075BCT092

LAB4: Mid-Point Ellipse Drawing Algorithm.

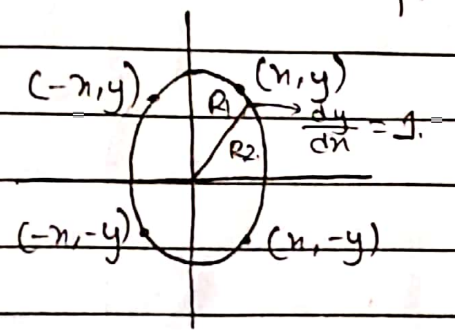
THEORY

All ellipse is defined as the set of points such that the sum of the distances from two fixed point is same for all points.

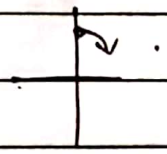
general equation of ellipse having center at origin.

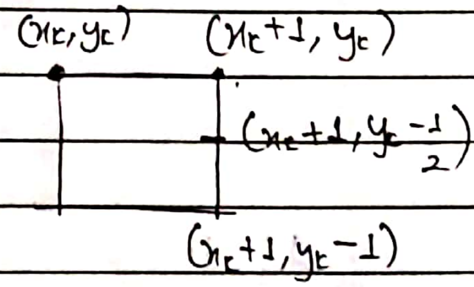
$$\frac{(x-0)^2}{r_x^2} + \frac{(y-0)^2}{r_y^2} = 1$$

Let us consider the following figure:



Case: Region I





x increases,

y decreases.

sampling in x-axis.

Starting pixel → (x_k, y_k).

next pixel i.e. $(x_{k+1}, y_{k+1}) \rightarrow (x_k+1, \; y_k$ or $y_k-1)$

Decision parameter $P_k = f_e(x_k+1, y_k-1/2)$

Next successive Decision parameter $P_{k+1} = f_e(x_{k+1}+1, y_{k+1}-1/2)$

Subtracting

$$P_{k+1} - P_k = f_e(x_{k+1}+1, y_{k+1} - 1/2) - f_e(x_k+1, y_k - 1/2)$$

$$P_{k+1} - P_k = r_y^2(x_{k+1}+1)^2 + r_x^2(y_{k+1} - 1/2)^2 - r_x^2 r_y^2 - r_y^2(x_k+1)^2 - r_x^2(y_k - 1/2)^2 + r_x^2 r_y^2.$$

$$P_{k+1} - P_k = r_y^2(x_k+2)^2 + r_x^2\left((y_{k+1})^2 - y_{k+1} + \frac{1}{4}\right) - r_y^2(x_k+1)^2 - r_x^2\left(y_k^2 - y_k + \frac{1}{4}\right).$$

$$P_{k+1} - P_k = r_y^2\left((x_k+1)^2 + 2(x_k+1) + 1\right) + r_x^2\left((y_{k+1})^2 - y_{k+1} + \frac{1}{4}\right) - r_y^2(x_k+1)^2 - r_x^2\left(y_k^2 - y_k + \frac{1}{4}\right).$$

$$P_{k+1} - P_k = 2r_y^2(x_k+1) + r_y^2 + r_x^2\left((y_{k+1})^2 - y_{k+1} + \frac{1}{4}\right) - r_x^2\left(y_k^2 - y_k + \frac{1}{4}\right)$$

If $p_k < 0$ i.e. $y_{k+1} = y_k$

$$P_{k+1} - P_k = 2r_y^2(x_k+1) + r_y^2 + r_x^2(y_k^2 - y_k + 1/4) - r_x^2(y_k^2 - y_k + 1/4)$$

$$\boxed{\therefore P_{k+1} = P_k + 2r_y^2(x_{k+1}) + r_y^2.}$$

If $p_k > 0$ i.e. $y_{k+1} = y_k - 1.$

$$P_{k+1} - P_k = 2r_y^2(x_k+1) + r_y^2 + r_x^2\left((y_k-1)^2 - y_k + 1\right) - r_x^2(y_k^2 - y_k$$

