Imrohoroglu (1989) - Cost of Business Cycles with Indivisibilites and Liquidity Constraints

The model is the first ever to numerically solve a heterogenous agent model. It is a partial equilibrium heterogenous agent model with idiosyncratic and aggregate risk. The household problem has one endogenous state (assets, a), and two exogenous shocks (one idiosyncratic s, one aggregate z; s is employed (or unemployed), z is good times (or bad times)). The value function problem to be solved is

$$\begin{aligned} V(a,s,z) &= \max_{c,a'} \frac{c^{1-\mu}}{1-\mu} + \beta E[V(a',s',z')|s,z] \\ \text{s.t.} \quad & c + a'/(1+r) = a + e \\ & a' \geq \underline{a} \end{aligned}$$

where (s,z) follows a markov process with two states for s and two states for z. The transition matrix

$$\begin{aligned} \pi(s',z'|s,z) = &[0.9141, 0.0234, 0.0587, 0.0038; 0.5625, 0.3750, 0.0269, 0.0356; \\ &0.0608, 0.0016, 0.8813, 0.0563; 0.0375, 0.0250, 0.4031, 0.5344] \end{aligned}$$

allows for transition of s to depend on z. $r = r_l$ if $a' > 0$, $r = r_b$ if $a' < 0$. $e = y$ if s is employed, $e = \theta y$ if s is unemployed.

Calibration is $\beta = 0.995$, $r_l = 0$, $r_b = 0.08$, $\theta = 0.25$, $y = 1$, $\sigma = 1.5$. y is a normalization, the value to which y is normalized is not explicitly given in Imrohoroglu (1989) and I am guessing that it is to 1. Paper considers alternative asset/insurance environments and in one of these $\underline{a} = 0$; in the others there is no $\underline{a}$ (viewed another way it is negative infinity). In one alternative calibration $\sigma = 6.2$. Paper also considers an alternative in which there is only one z state, in this case $\pi(s',z'|s,z) = \pi(s'|s) = [0.9565, 0.0435; 0.5, 0.5]$.

Solving the model involves solving this value function and then finding the stationary agent distribution, all in standard manner. All model output is just functions of these.

Find that Imrohoroglu (1989) gets the correct policy functions, and largely accurate agents distribution, except for identifying two spikes which are an artifact of numerical error (this is clear as they disappear as the grid size is increased; ie., grid spacing reduced). The numbers for Table 2 appear accurate for storage only economy, but those for intermediation economy appear incorrect; changing the parameter values suggests it is likely related to an error relating to the interest rate on borrowing, but without original codes it is unclear. Numbers from Table 1 relating to welfare evaluations do not appear accurate. As discussed in body of this paper it seems likely that this is due to original paper getting the policy functions sufficiently accurate, but not the value function.

The formula for the welfare calculations is (as confirmed/provided by Ayse Imrohoroglu by email):
For the welfare results here is the way to calculate them: Cost of businsess cycles is found by the consumption equivalence (change in welfare in terms of %-change in consumption across all possible states)
$CE = (V1/V0)^(1/(1-\sigma)) - 1.$
where V0 is average value of no-BC and V1 is that of BC

Table 1: Table 1 of Imrohoroglu (1989)
Cost of Business Cycles as a Percentage of Consumption

Risk Aversion Parameter	For Economies with Perfect Insurance	For Economies with Only a Storage Technology
$\sigma = 1.5$	0.051	0.102
$\sigma = 6.2$	0.065	0.268

Replication of Table 1 of Imrohoroglu (1989) using grid size $n_a = 1501$, $n_s = 2$, $n_z = 2$.
Note that these are simply evaluated at the mean of the utility, not as an expectation across agent distribution of their individual compensating variation based on value function. The later would better capture the heterogeneity of the costs

Table 2: Original Table 1 of Imrohoroglu (1989)

COST OF BUSINESS CYCLES AS A PERCENTAGE OF CONSUMPTION

Risk Aversion Parameter	For Economies with Perfect Insurance	For Economies with Only a Storage Technology
$\sigma = 1.5$	.080	.300
$\sigma = 6.2$	.300	1.500

In the storage only case (risk aversion 1.5)
V1= -414.29
V0=-414.06
CE=0.1%

Ayse Imrohoroglu and Kanika Aggarwal have performed another independent replication of this paper and get the same results as this replication (communication by email from Ayse Imrohoroglu). We conclude that the results of this replication can be considered correct.

