# Electromagnetism from relativistic fluid dynamics

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ABSTRACT: We present a matter-space framework characterizing particles and establish its compatibility with electromagnetism. In this approach, matter, such as photons, is considered to reside in a three-dimensional matter space, with the electromagnetic fields observed in four-dimensional spacetime interpreted as projections from this space. By imposing gauge symmetry through constraint equations, we derive the relationship between the vector field  $A_a$  and the antisymmetric tensor  $F_{ab}$ , forming part of Maxwell's equations. The remaining Maxwell equation is obtained through the action principle in relativistic fluid dynamics. Notably, we demonstrate that this imposition of the gauge symmetry and constraints develop the dynamics. This framework offers a fresh perspective on particle-field interactions and deepens the theoretical foundation of relativistic fluid dynamics.

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## 1 Introduction

Astronomical magnetic fields play crucial roles in the formation and evolution of galaxies, stars, and planets. Their origin and amplification are generally explained by the dynamo mechanism (for a comprehensive review, see [1]), providing a theoretical framework for understanding the galactic magnetic fields observed today [2]. Magnetohydrodynamics (MHD) uses the electromagnetism and fluid dynamics to describe the interaction between electromagnetic fields and conductive fluids [3]. Despite its success in providing phenomenological explanations, MHD faces significant challenges. Observational evidence for the initial seed magnetic fields remains elusive, and the theoretical mechanisms proposed for their generation are not yet fully understood [4]. Furthermore, critical issues persist, such as understanding how the back-reaction of magnetic fields limits magnetic amplification in turbulent flow and the mechanisms reasonable for the generation/maintenance of large-scale galactic magnetic fields [5]. These challenges partly stem from the fact that, in MHD, electromagnetic fields are described by using particle dynamics based on field theory, in contrast to the description of fluids using hydrodynamics. As a result, different theoretical frameworks are employed, leading to a lack of a unified approach, which poses a major obstacle to achieving a consistent and comprehensive interpretation of the underlying physics.

In this paper, we propose a novel theoretical approach to address these challenges. Our research aims to develop a unified framework that integrates electromagnetism and fluid dynamics, thereby transcending their conventional separate treatment in MHD. We anticipate that this unified framework will not only provide a more fundamental understanding of the interaction between electromagnetic fields and fluids but also offer a relativistic extension necessary for explaining high-energy astrophysical phenomena [6] such as supernova explosions, gamma-ray bursts, and black hole jets in addition to nuclear fusion.

Before delving further, it is important to outline a theoretical motivation for this study. In modern physics, particles are described as fields inhabiting spacetime, endowed with specific symmetries and properties such as spin, charge, and mass. The field-theoretic approach to matter and particles originates from the electromagnetism, where electric and magnetic interactions were unified within the framework of a local U(1) gauge theory. Quantum field

theory, built on this foundation, has been remarkably successful in describing fundamental interactions, including electromagnetic, weak, and strong forces. This framework has also been applied successfully to gravity, at least in classical regime. However, limitations in existing paradigms, such as the treatment of singularities, have driven the exploration of alternative frameworks like string theory [7], loop quantum gravity [8], and the emergent point of view for gravity [9].

On the other hand, the relativistic fluid and thermodynamic approach assumes that matter or particles reside in a three-dimensional Euclidean space, referred to as "matter space," with their dynamics governed by mappings between this matter space and four-dimensional spacetime, illustrated in Fig. 1, [10–16]. This perspective provides a geometrically and physically intuitive framework for describing the motion and properties of matter in spacetime.

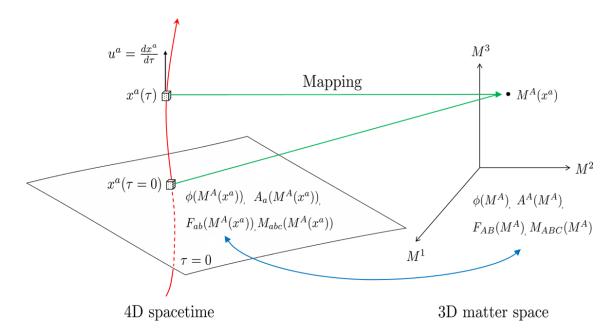


Figure 1. An illustration of the pull-back description between 4-dimensional spacetime and 3-dimensional matter space. Here, the green solid lines denote the mapping between the two spaces. A point (black dot) in the matter space labeled by coordinates  $M^A$  is mapped to the worldline (red solid curve) in the spacetime. Each small square represents a single fluid element, not an individual particle.

