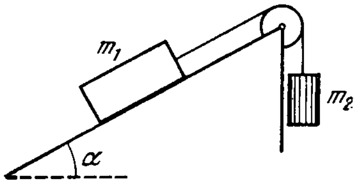

Fig. 1.11.

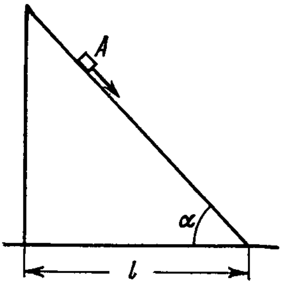

Fig. 1.12.

ficient of friction is equal to k. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be minimum. What is it equal to?

1.68. At the moment $t = 0$ the force $F = at$ is applied to a small body of mass m resting on a smooth horizontal plane (a is a constant).

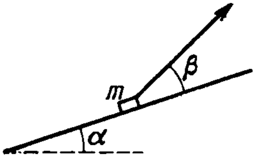

Fig. 1.13.

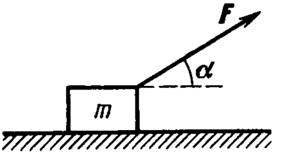

Fig. 1.14.

The permanent direction of this force forms an angle α with the horizontal (Fig. 1.14). Find:

(a) the velocity of the body at the moment of its breaking off the plane;

(b) the distance traversed by the body up to this moment.

1.69. A bar of mass m resting on a smooth horizontal plane starts moving due to the force $F = mg/3$ of constant magnitude. In the process of its rectilinear motion the angle α between the direction of this force and the horizontal varies as $\alpha = as$, where a is a constant, and s is the distance traversed by the bar from its initial position. Find the velocity of the bar as a function of the angle α.

1.70. A horizontal plane with the coefficient of friction k supports two bodies: a bar and an electric motor with a battery on a block. A thread attached to the bar is wound on the shaft of the electric motor. The distance between the bar and the electric motor is equal to l. When the motor is switched on, the bar, whose mass is twice as great as that of the other body, starts moving with a constant acceleration w. How soon will the bodies collide?

1.71. A pulley fixed to the ceiling of an elevator car carries a thread whose ends are attached to the loads of masses m_1 and m_2. The car starts going up with an acceleration $\mathbf{w}_0$. Assuming the masses of the pulley and the thread, as well as the friction, to be negligible find:

(a) the acceleration of the load m_1 relative to the elevator shaft and relative to the car;

(b) the force exerted by the pulley on the ceiling of the car.

1.72. Find the acceleration $\mathbf{w}$ of body *2* in the arrangement shown in Fig. 1.15, if its mass is η times as great as the mass of bar *1* and

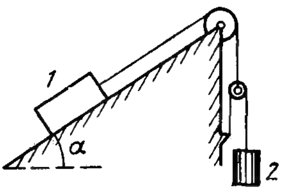

Fig. 1.15.

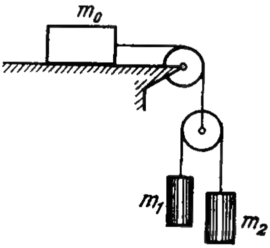

Fig. 1.16.

the angle that the inclined plane forms with the horizontal is equal to α. The masses of the pulleys and the threads, as well as the friction, are assumed to be negligible. Look into possible cases.

1.73. In the arrangement shown in Fig. 1.16 the bodies have masses m_0, m_1, m_2, the friction is absent, the masses of the pulleys and the threads are negligible. Find the acceleration of the body m_1. Look into possible cases.

1.74. In the arrangement shown in Fig. 1.17 the mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting

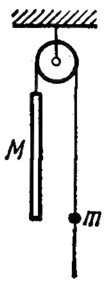

Fig. 1.17.

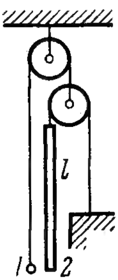

Fig. 1.18.

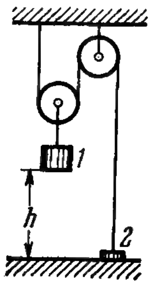

Fig. 1.19.

it to slide along the thread with some friction. The mass of the pulley and the friction in its axle are negligible. At the initial moment the ball was located opposite the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if t seconds after the beginning of motion the ball got opposite the upper end of the rod. The rod length equals l.

1.75. In the arrangement shown in Fig. 1.18 the mass of ball *1* is $\eta = 1.8$ times as great as that of rod *2*. The length of the latter is $l = 100$ cm. The masses of the pulleys and the threads, as well as the friction, are negligible. The ball is set on the same level as the lower end of the rod and then released. How soon will the ball be opposite the upper end of the rod?

1.76. In the arrangement shown in Fig. 1.19 the mass of body *1* is $\eta = 4.0$ times as great as that of body *2*. The height $h = 20$ cm. The masses of the pulleys and the threads, as well as the friction, are negligible. At a certain moment body *2* is released and the arrangement set in motion. What is the maximum height that body *2* will go up to?

1.77. Find the accelerations of rod A and wedge B in the arrangement shown in Fig. 1.20 if the ratio of the mass of the wedge to that of the rod equals η, and the friction between all contact surfaces is negligible.

1.78. In the arrangement shown in Fig. 1.21 the masses of the wedge M and the body m are known. The appreciable friction exists

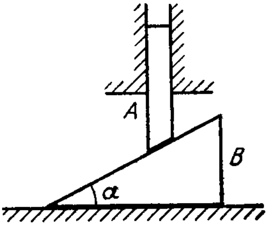

Fig. 1.20.

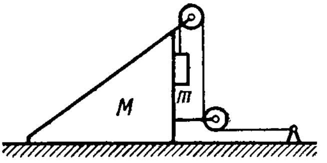

Fig. 1.21.

only between the wedge and the body m, the friction coefficient being equal to k. The masses of the pulley and the thread are negligible. Find the acceleration of the body m relative to the horizontal surface on which the wedge slides.

1.79. What is the minimum acceleration with which bar A (Fig. 1.22) should be shifted horizontally to keep bodies *1* and *2* stationary relative to the bar? The masses of the bodies are equal, and the coefficient of friction between the bar and the bodies is equal to k. The masses of the pulley and the threads are negligible, the friction in the pulley is absent.

1.80. Prism *1* with bar *2* of mass m placed on it gets a horizontal acceleration w directed to the left (Fig. 1.23). At what maximum value of this acceleration will the bar be still stationary relative to the prism, if the coefficient of friction between them $k < \cot \alpha$?

1.81. Prism *1* of mass m_1 and with angle α (see Fig. 1.23) rests on a horizontal surface. Bar *2* of mass m_2 is placed on the prism. Assuming the friction to be negligible, find the acceleration of the prism.

1.82. In the arrangement shown in Fig. 1.24 the masses m of the bar and M of the wedge, as well as the wedge angle α, are known.

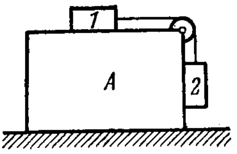

Fig. 1.22.

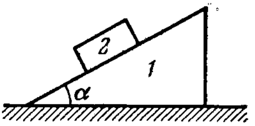

Fig. 1.23.

The masses of the pulley and the thread are negligible. The friction is absent. Find the acceleration of the wedge M.

1.83. A particle of mass m moves along a circle of radius R. Find the modulus of the average vector of the force acting on the particle over the distance equal to a quarter of the circle, if the particle moves

(a) uniformly with velocity v;

(b) with constant tangential acceleration w_τ, the initial velocity being equal to zero.

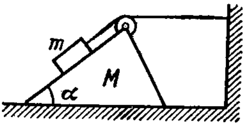

Fig. 1.24.

1.84. An aircraft loops the loop of radius $R = 500$ m with a constant velocity $v = 360$ km per hour. Find the weight of the flyer of mass $m = 70$ kg in the lower, upper, and middle points of the loop.

1.85. A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released. Find:

(a) the total acceleration of the sphere and the thread tension as a function of θ, the angle of deflection of the thread from the vertical;

(b) the thread tension at the moment when the vertical component of the sphere's velocity is maximum;

(c) the angle θ between the thread and the vertical at the moment when the total acceleration vector of the sphere is directed horizontally.

1.86. A ball suspended by a thread swings in a vertical plane so that its acceleration values in the extreme and the lowest position are equal. Find the thread deflection angle in the extreme position.

1.87. A small body A starts sliding off the top of a smooth sphere of radius R. Find the angle θ (Fig. 1.25) corresponding to the point at which the body breaks off the sphere, as well as the break-off velocity of the body.

1.88. A device (Fig. 1.26) consists of a smooth L-shaped rod located in a horizontal plane and a sleeve A of mass m attached by a weightless spring to a point B. The spring stiffness is equal to $\varkappa$. The whole system rotates with a constant angular velocity ω about a vertical axis passing through the point O. Find the elongation of the spring. How is the result affected by the rotation direction?

1.89. A cyclist rides along the circumference of a circular horizontal plane of radius R, the friction coefficient being dependent only on

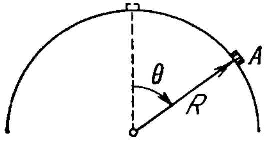

Fig. 1.25.

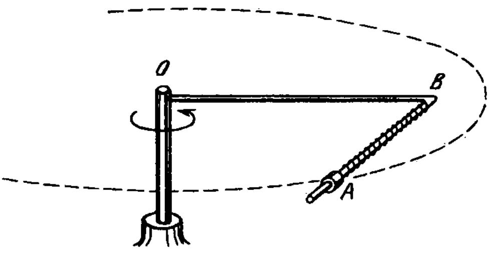

Fig. 1.26.

distance r from the centre O of the plane as $k = k_0(1 - r/R)$, where k_0 is a constant. Find the radius of the circle with the centre at the point along which the cyclist can ride with the maximum velocity. What is this velocity?

1.90. A car moves with a constant tangential acceleration $w_\tau = 0.62\ \text{m/s}^2$ along a horizontal surface circumscribing a circle of radius $R = 40$ m. The coefficient of sliding friction between the wheels of the car and the surface is $k = 0.20$. What distance will the car ride without sliding if at the initial moment of time its velocity is equal to zero?

1.91. A car moves uniformly along a horizontal sine curve $y = a \sin (x/\alpha)$, where a and α are certain constants. The coefficient of friction between the wheels and the road is equal to k. At what velocity will the car ride without sliding?

1.92. A chain of mass m forming a circle of radius R is slipped on a smooth round cone with half-angle θ. Find the tension of the chain if it rotates with a constant angular velocity ω about a vertical axis coinciding with the symmetry axis of the cone.

1.93. A fixed pulley carries a weightless thread with masses m_1 and m_2 at its ends. There is friction between the thread and the pulley. It is such that the thread starts slipping when the ratio $m_2/m_1 = \eta_0$. Find:

(a) the friction coefficient;

(b) the acceleration of the masses when $m_2/m_1 = \eta > \eta_0$.

1.94. A particle of mass m moves along the internal smooth surface of a vertical cylinder of radius R. Find the force with which the particle acts on the cylinder wall if at the initial moment of time its velocity equals v_0 and forms an angle α with the horizontal.

1.95. Find the magnitude and direction of the force acting on the particle of mass m during its motion in the plane xy according to the law $x = a \sin \omega t$, $y = b \cos \omega t$, where a, b, and ω are constants.

1.96. A body of mass m is thrown at an angle to the horizontal with the initial velocity $\mathbf{v}_0$. Assuming the air drag to be negligible, find:

(a) the momentum increment $\Delta\mathbf{p}$ that the body acquires over the first t seconds of motion;

(b) the modulus of the momentum increment $\Delta\mathbf{p}$ during the total time of motion.

1.97. At the moment $t = 0$ a stationary particle of mass m experiences a time-dependent force $\mathbf{F} = \mathbf{a}t(\tau - t)$, where $\mathbf{a}$ is a constant vector, τ is the time during which the given force acts. Find:

(a) the momentum of the particle when the action of the force discontinued;

(b) the distance covered by the particle while the force acted.

1.98. At the moment $t = 0$ a particle of mass m starts moving due to a force $\mathbf{F} = \mathbf{F}_0 \sin \omega t$, where $\mathbf{F}_0$ and ω are constants. Find the distance covered by the particle as a function of t. Draw the approximate plot of this function.

1.99. At the moment $t = 0$ a particle of mass m starts moving due to a force $\mathbf{F} = \mathbf{F}_0 \cos \omega t$, where $\mathbf{F}_0$ and ω are constants. How long will it be moving until it stops for the first time? What distance will it traverse during that time? What is the maximum velocity of the particle over this distance?

1.100. A motorboat of mass m moves along a lake with velocity v_0. At the moment $t = 0$ the engine of the boat is shut down. Assuming the resistance of water to be proportional to the velocity of the boat $\mathbf{F} = -r\mathbf{v}$, find:

(a) how long the motorboat moved with the shutdown engine;

(b) the velocity of the motorboat as a function of the distance covered with the shutdown engine, as well as the total distance covered till the complete stop;

(c) the mean velocity of the motorboat over the time interval (beginning with the moment $t = 0$), during which its velocity decreases η times.

1.101. Having gone through a plank of thickness h, a bullet changed its velocity from v_0 to v. Find the time of motion of the bullet in the plank, assuming the resistance force to be proportional to the square of the velocity.

1.102. A small bar starts sliding down an inclined plane forming an angle α with the horizontal. The friction coefficient depends on the distance x covered as $k = ax$, where a is a constant. Find the distance covered by the bar till it stops, and its maximum velocity over this distance.

1.103. A body of mass m rests on a horizontal plane with the friction coefficient k. At the moment $t = 0$ a horizontal force is applied to it, which varies with time as $\mathbf{F} = \mathbf{a}t$, where $\mathbf{a}$ is a constant vector.

Find the distance traversed by the body during the first t seconds after the force action began.

1.104. A body of mass m is thrown straight up with velocity v_0. Find the velocity v' with which the body comes down if the air drag equals kv^2, where k is a constant and v is the velocity of the body.

1.105. A particle of mass m moves in a certain plane P due to a force F whose magnitude is constant and whose vector rotates in that plane with a constant angular velocity ω. Assuming the particle to be stationary at the moment $t = 0$, find:

(a) its velocity as a function of time;

(b) the distance covered by the particle between two successive stops, and the mean velocity over this time.

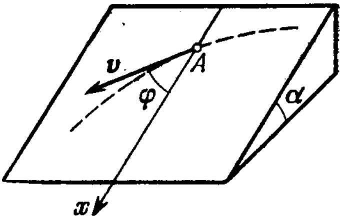

Fig. 1.27.

1.106. A small disc A is placed on an inclined plane forming an angle α with the horizontal (Fig. 1.27) and is imparted an initial velocity v_0. Find how the velocity of the disc depends on the angle φ if the friction coefficient $k = \tan \alpha$ and at the initial moment $\varphi_0 = \pi/2$.

1.107. A chain of length l is placed on a smooth spherical surface of radius R with one of its ends fixed at the top of the sphere. What will be the acceleration w of each element of the chain when its upper end is released? It is assumed that the length of the chain $l < \frac{1}{2}\pi R$.

1.108. A small body is placed on the top of a smooth sphere of radius R. Then the sphere is imparted a constant acceleration $\mathbf{w}_0$ in the horizontal direction and the body begins sliding down. Find:

(a) the velocity of the body relative to the sphere at the moment of break-off;

(b) the angle θ_0 between the vertical and the radius vector drawn from the centre of the sphere to the break-off point; calculate θ_0 for $w_0 = g$.

