Castaneda, Diaz-Gimenez, and Rios-Rull (2003) - Accounting for the U.S. Earnings and Wealth Inequality

Castaneda, Díaz-Giménez, and Ríos-Rull (2003) presents a model capable of capturing the inequality in the US earnings and wealth distributions, the later being a breakthrough at that time. Key to the model is the process of labour efficiency units combined with use of dynasties (infinite horizon value functions). Very high income dynasties expect their descendents to be much lower income and so save large fractions of their income, this leads to high levels of wealth inequality.

Only the first 9 Tables are reproduced here. Tables 10 and up are based on alternative calibrations and the parameter values underlying those alternatives have not survived and nor have the weights used as part of those alternative calibrations. Hence they could not be replicated. The replication codes do include code that implements alternative calibrations, and which work for a given combination of weights, but without the 'correct' weights they are not actually used for anything.

The model to be solved is now given, starting with the value function problem of the household.

$$V(a,s) = \max_{\substack{c\geq 0 \\ \tilde{a}\in\mathcal{A} \\ 0\leq h\leq \ell}} \frac{c^{1-\sigma_1}}{1-\sigma_1} + \frac{(\ell-h)^{1-\sigma_2}}{1-\sigma_2} + \beta \sum_{s'\in S} \Gamma_{ss'}\, V[a'(\tilde{a}), s'], \tag{1}$$

$$s.t. \quad c + \tilde{a} = y - \tau(y) + a, \tag{2}$$

$$y = a\,r + e(s)\,h\,w + \omega(s), \tag{3}$$

$$\tau(y) = a_0[y - (y^{-a_1} + a_2)^{-1/a_1}] + a_3 y \tag{4}$$

$$a'(\tilde{a}) = \begin{cases} \tilde{a} - \tau_e(\tilde{a}) \text{ if } s\in\mathcal{R} \text{ and } s'\in\mathcal{E}, \\ \tilde{a} \text{ otherwise.} \end{cases} \tag{5}$$

$$\Gamma_{ss'} \tag{6}$$

Note: for VFI Toolkit this is a Case 2 value function problem (in nomenclature of SLP1989), as next period assets cannot be chosen directly due to existence of estate taxes. Notice that household income, which we denote by y, includes three terms: capital income, $y_k = a\,r$, labor income, $y_l = e(s)\,h\,w$, and retirement pensions, ω. Every household can earn capital income. Only workers can earn labor income. And only retirees receive retirement pensions. We denote labour supply by h, while CDGRR2003 use l. The estate taxes are

$$\tau_e(\tilde{a}) = \begin{cases} 0 \text{ for } \tilde{a} \leq \underline{a} \\ (a-\tilde{a})\tau_e \text{ for } \tilde{a} > \underline{a} \end{cases}$$

The retirement benefit $\omega(s)$ is zero for working age (first four states of s) and a constant for retirees (fifth to eigth states of s).

There is a representative firm with Cobb-Douglas production function (and perfect competition): $Y = K^\theta L^{1-\theta}$, where L is labour supply $L = \int he(s)d\mu$ (μ is agent distribution), as distinct from hours worked $H = \int hd\mu$. (Note: I depart from CDGRR2003 in that I use the reverse the notation of l and H relative to them.)

Since the actual definition for a stationary competitive equilibrium in this economy is essentially the same as that of Aiyagari (1994) I largely omit it here (I skip the definitions of some aggregates, and the law-of-motion for the agent distribution). I do provide the general equilibrium conditions. There are two. The first general equilibrium condition is the 'same' as that of Aiyagari (1994), namely that the interest rate (the rate of return on capital) is equal to the marginal product of capital (minus the depreciation rate, as it is the net return); $r = \theta K^{\theta-1} L^{1-\theta} - \delta$.[1] The second general equilibrium condition is the Government budget constraint, $Tr + G = T$, where T is tax revenue (from income and estate taxes), and Tr is transfers (the retiree benefits ω). Finding the general equilibrium amounts to finding r and G that satisfy these two general equilibrium conditions.

