Conesa and Krueger (1999) - Social Security Reform with Heterogeneous Agents

Conesa and Krueger (1999) study the economic implications of three possible reforms of the US Social Security system, and whether they might be politically feasible. This is done using a stochastic OLG model, and importantly includes the general equilibrium transition paths.

Tables 1 to 4 are just the model parameters. Two challenges when replicating were the age-conditional survival probabilities and the producitivity as a deterministic function of age (s_j and ϵ_j in my notation). Conesa and Krueger (1999) take their age-specific conditional probabilities of survival from Faber (1982). This document does not exist online but in an email communication Juan Carlos Conesa sent codes that contain their original numbers. Conesa and Krueger (1999) take their age-specific deterministic labor productivity from Hansen (1993). I got the numbers from Hansen (1993) but the exact process by which Conesa and Krueger (1999) interpolate these into a function over age is not described in their paper. Again, an email from Juan Carlos Conesa he provided codes that contain their original numbers. My replication codes have therefore used their exact numbers.

Brief description of model is as follows. I present it in the form relevant to the replication codes, a fuller description is provided by Conesa and Krueger (1999).

The model is a general equilibrium OLG model. The finite-horizon value function problem has one exogenous state (idiosyncratic labor productivity), one endogenous state (assets), and 66 periods. The household value function problem is given by

$$\begin{aligned}
V(a,\eta,j) = & \max_{c,a'} \frac{(c^{1-\gamma}(1-l)^{\gamma})^{1-\sigma}}{1-\sigma} + \beta s_j E_j[V(a',\eta',j)|\eta] \\
\text{subject to } & c + a' \leq (1+r)(a+Tr) + (1-\tau)wl\epsilon_j\eta\mathbb{I}_{(j<J_r)} + +SS\mathbb{I}_{(j\geq J_r)} \\
& k' \geq 0
\end{aligned}$$

There are $J = 66$ periods and $V(a,\eta,J+1) = 0$ for all a, & η. So household faces idiosyncratic labor productivity shocks (η) and solves both a consumption-savings problem and a consumption-leisure problem involving choosing consumption c, labour supply l, and next period assets a'.

For most calibrations the earnings process η consists of two states, a high labor producitivity state and a low labor producitivity state which follow a markov process. ϵ_j is a deterministic spline of earnings in terms of age and is used to generate the age profile of earnings.

The initial distribution of agents at birth is for them to have zero assets and the stationary distribution of shocks.

The social security payroll tax, τ, pays for social security (pension) benefits SS.

The model has four general equilibrium constraints, the first is that the interest rate r equals the marginal product of capital minus the depreciation rate δ. The next two are fiscal: that the payroll tax pays for social security, and that the replacement rate of social security is as intended. The fourth is that the the (total across the population of the) lump-sum transfer of accidental bequests Tr much equal the assets left behind by people on dying.

Much of the replication involves solving stationary general equilibrium problems (finding r, b, SS, and Tr), and then general equilibrium transition paths (path for the same four) for a change/path in the social security system. See CK1999 paper for details. The full replication involves 7 different setups in terms of the idiosyncratic labor producitivity shocks, η, and three different reforms to social security. The majority of the paper is about the first three of the different setups in terms of the idiosyncratic labor productivity shocks, referred to as 'no heterogeneity' (a deterministic model with no idiosyncratic labor productivity shocks), a 'symmetric heterogeneity' model, and an 'asymmetric heterogeneity' model.

Conesa and Krueger (1999), when doing the 'symmetric' shocks case say that their Markov is approximating an AR(1) plus an iid. This is internally inconsistent with the actual model as if it were true then the agent's exogenous state should contain each of the AR(1) and the iid seperately as two different exogenous states (not just their sum eta). Further calculations suggest that in fact this is not what happened, and CK1999 simply set eta in their model equal to exp(z) from equation (4.1) on page 767; another way to say this is that eta in equation (4.1) is a different eta from the eta in the model. Essentially, eta in eqn (4.1) is a typo; and exp(z) in eqn (4.1) is what in fact corresponds to eta in the model. These codes use exp(z) in eqn (4.1) as the process that is approximated to get eta. (Note: This is anyway irrelevant except of the results in Table 6.) What CK1999 do, in ignoring epsilon and using their z from eqn (4.1) is standard practice based on interpretation of the iid epsilon as measurement error, it is simply notationally very confusing that eta is not eta!

Conesa and Krueger (1999) provide the exact grids and transition matrices for the exogenous shocks, η for the two-state cases. The paper refers to discretizing the versions with more states using the Tauchen method, but the references of the paper cite Tauchen-Hussey paper. Email from the authors confirms that they used Tauchen-Hussey, and so my replication codes follow this. Note that since Tauchen-Hussey is not part of VFI Toolkit an implementation is included alongside the replication codes.

Note that equation (2.1) of Conesa and Krueger (1999) includes the conditional probability of survival in the expectations operator.

I make a minor notational different to Conesa and Krueger (1999). I call the age-conditional survival probability s_j, while they called it ϕ_j.

Everything replicates fine. There are some numerical differences, but nothing of substance although the reforms are more popular it is most often remains not enough to get a median voter to approve of them. I have aimed to get all replication figures use the same y-axis as the original so as to make comparison easier.

Figure 3, 9 and 12 in the bottom right subplot are titled 'Evolution of Labor Supply'. It was clear from the y-axes that they show the 'Evolution of Hours Worked'. I mention this so as to clearly distinguish it from the labor supply in effective units of labor that enter the production fn.

Figure 11, note that the ages are different to all the other figures. All other figures use ages 20, 30, & 60. Figure 11 uses ages 20, 45 & 81.

Figure 14. Unclear what to do with age dimension? CK1999 do not appear to describe anywhere what is being plotted in terms of the age dimension. I tried taking means of EV over the age dimension to eliminate it but this gave weird answers. It is unclear what CK1999 did. Does not appear to be any mention in text (their previous similar Figures are all for specific ages). I have therefore ended up just drawing the age 20 version as out of the age 20, 30 & 60 versions this was the one that looked most like that of CK1999. In a later communication by email Dirk Krueger

Table 1: Table 1 of Conesa & Krueger (1999)

Preference Parameters

Parameter	Value
σ	2.00
β	0.97
γ	0.42

Table 2: Original Table 1 of Conesa & Krueger (1999)

TABLE I

Preference Parameters

Parameter	Value
σ	2.00
β	0.97
γ	0.42

confirms that he is confident that the original was of age 20.

