DSE 2256 DESIGN & ANALYSIS OF ALGORITHMS

Lecture 16

Decrease-and-Conquer:

Topological Sorting

Courtesy : www.alamy.com

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Recap of L14 & L15

Decrease-and-Conquer

1. Reduce problem instance to smaller instance of the same problem

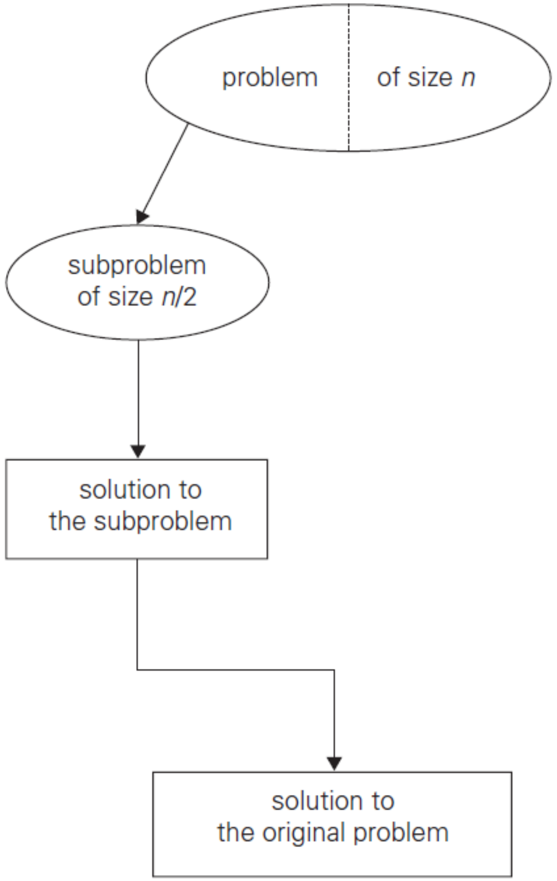
2. Solve smaller instance.

3. Extend solution of smaller instance to obtain solution to original instance

Recap of L14 & L15

Types of Decrease-and-conquer techniques:

1. Decrease by a constant (usually by 1):
   - ✓ Insertion sort
   - ✓ Topological sorting
2. Decrease by a constant factor (usually by half):
   - ✓ Binary search
3. Variable-size decrease
   - ✓ Euclid's algorithm



Recap of L14 & L15

Many problems require processing all graph vertices (and edges) in systematic fashion

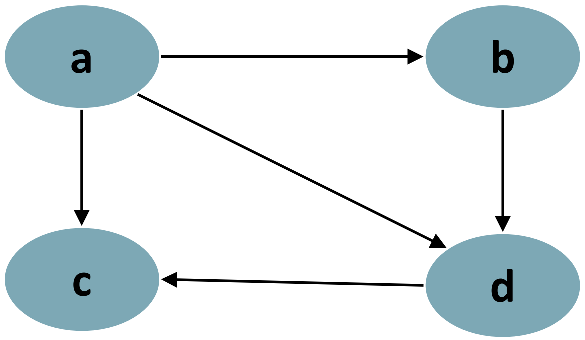
Graph traversal algorithms:

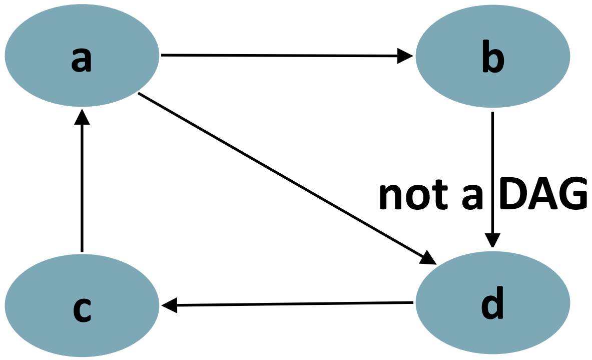
- **Depth-first search (DFS)**
  - Data structure used -Stack
- **Breadth-first search (BFS)**
  - Data structure used - Queue

Directed Acyclic Graph (DAG)

A *dag*: a directed acyclic graph, i.e. a directed graph with no (directed) cycles

a DAG





not a DAG

- Arise in modeling many problems that involve prerequisite constraints (construction projects, document version control)
- **Vertices of a dag can be linearly ordered so that for every edge, its starting vertex is listed before its ending vertex (*topological sorting*).**
- Being a dag is also a necessary condition for topological sorting to be possible.