An important part of this 'infinite-horizon dynasties' setup is the transition process on s which determines 'stochastic aging' and also 'productivity level'. s has eight states, four producitivity levels crossed with two 'ages' (worker and retiree). The eight-by-eight transition matrix $\Gamma_{ss'}$ plays an important role in this model and is described in full below. This description also contains a correction to a typo in the original paper (in the equations relating to ϕ_1 and ϕ_2.

We use the one-dimensional shock, s, to denote the household's random age and random endowment of efficiency labor units jointly.[2]

The process on s is independent and identical across households, and follows a finite state Markov chainwith conditional transition probabilities given by $\Gamma = \Gamma(s' \mid s) = Pr\{s_{t+1} = s' \mid s_t = s\}$, where s and $s' \in S$. We assume that s takes values in one of two possible J–dimensional sets, $\mathcal{E}$ and $\mathcal{R}$. Therefore the formal description of set S is $S = \mathcal{E} \cup \mathcal{R} = \{1, 2, \ldots, J\} \cup \{J+1, J+2, \ldots, 2J\}$. When a household draws shock $s \in \mathcal{E}$, it is a worker and its endowment ofefficiency labor units is $e(s) > 0$. When a household draws shock $s \in \mathcal{R}$ it is a retiree. When a household's shock changes from $s \in \mathcal{E}$ to $s' \in \mathcal{R}$, we say that it has retired and when it changes from $s \in \mathcal{R}$ to $s' \in \mathcal{E}$, we say that it has died and has been replaced by a working-age descendant. When a household dies, its estate is liquidated, and its descendant inherits a fraction $1 - \tau_e(\tilde{a})$ of the estate, where $\tilde{a}$ denotes the value of the household's stock of wealth at the end of the period, and $\tau_e(\tilde{a})$ represents estate taxes.

This specification of the joint age and endowment process implies that the transition probability matrix, Γ, controls the demographics of the model economy, the life-cycle profile of earnings, and their intergenerational persistence (in combination with hours worked choices). When we come to calibrating this markov process it is done based on these issues; demographics, life-cycle profile of earnings, and intergenerational persistence of earnings.

To specify the process on s (and the values for $e(s)$) we must choose the values of $(2J)^2 + J$ parameters, of which $(2J)^2$ are the conditional transition probabilities and the remaining J are the values of the endowment of efficiency labor units. To reduce this large number of parameters, we impose some additional restrictions on matrix Γ. To understand these restrictions better, it helps to consider the following partition of matrix Γ:

$$\Gamma = \begin{bmatrix} \Gamma_{\mathcal{E}\mathcal{E}} & \Gamma_{\mathcal{E}\mathcal{R}} \\ \Gamma_{\mathcal{R}\mathcal{E}} & \Gamma_{\mathcal{R}\mathcal{R}} \end{bmatrix}$$

[1] Note that unlike Aiyagari (1994), labour supply is endogenous and so actually has to be calculated as part of evaluating this.

[2] To ease interpretation, we use $e(s)$ to denote the endowment of efficiency labour units, and simply have s take integer values. In principle, s and $e(s)$ could be combined to be one object. By seperating the efficiency labor units, $e(s)$, from the actual values taken by s, it is easier to see how earnings ability is transferred from one generation to the next, the transitions of s, without being confused by the fact that $e(s) = 0$ in all of the retirement states.

Submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$ contains the transition probabilities of working-age households that are still of working-age one period later. Since we impose no restrictions on these transitions, to characterize $\Gamma_{\mathcal{E}\mathcal{E}}$ we must choose the values of J^2 parameters.

