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075BCT092.

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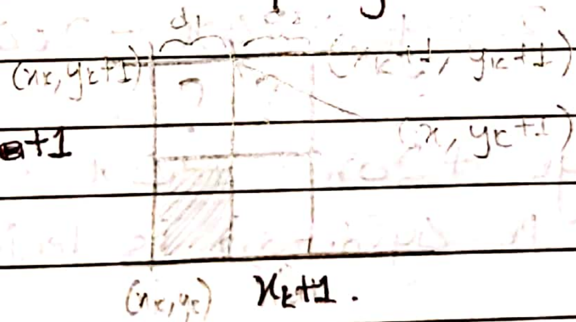
LAB-2 : Bresenham's Line - Drawing Algorithm.

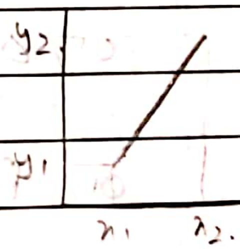
THEORY

Bresenham's line algorithm is an accurate and efficient line drawing algorithm. It uses only integer arithmetic to find the next position to be plotted. The major concept of Bresenham's algorithm is to determine the nearest pixel position. We calculate the decision parameter which decides which pixel to select and which function is used for next decision parameter.

2. Left to Right, +ve slope, slope>1.

As |slope| > 1, sampling is done in y axis.





$x_{k+1} = x_k$ or x_k+1

$y_{k+1} = y_k + 1$.

Assuming the pixel at (x_k, y_k) to be displayed is determined, we next to decide with pixel to plot in. row y_{k+1}, our choices are the pixels at positions.

(x_k, y_k+1) and (x_k+1, y_k+1)

At sampling position y_{k+1} we label horizontal pixel separations from the path d_1 and d_2.
The x-co-ordinate on the mathematical at pixel row y_k+1 is calculation.

(2)

$$y_k + 1 = mx + c.$$
$$d_1 = x - x_k$$
$$d_2 = x_k + 1 - x$$

$$\therefore d_1 - d_2 = x - x_k - (x_k + 1 - x)$$
$$= 2x - x_k - (x_k + 1)$$
$$= 2x - x_k - (x_k + 1)$$
$$= 2x - 2x_k - 1.$$
$$= 2\left(\frac{y_k + 1 - c}{m}\right) - 2x_k - 1.$$
$$= \frac{2y_k}{m} + \frac{2}{m} - \frac{2c}{m} - 2x_k - 1$$
$$= 2y_k \cdot \frac{\Delta x}{\Delta y} + \frac{2 \cdot \Delta x}{\Delta y} - 2c\frac{\Delta x}{\Delta y} - 2x_k - 1$$

or, $\Delta y(d_1 - d_2) = 2\Delta x y_k + 2\Delta x - 2c\Delta x - 2x_k\Delta y - \Delta y$

or, $P_k = 2\Delta x y_k + 2\Delta x - 2c\Delta x - 2\Delta y x_k - \Delta y$ —(i)

where $P_k = \Delta y(d_1 - d_2)$ is decision parameter.

For next step.

$$P_{k+1} = 2\Delta x y_{k+1} + 2\Delta x - 2c\Delta x - 2\Delta y x_{k+1} - \Delta y$$

Subtracting.

$$P_{k+1} - P_k = 2\Delta x y_{k+1} + \cancel{2\Delta x} - \cancel{2c\Delta x} - 2\Delta y . x_{k+1} - \cancel{\Delta y}$$
$$- 2\Delta x y_k - \cancel{2\Delta x} + \cancel{2c\Delta x} + 2\Delta y x_k + \cancel{\Delta y}$$

$$= 2\Delta x(y_{k+1} - y_k) - 2\Delta y(x_{k+1} - x_k) \quad \text{—(ii)}$$

If P_k is '-'ve i.e. if $P_k < 0$

$x_{k+1} = x_k$, $y_{k+1} = y_k + 1$.

from (II)

$P_{k+1} - P_k = 2\Delta x(y_k + 1 - y_k) - 2\Delta y(x_k - x_k)$

$\therefore P_{k+1} = P_k + 2\Delta x$.