References

Ayse Imrohoroglu. Cost of business cycles with indivisibilities and liquidity constraints. Journal of Political Economy, 97(6):1368–1383, 1989.

Table 3: Table 2 of Imrohoroglu (1989)
Properties of the Equilibrium

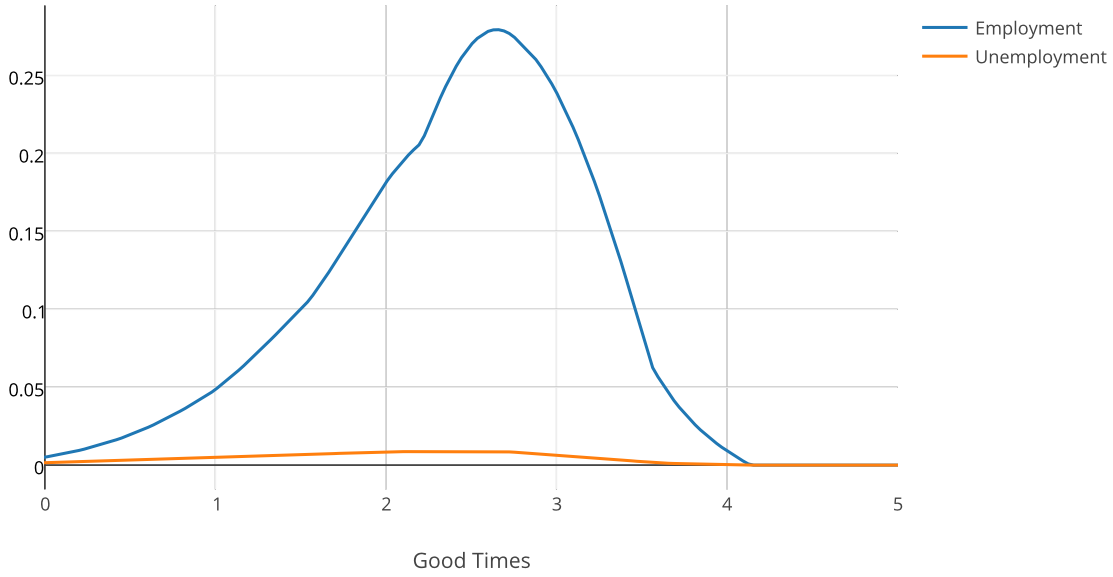
Time Average of	Economies with an Intermediation Technology	Economies with Only a Storage Technology
Assets Borrowed	0.614	-0.000
Assets Stored	-0.437	2.346
Assets Saved	0.177	2.346
Income	0.934	0.940
Consumption	0.934	0.940

Replication of Table 2 of Imrohoroglu (1989) using grid sizes $n_a = 1501$, $n_s = 2$, $n_z = 2$.

Table 4: Original Table 2 of Imrohoroglu (1989)

PROPERTIES OF THE EQUILIBRIUM

Time Average of	Economies with an Intermediation Technology	Economies with Only a Storage Technology
Assets borrowed	.480	.000
Assets stored	.220	2.400
Assets saved	.700	2.400
Income	.940	.940
Consumption	.935	.940