Table 3: Table 2 of Conesa & Krueger (1999)
Demographics

Parameter	Value
J	66
j_r	46
s_j	Bell and Miller (2005)
n	0.011

Note: Renamed the age-conditional surivial probability as s_j, while CK1999 call it ψ_j.

Table 4: Original Table 2 of Conesa & Krueger (1999)
TABLE II
Demographics

Parameter	Value
J	66
jr	46
ψ_j	Faber [11]
n	0.011

Table 5: Table 3 of Conesa & Krueger (1999)
Technology Parameters

Parameter	Value
α	0.36
δ	0.06
θ	1

Table 6: Original Table 3 of Conesa & Krueger (1999)
TABLE III
Technology Parameters

Parameter	Value
α	0.36
δ	0.06
θ	1

Table 7: Table 4 of Conesa & Krueger (1999)
Individual Productivity Parameters

Parameter	Diaz-Jimenez et al.	Storeslettern et al.
η_1	0.5	0.73
η_2	3.0	1.27
π_1	0.9811	0.82
π_2	0.9261	0.82
ϵ_j	Hansen (1993)	Hansen (1993)

Table 8: Original Table 4 of Conesa & Krueger (1999)

TABLE IV
Individual Productivity

Parameter	Diaz-Jimenez *et al.*	Storesletten *et al.*
η_1	0.5	0.73
η_2	3.0	1.27
π_1	0.9811	0.82
π_2	0.9261	0.82
ε_j	Hansen [16]	Hansen [16]

References

Juan Carlos Conesa and Dirk Krueger. Social security reform with heterogeneous agents. Review of Economic Dynamics, 2(4):757–795, 1999.

Table 9: Table 5 of Conesa & Krueger (1999)
Steady-state Results

Variables	No heterogeneity		Het. (sym. case)		Het. (asym. case)	
	Init. St. St.	Fin. St. St.	Init. St. St.	Fin. St. St.	Init. St. St.	Fin. St. St.
b	50 %	0 %	50 %	0 %	50 %	0 %
r	6.0 %	4.9 %	5.5 %	4.3 %	3.4 %	2.0 %
w	1.18	1.25	1.21	1.30	1.36	1.49 %
h	39.8 %	41.9 %	37.9 %	40.3 %	35.7 %	37.7 %
K/Y	2.99	3.30	3.12	3.50	3.83	4.48
y	1.05	1.17	1.08	1.22	1.31	1.51
SS/y	38.9 %	0 %	38.9 %	0 %	38.9 %	0 %
cv(lab)	0.16	0.08	0.31	0.23	0.44	0.41
cv(weal)	0.79	0.91	0.90	0.93	1.71	1.57
EV^{ss}	–	12.68 %	–	12.72 %	–	11.32 %

Table 10: Original Table 5 of Conesa & Krueger (1999)

TABLE V
Steady-State Results

Var.	No heterogeneity		Het. (sym. case)		Het. (asym. case)	
	In. St.St.	Fi. St.St.	In. St.St.	Fi. St.St.	In. St.St.	Fi. St.St.
b	50%	0%	50%	0%	50%	0%
r	6.0%	4.9%	5.5%	4.3%	3.4%	2.0%
w	1.18	1.25	1.21	1.30	1.36	1.49
h	32.8%	34.5%	31.3%	33.2%	29.4%	31.0%
K/Y	2.98	3.30	3.12	3.51	3.84	4.49
y	1.04	1.17	1.08	1.22	1.31	1.51
SS/y	38.9%	0	38.9%	0	38.9%	0
$cv(lab)$	0.52	0.51	0.71	0.68	1.39	1.38
$cv(weal)$	0.81	0.93	0.92	0.94	1.17	1.58
EV^{SS}	—	12.7%	—	12.8%	—	11.2%

Table 11: Table 6 of Conesa & Krueger (1999)
Dispersion of labor earnings, wealth, votes for Reform A

	Sym. 2 States	Sym. 3 States	Sym. 5 States	Asym. 2 States
cv(lab)	0.27	0.24	0.35	0.41
cv(weal)	0.92	1.00	1.14	1.57
Votes	39.49 %	37.62 %	34.87 %	26.23 %

Table 12: Original Table 6 of Conesa & Krueger (1999)

TABLE VI
Dispersion of labor earnings, wealth, votes for reform A

	Sym, 2 states	Sym, 3 states	Sym, 5 states	Asym, 2 states
$cv(lab)$	0.71	0.81	0.92	1.39
$cv(weal)$	0.92	1.00	1.13	1.71
Votes	36.4%	33.8%	30.4%	21.3%

