Data Security and Privacy DSE 3258

L4 -BLOCK CIPHER

- A stream cipher is one that encrypts a digital data stream one bit or one byte at a time
- However, the keystream must be provided to both users in advance via some independent and secure channel. This introduces insurmountable logistical problems if the intended data traffic is very large.
- Accordingly, for practical reasons, the bit-stream generator must be implemented as an algorithmic procedure, so that the cryptographic bit stream can be produced by both users. In this approach ,the bit-stream generator is a key-controlled algorithm and must produce a bit stream that is cryptographically strong..
- A block cipher is one in which a block of plaintext is treated as a whole and used to produce a ciphertext block of equal length.
- Typically, a block size of 64 or 128 bits is used. As with a stream cipher, the two users share a symmetric encryption key

Confusion and Diffusion

Two basic building blocks for any cryptographic system

Confusion:

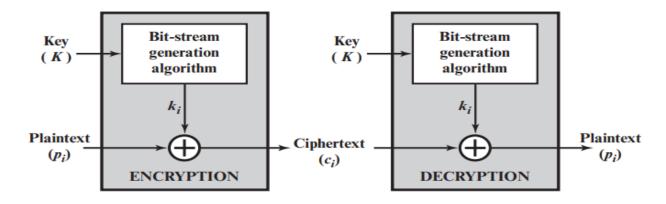
- Making the relationship between encryption key and the cipher text as complex as possible.
- Relationship between PT and CT is obscured.
- That is given CT no information about PT, Key, encryption algorithm.
- Thus, even if the attacker can get some handle on the statistics of the ciphertext, the way in which the key was used to produce that ciphertext is so complex as to make it difficult to deduce the key. This is achieved by the use of a complex substitution algorithm.
- Ex: substitution

Diffusion:

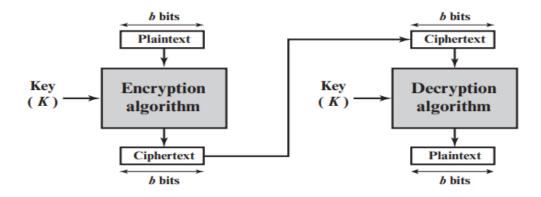
- Making each plaintext bit affect as many CT bits as possible
- One bit change in PT has significant change in CT. This is equivalent to having each ciphertext digit be affected by many plaintext digits
- An example of diffusion is to encrypt a message M = m1, m2, m3, c of characters with an averaging operation: $y_n = \left(\sum_{i=1}^k m_{n+i}\right) \bmod 26$

• Ex: permutation or transposition

Block Cipher	Stream Cipher
Chunk of PT is taking as input and converted to CT	Bit-by-bit PT is converted to CT
PT size is 64 bits or more (multiple of 64)	PT size is 8 bits
Simple operations	Complex operations
Uses confusion and diffusion	Uses only confusion
Decipher is hard	Decipher is easy
Block cipher works on transposition techniques like rail-fence technique, columnar transposition technique, etc.	While stream cipher works on substitution techniques like Caesar cipher, polygram substitution cipher, etc.
Block cipher is slow as compared to a stream cipher.	While stream cipher is fast in comparison to block cipher.



(a) Stream cipher using algorithmic bit-stream generator



(b) Block cipher

Figure 4.1 Stream Cipher and Block Cipher

Block Cipher

Plaintext and ciphertext consists of fixed sized blocks
Ciphertext obtained from plaintext by iterating a round function
Input to round function consists of key and the output of previous round

- Feistel proposed [FEIS73] block cipher by utilizing the concept of a product cipher, which is the execution of two or more simple ciphers in sequence in such a way that the final result or product is cryptographically stronger than any of the component ciphers.
- The essence of the approach is to develop a block cipher with a key length of k bits and a block length of n bits, allowing a total of 2^k possible transformations.
- Feistel proposed the use of a cipher that alternates substitutions and permutations, where these terms are defined as follows:
- **■** Substitution: Each plaintext element or group of elements is uniquely replaced by a corresponding ciphertext element or group of elements.
- Permutation: A sequence of plaintext elements is replaced by a permutation of that sequence. That is, no elements are added or deleted or replaced in the sequence, rather the order in which the elements appear in the sequence is changed

- Plain text is divided into two equal halves and processed through the algorithm.
- Feistel cipher refers to a type of block cipher design, not a specific cipher

Encryption:

- Split plaintext block into left and right halves: Plaintext = (L_0,R_0)
- For each round i=1,2,...,n, compute

$$\begin{split} L_i &= R_{i-1} \\ R_i &= L_{i-1} \oplus F(R_{i-1}, K_i) \\ \text{where } F \text{ is round function and } K_i \text{ is subkey} \end{split}$$

• Ciphertext = (L_n, R_n)

Feistel Cipher

Decryption:

Ciphertext = (L_n, R_n)