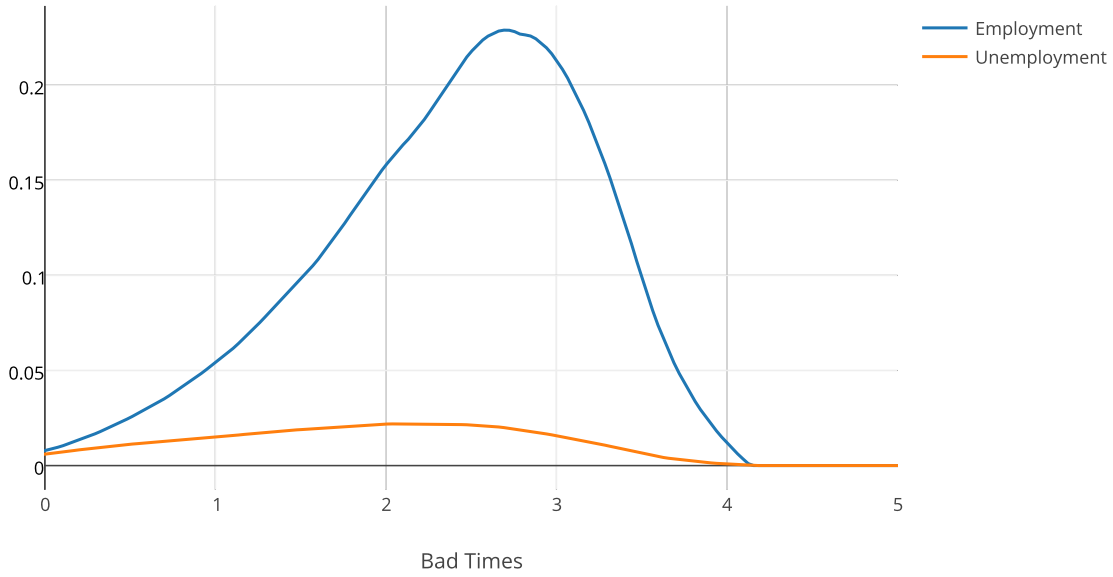


Figure 1: Figure 1 of Imrohoroglu (1989)

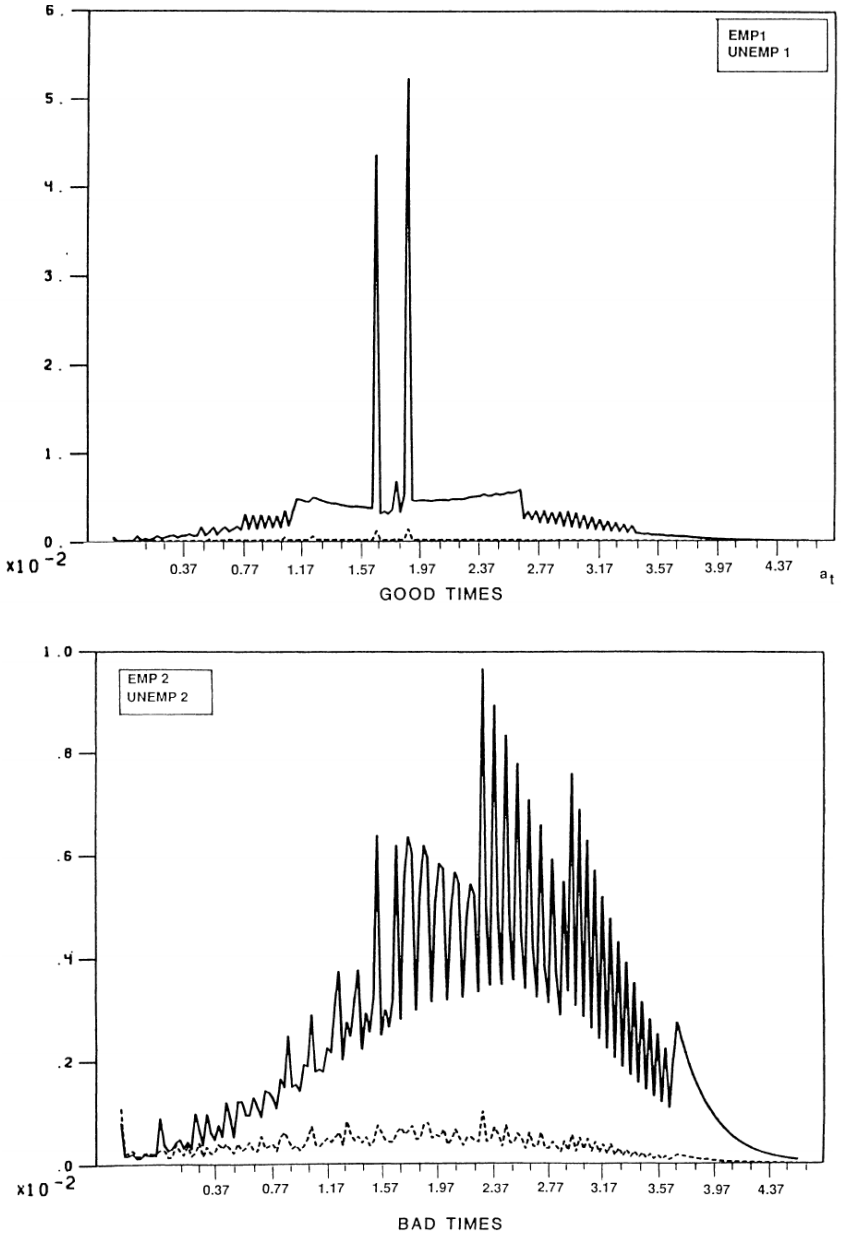
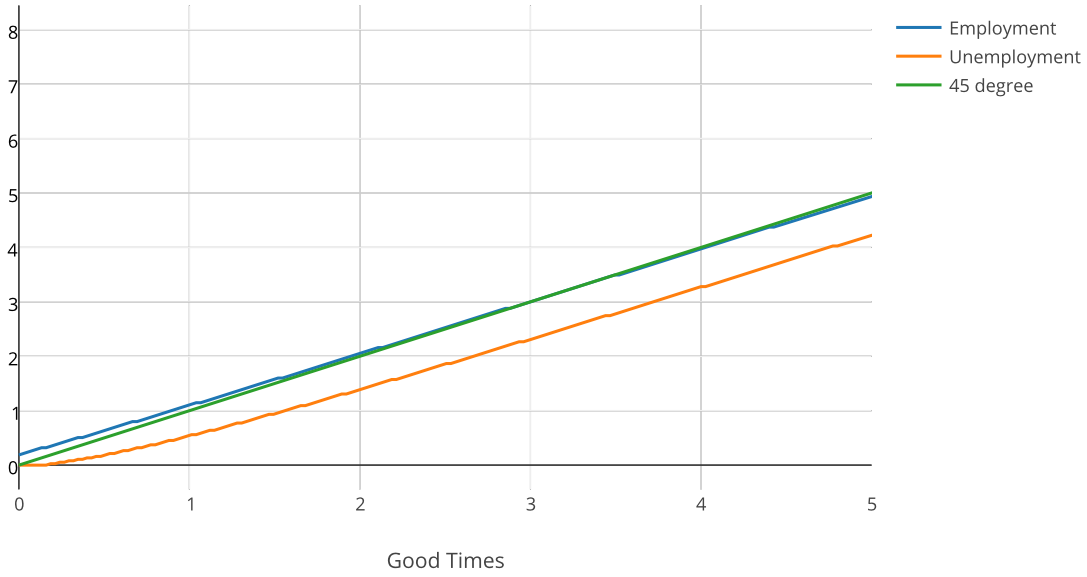