Topological Sorting

Vertices of a DAG can be linearly ordered so that for every edge its starting vertex is listed before its ending vertex is called topological sorting.

- **Two methods to implement: DFS based topological sorting, Source removal algorithm**

Example:

{C1,C2,C3,C4,C5} – are set of five courses a part-time student has to take in the degree program

Courses should meet the following prerequisites:

- C1 and C2 have no prerequisites
- C3 requires C1 and C2
- C4 requires C3
- C5 requires C3 and C4
- Student can take only one course per term.

In which order should the student take the courses?

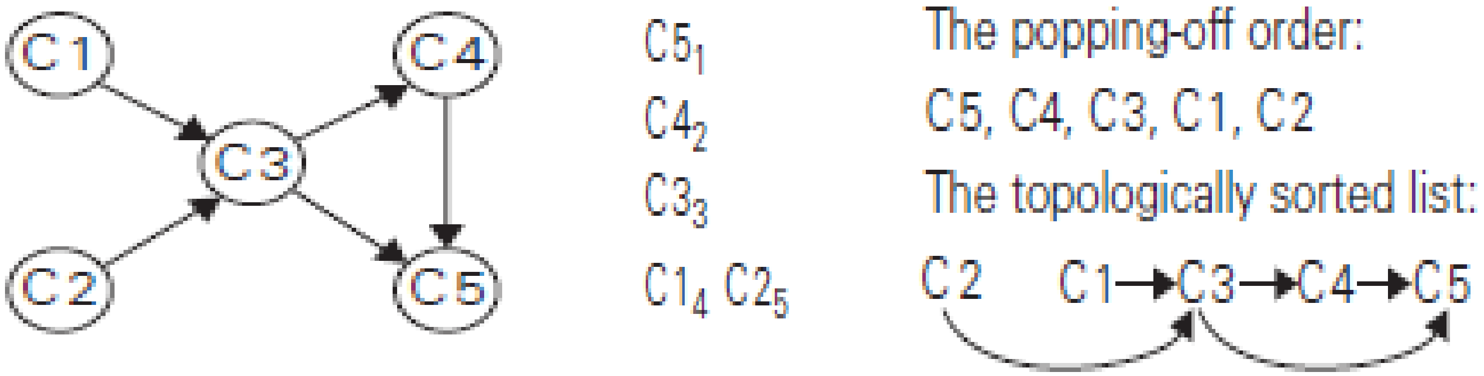
DFS-based algorithm for topological sorting

Solution:

- **Perform DFS traversal, noting the order of vertices, popped off the traversal stack**
- **Reverse order solves topological sorting problem**
- **Back edges encountered?→ NOT a dag!**

Prerequisites:

- C1 and C2 have no prerequisites
- C3 requires C1 and C2
- C4 requires C3
- C5 requires C3 and C4



$C5_1$

$C4_2$

$C3_3$

$C1_4$ $C2_5$

The popping-off order:

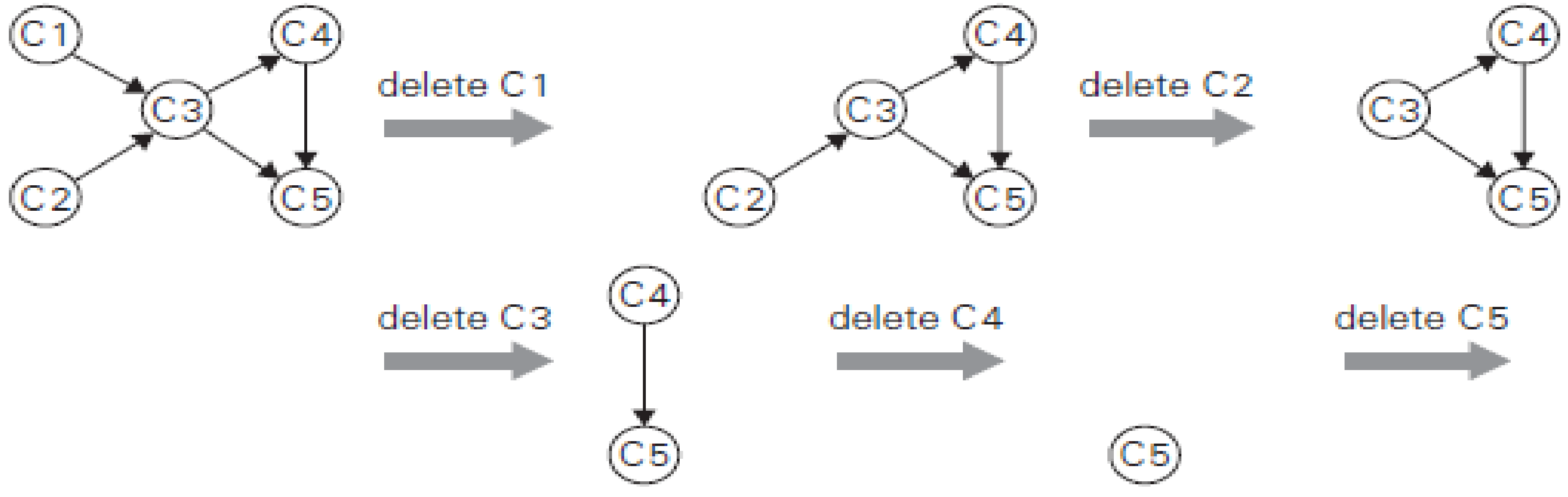
C5, C4, C3, C1, C2

The topologically sorted list:

C2 C1→C3→C4→C5

Source removal algorithm for Topological Sorting

- Repeatedly identify and remove a *source* (a vertex with no incoming edges) and all the edges incident to it until either no vertex is left or there is no source among the remaining vertices (not a dag)



The solution obtained is C1, C2, C3, C4, C5

Notes on Topological Sorting

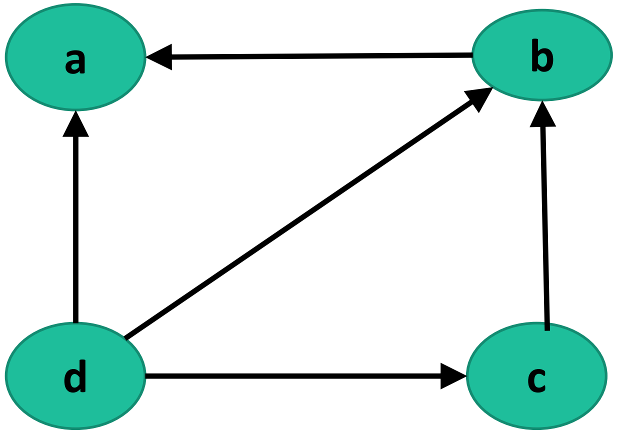
- Algorithm for directed graph or digraph
- A directed graph should be DAG (Directed Acyclic Graph)

- Topological sorting implemented by two methods
    - DFS based algorithm
    - Source removal algorithm

- Topological sorting may have several alternative solutions.
    - Solution obtained by the DFA based algorithm and source removal are different.

Problem

Apply the DFS-based algorithm and Source removal algorithm to solve the topological sorting problem for the following digraph.



Thank you!

Any queries?