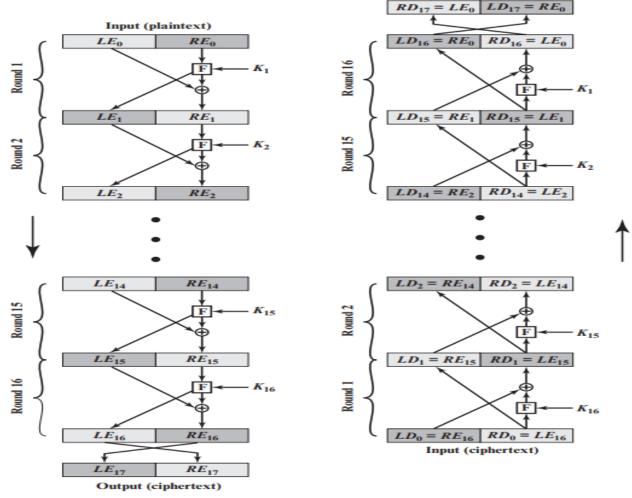
• For each round i=n,n-1,...,1, compute

$$R_{i-1} = L_i$$

$$L_{i-1} = R_i \oplus F(R_{i-1}, K_i)$$

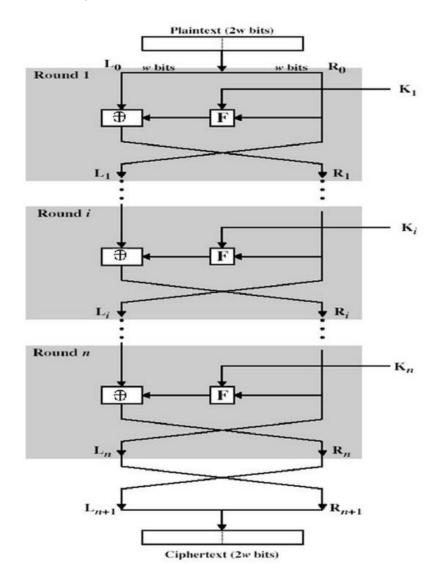
where F is round function and K_i is subkey

- Plaintext = (L_0, R_0)
- Formula "works" for any function F
- But only secure for certain functions F



Output (plaintext)

Figure 4.3 Feistel Encryption and Decryption (16 rounds)



A Feistel network depends on the choice of the following parameters and design features:

- **Block size:** Larger block sizes mean greater security but reduced encryption/decryption speed for a given algorithm. The greater security is achieved by greater diffusion. Traditionally, a block size of 64 bits has been considered a reasonable tradeoff and was nearly universal in block cipher design. However, the new AES uses a 128-bit block size.
- **Key Size:** Larger key size means greater security but may decrease encryption/ decryption speed. The greater security is achieved by greater resistance to brute-force attacks and greater confusion. Key sizes of 64 bits or less are now widely considered to be inadequate, and 128 bits has become a common size.
- **Number of rounds:** multiple rounds offer increasing security. Typical is 16 rounds as a single round provide inadequate security.
- Subkey generation algorithm: greater complexity will lead to greater difficulty of cryptanalysis.
- Round function F greater complexity generally means greater resistance to cryptanalysis.

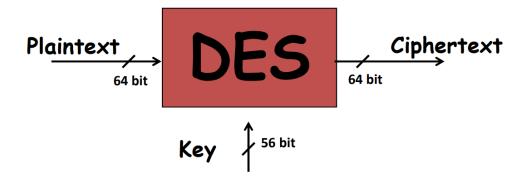
There are two other considerations in the design of a Feistel cipher:

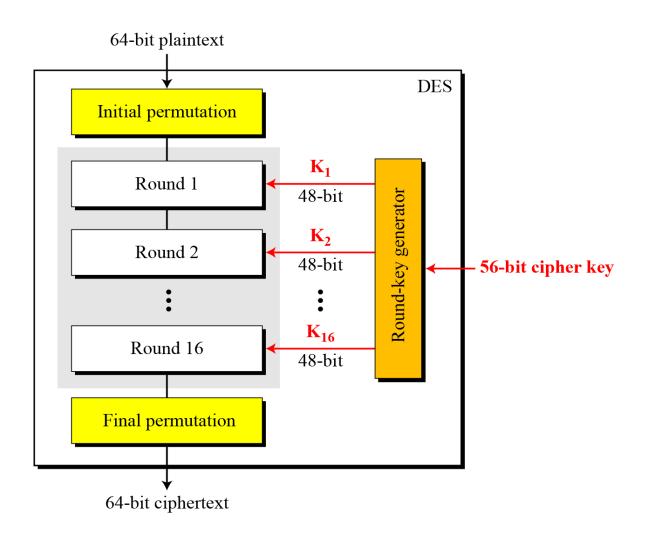
- Fast software encryption/decryption: In many cases, encryption is embedded in applications or utility functions in such a way as to preclude a hardware implementation. Accordingly, the speed of execution of the algorithm becomes a concern.
- Ease of analysis: Although we would like to make our algorithm as difficult as possible to cryptanalyze, there is great benefit in making the algorithm easy to analyze. That is, if the algorithm can be concisely and clearly explained, it is easier to analyze that algorithm for cryptanalytic vulnerabilities and therefore develop a higher level of assurance as to its strength.

