Hopenhayn and Rogerson (1993) - Job Turnover and Policy Evaluation: A General Equilibrium Analysis

Hopenhayn and Rogerson (1993) study the impact of factor misallocation, specifically the misallocation of labor due to firing costs, in a model of heterogeneous firms. The model has both endogenous entry and endogenous exit of firms.

The problem of an existing firm is to choose employment to maximize (present discounted value of expected) profits, subject to firing costs which must be payed if employment is lower this period than last period. Hiring is costless. Firms have a decreasing-returns-to-scale production function, with labor being the only factor of production, and face a fixed cost of production. Alternatively, a firm can decide to exit in which case it incurs the firing costs of all remaining employees and ceases to exist. The technology level in the production function, z, follows an AR(1) process in logs.

The (existing) firm's problem is thus,

$$V(n,z) = \max_{n',x} \mathbb{1}_{\{x==0\}}(pz(n')^{\alpha} - wn' - pc_f - \tau(\mathbb{1}_{n'<n}(n-n'))) - \mathbb{1}_{\{x==1\}}\tau n + \beta \mathbb{1}_{\{x==0\}} E[V(n',z')|z] \quad (1)$$

where n is 'lag' of employment (last period employment; so n' is this employment this period); z is (idiosyncratic) technology level; $x \in \{0,1\}$ is the exit decision; p is the price level; w is the wage; c_f is the fixed-cost of production.

The problem facing potential entrant firms is to decide whether or not to enter. A firm that decides to enter must pay a fixed cost of entry, c_e, and will enter as with technology level z drawn from a distribution of entrants η calibrated as a uniform distribution over the lower two-thirds of the range of possible z values, and a 'lagged employment' value of zero.[1] "Once this [entry] cost has been paid, the entrant is in the same position as an incumbent that has chosen to remain in the productive sector and had zero employees in the previous period", according to Hopenhayn and Rogerson (1993) on pg 919; however, in direct conflict with this their Footnote 5 on page 922 states that "Note that we are assuming that a new entrant bears only the fixed cost of entry and does not pay the cost cf"; meaning need to make a slight modification to the existing firm's problem for new entrants (I will ignore this dependence notationally elsewhere). This replication reports the results with Footnote 5 imposed; the codes allow for it to be turned on and off. The mass of new entrants is given by parameter N_e. The problem faced by potential entrants is thus to choose to enter or not based on whether or not $E_{\eta}[V] > c_e$.

The model also includes a representative household with utility function $\sum_{t=1}^{\infty} \beta^t [log(c) - aN]$; note that there is no aggregate uncertainty in this model. This model gives us the 'demand function' (or equally, the condition for goods market clearance), which enters the model as a general equilibrium condition. It provides household-side assumptions that deliver the infinitely elastic labor-supply (guaranteeing that the labour market clears, regardless of labour demand), and allows for welfare interpretations. It does necessitate additional calibrations, namely A and β to

[1] "We found that a uniform distribution on the lower part of the interval in which realizations of z lie produced a reasonable fit.", pg 930 of Hopenhayn & Rogerson (1993), but no mention of what constitutes 'lower part'. Martin Flodén concludes that roughly the bottom 2/3 (0.65 to be precise) is a good definiting of 'lower' (pg 5): http://martinfloden.net/files/macrolab.pdf Also, "Once this cost has been paid the entrant is in the same position as an incumbent that has chosen to remain in the productive sector and had zero employees in the previous period.", pg 919 of Hopenhayn & Rogerson (1993) tells us that all entrants have zero 'lagged employees'.

deliver the model values of r and fraction-of-time-worked.[2]

A stationary competitive equilibrium of this model is given by,

Definition 1. *A Stationary Competitive Equilibrium is an agents value function V; agents policy function g; agents exit decision g^x; price of goods p; mass of new entrants N_e; and measure of agents μ; such that*

1. *Given interest rate r, the agents value function V, policy function g, and exit decision g^x, solve the agents problem, as given by equation (1).*
2. *Equilibrium in goods market: $p = \frac{A}{\int z(g^n)^\alpha d\mu}$.*
3. *Free-entry condition: $\int V\eta(n,z)dndz - c_e = 0$*
4. *The measure of agents is invariant:*

$$\mu_(n,z) = \int\int\left[\int 1_{n=g(\hat{n},z)}1_{g^x(\hat{n},z)=0}\mu(\hat{n},z)Q(z,dz')\right]d\hat{n}dz + N_e\eta(n,z) \tag{2}$$

Note that the measure of agents μ is not a probability density function, as it includes the mass, N, of agents. Also note that this stationary competitive equilibrium condition can also be defined in terms of finding c_e and treating p as exogenous; Hopenhayn and Rogerson (1993) follow this later in baseline case, and the alternative (p) in other two calibrations with positive values of τ that they solve.

Parameter $a = 0.078$, Hopenhayn & Rogerson (1993) do not report this, but Martin Flodén figures out the following (pg 5): martinfloden.net/files/macrolab.pdf. Hopenhayn & Rogerson (1993) state that they use Tauchen method with 20 grid points to discretize z (which they call s), but do not report the value of the hyperparameter used; $q = 4$ seems to be appropriate based on Table 4 otherwise don't get values of z anywhere near as high as 27.3.