1.109. A particle moves in a plane under the action of a force which is always perpendicular to the particle's velocity and depends on a distance to a certain point on the plane as $1/r^n$, where n is a constant. At what value of n will the motion of the particle along the circle be *steady*?

1.110. A sleeve A can slide freely along a smooth rod bent in the shape of a half-circle of radius R (Fig. 1.28). The system is set in rotation with a constant angular velocity ω about a vertical axis OO'. Find the angle θ corresponding to the steady position of the sleeve.

1.111. A rifle was aimed at the vertical line on the target located precisely in the northern direction, and then fired. Assuming the air drag to be negligible, find how much off the line, and in what direction, will the bullet hit the target. The shot was fired in the horizontal direction at the latitude $\varphi = 60°$, the bullet velocity $v = 900$ m/s, and the distance from the target equals $s = 1.0$ km.

1.112. A horizontal disc rotates with a constant angular velocity $\omega = 6.0$ rad/s about a vertical axis passing through its centre. A small body of mass $m = 0.50$ kg moves along a diameter of the disc with a velocity $v' = 50$ cm/s which is constant relative to the disc. Find the force that the disc exerts on the body at the moment when it is located at the distance $r = 30$ cm from the rotation axis.

1.113. A horizontal smooth rod AB rotates with a constant angular velocity $\omega = 2.00$ rad/s about a vertical axis passing through its end A. A freely sliding sleeve of mass $m = 0.50$ kg moves along the rod from the point A with the initial velocity $v_0 = 1.00$ m/s. Find the Coriolis force acting on the sleeve (in the reference frame fixed to the rotating rod) at the moment when the sleeve is located at the distance $r = 50$ cm from the rotation axis.

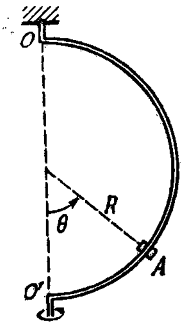

Fig. 1.28.

1.114. A horizontal disc of radius R rotates with a constant angular velocity ω about a stationary vertical axis passing through its edge. Along the circumference of the disc a particle of mass m moves with a velocity that is constant relative to the disc. At the moment when the particle is at the maximum distance from the rotation axis, the resultant of the inertial forces F_{in} acting on the particle in the reference frame fixed to the disc turns into zero. Find:

(a) the acceleration w' of the particle relative to the disc;

(b) the dependence of F_{in} on the distance from the rotation axis.

1.115. A small body of mass $m = 0.30$ kg starts sliding down from the top of a smooth sphere of radius $R = 1.00$ m. The sphere rotates with a constant angular velocity $\omega = 6.0$ rad/s about a vertical axis passing through its centre. Find the centrifugal force of inertia and the Coriolis force at the moment when the body breaks off the surface of the sphere in the reference frame fixed to the sphere.

1.116. A train of mass $m = 2000$ tons moves in the latitude $\varphi = 60°$ North. Find:

(a) the magnitude and direction of the lateral force that the train exerts on the rails if it moves along a meridian with a velocity $v = 54$ km per hour;

(b) in what direction and with what velocity the train should move for the resultant of the inertial forces acting on the train in the reference frame fixed to the Earth to be equal to zero.

1.117. At the equator a stationary (relative to the Earth) body falls down from the height $h = 500$ m. Assuming the air drag to be negligible, find how much off the vertical, and in what direction, the body will deviate when it hits the ground.

1.3. LAWS OF CONSERVATION OF ENERGY, MOMENTUM, AND ANGULAR MOMENTUM

• Work and power of the force **F**:

$$A = \int \mathbf{F}\, d\mathbf{r} = \int F_s\, ds, \quad P = \mathbf{F}\mathbf{v}. \tag{1.3a}$$

• Increment of the kinetic energy of a particle:

$$T_2 - T_1 = A, \tag{1.3b}$$

where A is the work performed by the resultant of *all* the forces acting on the particle.

• Work performed by the forces of a field is equal to the decrease of the potential energy of a particle in the given field:

$$A = U_1 - U_2. \tag{1.3c}$$

• Relationship between the force of a field and the potential energy of a particle in the field:

$$\mathbf{F} = -\nabla U, \tag{1.3d}$$

i.e. the force is equal to the antigradient of the potential energy.

• Increment of the total mechanical energy of a particle in a given potential field:

$$E_2 - E_1 = A_{extr} \tag{1.3e}$$

where A_{extr} is the algebraic sum of works performed by all *extraneous* forces, that is, by the forces not belonging to those of the *given* field.

• Increment of the total mechanical energy of a system:

$$E_2 - E_1 = A_{ext} + A_{int}^{noncons}, \tag{1.3f}$$

where $E = T + U$, and U is the *inherent* potential energy of the system.

• Law of momentum variation of a system:

$$d\mathbf{p}/dt = \mathbf{F}, \tag{1.3g}$$

where **F** is the resultant of all *external* forces.

• Equation of motion of the system's centre of inertia:

$$m\frac{d\mathbf{v}_C}{dt} = \mathbf{F}, \tag{1.3h}$$

where **F** is the resultant of all *external* forces.

• Kinetic energy of a system

$$T = \widetilde{T} + \frac{mv_C^2}{2}, \tag{1.3i}$$

where $\widetilde{T}$ is its kinetic energy in the system of centre of inertia.

• Equation of dynamics of a body with variable mass:

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F} + \frac{dm}{dt}\mathbf{u}, \tag{1.3j}$$

where **u** is the velocity of the separated (gained) substance relative to the body considered.

• Law of angular momentum variation of a system:

$$\frac{d\mathbf{M}}{dt}=\mathbf{N}, \tag{1.3k}$$

where **M** is the angular momentum of the system, and **N** is the total moment of all *external* forces.

• Angular momentum of a system:

$$\mathbf{M} = \tilde{\mathbf{M}} + [\mathbf{r}_C\mathbf{p}], \tag{1.3l}$$

where $\tilde{\mathbf{M}}$ is its angular momentum in the system of the centre of inertia, $\mathbf{r}_C$ is the radius vector of the centre of inertia, and **p** is the momentum of the system.

1.118. A particle has shifted along some trajectory in the plane xy from point *1* whose radius vector $\mathbf{r}_1 = \mathbf{i} + 2\mathbf{j}$ to point *2* with the radius vector $\mathbf{r}_2 = 2\mathbf{i} - 3\mathbf{j}$. During that time the particle experienced the action of certain forces, one of which being $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j}$. Find the work performed by the force **F**. (Here r_1, r_2, and F are given in SI units).

1.119. A locomotive of mass m starts moving so that its velocity varies according to the law $v = a\sqrt{s}$, where a is a constant, and s is the distance covered. Find the total work performed by all the forces which are acting on the locomotive during the first t seconds after the beginning of motion.

1.120. The kinetic energy of a particle moving along a circle of radius R depends on the distance covered s as $T = as^2$, where a is a constant. Find the force acting on the particle as a function of s.

1.121. A body of mass m was slowly hauled up the hill (Fig. 1.29) by a force **F** which at each point was directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is h, the length of its base l, and the coefficient of friction k.

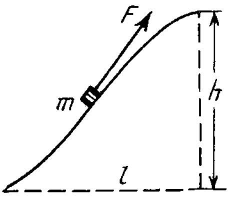

Fig. 1.29.

1.122. A disc of mass $m = 50$ g slides with the zero initial velocity down an inclined plane set at an angle $\alpha = 30°$ to the horizontal; having traversed the distance $l = 50$ cm along the horizontal plane, the disc stops. Find the work performed by the friction forces over the whole distance, assuming the friction coefficient $k = 0.15$ for both inclined and horizontal planes.

1.123. Two bars of masses m_1 and m_2 connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between the bars and the surface is equal to k. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar?

1.124. A chain of mass $m = 0.80$ kg and length $l = 1.5$ m rests on a rough-surfaced table so that one of its ends hangs over the edge. The chain starts sliding off the table all by itself provided the overhanging part equals $\eta = 1/3$ of the chain length. What will be the total work performed by the friction forces acting on the chain by the moment it slides completely off the table?

1.125. A body of mass m is thrown at an angle α to the horizontal with the initial velocity v_0. Find the mean power developed by gravity over the whole time of motion of the body, and the instantaneous power of gravity as a function of time.

1.126. A particle of mass m moves along a circle of radius R with a normal acceleration varying with time as $w_n = at^2$, where a is a constant. Find the time dependence of the power developed by all the forces acting on the particle, and the mean value of this power averaged over the first t seconds after the beginning of motion.

1.127. A small body of mass m is located on a horizontal plane at the point O. The body acquires a horizontal velocity v_0. Find:

(a) the mean power developed by the friction force during the whole time of motion, if the friction coefficient $k = 0.27$, $m = 1.0$ kg, and $v_0 = 1.5$ m/s;

(b) the maximum instantaneous power developed by the friction force, if the friction coefficient varies as $k = \alpha x$, where α is a constant, and x is the distance from the point O.

1.128. A small body of mass $m = 0.10$ kg moves in the reterence frame rotating about a stationary axis with a constant angular velocity $\omega = 5.0$ rad/s. What work does the centrifugal force of inertia perform during the transfer of this body along an arbitrary path from point *1* to point *2* which are located at the distances $r_1 = 30$ cm and $r_2 = 50$ cm from the rotation axis?

1.129. A system consists of two springs connected in series and having the stiffness coefficients k_1 and k_2. Find the minimum work to be performed in order to stretch this system by Δl.

1.130. A body of mass m is hauled from the Earth's surface by applying a force $\mathbf{F}$ varying with the height of ascent y as $\mathbf{F} = 2(ay - 1)m\mathbf{g}$, where a is a positive constant. Find the work performed by this force and the increment of the body's potential energy in the gravitational field of the Earth over the first half of the ascent.

1.131. The potential energy of a particle in a certain field has the form $U = a/r^2 - b/r$, where a and b are positive constants, r is the distance from the centre of the field. Find:

(a) the value of r_0 corresponding to the equilibrium position of the particle; examine whether this position is steady;

(b) the maximum magnitude of the attraction force; draw the plots $U(r)$ and $F_r(r)$ (the projections of the force on the radius vector $\mathbf{r}$).

1.132. In a certain two-dimensional field of force the potential energy of a particle has the form $U = \alpha x^2 + \beta y^2$, where α and β are positive constants whose magnitudes are different. Find out:

(a) whether this field is central;

(b) what is the shape of the equipotential surfaces and also of the surfaces for which the magnitude of the vector of force $F = \text{const}$.

1.133. There are two stationary fields of force $\mathbf{F} = ay\mathbf{i}$ and $\mathbf{F} =$

$= ax\mathbf{i} + by\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y axes, and a and b are constants. Find out whether these fields are potential.

1.134. A body of mass m is pushed with the initial velocity v_0 up an inclined plane set at an angle α to the horizontal. The friction coefficient is equal to k. What distance will the body cover before it stops and what work do the friction forces perform over this distance?

1.135. A small disc A slides down with initial velocity equal to zero from the top of a smooth hill of height H having a horizontal portion (Fig. 1.30). What must be the height of the horizontal portion h to ensure the maximum distance s covered by the disc? What is it equal to?

1.136. A small body A starts sliding from the height h down an inclined groove passing into a half-circle of radius $h/2$ (Fig. 1.31).

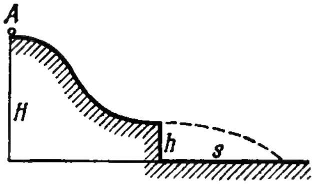

Fig. 1.30.

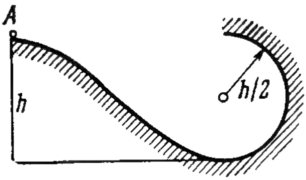

Fig. 1.31.

Assuming the friction to be negligible, find the velocity of the body at the highest point of its trajectory (after breaking off the groove).

1.137. A ball of mass m is suspended by a thread of length l. With what minimum velocity has the point of suspension to be shifted in the horizontal direction for the ball to move along the circle about that point? What will be the tension of the thread at the moment it will be passing the horizontal position?

1.138. A horizontal plane supports a stationary vertical cylinder of radius R and a disc A attached to the cylinder by a horizontal thread AB of length l_0 (Fig. 1.32, top view). An initial velocity v_0

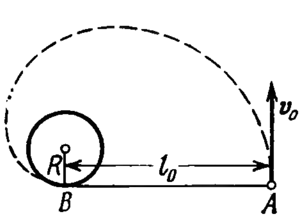

Fig. 1.32.

Fig. 1.33.

is imparted to the disc as shown in the figure. How long will it move along the plane until it strikes against the cylinder? The friction is assumed to be absent.

1.139. A smooth rubber cord of length l whose coefficient of elasticity is k is suspended by one end from the point O (Fig. 1.33). The other end is fitted with a catch B. A small sleeve A of mass m starts falling from the point O. Neglecting the masses of the thread and the catch, find the maximum elongation of the cord.

1.140. A small bar A resting on a smooth horizontal plane is attached by threads to a point P (Fig. 1.34) and, by means of a weightless pulley, to a weight B possessing the same mass as the bar itself.

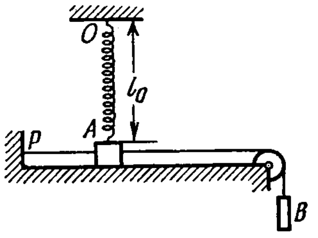

Fig. 1.34.

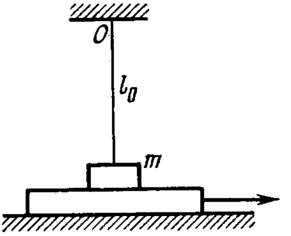

Fig. 1.35.

Besides, the bar is also attached to a point O by means of a light non-deformed spring of length $l_0 = 50$ cm and stiffness $\varkappa = 5\ mg/l_0$, where m is the mass of the bar. The thread PA having been burned, the bar starts moving. Find its velocity at the moment when it is breaking off the plane.

1.141. A horizontal plane supports a plank with a bar of mass $m = 1.0$ kg placed on it and attached by a light elastic non-deformed cord of length $l_0 = 40$ cm to a point O (Fig. 1.35). The coefficient of friction between the bar and the plank equals $k = 0.20$. The plank is slowly shifted to the right until the bar starts sliding over it. It occurs at the moment when the cord deviates from the vertical by an angle $\theta = 30°$. Find the work that has been performed by that moment by the friction force acting on the bar in the reference frame fixed to the plane.

1.142. A smooth light horizontal rod AB can rotate about a vertical axis passing through its end A. The rod is fitted with a small sleeve of mass m attached to the end A by a weightless spring of length l_0 and stiffness $\varkappa$. What work must be performed to slowly get this system going and reaching the angular velocity ω?

1.143. A pulley fixed to the ceiling carries a thread with bodies of masses m_1 and m_2 attached to its ends. The masses of the pulley and the thread are negligible, friction is absent. Find the acceleration $\mathbf{w}_C$ of the centre of inertia of this system.

1.144. Two interacting particles form a closed system whose centre of inertia is at rest. Fig. 1.36 illustrates the positions of both particles at a certain moment and the trajectory of the particle of mass m_1. Draw the trajectory of the particle of mass m_2 if $m_2 = m_1/2$.

1.145. A closed chain A of mass $m = 0.36$ kg is attached to a vertical rotating shaft by means of a thread (Fig. 1.37), and rotates with a constant angular velocity $\omega = 35$ rad/s. The thread forms an angle $\theta = 45°$ with the vertical. Find the distance between the chain's centre of gravity and the rotation axis, and the tension of the thread.

Fig. 1.36.