In this work, we adopt this point of view. Specifically, we consider matter (in this case, photons) to reside in a three-dimensional matter space, while the electromagnetic fields observed in four-dimensional spacetime are projections of this system. This description not only ensures consistency with electromagnetism but also establishes a unified theoretical basis for treating electromagnetism and fluid dynamics within a single framework. Such an approach offers promising insights into the interplay between matter and fields.

In a certain sense, our approach is analogous with the emergent concept of gravity proposed by [9], which interprets gravity as a phenomenon arising from thermodynamic

principles. Similarly, our framework suggests that the dynamics of electromagnetic fields can be understood as an emergent phenomenon resulting from the pull-back mechanism depicted in Fig. 1. This approach emphasizes that describing collective phenomena through fluid dynamics provides a more natural representation of physical systems.

In Sec. 2, we demonstrate how gauge symmetry is achieved in the matter-space formulation and discuss its consequences. In Sec. 3, we introduce the action and shows the variational equations. In Sec. 4, we summarize the results and discuss various aspects of the theory.

## 2 Gauge symmetry and matter space formulation

In this work, we consider one-fluid system incorporating one matter space. Let the three-dimensional matter space have coordinates  $-\infty < M^A < \infty$ , where A=1,2,3, as illustrated in Fig. 1. The coordinates  $M^A(x^a)$  co-move with their respective world-lines in four-dimensional spacetime. In this work, we consider only form fields to be associated with matter, which are anti-symmetric tensor fields, rather than symmetric ones. Since the matter space is three-dimensional, the following form fields may exist: a 0-form field  $\phi(M^A)$ , a 1-form field  $A=A_A(M^A)\,dM^A$ , a 2-form field  $F=\frac{1}{2!}F_{AB}(M^A)\,dM^A\wedge dM^B$ , and a 3-form field  $M=\frac{1}{3!}M_{ABC}(M^A)\,dM^A\wedge dM^B\wedge dM^C$ . We assume that these forms characterize the matter itself.

As we are primarily interested in electromagnetism, we disregard the 3-form field, which is often associated with the conservation of particle number density. We reconstruct electromagnetism based on the behavior of the remaining fields. Later in this work, we will present a short comment on the 3-form field in a different context. The corresponding fields induced in spacetime, through the mapping in Fig. 1, are given by

$$\phi(x^a) \equiv \phi(M^A(x^a)),$$

$$A_a(x^a) \equiv (\nabla_a M^A(x^a)) A_A(M^B(x^a)),$$

$$F_{ab}(x^a) \equiv (\nabla_a M^A(x^a)) (\nabla_b M^B(x^a)) F_{AB}(M^C(x^a)).$$
(2.1)

Since the matter space is lower dimensional than the physical spacetime, a seemingly trivial relation in the matter space can yield a non-trivial result when projected onto spacetime. For example,  $d\phi \wedge dF$  vanishes trivially in the matter space but its spacetime projection,

$$0 = d\phi \wedge dF$$

$$= \frac{1}{2} \frac{\partial \phi}{\partial M^A} \frac{\partial F_{CD}}{\partial M^B} dM^A \wedge dM^B \wedge dM^C \wedge dM^D$$

$$= \frac{1}{4!} (\nabla_{[a}\phi)(\nabla_b F_{cd]}) dx^a \wedge dx^b \wedge dx^c \wedge dx^d, \qquad (2.2)$$

presents a nontrivial relation:

$$\epsilon^{abcd}(\nabla_{[a}\phi)(\nabla_b F_{cd]}) = 0.$$
(2.3)

