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Homework 2

CS 471

Using Newton method to find the roots of function

Introduction:

In this assignment we use Perl scripting to automate computation to find the roots of function f(x) using
Newtons method. We test different functions and determine their rate of convergence.

Functions

$$\mathbf{1)}\quad \boldsymbol{f(x) = x}$$

This is a linear function; the Newton method does not provide good approximation for the root of the function.

$$\mathbf{2)}\quad \boldsymbol{f(x) = x^2}$$

Newton's Method does converge to the root r = 0. The error formula is $e_i + 1 = \frac{e_i}{2}$, so the convergence is linear with convergence proportionality constant $S = \frac{1}{2}$

$$\mathbf{3)}\quad \boldsymbol{f(x) = Sin(x) + cos(x^2)}$$

Once convergence starts to take hold, the number of correct places in Xi approximately doubles on each iteration. So, the given function converges quadratically and Newton method provides best approximation for the function.

Modified Newton's Method

If f is $(m + 1)$ times continuously differentiable on $[a, b]$, which contains a root r of multiplicity $m > 1$,
then Modified Newton's Method

$$x_i + 1 = x_i - \frac{f(x_i)}{f'(x_i)} m$$

Converges locally and quadratically to r

Code:

The modified code is in HW2 sub directory. The file name is newtonS.pl.

Output:

The output of the code is in the temp.txt file and we also have some major values in the table below.

Convergence data

```
x,     01 , 0.0000000000000000E+00 ,  0.5000000000000000E+00 , 0.5000000000000000E-02 ,    0.5000000000000000E-04
 x,    02 , 0.0000000000000000E+00 , -0.0000000000000000E+00, -0.0000000000000000E+00 , -0.0000000000000000E+00
 x*x,  01 , -0.2500000000000000E+00 ,  0.2500000000000000E+00, 0.2500000000000000E-02 ,    0.2500000000000000E-04
 x*x,  02 , -0.1250000000000000E+00 ,  0.1250000000000000E+00, 0.5000000000000000E+00 ,    0.2000000000000000E+01
 x*x,  13 , -0.6103515625000000E-04 ,  0.6103515625000000E-04 ,  0.5000000000000000E+00 ,  0.4096000000000000E+04
x*x,   23 , -0.5960464477539062E-07 ,  0.5960464477539062E-07,   0.5000000000000000E+00 ,  0.4194304000000000E+07
 x*x,  24 , -0.2980232238769531E-07 ,  0.2980232238769531E-07,   0.5000000000000000E+00 ,  0.8388608000000000E+07
 x*x,  25 , -0.1490116119384766E-07 ,  0.1490116119384766E-07,   0.5000000000000000E+00 ,  0.1677721600000000E+08
 x*x,  46 , -0.7105427357601002E-14 ,  0.7105427357601002E-14,   0.5000000000000000E+00 ,  0.3518437208883200E+14
 x*x,  47 , -0.3552713678800501E-14 ,  0.3552713678800501E-14,   0.5000000000000000E+00 ,  0.7036874417766400E+14
 x*x,  48 , -0.1776356839400250E-14 ,  0.1776356839400250E-14,   0.5000000000000000E+00 ,  0.1407374883553280E+15
 x*x,  49 , -0.8881784197001252E-15 ,  0.8881784197001252E-15,   0.5000000000000000E+00 ,  0.2814749767106560E+15


sin(x)+cos(x*x),  01 , -0.9351046647281536E+00 , -0.4351046647281536E+00,  -0.4351046647281536E-02 ,  -0.4351046647281536E-04
 sin(x)+cos(x*x),  02 , -0.8546415960180649E+00 ,  0.8046306871008869E-01 ,  -0.1849280764672122E+00 ,  0.4250197514723320E+00
 sin(x)+cos(x*x),  03 , -0.8493901358009869E+00 ,  0.5251460217077986E-02,    0.6526547273506537E-01 ,  0.8111233362254577E+00
 sin(x)+cos(x*x),  04 , -0.8493688627401133E+00 ,  0.2127306087358230E-04,    0.4050884895671750E-02 ,  0.7713825732694478E+00
 sin(x)+cos(x*x),  05 , -0.8493688623926730E+00 ,  0.3474402480610000E-09,    0.1633240510736584E-04 ,  0.7677505933172054E+00
 sin(x)+cos(x*x),  06 , -0.8493688623926731E+00 , -0.6228807177220662E-16,   -0.1792770760435064E-06 ,  -0.5159939789475131E+03
```