If P_k is '+ve' i.e. if $P_k > 0$.

$x_{k+1} = x_k + 1$.

$y_{k+1} = y_k + 1$.

from (II)

$P_{k+1} - P_k = 2\Delta x(y_k + 1 - y_k) - 2\Delta y(x_k + 1 - x_k)$.

$P_{k+1} = P_k + 2(\Delta x - \Delta y)$

Initial decision parameter from (I).

$p_0 = 2\Delta x y_0 + 2\Delta x - 2c\Delta x - 2\Delta y x_0 - \Delta y$.

$= 2\Delta x y_0 - 2c\Delta x - 2\Delta y x_0 + 2\Delta x - \Delta y$.

$= 2\Delta x y_0 - 2(\Delta y x_0 + c\Delta x) + 2\Delta x - \Delta y$

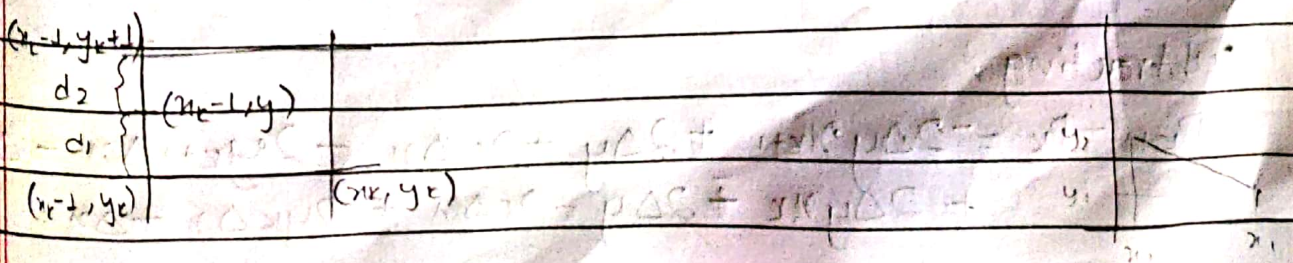
$= 2\Delta x y_0 - 2\Delta x\left(\frac{\Delta y}{\Delta x} x_0 + c\right) + (2\Delta x - \Delta y$.

$= \cancel{2\Delta x y_0} - \cancel{2\Delta x y_0} + 2\Delta x - \Delta y$

$= 2\Delta x - \Delta y$

4. Right to left, '-ve' slope, slope < 1.

→ As |slope| < 1, sampling is done in x.



④

Assuming the pixel at (x_k, y_k) to be displayed is determined, we next to decide with pixel to plot in column $x_k - 1$, the choices are.

$(x_k - 1, y_k)$ and $(x_k - 1, y_k + 1)$

weknow from the graph,

$$y = m(x_k - 1) + c$$
$$d_1 = y - y_k$$
$$d_2 = y_k + 1 - y$$

$$
\begin{aligned}
d_1 - d_2 &= y - y_k - (y_k + 1 - y) \\
&= y - y_k - y_k - 1 + y \\
&= 2y - 2y_k - 1 \\
&= 2(-m(x_k - 1) + c) - 2y_k - 1 \\
&= -2mx_k + 2m + 2c - 2y_k - 1 \\
&= -2\frac{\Delta y}{\Delta x} x_k + 2\frac{\Delta y}{\Delta x} + 2c - 2y_k - 1
\end{aligned}
$$

or, $\Delta x(d_1 - d_2) = -2\Delta y x_k + 2\Delta y + 2c\Delta x - 2y_k\Delta x - \Delta x$.

where, $\Delta x(d_1 - d_2) = P_k$ = decision parameter.

$$P_k = -2\Delta y x_k + 2\Delta y + 2c\Delta x - 2y_k\Delta x - \Delta x$$

For next step.

$$P_{k+1} = -2\Delta y x_{k+1} + 2\Delta y + 2c\Delta x - 2y_{k+1}\Delta x - \Delta x.$$

Subtracting,

$$
\begin{aligned}
P_{k+1} - P_k = &-2\Delta y x_{k+1} + 2\Delta y + 2c\Delta x - 2y_{k+1}\Delta x - \Delta x \\
&+ 2\Delta y x_k - 2\Delta y - 2c\Delta x + 2y_k\Delta x + \Delta x.
\end{aligned}
$$

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$$P_{k+1} - P_k = -2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k).$$

If $P_k < 0$. i.e. $d_1 > d_2$ as slope is '-ve'

$x_{k+1} = x_k - 1$ $y_{k+1} = y_k + 1$.

