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# Lab 5: Twodimensional transformations:

triangular object fixed point scaling and rotation about pivot point.

## THEORY

Scaling: A scaling is a basic transformation that alters the size of object. Points can be scaled by $S_x$ along x axis and $S_y$ along y axis in new points. Transformation equations are:

$$x' = x \cdot S_x \quad , \quad y' = y \cdot S_y$$

In matrix form.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation:

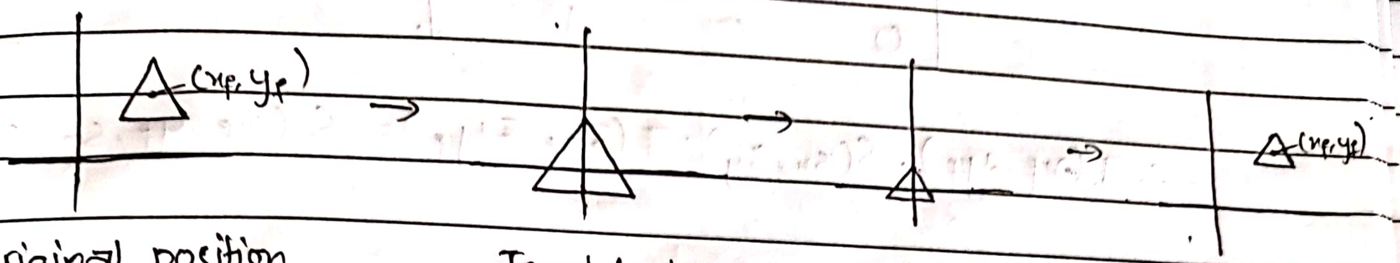
Rotation repositions an object along a circular path in the xy plane. To generate rotation, we specify a rotation angle $\theta$ and the position $(x_r, y_r)$ of the rotation point about which the object is to be rotated.

Transformation equations are in matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Q. Find general Fixed point scaling composite matrix.

Sol^n,




original position fixedpoint.

Translate to origin

Scale object

Translate to fixed point position.

1. Translate object so that the fixed point coincides with coordinate origin. $(t_x = -x_f \ , \ t_y = -y_f)$.
2. Scale the object with respect to the coordinate origin.
3. Use inverse translation of step 1 to return the object to its original position. $(t_x = x_f \ , \ t_y = y_f)$

So, composite transformation matrix $= \{[3^{rd}] \times [2^{nd}]\} \times [1^{st}]$.

$$= \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & x_f \\ 0 & S_y & y_f \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & -S_x x_f + 0 + x_f \\ 0 & S_y & -S_y y_f + y_f \\ 0 & 0 & 1. \end{bmatrix}.$$

$$= \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\therefore T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f) = S(x_f, y_f, s_x, s_y)$$

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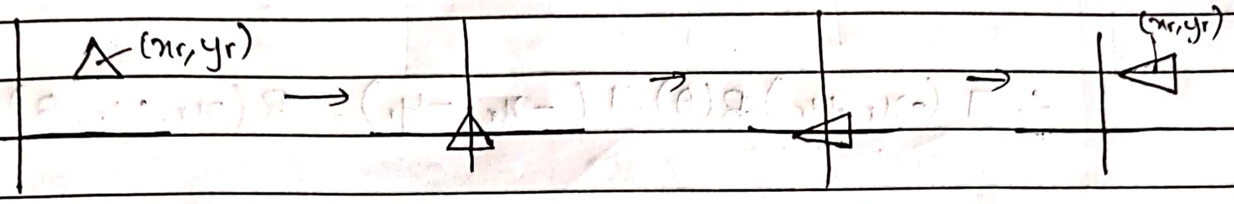
Q. Obtain the composite matrix for rotation about any arbitrary point (Rotation in anticlockwise direction).

Soln,




original position pivot point — Translate to origin — rotation about origin. — Translate to original position.

1. Translate object so that the pivot point position is moved to the coordinate origin ($t_x = -x_r$, $t_y = -y_r$).
2. Rotate the object about the coordinate origin.
3. Translate the object so that the pivot point is returned to its original position. ($t_x = x_r$, $t_y = y_r$)

So, the composite matrix = $\{[3^{rd}] \times [2^{nd}]\} \times [1^{st}]$

$$= \left\{ \begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \times \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r \\ \sin\theta & \cos\theta & y_r \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & -x_r\cos\theta + y_r\sin\theta + x_r \\ \sin\theta & \cos\theta & -x_r\sin\theta - y_r\cos\theta + y_r \\ 0 & 0 & 1 \end{bmatrix}$$

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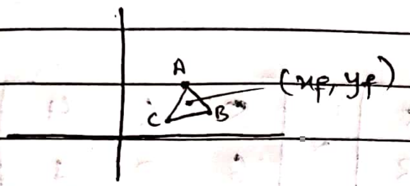
$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T(x_r, y_r) \cdot R(\theta) \cdot T(-x_r, -y_r) = R(x_r, y_r, \theta)$$

Example.

