Restuccia and Urrutia (2004) - Intergenerational Persistence of Earnings: The Role of Early and College Education

Restuccia and Urrutia (2004) present a general equilibrium 2-period OLG model to study how much of cross-country differences in the intergenerational persistence of earnings can potentially be explained by the interactions between innate ability, income, and private eduction (high income parents being able to spend more on the private education of their child). They also consider how private education can be crowded out by public education, and how this might effect inetergenerational persistence of earnings and economic efficiency. From perspective of computation the complication is that they have dynasties (so care directly about value function of child) and that the problems to be solved by young (1st period) and old (2nd period) households look very different. Using the VFI toolkit to solve this model requires rewriting in an equivalent, but slightly different looking, mathematical form, as described here.

Compared to the original paper there are two main surface/notational differences: variables h, $\hat{b}$ and b are called b, $\hat{\pi}$, and π by Restuccia and Urrutia (2004). And the way the decision to attend college and whether student completes or drops out is determined: here $s \in \{0,1\}$ is the decision to attend, and completion/dropout is determined by exogenous state (shock) $\hat{\theta} \sim U[0,1]$, while in Restuccia and Urrutia (2004) $\theta \in \{0,1\}$ is the decision to attend, and the actual complete/dropout shock value is never denoted as it does not need to be kept.

Table 1 reports σ_b as 0.48, and first eqn on pg 1359 describes σ_b as the standard deviation of the innovations to the AR(1) in logs process on innate ability b. This is incorrect, and the reported value of σ_b in Table 1 corresponds to the standard deviation of log(b); the value for the innovations is just $(1-\rho)^2\sigma_b$. (In their notation, σ_b is called σ_π.) In this replication I define σ_b as the standard deviation of $\log(b)$, thus meaning that the same values are reported for Tables 1 and 2, but it's definition in the model is corrected to fit these. Relatedly, Restuccia and Urrutia (2004) are not quite explicit on evolution of b, but it appears that b is constant between ages $j=1$ and $j=2$, and then follows the aforementioned AR(1)-in-logs between $j=2$ and $j=1$.

Note that the 'college dropout premium', $\underline{\text{p}}$, is less than one. So model does imply college dropouts receive lower wages than those who do not attend (lower wage-per-unit-of-time, not just lower earnings). This is likely counterfactual.

0.0.1 The Household problem

The household lives for two-periods, with different problems at each age, and with intergenerational linkages.

From the perspective of VFI Toolkit there are two decision (d) variables, three endogenous state (a) variables, and two exogenous state (z) variables. But at each of the two ages, j=1 and j=2, only a certain few of these are actually relevant. (A third exogenous state variable is required later in the paper as part of the exercise underlying Table 7.)

The two decision variables are: $\hat{b}$ acquired ability of the child, and s a schooling choice (decision to send child to college); both decision variables are only relevant at age j=1. The three exogenous state variables are: h human capital, $\hat{b}$ acquired ability of the child, and s whether the child is attending school (college); these are relevant for both ages, only age $j=2$, and only age $j=2$ respectively. The two exogenous state variables are: b innate ability of the child, and $\hat{\theta}$ which determines whether a child attending school completes college or drops out; b is relevant at both ages, $\hat{\theta}$ is only relevant at age $j = 2$.

The value function problem for young (age $j = 1$) households is given by

$$
\begin{aligned}
V_1(h, b) = & \max_{c \geq 0,\, e \geq 0, s, \hat{b}} \left\{ \frac{c^{1-\sigma} - 1}{1 - \sigma} + \beta E[V(h', \hat{b}, s, b', \hat{\theta})] \right\} \\
& \text{s.t. } c + e = (1 - \tau) w h \\
& \quad \hat{b} = b(e + g)^{\gamma}, \quad 0 \leq \gamma \leq 1 \\
& \quad s \in \{0, 1\} \\
& \quad h' = \zeta h \\
& \quad b' = b \\
& \quad \hat{\theta} \sim U[0, 1]
\end{aligned}
$$

where w, τ, g, σ, β, ζ and γ are all parameters from the household's perspective. Notice that e is determined by choice of $\hat{b}$ (given b); so VFI Toolkit takes advantage of this and considers that it is $\hat{b}$ that is chosen rather than e (b is multiplying the $(e + g)^{\gamma}$ term).[1]