If $p_k < 0$ then $d_1 > d_2$, which implies pixel at y_{k+1} is nearer than pixel (y_k). So, next pixel co-ordinate is $(x_k - 1, y_{k+1})$

$$P_{k+1} - P_k = -2\Delta y(x_k - 1 - x_k) - 2\Delta x(y_k - y_k + 1)$$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x.$$

If $p_k > 0$ ie. $d_1 < d_2$ as slope is '-ve' which implies pixel at (y_k) is nearer than pixel y_{k+1} so next pixel is.

$x_{k+1} = x_k - 1$ $y_{k+1} = y_k$.

$$P_{k+1} - P_k = -2\Delta y(x_k - 1 - x_k) - 2\Delta x(y_k - y_k)$$

$$P_{k+1} = \cancel{P_k - 2(\Delta y + \Delta x)} \quad P_k + 2\Delta y.$$

Initial decision parameter.

$$P_0 = -2\Delta y x_0 + 2\Delta y + 2c\Delta x - 2y_0\Delta x - \Delta x.$$

$$= 2(\Delta y x_0 + c\Delta x) - 2y_0\Delta x + 2\Delta y - \Delta x.$$

$$= 2\Delta x\left(\frac{\Delta y}{\Delta x} x_0 + c\right) - 2y_0\Delta x + 2\Delta y - \Delta x$$

$$= 2\Delta x y_0 - 2y_0\Delta x + 2\Delta y - \Delta x$$

$$= +2\Delta y - \Delta x$$

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Algorithm.

1. start
2. Input $(x_1, y_1), (x_2, y_2)$
3. $\Delta x = abs(x_2 - x_1)$; $\Delta y = abs(y_2 - y_1)$.
4. If $(x_2 > x_1)$

   $xinc = 1$

   else

   $xinc = -1$.
5. If $(y_2 > y_1)$

   $yinc = 1$

   else

   $yinc = -1$.
6. ~~plot~~ $x = x_1$, $y = y_1$
7. plot (x, y)
8. If $(\Delta x > \Delta y)$

   $p_0 = 2\Delta y - \Delta x$.

   for $k = 0$ to Δx:

   if $(p_k < 0)$

   $x = x + xinc$

   $y = y$

   $p_{k+1} = p_k + 2\Delta y$

   Else

   $x = x + xinc$

   $y = y + yinc$

   $p_{k+1} = p_k + 2\Delta y - 2\Delta x$.

   Plot (x, y)

   else

   $p_0 = 2\Delta x - \Delta y$.

   for $k = 0$ to Δy

Scanned with CamScanner

(7)

if (pk < 0)

$x = x$.

$y = y + yinc$.

$p_{k+1} = p_k + 2\Delta x$.

else.

$x = x + xinc$

$y = y + yinc$.

$p_{k+1} = p_k + 2\Delta x - 2\Delta y$.

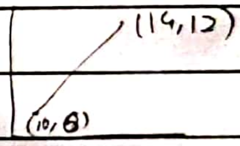
plot (x, y)

9. End.

Example.

L-R, slope > 0, slope = +ve.

(10, 6) (14, 12.)