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$$= 2r_y^2(x_k+1) + r_y^2 + r_x^2 y_k^2 - 2r_x^2 y_k + r_x^2 - y_k r_x^2 + r_x^2 - r_x^2 y_k^2 + r_x^2 y_k.$$

$$= 2r_y^2(x_k+1) + r_y^2 + 2r_x^2 - 2r_x^2 y_k$$
$$= 2r_y^2(x_k+1) + r_y^2 + 2r_x^2(-(y_k-1))$$

$$\boxed{\therefore P_{k+1} = P_k + 2r_y^2(x_{k+1}) + r_y^2 - 2r_x^2 y_{k+1}.}$$

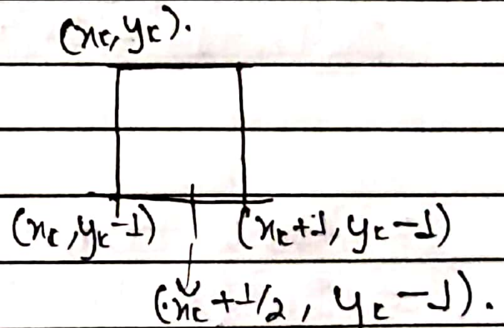
The initial decision parameter is evaluated at start position $(x_0, y_0) = (0, r_y)$.

$$P_0 = f_e(0+1, r_y - 1/2)$$
$$= r_y^2 + r_x^2(r_y - 1/2)^2 - r_x^2 r_y^2.$$
$$= r_y^2 + \cancel{r_x^2 r_y^2} - r_x^2 r_y + \frac{r_x^2}{4} - \cancel{r_x^2 r_y^2}.$$

$$\boxed{P_0 = r_y^2 - r_x^2 r_y + \frac{r_x^2}{4}}$$

Region II :

here, sampling is done in y-axis.



The ending point of Region I is the starting point of Region II i.e. $(x_k+1/2, y_k-1)$, here, $y_{k+1} = y_k - 1$.

$x_{k+1} = x_k$ or x_{k+1}

$$P_k = f_e(x_k + 1/2, y_k - 1)$$
$$= r_y^2(x_k + 1/2)^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2.$$

$$P_{k+1} = f_e(x_{k+1} + 1/2, y_{k+1} - 1)$$
$$= r_y^2(x_{k+1} + 1/2)^2 + r_x^2(y_{k+1} - 1)^2 - r_x^2 r_y^2.$$

$$P_{k+1} - P_k = r_y^2 (x_{k+1} + 1/2)^2 + r_x^2 (y_{k+1} - 1)^2 - r_x^2 r_y^2 - r_y^2 (x_k + 1/2)^2$$
$$- r_x^2 (y_k - 1)^2 + r_x^2 r_y^2.$$

If $p_k < 0$ then $x_{k+1} = x_k + 1$

$$P_{k+1} - P_k = r_y^2 (x_k + 3/2)^2 + r_x^2 (y_{k+1} - 1)^2 - r_y^2 (x_k + 1/2)^2$$
$$- r_x^2 (y_k - 1)^2$$

$$= r_y^2 (x_k + 3/2)^2 + r_x^2 (y_k - 1 - 1)^2 - r_y^2 (x_k + 1/2)^2 - r_x^2 (y_k - 1)^2.$$

$$= r_y^2 x_k^2 + 3 r_y^2 x_k + \frac{9}{4} r_y^2 + r_x^2 [(y_k - 1) - 1]^2 - r_y^2 (x_k + 1/2)^2 - r_x^2 (y_k - 1)^2.$$

$$= \cancel{r_y^2 x_k^2} + 3 r_y^2 x_k + \frac{9}{4} r_y^2 + \cancel{r_x^2 (y_k - 1)^2} - 2 r_x^2 (y_k - 1) + r_x^2 - \cancel{r_y^2 \{ x_k^2} - r_y^2 x_k - \frac{r_y^2}{4} - \cancel{r_x^2 (y_k - 1)^2}.$$

$$= 2 r_y^2 x_k + 2 r_y^2 - 2 r_x^2 y_{k+1} + r_x^2.$$

$$p_{k+1} - p_k = 2 r_y^2 x_{k+1} - 2 r_x^2 y_{k+1} + r_x^2.$$

If $p_k > 0$ then $x_{k+1} = x_k$.

$$p_{k+1} - p_k = r_y^2 \cancel{(x_k + 1/2)^2} + r_x^2 (y_{k+1} - 1)^2 - r_x^2 r_y^2 - \cancel{r_y^2 (x_k + 1/2)^2} - \cancel{r_x^2 (y_k - 1)^2} + r_x^2 r_y^2$$

$$p_{k+1} - p_k = - 2 r_x^2 y_{k+1} + r_x^2$$

Algorithm.