Since  $\phi$  is considered to represent the gauge degrees of freedom, this identity must hold for any function  $\phi$ . Consequently, this equation naturally gives rise to the Bianchi identity,

$$\nabla_{[a}F_{bc]} = 0 , \qquad (2.4)$$

which leads  $F_{bc} = \nabla_b B_c - \nabla_c B_b$  with a one-form field  $\mathbf{B}$ . In a similar manner, the relationships  $d\mathbf{A} \wedge \mathbf{F} = 0$  and  $\mathbf{A} \wedge d\mathbf{F} = 0$  present

$$\epsilon^{abcd}(\nabla_{[a}A_{b]})F_{cd} = *F^{ab}(\nabla_{[a}A_{b]}) = 0,$$
  
$$\epsilon^{abcd}A_{a}\nabla_{[b}F_{cd]} = 0,$$
(2.5)

where  $*F_{ab} \equiv \epsilon_{abcd} F^{cd}$ . In other words,  $*F_{ab}$  is orthogonal to  $\nabla_{[a} A_{b]}$ , i.e.,  $*F^{ab}(\nabla_{[a} A_{b]}) = 0$ . Furthermore, the constraint equation,  $\mathbf{F} \wedge \mathbf{F} = 0$ , which is a 4-form field, presents

$$*F^{ab}F_{ab} = \epsilon^{abcd}F_{ab}F_{cd} = 0. (2.6)$$

To summarize, the spacetime-induced fields satisfy the following relations:

$$\frac{(A) \qquad (B)}{\epsilon^{abcd}(\nabla_{[a}\phi)(\nabla_{b}F_{cd]}) = 0 \quad \epsilon^{abcd}(\nabla_{[a}A_{b]})F_{cd} = 0}}{(C) \qquad (D)}$$

$$\frac{\epsilon^{abcd}A_{a}(\nabla_{[b}F_{cd]}) = 0 \qquad \epsilon^{abcd}F_{ab}F_{cd} = 0}$$

Table 1.

Because of the Bianchi identity (2.4), equation (C) in Table 1 is automatic. On the other hand, equations (B) and (D) impose constraints on the relationship between the two 1-forms,  $B_a$  and  $A_a$ . Although an additional equation is required to fully determine the relation between the two 1-forms up to a gauge transformation, we conjecture that  $B_a = A_a + \nabla_a \phi$  for some scalar  $\phi$ , given that we are dealing with a one-fluid system <sup>1</sup>. This postulate leads to the following expression:

$$F_{ab} = \nabla_a A_b - \nabla_b A_a \,\,, \tag{2.7}$$

where  $F_{ab}$  satisfies all the constraints listed in Table 1 and represents a subset of Maxwell's equations.

We now turn to the Lagrangian formulation of relativistic fluid dynamics and derive the remaining Maxwell equations. Our framework focuses on fluid of matter characterized by the vector field  $A_a$  and the antisymmetric field  $F_{ab}$  without introducing additional fields. We consider terms up to quadratic order in the fields but intentionally omit the  $A^2$  term, which is typically associated with a "mass" term. Since the scalar is regarded as representing gauge degrees of freedom, it is not included explicitly. Additionally, we do not explicitly include kinetic terms arising from gradients of the field values as usual in fluid mechanics.

<sup>&</sup>lt;sup>1</sup>In the discussion of this letter, we further demonstrate that the introduction of a 3-form field  $M_{ABC}$  supplements the additional equation for the two 1-forms.

## 3 Electromagnetism from relativistic fluid dynamics

The relativistic Lagrangian, which is a scalar in (3+1)-dimensional spacetime, depends on the field with properties mentioned above and the spacetime metric  $g_{ab}$ , is given by

$$\Lambda(A_a, F_{ab}, g_{ab}) = -\frac{1}{4} F^{ab} F_{ab} - j^a A_a, \tag{3.1}$$

where  $j^a$  denotes an external current vector, added manually. In this formulation (3.1),  $A_a$  and  $F_{ab}$  are treated as independent fields, and the Lagrangian itself does not inherently encode their dynamics. However, as aforementioned, imposition of the constraint equations listed in Table 1 establishes the connection between  $A_a$  and  $F_{ab}$  and hence develops dynamics in the resulting fields.

Introducing a Lagrangian displacement vector  $\xi^a$ , the relationship between the Lagrangian variation  $\Delta$  and the Eulerian variation  $\delta$  is given by

$$\Delta = \delta + \mathcal{L}_{\xi},\tag{3.2}$$

where  $\mathcal{L}_{\xi}$  denotes the Lie derivative with respect to  $\xi^{a}$ . Here,  $\Delta$  and  $\delta$  capture changes relative to a reference configuration and variations with respect to spacetime fields, respectively. Since the Lagrangian variation of the matter space coordinates  $M^{A}$  vanishes, i.e.,  $\Delta M^{A} = 0$ , we obtain:

$$\delta M^A = -\mathcal{L}_{\xi} M^A. \tag{3.3}$$

This relation allows the variation of the action to be expressed in terms of the displacement  $\xi^a$  rather than the flux.