The value function problem for old (age $j = 2$) households is given by

$$
\begin{aligned}
V_2(h, \hat{b}, s, b, \hat{\theta}) = & \max_{c \geq 0,\, e \geq 0} \left\{ \frac{c^{1-\sigma} - 1}{1 - \sigma} + \beta E[V_1(h', b')] \right\} \\
& \text{s.t. } q(\hat{b}) = min\{\psi_0 (1 + \hat{b})^{\psi_1}, 1\} \\
& \quad \text{if } s = 0 : \; c = (1 - \tau)(wh + wh') \\
& \quad \text{if } s = 1 \,\&\, q(\hat{b}) < \hat{\theta} : \; c + (1 - \kappa(wh)) f \underline{n} = (1 - \tau)(wh + wh'(1 - \underline{n})) \\
& \quad \text{if } s = 1 \,\&\, q(\hat{b}) > \hat{\theta} : \; c + (1 - \kappa(wh)) f \bar{n} = (1 - \tau)(wh + wh'(1 - \bar{n})) \\
& \quad \text{if } s = 0 : h' = \bar{b} \\
& \quad \text{if } s = 1 \,\&\, q(\hat{b}) < \hat{\theta} : \; h' = \underline{p} \bar{b} \\
& \quad \text{if } s = 1 \,\&\, q(\hat{b}) > \hat{\theta} : \; h' = \bar{p} \bar{b} \\
& \quad \kappa(wh) = \min\{\max\{\kappa_1 - \kappa_0 wh, 0\}, 1\} \\
& \quad log(b') = \rho_b log(b) + \epsilon_b, \quad \epsilon \sim N(0, \sigma^2_{\epsilon_b})
\end{aligned}
$$

where f, σ, β, ψ_0, ψ_1, τ, w, $\underline{p}$, $\bar{p}$, $\underline{n}$, $\bar{n}$ κ_0, κ_1 are all parameters from the household's perspective. Note that s is whether they attend college, and $q(\hat{b}) \lesseqgtr \hat{\theta}$ determines whether they complete ($>$) or dropout ($<$).[2]

[1] Consumption c is similarly residually determined by the budget constraint and so not really one of the decision variables.

[2] Thus, e.g., $s = 1$ and $q(\hat{b}) < \hat{\theta}$ corresponds to $\theta = 0$ in notation of Restuccia and Urrutia (2004) (they denote different value functions for $s = 0, 1$, while here were it is simply treated as an endogenous state variable). I use $\hat{\theta}$ notation specifically to emphasize that it is not the same as their θ, although it serves a similar role in the model in determining complete/dropout for university.

0.0.2 Rest of the Model Economy and General Equilibrium

All sectors of the economy are model as price-taking perfect competition. There is a representative firm with linear production function that solves the static profit maximization problem,

$$\max_{H^f \geq 0} \{Y - wH^f\} \text{ subject to } Y = AH^f$$

The labour market clearance requires

$$H^f = H \equiv \int h d\mu + \int [(1-s)\hat{b} + s(p\hat{b}(1-n))] d\mu$$

Government budget balance, tax revenues equals pulic education expenditure, requires

$$g + \kappa F = \tau Y$$

where $\kappa F \equiv f \int \kappa(wh) snd\mu_{j=2}$. Not needed for computation is that the aggregate resource constraint is given by $Y = AH = C + E + F + g$ where $F \equiv f \int snd\mu_{j=2}$.

Two observations that make this easier to compute, first is that $w = A = 1$ (follows from perfect competition so wage equals marginal product of labour). Second, since production function is $Y = AH$ the demand for labour is $H^f \in [0, \infty)$ and so general equilibrium for the labour market is trivially satisfied.