Table 1 relates to data, and so is not part of this replication.

Table 2, I do not replicate 'Co-worker Mean' as I do not know what this means. Hopenhayn and Rogerson (1993) do not provide formulae for any of the statistics reported in the Tables and so I report my interpretation of what their verbal descriptions and the names of the statistics (in some cases they are obvious, the codes implementing the replication provide numerous comments wherever I have made assumptions about how to do this, mostly relating to how to treat entrants and exits). The 0 value for 'Hazard rates by cohort - 1 period' follows directly from imposing footnote 5 of Hopenhayn and Rogerson (1993) (see discussion earlier in this appendix).

The first row of Table 4 contains 'nan' values in replication. This is because according to Hopenhayn and Rogerson (1993) results this was presumably just above the 'exit cutoff', while for me this falls below the 'exit cutoff', so since all firms with this value of z will choose to exit in my replication and the concepts of n_l and n_u are thus undefined. If the grid on n used by Hopenhayn

[2] To derive the goods-market clearance condition you can first derive $A/C = p$ from the household problem and then combine this with $C =$ real output, and definition of real output as $\int zn^\alpha d\mu$ (note: Y is nominal output in notation of Hopenhayn and Rogerson (1993), so real output can also be found from Y/p. For household side you can get $A/C = p$ from solving $\max_{c,N}[log(c) - AN]$ $s.t.$ $pC = wN + T$, where T is just all lump-sum transfers and other wealth (is not part of standard model notation); note that it is just the standard intratemporal labour/leisure tradeoff condition $-\frac{u_c}{u_N} = \frac{p}{w}$, together with normalization of $w = 1$.

and Rogerson (1993) was truely log-spaced as they indicate, then the numbers they report in Table 4 are rounded to whole numbers in the lower three rows (but not top two rows); I choose not to round these from my actual grid values and instead report all to two decimal places (no good reason for this, the rounding makes just as much sense).

The results of the paper largely replicate, with two main issues. I find a much smaller role of firms with 500+ employees, and I find that firms with 1-19 employees play a smaller role relative to 20-99; smaller is to be understood as relative to original findings. In both cases I beleive this to be because the original paper had 250 points log-spaced from 0 to 5000 employees for a grid on n. I use more than double this. The role of this in creating many more points near 500 employees is clear. As to why it changes relative importance of 1-19 vs 20-99, I suspect that original grid had a point just below 19 and none just above 20, leading firms who might normally choose just above 20 to be forced to choose just below (my grid includes every one of the 101 points from 0 to 100 employees, and then uses log-spaced points from 101 to 5000). Note that with 25+ years of extra computational power it was easy for this replication to use many more points than the original, so I caution against reading too much into this.

Relatedly, Hopenhayn and Rogerson (1993) used 201 points log-spaced from 0 to 5000 for the number of employees of a firm. Together with their use of pure discretization as the solution method this (implicitly) imposes a minimum adjustment size and so their implemented model behaves like one with a (proportional) fixed cost of adjustment plus a linear cost of adjustment, instead of just a linear cost of adjustment in the model itself. The replication uses many more points and so behaves like the linear cost of adjustment model. This intuition explains some of how and where the replication results differ quantitatively from the original. Another explanation is that the limited number of grid points for the number of employees of a firm interacts with the finite nature of productivity z (which takes 20 values) causing productivity distortions; this is largely absent from the replication so the misallocation is smaller.

The utility-adjusted consumption is calculated as $100 * exp(U_i - U_0)$, this formula can be derived definition of consumption-equivalent utility variation as $\bar{c}$ that solves $log(\bar{c}c_0) - aN_0 = log(c_i) - aN_i$, where i is intended to represent the relevate case/tax-rate; note that this ignores the possibility of compensating part of the utility variation as leisure, the present day consensus is to allow for this adjustment of leisure as part of the calculation.[3] This my interpretation of 'The figure for utility-adjusted compensation shows the amount by which consumption would have to be increased in order for utility to reach the same level attained when $\tau = 0$. This measure takes into account the fact that leisure is higher when τ is positive.' (HR1993, pg 934-5).

References

Hugo Hopenhayn and Richard Rogerson. Job turnover and policy evaluation: A general equilibrium analysis. Journal of Political Economy, 101(5):915–938, 1993.

[3]The 100$*$ comes from the reporting of the numbers as relative to the $\tau = 0$ case; see footnote to replicated Table 3, here Table 3.