The Eulerian variation of  $F_{ab}$  is given by

$$\delta F_{ab} = \delta F_{AB}(\nabla_a M^A)(\nabla_b M^B)$$

$$+ F_{AB} \left[ (\nabla_a \delta M^A) \nabla_b M^B + (\nabla_a M^A)(\nabla_b \delta M^B) \right]$$

$$= -\xi^c \nabla_c F_{ab} - (\nabla_a \xi^c) F_{cb} - (\nabla_b \xi^c) F_{ac}$$

$$= -\mathcal{L}_{\mathcal{E}} F_{ab}. \tag{3.4}$$

Similarly, for the vector field  $A_a$ , the variation takes the form

$$\delta A_a = -\mathcal{L}_{\xi} A_a. \tag{3.5}$$

As noted above, Eqs. (3.4) and (3.5) are merely restatements of  $\Delta F_{ab} = 0$  and  $\Delta A_a = 0$ . By performing a complete variation of the Lagrangian  $\Lambda(A, F)$  with respect to **both** 

 $F_{ab}$  and  $A_a$ , and utilizing the variations in Eqs. (3.4) and (3.5), we get

$$\frac{\delta(\sqrt{-g}\Lambda)}{\sqrt{-g}} = \delta\Lambda + \frac{1}{2}\Lambda g^{ab}\delta g_{ab}$$

$$= \Pi^{ab} \left( -\mathcal{L}_{\xi} F_{ab} \right) + j^{a} \left( -\mathcal{L}_{\xi} A_{a} \right) + \left( \frac{\partial\Lambda}{\partial g_{ab}} + \frac{1}{2}\Lambda g^{ab} \right) \delta g_{ab}$$

$$= \left[ A_{e} \nabla_{a} j^{a} - 3\Pi^{ab} \nabla_{[e} F_{ab]} + 2F_{ea} \nabla_{b} \Pi^{ba} + 2j^{a} \nabla_{[a} A_{e]} \right] \xi^{e}$$

$$+ \left[ \frac{\partial\Lambda}{\partial g_{ab}} + \frac{1}{2}\Lambda g^{ab} \right] \delta g_{ab} + \text{total derivatives.} \tag{3.6}$$

Here, the conjugate to  $F_{ab}$  is

$$\Pi^{ab} = \frac{\partial \Lambda}{\partial F_{ab}} = \left(\frac{\partial F}{\partial F_{ab}}\right) \left(\frac{\partial \Lambda}{\partial F}\right) = \frac{F^{ab}}{F} \Pi,$$
(3.7)

where  $\Pi \equiv \partial \Lambda/\partial F$ . With respect to the Lagrangian (3.1), we get  $\Pi/F = -1/2$  and  $\Pi^{ab} = -F^{ab}/2$ . Finally, requiring the charge conservation associated with the external source,

$$\nabla_a j^a = 0, (3.8)$$

we obtain field equations from (3.6), which are given by

$$3F^{ab}\nabla_{[e}F_{ab]} - 2\left(\nabla_{b}F^{ba}\right)F_{ea} + 4j^{a}\nabla_{[a}A_{e]} = 0.$$
(3.9)

Substituting the equation (2.7) into the field equation (3.9) gives

$$F_{ea} \left[ \nabla_b F^{ab} - j^a \right] = 0. \tag{3.10}$$

The equation (3.10) leads to <sup>2</sup>

$$\nabla_b F^{ab} = j^a. (3.11)$$

This equation is the last piece of the Maxwell equation other than Eq. (2.4). Consequently, it becomes evident that equations (2.4) and (3.11) consist the Maxwell equations describing electromagnetism, where  $F_{ab}$  is interpreted as the electromagnetic field tensor and  $A^a$  as the vector potential.

#### 4 Summary and discussions

In summary, we began with a Euclidean matter space  $\{M^A\}$  (A=1,2,3), incorporating 0-, 1-, and 2-form fields. These form fields were subsequently mapped onto spacetime, yielding the constraint equations summarized in Table 1, including the Bianchi identity, which arises from the local U(1) symmetry. To complete the framework, we employed a Lagrangian formulation for relativistic fluid dynamics. The proposed model, governed by the Lagrangian (3.1), inherently lacks intrinsic dynamics and is formulated in terms of the induced fields up to quadratic order. By substituting the constraint equations into the equations of motion, we derived the remaining Maxwell's equations (3.11). As a result, thanks to the constraint equations arising from the U(1) symmetry, the dynamics of fields has been generated.