1.146. A round cone A of mass $m = 3.2$ kg and half-angle $\alpha = 10°$ rolls uniformly and without slipping along a round conical surface B so that its apex O remains stationary (Fig. 1.38). The centre of gravity of the cone A is at the same level as the point O and at a distance $l = 17$ cm from it. The cone's axis moves with angular velocity ω. Find:

(a) the static friction force acting on the cone A, if $\omega = 1.0$ rad/s;

(b) at what values of ω the cone A will roll without sliding, if the coefficient of friction between the surfaces is equal to $k = 0.25$.

1.147. In the reference frame K two particles travel along the x axis, one of mass m_1 with velocity $\mathbf{v}_1$, and the other of mass m_2 with velocity $\mathbf{v}_2$. Find:

(a) the velocity $\mathbf{V}$ of the reference frame K' in which the cumulative kinetic energy of these particles is minimum;

(b) the cumulative kinetic energy of these particles in the K' frame.

1.148. The reference frame, in which the centre of inertia of a given system of particles is at rest, translates with a velocity $\mathbf{V}$ relative

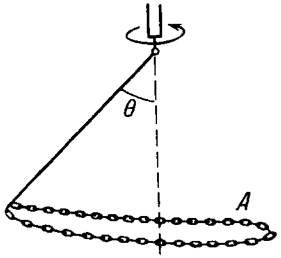

Fig. 1.37.

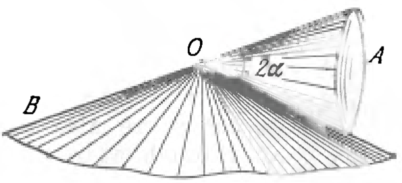

Fig. 1.38.

to an inertial reference frame K. The mass of the system of particles equals m, and the total energy of the system in the frame of the centre of inertia is equal to $\tilde{E}$. Find the total energy E of this system of particles in the reference frame K.

1.149. Two small discs of masses m_1 and m_2 interconnected by a weightless spring rest on a smooth horizontal plane. The discs are set in motion with initial velocities v_1 and v_2 whose directions are mutually perpendicular and lie in a horizontal plane. Find the total energy $\tilde{E}$ of this system in the frame of the centre of inertia.

1.150. A system consists of two small spheres of masses m_1 and m_2 interconnected by a weightless spring. At the moment $t = 0$ the spheres are set in motion with the initial velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ after which the system starts moving in the Earth's uniform gravitational field. Neglecting the air drag, find the time dependence of the total momentum of this system in the process of motion and of the radius vector of its centre of inertia relative to the initial position of the centre.

1.151. Two bars of masses m_1 and m_2 connected by a weightless spring of stiffness $\varkappa$ (Fig. 1.39) rest on a smooth horizontal plane.

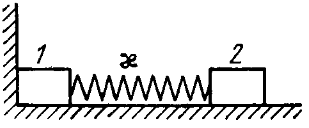

Fig. 1.39.

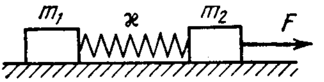

Fig. 1.40.

Bar *2* is shifted a small distance x to the left and then released. Find the velocity of the centre of inertia of the system after bar *1* breaks off the wall.

1.152. Two bars connected by a weightless spring of stiffness $\varkappa$ and length (in the non-deformed state) l_0 rest on a horizontal plane. A constant horizontal force F starts acting on one of the bars as shown in Fig. 1.40. Find the maximum and minimum distances between the bars during the subsequent motion of the system, if the masses of the bars are:

(a) equal;

(b) equal to m_1 and m_2, and the force F is applied to the bar of mass m_2.

1.153. A system consists of two identical cubes, each of mass m, linked together by the compressed weightless spring of stiffness $\varkappa$ (Fig. 1.41). The cubes are also connected by a thread which is burned through at a certain moment. Find:

(a) at what values of Δl, the initial compression of the spring, the lower cube will bounce up after the thread has been burned through;

(b) to what height h the centre of gravity of this system will rise if the initial compression of the spring $\Delta l = 7\ mg/\varkappa$.

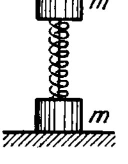

Fig. 1.41.

1.154. Two identical buggies *1* and *2* with one man in each move without friction due to inertia along the parallel rails toward each other. When the buggies get opposite each other, the men exchange their places by jumping in the direction perpendicular to the motion direction. As a consequence, buggy

1 stops and buggy *2* keeps moving in the same direction, with its velocity becoming equal to $\mathbf{v}$. Find the initial velocities of the buggies $\mathbf{v}_1$ and $\mathbf{v}_2$ if the mass of each buggy (without a man) equals M and the mass of each man m.

1.155. Two identical buggies move one after the other due to inertia (without friction) with the same velocity $\mathbf{v}_0$. A man of mass m rides the rear buggy. At a certain moment the man jumps into the front buggy with a velocity $\mathbf{u}$ relative to his buggy. Knowing that the mass of each buggy is equal to M, find the velocities with which the buggies will move after that.

1.156. Two men, each of mass m, stand on the edge of a stationary buggy of mass M. Assuming the friction to be negligible, find the velocity of the buggy after both men jump off with the same horizontal velocity $\mathbf{u}$ relative to the buggy: (1) simultaneously; (2) one after the other. In what case will the velocity of the buggy be greater and how many times?

1.157. A chain hangs on a thread and touches the surface of a table by its lower end. Show that after the thread has been burned through, the force exerted on the table by the falling part of the chain at any moment is twice as great as the force of pressure exerted by the part already resting on the table.

1.158. A steel ball of mass $m = 50$ g falls from the height $h = 1.0$ m on the horizontal surface of a massive slab. Find the cumulative momentum that the ball imparts to the slab after numerous bounces, if every impact decreases the velocity of the ball $\eta = 1.25$ times.

1.159. A raft of mass M with a man of mass m aboard stays motionless on the surface of a lake. The man moves a distance $\mathbf{l}'$ relative to the raft with velocity $\mathbf{v}'(t)$ and then stops. Assuming the water resistance to be negligible, find:

(a) the displacement of the raft $\mathbf{l}$ relative to the shore;

(b) the horizontal component of the force with which the man acted on the raft during the motion.

1.160. A stationary pulley carries a rope whose one end supports a ladder with a man and the other end the counterweight of mass M. The man of mass m climbs up a distance $\mathbf{l}'$ with respect to the ladder and then stops. Neglecting the mass of the rope and the friction in the pulley axle, find the displacement $\mathbf{l}$ of the centre of inertia of this system.

1.161. A cannon of mass M starts sliding freely down a smooth inclined plane at an angle α to the horizontal. After the cannon covered the distance l, a shot was fired, the shell leaving the cannon in the horizontal direction with a momentum $\mathbf{p}$. As a consequence, the cannon stopped. Assuming the mass of the shell to be negligible, as compared to that of the cannon, determine the duration of the shot.

1.162. A horizontally flying bullet of mass m gets stuck in a body of mass M suspended by two identical threads of length l (Fig. 1.42).

As a result, the threads swerve through an angle θ. Assuming $m \ll M$, find:

(a) the velocity of the bullet before striking the body;

(b) the fraction of the bullet's initial kinetic energy that turned into heat.

1.163. A body of mass M (Fig. 1.43) with a small disc of mass m placed on it rests on a smooth horizontal plane. The disc is set in

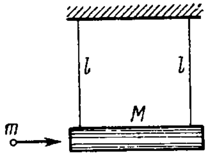

Fig. 1.42.

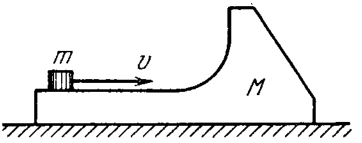

Fig. 1.43.

motion in the horizontal direction with velocity v. To what height (relative to the initial level) will the disc rise after breaking off the body M? The friction is assumed to be absent.

1.164. A small disc of mass m slides down a smooth hill of height h without initial velocity and gets onto a plank of mass M lying on

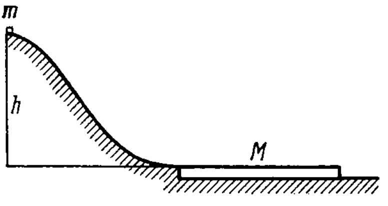

Fig. 1.44.

the horizontal plane at the base of the hill (Fig. 1.44). Due to friction between the disc and the plank the disc slows down and, beginning with a certain moment, moves in one piece with the plank.

(1) Find the total work performed by the friction forces in this process.

(2) Can it be stated that the result obtained does not depend on the choice of the reference frame?

1.165. A stone falls down without initial velocity from a height h onto the Earth's surface. The air drag assumed to be negligible, the stone hits the ground with velocity $v_0 = \sqrt{2gh}$ relative to the Earth. Obtain the same formula in terms of the reference frame "falling" to the Earth with a constant velocity v_0.

1.166. A particle of mass 1.0 g moving with velocity $\mathbf{v}_1 = 3.0\mathbf{i} - 2.0\mathbf{j}$ experiences a perfectly inelastic collision with another particle of mass 2.0 g and velocity $\mathbf{v}_2 = 4.0\mathbf{j} - 6.0\mathbf{k}$. Find the velocity of the formed particle (both the vector $\mathbf{v}$ and its modulus), if the components of the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are given in the SI units.

1.167. Find the increment of the kinetic energy of the closed system comprising two spheres of masses m_1 and m_2 due to their perfectly inelastic collision, if the initial velocities of the spheres were equal to $\mathbf{v}_1$ and $\mathbf{v}_2$.

1.168. A particle of mass m_1 experienced a perfectly elastic collision with a stationary particle of mass m_2. What fraction of the kinetic energy does the striking particle lose, if

(a) it recoils at right angles to its original motion direction;

(b) the collision is a head-on one?

1.169. Particle *1* experiences a perfectly elastic collision with a stationary particle *2*. Determine their mass ratio, if

(a) after a head-on collision the particles fly apart in the opposite directions with equal velocities;

(b) the particles fly apart symmetrically relative to the initial motion direction of particle *1* with the angle of divergence $\theta = 60°$.

1.170. A ball moving translationally collides elastically with another, stationary, ball of the same mass. At the moment of impact the angle between the straight line passing through the centres of the balls and the direction of the initial motion of the striking ball is equal to $\alpha = 45°$. Assuming the balls to be smooth, find the fraction η of the kinetic energy of the striking ball that turned into potential energy at the moment of the maximum deformation.

1.171. A shell flying with velocity $v = 500$ m/s bursts into three identical fragments so that the kinetic energy of the system increases $\eta = 1.5$ times. What maximum velocity can one of the fragments obtain?

1.172. Particle *1* moving with velocity $v = 10$ m/s experienced a head-on collision with a stationary particle *2* of the same mass. As a result of the collision, the kinetic energy of the system decreased by $\eta = 1.0\%$. Find the magnitude and direction of the velocity of particle *1* after the collision.

1.173. A particle of mass m having collided with a stationary particle of mass M deviated by an angle $\pi/2$ whereas the particle M recoiled at an angle $\theta = 30°$ to the direction of the initial motion of the particle m. How much (in per cent) and in what way has the kinetic energy of this system changed after the collision, if $M/m = 5.0$?

1.174. A closed system consists of two particles of masses m_1 and m_2 which move at right angles to each other with velocities v_1 and v_2. Find:

(a) the momentum of each particle and

(b) the total kinetic energy of the two particles in the reference frame fixed to their centre of inertia.

1.175. A particle of mass m_1 collides elastically with a stationary particle of mass m_2 $(m_1 > m_2)$. Find the maximum angle through which the striking particle may deviate as a result of the collision.

1.176. Three identical discs A, B, and C (Fig. 1.45) rest on a smooth horizontal plane. The disc A is set in motion with velocity $\mathbf{v}$ after which it experiences an elastic collision simultaneously with the discs B and C. The distance between the centres of the latter discs prior to the collision is η times greater than the diameter of each disc. Find the velocity of the disc A after the collision. At what value of η will the disc A recoil after the collision; stop; move on?

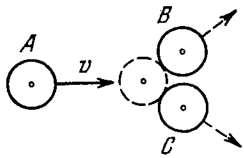

Fig. 1.45.

1.177. A molecule collides with another, stationary, molecule of the same mass. Demonstrate that the angle of divergence

(a) equals 90° when the collision is ideally elastic;

(b) differs from 90° when the collision is inelastic.

1.178. A rocket ejects a steady jet whose velocity is equal to $\mathbf{u}$ relative to the rocket. The gas discharge rate equals μ kg/s. Demonstrate that the rocket motion equation in this case takes the form

$$m\mathbf{w} = \mathbf{F} - \mu\mathbf{u},$$

where m is the mass of the rocket at a given moment, $\mathbf{w}$ is its acceleration, and $\mathbf{F}$ is the external force.

1.179. A rocket moves in the absence of external forces by ejecting a steady jet with velocity $\mathbf{u}$ constant relative to the rocket. Find the velocity $\mathbf{v}$ of the rocket at the moment when its mass is equal to m, if at the initial moment it possessed the mass m_0 and its velocity was equal to zero. Make use of the formula given in the foregoing problem.

1.180. Find the law according to which the mass of the rocket varies with time, when the rocket moves with a constant acceleration w, the external forces are absent, the gas escapes with a constant velocity u relative to the rocket, and its mass at the initial moment equals m_0.

1.181. A spaceship of mass m_0 moves in the absence of external forces with a constant velocity $\mathbf{v}_0$. To change the motion direction, a jet engine is switched on. It starts ejecting a gas jet with velocity u which is constant relative to the spaceship and directed at right angles to the spaceship motion. The engine is shut down when the mass of the spaceship decreases to m. Through what angle α did the motion direction of the spaceship deviate due to the jet engine operation?

1.182. A cart loaded with sand moves along a horizontal plane due to a constant force $\mathbf{F}$ coinciding in direction with the cart's velocity vector. In the process, sand spills through a hole in the bottom with a constant velocity μ kg/s. Find the acceleration and the velocity of the cart at the moment t, if at the initial moment $t = 0$ the cart with loaded sand had the mass m_0 and its velocity was equal to zero. The friction is to be neglected.

1.183. A flatcar of mass m_0 starts moving to the right due to a constant horizontal force $\mathbf{F}$ (Fig. 1.46). Sand spills on the flatcar from a stationary hopper. The velocity of loading is constant and equal to μ kg/s. Find the time dependence of the velocity and the acceleration of the flatcar in the process of loading. The friction is negligibly small.

1.184. A chain AB of length l is located in a smooth horizontal tube so that its fraction of length h hangs freely and touches the surface of the table with its end B (Fig. 1.47). At a certain moment

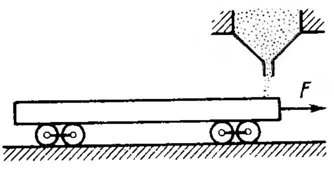

Fig. 1.46.

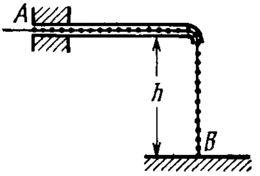

Fig. 1.47.

the end A of the chain is set free. With what velocity will this end of the chain slip out of the tube?

1.185. The angular momentum of a particle relative to a certain point O varies with time as $\mathbf{M} = \mathbf{a} + \mathbf{b}t^2$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors, with $\mathbf{a} \perp \mathbf{b}$. Find the force moment $\mathbf{N}$ relative to the point O acting on the particle when the angle between the vectors $\mathbf{N}$ and $\mathbf{M}$ equals 45°.

1.186. A ball of mass m is thrown at an angle α to the horizontal with the initial velocity v_0. Find the time dependence of the magnitude of the ball's angular momentum vector relative to the point from which the ball is thrown. Find the angular momentum M at the highest point of the trajectory if $m = 130$ g, $\alpha = 45°$, and $v_0 = 25$ m/s. The air drag is to be neglected.