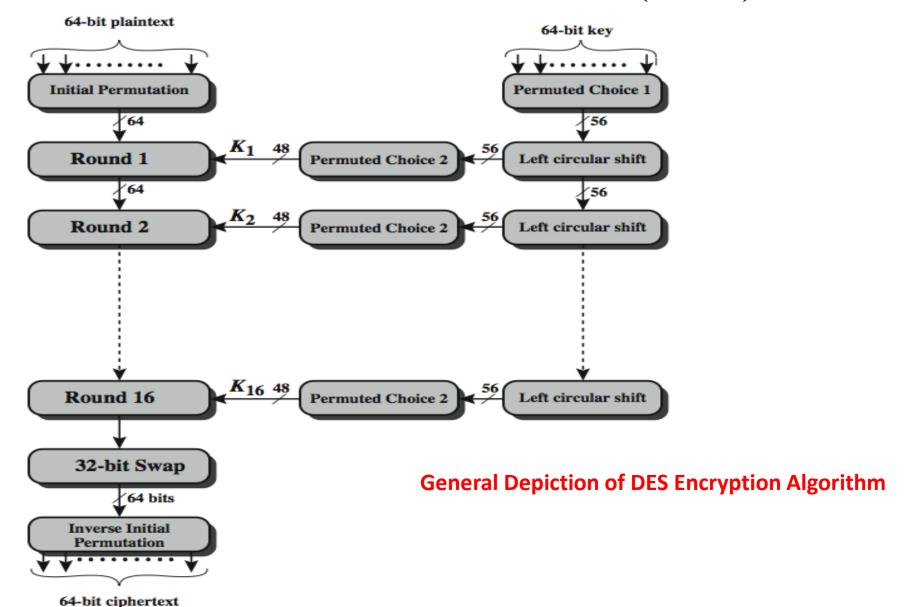
- >Symmetric block cipher.
- most widely used block cipher in world
- ➤ adopted in 1977 by National Bureau of Standards (NBS), now the National Institute of Standards and Technology (NIST
- > encrypts 64-bit data using 56-bit key
- has widespread use
- The algorithm transforms 64-bit input in a series of steps into a 64-bit output. The same steps, with the same key, are used to reverse the encryption

- Follows Feistel structure.
- Block size = 64 bits of plain text
- No of rounds = 16 rounds
- Key size = 56 bits
- No of subkeys = 16 subkeys (16 rounds)
- Sub key size = 48 bits
- Cipher text size = 64 bits

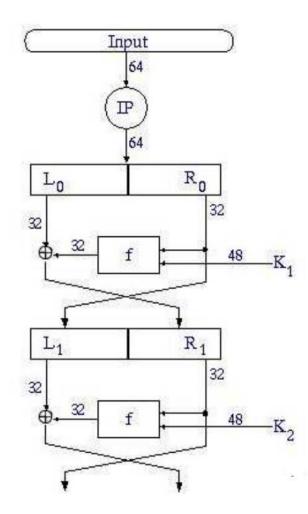
the function expects a 64-bit key as input. However, only 56 of these bits are ever used; the other 8 bits can be used as parity bits or simply set arbitrarily.



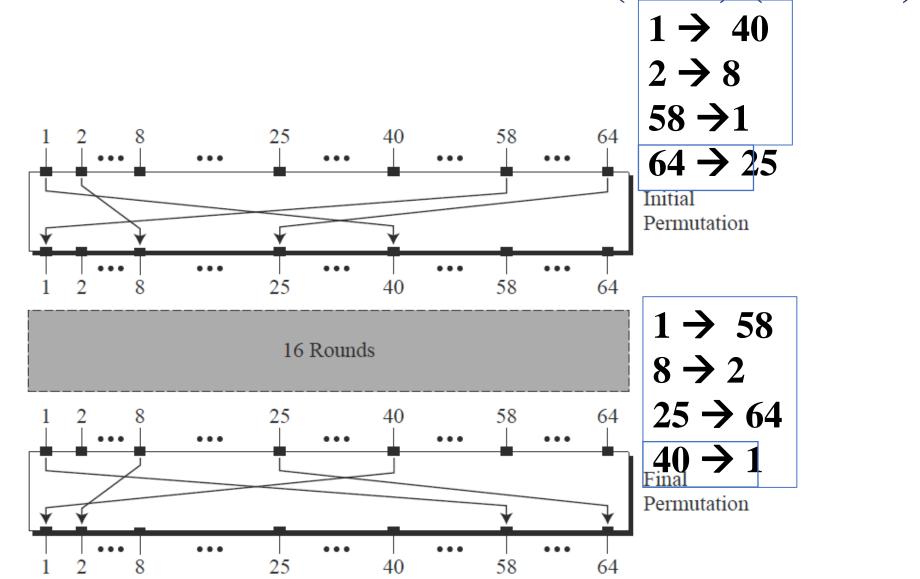




- $IP(x) = L_0R_0$
- $L_i = R_{i-1}$
- $R_i = L_{i-1} \bigoplus f(R_{i-1}, K_i)$
- $y = IP^{-1}(R_{16}L_{16})$

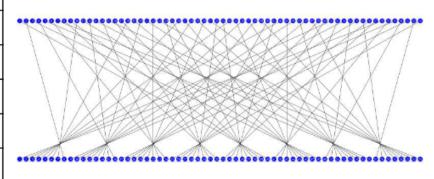


- First, the 64-bit plaintext passes through an initial permutation (IP) that rearranges the bits to produce the permuted input.
- This is followed by a phase consisting of sixteen rounds of the same function, which involves both permutation and substitution functions.
- The output of the last (sixteenth) round consists of 64 bits that are a function of the input plaintext and the key.
- The left and right halves of the output are swapped to produce the preoutput.
- Finally, the preoutput is passed through a permutation [IP-1] that is the inverse of the initial permutation function, to produce the 64-bit ciphertext.
- With the exception of the initial and final permutations, DES has the exact structure of a Feistel cipher.



