1 Number Representation

1> -243 = 1111111100001101 = 0XFF0D

Solution:

$243 = 2^7 + 2^6 + 2^5 + 2^4 + 2^1 + 2^0 = 0000000011110011$

$$\begin{array}{r} 0000000011110011 \\ +\ 1111111100001100 \\ \hline 1111111111111111 \end{array}$$

$$\begin{array}{r} 1111111111111111 \\ +\ 1 \\ \hline 0000000000000000 \end{array}$$

$$\begin{array}{r} 1111111100001100 \\ +\ 1 \\ \hline 1111111100001101 \end{array}$$

2> 728 = 0000001011011000 = 0X02D8

Solution:

$728 = 2^9 + 2^7 + 2^6 + 2^4 + 2^3 = 0000001011011000$

3> 13.4375 = 1101.0111 = 0XD.7

Solution:

$1101.0111 = 1\times2^3 + 1\times2^2 + 1\times2^0 + 1\times2^{-2} + 1\times2^{-3} + 1\times2^{-4}$

$= 8 + 4 + 1 + 0.25 + 0.125 + 0.0625$

$= 13.4375$

4> 220 = 11011100 = 0XDC

Solution:

$11011100 = 1\times2^7 + 1\times2^6 + 1\times2^4 + 1\times2^3 + 1\times2^2$

$= 128 + 64 + 16 + 8 + 4$

$= 220$

5> 125.5 = 01111101.1000 = 7D.8

Solution:

7D.8 = 01111101.1000

$01111101.1000 = 1\times2^6 + 1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^0 + 1\times2^{-1}$

$= 64 + 32 + 16 + 8 + 4 + 1 + 0.5$

$= 125.5$

6> 437.3125 = 0001 1011 0101 = 1B5

Solution:

1B5 = 000110110101

$000110110101 = 1\times2^8 + 1\times2^7 + 1\times2^5 + 1\times2^4 + 1\times2^2 + 1\times2^0 + 1\times2^{-2} + 1\times2^{-4}$

$= 256 + 128 + 32 + 16 + 4 + 1 + 0.25 + 0.0625$

$= 437.3125$

2 Floating Point Number Representation

1. $-76.678595 = 1\ 10000110\ 00110010101101101110001$

   $= 0XC2995B71$

   Solution:

   $$76.678595 = 2^{6} + 2^{3} + 2^{2} + 2^{-1} + 2^{-3} + 2^{-5} + 2^{-6} + 2^{-8} + 2^{-9} + 2^{-11} + 2^{-12} + 2^{-13} + 2^{-18} + 2^{-19}$$

   $$= 1001100.10101101101110000011$$

Since significant is more than 23 bits, we round -76.678595 to -76.6786

$$76.6786 = 2^{6} + 2^{3} + 2^{2} + 2^{-1} + 2^{-3} + 2^{-5} + 2^{-6} + 2^{-8} + 2^{-9} + 2^{-11} + 2^{-12} + 2^{-13} + 2^{-17}$$

$$= 1.00110010101101101110001 \times 2^{6}$$

$$6 + 127 = 133 = 2^{7} + 2^{2} + 2^{0} = 10000101$$

$\therefore$ IEEE : $1\ 10000101\ 00110010101101101110001$

hexadecimal : $0XC2995B71$

2. $19.459931 = 0\ 10000011\ 00110111010110111110000$

   $= 0X419BADF0$

   Solution:

   $$19.459931 = 2^{4} + 2^{1} + 2^{0} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-6} + 2^{-8} + 2^{-9} + 2^{-11} + 2^{-12} + 2^{-13} + 2^{-14} + 2^{-15} + 2^{-20}$$

   $$= 10011.01110101101111100001$$

   $$= 1.00110111010110111110000 1 \times 2^{4}$$

   since significand is more than 23 bits,

   we round 19.459931 off to 19.45993

   $$\therefore 19.45993 = 2^{4} + 2^{1} + 2^{0} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-6} + 2^{-8} + 2^{-9} + 2^{-11} + 2^{-12} + 2^{-13} + 2^{-14} + 2^{-15}$$