1.187. A disc A of mass m sliding over a smooth horizontal surface with velocity v experiences a perfectly elastic collision with a smooth stationary wall at a point O (Fig. 1.48). The angle between the motion direction of the disc and the normal of the wall is equal to α. Find:

(a) the points relative to which the angular momentum $\mathbf{M}$ of the disc remains constant in this process;

(b) the magnitude of the increment of the vector of the disc's angular momentum relative to the point O' which is located in the plane of the disc's motion at the distance l from the point O.

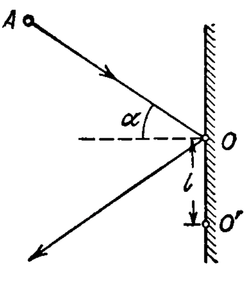

Fig. 1.48.

1.188. A small ball of mass m suspended from the ceiling at a point O by a thread of length l moves along a horizontal circle with a constant angular velocity ω. Relative to which points does the angular momentum $\mathbf{M}$ of the ball remain constant? Find the magnitude of the increment of the vector of the ball's angular momentum relative to the point O picked up during half a revolution.

1.189. A ball of mass m falls down without initial velocity from a height h over the Earth's surface. Find the increment of the ball's angular momentum vector picked up during the time of falling (relative to the point O of the reference frame moving translationally in a horizontal direction with a velocity V). The ball starts falling from the point O. The air drag is to be neglected.

1.190. A smooth horizontal disc rotates with a constant angular velocity ω about a stationary vertical axis passing through its centre, the point O. At a moment $t = 0$ a disc is set in motion from that

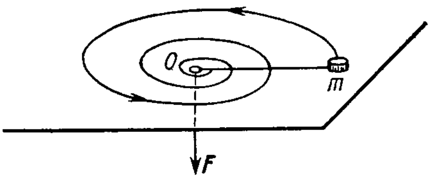

Fig. 1.49.

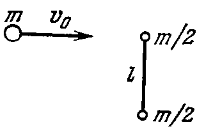

Fig. 1.50.

point with velocity v_0. Find the angular momentum $M(t)$ of the disc relative to the point O in the reference frame fixed to the disc. Make sure that this angular momentum is caused by the Coriolis force.

1.191. A particle moves along a closed trajectory in a central field of force where the particle's potential energy $U = kr^2$ (k is a positive constant, r is the distance of the particle from the centre O of the field). Find the mass of the particle if its minimum distance from the point O equals r_1 and its velocity at the point farthest from O equals v_2.

1.192. A small ball is suspended from a point O by a light thread of length l. Then the ball is drawn aside so that the thread deviates through an angle θ from the vertical and set in motion in a horizontal direction at right angles to the vertical plane in which the thread is located. What is the initial velocity that has to be imparted to the ball so that it could deviate through the maximum angle $\pi/2$ in the process of motion?

1.193. A small body of mass m tied to a non-stretchable thread moves over a smooth horizontal plane. The other end of the thread is being drawn into a hole O (Fig. 1.49) with a constant velocity. Find the thread tension as a function of the distance r between the body and the hole if at $r = r_0$ the angular velocity of the thread is equal to ω_0.

1.194. A light non-stretchable thread is wound on a massive fixed pulley of radius R. A small body of mass m is tied to the free end of the thread. At a moment $t = 0$ the system is released and starts moving. Find its angular momentum relative to the pulley axle as a function of time t.

1.195. A uniform sphere of mass m and radius R starts rolling without slipping down an inclined plane at an angle α to the horizontal. Find the time dependence of the angular momentum of the sphere relative to the point of contact at the initial moment. How will the obtained result change in the case of a perfectly smooth inclined plane?

1.196. A certain system of particles possesses a total momentum $\mathbf{p}$ and an angular momentum $\mathbf{M}$ relative to a point O. Find its angular momentum $\mathbf{M}'$ relative to a point O' whose position with respect to the point O is determined by the radius vector $\mathbf{r}_0$. Find out when the angular momentum of the system of particles does not depend on the choice of the point O.

1.197. Demonstrate that the angular momentum $\mathbf{M}$ of the system of particles relative to a point O of the reference frame K can be represented as

$$\mathbf{M} = \widetilde{\mathbf{M}} + [\mathbf{r}_C \mathbf{p}],$$

where $\widetilde{\mathbf{M}}$ is its proper angular momentum (in the reference frame moving translationally and fixed to the centre of inertia), $\mathbf{r}_C$ is the radius vector of the centre of inertia relative to the point O, $\mathbf{p}$ is the total momentum of the system of particles in the reference frame K.

1.198. A ball of mass m moving with velocity v_0 experiences a head-on elastic collision with one of the spheres of a stationary rigid dumbbell as whown in Fig. 1.50. The mass of each sphere equals $m/2$, and the distance between them is l. Disregarding the size of the spheres, find the proper angular momentum $\widetilde{M}$ of the dumbbell after the collision, i.e. the angular momentum in the reference frame moving translationally and fixed to the dumbbell's centre of inertia.

1.199. Two small identical discs, each of mass m, lie on a smooth horizontal plane. The discs are interconnected by a light non-deformed spring of length l_0 and stiffness $\varkappa$. At a certain moment one of the discs is set in motion in a horizontal direction perpendicular to the spring with velocity v_0. Find the maximum elongation of the spring in the process of motion, if it is known to be considerably less than unity.

1.4. UNIVERSAL GRAVITATION

- Universal gravitation law

$$F = \gamma \frac{m_1 m_2}{r^2}. \tag{1.4a}$$

- The squares of the periods of revolution of any two planets around the Sun are proportional to the cubes of the major semiaxes of their orbits (Kepler):

$$T^2 \propto a^3. \tag{1.4b}$$

- Strength G and potential φ of the gravitational field of a mass point:

$$\mathbf{G} = -\gamma \frac{m}{r^3}\mathbf{r}, \quad \varphi = -\gamma \frac{m}{r}. \tag{1.4c}$$

- Orbital and escape velocities:

$$v_1 = \sqrt{gR}, \quad v_2 = \sqrt{2}\, v_1. \tag{1.4d}$$

1.200. A planet of mass M moves along a circle around the Sun with velocity $v = 34.9$ km/s (relative to the heliocentric reference frame). Find the period of revolution of this planet around the Sun.

1.201. The Jupiter's period of revolution around the Sun is 12 times that of the Earth. Assuming the planetary orbits to be circular, find:

(a) how many times the distance between the Jupiter and the Sun exceeds that between the Earth and the Sun;

(b) the velocity and the acceleration of Jupiter in the heliocentric reference frame.

1.202. A planet of mass M moves around the Sun along an ellipse so that its minimum distance from the Sun is equal to r and the maximum distance to R. Making use of Kepler's laws, find its period of revolution around the Sun.

1.203. A small body starts falling onto the Sun from a distance equal to the radius of the Earth's orbit. The initial velocity of the body is equal to zero in the heliocentric reference frame. Making use of Kepler's laws, find how long the body will be falling.

1.204. Suppose we have made a model of the Solar system scaled down in the ratio η but of materials of the same mean density as the actual materials of the planets and the Sun. How will the orbital periods of revolution of planetary models change in this case?

1.205. A double star is a system of two stars moving around the centre of inertia of the system due to gravitation. Find the distance between the components of the double star, if its total mass equals M and the period of revolution T.

1.206. Find the potential energy of the gravitational interaction

(a) of two mass points of masses m_1 and m_2 located at a distance r from each other;

(b) of a mass point of mass m and a thin uniform rod of mass M and length l, if they are located along a straight line at a distance a from each other; also find the force of their interaction.

1.207. A planet of mass m moves along an ellipse around the Sun so that its maximum and minimum distances from the Sun are equal to r_1 and r_2 respectively. Find the angular momentum M of this planet relative to the centre of the Sun.

1.208. Using the conservation laws, demonstrate that the total mechanical energy of a planet of mass m moving around the Sun along an ellipse depends only on its semi-major axis a. Find this energy as a function of a.

1.209. A planet A moves along an elliptical orbit around the Sun. At the moment when it was at the distance r_0 from the Sun its velocity was equal to v_0 and the angle between the radius vector $\mathbf{r}_0$ and the velocity vector $\mathbf{v}_0$ was equal to α. Find the maximum and minimum distances that will separate this planet from the Sun during its orbital motion.

1.210. A cosmic body A moves to the Sun with velocity v_0 (when far from the Sun) and aiming parameter l the arm of the vector $\mathbf{v}_0$

relative to the centre of the Sun (Fig. 1.51). Find the minimum distance by which this body will get to the Sun.

1.211. A particle of mass m is located outside a uniform sphere of mass M at a distance r from its centre. Find:

(a) the potential energy of gravitational interaction of the particle and the sphere;

(b) the gravitational force which the sphere exerts on the particle.

1.212. Demonstrate that the gravitational force acting on a particle A inside a uniform spherical layer of matter is equal to zero.

1.213. A particle of mass m was transferred from the centre of the base of a uniform hemisphere of mass M and radius R into infinity.

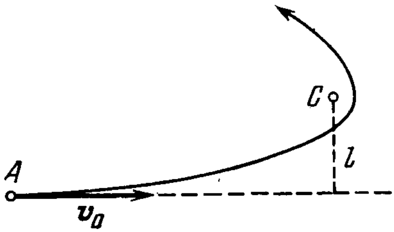

Fig. 1.51.

What work was performed in the process by the gravitational force exerted on the particle by the hemisphere?

1.214. There is a uniform sphere of mass M and radius R. Find the strength **G** and the potential φ of the gravitational field of this sphere as a function of the distance r from its centre (with $r < R$ and $r > R$). Draw the approximate plots of the functions $G(r)$ and $\varphi(r)$.

1.215. Inside a uniform sphere of density ρ there is a spherical cavity whose centre is at a distance **l** from the centre of the sphere. Find the strength **G** of the gravitational field inside the cavity.

1.216. A uniform sphere has a mass M and radius R. Find the pressure p inside the sphere, caused by gravitational compression, as a function of the distance r from its centre. Evaluate p at the centre of the Earth, assuming it to be a uniform sphere.

1.217. Find the proper potential energy of gravitational interaction of matter forming

(a) a thin uniform spherical layer of mass m and radius R;

(b) a uniform sphere of mass m and radius R (make use of the answer to Problem 1.214).

1.218. Two Earth's satellites move in a common plane along circular orbits. The orbital radius of one satellite $r = 7000$ km while that of the other satellite is $\Delta r = 70$ km less. What time interval separates the periodic approaches of the satellites to each other over the minimum distance?

1.219. Calculate the ratios of the following accelerations: the acceleration w_1 due to the gravitational force on the Earth's surface, the acceleration w_2 due to the centrifugal force of inertia on the Earth's equator, and the acceleration w_3 caused by the Sun to the bodies on the Earth.

1.220. At what height over the Earth's pole the free-fall acceleration decreases by one per cent; by half?

1.221. On the pole of the Earth a body is imparted velocity v_0 directed vertically up. Knowing the radius of the Earth and the free-fall acceleration on its surface, find the height to which the body will ascend. The air drag is to be neglected.

1.222. An artificial satellite is launched into a circular orbit around the Earth with velocity v relative to the reference frame moving translationally and fixed to the Earth's rotation axis. Find the distance from the satellite to the Earth's surface. The radius of the Earth and the free-fall acceleration on its surface are supposed to be known.

1.223. Calculate the radius of the circular orbit of a stationary Earth's satellite, which remains motionless with respect to its surface. What are its velocity and acceleration in the inertial reference frame fixed at a given moment to the centre of the Earth?

1.224. A satellite revolving in a circular equatorial orbit of radius $R = 2.00 \cdot 10^4$ km from west to east appears over a certain point at the equator every $\tau = 11.6$ hours. Using these data, calculate the mass of the Earth. The gravitational constant is supposed to be known.

1.225. A satellite revolves from east to west in a circular equatorial orbit of radius $R = 1.00 \cdot 10^4$ km around the Earth. Find the velocity and the acceleration of the satellite in the reference frame fixed to the Earth.

1.226. A satellite must move in the equatorial plane of the Earth close to its surface either in the Earth's rotation direction or against it. Find how many times the kinetic energy of the satellite in the latter case exceeds that in the former case (in the reference frame fixed to the Earth).

1.227. An artificial satellite of the Moon revolves in a circular orbit whose radius exceeds the radius of the Moon η times. In the process of motion the satellite experiences a slight resistance due to cosmic dust. Assuming the resistance force to depend on the velocity of the satellite as $F = \alpha v^2$, where α is a constant, find how long the satellite will stay in orbit until it falls onto the Moon's surface.

1.228. Calculate the orbital and escape velocities for the Moon. Compare the results obtained with the corresponding velocities for the Earth.

1.229. A spaceship approaches the Moon along a parabolic trajectory which is almost tangent to the Moon's surface. At the moment of the maximum approach the brake rocket was fired for a short time interval, and the spaceship was transferred into a circular orbit of a Moon satellite. Find how the spaceship velocity modulus increased in the process of braking.

1.230. A spaceship is launched into a circular orbit close to the

Earth's surface. What additional velocity has to be imparted to the spaceship to overcome the gravitational pull?

1.231. At what distance from the centre of the Moon is the point at which the strength of the resultant of the Earth's and Moon's gravitational fields is equal to zero? The Earth's mass is assumed to be $\eta = 81$ times that of the Moon, and the distance between the centres of these planets $n = 60$ times greater than the radius of the Earth R.

1.232. What is the minimum work that has to be performed to bring a spaceship of mass $m = 2.0 \cdot 10^3$ kg from the surface of the Earth to the Moon?

1.233. Find approximately the third cosmic velocity v_3, i.e. the minimum velocity that has to be imparted to a body relative to the Earth's surface to drive it out of the Solar system. The rotation of the Earth about its own axis is to be neglected.

1.5. DYNAMICS OF A SOLID BODY

- Equation of dynamics of a solid body rotating about a stationary axis z:

$$I\beta_z = N_z, \tag{1.5a}$$

where N_z is the algebraic sum of the moments of external forces relative to the z axis.

- According to Steiner's theorem:

$$I = I_C + ma^2. \tag{1.5b}$$

- Kinetic energy of a solid body rotating about a stationary axis:

$$T = \frac{1}{2} I\omega^2. \tag{1.5c}$$

- Work performed by external forces during the rotation of a solid body about a stationary axis:

$$A = \int N_z \, d\varphi. \tag{1.5d}$$

- Kinetic energy of a solid body in plane motion:

$$T = \frac{I_C\omega^2}{2} + \frac{mv_C^2}{2}. \tag{1.5e}$$

- Relationship between the angular velocity $\boldsymbol{\omega}'$ of gyroscope precession, its angular momentum $\mathbf{M}$ equal to $I\omega$, and the moment $\mathbf{N}$ of the external forces:

$$[\boldsymbol{\omega}'\mathbf{M}] = \mathbf{N}. \tag{1.5f}$$

1.234. A thin uniform rod AB of mass $m = 1.0$ kg moves translationally with acceleration $w = 2.0$ m/s² due to two antiparallel forces $\mathbf{F}_1$ and $\mathbf{F}_2$ (Fig. 1.52). The distance between the points at which these forces are applied is equal to $a = 20$ cm. Besides, it is known that $F_2 = 5.0$ N. Find the length of the rod.