Initial Permutation (IP)

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

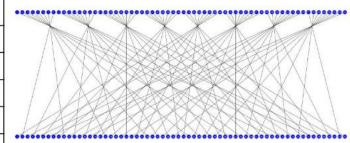


This table specifies the input permutation on a 64-bit block.

- The meaning is as follows:
- the first bit of the output is taken from the 58th bit of the input; the second bit from the 50th bit, and so on, with the last bit of the output taken from the 7th bit of the input.
- This information is presented as a table for ease of presentation:
- it is a vector, not a matrix.

Final Permutation (IP-1)

	307	3.2	0.00	131	77.0	33.2	
40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25



The final permutation is the inverse of the initial permutation; the table is interpreted similarly.

• That is, the output of the Final Permutation has bit 40 of the preoutput block as its first bit, bit 8 as its second bit, and so on, until bit 25 of the preoutput block is the last bit of the output

Example 1

- Example:
 - Find the output of the initial permutation box when the input is given in hexadecimal as:

0x0002 0000 0000 0001

- The input has only two 1s (bit 15 and bit 64); the output must also have only two 1s (the nature of straight permutation).
- Using Table, we can find the output related to these two bits.
- Bit 15 in the input becomes bit 63 in the output.
- Bit 64 in the input becomes bit 25 in the output.
- So the output has only two 1s, bit 25 and bit 63. The result in hexadecimal is

0x0000 0080 0000 0002

```
0: 0000 (bits 1-4)
0: 0000 (bits 5-8)
0: 0000 (bits 9-12)
2: 0010 <--bit 15 is 1 (bits 13-16)
```

```
0: 0000 (bits 49-52)
0: 0000 (bits 53-56)
0: 0000 (bits 57-60)
1: 0001 <--bit 64 is 1 (bits 61-64)
```

Example 2

Find the output of the initial permutation box when the input is given in hexadecimal as:

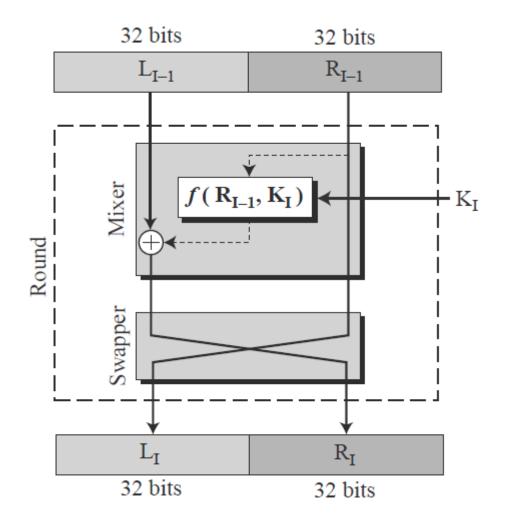
0x0000 0080 0000 0002

Solution

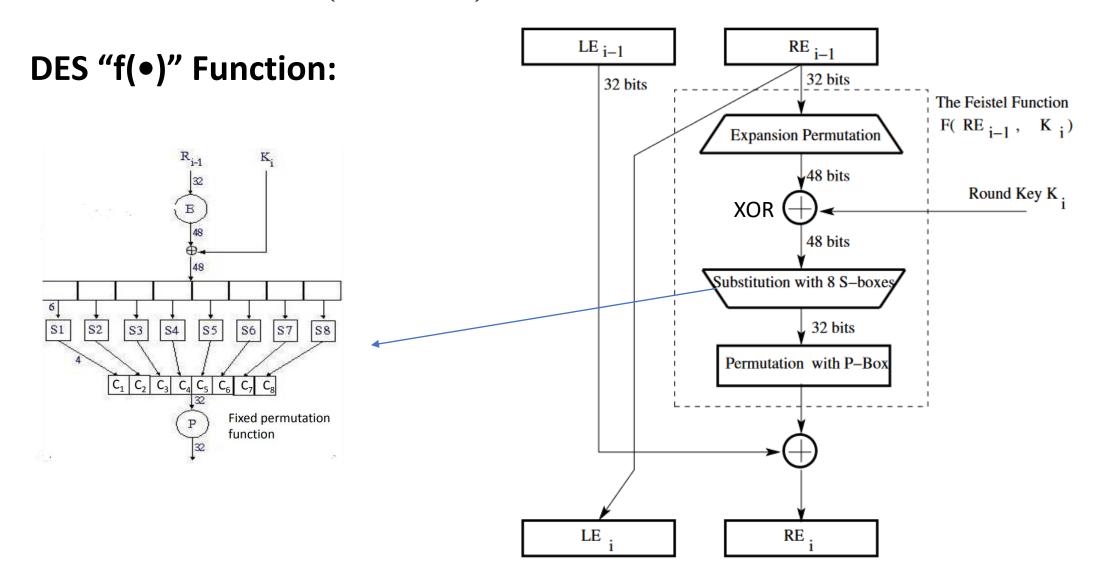
Only bit 25 and bit 64 are 1s; the other bits are 0s. In the final permutation, bit 25 becomes bit 64 and bit 63 becomes bit 15. The result is

0x0002 0000 0000 0001

DES Rounds:



A round in DES (encryption site)



DES "f(•)" Function:

Expansion Function (E):

E is an expansion function which takes a block of 32 bits as input and produces a block of 48 bits as output

E-step entails the following:

- first divide the 32-bit block into eight 4-bit words
- attach an additional bit on the left to each 4-bit word that is the last bit of the previous 4-bit word
- attach an additional bit to the right of each 4-bit word that is the beginning bit of the next 4-bit

word.

