Problem Set #3 - Random Variables

The following problems will cover material discussed in Week 3. You must answer each question correctly to earn credit on the question. You have three attempts per question.

If not specified in a particular question, please round your numerical answers to three decimal places.

Part 1

The following probability is the mass function for the number of times a certain computer program will malfunction over an interval of time:

x	0	1	2	3	4	5
p(x)	0.08	0.27	0.34	0.19	0.09	0.03

Question 1
Review

Compute $E(X)$.

Type your numeric answer and submit

Show Submitted Answer | Show Correct Answer | Check My Answer

Question 2
Review

Compute $\mathrm{Var}(X)$.

Type your numeric answer and submit

Show Submitted Answer | Show Correct Answer | Check My Answer

Part 2

A consumer organization that evaluates new motorcycles customarily reports the number of major defects in each motorcycle examined. Let *X* denote the number of major defects in a randomly selected motorcycle of a certain type. Recall the *cumulative density function*, or "cdf", is a function for x that calculates the probability of the value x and all values below, F(x) = P(X≤x).

The cdf of X is as follows:

x	0	1	2	3	4	5	6
F(x)	0.05	0.21	0.45	0.63	0.85	0.98	1

Question 3
Review

Fill in the Blanks

Type your answers in all of the blanks and submit

X_2 X^2 Ω

Calculate the following probabilities from the cdf: (Round to two decimal places)

(a) $F(2)$, that is, $P(X \leq 2)$ Type your answer here

(b) $P(X > 3)$ Type your answer here

(c) $P(2 \leq X \leq 5)$ Type your answer here

(d) $P(2 < X < 5)$ Type your answer here

Correct Answers:

Calculate the following probabilities from the cdf: (Round to two decimal places) (a) [math]F(2)[/math], that is, [math] P(X\leq2)[/math]

✓ 0.45

...(b) [math]P(X>3) [/math]

✓ 0.37

...(c) [math] P(2 \leq X \leq 5) [/math]

✓ 0.77

...(d) [math] P(2 < X < 5) [/math]

✓ 0.4

Show Submitted Answer | Hide Correct Answer | Check My Answer

Question 4

Review

Fill in the Blanks

Type your answers in all of the blanks and submit

X_2 X^2 Ω

What is the probability mass function, $P(X = x)$ for X? (Round to two decimal places)

$P(X = 0) =$ Type your answer here

$P(X = 1) =$ Type your answer here

$P(X = 2) =$ Type your answer here

$P(X = 3) =$ Type your answer here

$P(X = 4) =$ Type your answer here

$P(X = 5) =$ Type your answer here

$P(X = 6) =$ Type your answer here

Correct Answers:

What is the probability mass function, [math] P(X=x)[/math] for [math] X[/math]? (Round to two decimal places)
[math]P(X=0) = [/math]

✓ 0.05

...[math]P(X=1) = [/math]

✓ 0.16

...[math]P(X=2) = [/math]

✓ 0.24

...[math]P(X=3) = [/math]

✓ 0.18

...[math]P(X=4) = [/math]

✓ 0.22

...[math]P(X=5) = [/math]

✓ 0.13

...[math]P(X=6) = [/math]

✓ 0.02

Show Submitted Answer | Hide Correct Answer | Check My Answer

Part 3

Question 5
Review

Fill in the Blanks

Type your answers in all of the blanks and submit

X_2 X^2 Ω

Suppose 80% of all students taking a beginning programming course fail to get their first program to run on the first submission. Consider a group of 5 such students, where each student's success is independent from the other and the chance each student fails on their first try is consistent. (Round answers to three decimal places.)

If X is the number of students whose program fails on the first run, then X comes from a **binomial** distribution with parameters:

$n =$ Type your answer here

$p =$ Type your answer here

The binomial probability mass function is $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$. Use this function to calculate the following probabilities. You may verify the outcome of the function using the R command dbinom(x,n,p).

What is the probability exactly 4 fail on their first submissions? Type your answer here

What is the probability 4 or fewer fail on their first submissions? Type your answer here

Hint: Find the P(X≤x) in R using pbinom(x,n,p).

How many students should be expected to fail? $\mu_x =$ Type your answer here

What is the standard deviation? $\sigma_x =$ Type your answer here

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Correct Answers:

Suppose 80% of all students taking a beginning programming course fail to get their first program to run on the first submission. Consider a group of 5 such students, where each student's success is independent from the other and the chance each student fails on their first try is consistent. (Round answers to

three decimal places.)   If [math]X[/math] is the number of students whose program fails on the first run, then [math]X[/math] comes from a **binomial** distribution with parameters: [math]n=[/math]

✓ 5

...[math]p=[/math]

✓ 0.8

... The binomial probability mass function is [math] P(X=x) = {n\choose x}p^x (1-p)^{n-x}[/math]. Use this function to calculate the following probabilities. You may verify the outcome of the function using the R command dbinom(x,n,p). What is the probability exactly 4 fail on their first submissions?

✓ 0.41

...What is the probability 4 or fewer fail on their first submissions?

✓ 0.672

...*Hint: Find the P(X≤x) in R using pbinom(x,n,p).*   How many students should be expected to fail? [math]\mu_x=[/math]

✓ 4

...What is the standard deviation? [math]\sigma_x=[/math]

Show Submitted Answer | Hide Correct Answer | Check My Answer

Part 4

Question 6
Review

Fill in the Blanks

Type your answers in all of the blanks and submit

X_a X^a Ω

Let X be the number of material anomalies occurring in a particular region of an aircraft gas-turbine disk. A researcher proposes a Poisson distribution for X. Suppose that $\lambda = 4$.