0.0.3 Replication Results

The resulting Tables and Figures of the replication, together with those of the original, are now presented.

References

Diego Restuccia and Carlos Urrutia. Intergenerational persistence of earnings: The role of early and college education. American Economic Review, 94(5):1354–1378, 2004.

Table 1: Table 1 of Restuccia and Urrutia (2004)
Calibration of the Benchmark Economy

Target	Data	Model	Parameter	Value
(i) Fraction of non-college	0.54	0.51	ψ_0	0.27
(ii) Dropout rate	0.50	0.53	ψ_1	1.02
(iii) Early education/GDP	0.044	0.09	γ	0.24
(iv) College/GDP	0.028	0.02	f	0.64
(v) Public/Total college	0.64	0.58	κ_0	0.36
(vi) Average dropout premium	1.41	2.04	$\underline{p}$	0.86
(vii) Average college premium	2.33	2.52	$\bar{p}$	1.48
(viii) std(log earnings)	0.60	0.80	σ_b	0.48
(ix) Intergenerational correlation of earnings	0.40	0.52	ρ_b	0.20

Note: Data column is copy of original from Restuccia & Urrutia (2004), is not part of the replication.

Table 2: Original Table 1 of Restuccia and Urrutia (2004)

TABLE 1—CALIBRATION OF THE BENCHMARK ECONOMY

Target	Data	Model	Parameter	Value
(i) Fraction of non-college	0.54	0.54	ψ_0	0.27
(ii) Dropout rate	0.50	0.50	ψ_1	1.02
(iii) Early education/GDP	0.044	0.043	γ	0.24
(iv) College/GDP	0.028	0.027	f	0.64
(v) Public/total college	0.64	0.64	κ_0	0.36
(vi) Average dropout premium	1.41	1.38	$\underline{p}$	0.86
(vii) Average college premium	2.33	2.37	$\bar{p}$	1.48
(viii) std(log earnings)	0.60	0.60	σ_π	0.48
(ix) Intergenerational correlation of earnings	0.40	0.40	ρ	0.20

Table 3: Table 2 of Restuccia and Urrutia (2004)
Disparity and Persistence in the Benchmark Economy

	Innate Ability	Acquired Ability	Earnings
Cross-sectional disparity: std(log x)	0.48	0.55	0.80
Intergenerational correlation:	0.24	0.21	0.53

Note: In model notation these columns are: b, $\hat{b}$, and wh. I follow Restuccia & Urrutia (2004) in reporting as cross-sectional the number conditional on being an elderly household, not cross-sectional over the whole model economy. Restuccia & Urrutia (2004) explain calculation of intergenerational correlation of earnings at bottom of pg 1363. I assume the intergeneration correlations of (log) innate and acquired ability are calculated by the analagous regressions (with modification for acquired ability as is only observed for old).

Table 4: Original Table 2 of Restuccia and Urrutia (2004)

TABLE 2—DISPARITY AND PERSISTENCE IN THE BENCHMARK ECONOMY

	Innate ability	Acquired ability	Earnings
Cross-sectional disparity: std(log x)	0.48	0.51	0.60
Intergenerational correlation	0.20	0.41	0.40

Table 5: Table 3 of Restuccia and Urrutia (2004)
Decision Rules by Parents Earnings and Childs Ability

Panel A: Expenditures in Early Education			
	Young Parent Earnings Tercile		
Child Innate Ability:	I	II	III
Low:	0.0027	0.0113	0.1620
Medium:	0.0010	0.0090	0.1429
High:	0.0028	0.0095	0.1404
Panel B: Childs Acquired Ability			
	Young Parent Earnings Tercile		
Child Innate Ability:	I	II	III
Low:	0.245	0.266	0.359
Medium:	0.412	0.432	0.565
High:	0.714	0.770	1.034
Panel C: College Enrollment Rate (percentage)			
	Old Parent Earnings Tercile		
Child Acquired Ability:	I	II	III
Low:	0.00	0.00	2.52
Medium:	NaN	0.00	45.15
High:	NaN	31.34	88.97

Note: Earnings, Innate ability, and aquired ability are *wh*, *b* and *bhat*. The three panels report *e*, *bhat* and *s* respectively.