Figure 2: Original Figure 1 of Imrohoroglu (1989)



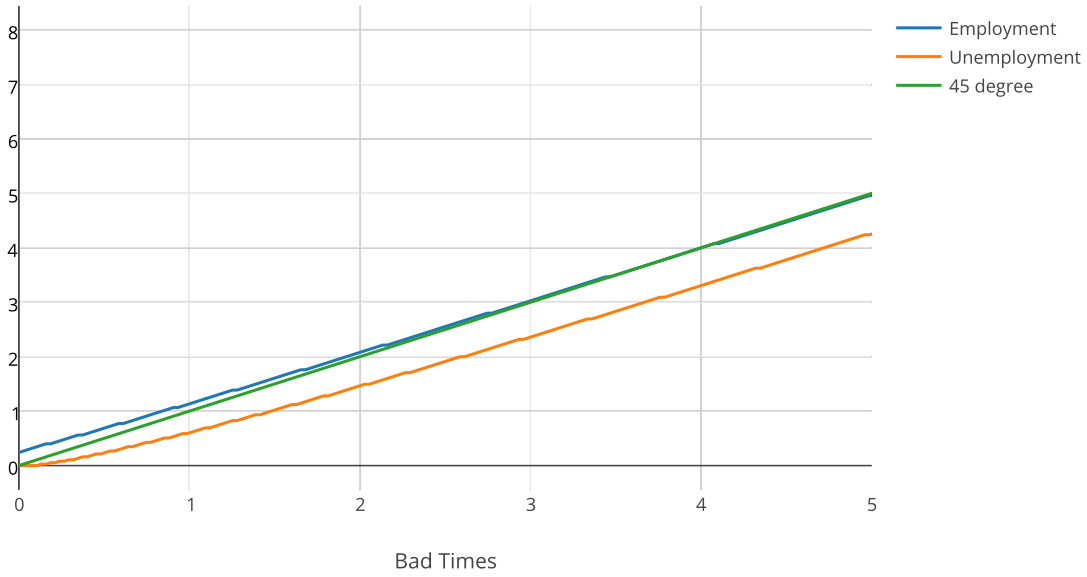


Figure 3: Figure A1 of Imrohoroglu (1989)