Submatrix $\Gamma_{\mathcal{E}\mathcal{R}}$ describes the transitions from the working-age states into the retirement states. The value of this submatrix is $\Gamma_{\mathcal{E}\mathcal{R}} = p_{e\varrho} I$, where $p_{e\varrho}$ is the probability of retiring and I is the identity matrix. This is because we assume that every working-age household faces the same probability of retiring, and because we use only the last realization of the working-age shock to keep track of the earnings ability of retirees. Consequently, to characterize $\Gamma_{\mathcal{E}\mathcal{R}}$ we must choose the value of only one parameter.

Submatrix $\Gamma_{\mathcal{R}\mathcal{E}}$ describes the transitions from the retirement states into the working-age states that take place when a retiree exits the economy and is replaced by a working-age descendant. The rows of this submatrix contain a two parameter transformation of the stationary distribution of $s \in \mathcal{E}$, which we denote by $\gamma^*_{\mathcal{E}}$. This transformation allows us to control both the life-cycle profile of earnings and its intergenerational correlation. Intuitively, the transformation amounts to shifting the probability mass from $\gamma^*_{\mathcal{E}}$ towards both the first row of $\Gamma_{\mathcal{R}\mathcal{E}}$ and towards its diagonal. The exact description of this is given below. Consequently, to characterize $\Gamma_{\mathcal{R}\mathcal{E}}$ we must choose the value of the two shift parameters.

Finally, submatrix $\Gamma_{\mathcal{R}\mathcal{R}}$ contains the transition probabilities of retired households that are still retired one period later. The value of this submatrix is $\Gamma_{\mathcal{R}\mathcal{R}} = p_{\varrho\varrho} I$, where $(1 - p_{\varrho\varrho})$ is the probability of exiting the economy. This is because the type of retired households never changes, and because we assume that every retired household faces the same probability of exit. Therefore, to identify this submatrix we must choose the value of only one parameter.

To keep the dimension of the process on s as small as possible while still being able to achieve our calibration targets, we choose $J = 4$. Therefore, to characterize the process on s (and the values of $e(s)$), we must choose the values of $(J^2 + 4) + J = 24$ parameters. Notice that we have not yet imposed that Γ must be a Markov matrix. When we do this, the number of free parameters is reduced to 20.

In practice the easiest is actually to normalize $\Gamma_{\mathcal{E}\mathcal{E}}$, with normalization of Γ following trivially from this. The choice of the normalization is, for computational reasons best done to the diagonals of $\Gamma_{\mathcal{E}\mathcal{E}}$; e.g., set $p^{ee}_{22} = 1 - p^{ee}_{21} - p^{ee}_{23} - p^{ee}_{24}$; where $\Gamma_{\mathcal{E}\mathcal{E}} = (1 - p_{e\varrho}) * [p^{ee}_{ij}]$. Since the non-diagonals are smaller by having the diagonals given by whatever was leftover to make the row sum up to one we avoided the problem that they may end up being negative — something that occoured in an earlier version of the codes where we used the last element of each row for the normalization. This saves throwing out many 'evaluations' of parametrizations (during calibration) due to that specific vector of parameters containing negative elements in the transition matrix. (Note: this normalization of diagonals is part of the replication, and is different (improved) from that of the original paper.)

The following explains the definition of parameters ϕ_1 and ϕ_2, and how they affect the transition matrix. Let p_{ij} denote the transition probability from $i \in \mathcal{R}$ to $j \in \mathcal{E}$, let γ^*_i be the invariant measure of households that receive shock $i \in \mathcal{E}$, and let ϕ_1 and ϕ_2 be the two parameters that shift the probability mass towards the diagonal and towards the first column of submatrix $\Gamma_{\mathcal{E}\mathcal{E}}$, then the recursive procedure that we use to compute the p_{ij} is the following:

• Step 1: First, we use parameter ϕ_1 to shift the probability mass from a matrix with vector $\gamma^*_{\mathcal{E}} = (\gamma^*_1, \gamma^*_2, \gamma^*_3, \gamma^*_4)$ in every row towards its diagonal, as follows:

$$
\begin{aligned}
p_{51} &= \gamma_1^* + \phi_1\gamma_2^* + \phi_1^2\gamma_3^* + \phi_1^3\gamma_4^* \\
p_{52} &= (1-\phi_1)[\gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^*] \\
p_{53} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{54} &= (1-\phi_1)\gamma_4^* \\
p_{61} &= (1-\phi_1)\gamma_1^* \\
p_{62} &= \phi_1\gamma_1^* + \gamma_2^* + \phi_1\gamma_3^* + \phi_1^2\gamma_4^* \\
p_{63} &= (1-\phi_1)[\gamma_3^* + \phi_1\gamma_4^*] \\
p_{64} &= (1-\phi_1)\gamma_4^* \\
p_{71} &= (1-\phi_1)\gamma_1^* \\
p_{72} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{73} &= \phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^* + \phi_1\gamma_4^* \\
p_{74} &= (1-\phi_1)\gamma_4^* \\
p_{81} &= (1-\phi_1)\gamma_1^* \\
p_{82} &= (1-\phi_1)[\phi_1\gamma_1^* + \gamma_2^*] \\
p_{83} &= (1-\phi_1)[\phi_1^2\gamma_1^* + \phi_1\gamma_2^* + \gamma_3^*] \\
p_{84} &= \phi_1^3\gamma_1^* + \phi_1^2\gamma_2^* + \phi_1\gamma_3^* + \gamma_4^*
\end{aligned}
$$

• Step 2: Then for $i = 5, 6, 7, 8$ we use parameter ϕ_2 to shift the resulting probability mass towards the first column as follows:

$$
\begin{aligned}
p_{i1} &= p_{i1} + \phi_2 p_{i2} + \phi_2^2 p_{i3} + \phi_2^3 p_{i4} \\
p_{i2} &= (1-\phi_2)[p_{i2} + \phi_2 p_{i3} + \phi_2^2 p_{i4}] \\
p_{i3} &= (1-\phi_2)[p_{i3} + \phi_2 p_{i4}] \\
p_{i4} &= (1-\phi_2)p_{i4}
\end{aligned}
$$

References

S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. Quarterly Journal of Economics, 109(3):659–684, 1994.

Ana Castaneda, Javier Díaz-Giménez, and Jose Victor Ríos-Rull. Accounting for the U.S. earnings and wealth inequality. Journal of Political Economy, 111(4):818–857, 2003.

Table 1: Table 3 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)
Parameter Values for the Benchmark Model Economy

Preferences		
Time discount factor	β	0.942
Curvature of consumption	σ_1	1.500
Curvature of leisure	σ_2	1.016
Relative share of consumption and leisure	χ	1.138
Endowment of productive time	ℓ	3.200
Age and endowment process		
Common probability of retiring	$p_{e\varrho}$	0.022
Common probability of dying	$1 - p_{\varrho\varrho}$	0.066
Life cycle earnings profile	ϕ_1	0.969
Intergenerational persistence of earnings	ϕ_2	0.525
Technology		
Capital share of income	θ	0.376
Capital depreciation rate	δ	0.059
Fiscal policy		
Government consumption	G	0.296
Normalized Retirement pensions	ω	0.696
Income tax function parameters	a_0	0.258
	a_1	0.768
	a_2	0.491
	a_3	-0.058
Estate tax function parameters:		
Tax-exempt level	$\underline{z}$	14.101
Marginal tax rate	τ_E	0.160

Some minor changes to the precise description of the parameters are made from the original.