Table 6: Original Table 3 of Restuccia and Urrutia (2004)

TABLE 3—DECISION RULES BY PARENTS' EARNINGS AND CHILD'S ABILITY

Panel A: Expenditures in early education			
	Young parent earnings tercile		
Child innate ability	I	II	III
Low	0.0033	0.0188	0.0977
Medium	0.0024	0.0113	0.0952
High	0.0030	0.0142	0.0989
Panel B: Child's acquired ability			
	Young parent earnings tercile		
Child innate ability	I	II	III
Low	0.264	0.289	0.350
Medium	0.432	0.453	0.554
High	0.745	0.814	1.022
Panel C: College enrollment rate (percentage)			
	Old parent earnings tercile		
Child acquired ability	I	II	III
Low	1.15	0.70	8.98
Medium	22.29	26.18	64.86
High	99.74	82.66	82.05

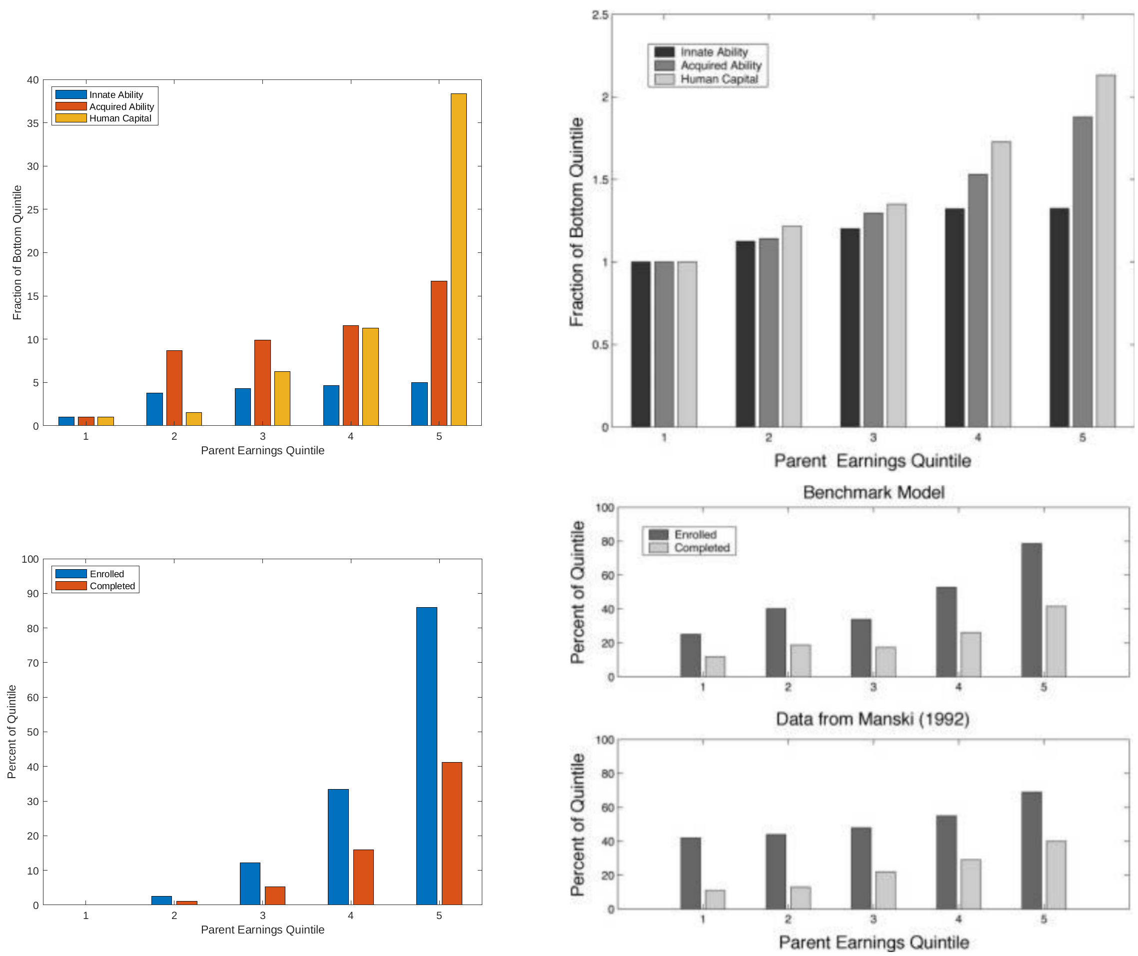

Figure 1: Figures 1 and 2 of Restuccia and Urrutia (2004)

Table 7: Table 4 of Restuccia and Urrutia (2004)
Uniform Transfer Experiment

Transfer to	Young Parents	Old Parents
Increase in education expenditures: (percent of transfer)	197.60	0.00
Parents changing education decisions: (percent of parents in age group)	24.56	NaN

Note: Clockwise from top-left, these are percent change in e, F-kappaF, s, and bhat.

Table 8: Original Table 4 of Restuccia and Urrutia (2004)

TABLE 4—UNIFORM TRANSFER EXPERIMENT

Transfer to:	Young parents	Old parents
Increase in education expenditures (percentage of transfer)	13.96	1.02
Parents changing education decisions (percentage of parents in age group)	29.23	2.54

Note: Uniform lump-sum transfer of 30 percent of the total college tuition cost to either young or old parents, keeping decision rules constant.