1. Start.
2. Input (x_c, y_c), r_x, r_y.
3. Initialize $x=0$, $y=r_y$
4. Calculate initial value of decision parameter of region I

$$p_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2.$$

5. do while loop.

plot (x_c, y)

If $p<0$.

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k$.

$p_{k+1} = p_k + 2r_y^2 x_{k+1} + r_y^2$.

else.

$x_{k+1} = x_k + 1$.

$y_{k+1} = y_k - 1$.

$p_{k+1} = p_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$.

continue loop until $2r_y^2 x > 2r_x^2 y$

6. $p = r_y^2 (x+0.5)^2 + r_x^2 (y-1)^2 - r_x^2 r_y^2$.
7. do while loop.

plot (x_c, y_c)

If $p<0$.

$x_{k+1} = x_k + 1$.

$y_{k+1} = y_k - 1$.

$p_{k+1} = p_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$.

Else.

$x_{k+1} = x_k$

$y_{k+1} = y_k - 1$

$p_{k+1} = p_k - 2r_x^2 y_{k+1} + r_x^2$.

continue loop unit $y=0$

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8. Determine the symmetry points in the other three quadrants.
8. Plot the coordinates value
8. Continue loop untill $y=0$.
9. Stop

Example:

Given, $r_x = 8$, $r_y = 5$ and centered at ~~origin~~ (10,10)

Initial values and increments for the decision parameter calculations are $2r_y^2 x = 0$, $2r_x^2 y = 2r_x^2 r_y$

here, $(x_c, y_c) = (10, 10)$

Region I:

The initial point for the ellipse centered on the ~~origin~~ (10,10) is $(x_0, y_0) = (0, 5)$.

Initial decision parameter value is.

$$P_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

$$= -279.$$

Successive decision parameter.

K	x_{k+1}	y_{k+1}	P_{k+1}	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	1	5	-204.	50.	640.
1.	2	5	-79	100	640
2.	3	5	96	150.	640
3	4	4	-191	200	512.
4	5	4	84	250	512
5	6	3	25	300	384
6	7	2	144	350	256.

Now, we move out of region I, since $2r_y^2x > 2r_x^2y$.

For region 2., initial point is $(x_0, y_0) = (7,2)$.
and initial decision parameter is.

