Report

Homework 3

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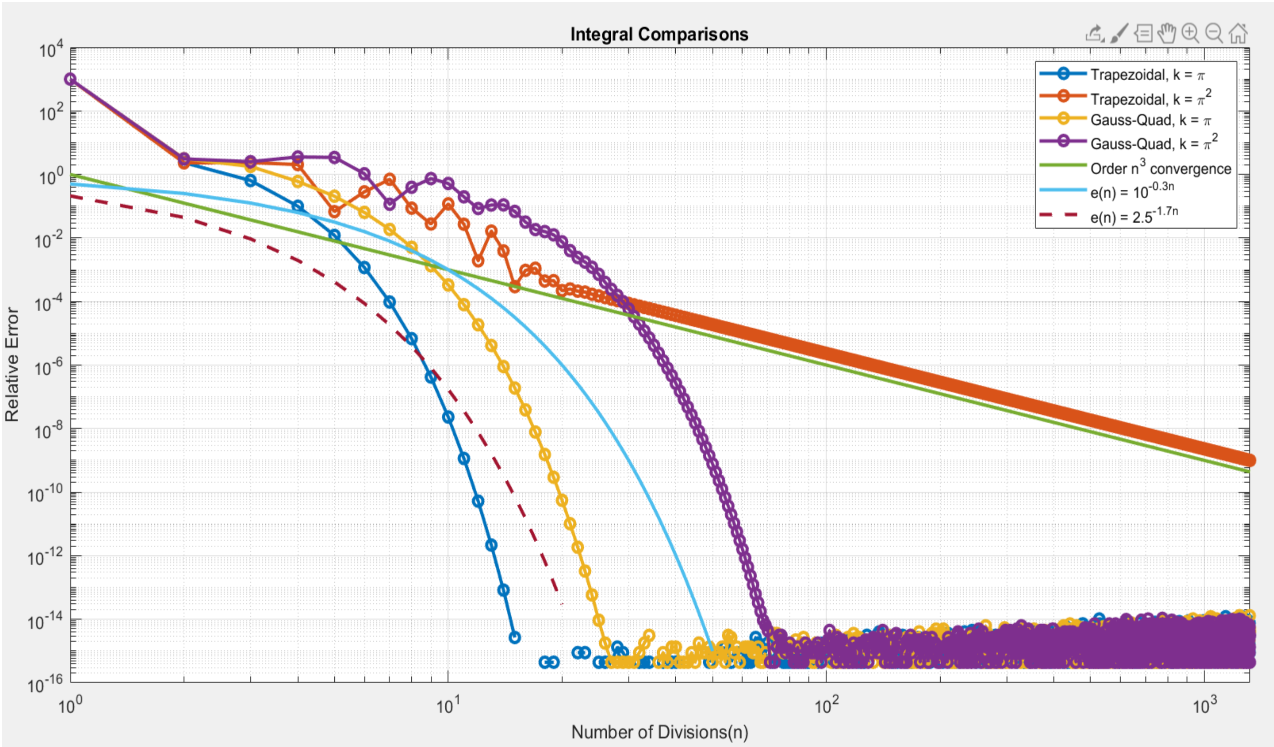
Experimentation with two different ways of computing approximate values of integral

For this assignment we use trapezoidal and Gaussian quadrature method to calculate the rate of convergence of the numerical integration of two functions.

$$f(x) = e^{\cos(\pi x)} and g(x) = e^{\cos\left(\pi^2 x\right)}$$

The plot below represents the error against n using a logarithmic scale for both axes.



1) **Trapezoidal Rule**

The trapezoidal rule belongs to a class of Newton-Cotes quadrature rules that approximate integrals using equidistant grids. The function $g(x)$ converges algebraically with $O(n\hat{} - 3)$. As we expect the absolute error to converge $O(n\hat{}2)$ the discrepancy in the given plot is caused because the error shown is relative error.

For f(x) the Trapezoid rule converges faster, as the result of Euler McLauren sum formula. Since the function itself is period, each of its derivatives are period on same interval which implies not only does $f‘$ approaches zero but as subsequent terms of higher order involve differences between the function's derivatives as xN and $x0$ this implies integral is exactly equal to sum. However, rather than this being true different order of convergence takes place that requires different analysis than Trapezoidal rule.

2) **Gaussian Quadrature**

In Newton–Cotes quadrature rules, it is assumed that the value of the integrand is known at equally spaced points. As shown in the plot Gaussian Quadrature showed a better rate of convergence than $O\left(h\hat{}n\right)$, they converge like $e\left(n\right) = c\hat{}\left(-an\right)$. As the exact form of convergence was not derived, two sample curves of the same function form were given as evidence of the approximate functional form for the convergence of Gaussian Quadrature method. As we can see in the k=pi(plot) the convergence is faster for gaussian quadrature too.