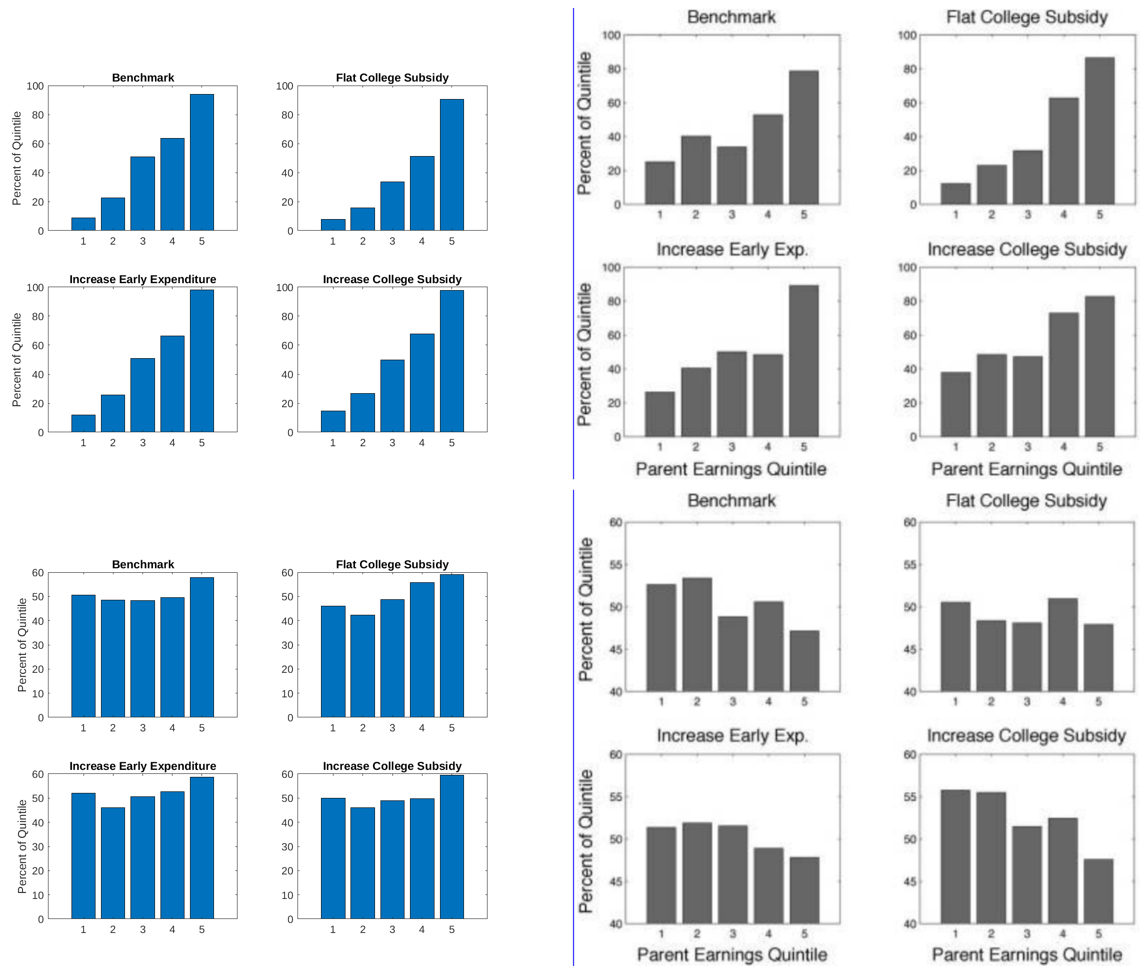

Figure 2: Figures 3 and 4 of Restuccia and Urrutia (2004): Fig 3 shows Enrollment Rates, Fig 4 shows Dropout Rates.

Table 9: Table 5 of Restuccia and Urrutia (2004)
Sensitivity Analysis with Respect to ρ and σ_π

	Benchmark $\rho = 0.2$ $\sigma_\pi = 0.48$	$\rho = 0.1$	$\rho = 0.3$	$\sigma_\pi = 0.4$	$\sigma_\pi = 0.6$
Intergenerational correlation					
Innate ability	0.27	0.15	0.34	0.22	0.24
Acquired ability	0.00	0.00	0.00	0.00	0.00
Earnings	0.70	0.71	0.70	0.58	0.88
Disparity: std(log x)					
Innate ability	0.48	0.48	0.48	0.40	0.60
Acquired ability	0.55	0.54	0.56	0.42	0.73
Earnings	0.80	0.77	0.83	0.68	0.96
Other aggregate statistics					
College enrollment	0.49	0.49	0.53	0.43	0.58
Private early/GDP (as %)	6.00	5.64	6.50	4.42	8.12

Note: All other parameters are the same as in the benchmark economy; public expenditures in early education adjusted to balance the government budget.

Note: In model notation these columns are: b, $\hat{b}$, and wh. I follow Restuccia & Urrutia (2004) in reporting as cross-sectional the number conditional on being an elderly household, not cross-sectional