Table 2: Original Table 3 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 3
PARAMETER VALUES FOR THE BENCHMARK MODEL ECONOMY

	Parameter	Value
Preferences:		
Time discount factor	β	.924
Curvature of consumption	σ_1	1.500
Curvature of leisure	σ_2	1.016
Relative share of consumption and leisure	χ	1.138
Productive time	ℓ	3.200
Age and employment process:		
Common probability of retiring	$p_{e\varrho}$	.022
Common probability of dying	$1 - p_{\varrho\varrho}$	.066
Earnings life cycle controller	ϕ_1	.969
Intergenerational earnings persistence controller	ϕ_2	.525
Technology:		
Capital share	θ	.376
Capital depreciation rate	δ	.059
Government policy:		
Government expenditures	G	.296
Normalized transfers to retirees	ω	.696
Income tax function parameters	a_0	.258
	a_1	.768
	a_2	.491
	a_3	.144
Estate tax function parameters:		
Tax-exempt level	$\underline{z}$	14.101
Marginal tax rate	τ_E	.160

Table 3: Tables 4 and 5 of Castaneda, Diaz-Gimenez and Rios-Rull (2003) Transition Probabilities of the Process on the Endowment of Efficiency Labor Units for Working-Age Households That Remain at Working Age One Period Later $\Gamma_{\mathcal{E}\mathcal{E}}$ (%)

			$\Gamma_{\mathcal{E}\mathcal{E}}$ (%) From s To s'			
	$e(s)$	γ^*_s (%)	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	1.00	69.15	98.43	1.17	0.40	0.01
$s = 2$	3.15	19.67	3.14	96.48	0.38	0.00
$s = 3$	9.78	11.13	1.53	0.44	98.01	0.02
$s = 4$	1061.00	0.05	10.90	0.50	6.25	82.35

Table 4: Original Tables 4 and 5 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 4

Transition Probabilities of the Process on the Endowment of Efficiency Labor Units for Working-Age Households That Remain at Working Age One Period Later, $\boldsymbol{\Gamma}_{\varepsilon\varepsilon}$ (%)

	To s'			
From s	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.24	1.14	.39	.006
$s = 2$	3.07	94.33	.37	.000
$s = 3$	1.50	.43	95.82	.020
$s = 4$	10.66	.49	6.11	80.51

TABLE 5

Relative Endowments of Efficiency Labor Units, $e(s)$, and the Stationary Distribution of Working-Age Households, $\boldsymbol{\gamma}^*_{\varepsilon}$

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.15	9.78	1,061.00
$\boldsymbol{\gamma}^*_{\varepsilon}$ (%)	61.11	22.35	16.50	.0389

Table 5: Table 6 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

Values of the Targeted Ratios and Aggregates in the United States and in the Benchmark Model Economies

	K/Y	I/Y	G/Y	Tr/Y	T_E/Y	$mean(h)$	CV_C/CV_H	$e_{40/20}$	$ho(f,s)$
Target (USA)	3.13	18.6%	20.2%	4.9%	0.20%	30.0%	3.00	1.30	0.40
Benchmark	4.30	25.9%	6.8%	4.0%	0.51%	23.7%	0.35	1.02	0.74

Note: Variable $mean(h)$ (column 6) denotes the average share of disposable time allocated to the market. The statistic CV_c/CV_h (column 7) is the ratio of the coefficients of variation of consumption and of hours worked. $e_{40/20}$ is the ratio of average earnings of 40 year old to 20 year old. $ho(f,s)$ the intergenerational correlation coefficient between lifetime earnings of father and so. Note that model actually has households, while data is individuals.

Table 6: Original Table 6 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 6

Values of the Targeted Ratios and Aggregates in the United States and in the Benchmark Model Economies

	K/Y (1)	I/Y (2)	G/Y (3)	Tr/Y (4)	T_E/Y (5)	h (6)	CV_c/CV_l (7)	$e_{40/20}$ (8)	$\rho(f, s)$ (9)
Target (United States)	3.13	18.6%	20.2%	4.9%	.20%	30.0%	3.00	1.30	.40
Benchmark	3.06	18.1%	20.8%	4.4%	.20%	31.2%	3.25	1.09	.25

NOTE.—Variable h (col. 6) denotes the average share of disposable time allocated to the market. The statistic CV_c/CV_l (col. 7) is the ratio of the coefficients of variation of consumption and of hours worked.