1.235. A force $\mathbf{F} = A\mathbf{i} + B\mathbf{j}$ is applied to a point whose radius vector relative to the origin of coordinates O is equal to $\mathbf{r} = a\mathbf{i} + b\mathbf{j}$, where a, b, A, B are constants, and $\mathbf{i}$, $\mathbf{j}$ are the unit vectors of the x and y axes. Find the moment **N** and the arm l of the force **F** relative to the point O.

1.236. A force $\mathbf{F}_1 = A\mathbf{j}$ is applied to a point whose radius vector $\mathbf{r}_1 = a\mathbf{i}$, while a force $\mathbf{F}_2 = B\mathbf{i}$ is applied to the point whose radius vector $\mathbf{r}_2 = b\mathbf{j}$. Both radius vectors are determined relative to the origin of coordinates O, $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors of the x and y

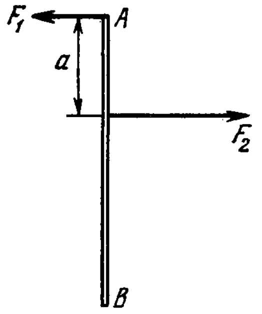

Fig. 1.52.

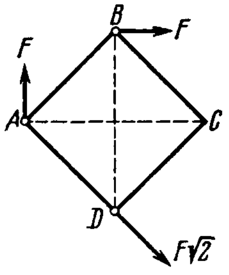

Fig. 1.53.

axes, a, b, A, B are constants. Find the arm l of the resultant force relative to the point O.

1.237. Three forces are applied to a square plate as shown in Fig. 1.53. Find the modulus, direction, and the point of application of the resultant force, if this point is taken on the side BC.

1.238. Find the moment of inertia

(a) of a thin uniform rod relative to the axis which is perpendicular to the rod and passes through its end, if the mass of the rod is m and its length l;

(b) of a thin uniform rectangular plate relative to the axis passing perpendicular to the plane of the plate through one of its vertices, if the sides of the plate are equal to a and b, and its mass is m.

1.239. Calculate the moment of inertia

(a) of a copper uniform disc relative to the symmetry axis perpendicular to the plane of the disc, if its thickness is equal to $b = 2.0$ mm and its radius to $R = 100$ mm;

(b) of a uniform solid cone relative to its symmetry axis, if the mass of the cone is equal to m and the radius of its base to R.

1.240. Demonstrate that in the case of a thin plate of arbitrary shape there is the following relationship between the moments of inertia: $I_1 + I_2 = I_3$, where subindices 1, 2, and 3 define three mutually perpendicular axes passing through one point, with axes 1 and 2 lying in the plane of the plate. Using this relationship, find the moment of inertia of a thin uniform round disc of radius R and mass m relative to the axis coinciding with one of its diameters.

1.241. A uniform disc of radius $R = 20$ cm has a round cut as shown in Fig. 1.54. The mass of the remaining (shaded) portion of the disc equals $m = 7.3$ kg. Find the moment of inertia of such a disc relative to the axis passing through its centre of inertia and perpendicular to the plane of the disc.

1.242. Using the formula for the moment of inertia of a uniform sphere, find the moment of inertia of a thin spherical layer of mass m and radius R relative to the axis passing through its centre.

1.243. A light thread with a body of mass m tied to its end is wound on a uniform solid cylinder of mass M and radius R (Fig. 1.55). At a moment $t = 0$ the system is set in motion. Assuming the friction in the axle of the cylinder to be negligible, find the time dependence of

(a) the angular velocity of the cylinder;
(b) the kinetic energy of the whole system.

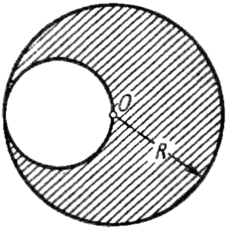

Fig. 1.54.

1.244. The ends of thin threads tightly wound on the axle of radius r of the Maxwell disc are attached to a horizontal bar. When the disc unwinds, the bar is raised to keep the disc at the same height. The mass of the disc with the axle is equal to m, the moment of inertia of the arrangement relative to its axis is I. Find the tension of each thread and the acceleration of the bar.

1.245. A thin horizontal uniform rod AB of mass m and length l can rotate freely about a vertical axis passing through its end A. At a certain moment the end B starts experiencing a constant force

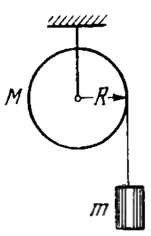

Fig. 1.55.

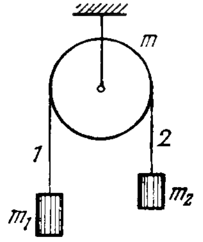

Fig. 1.56.

F which is always perpendicular to the original position of the stationary rod and directed in a horizontal plane. Find the angular velocity of the rod as a function of its rotation angle φ counted relative to the initial position.

1.246. In the arrangement shown in Fig. 1.56 the mass of the uniform solid cylinder of radius R is equal to m and the masses of two bodies are equal to m_1 and m_2. The thread slipping and the friction in the axle of the cylinder are supposed to be absent. Find the angular acceleration of the cylinder and the ratio of tensions T_1/T_2 of the vertical sections of the thread in the process of motion.

1.247. In the system shown in Fig. 1.57 the masses of the bodies are known to be m_1 and m_2, the coefficient of friction between the body m_1 and the horizontal plane is equal to k, and a pulley of mass m is assumed to be a uniform disc. The thread does not slip over the pulley. At the moment $t = 0$ the body m_2 starts descending. Assuming the mass of the thread and the friction in the axle of the pulley to be negligible, find the work performed by the friction forces acting on the body m_1 over the first t seconds after the beginning of motion.

1.248. A uniform cylinder of radius R is spinned about its axis to the angular velocity ω_0 and then placed into a corner (Fig. 1.58).

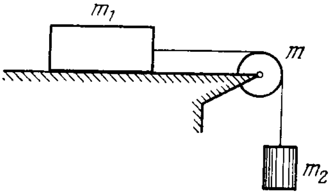

Fig. 1.57.

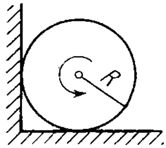

Fig. 1.58.

The coefficient of friction between the corner walls and the cylinder is equal to k. How many turns will the cylinder accomplish before it stops?

1.249. A uniform disc of radius R is spinned to the angular velocity ω and then carefully placed on a horizontal surface. How long will the disc be rotating on the surface if the friction coefficient is equal to k? The pressure exerted by the disc on the surface can be regarded as uniform.

1.250. A flywheel with the initial angular velocity ω_0 decelerates due to the forces whose moment relative to the axis is proportional to the square root of its angular velocity. Find the mean angular velocity of the flywheel averaged over the total deceleration time.

1.251. A uniform cylinder of radius R and mass M can rotate freely about a stationary horizontal axis O (Fig. 1.59). A thin cord of length l and mass m is wound on the cylinder in a single layer. Find the angular acceleration of the cylinder as a function of the length x of the hanging part of the cord. The wound part of the cord is supposed to have its centre of gravity on the cylinder axis.

1.252. A uniform sphere of mass m and radius R rolls without slipping down an inclined plane set at an angle α to the horizontal. Find:

(a) the magnitudes of the friction coefficient at which slipping is absent;

(b) the kinetic energy of the sphere t seconds after the beginning of motion.

1.253. A uniform cylinder of mass $m = 8.0$ kg and radius $R = 1.3$ cm (Fig. 1.60) starts descending at a moment $t = 0$ due to gravity. Neglecting the mass of the thread, find:

(a) the tension of each thread and the angular acceleration of the cylinder;

(b) the time dependence of the instantaneous power developed by the gravitational force.

1.254. Thin threads are tightly wound on the ends of a uniform solid cylinder of mass m. The free ends of the threads are attached to

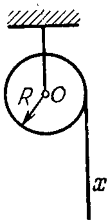

Fig. 1.59.

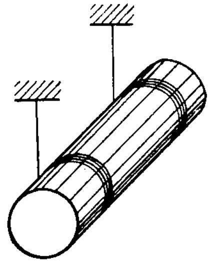

Fig. 1.60.

the ceiling of an elevator car. The car starts going up with an acceleration $\mathbf{w}_0$. Find the acceleration $\mathbf{w}'$ of the cylinder relative to the car and the force $\mathbf{F}$ exerted by the cylinder on the ceiling (through the threads).

1.255. A spool with a thread wound on it is placed on an inclined smooth plane set at an angle $\alpha = 30°$ to the horizontal. The free end of the thread is attached to the wall as shown in Fig. 1.61. The mass of the spool is $m = 200$ g, its moment of inertia relative to its own axis $I = 0.45\ \text{g}\cdot\text{m}^2$, the radius of the wound thread layer $r = 3.0$ cm. Find the acceleration of the spool axis.

1.256. A uniform solid cylinder of mass m rests on two horizontal planks. A thread is wound on the cylinder. The hanging end of the thread is pulled vertically down with a constant force F (Fig. 1.62).

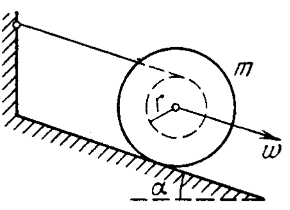

Fig. 1.61.

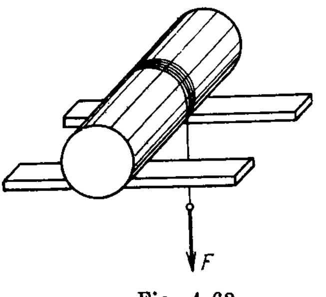

Fig. 1.62.

Find the maximum magnitude of the force F which still does not bring about any sliding of the cylinder, if the coefficient of friction between the cylinder and the planks is equal to k. What is the acceleration w_{max} of the axis of the cylinder rolling down the inclined plane?

1.257. A spool with thread wound on it, of mass m, rests on a rough horizontal surface. Its moment of inertia relative to its own axis is equal to $I = \gamma m R^2$, where γ is a numerical factor, and R is the outside radius of the spool. The radius of the wound thread layer is equal

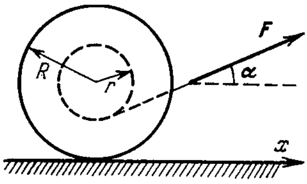

Fig. 1.63.

to r. The spool is pulled without sliding by the thread with a constant force F directed at an angle α to the horizontal (Fig. 1.63). Find:

(a) the projection of the acceleration vector of the spool axis on the x-axis;

(b) the work performed by the force **F** during the first t seconds after the beginning of motion.

1.258. The arrangement shown in Fig. 1.64 consists of two identical uniform solid cylinders, each of mass m, on which two light threads

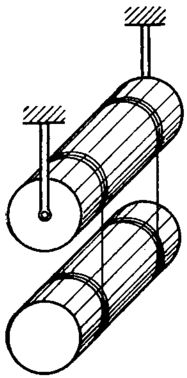

Fig. 1.64.

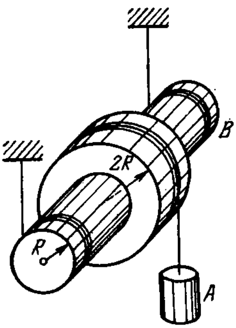

Fig. 1.65.

are wound symmetrically. Find the tension of each thread in the process of motion. The friction in the axle of the upper cylinder is assumed to be absent.

1.259. In the arrangement shown in Fig. 1.65 a weight A possesses mass m, a pulley B possesses mass M. Also known are the moment of inertia I of the pulley relative to its axis and the radii of the pulley

R and $2R$. The mass of the threads is negligible. Find the acceleration of the weight A after the system is set free.

1.260. A uniform solid cylinder A of mass m_1 can freely rotate about a horizontal axis fixed to a mount B of mass m_2 (Fig. 1.66). A constant horizontal force F is applied to the end K of a light thread tightly wound on the cylinder. The friction between the mount and the supporting horizontal plane is assumed to be absent. Find:

(a) the acceleration of the point K;

(b) the kinetic energy of this system t seconds after the beginning of motion.

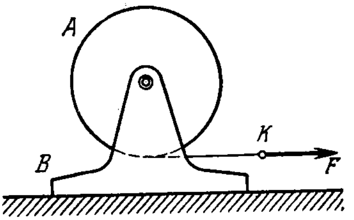

Fig. 1.66.

1.261. A plank of mass m_1 with a uniform sphere of mass m_2 placed on it rests on a smooth horizontal plane. A constant horizontal force F is applied to the plank. With what accelerations will the plank and the centre of the sphere move provided there is no sliding between the plank and the sphere?

1.262. A uniform solid cylinder of mass m and radius R is set in rotation about its axis with an angular velocity ω_0, then lowered with its lateral surface onto a horizontal plane and released. The coefficient of friction between the cylinder and the plane is equal to k. Find:

(a) how long the cylinder will move with sliding;

(b) the total work performed by the sliding friction force acting on the cylinder.

1.263. A uniform ball of radius r rolls without slipping down from the top of a sphere of radius R. Find the angular velocity of the ball at the moment it breaks off the sphere. The initial velocity of the ball is negligible.

1.264. A uniform solid cylinder of radius $R = 15$ cm rolls over a horizontal plane passing into an inclined plane forming an angle

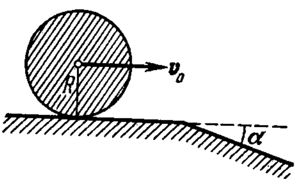

Fig. 1.67.

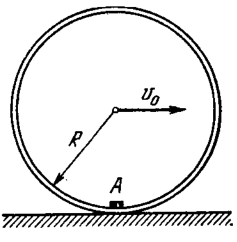

Fig. 1.68.

$\alpha = 30°$ with the horizontal (Fig. 1.67). Find the maximum value of the velocity v_0 which still permits the cylinder to roll onto the inclined plane section without a jump. The sliding is assumed to be absent.

1.265. A small body A is fixed to the inside of a thin rigid hoop of radius R and mass equal to that of the body A. The hoop rolls without slipping over a horizontal plane; at the moments when the body A gets into the lower position, the centre of the hoop moves with velocity

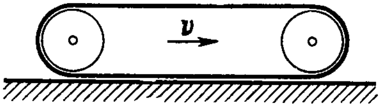

Fig. 1.69.

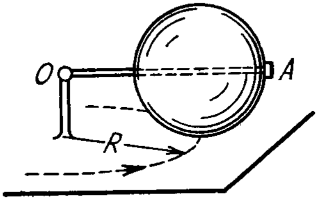

Fig. 1.70.

v_0 (Fig. 1.68). At what values of v_0 will the hoop move without bouncing?

1.266. Determine the kinetic energy of a tractor crawler belt of mass m if the tractor moves with velocity v (Fig. 1.69).

1.267. A uniform sphere of mass m and radius r rolls without sliding over a horizontal plane, rotating about a horizontal axle OA (Fig. 1.70). In the process, the centre of the sphere moves with velocity v along a circle of radius R. Find the kinetic energy of the sphere.

1.268. Demonstrate that in the reference frame rotating with a constant angular velocity ω about a stationary axis a body of mass m experiences the resultant

(a) centrifugal force of inertia $\mathbf{F}_{cf} = m\omega^2\mathbf{R}_C$, where $\mathbf{R}_C$ is the radius vector of the body's centre of inertia relative to the rotation axis;

(b) Coriolis force $\mathbf{F}_{cor} = 2m[\mathbf{v}'_C\boldsymbol{\omega}]$, where $\mathbf{v}'_C$ is the velocity of the body's centre of inertia in the rotating reference frame.

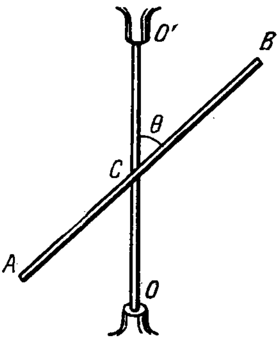

Fig. 1.71.