$= 10011.01110101101111$

$= 1.001101110101101111\,1 \times 2^4$

$4 + 127 = 131 = 2^7 + 2^1 + 2^0 = 10000011$

$\therefore$ IEEE = 0 10000011 00110111010110111110000

hexadecimal = 0X419BADF0

3. Sum is : 1000110.11001000000001011

Solution:

$76.678595 = 1.00110010101101101110001 \times 2^6$

$$
\begin{array}{r}
0.11001101010010010001110 \times 2^6 \\
+\ 0.00000000000000000000001 \\
\hline
\end{array}
$$

$-76.678595 = 0.11001101010010010001111 \times 2^6$

$= 011.00110101001001000 1111 \times 2^4$

$19.459931 = 1.00110111010110111111 \times 2^4$

$-76.678595 + 19.459931 =$

$$
\begin{array}{r}
11.001101010010010001111 \times 2^4 \\
+\ 1.00110111010110111111 \times 2^4 \\
\hline
100.01101100100000000001011 \times 2^4
\end{array}
$$

$\Downarrow$

1000110.11001000000001011

3 Boolean Algebra

a> $F(A, B, C, D) = (\bar{A} + B \cdot \bar{D}) \cdot (C \cdot B \cdot A + \bar{C} \cdot D)$

i>

A	$\bar{A}$	B	C	$\bar{C}$	D	$\bar{D}$	$B*\bar{D}$	$\bar{A}+B*\bar{D}$	C*B	C*B*A	$\bar{C}*D$	$C*B*A+\bar{C}*D$	$(\bar{A}+B*\bar{D})(C*B*A+\bar{C}*D)$
0	1	0	0	1	0	1	0	1	0	0	0	0	0
0	1	0	0	1	1	0	0	1	0	0	1	1	1
0	1	0	1	0	0	1	0	1	0	0	0	0	0
0	1	0	1	0	1	0	0	1	0	0	0	0	0
0	1	1	0	1	0	1	1	1	0	0	0	0	0
0	1	1	0	1	1	0	0	1	0	0	1	1	1
0	1	1	1	0	0	1	1	1	1	0	0	0	0
0	1	1	1	0	1	0	0	1	1	0	0	0	0
1	0	0	0	1	0	1	0	0	0	0	0	0	0
1	0	0	0	1	1	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0	0	0	0	0
1	0	1	0	1	0	1	1	1	0	0	0	0	0
1	0	1	0	1	1	0	0	0	0	0	1	1	0
1	0	1	1	0	0	1	1	1	1	1	0	1	1
1	0	1	1	0	1	0	0	0	1	1	0	1	0

ii> $F(A, B, C, D) = (\bar{A} + B \cdot \bar{D}) \cdot (C \cdot B \cdot A + \bar{C} \cdot D)$

$$= \bar{A} \cdot ABC + \bar{A}\bar{C}D + A \cdot B \cdot B \cdot C\bar{D} + B\bar{C}D\bar{D}$$

$$= \bar{A} \cdot \bar{C} \cdot D + A \cdot C \cdot \bar{D} \cdot B \cdot B$$

iii> $F(A, B, C, D) = \overline{\overline{(\bar{A} + B \cdot \bar{D})(A \cdot B \cdot C + \bar{C} \cdot D)}}$

$$= \overline{\overline{(\bar{A} + B\bar{D})} + \overline{(A \cdot B \cdot C + \bar{C} \cdot D)}}$$

$$= \overline{A(\bar{B} + D) + (\bar{A} + \bar{B} + \bar{C})(C + \bar{D})}$$

$$= \overline{A\bar{B} + AD + \bar{A}C + \bar{A}\bar{D} + \bar{B}C + \bar{B}\bar{D} + \bar{C}C + \bar{C}\bar{D}}$$

$$= (\bar{A} + B)(\bar{A} + \bar{D})(A + \bar{C})(A + D)(B + \bar{C})(B + D)(C + D)$$

