Assignment (4) 2301010123

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Q1 :- Rank =]

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_4} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & -4 & -11 & 5 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_4} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 3

2) for dim of W

∵ Symmetric matrix has 2 independent value

∴ dim (W) = 3

Rank of T :-

T has all polynomials in P that can come by using mapping to all polynomial of degree 2

∴ Rank of T = 3

using Rank Nullity theorem

Nullity = dim (W) − rank (T)

= 3 − 3 = 0

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3:- So A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Characteristic eq for this

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$$

$$(2-\lambda)^2 - 1 = 0$$

$$\lambda^2 + 4 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow \lambda(\lambda-3) - 1(\lambda-3) = 0$$

$$(\lambda-1)(\lambda-3) = 0 \Rightarrow \lambda = 1, \lambda = 3$$

∴ Eigen Values are 1, 3

For $\lambda = 1$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\left.\begin{aligned} x - y &= 0 \\ -x + y &= 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} K \\ K \end{bmatrix}$$

& if $\lambda = 3$ $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$$-x - y = 0$$

$$-x - y = 0$$

$$\Rightarrow \begin{bmatrix} K \\ -K \end{bmatrix}$$

∴ As we know,
If A is invertible, Eigen Vector (A^{-1}) = Eigen Vector (A)

& Eigen Vector of $(A + 4I)$ = Eigenvector (A)

∴ Eigen Values of A = 1, 3

& $A^{-1} = 1, \frac{1}{3}$

& $A + 4I = 5, 7$

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4 :- $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

with initial values $x(0) = 0,\ y(0) = 0,$

$z(0) = 0$

∴ from these 3 eqⁿ

$z = \dfrac{71.4 - 0.3x - 0.2y}{10}$

$y = \dfrac{-19.3 - 0.1x + 0.3z}{7}$

$x = \dfrac{7.85 + 0.1y + 0.2z}{3}$

For first iterⁿ,

$x = \dfrac{7.85}{3} = 2.61$

& $y = \left(\dfrac{-19.3 + 0.1(2.61)}{7}\right)$

$= 19.56$

$z \Rightarrow \dfrac{71.4 - 0.3(2.61) - 0.2(19.56)}{10}$

$= \dfrac{71.4 - 0.785 - 3.912}{10}$

$z = 6.67$

for second one,

base case $(x, y, z) = (2.61, 19.56, 6.67)$

$x = \dfrac{7.85 + 0.1(19.56) + 0.2(6.67)}{3}$

$= 7.71$

$y = \dfrac{-19.3 + 0.1(7.71) + 0.3(6.67)}{7}$

$= -2.36$

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$$z = \frac{72.4 - 0.3(7.716) - 0.2(-2.36)}{10}$$

$$= 6.956$$

for third one,

Base case $(x, y, z) = (7.72, -2.36, 6.95)$

$$x = \frac{7.85 + 0.1(-2.36) + 0.2(6.956)}{3}$$

$$x = 3.002$$

$$y = \frac{-19.3 + 0.1(3.002) + 0.3(6.956)}{7}$$

$$y = -2.45$$

$$z = \frac{72.4 - 0.3(3.002) - 0.2(-2.45)}{10}$$

$$z = 7.099$$

5:- $A:B = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) \neq \rho(AB) \neq n$

Inconsistent Sol^n

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6) $T(a+bx+cx^2) = (a+1) + (b+1)x + (c+1)x^2$

for Additivity $T(u+v) = T(u) + T(v)$

Homogeneity $T(cu) = c(T(u))$

(1) $T(u) = (a+1) + (b+1)x + (c+1)x^2$

assume $T(v) = (\bar{a}+1) + (\bar{b}+1)x + (\bar{c}+1)x^2$

$T(u+v) = (a+\bar{a}+2) + (b+\bar{b}+2)x + (c+\bar{c}+2)x^2$

∴ Satisfies additivity

(2) Let $du = da + dbx + dcx^2$

$T(du) = T(da + dbx + dcx^2)$

$d(T(u)) = d(a+1) + d(b+1)x + d(c+1)x^2$

∴ Satisfies homogeneity

∴ It is a linear transformation

9) One application is image filtering. Filters are small matrices that are applied to an image to modify the pixel values. This can be done to achieve various effects, such as blurring images, sharpening edges or removing noise.

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ex:- If a 3x3 image where each pixels is value is intensity (0 → black, 255 white).

Blurring → { [1/9, 1/9, 1/9],
[1/9, 1/9, 1/9],
[1/9, 1/9, 1/9] }

→ We perform element-wise multiplication b/w the filter & image & then take the average of the resulting values

10 → Linear Transformation are a powerful method for rotating 2D images in Computer Vision. They work by applying a rotation matrix to the image co-ordinates. This matrix based on the rotation angle, essentially scales & rotates each point in the image.