Note that what gets prefixed to the first 4-bit block is the last bit of the last 4-bit block. By the same token, what gets appended to the last 4-bit block is the first bit of the first 4-bit block

1	2	3	4		
5	6	7	8		
9	10	11	12		
13	14	15	16		
17	18	19	20		
21	22	23	24	,	
25	26	27	28	(
20	20	21	22		

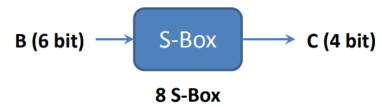
32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

DES "f(•)" Function:

The S-Box for the Substitution Step in Each Round

S-box is to introduce diffusion in the generation of the output from the input.

The 48-bit input word is divided into eight 6-bit words and each 6-bit word fed into a separate S-box. Each S-box produces a 4-bit output. Therefore, the 8 S-boxes together generate a 32-bit output.

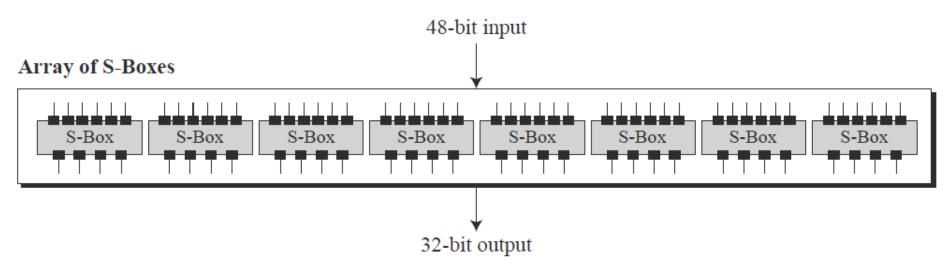


Each of the eight S-boxes consists of a 4×16 table lookup for an output 4-bit word.

- S = matrix 4x16, values from 0 to 15
- B (6 bit long) = $b_1b_2b_3b_4b_5b_6$
 - $-b_1b_6$ → r = row of the matrix (2 bits: 0,1,2,3)
 - $-b_2b_3b_4b_5$ \rightarrow c = column of the matrix (4 bits:0,1,...15)
- C (4 bit long) = Binary representation of S(r, c)

DES "f(•)" Function:

There are 8 S-boxes



Each row in all eights tables is a random permutation of the 16 integers, 0 through 15, and no two permutations are the same in all of the eight tables taken together.

DES "f(•)" Function(Feistel Function):

Example S-boxes

Row #	S_1	1	2	3				7								15	Column #
0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7	
1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8	
2	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0	
3	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13	

 $S(i, j) \le 16$, can be represented with 4 bits

Example: B = 101111

$$b_1b_6 = 11 = row 3$$

$$b_2b_3b_4b_5 = 0111 = column 7$$



Assignment: B=011011, C=?

	The 4×16 substitution table for S-box S_1														
14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13
								$\times S_2$							
15	1	8	14	6	11	3	4	9	7	2	13	12	O	5	10
3	13	4	7	15	2	8	14	12	O	1	10	6	9	11	5
0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9
								$\propto S_3$							
10 0 9 14 6 3 15 5 1 13 12 7 11 4 2 8															
13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12
			_		_	_		$\propto S_4$	_					_	
7	13	14	3	0	6	9	10	1	2	8	5	11	12	-4	15
13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
10	6	9	0	12	11	7	13	15	1	3	14	.5	2	8	4
3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14
	10		-	-	10		S-bo		_			- 10			
2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3
10	-1	10	45				S-bo		4.2	- 2		1.4	7	_	
12 10	1 15	10	15 2	9	2 12	6	8 5	0 6	13	3 13	4 14	14	7	5 3	11
	14	15	5	2		9	3	7	1	4	10	0	11 13		8 6
9	3	15 2	12	9	8 5	12 15	10	11	0 14	1	7	1 6	0	11 8	13
-1	-3		12	9	- 3	13	S-bo		1.4	1	-	0	U	0	13
4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
1	4	11	13	12	3	7	14	10	15	6	8	o	5	9	2
6	11	13	8	1	4	10	7	9	5	ő	15	14	2	3	12
_		10		_	_	10		$\propto S_8$	-		10		2		
13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
1	15	13	8	10	3	7	4	12	5	6	11	o	14	9	2
7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11
	_			_											

Example

The input to S-box 1 is $\underline{1}0001\underline{1}$. What is the output?

Solution If we write the first and the sixth bits together, we get 11 in binary, which is 3 in decimal. The remaining bits are 0001 in binary, which is 1 in decimal. We look for the value in row 3, column 1, in Table 6.3 (S-box 1). The result is 12 in decimal, which in binary is 1100. So the input 100011 yields the output 1100.

The input to S-box 8 is $\underline{\mathbf{0}}$ 0000 $\underline{\mathbf{0}}$. What is the output?