1.269. A midpoint of a thin uniform rod AB of mass m and length l is rigidly fixed to a rotation axle OO' as shown in Fig. 1.71. The rod is set into rotation with a constant angular velocity ω. Find the resultant moment of the centrifugal forces of inertia relative to the point C in the reference frame fixed to the axle OO' and to the rod.

1.270. A conical pendulum, a thin uniform rod of length l and mass m, rotates uniformly about a vertical axis with angular velocity ω (the upper end of the rod is hinged). Find the angle θ between the rod and the vertical.

1.271. A uniform cube with edge a rests on a horizontal plane whose friction coefficient equals k. The cube is set in motion with an initial velocity, travels some distance over the plane and comes to a stand-still. Explain the disappearance of the angular momentum of the cube relative to the axis lying in the plane at right angles to the cube's motion direction. Find the distance between the resultants of gravitational forces and the reaction forces exerted by the supporting plane.

1.272. A smooth uniform rod AB of mass M and length l rotates freely with an angular velocity ω_0 in a horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m starts sliding along the rod from the point A. Find the velocity v' of the sleeve relative to the rod at the moment it reaches its other end B.

1.273. A uniform rod of mass $m = 5.0$ kg and length $l = 90$ cm rests on a smooth horizontal surface. One of the ends of the rod is struck with the impulse $J = 3.0$ N·s in a horizontal direction perpendicular to the rod. As a result, the rod obtains the momentum $p = 3.0$ N·s. Find the force with which one half of the rod will act on the other in the process of motion.

1.274. A thin uniform square plate with side l and mass M can rotate freely about a stationary vertical axis coinciding with one of its sides. A small ball of mass m flying with velocity $\mathbf{v}$ at right angles to the plate strikes elastically the centre of it. Find:

(a) the velocity of the ball $\mathbf{v}'$ after the impact;

(b) the horizontal component of the resultant force which the axis will exert on the plate after the impact.

1.275. A vertically oriented uniform rod of mass M and length l can rotate about its upper end. A horizontally flying bullet of mass m strikes the lower end of the rod and gets stuck in it; as a result, the rod swings through an angle α. Assuming that $m \ll M$, find:

(a) the velocity of the flying bullet;

(b) the momentum increment in the system "bullet-rod" during the impact; what causes the change of that momentum;

(c) at what distance x from the upper end of the rod the bullet must strike for the momentum of the system "bullet-rod" to remain constant during the impact.

1.276. A horizontally oriented uniform disc of mass M and radius R rotates freely about a stationary vertical axis passing through its centre. The disc has a radial guide along which can slide without friction a small body of mass m. A light thread running down through the hollow axle of the disc is tied to the body. Initially the body was located at the edge of the disc and the whole system rotated with an angular velocity ω_0. Then by means of a force F applied to the lower end of the thread the body was slowly pulled to the rotation axis. Find:

(a) the angular velocity of the system in its final state;

(b) the work performed by the force F.

1.277. A man of mass m_1 stands on the edge of a horizontal uniform disc of mass m_2 and radius R which is capable of rotating freely about a stationary vertical axis passing through its centre. At a certain moment the man starts moving along the edge of the disc; he shifts over an angle φ' relative to the disc and then stops. In the process of motion the velocity of the man varies with time as $v'(t)$. Assuming the dimensions of the man to be negligible, find:

(a) the angle through which the disc had turned by the moment the man stopped;

(b) the force moment (relative to the rotation axis) with which the man acted on the disc in the process of motion.

1.278. Two horizontal discs rotate freely about a vertical axis passing through their centres. The moments of inertia of the discs relative to this axis are equal to I_1 and I_2, and the angular velocities to $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$. When the upper disc fell on the lower one, both discs began rotating, after some time, as a single whole (due to friction). Find:

(a) the steady-state angular rotation velocity of the discs;

(b) the work performed by the friction forces in this process.

1.279. A small disc and a thin uniform rod of length l, whose mass is η times greater than the mass of the disc, lie on a smooth horizontal plane. The disc is set in motion, in horizontal direction and perpendicular to the rod, with velocity $\mathbf{v}$, after which it elastically collides with the end of the rod. Find the velocity of the disc and the angular velocity of the rod after the collision. At what value of η will the velocity of the disc after the collision be equal to zero? reverse its direction?

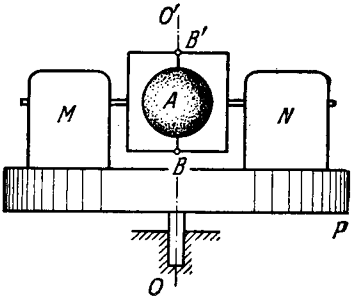

Fig. 1.72.

1.280. A stationary platform P which can rotate freely about a vertical axis (Fig. 1.72) supports a motor M and a balance weight N. The moment of inertia of the platform with the motor and the balance weight relative to this axis is equal to I. A light frame is fixed to the motor's shaft with a uniform sphere A rotating freely with an angular velocity ω_0 about a shaft BB' coinciding with the axis OO'. The moment of inertia of the sphere relative to the rotation axis is equal to I_0. Find:

(a) the work performed by the motor in turning the shaft BB' through 90°; through 180°;

(b) the moment of external forces which maintains the axis of the arrangement in the vertical position after the motor turns the shaft BB' through 90°.

1.281. A horizontally oriented uniform rod AB of mass $m = 1.40$ kg and length $l_0 = 100$ cm rotates freely about a stationary vertical axis OO' passing through its end A. The point A is located at the middle of the axis OO' whose length is equal to $l = 55$ cm. At what angular velocity of the rod the horizontal component of the force acting on the lower end of the axis OO' is equal to zero? What is in this case the horizontal component of the force acting on the upper end of the axis?

1.282. The middle of a uniform rod of mass m and length l is rigidly fixed to a vertical axis OO' so that the angle between the rod and the axis is equal to θ (see Fig. 1.71). The ends of the axis OO' are provided with bearings. The system rotates without friction with an angular velocity ω. Find:

(a) the magnitude and direction of the rod's angular momentum $\mathbf{M}$ relative to the point C, as well as its angular momentum relative to the rotation axis;

(b) how much the modulus of the vector $\mathbf{M}$ relative to the point C increases during a half-turn;

(c) the moment of external forces N acting on the axle OO' in the process of rotation.

1.283. A top of mass $m = 0.50$ kg, whose axis is tilted by an angle $\theta = 30°$ to the vertical, precesses due to gravity. The moment of inertia of the top relative to its symmetry axis is equal to $I = 2.0\ \text{g}\cdot\text{m}^2$, the angular velocity of rotation about that axis is equal to $\omega = 350$ rad/s, the distance from the point of rest to the centre of inertia of the top is $l = 10$ cm. Find:

(a) the angular velocity of the top's precession;

(b) the magnitude and direction of the horizontal component of the reaction force acting on the top at the point of rest.

1.284. A gyroscope, a uniform disc of radius $R = 5.0$ cm at the end of a rod of length $l = 10$ cm (Fig. 1.73), is mounted on the floor of an elevator car going up with a constant acceleration $w = 2.0\ \text{m/s}^2$. The other end of the rod is hinged at the point O. The gyroscope precesses with an angular velocity $n = 0.5$ rps. Neglecting the friction and the mass of the rod, find the proper angular velocity of the disc.

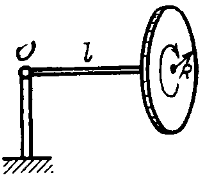

Fig. 1.73.

1.285. A top of mass $m = 1.0$ kg and moment of inertia relative to its own axis $I = 4.0\ \text{g}\cdot\text{m}^2$ spins with an angular velocity $\omega = 310$ rad/s. Its point of rest is located on a block which is shifted in a horizontal direction with a constant acceleration $w = 1.0\ \text{m/s}^2$. The distance between the point of rest and the centre of inertia of the top equals $l = 10$ cm. Find the magnitude and direction of the angular velocity of precession $\boldsymbol{\omega}'$.

1.286. A uniform sphere of mass $m = 5.0$ kg and radius $R = 6.0$ cm rotates with an angular velocity $\omega = 1250$ rad/s about a horizontal axle passing through its centre and fixed on the mounting base by means of bearings. The distance between the bearings equals $l = 15$ cm. The base is set in rotation about a vertical axis with an angular velocity $\omega' = 5.0$ rad/s. Find the modulus and direction of the gyroscopic forces.

1.287. A cylindrical disc of a gyroscope of mass $m = 15$ kg and radius $r = 5.0$ cm spins with an angular velocity $\omega = 330$ rad/s.

The distance between the bearings in which the axle of the disc is mounted is equal to $l = 15$ cm. The axle is forced to oscillate about a horizontal axis with a period $T = 1.0$ s and amplitude $\varphi_m = 20°$. Find the maximum value of the gyroscopic forces exerted by the axle on the bearings.

1.288. A ship moves with velocity $v = 36$ km per hour along an arc of a circle of radius $R = 200$ m. Find the moment of the gyroscopic forces exerted on the bearings by the shaft with a flywheel whose moment of inertia relative to the rotation axis equals $I = 3.8 \cdot 10^3$ kg·m² and whose rotation velocity $n = 300$ rpm. The rotation axis is oriented along the length of the ship.

1.289. A locomotive is propelled by a turbine whose axle is parallel to the axes of wheels. The turbine's rotation direction coincides with that of wheels. The moment of inertia of the turbine rotor relative to its own axis is equal to $I = 240$ kg·m². Find the additional force exerted by the gyroscopic forces on the rails when the locomotive moves along a circle of radius $R = 250$ m with velocity $v = 50$ km per hour. The gauge is equal to $l = 1.5$ m. The angular velocity of the turbine equals $n = 1500$ rpm.

1.6. ELASTIC DEFORMATIONS OF A SOLID BODY

• Relation between tensile (compressive) strain ε and stress σ:

$$\varepsilon = \sigma/E, \tag{1.6a}$$

where E is Young's modulus.

• Relation between lateral compressive (tensile) strain ε' and longitudinal tensile (compressive) strain ε:

$$\varepsilon' = -\mu\varepsilon, \tag{1.6b}$$

where μ is Poisson's ratio.

• Relation between shear strain γ and tangential stress τ:

$$\gamma = \tau/G, \tag{1.6c}$$

where G is shear modulus.

• Compressibility:

$$\beta = -\frac{1}{V}\frac{dV}{dp}. \tag{1.6d}$$

• Volume density of elastic strain energy:

$$u = E\varepsilon^2/2, \quad u = G\gamma^2/2. \tag{1.6e}$$

1.290. What pressure has to be applied to the ends of a steel cylinder to keep its length constant on raising its temperature by 100 °C?

1.291. What internal pressure (in the absence of an external pressure) can be sustained

(a) by a glass tube; (b) by a glass spherical flask, if in both cases the wall thickness is equal to $\Delta r = 1.0$ mm and the radius of the tube and the flask equals $r = 25$ mm?

1.292. A horizontally oriented copper rod of length $l = 1.0$ m is rotated about a vertical axis passing through its middle. What is the number of rps at which this rod ruptures?

1.293. A ring of radius $r = 25$ cm made of lead wire is rotated about a stationary vertical axis passing through its centre and perpendicular to the plane of the ring. What is the number of rps at which the ring ruptures?

1.294. A steel wire of diameter $d = 1.0$ mm is stretched horizontally between two clamps located at the distance $l = 2.0$ m from each other. A weight of mass $m = 0.25$ kg is suspended from the midpoint O of the wire. What will the resulting descent of the point O be in centimetres?

1.295. A uniform elastic plank moves over a smooth horizontal plane due to a constant force F_0 distributed uniformly over the end face. The surface of the end face is equal to S, and Young's modulus of the material to E. Find the compressive strain of the plank in the direction of the acting force.

1.296. A thin uniform copper rod of length l and mass m rotates uniformly with an angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends. Determine the tension in the rod as a function of the distance r from the rotation axis. Find the elongation of the rod.

1.297. A solid copper cylinder of length $l = 65$ cm is placed on a horizontal surface and subjected to a vertical compressive force $F = 1000$ N directed downward and distributed uniformly over the end face. What will be the resulting change of the volume of the cylinder in cubic millimetres?

1.298. A copper rod of length l is suspended from the ceiling by one of its ends. Find:

(a) the elongation Δl of the rod due to its own weight;

(b) the relative increment of its volume $\Delta V/V$.

1.299. A bar made of material whose Young's modulus is equal to E and Poisson's ratio to μ is subjected to the hydrostatic pressure p. Find:

(a) the fractional decrement of its volume;

(b) the relationship between the compressibility β and the elastic constants E and μ.

Show that Poisson's ratio μ cannot exceed 1/2.

1.300. One end of a steel rectangular girder is embedded into a wall (Fig. 1.74). Due to gravity it sags slightly. Find the radius of curvature of the neutral layer (see the dotted line in the figure) in the vicinity of the point O if the length of the protruding section of

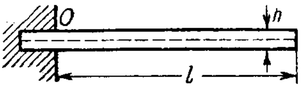

Fig. 1.74.

the girder is equal to $l = 6.0$ m and the thickness of the girder equals $h = 10$ cm.

1.301. The bending of an elastic rod is described by the *elastic curve* passing through centres of gravity of rod's cross-sections. At small bendings the equation of this curve takes the form

$$N(x) = EI\frac{d^2y}{dx^2},$$

where $N(x)$ is the bending moment of the elastic forces in the cross-section corresponding to the x coordinate, E is Young's modulus, I is the *moment of inertia* of the cross-section relative to the axis passing through the neutral layer ($I = \int z^2 dS$, Fig. 1.75).

Suppose one end of a steel rod of a square cross-section with side a is embedded into a wall, the protruding section being of length l

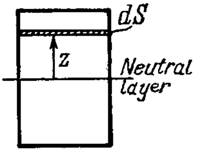

Fig. 1.75.

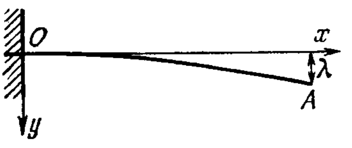

Fig. 1.76.

(Fig. 1.76). Assuming the mass of the rod to be negligible, find the shape of the elastic curve and the deflection of the rod λ, if its end A experiences

(a) the bending moment of the couple N_0;
(b) a force F oriented along the y axis.

1.302. A steel girder of length l rests freely on two supports (Fig. 1.77). The moment of inertia of its cross-section is equal to I (see the foregoing problem). Neglecting the mass of the girder and assuming the sagging to be slight, find the deflection λ due to the force F applied to the middle of the girder.

1.303. The thickness of a rectangular steel girder equals h. Using the equation of Problem 1.301, find the deflection λ caused by the weight of the girder in two cases:

(a) one end of the girder is embedded into a wall with the length of the protruding section being equal to l (Fig. 1.78*a*);
(b) the girder of length $2l$ rests freely on two supports (Fig. 1.78*b*).

1.304. A steel plate of thickness h has the shape of a square whose side equals l, with $h \ll l$. The plate is rigidly fixed to a vertical axle

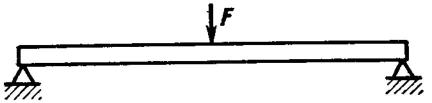

Fig. 1.77.

OO which is rotated with a constant angular acceleration β (Fig. 1.79). Find the deflection λ, assuming the sagging to be small.

1.305. Determine the relationship between the torque N and the torsion angle φ for

(a) the tube whose wall thickness Δr is considerably less than the tube radius;

(b) for the solid rod of circular cross-section. Their length l, radius r, and shear modulus G are supposed to be known.

