

Lab Sheet - 4

Simulation of the chemical reaction (continuous system)

Objectives:

- To develop the mathematical modeling of the (continuous system) mass spring damper system.
- To determine the state of the system i.e. x , distance moved at different point of time.

Theory

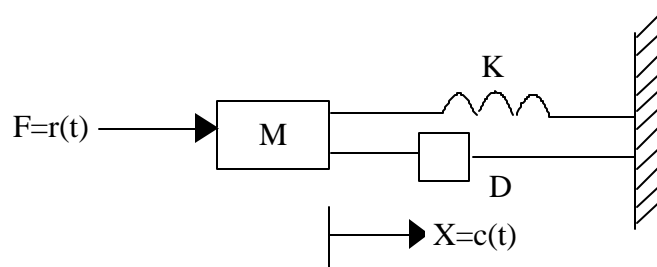
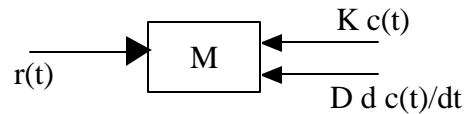


Figure 1: A mass-spring damper system

The free body diagram of the above system is



According to Newton's law of motion, we have

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

The motion of the system is described by the following differential equation:

$$M \times \frac{d^2 C(t)}{dt^2} = Kr(t) - \left(D \frac{dC(t)}{dt} + KC(t) \right)$$

$$M \times \frac{d^2 C(t)}{dt^2} + D \frac{dC(t)}{dt} + KC(t) = Kr(t)$$

Therefore, the mathematical modeling of a mass spring damper system with mass M , force $r(t)$, damper coefficient D , spring coefficient K and displacement $C(t)$ is given by

$$M \times \frac{d^2 C(t)}{dt^2} + D \frac{dC(t)}{dt} + KC(t) = Kr(t) \quad \text{-----[I]}$$

This equation can be re-written as

$$\frac{d^2 C(t)}{dt^2} + \frac{D}{M} \frac{dC(t)}{dt} + \frac{K}{M} C(t) = \frac{1}{M} Kr(t) \quad \text{-----[II]}$$

Equation [I] is the mathematical model of the given Mathematical system.
Comparing equation [II] with general second order partial equation.

$$\frac{d^2 x(t)}{dt^2} + 2\xi W \frac{dx(t)}{dt} + W^2 x(t) = W^2 F$$

Where,

Damping ration $\xi = D/2.M.W$

Angular frequency of oscillation $W = \sqrt{K / M}$

The response of this system with unit step function depends on the value of ξ . The condition for the motion to occur without oscillation requires that $\xi \geq 1$. Show how x varies in response to a steady force applied at time $t=0$ for the various values of ξ .