over the whole model economy. Restuccia & Urrutia (2004) explain calculation of intergenerational correlation of earnings at bottom of pg 1363. I assume the intergeneration correlations of (log) innate and acquired ability are calculated by the analagous regressions (with modification for acquired ability as is only observed for old).

Table 10: Original Table 5 of Restuccia and Urrutia (2004)

TABLE 5—SENSITIVITY ANALYSIS WITH RESPECT TO ρ AND σ_π

	Benchmark $\rho = 0.20$ $\sigma_\pi = 0.48$	$\rho = 0.1$	$\rho = 0.3$	$\sigma_\pi = 0.4$	$\sigma_\pi = 0.6$
Intergenerational correlation					
Innate ability	0.20	0.10	0.30	0.20	0.20
Acquired ability	0.41	0.33	0.47	0.36	0.38
Earnings	0.40	0.33	0.48	0.41	0.38
Disparity: std(log x)					
Innate ability	0.48	0.48	0.48	0.40	0.60
Acquired ability	0.51	0.50	0.54	0.45	0.65
Earnings	0.60	0.58	0.63	0.53	0.76
Other aggregate statistics					
College enrollment	0.46	0.47	0.42	0.32	0.44
Private early/GDP	2.13	2.22	1.99	1.73	2.47

Note: All other parameters are the same as in the benchmark economy; public expenditures in early education adjusted to balance the government's budget.