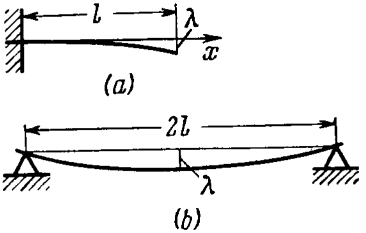

Fig. 1.78.

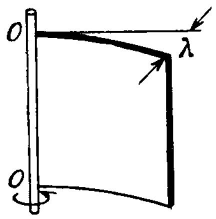

Fig. 1.79.

1.306. Calculate the torque N twisting a steel tube of length $l = 3.0$ m through an angle $\varphi = 2.0°$ about its axis, if the inside and outside diameters of the tube are equal to $d_1 = 30$ mm and $d_2 = 50$ mm.

1.307. Find the maximum power which can be transmitted by means of a steel shaft rotating about its axis with an angular velocity $\omega = 120$ rad/s, if its length $l = 200$ cm, radius $r = 1.50$ cm, and the permissible torsion angle $\varphi = 2.5°$.

1.308. A uniform ring of mass m, with the outside radius r_2, is fitted tightly on a shaft of radius r_1. The shaft is rotated about its axis with a constant angular acceleration β. Find the moment of elastic forces in the ring as a function of the distance r from the rotation axis.

1.309. Find the elastic deformation energy of a steel rod of mass $m = 3.1$ kg stretched to a tensile strain $\varepsilon = 1.0 \cdot 10^{-3}$.

1.310. A steel cylindrical rod of length l and radius r is suspended by its end from the ceiling.

(a) Find the elastic deformation energy U of the rod.

(b) Define U in terms of tensile strain $\Delta l/l$ of the rod.

1.311. What work has to be performed to make a hoop out of a steel band of length $l = 2.0$ m, width $h = 6.0$ cm, and thickness $\delta = 2.0$ mm? The process is assumed to proceed within the elasticity range of the material.

1.312. Find the elastic deformation energy of a steel rod whose one end is fixed and the other is twisted through an angle $\varphi = 6.0°$. The length of the rod is equal to $l = 1.0$ m, and the radius to $r = 10$ mm.

1.313. Find how the volume density of the elastic deformation energy is distributed in a steel rod depending on the distance r from its axis. The length of the rod is equal to l, the torsion angle to φ.

1.314. Find the volume density of the elastic deformation energy in fresh water at the depth of $h = 1000$ m.

1.7. HYDRODYNAMICS

• The fundamental equation of hydrodynamics of ideal fluid (Eulerian equation):

$$\rho \frac{d\mathbf{v}}{dt} = \mathbf{f} - \nabla p, \tag{1.7a}$$

where ρ is the fluid density, $\mathbf{f}$ is the volume density of mass forces ($\mathbf{f} = \rho\mathbf{g}$ in the case of gravity), ∇p is the pressure gradient.

• Bernoulli's equation. In the steady flow of an ideal fluid

$$\frac{\rho v^2}{2} + \rho g h + p = \text{const} \tag{1.7b}$$

along any streamline.

• Reynolds number defining the flow pattern of a viscous fluid:

$$\text{Re} = \rho v l/\eta, \tag{1.7c}$$

where l is a characteristic length, η is the fluid viscosity.

• Poiseuille's law. The volume of liquid flowing through a circular tube (in m³/s):

$$Q = \frac{\pi R^4}{8\eta} \frac{p_1 - p_2}{l}, \tag{1.7d}$$

where R and l are the tube's radius and length, $p_1 - p_2$ is the pressure difference between the ends of the tube.

• Stokes' law. The friction force on the sphere of radius r moving through a viscous fluid:

$$F = 6\pi\eta r v. \tag{1.7e}$$

1.315. Ideal fluid flows along a flat tube of constant cross-section, located in a horizontal plane and bent as shown in Fig. 1.80 (top view). The flow is steady. Are the pressures and velocities of the fluid equal at points *1* and *2*? What is the shape of the streamlines?

1.316. Two manometric tubes are mounted on a horizontal pipe of varying cross-section at the sections S_1 and S_2 (Fig. 1.81). Find the volume of water flowing across the pipe's section per unit time if the difference in water columns is equal to Δh.

1.317. A Pitot tube (Fig. 1.82) is mounted along the axis of a gas pipeline whose cross-sectional area is equal to S. Assuming the viscosity to be negligible, find the volume of gas flowing across the

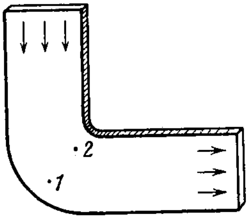

Fig. 1.80.

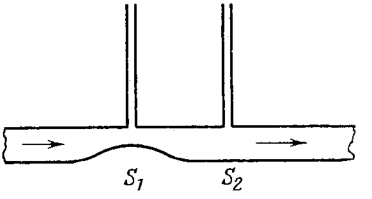

Fig. 1.81.

section of the pipe per unit time, if the difference in the liquid columns is equal to Δh, and the densities of the liquid and the gas are ρ_0 and ρ respectively.

1.318. A wide vessel with a small hole in the bottom is filled with water and kerosene. Neglecting the viscosity, find the velocity of the water flow, if the thickness of the water layer is equal to $h_1 = 30$ cm and that of the kerosene layer to $h_2 = 20$ cm.

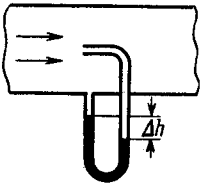

Fig. 1.82.

1.319. A wide cylindrical vessel 50 cm in height is filled with water and rests on a table. Assuming the viscosity to be negligible, find at what height from the bottom of the vessel a small hole should be perforated for the water jet coming out of it to hit the surface of the table at the maximum distance l_{max} from the vessel. Find l_{max}.

1.320. A bent tube is lowered into a water stream as shown in Fig. 1.83. The velocity of the stream relative to the tube is equal to $v = 2.5$ m/s. The closed upper end of the tube located at the height $h_0 = 12$ cm has a small orifice. To what height h will the water jet spurt?

1.321. The horizontal bottom of a wide vessel with an ideal fluid has a round orifice of radius R_1 over which a round closed cylinder is mounted, whose radius $R_2 > R_1$ (Fig. 1.84). The clearance between the cylinder and the bottom of the vessel is very small, the fluid density is ρ. Find the static pressure of the fluid in the clearance as a function of the distance r from the axis of the orifice (and the cylinder), if the height of the fluid is equal to h.

1.322. What work should be done in order to squeeze all water from a horizontally located cylinder (Fig. 1.85) during the time t by means of a constant force acting on the piston? The volume of water in the cylinder is equal to V, the cross-sectional area of the orifice to s, with s being considerably less than the piston area. The friction and viscosity are negligibly small.

1.323. A cylindrical vessel of height h and base area S is filled with water. An orifice of area $s \ll S$ is opened in the bottom of the vessel. Neglecting the viscosity of water, determine how soon all the water will pour out of the vessel.

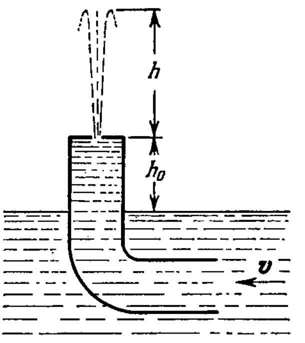

Fig. 1.83.

1.324. A horizontally oriented tube AB of length l rotates with a constant angular velocity ω about a stationary vertical axis OO' passing through the end A (Fig. 1.86). The tube is filled with an ideal fluid. The end A of the tube is open, the closed end B has a very small orifice. Find the velocity of the fluid relative to the tube as a function of the column "height" h.

1.325. Demonstrate that in the case of a steady flow of an ideal fluid Eq. (1.7a) turns into Bernoulli equation.

1.326. On the opposite sides of a wide vertical vessel filled with water two identical holes are opened, each having the cross-sectional

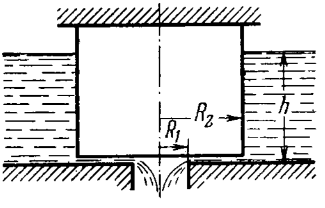

Fig. 1.84.

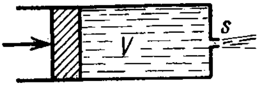

Fig. 1.85.

area $S = 0.50$ cm^2. The height difference between them is equal to $\Delta h = 51$ cm. Find the resultant force of reaction of the water flowing out of the vessel.

1.327. The side wall of a wide vertical cylindrical vessel of height $h = 75$ cm has a narrow vertical slit running all the way down to the bottom of the vessel. The length of the slit is $l = 50$ cm and the width $b = 1.0$ mm. With the slit closed, the vessel is filled with water. Find the resultant force of reaction of the water flowing out of the vessel immediately after the slit is opened.

1.328. Water flows out of a big tank along a tube bent at right angles; the inside radius of the tube is equal to $r = 0.50$ cm (Fig. 1.87). The length of the horizontal section of the tube is equal to $l = 22$ cm. The water flow rate is $Q = 0.50$ litres per second. Find the moment of reaction forces of flowing water, acting on the tube's walls, relative to the point O.

1.329. A side wall of a wide open tank is provided with a narrowing tube (Fig. 1.88) through which water flows out. The cross-sectional area of the tube decreases from $S = 3.0$ cm^2 to $s = 1.0$ cm^2. The water level in the tank is $h = 4.6$ m higher than that in the tube.

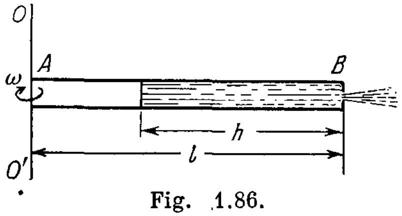

Fig. 1.86.

Neglecting the viscosity of the water, find the horizontal component of the force tending to pull the tube out of the tank.

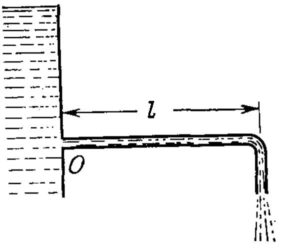

Fig. 1.87.

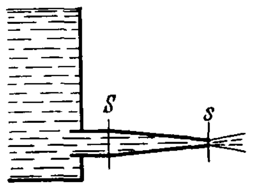

Fig. 1.88.

1.330. A cylindrical vessel with water is rotated about its vertical axis with a constant angular velocity ω. Find:

(a) the shape of the free surface of the water;

(b) the water pressure distribution over the bottom of the vessel along its radius provided the pressure at the central point is equal to p_0.

1.331. A thin horizontal disc of radius $R = 10$ cm is located within a cylindrical cavity filled with oil whose viscosity $\eta = 0.08$ P (Fig. 1.89). The clearance between the disc and the horizontal planes

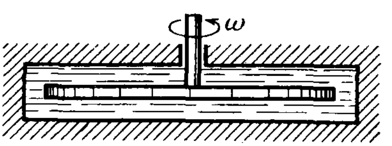

Fig. 1.89.

of the cavity is equal to $h = 1.0$ mm. Find the power developed by the viscous forces acting on the disc when it rotates with the angular velocity $\omega = 60$ rad/s. The end effects are to be neglected.

1.332. A long cylinder of radius R_1 is displaced along its axis with a constant velocity v_0 inside a stationary co-axial cylinder of radius R_2. The space between the cylinders is filled with viscous liquid. Find the velocity of the liquid as a function of the distance r from the axis of the cylinders. The flow is laminar.

1.333. A fluid with viscosity η fills the space between two long co-axial cylinders of radii R_1 and R_2, with $R_1 < R_2$. The inner cylinder is stationary while the outer one is rotated with a constant angular velocity ω_2. The fluid flow is laminar. Taking into account that the friction force acting on a unit area of a cylindrical surface of radius r is defined by the formula $\sigma = \eta r\,(\partial\omega/\partial r)$, find:

(a) the angular velocity of the rotating fluid as a function of radius r;

(b) the moment of the friction forces acting on a unit length of the outer cylinder.

1.334. A tube of length l and radius R carries a steady flow of fluid whose density is ρ and viscosity η. The fluid flow velocity depends on the distance r from the axis of the tube as $v = v_0\,(1 - r^2/R^2)$. Find:

(a) the volume of the fluid flowing across the section of the tube per unit time;

(b) the kinetic energy of the fluid within the tube's volume;

(c) the friction force exerted on the tube by the fluid;

(d) the pressure difference at the ends of the tube.

1.335. In the arrangement shown in Fig. 1.90 a viscous liquid whose density is $\rho = 1.0$ g/cm^3 flows along a tube out of a wide tank

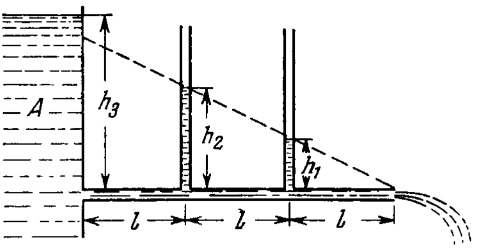

Fig. 1.90.

A. Find the velocity of the liquid flow, if $h_1 = 10$ cm, $h_2 = 20$ cm, and $h_3 = 35$ cm. All the distances l are equal.

1.336. The cross-sectional radius of a pipeline decreases gradually as $r = r_0 e^{-\alpha x}$, where $\alpha = 0.50$ m^{-1}, x is the distance from the pipeline inlet. Find the ratio of Reynolds numbers for two cross-sections separated by $\Delta x = 3.2$ m.

1.337. When a sphere of radius $r_1 = 1.2$ mm moves in glycerin, the laminar flow is observed if the velocity of the sphere does not exceed $v_1 = 23$ cm/s. At what minimum velocity v_2 of a sphere of radius $r_2 = 5.5$ cm will the flow in water become turbulent? The viscosities of glycerin and water are equal to $\eta_1 = 13.9$ P and $\eta_2 = 0.011$ P respectively.

1.338. A lead sphere is steadily sinking in glycerin whose viscosity is equal to $\eta = 13.9$ P. What is the maximum diameter of the sphere at which the flow around that sphere still remains laminar? It is known that the transition to the turbulent flow corresponds to Reynolds number Re $= 0.5$. (Here the characteristic length is taken to be the sphere diameter.)

1.339. A steel ball of diameter $d = 3.0$ mm starts sinking with zero initial velocity in olive oil whose viscosity is $\eta = 0.90$ P. How soon after the beginning of motion will the velocity of the ball differ from the steady-state velocity by $n = 1.0\%$?

1.8. RELATIVISTIC MECHANICS

- Lorentz contraction of length and slowing of a moving clock:

$$l = l_0\sqrt{1-(v/c)^2}, \qquad \Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}}, \tag{1.8a}$$

where l_0 is the proper length and Δt_0 is the proper time of the moving clock.

- Lorentz transformation*:

$$x' = \frac{x - Vt}{\sqrt{1-(V/c)^2}}, \qquad y' = y, \qquad t' = \frac{t - xV/c^2}{1-(V/c)^2}. \tag{1.8b}$$

- Interval s_{12} is an invariant:

$$s_{12}^2 = c^2t_{12}^2 - l_{12}^2 = \text{inv}, \tag{1.8c}$$

where t_{12} is the time interval between events 1 and 2, l_{12} is the distance between the points at which these events occurred.

- Transformation of velocity*:

$$v'_x = \frac{v_x - V}{1 - v_xV/c^2}, \qquad v'_y = \frac{v_y\sqrt{1-(V/c)^2}}{1 - v_xV/c^2}. \tag{1.8d}$$

- Relativistic mass and relativistic momentum:

$$m = \frac{m_0}{\sqrt{1-(v/c)^2}}, \qquad \mathbf{p} = m\mathbf{v} = \frac{m_0\mathbf{v}}{\sqrt{1-(v/c)^2}}, \tag{1.8e}$$

where m_0 is the rest mass, or, simply, the mass.