Soln,

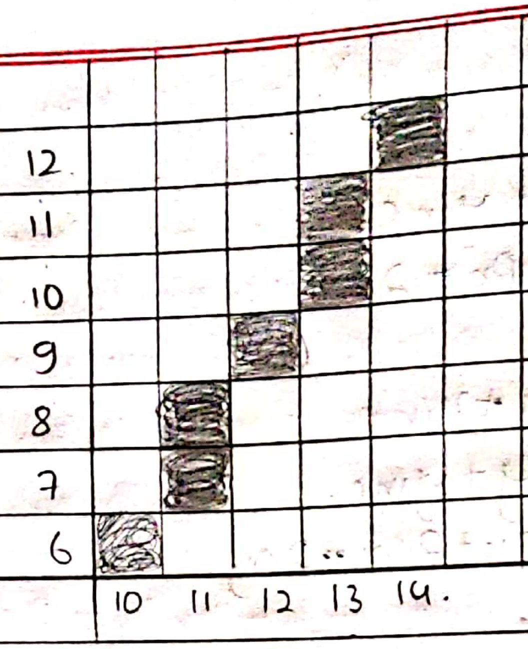
$\Delta x = 4$, $\Delta y = 6$.

(10, 6) → First pixel.

Initial decision parameter $(p_0) = 2\Delta x - \Delta y$.

$= 2 \times 4 - 6 = 2$.

K	x_{k+1}	y_{k+1}	P_{k+1}	
0	11	7	-2	$P_{k+1} = P_k + 2\Delta x - 2\Delta y = 2 + 2*4 - 2*6 = -2$
1.	11	8	6.	$P_{k+1} = P_k + 2\Delta x = -2 + 2*4 = 6$.
2	12.	9	2.	$P_{k+1} = P_k + 2\Delta x - 2\Delta y = 2$.
3	13	10	-2.	$P_{k+1} = P_k + 2\Delta x - 2\Delta y = -2$.
4	13	11	6.	$P_{k+1} = -2 + 2\Delta x = 6$.
5	14	12.	2.	$P_{k+1} = 6 + 2*4 - 2*6 = 2$.

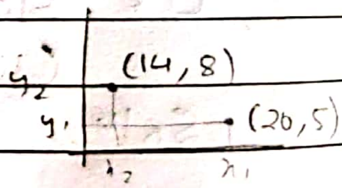


Pixel Plot.

Example:

R-L , '-ve' slope , slope < 1.

(20,5) , (14,8)



Sol^n,

$\Delta x = 14 - 20 = -6 = |\Delta x|$

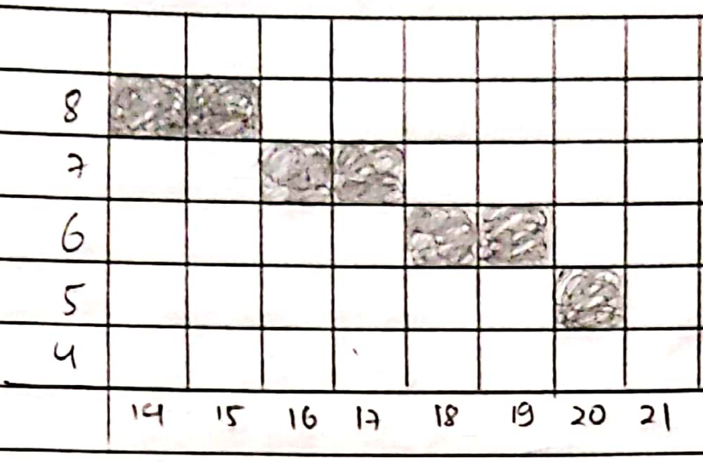
$\Delta y = 8 - 5 = 3.$

(20,5) → First Pixel.

Initial decision parameter $(P_0) = 2\Delta y - \Delta x.$

$= 2 \times 3 - 6 = 0$

K	x_{k+1}	y_{k+1}	P_{k+1}	
0	19	6.	-6.	$P_{k+1} = P_k + 2\Delta y - 2\Delta x$
1.	18	6.	0	$P_{k+1} = P_k + 2\Delta y$
2.	17	7	-6.	$P_{k+1} = P_k + 2\Delta y - 2\Delta x$
3.	16	7	0	$P_{k+1} = P_k + 2\Delta y$
4.	15	8	-6	$P_{k+1} = P_k + 2\Delta y - 2\Delta x$
5	14	8	0	$P_{k+1} = P_k + 2\Delta y$



Pixel Plot.

Discussion and conclusion.

In this lab we analyse the line drawing process using Bresenhams algorithm which was more efficient, accurate than DDA algorithm. We derived the formula, wrote the algorithm and implemented. by solving the examples.