Table 11: Table 6 of Restuccia and Urrutia (2004)
Sensitivity Analysis with Respect to γ and $\bar{p}$

	Benchmark $\gamma = 0.24$ $\bar{p} = 1.48$	$\gamma = 0.1$	$\gamma = 0.4$	$\bar{p} = 1.4$	$\bar{p} = 1.6$
Intergenerational correlation					
Innate ability	0.27	0.27	0.22	0.26	0.26
Acquired ability	0.00	0.00	0.00	0.00	0.00
Earnings	0.70	0.74	0.67	0.69	0.72
Disparity: std(log x)					
Innate ability	0.48	0.48	0.48	0.48	0.48
Acquired ability	0.55	0.46	0.79	0.54	0.55
Earnings	0.80	0.72	0.88	0.77	0.82
Other aggregate statistics					
College enrollment	0.49	0.88	0.21	0.34	0.72
Private early/GDP (as %)	6.00	1.91	12.81	5.55	6.39

Note: All other parameters are the same as in the benchmark economy; public expenditures in early education adjusted to balance the government budget.
Note: In model notation these columns are: b, $\hat{b}$, and wh. I follow Restuccia & Urrutia (2004) in reporting as cross-sectional the number conditional on being an elderly household, not cross-sectional over the whole model economy. Restuccia & Urrutia (2004) explain calculation of intergenerational correlation of earnings at bottom of pg 1363. I assume the intergeneration correlations of (log) innate and acquired ability are calculated by the analagous regressions (with modification for acquired ability as is only observed for old).

Table 12: Original Table 6 of Restuccia and Urrutia (2004)

TABLE 6—SENSITIVITY ANALYSIS WITH RESPECT TO γ AND $\bar{p}$

	Benchmark $\gamma = 0.24$ $\bar{p} = 1.48$	$\gamma = 0.1$	$\gamma = 0.4$	$\bar{p} = 1.4$	$\bar{p} = 1.6$
Intergenerational correlation					
Innate ability	0.20	0.20	0.20	0.20	0.20
Acquired ability	0.41	0.23	0.54	0.36	0.45
Earnings	0.40	0.22	0.55	0.37	0.43
Disparity: std(log x)					
Innate ability	0.48	0.48	0.48	0.48	0.48
Acquired ability	0.51	0.49	0.56	0.51	0.52
Earnings	0.60	0.60	0.61	0.57	0.64
Other aggregate statistics					
College enrollment	0.46	0.75	0.20	0.25	0.66
Private early/GDP	2.13	0.51	3.57	1.57	2.75

Note: All other parameters are the same as in the benchmark economy; public expenditures in early education adjusted to balance the government's budget.

Table 13: Table 7 of Restuccia and Urrutia (2004)
Adding Post-College Permanent Earnings Shocks

	Benchmark Economy	Small shock $z = \{0.8, 1.2\}$	Large shock $z = \{0.5, 1.5\}$
Intergenerational correlation			
Innate ability	0.27	0.24	0.24
Acquired ability	0.00	0.00	0.00
Earnings	0.70	0.16	0.18
Disparity: std(log x)			
Innate ability	0.48	0.48	0.48
Acquired ability	0.55	0.53	0.54
Earnings	0.80	1.32	1.32
Correlation with log acquired ability			
College enrollment	0.64	0.64	0.64
Educational attainment	0.25	0.24	0.24
Log earnings	NaN	NaN	NaN
Other aggregate statistics			
College enrollment	0.49	0.50	0.50
Private early/GDP (as %)	6.00	4.96	5.41
Average College Premium	2.52	2.24	2.80

Note: Same parameters as in the benchmark economy; public expenditures in early education adjusted to balance the government budget.