- Relativistic equation of dynamics for a particle:

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}, \tag{1.8f}$$

where $\mathbf{p}$ is the relativistic momentum of the particle.

- Total and kinetic energies of a relativistic particle:

$$E = mc^2 = m_0c^2 + T, \quad T = (m - m_0)c^2. \tag{1.8g}$$

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* The reference frame K' is assumed to move with a velocity V in the positive direction of the x axis of the frame K, with the x' and x axes coinciding and the y' and y axes parallel.

• Relationship between the energy and momentum of a relativistic particle

$$E^2-p^2c^2=m_0^2c^4, \qquad pc=\sqrt{T\,(T+2m_0c^2)}. \tag{1.8h}$$

• When considering the collisions of particles it helps to use the following invariant quantity:

$$E^2-p^2c^2=m_0^2c^4, \tag{1.8i}$$

where E and p are the total energy and momentum of the system prior to the collision, and m_0 is the rest mass of the particle (or the system) formed.

1.340. A rod moves lengthwise with a constant velocity v relative to the inertial reference frame K. At what value of v will the length of the rod in this frame be $\eta = 0.5\%$ less than its proper length?

1.341. In a triangle the proper length of each side equals a. Find the perimeter of this triangle in the reference frame moving relative to it with a constant velocity V along one of its

(a) bisectors; (b) sides.

Investigate the results obtained at $V \ll c$ and $V \to c$, where c is the velocity of light.

1.342. Find the proper length of a rod if in the laboratory frame of reference its velocity is $v = c/2$, the length $l = 1.00$ m, and the angle between the rod and its direction of motion is $\theta = 45°$.

1.343. A stationary upright cone has a taper angle $\theta = 45°$, and the area of the lateral surface $S_0 = 4.0$ m^2. Find: (a) its taper angle; (b) its lateral surface area, in the reference frame moving with a velocity $v = (4/5)c$ along the axis of the cone.

1.344. With what velocity (relative to the reference frame K) did the clock move, if during the time interval $t = 5.0$ s, measured by the clock of the frame K, it became slow by $\Delta t = 0.10$ s?

1.345. A rod flies with constant velocity past a mark which is stationary in the reference frame K. In the frame K it takes $\Delta t = 20$ ns for the rod to fly past the mark. In the reference frame fixed to the rod the mark moves past the rod for $\Delta t' = 25$ ns. Find the proper length of the rod.

1.346. The proper lifetime of an unstable particle is equal to $\Delta t_0 = 10$ ns. Find the distance this particle will traverse till its decay in the laboratory frame of reference, where its lifetime is equal to $\Delta t = 20$ ns.

1.347. In the reference frame K a muon moving with a velocity $v = 0.990c$ travelled a distance $l = 3.0$ km from its birthplace to the point where it decayed. Find:

(a) the proper lifetime of this muon;

(b) the distance travelled by the muon in the frame K "from the muon's standpoint".

1.348. Two particles moving in a laboratory frame of reference along the same straight line with the same velocity $v = (3/4)c$ strike against a stationary target with the time interval $\Delta t = 50$ ns. Find the proper distance between the particles prior to their hitting the target.

1.349. A rod moves along a ruler with a constant velocity. When the positions of both ends of the rod are marked simultaneously in the reference frame fixed to the ruler, the difference of readings on the ruler is equal to $\Delta x_1 = 4.0$ m. But when the positions of the rod's ends are marked simultaneously in the reference frame fixed to the rod, the difference of readings on the same ruler is equal to $\Delta x_2 = 9.0$ m. Find the proper length of the rod and its velocity relative to the ruler.

1.350. Two rods of the same proper length l_0 move toward each other parallel to a common horizontal axis. In the reference frame fixed to one of the rods the time interval between the moments, when the right and left ends of the rods coincide, is equal to Δt. What is the velocity of one rod relative to the other?

1.351. Two unstable particles move in the reference frame K along a straight line in the same direction with a velocity $v = 0.990c$. The distance between them in this reference frame is equal to $l = 120$ m. At a certain moment both particles decay simultaneously in the reference frame fixed to them. What time interval between the moments of decay of the two particles will be observed in the frame K? Which particle decays later in the frame K?

1.352. A rod AB oriented along the x axis of the reference frame K moves in the positive direction of the x axis with a constant velocity v. The point A is the forward end of the rod, and the point B its rear end. Find:

(a) the proper length of the rod, if at the moment t_A the coordinate of the point A is equal to x_A, and at the moment t_B the coordinate of the point B is equal to x_B;

(b) what time interval should separate the markings of coordinates of the rod's ends in the frame K for the difference of coordinates to become equal to the proper length of the rod.

1.353. The rod $A'B'$ moves with a constant velocity v relative to the rod AB (Fig. 1.91). Both rods have the same proper length l_0 and

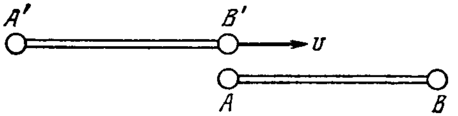

Fig. 1.91.

at the ends of each of them clocks are mounted, which are synchronized pairwise: A with B and A' with B'. Suppose the moment when the clock B' gets opposite the clock A is taken for the beginning of the time count in the reference frames fixed to each of the rods. Determine:

(a) the readings of the clocks B and B' at the moment when they are opposite each other;

(b) the same for the clocks A and A'.

1.354. There are two groups of mutually synchronized clocks K and K' moving relative to each other with a velocity v as shown in Fig. 1.92. The moment when the clock A' gets opposite the clock A

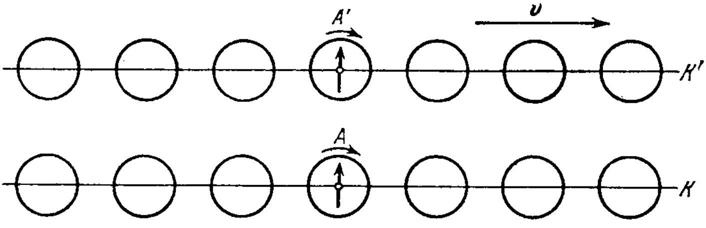

Fig. 1.92.

is taken for the beginning of the time count. Draw the approximate position of hands of all the clocks at this moment "in terms of the K clocks"; "in terms of the K' clocks".

1.355. The reference frame K' moves in the positive direction of the x axis of the frame K with a relative velocity V. Suppose that at the moment when the origins of coordinates O and O' coincide, the clock readings at these points are equal to zero in both frames. Find the displacement velocity $\dot{x}$ of the point (in the frame K) at which the readings of the clocks of both reference frames will be permanently identical. Demonstrate that $\dot{x} < V$.

1.356. At two points of the reference frame K two events occurred separated by a time interval Δt. Demonstrate that if these events obey the cause-and-effect relationship in the frame K (e.g. a shot fired and a bullet hitting a target), they obey that relationship in any other inertial reference frame K'.

1.357. The space-time diagram of Fig. 1.93 shows three events A, B, and C which occurred on the x axis of some inertial reference frame. Find:

(a) the time interval between the events A and B in the reference frame where the two events occurred at the same point;

(b) the distance between the points at which the events A and C occurred in the reference frame where these two events are simultaneous.

1.358. The velocity components of a particle moving in the xy plane of the reference frame K are equal to v_x and v_y. Find the velocity v' of this particle in the frame K' which moves with the velocity V relative to the frame K in the positive direction of its x axis.

1.359. Two particles move toward each other with velocities $v_1 = 0.50c$ and $v_2 = 0.75c$ relative to a laboratory frame of reference. Find:

(a) the approach velocity of the particles in the laboratory frame of reference;

(b) their relative velocity.

1.360. Two rods having the same proper length l_0 move lengthwise toward each other parallel to a common axis with the same velocity

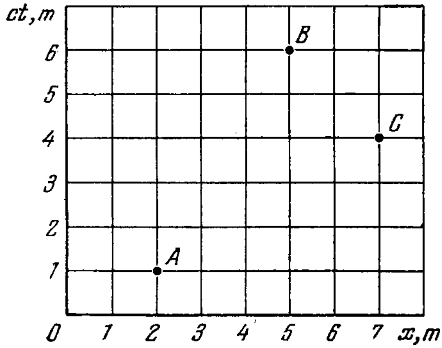

Fig. 1.93.

v relative to the laboratory frame of reference. What is the length of each rod in the reference frame fixed to the other rod?

1.361. Two relativistic particles move at right angles to each other in a laboratory frame of reference, one with the velocity v_1 and the other with the velocity v_2. Find their relative velocity.

1.362. An unstable particle moves in the reference frame K' along its y' axis with a velocity v'. In its turn, the frame K' moves relative to the frame K in the positive direction of its x axis with a velocity V. The x' and x axes of the two reference frames coincide, the y' and y axes are parallel. Find the distance which the particle traverses in the frame K, if its proper lifetime is equal to Δt_0.

1.363. A particle moves in the frame K with a velocity v at an angle θ to the x axis. Find the corresponding angle in the frame K' moving with a velocity V relative to the frame K in the positive direction of its x axis, if the x and x' axes of the two frames coincide.

1.364. The rod AB oriented parallel to the x' axis of the reference frame K' moves in this frame with a velocity v' along its y' axis. In its turn, the frame K' moves with a velocity V relative to the frame K as shown in Fig. 1.94. Find the angle θ between the rod and the x axis in the frame K.

1.365. The frame K' moves with a constant velocity $\mathbf{V}$ relative to the frame K. Find the acceleration w' of a particle in the frame K', if in the frame K this particle moves with a velocity v and acceleration w along a straight line

(a) in the direction of the vector $\mathbf{V}$;

(b) perpendicular to the vector $\mathbf{V}$.

1.366. An imaginary space rocket launched from the Earth moves with an acceleration $w' = 10g$ which is the same in every instantaneous co-moving inertial reference frame. The boost stage lasted

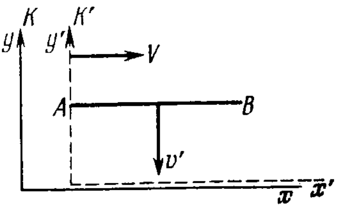

Fig. 1.94.

$\tau = 1.0$ year of terrestrial time. Find how much (in per cent) does the rocket velocity differ from the velocity of light at the end of the boost stage. What distance does the rocket cover by that moment?

1.367. From the conditions of the foregoing problem determine the boost time τ_0 in the reference frame fixed to the rocket. Remember that this time is defined by the formula

$$\tau_0 = \int_0^{\tau} \sqrt{1-(v/c)^2}\, dt,$$

where dt is the time in the geocentric reference frame.

1.368. How many times does the relativistic mass of a particle whose velocity differs from the velocity of light by 0.010% exceed its rest mass?

1.369. The density of a stationary body is equal to ρ_0. Find the velocity (relative to the body) of the reference frame in which the density of the body is $\eta = 25\%$ greater than ρ_0.

1.370. A proton moves with a momentum $p = 10.0$ GeV/c, where c is the velocity of light. How much (in per cent) does the proton velocity differ from the velocity of light?

1.371. Find the velocity at which the relativistic momentum of a particle exceeds its Newtonian momentum $\eta = 2$ times.

1.372. What work has to be performed in order to increase the velocity of a particle of rest mass m_0 from 0.60 c to 0.80 c? Compare the result obtained with the value calculated from the classical formula.

1.373. Find the velocity at which the kinetic energy of a particle equals its rest energy.

1.374. At what values of the ratio of the kinetic energy to rest energy can the velocity of a particle be calculated from the classical formula with the relative error less than $\varepsilon = 0.010$?

1.375. Find how the momentum of a particle of rest mass m_0 depends on its kinetic energy. Calculate the momentum of a proton whose kinetic energy equals 500 MeV.

1.376. A beam of relativistic particles with kinetic energy T strikes against an absorbing target. The beam current equals I, the charge and rest mass of each particle are equal to e and m_0 respectively. Find the pressure developed by the beam on the target surface, and the power liberated there.

1.377. A sphere moves with a relativistic velocity v through a gas whose unit volume contains n slowly moving particles, each of mass m. Find the pressure p exerted by the gas on a spherical surface element perpendicular to the velocity of the sphere, provided that the particles scatter elastically. Show that the pressure is the same both in the reference frame fixed to the sphere and in the reference frame fixed to the gas.

1.378. A particle of rest mass m_0 starts moving at a moment $t = 0$ due to a constant force $\mathbf{F}$. Find the time dependence of the particle's velocity and of the distance covered.

1.379. A particle of rest mass m_0 moves along the x axis of the frame K in accordance with the law $x = \sqrt{a^2 + c^2t^2}$, where a is a constant, c is the velocity of light, and t is time. Find the force acting on the particle in this reference frame.

1.380. Proceeding from the fundamental equation of relativistic dynamics, find:

(a) under what circumstances the acceleration of a particle coincides in direction with the force $\mathbf{F}$ acting on it;

(b) the proportionality factors relating the force $\mathbf{F}$ and the acceleration $\mathbf{w}$ in the cases when $\mathbf{F} \perp \mathbf{v}$ and $\mathbf{F} \parallel \mathbf{v}$, where $\mathbf{v}$ is the velocity of the particle.

1.381. A relativistic particle with momentum p and total energy E moves along the x axis of the frame K. Demonstrate that in the frame K' moving with a constant velocity V relative to the frame K in the positive direction of its axis x the momentum and the total energy of the given particle are defined by the formulas:

$$p'_x = \frac{p_x - EV/c^2}{\sqrt{1-\beta^2}}, \qquad E' = \frac{E - p_xV}{\sqrt{1-\beta^2}}$$

where $\beta = V/c$.

1.382. The photon energy in the frame K is equal to ε. Making use of the transformation formulas cited in the foregoing problem, find the energy ε' of this photon in the frame K' moving with a velocity V relative to the frame K in the photon's motion direction. At what value of V is the energy of the photon equal to $\varepsilon' = \varepsilon/2$?

1.383. Demonstrate that the quantity $E^2 - p^2c^2$ for a particle is an invariant, i.e. it has the same magnitude in all inertial reference frames. What is the magnitude of this invariant?

1.384. A neutron with kinetic energy $T = 2m_0c^2$, where m_0 is its rest mass, strikes another, stationary, neutron. Determine:

(a) the combined kinetic energy $\widetilde{T}$ of both neutrons in the frame of their centre of inertia and the momentum $\widetilde{p}$ of each neutron in that frame;

(b) the velocity of the centre of inertia of this system of particles.

Instruction. Make use of the invariant $E^2 - p^2c^2$ remaining constant on transition from one inertial reference frame to another (E is the total energy of the system, p is its composite momentum).

1.385. A particle of rest mass m_0 with kinetic energy T strikes a stationary particle of the same rest mass. Find the rest mass and the velocity of the compound particle formed as a result of the collision.

1.386. How high must be the kinetic energy of a proton striking another, stationary, proton for their combined kinetic energy in the frame of the centre of inertia to be equal to the total kinetic energy of two protons moving toward each other with individual kinetic energies $T = 25.0$ GeV?

1.387. A stationary particle of rest mass m_0 disintegrates into three particles with rest masses m_1, m_2, and m_3. Find the maximum total energy that, for example, the particle m_1 may possess.

1.388. A relativistic rocket emits a gas jet with non-relativistic velocity $\mathbf{u}$ constant relative to the rocket. Find how the velocity v of the rocket depends on its rest mass m if the initial rest mass of the rocket equals m_0.