CS 471

HW 5

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1) Abstract

In this assignment we model and simulate the flocking behavior of birds in a two-dimensional space. We derive a set of ODEs, as a mathematical model, that describes the flocking behavior of a flock of N birds.

2) Parameters

$Gamma\ 1$	Leader's attraction to feeder
$Gamma\ 2$	attraction of flock to leader
$Kappa$	likeliness of birds to stay in compact flock
$Lamda$	distance between birds in a flock
rho	repellent force between birds in a flock

3) Method

Let $B_k(t) = \big(x_k(t), y_k(t)\big)$ be the location of the k-th birdat time t, where k=1, 2 ,..., N. $B_1(t)$ is the location of the bird interested only in bird feeder at $C_k(t) = \big(x_c(t), y_c(t)\big)$. At any instance of time $B_1(t)$ we proceed as by using ODEs.

$$B_1'(t) = \gamma_1(C(t) - B_1(t))$$

Where $\gamma_1 > 0$ is a positive constant whose magnitude depends on food quality and the value is input by us. The rest of the birds try to stay close to the leader, therefore, they are governed by

$$B_k'(t) = \gamma_2(B_1(t) - B_k(t)), \qquad k = 2, \dots, N$$

Where $\gamma_2 > 0$ is also a positive constant whose magnitude is decided by us.

In order to maintain the safety of the flock the birds try to be as close as possible to the center if the flock, which is denoted by $\overline{B(t)} = \frac{\sum_{k=1}^{N} B_k(t)}{N}$. This can be described by a flocking force that holds the birds together:

$$F_k^{fl}(t) = \kappa(\overline{B(t)} - B_k(t)), \qquad k = 2, \dots, N$$

Where $\kappa > 0$ is also a positive constant whose magnitude is decided by us. This flocking force can be added to the right-hand side of the ODEs system.

If the birds get too close to each other there will be a strong repelling force. If the k-th bird B_k is repelled by its L closest neighbors, we add to the right-hand sides

$$F_k^{rep}(t) = \sum_{l=1}^{L} \rho \frac{(B_k(t) - B_l(t)}{(B_k(t) - B_l(t))^2 + \delta}, \quad k = 2, \dots, N$$

Where $\rho > 0, \delta > 0$.

For the 'smelly bird' the repelling force is different than the other birds. The nature of smelly bird is that it wants to be at the center of the flock, however the repelling force of smelly bird is larger and acts towards the other birds. So, as the smelly birds comes in play the flock moves in the direction to avoid the smelly. We have increased the value of ***rho (ρ)*** form the smelly and generated the video for the flock.

4) Conclusion

1.

$$B_1'(t) = \gamma_1(C(t) - B_1(t))$$
$$B_k'(t) = \gamma_2\big(B_1(t) - B_k(t)\big) + F_k^{fl}(t) + F_k^{rep}(t), \text{k} = 2, 3, \dots, \text{N}$$

The above equation above gives us the location of the birds and their flocking intimidation. $\boldsymbol{x_c} > \boldsymbol{x_1}$, means that the attraction force of the leader is greater than the force of the feeder. So, the birds are more attracted to the leader. However, these forces are comparable resulting in the leader being attracted to the feeder and rest of the birds to the leader.

2.

The number N of the birds affects the flocking behavior of the birds. When we have $N = 10$ the overall flocking pattern is loose which means the distance between individual birds is bigger. So, we can see

more scattered flocking pattern. As N increases the flocking pattern becomes denser, and the distance between the birds gets smaller. So, we can see smaller or more dense flock (documented in the movie).

$C(t) = (\sin(\alpha t), \cos(\alpha t))$ gives us a pattern with the leader moving in sin and cos direction attracting other birds in the same direction too. This shows that when the leader is moving other birds follow the leader. However, as $C(t) = (0, 0)$ the leader remains in the fixed direction with other birds only being affected by the repellent force between each other. Similarly, other birds continue in motion in a fixed boundary, i.e they are attracted to the leader but when they come very close to each other there is a small repelling force that makes them continue motion in fixed diameter around the leader(documented in the movie).

3.

Flock diameter changes in different scenarios, when the size of the birds is small the i.e. N is smaller, we can observe large flock diameter, i.e. birds are more scattered, as the value of N increases the diameter is larger initially but with time the flock diameter becomes small and the birds are gathered together, making the distance between each bird small.

Also, we concluded that the overall diameter of the flock seemed to converge with time, however there are outliers in our calculation i.e. when $C(t) = (0, 0)$ the overall diameter did not change over time.

4.

The nature of smelly bird is that it wants to be at the center of the flock. However, the repelling force of the smelly bird is larger and acts towards the other birds. So, as the smelly birds comes in play, the flock moves in the direction to avoid the smelly bird (documented in movie).