Let us examine various aspects of the theory. By applying the constraint equations discussed in the Table 1, we demonstrate that the equations of motion (2.4) and (3.11) respect the local U(1) gauge symmetry,

$$A_a \to A_a + \nabla_a \phi.$$
 (4.1)

<sup>&</sup>lt;sup>2</sup>This choice of equation involves an ambiguity, which will be addressed in the discussion.

In other words, the symmetry inherent in our model, as defined by (3.1), and supported by the conditions arising from matter space including form fields, is the local U(1) symmetry, indicating that our model effectively describes electromagnetism.

The charge conservation relation (3.8), necessitated by gauge symmetry, was imposed manually in our framework. However, this conservation law arises naturally if one introduces a 3-form field,  $\mathbf{N} = \frac{1}{3!} N_{ABC}(N^A) dN^A \wedge dN^B \wedge dN^C$ , defined within a charge-carrier matter space  $\{N^A\}$ , where A = 1, 2, 3. Let us define the number flow  $n^a \equiv \frac{1}{3!} \epsilon^{abcd} N_{bcd}$ , dual to the induced field  $N_{abc} \equiv (\nabla_a N^A)(\nabla_b N^B)(\nabla_c N^C)N_{ABC}$ . Then, let the current

$$j^a \equiv q n^a \tag{4.2}$$

act as a source. Since the matter space is three-dimensional, the exterior derivative of the 3-form field vanishes, dN = 0. This condition simplifies to the conservation equation (3.8), [14] where  $j^a$  corresponds to the charge flux 4-vector in this case. Consequently, the equation (3.8), arising as a spacetime constraint induced from the matter space, describes the conservation of the particle number associated with the external source. This result reinforces the U(1) symmetry in the matter space description of electromagnetism.

Until now, we have disregarded the 3-form field  $M_{ABC}$  defined on the matter space  $\{M^A\}$ . Similar to the case of  $N_{ABC}$ , this 3-form field introduces a conservation relation of the form  $\nabla_a m^a = 0$ , where  $m^a \equiv \frac{1}{3!} \epsilon^{abcd} \nabla_b M^B \nabla_c M^C \nabla_d M^D M_{BCD}$ . Using the same method employed to derive Eq. (2.2), we obtain four independent constraints:

$$m^a \nabla_a \phi = 0, \qquad m^a A_a = 0, \qquad m^a F_{ab} = 0.$$
 (4.3)

Three of these constraints determine the direction of  $m^a$ . The scale of  $m^a$  can then be specified by the conservation relation  $\nabla_a m^a = 0$ , up to a multiplicative constant. The remaining constraint in (4.3) can be used to further specify the relation (2.7) between  $F_{ab}$  and  $A_b$  as follows: As noted in Eq. (2.4), the 2-form field  $F_{ab}$  can be expressed as  $F_{ab} = \nabla_a B_b - \nabla_b B_a$  with a 1-form field  $B_b$ . However, the form  $B_b = A_b + \nabla_b \phi$  cannot be yet asserted because Table (1) provides only two independent relations satisfied by the  $B_a$  field. To fully specify this field, an additional equation is required, which arises from the remaining constraint after determining the direction of the vector  $m^a$ .

Combining the two equations in Eq. (2.5) and defining

$$C^a \equiv \epsilon^{abcd} A_{[b} F_{cd]} , \qquad (4.4)$$

we derive another nontrivial equation:

$$\nabla_a C^a = 0. (4.5)$$

The vector  $C^a$  satisfies  $C^a \nabla_a \phi = 0$  and is orthogonal to both  $A_a$  and  $F_{ab}$ . These relations are identical to the equations satisfied by  $m^a$ . Consequently, it is evident that  $C^a \propto m^a$ . Since  $m^a$  is orthogonal to the electromagnetic field and conserved, it can be identified as representing the energy flux, specifically the Poynting four-vector. Thus, Eq. (4.5) can be interpreted as the Poynting theorem. In other words, the conservation of  $m^a$  corresponds

to a fundamental conservation law, such as the energy-momentum conservation of photons in pature