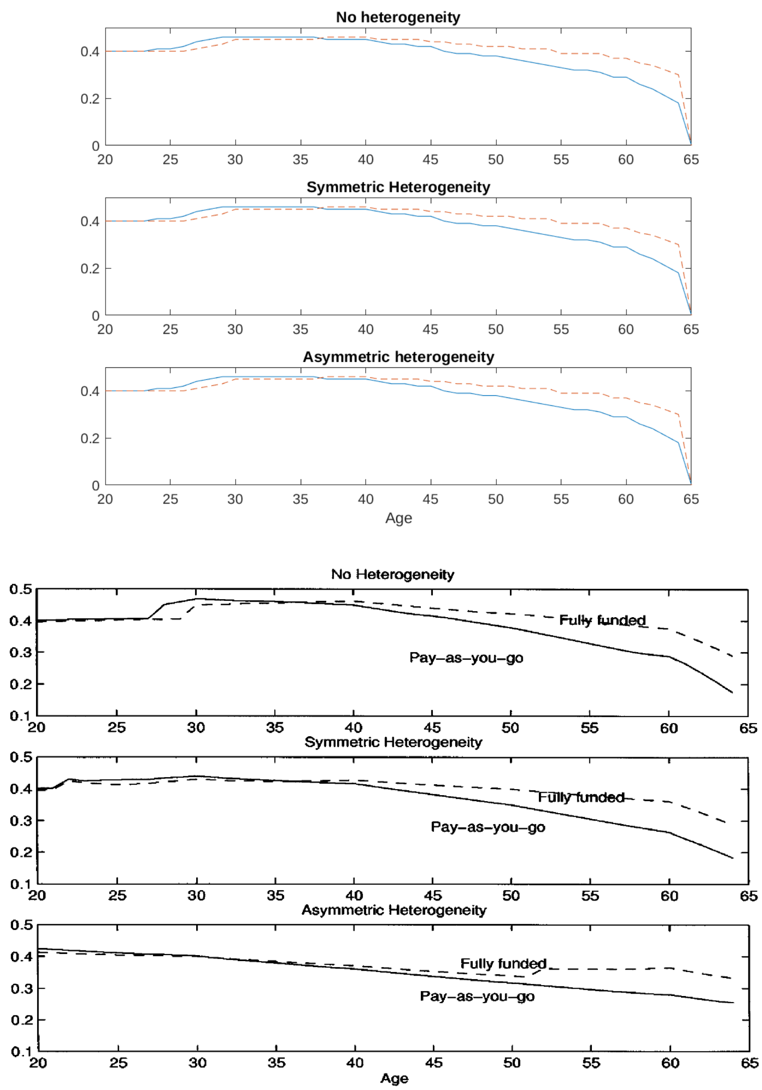
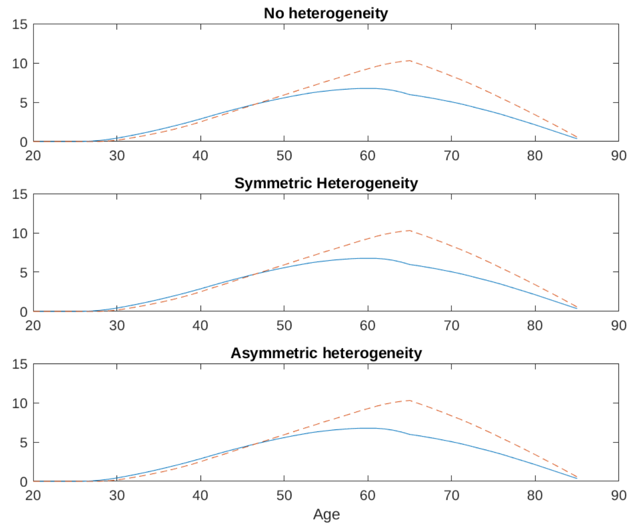


FIG. 1. Age profile of average hours worked.

Figure 1: Figure 1 of Conesa & Krueger (1999)



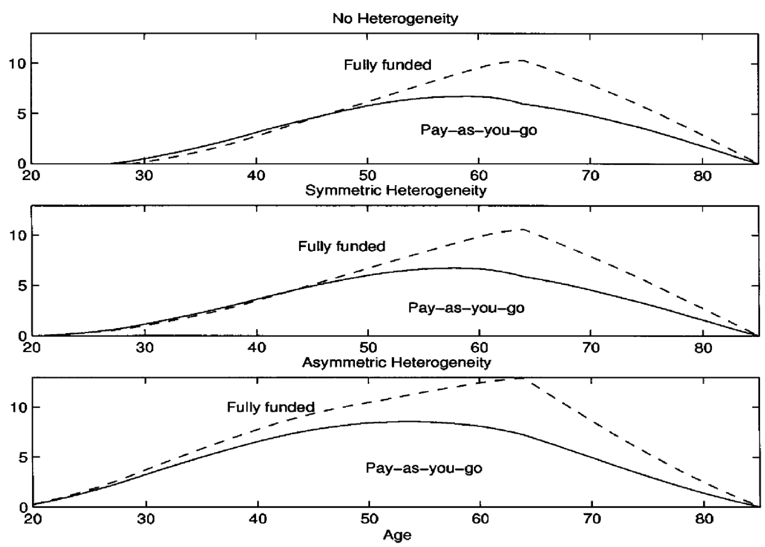


FIG. 2. Age profile of average asset holdings.

Figure 2: Figure 2 of Conesa & Krueger (1999)