Solution If we write the first and the sixth bits together, we get 00 in binary, which is 0 in decimal. The remaining bits are 0000 in binary, which is 0 in decimal. We look for the value in row 0, column 0, in Table 6.10 (S-box 8). The result is 13 in decimal, which is 1101 in binary. So the input 000000 yields the output 1101.

DES Rounds:

The P-Box Permutation in the Feistel Function

- The last operation in the Feistel function is a permutation with a "Permutation with P-Box"--- 32-bit input and a 32-bit output.
- The input/output relationship for this operation is shown in Table and follows the same general rule as previous tables.

	P-Box Permutation											
15	15 6 19 20 28 11 27 16											
0	14	22	25	4	17	30	9					
1	1 7 23 13 31 26 2 8											
18	12	29	5	21	10	3	24					

This permutation 'table' says that the 0th output bit will be the 15th bit of the input, the 1st output bit the 6th bit of the input, and so on, for all of the 32 bits of the output that are obtained from the 32 bits of the input.

The DES Key Schedule: Generating the Round Keys

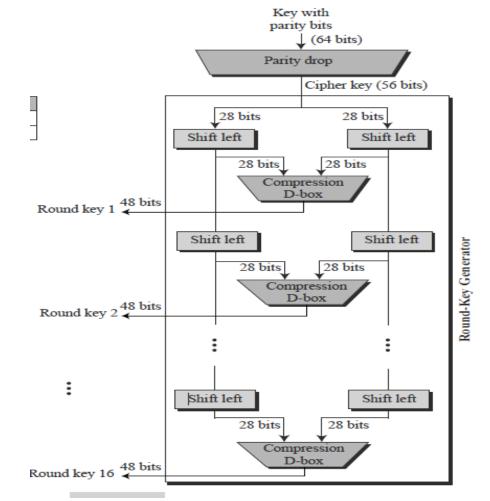
The **round-key generator** creates sixteen 48-bit keys out of a 56-bit cipher key.

However, the cipher key is normally given as a 64-bit key in which 8 extra bits are the parity bits, which are dropped before the actual keygeneration process.

At the beginning of each round, we divide the 56 relevant key bits into two 28 bit halves and circularly shift to the left each half by one or two bits, depending on the round, as shown in the table

Shifting

Rounds	Shift
1, 2, 9, 16	one bit
Others	two bits



The DES Key Schedule: Generating the Round Keys

- The 56-bit encryption key is represented by 8 bytes, with the last bit (the least significant bit)
 of each byte used as a parity bit.
- The relevant 56 bits are subject to a permutation at the beginning before any round keys are generated. This is referred to as Key Permutation 1 (PC1)
- At the beginning of each round, we divide the 56 relevant key bits into two 28 bit halves and circularly shift to the left each half by one or two bits, depending on the round,
- For generating the round key, we join together the two halves and apply a 56 bit to 48 bit contracting permutation this is referred to as **Key Permutation 2 (PC2)**, The resulting 48 bits constitute round key.
- The contraction permutation in **Key Permutation 2**, along with the one-bit or two-bit rotation of the two key halves prior to each round, is meant to ensure that each bit of the original encryption key is used in roughly 14 of the 16 rounds.

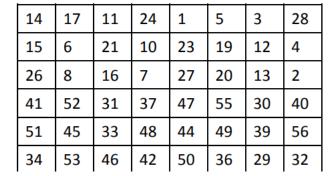
The DES Key Schedule: Generating the Round Keys

Parity-check bits (namely, bits 8,16, 4,32,40,48,56,64) are not chosen, they do not appear in **PC-1**



Left											
57	49	41	33	25	17	9					
1	58	50	42	34	26	18					
10	2	59	51	43	35	27					
19	11	3	60	52	44	36					
	Right										
63	55	47	39	31	23	15					
7	62	54	46	38	30	22					
14	6	61	53	45	37	29					
21	13	5	28	20	12	4					

I aft





PC-2 selects the 48-bit subkey for each round from the 56-bit key-schedule state

The permutation order for the bits is given by reading the entries shown from the upper left corner to the lower right corner.

This permutation tells us that the 0th bit of the output will be the 57 th bit of the input (in a 64 bit representation of the 56-bit encryption key), the 1st bit of the output the 49th bit of the input, and so on

THE STRENGTH OF DES

1. The Use of 56-Bit Keys

- With a key length of 56 bits, there are 256 possible keys, which is approximately 7.2 * 1016 keys. Thus, on the face of it, a brute-force attack appears impractical.
- Assuming that, on average, half the key space has to be searched, a single machine

2. Timing attack

- The authors conclude that DES appears to be fairly resistant to a successful timing attack but suggest some avenues to explore.
- A timing attack is one in which information about the key or the plaintext is obtained by observing how long it takes a given implementation to perform decryptions on various ciphertexts. A timing attack exploits the fact that an encryption or decryption algorithm often takes slightly different amounts of time on different inputs.

