

## 1. GEOMETRIC CONSTRUCTION

You can only use compass and straightedge which has no measure

### Basic construction of lines.

**Problem 1.1.** You are given a line segment with end points  $A$  and  $B$ , draw a line bisecting  $AB$  and perpendicular to  $AB$ .

This line is called the *perpendicular bisector* of  $AB$ , and the point these two lines meet is called the *midpoint* of  $AB$ .

**Problem 1.2.** You are given an angle  $\angle ABC$ , find a point  $X$  in the interior of  $\angle ABC$  such that  $\angle ABX = \angle CBX$  and draw the line  $BX$ .

The line  $BX$  is called the *angle bisector* of  $\angle ABC$ .

**The four special points of triangle.** In this part, you are given a triangle  $\triangle ABC$ .

We will consider the intersection of the following two kind of lines, the medians and the altitudes.

It turns out that every lines of the same kinds will meet at a unique point.

**Definition.** Let  $M_{AB}$  be the midpoint of  $AB$ , draw the line passing through  $M_{AB}$  and  $C$ .

Similarly, draw the others two lines.

These three lines are called the *median* of  $\triangle ABC$ .

**Definition.** Draw a line passing through  $A$  and perpendicular to  $BC$ , similar the other two lines.

These three lines are called the *altitude* of  $\triangle ABC$ .

**Theorem.** *The three medians of  $\triangle ABC$  are concurrent.*

*The three altitudes of  $\triangle ABC$  are concurrent.*

*The three perpendicular bisectors of  $\triangle ABC$  are concurrent.*

*The three angle bisectors of  $\triangle ABC$  are concurrent.*

**Definition.** The point that the three medians meet is called the *centroid* of  $\triangle ABC$ .

**Definition.** The point that the three altitudes meet is called the *orthocenter* of  $\triangle ABC$ .

**Problem 1.3.** Locate the center of the circle passing through the points  $A, B$  and  $C$ , and draw the circle.

The circle is called the *circumcircle* of  $\triangle ABC$ , and the center is called the *circumcenter*.

**Problem 1.4.** Locate the center of the circle which touches each side of the triangle, and draw the circle.

The circle is called the *incircle* of  $\triangle ABC$ , and the center is called the *incenter*.

**Problem 1.5.** Let  $G$  be the orthocenter of the triangle,  $\angle BAC = x^\circ$ , find  $\angle BGC$  in terms of  $x$ .