

## Problem Set 1: An Economy with Distortionary Taxation

### 1 The Setup

Consider the basic RBC model discussed in class and implemented in Dynare. Households own the capital stock and rent it out at rental rate  $R_t$  to firms. Moreover, firms rent labor services  $L_t$  at wage rate  $W_t$  from the households. The production function operated with by the firm is the same Cobb-Douglas function used in class. Households have the same power utility function and face the same law of motion in their capital investment decisions.

The basic model is expanded to include distortionary labor and capital taxation. Assume that the households have to pay labor taxes  $\tau_t^n$  on their wage income and capital taxes  $\tau_t^k$  on their capital income. The new household budget constraint is thus given by

$$C_t + I_t = (1 - \tau_t^n) W_t L_t + (1 - \tau_t^k) R_t K_t - T_t, \quad (1)$$

where  $T_t$  are lump-sum taxes/transfers levied by the government. The budget constraint of the government is given by

$$G_t = T_t + \tau_t^n W_t L_t + \tau_t^k R_t K_t, \quad (2)$$

where  $G_t$  is government spending, which follows the same process as before. This means that any income from labor and capital taxes reduces the amount of lump-sum taxes required to finance government spending (or even leads to transfers to households if the tax income is bigger than the expenditures). The tax-rate processes are given by

$$\tau_t^k = (1 - \rho^k) \bar{\tau}^k + \rho^k \tau_{t-1}^k + \varepsilon_t^k \quad (3)$$

$$\tau_t^n = (1 - \rho^n) \bar{\tau}^n + \rho^n \tau_{t-1}^n + \varepsilon_t^n \quad (4)$$

where bars denote steady state values, the  $\rho$  are autoregressive coefficients, and  $\varepsilon_t^i$  is a surprise tax shock that happens at time  $t$ .

	Std(x)	Std(x)/Std(y)	corr( $x_t, x_{t-1}$ )	corr( $x_t, y_t$ )
$y_t$	1.66	1.00	0.86	1.00
$c_t$	1.19	0.71	0.80	0.81
$i_t$	7.61	4.57	0.80	0.82
$g_t$	3.24	1.95	0.89	0.22
$h_t$	1.89	1.14	0.91	0.87
$y_t/h_t$	1.06	0.64	0.68	0.42
$w_t$	0.97	0.58	0.68	0.13
$A_t$	0.87	0.53	0.74	0.67
$r_t^*$	0.3	0.2	0.6	-0.35

Table 1: Business Cycle Moments: GDP and its Components, USA 1947Q1–2018Q1  
GDP-components deflated with GDP-Deflator and per capita; \* based on Stock-Watson (1998)-data until 1996

## 2 Your Tasks

Given this setup and your Dynare knowledge, do the following:

1. Solve the firm problem, i.e. find the first-order conditions.
2. Solve the household problem, i.e. find the first-order conditions.
3. Impose market clearing
4. Compute the steady state using the parameters given in Table 2. Calibrate the model to imply time devoted to work in steady state to be 1/3 and a capital-output ratio of 10.4.
5. Enter the model including the steady state into Dynare and generate IRFs for the two tax shocks. Interpret.
6. Compute HP-filtered theoretical moments from the model and compare them to the empirical moments displayed in Table 1.

Table 2: Parameters for Calibration

Parameter	Value	Target
$g_x$	0.0055	2.2% Output/Capita Growth
$n$	0.0027	1.1% Population Growth
$A^*$	1	Normalization
$\alpha$	0.33	Capital Share
$\delta$	1.58%	Investment/Output = 0.25
$\beta$	?	Capital/Output = 10.4
$\sigma$	1	Model-independent evidence
$\psi$	?	$l^* = 0.33$
$\rho_z$	0.97	Estimate
$\sigma_z$	0.0065	Estimate
$\chi$	0.2038	G/Y of 0.2038
$\rho_g$	0.989	Estimate
$\sigma_g$	0.010	Estimate
$\bar{\tau}_k$	0.387	Estimate
$\bar{\tau}_n$	0.207	Estimate
$\rho^n$	0.93	Rough Estimate
$\rho_k$	0.93	Rough Estimate
$\sigma_{\tau k}$	0.0093	Rough Estimate
$\sigma_{\tau n}$	0.0022	Rough Estimate