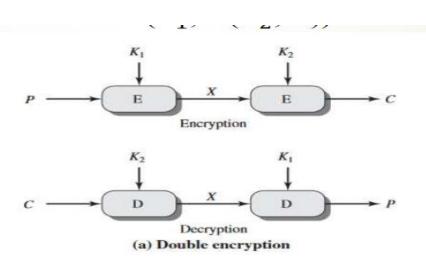
THE STRENGTH OF DES

3. The Nature of the DES Algorithm

Another concern is the possibility that cryptanalysis is possible by exploiting the characteristics of the DES algorithm. The focus of concern has been on the eight substitution tables, or S-boxes, that are used in each iteration. Because the design criteria for these boxes, and indeed for the entire algorithm, were not made public, there is a suspicion that the boxes were constructed in such a way that cryptanalysis is possible for an opponent.

Double DES

- DES uses a 56-bit key, this raised concerns about brute force attacks.
- One proposed solution: double DES.
- Apply DES twice using two keys, K1 and K2.
- - Encryption: $C = E_{K2} [E_{K1} [P]]$
- - Decryption: $P = D_{K2} [D_{K1} [C]]$
- This leads to a 2x56=112 bit key, so it is more secure than DES



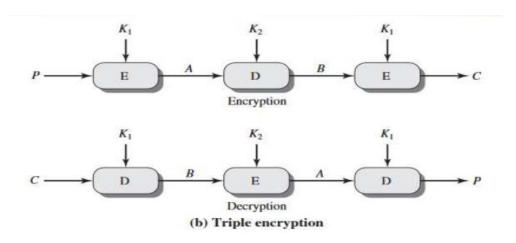
Triple DES

$$C = E(K_3, D(K_2, E(K_1, P)))$$

The encryption-decryption process is as follows –

- Encrypt the plaintext blocks using single DES with key K₁.
- Now decrypt the output of step 1 using single DES with key K_2 .
- Finally, encrypt the output of step 2 using single DES with key K_3 .
- The output of step 3 is the ciphertext.
- Decryption of a ciphertext is a reverse process. User first decrypt using K_{3} , then encrypt with K_{2} and finally decrypt with K_{1} .

Triple DES

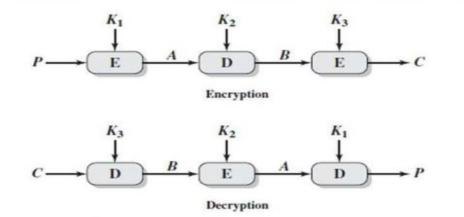


Triple DES with 2-key

The function follows an encrypt-decrypt-encrypt (EDE) sequence.

$$C = E(K1, D(K2, E(K1, P)))$$

$$P = D(K1, E(K2, D(K1, C)))$$



Triple DES with 3-key

3-key 3DES has an effective key length of 168 bits and is defined as

$$C = E(K3, D(K2, E(K1, P)))$$

$$P = D(K1, E(K2, D(K3, C))$$

Example (DES Round key generation)

- Generate round-1 key using the key in Hex K=0x133457799BBCDFF1
- Given data: PC1 and PC2=

	Left											
57	49	41	33	25	17	9						
1	58	50	42	34	26	18						
10	2	59	51	43	35	27						
19	19 11 3 60 52 44 36											
			Right									
63	55	47	39	31	23	15						
7	62	54	46	38	30	22						
14	6	61	53	45	37	29						
21	13	5	28	20	12	4						

14	17	11	24	1	5	3	28
15	6	21	10	23	19	12	4
26	8	16	7	27	20	13	2
41	52	31	37	47	55	30	40
51	45	33	48	44	49	39	56
34	53	46	42	50	36	29	32

- Solution:
- Step1: Convert Hex number to 64 bit binary format

Example:

Step2: Remove parity bits and apply PC1

Hex digi ts	Bit position	bits	bits	bits	bits	bi s	bits	bits	bits
13	1-2-3-4-5-6-7-8	0	0	0	1	0	0	1	1
34	9-10-11-12-13-14- 15-16	0	0	1	1	0	1	0	0
57	17-18-19-20-21-22- 23-24	0	1	0	1	0	1	1	<u>1</u>
79	25-26-27-28-29-30- 31-32	0	1	1	1	1	0	0	1
9B	33-34-35-36-37-38- 39-40	1	0	0	1	1	0	1	<u>1</u>
ВС	41-42-43-44-45-46- 47-48	1	0	1	1	1	0	0	0
DF	49-50-51-52-53-54- 55-56	1	1	0	1	1	1	1	<u>1</u>
F1	57-58-59-60-61-62- 63-64	1	1	1	1	0	0	0	1

- Parity bits
- Apply PC1: we get 56 bit key permutation
- 1111000 0110011 0010101 0101111 0101010 1010001 1001111 0001111

Left							
57	49	41	33	25	17	9	
1	58	50	42	34	26	18	
10	2	59	51	43	35	27	
19	11	3	60	52	44	36	
	Right						
63	55	47	39	31	23	15	
7	62	54	46	38	30	22	
14	6	61	53	45	37	29	
21	13	5	28	20	12	4	

1	1	1	1	0	0	0
0	1	1	0	0	1	1
0	0	1	0	1	0	1
0	1	0	1	1	1	1
0	1	0	1	0	1	0
1	0	1	0	0	0	1
1	0	0	1	1	1	1
0	0	0	1	1	1	1

Example

- Step3: Split this key into left and right halves, C_0 and D_0 , where each half has 28 bits.
- $C_o = 1111000 0110011 0010101 0101111$ $D_o = 0101010 1011001 1001111 0001111$
- Step4: Left shift → Round 1 i-shift
- C_o = 111000 0110011 0010101 01011111 D_o = 101010 1011001 1001111 00011110