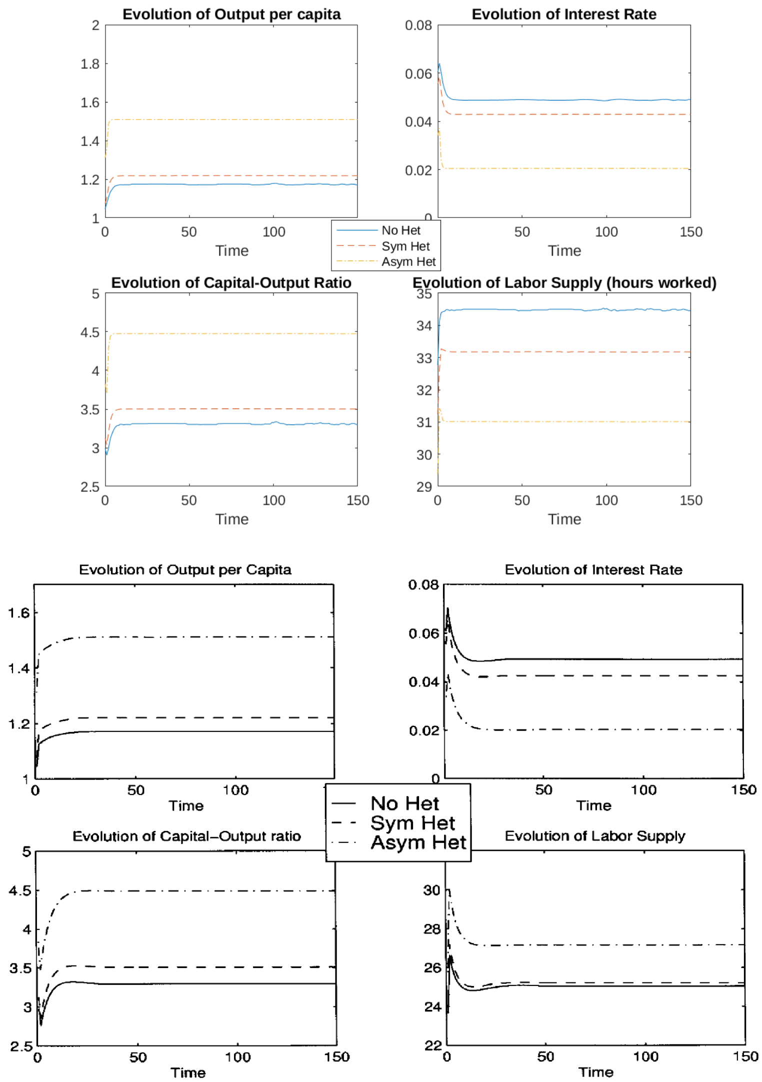
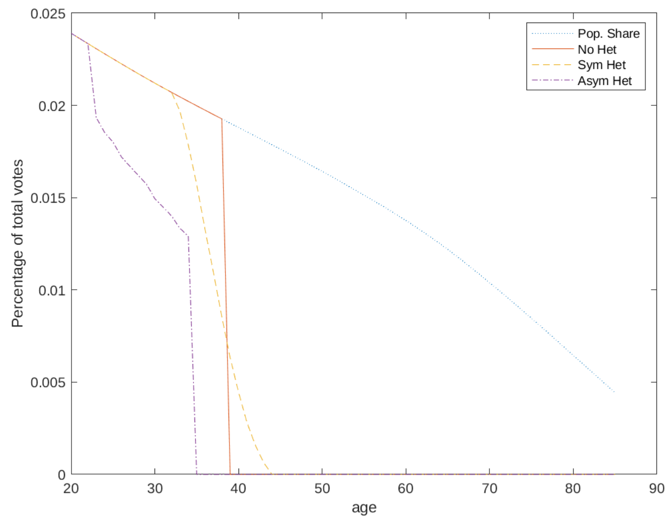


FIG. 3. Evolution of macroeconomic aggregates: reform A.

Figure 3: Figure 3 of Conesa & Krueger (1999)



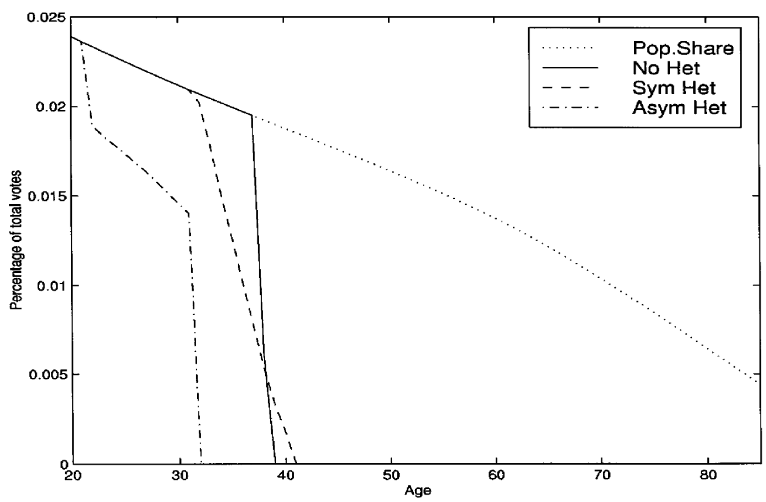
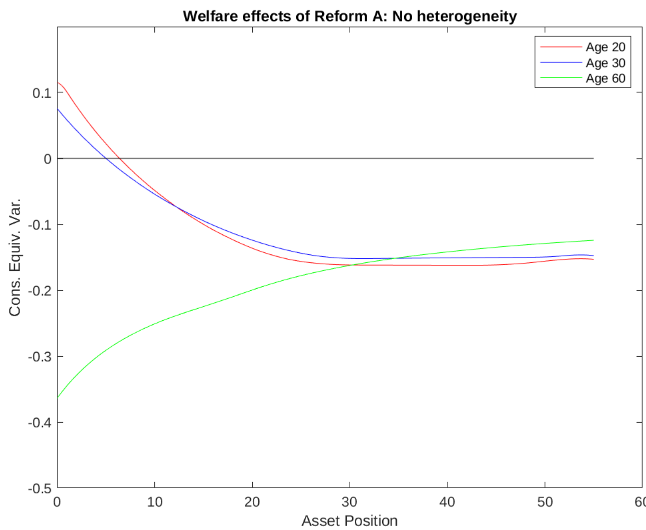


FIG. 4. Votes in favor of reform A.

Figure 4: Figure 4 of Conesa & Krueger (1999)



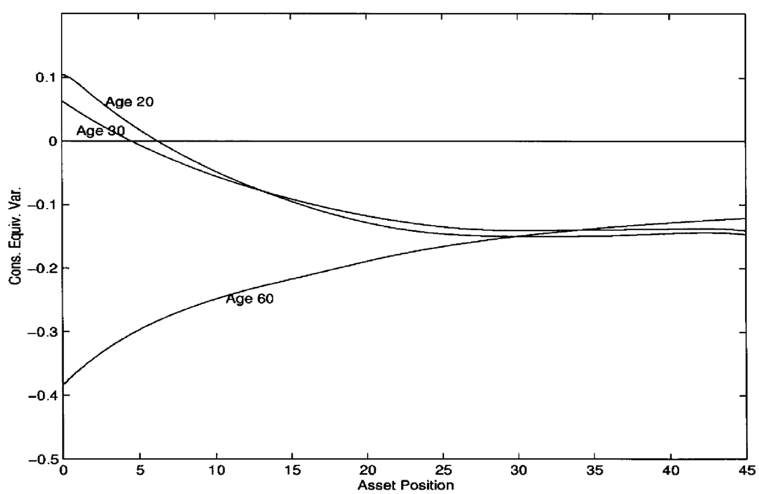
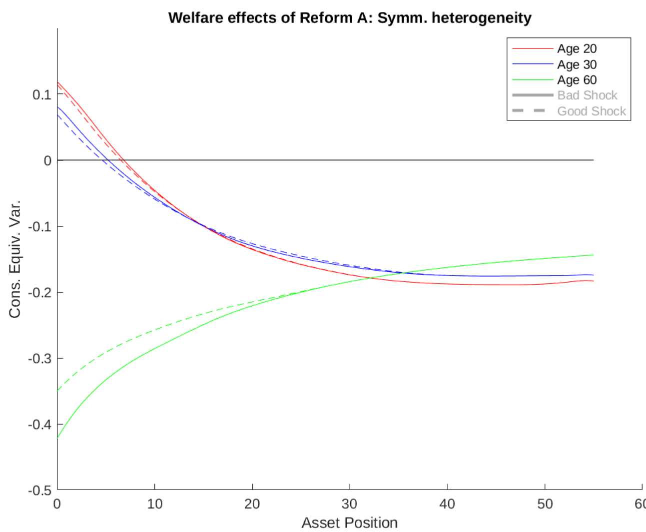