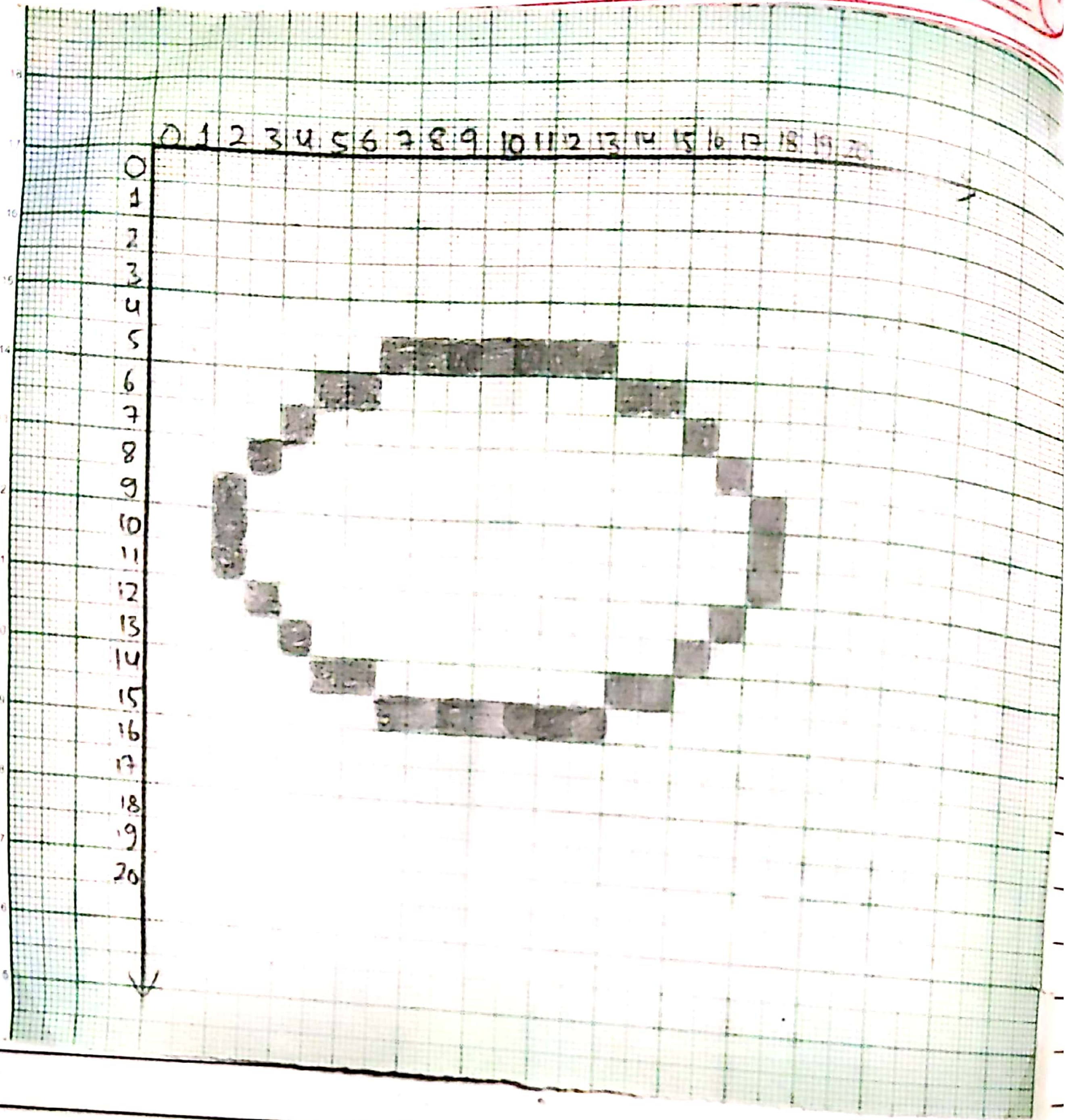
$$P_0 = f(7+1/2, 2-1) = f(15/2, 1)$$
$$= 5^2(15/2)^2 + 8^2 \times 1^2 - 5^2 \times 8^2$$
$$= -129.75 \simeq -130.$$

The remaining positions of ellipse path in the first quadrant are then calculated as.

k	x_{k+1}	y_{k+1}	P_{k+1}	$2r_y^2x_{k+1}$	$2r_x^2y_{k+1}$
0	8	1.	266.	400	128.
1.	8	0.	270.	400	0.

Therefore the overal points to plot are.

(x, y)	$(x+x_c, y+y_c)$	$(x+x_c, -y+y_c)$	$(-x+x_c, -y+y_c)$	$(-x+x_c, y+y_c)$
(1,5)	(11, 15)	(11, 5)	(9, 5)	(9, 15)
(2,5)	(12, 15)	(12, 5)	(8, 5)	(8, 15)
(3,5)	(13, 15)	(13, 5)	(7, 5)	(7, 15)
(4,4)	(14, 14)	(14, 6)	(6, 6)	(6, 14)
(5,4)	(15, 14)	(15, 6)	(5, 6)	(5, 14)
(6,3)	(16, 13)	(16, 7)	(4, 7)	(4, 13)
(7,2)	(17, 12)	(17, 8)	(3, 8)	(3, 12)
(8,1)	(18, 11)	(18, 9)	(2, 9)	(2, 11)
(8,0)	(18, 10)	(18, 10)	(2, 10)	(2, 10)



Discussion and conclusion.

Midpoint Ellipse Drawing Algorithm is a very efficient way to obtain ellipses with very little computation. We derived decision parameters, obtained algorithm and also saw an example we plotted the pixels for a quarter of ellipse and also wrote program for it!

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