Huggett (1993) - The Risk-Free Rate in Heterogeneous Agent Incomplete Insurance Economies

The model of Huggett (1993) is a heterogeneous agent model with idiosyncratic but no aggregate risk. The household problem has one exogenous state, a, and one exogenous state e, and is given the value function problem,

$$\begin{aligned} V(a,e) = \max_{c,a'} & \frac{c^{1-\mu}}{1-\mu} + \beta E[V(a',e')|e] \\ \text{s.t.} \quad & c + qa' = a + e \\ & a' \geq \underline{a} \end{aligned}$$

where e follows a markov process with two states, $e \in \{e_l, e_h\}$, and transition matrix $\pi(e) = [\pi_{e_l,e_l}, \pi_{e_h,e_l}; \pi_{e_l,e_h}, \pi_{e_h,e_h}]$.

The households choices of a' together with the exogenous markov process e imply a transition function P on the state (a,e). Let $\Psi(a,e)$ denote a distribution of agents over (a,e) in the state space $A \times E$.

Definition 1. *A stationary equilibrium for this economy is $c(a,e)$, $a'(a,e)$, q, Φ satisfying*

- *$c(a,e)$ & $a'(a,e)$ are optimal decision rules given q.*
- *Market clear: (i) $\int_{A\times E} c(a,e)d\Psi = \int_{A\times E} ed\Psi$, and (ii) $\int_{A\times E} a'(a,e)d\Psi = 0$.*
- *Agent distribution is stationary: $\Psi(a',e') = \int_{A\times E} P((a,e),(a',e'))d\Psi(a,e)$.*

Note: this definition is lazy on notation; both Φ and P should be defined on the σ-algebra, not on specific points.

So the market clearance (general equilibrium) requirement is for price q to balance borrowing and lending (ie. that integral of a over the stationary agent distribution is zero).[1] Baseline parameter values are given by: $\beta = 0.99322$, $\mu = 1.5$, $\underline{a} = -2$, $e_h = 1$, $e_l = 0.1$, $\pi_{e_h,e_h} = 0.925$, $\pi_{e_h,e_l} = 0.5$.

Replication involves two Figures, 1 & 2, and two Tables, 1 & 2. Everything replicates pretty accurately (minor numerical differences). The general equilibrium is almost certainly unique (graphs of market clearance condition not shown).

References

Mark Huggett. The risk-free rate in heterogeneous agent incomplete insurance economies. Journal of Economic Dynamics and Control, 17:953–969, 1993.

[1]The second clearance condition in the definition of stationary equilibrium will follow by Walras' law).

Table 1: Table 1 of Huggett (1993)
Coefficient of Relative Risk Aversion μ=1.5

Credit Limit (-$\underline{a}$)	Interest Rate (r)	Price (q)
-2	-7.3 %	1.0128
-4	1.2 %	0.9981
-6	3.1 %	0.9950
-8	3.8 %	0.9938

Replication of Table 1 of Huggett (1993) using grid sizes $n_a = 1024$, $n_e = 2$, $n_q = 1551$

Table 2: Table 2 of Huggett (1993)
Coefficient of Relative Risk Aversion μ=3.0

Credit Limit (-$\underline{a}$)	Interest Rate (r)	Price (q)
-2	-23.6 %	1.0459
-4	-4.4 %	1.0075
-6	0.8 %	0.9987
-8	2.8 %	0.9955

Replication of Table 2 of Huggett (1993) using grid sizes $n_a = 1024$, $n_e = 2$, $n_q = 1551$

Table 1
Coefficient of relative risk aversion $\sigma = 1.5$.

Credit limit ($\underline{a}$)	Interest rate (r)	Price (q)
− 2	− 7.1%	1.0124
− 4	2.3%	0.9962
− 6	3.4%	0.9944
− 8	4.0%	0.9935

Table 2
Coefficient of relative risk aversion $\sigma = 3.0$.