Table 7: Table 7 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

Distributions of Earnings and of Wealth in the United States and in the Benchmark Model Economies (%)

		QUINTILE					(TOP GROUPS (Percentile)		
ECONOMY	GINI	First	Second	Third	Fourth	Fifth	90th-95th	95th-99th	99th-100th
				A. Distribution of Earnings					
United States	0.63	-0.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	0.63	0.00	4.79	12.99	14.76	64.61	14.53	16.96	17.27
				B. Distribution of Wealth					
United States	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	0.84	0.05	0.27	0.64	12.55	85.10	17.33	17.62	35.82

Table 8: Original Table 7 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 7

Distributions of Earnings and of Wealth in the United States and in the Benchmark Model Economies (%)

		Quintile					Top Groups (Percentile)		
Economy	Gini	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
				A. Distributions of Earnings					
United States	.63	−.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	.63	.00	3.74	14.59	15.99	65.68	15.15	17.65	14.93
				B. Distributions of Wealth					
United States	.78	−.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	.79	.21	1.21	1.93	14.68	81.97	16.97	18.21	29.85

Table 9: Table 8 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

Distribution of Consumption in the United States and in the Benchmark Model Economies (%)

		QUINTILE					(TOP GROUPS (Percentile)		
ECONOMY	GINI	First	Second	Third	Fourth	Fifth	90th-95th	95th-99th	99th-100th
United States:									
Nondurables	0.32	6.87	12.27	17.27	23.33	40.27	9.71	10.30	4.83
Nondurables+*	0.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
Benchmark:									
Wealthiest 1% Excluded	0.38	6.00	12.43	12.51	19.95	45.36	13.66	13.81	3.86
Entire Sample	0.44	5.38	11.04	11.13	18.12	50.98	13.10	12.31	12.88

*: Includes imputed services of consumer durables.

Table 10: Original Table 8 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 8

DISTRIBUTIONS OF CONSUMPTION IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

		QUINTILE					TOP GROUPS (Percentile)		
ECONOMY	GINI	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
United States:									
Nondurables	.32	6.87	12.27	17.27	23.33	40.27	9.71	10.30	4.83
Nondurables+*	.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
Benchmark:									
Wealthiest 1% excluded	.40	5.23	12.96	13.55	20.41	47.85	12.77	14.89	3.83
Entire sample	.46	4.68	11.58	12.07	18.68	52.99	12.82	13.45	11.94

* Includes imputed services of consumer durables.

Table 11: Table 9 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)
Earnings and Wealth Persistence in the United States and in the Benchmark Model Economies:
Fraction of Households That Remain In The Same Quintile After Five Years

	QUINTILE				
ECONOMY	First	Second	Third	Fourth	Fifth
	A. Earnings Persistence				
United States	0.86	0.41	0.47	0.46	0.66
Benchmark	0.00	0.83	0.00	0.86	0.83
	A. Wealth Persistence				
United States	0.67	0.47	0.45	0.50	0.71
Benchmark	0.81	0.31	0.85	0.79	0.90

Note: Based on 10000000 simulations.

Table 12: Original Table 9 of Castaneda, Diaz-Gimenez and Rios-Rull (2003)

TABLE 9
EARNINGS AND WEALTH PERSISTENCE IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES: FRACTIONS OF HOUSEHOLDS THAT REMAIN IN THE SAME QUINTILE AFTER FIVE YEARS

	QUINTILE				
ECONOMY	First	Second	Third	Fourth	Fifth
	A. Earnings Persistence				
United States	.86	.41	.47	.46	.66
Benchmark	.76	.55	.65	.80	.80
	B. Wealth Persistence				
United States	.67	.47	.45	.50	.71
Benchmark	.81	.80	.80	.75	.89