FIG. 5. Welfare effects of reform A: no heterogeneity.

Figure 5: Figure 5 of Conesa & Krueger (1999)



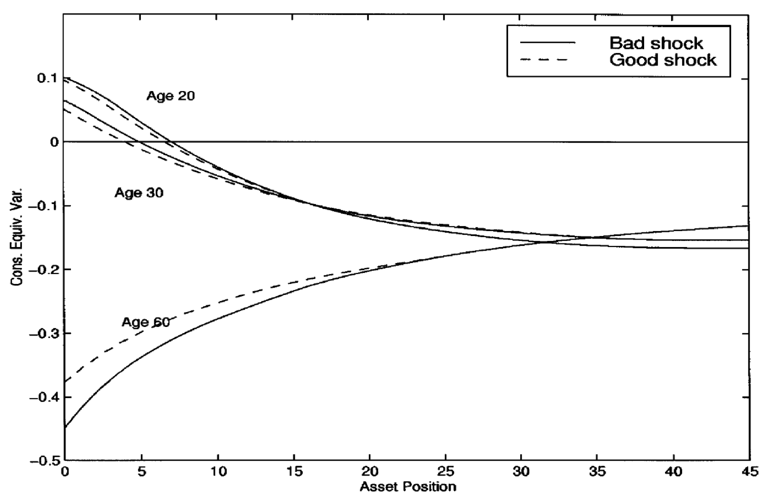
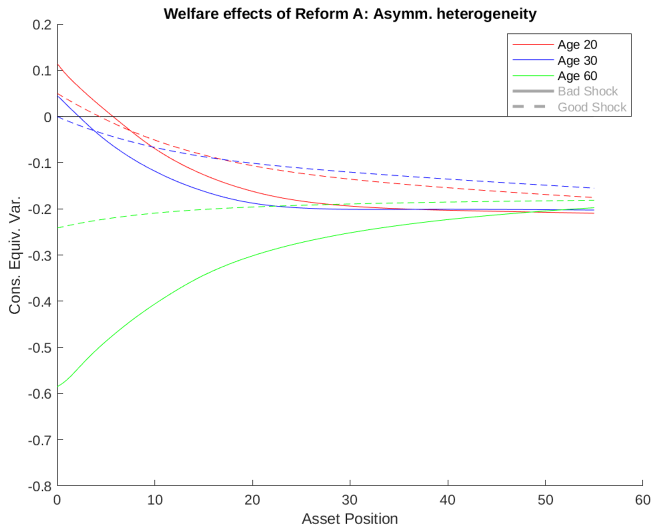


FIG. 6. Welfare effects of reform A: symm. heterogeneity.

Figure 6: Figure 6 of Conesa & Krueger (1999)



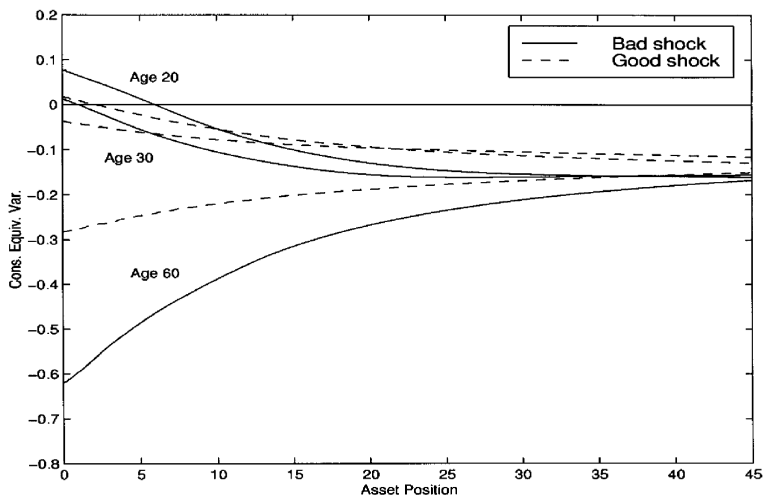
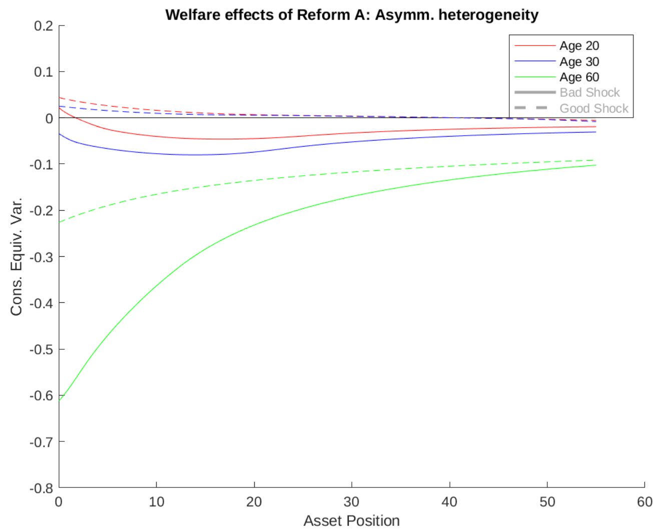


FIG. 7. Welfare effects of reform A: asymm. heterogeneity.