Credit limit ($\underline{a}$)	Interest rate (r)	Price (q)
− 2	− 23 %	1.0448
− 4	− 2.6%	1.0045
− 6	1.8%	0.9970
− 8	3.7%	0.9940

Table 3: Original Tables of Huggett (1993)

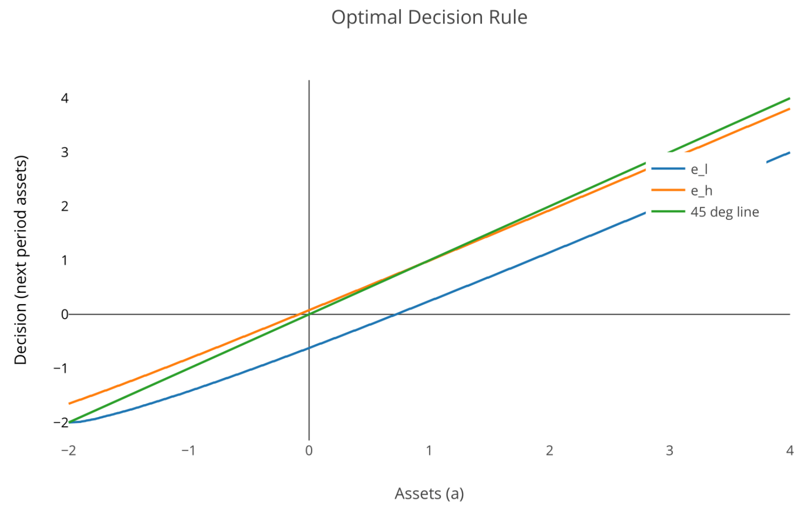

Figure 1: Figure 1 of Huggett (1993)

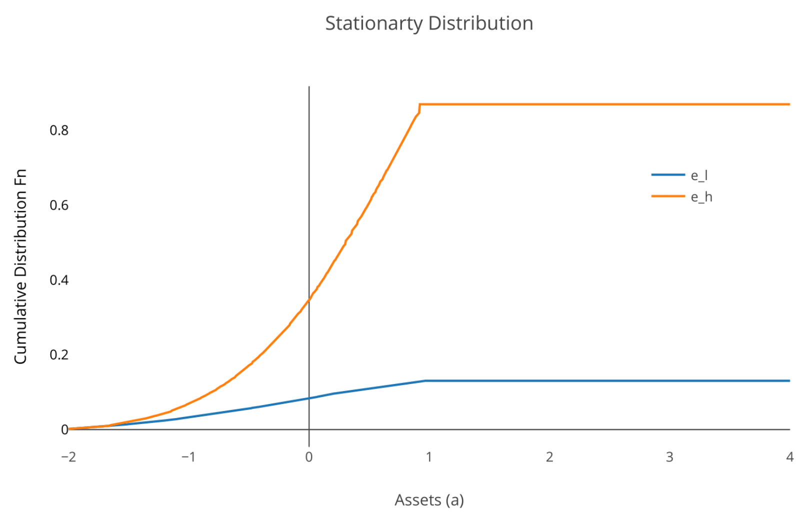

Figure 2: Figure 2 of Huggett (1993)

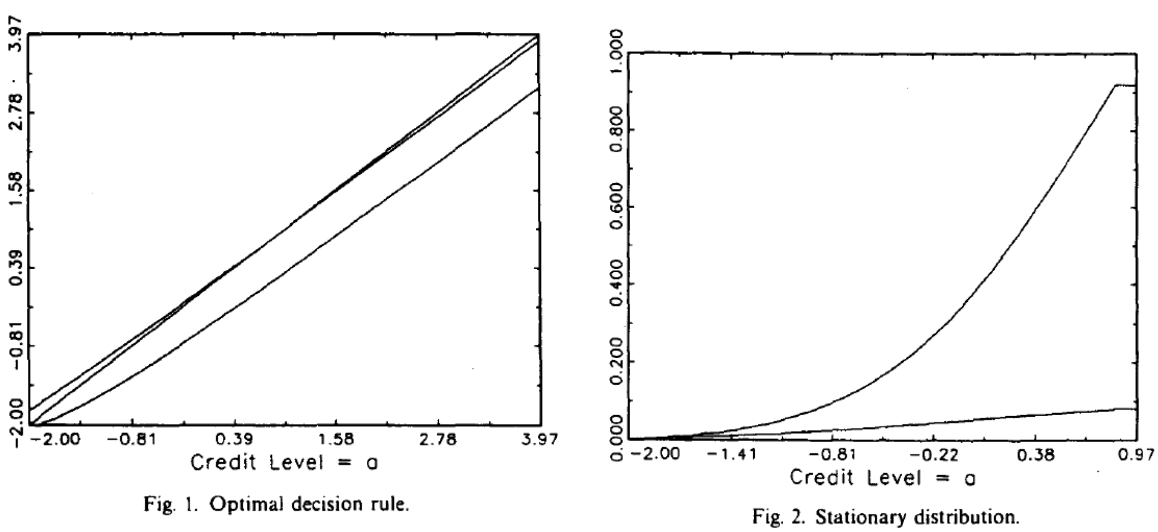

Figure 3: Original Figures of Huggett (1993)