Now, we are ready to address the ambiguity in Eq. (3.10). By comparing Eq. (4.3) with Eq. (3.10), we observe that a term proportional to  $m^a$  can be added to the right-hand side of the equation:

$$\nabla_b F^{ab} = q(n^a + \beta m^a) , \qquad (4.6)$$

where  $\beta$  is a scalar and we replaced  $j^a$  with  $qn^a$  defined in Eq. (4.2)<sup>3</sup>. When  $\beta=0$ , Eq. (4.6) reduces to the standard Maxwell equations, as described by Eqs. (2.7) and (3.11), which govern classical electromagnetism. According to Eq. (4.3),  $m^a$  can be expressed as a nonlinear function<sup>4</sup> of  $A^a$  and  $F_{ab}$ . Consequently, when  $\beta \neq 0$ , Eq. (4.6) can be interpreted as a set of nonlinear Maxwell equations, where the correction term is proportional to the energy flux. This nonlinear term can be related to the mass renormalization problem, akin to the case of Born-Infeld electrodynamics [17]. When  $\beta=0$ , the electron self energy problem appears as follows: The electric energy density of a static charge q located at r=0 is  $\frac{\epsilon_0 E_r^2}{2} = \frac{q^2}{32\pi^2\epsilon_0 r^4}$  where  $\epsilon_0$  is the electric permittivity of free space. Integrating over volume, the total electric energy becomes  $\frac{q^2}{8\pi\epsilon_0 \delta}$  where  $\delta$  denotes the characteristic size of the electron. Taking the infinitesimal limit  $\delta \to 0$ , the energy diverges. To compensate this divergence, one should choose the bare mass of electron to have negative infinite value. The divergence of the self-energy disappears when  $\beta>0$ . In this case, the solution to Eq. (4.6) is

$$E_r = \frac{q}{4\pi\epsilon_0 r^2 \left(1 + \frac{\beta q^2}{8\pi r}\right)}$$

with all other components vanish. Note that when  $r \gg r_c \equiv \frac{\beta q^2}{8\pi}$ , the formula reproduces the  $\beta = 0$  result. On the other hand, around  $r \sim 0$ , it presents nontrivial contributions so that the integration of the energy density

$$\frac{q^2}{32\pi^2\epsilon_0 r^4 (1 + \frac{\beta q^2}{8\pi r})^2}$$

over any volume be finite. Integrating over the whole space, the total energy stored in the electro-static field is  $\frac{1}{\beta\epsilon_0} = \frac{q^2}{8\pi\epsilon_0 r_c}$ . If we take this value to be the electron mass, we have  $r_c \sim 1.4\,\mathrm{fm}^5$ , which is comparable to the nuclear size and a bit smaller than the so-called classical electron radius, 2.8 fm [18]. These results show that the self-energy problem can be solved even at the classical level.

The present theory defines matter as a flow described by a fluid. Therefore, a single-fluid model used in this work describes an electromagnetic field which flows along the direction  $m^a$  at each points. A direct consequence is that the electric field must be orthogonal to the

<sup>&</sup>lt;sup>3</sup>Noting the electro-magnetic field  $F^{ab}$  change signature when  $q \to -q$ , we notice that the new term also change signature with the change.

<sup>&</sup>lt;sup>4</sup>Recall that energy is typically a quadratic function of the field.

<sup>&</sup>lt;sup>5</sup>Note that  $r_c$  does not denote the size of electron because the electric charge in this model is located at the center r = 0, which makes the electric field diverge there. Therefore, this result is consistent with the known upper bound of the electron size  $10^{-22}$  m.

magnetic field, or one of the two fields must vanish, as seen in Eq. (2.6). This result indicates that an electromagnetic field originating from a single source, and therefore belonging to a single fluid, must be orthogonal. Electromagnetic fields which have multiple origins do not satisfy this property and cannot be described by using the simplest model used in this work. They require multi-fluid models.

In this new framework, the observable corresponding to the particle concept is based on the matter space, where the particle's degrees of freedom is represented by the number of coordinates of this space. While the classical equations of motion within this framework are equivalent to those of conventional electromagnetism, this equivalence may not necessarily extend to the quantum regime. As a result, the process of quantization in matter space emerges as a significant and intriguing area for further exploration. Additionally, a key direction for future research will be to investigate how spinor fields and other symmetries can be realized within the context of matter space.

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