Step5: Apply PC2

1	1	1	<mark>0</mark>	<mark>0</mark>	O	0
<mark>1</mark>	1	O	<mark>0</mark>	<mark>1</mark>	<mark>1</mark>	0
O	1	O	<mark>1</mark>	<mark>0</mark>	<mark>1</mark>	0
1	<mark>0</mark>	1	1	<mark>1</mark>	1	<mark>1</mark>
1	0	1	0	1	0	1
0	1	0	0	0	1	1
0	0	1	1	1	1	0
0	0	1	1	1	1	0

14	17	11	24	1	5	3	28
15	6	21	10	23	19	12	4
26	8	16	7	27	20	13	2
41	52	31	37	47	55	30	40
51	45	33	48	44	49	39	56
34	53	46	42	50	36	29	32

1-7	
8-14	
15-21	
22-28	
29-35	
36-42	
43-49	
50-56	

1	1	1	0	0	0	0
1	1	0	0	1	1	0
<mark>0</mark>	1	<mark>0</mark>	1	<mark>O</mark>	<mark>1</mark>	0
1	0	1	1	<mark>1</mark>	<mark>1</mark>	1
1	0	1	0	1	0	1
0	1	0	0	0	1	1
0	0	1	1	1	1	0
0	0	1	1	1	1	0

0	0	0	1	1	0	1	1
0	0	0	0	0	0	1	0
1	1	1	0	1	1	1	1
1	1	1	1	1	1	0	0
0	1	1	1	0	0	0	0
0	1	1	1	0	0	1	0

 $K_1 = 000110 \ 110000 \ 001011 \ 101111 \ 111111 \ 000111 \ 000001 \ 110010----- \rightarrow 48 \ bit key$

$C_{\theta} = 11110000110011001010101011111$ $D_{\theta} = 01010101010110011100111110001111$
$C_{I} = 11100001100110010101010111111$ $D_{I} = 10101010110111001111100011110$
$C_2 = 1100001100110010101010111111$ $D_2 = 010101011011001111000111100$
$C_3 = 0000110011001010101011111111$ $D_3 = 0101011001100111100011110101$
$C_4 = 001100110010101010111111111100$ $D_4 = 01011001100111110001111010101$
$C_5 = 110011001010101011111111110000$ $D_5 = 01100110011111000111101010101$
$C_6 = 001100101010101111111111000011$ $D_6 = 10011001111100011110101010101$
$C_7 = 110010101010111111111100001100$ $D_7 = 01100111110001111010101010101$
$C_8 = 001010101011111111110000110011$ $D_8 = 1001111000111101010101010101$

```
D_q = 0011110001111010101010110011
C_{10} = 010101011111111110000110011001
D_{10} = 1111000111101010101011001100
C_{11} = 010101111111111000011001100101
D_{11} = 1100011110101010101100110011
C_{12} = 010111111111100001100110010101
D_{12} = 0001111010101010110011001111
C_{13} = 01111111110000110011001010101
D_{13} = 0111101010101011001100111100
C_{14} = 11111111000011001100101010101
D_{14} = 11101010101011001100111110001
C_{15} = 11111000011001100101010101111
D_{15} = 1010101010110011001111000111
C_{16} = 11110000110011001010101011111
D_{16} = 0101010101100110011110001111
```

 $C_{\mathbf{g}} = 010101010111111111100001100110$

 $K_2 = 011110\ 011010\ 111011\ 011001\ 110110\ 111100\ 100111\ 100101$ $K_3 = 010101\ 011111\ 110010\ 001010\ 010000\ 101100\ 111110\ 011001$ $K_4 = 011100\ 101010\ 110111\ 010110\ 110110\ 110011\ 010100\ 011101$ $K_5 = 011111 \ 001110 \ 110000 \ 000111 \ 111010 \ 110101 \ 001110 \ 101000$ $K_6 = 011000\ 111010\ 010100\ 111110\ 010100\ 000111\ 101100\ 101111$ $K_7 = 111011\ 001000\ 010010\ 110111\ 111101\ 100001\ 100010\ 1111100$ $K_R = 111101 \ 111000 \ 101000 \ 111010 \ 110000 \ 010011 \ 101111 \ 111011$ $K_{q} = 111000\ 001101\ 101111\ 101011\ 111011\ 011110\ 011110\ 000001$ $K_{10} = 101100\ 011111\ 001101\ 000111\ 101110\ 100100\ 011001\ 001111$ $K_{11} = 001000\ 010101\ 1111111\ 010011\ 110111\ 101101\ 001110\ 000110$ $K_{12} = 011101\ 010111\ 000111\ 110101\ 100101\ 000110\ 011111\ 101001$ $K_{13} = 100101\ 1111100\ 010111\ 010001\ 111110\ 101011\ 101001\ 000001$ $K_{14} = 010111 \ 110100 \ 001110 \ 110111 \ 111100 \ 101110 \ 011100 \ 111010$ $K_{15} = 101111 \ 111001 \ 000110 \ 001101 \ 001111 \ 010011 \ 111100 \ 001010$ $K_{16} = 110010 \ 110011 \ 110110 \ 001011 \ 000011 \ 100001 \ 011111 \ 110101$