Figure 7: Figure 7 of Conesa & Krueger (1999)



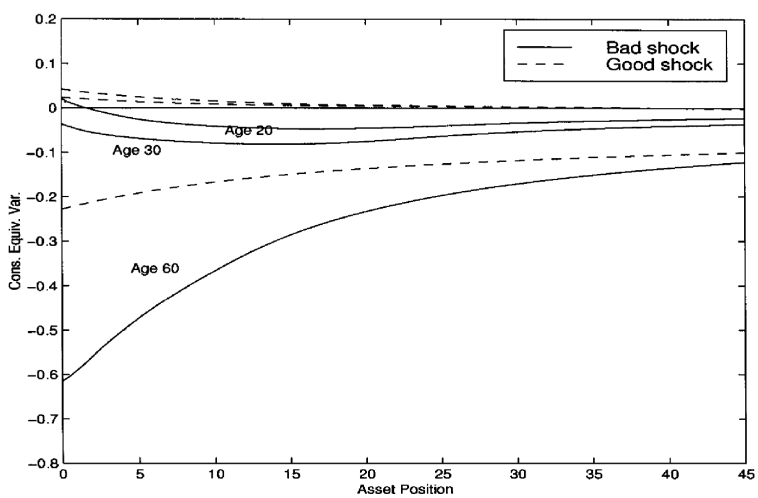
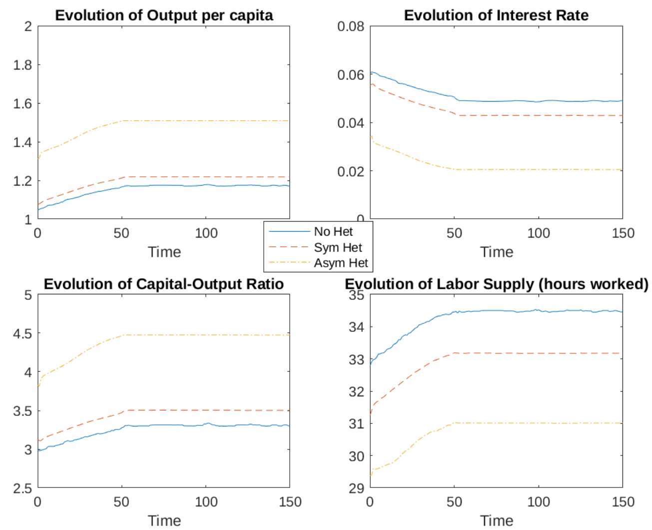


FIG. 8. Welfare effects of reform A: asymm. het., fixed prices.

Figure 8: Figure 8 of Conesa & Krueger (1999)



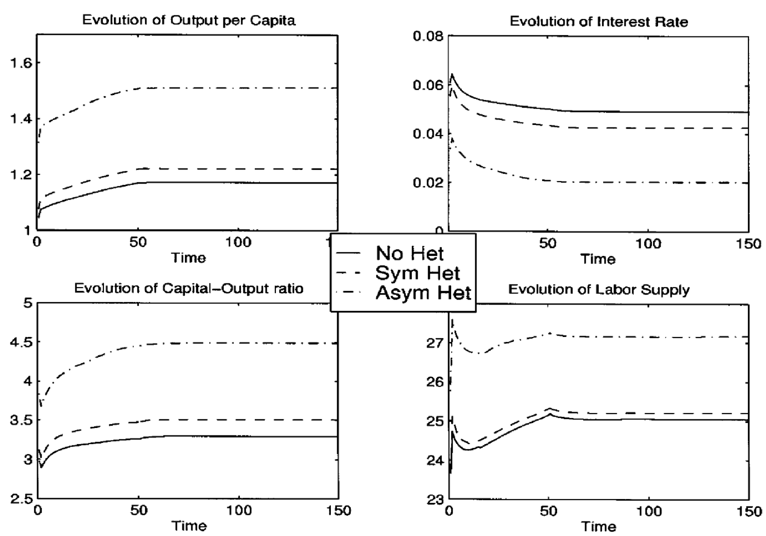
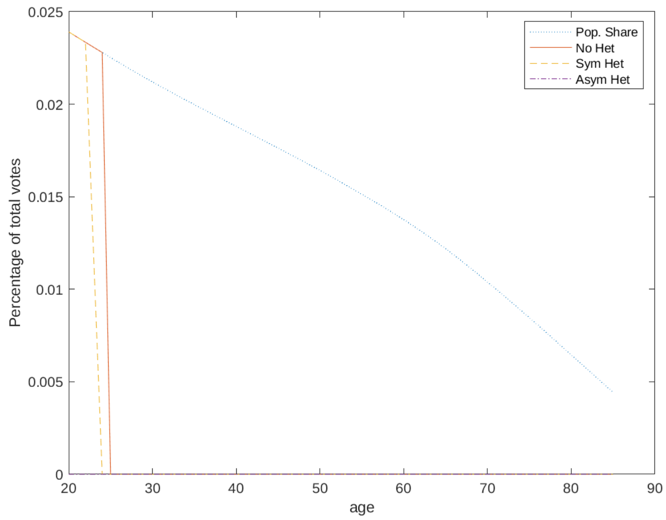


FIG. 9. Evolution of macroeconomic aggregates: reform B.