The Poisson probability mass function is:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ for } x = 0, 1, 2, \ldots.$$

Use the PMF to calculate the following probabilities. Verify these values in R using dpois(x,lambda).

(Round your answers to three decimal places.)

$P(X = 0) =$ Type your answer here

$P(X = 4) =$ Type your answer here

$P(X \leq 4) =$ Type your answer here

$P(X < 4)$ Type your answer here

$P(X > 4)$ Type your answer here

$P(3 \leq X \leq 7) =$ Type your answer here

Correct Answers:

Let X be the number of material anomalies occurring in a particular region of an aircraft gas-turbine disk. A researcher proposes a Poisson distribution for X. Suppose that [math] \lambda = 4[/math]. The Poisson probability mass function is: [math]P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} [/math] for [math] x = 0,1,2,... [/math].   Use the PMF to calculate the following probabilities. Verify these values in R using dpois(x,lambda). (Round your answers to three decimal places.) [math]P(X=0) = [/math]

✓ 0.018

...[math]P(X=4) = [/math]

✓ 0.195

...[math]P(X\leq 4) = [/math]

✓ 0.629

...[math] P(X<4)[/math]

✓ 0.433

...[math] P(X>4)[/math]

✓ 0.371

...[math] P(3 \leq X \leq 7) = [/math]

Show Submitted Answer | Hide Correct Answer | Check My Answer

Part 5 - Continuous Random Variables

Question 7
Review

Let X denote the amount of space occupied by an article placed in a 1-ft^3 packing container. The pdf of X is below.

$$f(x) = 56x^6(1-x) \text{ for } 0 \le x \le 1.$$

Obtain the CDF of X.

Select an answer and submit. For keyboard navigation, use the up/down arrow keys to select an answer.

a	$F(x) = 336x^5 - 392x^6 \text{ for } 0 \le x \le 1$
b	$F(x) = 336x^5(1-x) \text{ for } 0 \le x \le 1$
c	$F(x) = 8x^7(1-x) \text{ for } 0 \le x \le 1$
d	$F(x) = 8x^7 - 7x^8 \text{ for } 0 \le x \le 1$

Correct Answer:

✓ d - $F(x) = 8x^7 - 7x^8 \text{ for } 0 \le x \le 1$

Show Submitted Answer | Hide Correct Answer | Check My Answer

Question 8
Review

A continuous random variable Y has a pdf of $f(y) = \frac{6}{171}(y + y^2)$ where y is defined from 1 to 4. Which expression describes $F(y)$, the cumulative density function for Y?

Select an answer and submit. For keyboard navigation, use the up/down arrow keys to select an answer.

a	$F(y) = \frac{1}{171}(2y^3 + 3y^2 - 5) \text{ for } 1 \le y \le 4$

b	$F(y) = \frac{3y^2}{171} + \frac{2y^3}{171}$ for $1 \leq y \leq 4$
c	$F(y) = \frac{6y^2}{171} + \frac{6y^3}{171}$ for $1 \leq y \leq 4$
d	$F(y) = \infty$ for $1 \leq y \leq 4$

Correct Answer:

✓ a - $F(y) = \frac{1}{171}(2y^3 + 3y^2 - 5)$ for $1 \leq y \leq 4$

Show Submitted Answer | Hide Correct Answer | Check My Answer

Question 9
Review

Fill in the Blanks

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{25} & 0 \leq x \leq 5 \\ 1 & x > 5 \end{cases}$$

Type your answers in all of the blanks and submit

X_2 X^2 Ω

Let X denote the amount of time a book on two-hour reserve is actually checked out. The above equation represents the CDF of X.

Calculate the following probabilities. If necessary, round answers to three decimal places.

(a) $P(X \leq 2) =$ Type your answer here

(b) $P(1.5 \leq X \leq 2) =$ Type your answer here

(c) $P(X > 2.5) =$ Type your answer here

(d) What is the median check out duration? Hint: solve $F(X_{0.5}) = 0.5$ for $X_{0.5}$.

Type your answer here

Obtain the PDF of X, $f(x) = \frac{dF(x)}{dx}$. Use $f(x)$ to solve the following.

(e) Calculate $E(X) =$. Type your answer here

(f) Calculate $Var(X) =$. Type your answer here

Correct Answers:

Let [math] X [/math] denote the amount of time a book on two-hour reserve is actually checked out. The above equation represents the CDF of [math] X [/math]. Calculate the following probabilities. If necessary, round answers to three decimal places. (a) [math] P(X \leq 2) =   [/math]

✓ 0.16

...(b) [math] P(1.5 \leq X \leq 2) = [/math]

✓ 0.07

...(c) [math] P(X > 2.5) = [/math]

✓ 0.75

...(d) What is the median check out duration? Hint: solve [math] F(X_{0.5}) = 0.5 [/math] for [math] X_{0.5}[/math].

✓ 3.536

...Obtain the PDF of   [math] X [/math], [math] f(x) = \frac{dF(x)}{dx} [/math]. Use [math] f(x) [/math] to solve the following.

(e) Calculate [math] E(X) = [/math].

✓ 3.333

...(f) Calculate [math]Var(X) =

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