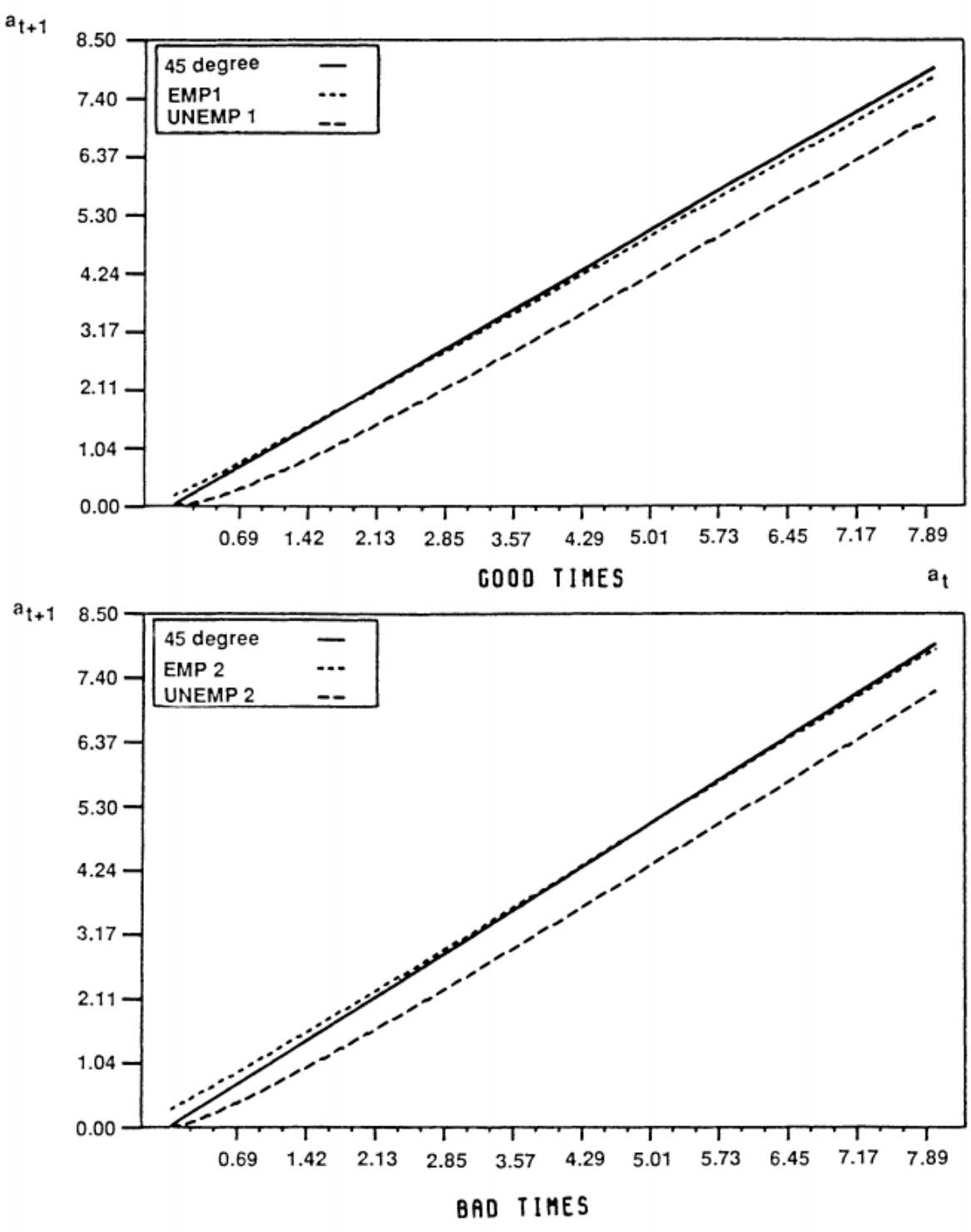


Figure 4: Original Figure A1 of Imrohoroglu (1989)