Figure 9: Figure 9 of Conesa & Krueger (1999)



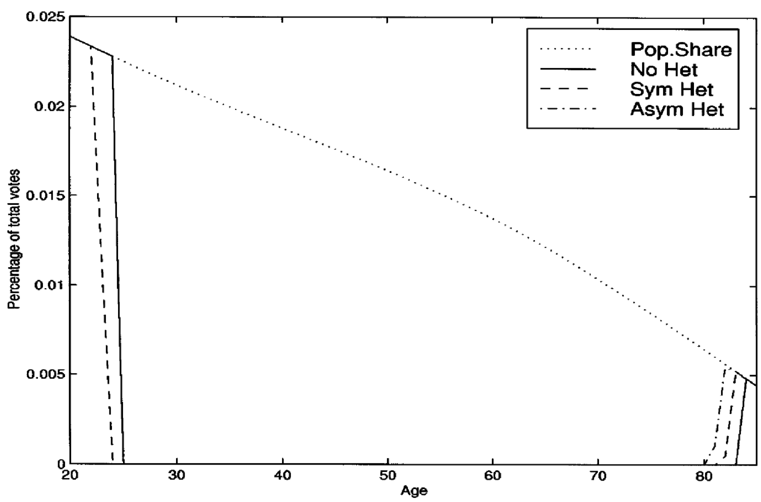
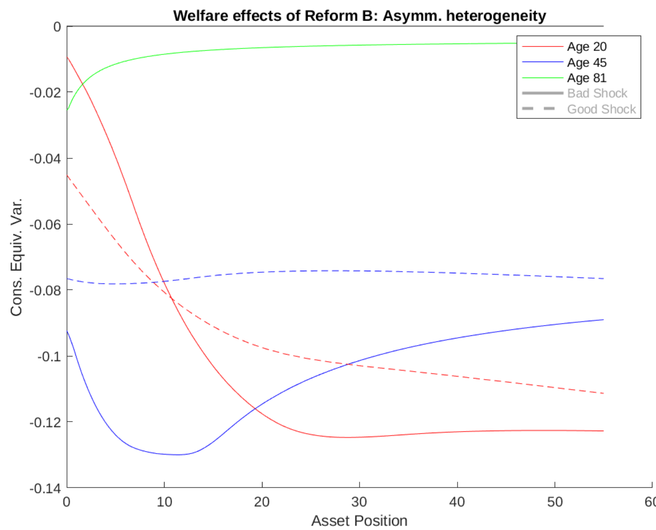


FIG. 10. Votes in favor of reform B.

Figure 10: Figure 10 of Conesa & Krueger (1999)



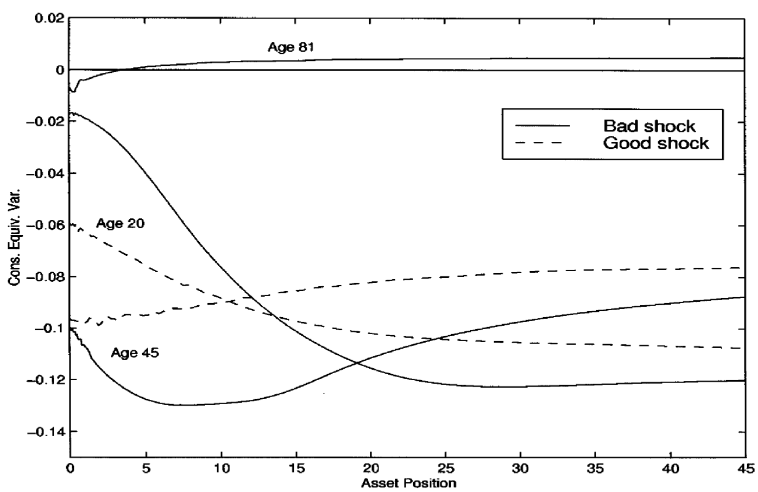


FIG. 11. Welfare effects of reform B: asymm. heterogeneity.

Figure 11: Figure 11 of Conesa & Krueger (1999)

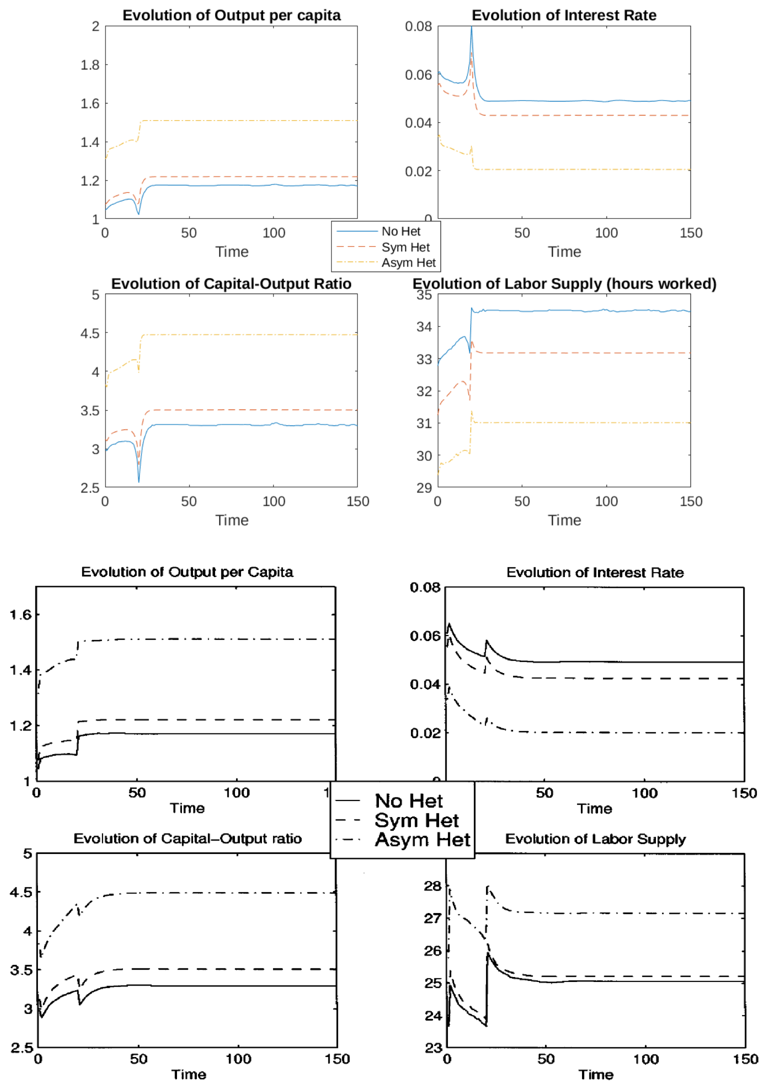
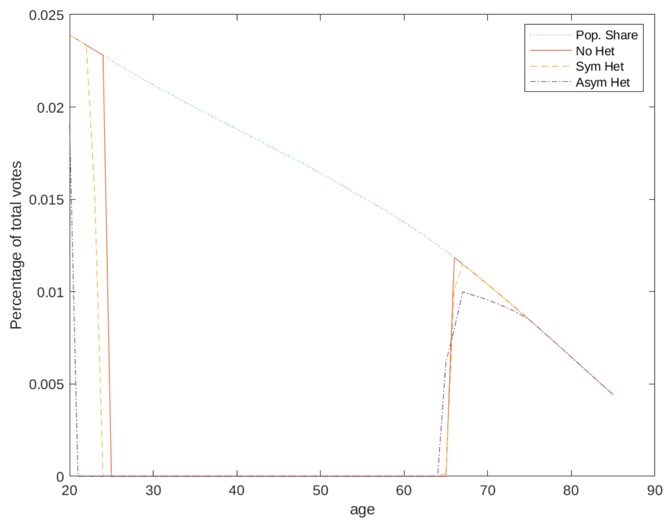