Table 1: Table 2 of Hopenhayn & Rogerson (1993)

A: Summary Statistics for Benchmark Model

Average firm size	25.18
Co-worker mean	-
Variance of growth rates (survivors)	0.44
Serial correlation in log(n) (survivors)	0.79
Exit rate of firms	0.30
Turnover rate of jobs	0.24
Fraction of hiring by new firms	0.06
Average size of new firms	5.02
Average size of exiting firms	4.64

B: Size Distribution

	1-19	20-99	100-499	500+
Firms	0.39	0.23	0.07	0.00
Employment	0.11	0.35	0.43	0.11
Hiring	0.14	0.32	0.41	0.13
Firing	0.18	0.40	0.37	0.06
By cohort:				
1 period	0.92	0.08	0.00	0.00
2 periods	0.52	0.46	0.02	0.00
5 periods	0.34	0.53	0.12	0.00
10 periods	0.24	0.56	0.20	0.00
Hazard rates by cohort:				
1 period	0.00			
2 periods	0.85			
5 periods	0.15			
10 periods	0.10			

Note: Do not attempt to replicate Co-worker mean as I do not know definition. Avg size of entering firms is defined in terms of nprime, while avg size of exiting firms is defined in terms of n.

Table 2: Original Table 2 of Hopenhayn & Rogerson (1993)

A. Summary Statistics for Benchmark Model

Average firm size	61.2
Co-worker mean	747
Variance of growth rates (survivors)	.55
Serial correlation in log n (survivors)	.92
Exit rate of firms	.39
Turnover rate of jobs	.30
Fraction of hiring by new firms	.15
Average size of new firm	7.5
Average size of existing firm	4.9

B. Size Distribution

	1–19	20–99	100–499	500+
Firms	.52	.37	.10	.01
Employment	.06	.24	.37	.33
Hiring	.05	.35	.41	.19
Firing	.12	.19	.34	.35
By cohort:				
1 period	.88	.12	.00	.00
2 periods	.54	.45	.01	.00
5 periods	.29	.58	.12	.01
10 periods	.20	.54	.20	.05
Hazard rates by cohort:				
1 period	.75			
2 periods	.32			
5 periods	.15			
10 periods	.10			

Table 3: Table 3 of Hopenhayn & Rogerson (1993)
Effect of Changes in τ (Benchmark Model)

	$\tau = 0$	$\tau = 0.1$	$\tau = 0.2$
Price	1.000	1.021	1.040
Consumption (output)	100.0	98.2	96.7
Average Productivity	100.0	99.4	98.3
Total Employment	100.0	98.8	98.4
Utility-adjusted consumption	100.0	98.2	96.8
Average firm size	25.2	25.7	26.6
Layoff costs/wage bill	0.0	0.009	0.013
Job turnover rate	0.24	0.21	0.18
Serial correlation in log(n)	0.78	0.83	0.86
Variance in growth rate	0.45	0.39	0.34

Consumption (output), Average Productivity, Total Employment and Utility-adjusted consumption are all reported relative to $\tau = 0$ (which is set equal to 100). Effectively also true of price which is normalized to one in the $\tau = 0$ case as part of calibration.

Table 4: Original Table 3 of Hopenhayn & Rogerson (1993)

Effect of Changes in τ (Benchmark Model)

	$\tau = 0$	$\tau = .1$	$\tau = .2$
Price	1.00	1.026	1.048
Consumption (output)	100	97.5	95.4
Average productivity	100	99.2	97.9
Total employment	100	98.3	97.5
Utility-adjusted consumption	100	98.7	97.2
Average firm size	61.2	61.8	65.1
Layoff costs/wage bill	0	.026	.044
Job turnover rate	.30	.26	.22
Serial correlation in log(n)	.92	.94	.94
Variance in growth rates	.55	.45	.39

Table 5: Table 4 of Hopenhayn & Rogerson (1993)
Effect of τ on Decision Rules

	$\tau = 0.1$		$\tau = 0.2$	
z	n_l	n_u	n_l	n_u
1.76	NaN	NaN	NaN	NaN
4.24	15.71	18.85	15.71	22.56
10.26	177.75	218.66	171.72	251.00
19.88	1102.50	1355.02	1029.25	1554.71
24.79	2046.63	2430.25	1910.71	2694.11

Note: Hopenhayn & Rogerson (1993) call the first column log(s), but it is clear from the values it should be s, which I call z (the idiosyncratic productivity). Difference in the first column, z, between original and replication represent differences in the grids.

Table 6: Original Table 4 of Hopenhayn & Rogerson (1993)

Effect of τ on Decision Rules

	$\tau = .1$		$\tau = .2$	
log s	n_l	n_u	n_l	n_u
1.83	1.36	1.78	1.18	1.98
4.75	21.7	26.7	21.0	32.8
10.5	194	238	181	282
19.9	1,110	1,410	1,036	1,617
27.3	2,610	3,316	2,522	3,935

Table 7: Table 5 of Hopenhayn & Rogerson (1993)
Absolute Deviations from MPL=1/p

	Fraction of Firms within Interval	
Size of Deviation (%)	$\tau = 0.1$	$\tau = 0.2$
0-3	0.32	0.00
3-5	0.38	0.11
5-10	0.27	0.56
10-15	0.00	0.07
>15	0.03	0.27

Table 8: Original Table 5 of Hopenhayn & Rogerson (1993)

ABSOLUTE DEVIATIONS FROM MPL = $1/p$

	FRACTION OF FIRMS WITHIN INTERVAL	
SIZE OF DEVIATION (%)	$\tau = .1$	$\tau = .2$
0–3	.30	.00
3–5	.45	.12
5–10	.15	.78
10–15	.00	.05
>15	.00	.05