Scale the triangle A(5,6), B(6,2), C(4,1) by 2. by keeping centroid fixed.

Soln,




centroid $(x_f, y_f) = \left(\frac{5+6+4}{3}, \frac{6+2+1}{3}\right) = (5,3)$

1. translate object as $t_x = -x_f$, $t_y = -y_f$.

$$= \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Scale the object with scale factor 2.

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. translate object as $t_x = x_f$, $t_y = y_f$.

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ composite transformation matrix

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix}.$$

Now, for final position,

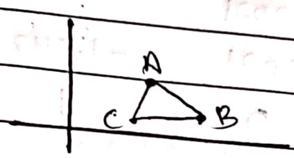
$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & 6 & 4 \\ 6 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 & 3 \\ 9 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

$\therefore$ The points after scaling is $A'(5,9)$, $B'(7,1)$, $C'(3,-1)$

Rotate a triangle A(5,6) B(6,2) C(4,1) by 45 degree about an arbitrary point (3,3).

Soln,




$(x_r, y_r) = (3,3)$

1. translate object as $t_x = -3$, $t_y = -3$.

$$= \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Rotate the object by 45°

$$= \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. translate the object as $t_x = 3$, $t_y = 3$.

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ composite transformation matrix

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

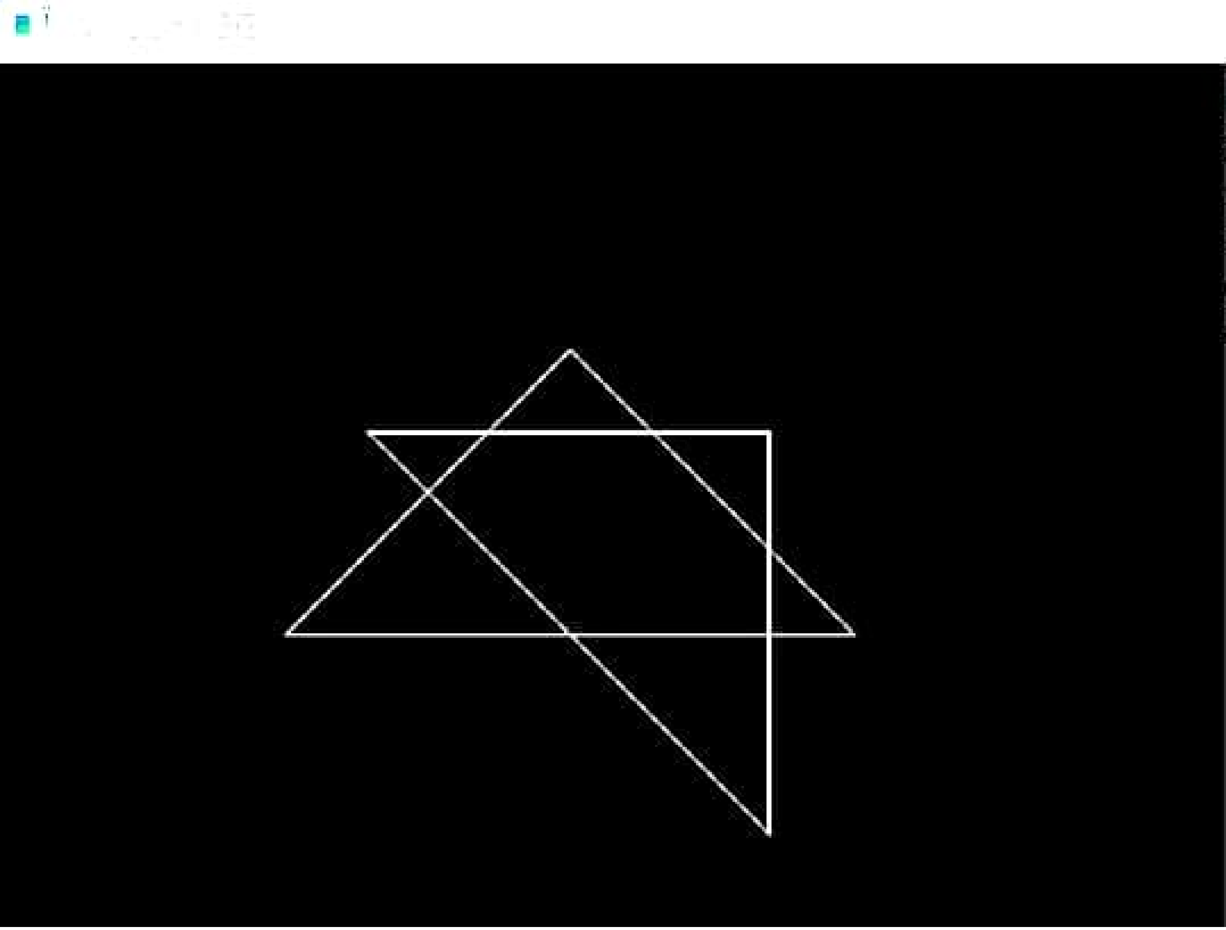
$$= \begin{bmatrix} 0.7071 & -0.7071 & 3 \\ 0.7071 & 0.7071 & -1.242 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, for final position