FIG. 12. Evolution of macroeconomic aggregates: reform C.

Figure 12: Figure 12 of Conesa & Krueger (1999)



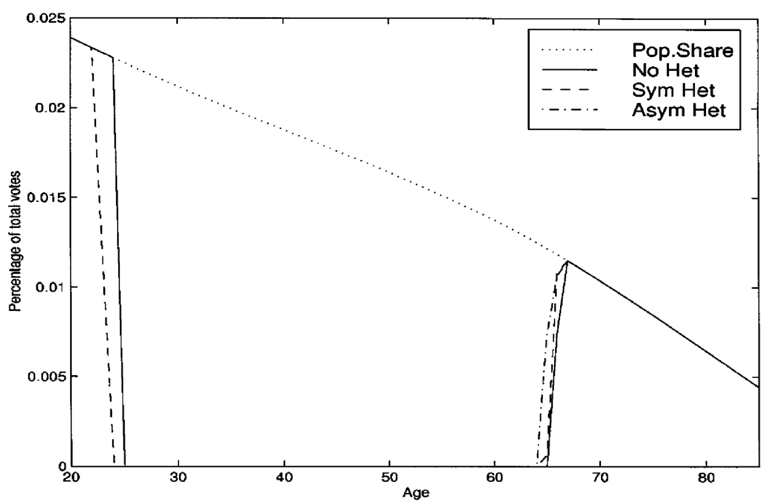


FIG. 13. Votes in favor of reform C.

Figure 13: Figure 13 of Conesa & Krueger (1999)

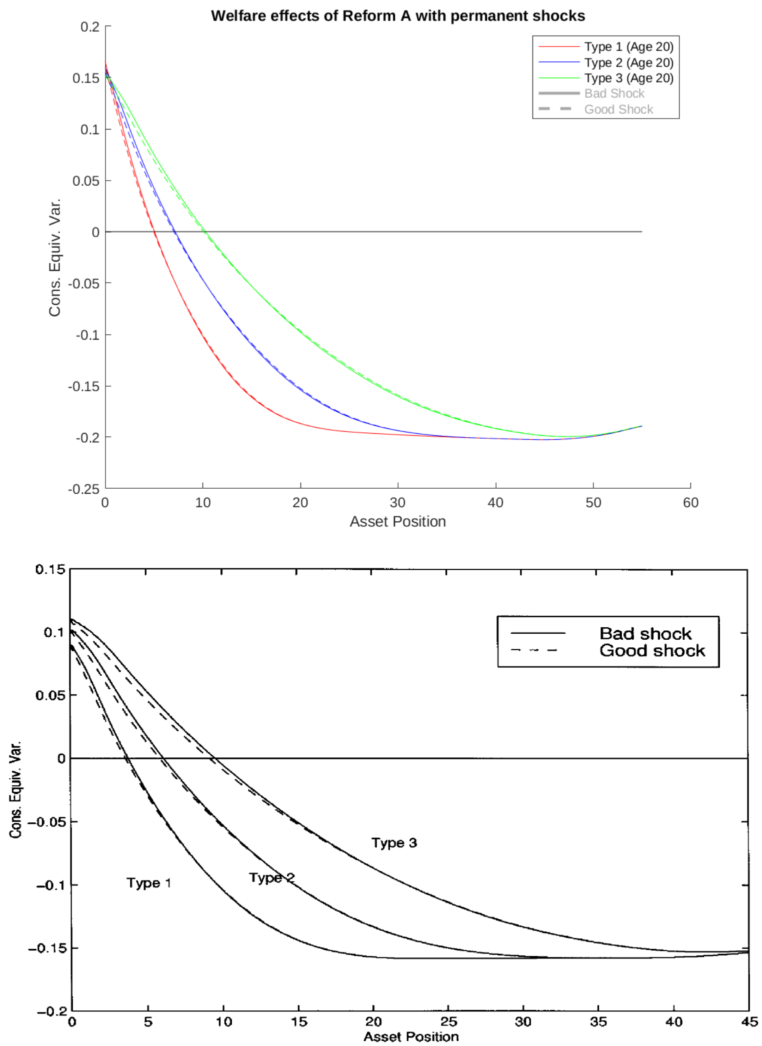


FIG. 14. Welfare effects of reform A with permanent effects.

Figure 14: Figure 14 of Conesa & Krueger (1999)

Table 13: Table 7 of Conesa & Krueger (1999)

Dispersion of labor earnings, wealth, votes for Reform A

	Sym. 2 States	Sym. 2 type, 2 States	Sym. 3 types, 2 States
cv(lab)	0.20	0.13	0.20
cv(weal)	0.84	0.53	0.67
Votes	41.69 %	43.74 %	29.16 %

Table 14: Original Table 7 of Conesa & Krueger (1999)

TABLE VII

Dispersion of Labor Earnings, Wealth, Votes for Reform A

	Sym, 2 states	Sym, 2 type 2 st	Sym, 3 types 2 st
$cv(lab)$	0.71	0.64	0.68
$cv(weal)$	0.92	0.96	1.06
Votes	36.4%	37.8%	37.4%