Note: In model notation these columns are: b, $\hat{b}$, and wh. I follow Restuccia & Urrutia (2004) in reporting as cross-sectional the number conditional on being an elderly household, not cross-sectional over the whole model economy. Restuccia & Urrutia (2004) explain calculation of intergenerational correlation of earnings at bottom of pg 1363. I assume the intergeneration correlations of (log) innate and acquired ability are calculated by the analagous regressions (with modification for acquired ability as is only observed for old).

Table 14: Original Table 7 of Restuccia and Urrutia (2004)

TABLE 7—ADDING POST-COLLEGE PERMANENT EARNINGS SHOCKS

	Benchmark economy	Small shock $z = \{0.8, 1.2\}$	Large shock $z = \{0.5, 1.5\}$
Intergenerational correlation			
Innate ability	0.20	0.20	0.20
Acquired ability	0.41	0.42	0.41
Earnings	0.40	0.38	0.29
Disparity: std(log *x*)			
Innate ability	0.47	0.47	0.47
Acquired ability	0.51	0.51	0.52
Earnings	0.60	0.63	0.82
Correlation with log acquired ability			
College enrollment	0.66	0.66	0.66
Educational attainment	0.64	0.64	0.64
Log earnings	0.95	0.91	0.76
Other aggregate statistics			
College enrollment	0.46	0.47	0.48
Private early/GDP	2.14	2.27	2.72
Average college premium	2.38	2.39	2.43

Note: Same parameters as in the benchmark economy; public expenditures in early education adjusted to balance the government's budget.

Table 15: Table 8 of Restuccia and Urrutia (2004)
Policy Experiments

	Benchmark	Increase in early expenditures	Increase in college subsidy	Flat college subsidy
Intergenerational correlation				
Earnings	0.70	0.70	0.71	0.69
Educational attainment	0.00	0.00	0.00	0.00
Consumption	0.00	0.00	0.00	0.00
Expenditures (percent of GDP)				
Private early education	6.00	5.69	5.73	5.80
Public early eduction	2.65	3.42	3.33	3.27
Private college education	0.91	0.95	0.92	1.13
Public college education	1.26	1.28	1.37	0.64
Other aggregate statistics				
College enrollment rate	0.49	0.51	0.52	0.40
College dropout rate	0.18	0.18	0.18	0.16
Aggregate human capital	1.06	1.06	1.06	1.06
Aggregate consumption	1.23	1.23	1.23	1.24

Note: All other parameters are the same as in the benchmark economy; public expenditures in early education adjusted to balance the government budget.

Note: In model notation these columns are: b, $\hat{b}$, and wh. I follow Restuccia & Urrutia (2004) in reporting as cross-sectional the number conditional on being an elderly household for b and wh (young for $\hat{b}$), not cross-sectional over the whole model economy.

Restuccia & Urrutia (2004) explain calculation of intergenerational correlation of earnings at bottom of pg 1363. I assume the intergeneration correlations of (log) innate and acquired ability are calculated by the analagous regressions (with modification for acquired ability as is only observed for old).

Table 16: Original Table 8 of Restuccia and Urrutia (2004)

TABLE 8—POLICY EXPERIMENTS

	Benchmark	Increase in early expenditures	Increase in college subsidy	Flat college subsidy
Intergenerational correlation				
Earnings	0.40	0.36	0.40	0.42
Educational attainment	0.35	0.28	0.26	0.45
Consumption	0.66	0.64	0.67	0.66
Expenditures (percentage of GDP)				
Private early education	2.13	1.77	2.16	2.23
Public early education	2.18	2.94	2.18	2.17
Private college education	0.96	1.03	0.70	0.69
Public college education	1.71	1.71	2.50	1.71
Other aggregate statistics				
College enrollment	0.46	0.50	0.56	0.42
Dropout rate	0.50	0.50	0.52	0.49
Aggregate human capital	2.06	2.15	2.07	2.06
Aggregate consumption	1.92	1.99	1.92	1.91

Note: All other parameters are the same as in the benchmark economy; public expenditures in early education adjusted to balance the government's budget.