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.7071 & -0.7071 & 3 \\ 0.7071 & 0.7071 & -1.242 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 6 & 4 \\ 6 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.2928 & 5.8283 & 5.1212 \\ 6.5361 & 4.4148 & 2.2935 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ The points are $A'(2.29, 6.53)$, $B'(5.82, 4.41)$, $C'(5.12, 2.29)$



"D:\5th sem\computer graphics\lab\lab5\rotation.exe"

```
Enter the coordinates of triangle:
200 100
100 200
300 200
enter the rotating angle:45
enter reference point:200 200
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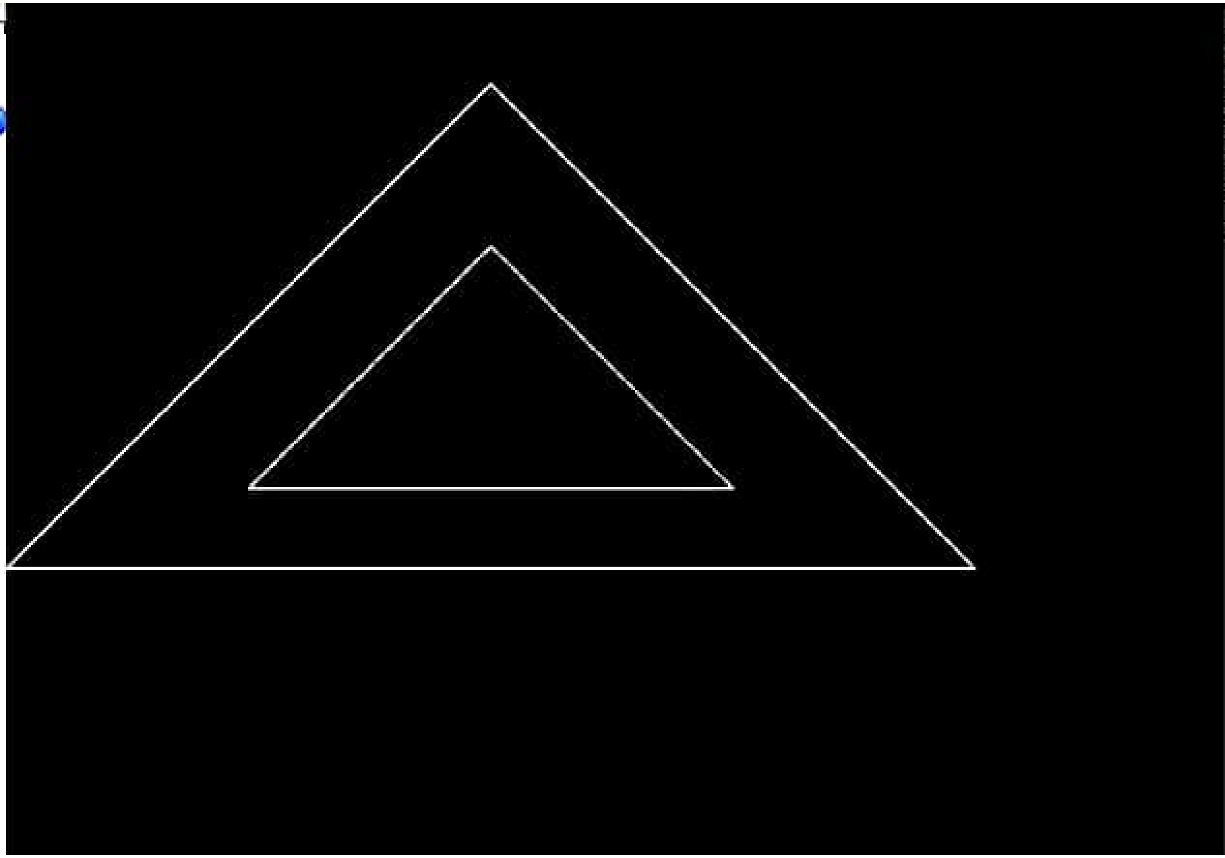

"D:\5th sem\computer graphics\lab\lab5\scaling.exe"

```
Enter the coordinates of triangle:
100 200
200 100
300 200
enter the scaling factor2 2
enter reference point200 167
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