b> $F(W,X,Y,Z)=\overline{(W+\bar{X})(Z\bar{Y}+X)}$

i>

W	X	$\bar{X}$	Y	$\bar{Y}$	Z	$W+\bar{X}$	$Z\bar{Y}$	ZY+X	$(W+\bar{X})(Z\bar{Y}+X)$	$\overline{(W+\bar{X})(Z\bar{Y}+X)}$
0	0	1	0	1	0	1	0	0	0	1
0	0	1	0	1	1	1	1	1	1	0
0	0	1	1	0	0	1	0	0	0	1
0	0	1	1	0	1	1	0	0	0	1
0	1	0	0	1	0	0	0	1	0	1
0	1	0	0	1	1	0	1	1	0	1
0	1	0	1	0	0	0	0	1	0	1
0	1	0	1	0	1	0	0	1	0	1
1	0	1	0	1	0	1	0	0	0	1
1	0	1	0	1	1	1	1	1	1	0
1	0	1	1	0	0	1	0	0	0	1
1	0	1	1	0	1	1	0	0	0	1
1	1	0	0	1	0	1	0	1	1	0
1	1	0	0	1	1	1	1	1	1	0
1	1	0	1	0	0	1	0	1	1	0
1	1	0	1	0	1	1	0	1	1	0

ii> $F(W,X,Y,Z)=\overline{(W+\bar{X})(Z\bar{Y}+X)}$

$=\overline{(W+\bar{X})}+\overline{(Z\bar{Y}+X)}$

$=\bar{W}\cdot X+\overline{(Z\bar{Y})}\cdot\bar{X}$

$=\bar{W}\cdot X+\bar{Z}\cdot\bar{X}+Y\cdot\bar{X}$

iii> $F(W,X,Y,Z)=\overline{(W+\bar{X})(Z\bar{Y}+X)}$

$=\overline{WZ\bar{Y}+WX+\bar{X}Z\bar{Y}+\bar{X}X}$

$=\overline{WZ\bar{Y}+WX+\bar{X}Z\bar{Y}}$

$=(\overline{WZ\bar{Y}})(\overline{WX})(\overline{\bar{X}Z\bar{Y}})$

$=(\bar{W}+\bar{Z}+Y)(\bar{W}+\bar{X})(X+\bar{Z}+Y)$

4 Circuit Design

4.1 Universal Gates

$$F(X, Y, Z, W) = (\bar{X}+\bar{Y})(Z+W)$$

$$= \overline{XY}\,(Z+W)$$

$$= \overline{XY}Z + \overline{XY}W$$

$$= \overline{\overline{(\overline{XY}Z)(\overline{XY}W)}}$$

$$= \overline{\overline{(\overline{XY}Z)}\;\overline{(\overline{XY}W)}}$$

4.2 Parity Counter

i)

A	B	C	D	F_2	F_1	F_0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	0	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	1	1
1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	0

ii) ① $F_2 = ABCD$

② for F_1

$CD \backslash AB$	00	01	10	11
00				1
01		1	1	1
10		1	1	1
11	1	1	1	

$$F_1 = \overline{AB}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + A\overline{BC}D + A\bar{B}C\bar{D} + A\bar{B}CD$$
$$+ AB\overline{CD} + AB\bar{C}D + ABC\bar{D}$$
$$= (AB\cdot\bar{C})(\bar{D}+D) + (A\cdot\bar{B}\cdot D)(\bar{C}+C) + (A\cdot\bar{B}\cdot C)(D+\bar{D}) +$$
$$(\bar{A}\cdot C\cdot D)(\bar{B}+B) + (B\cdot\bar{C}\cdot D)(A+\bar{A}) + (B\cdot C\cdot\bar{D})(A+\bar{A})$$

$$\therefore F_1 = A\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot D + A\cdot\bar{B}\cdot C + \bar{A}\cdot C\cdot D + B\cdot\bar{C}\cdot D + B\cdot C\cdot\bar{D}$$

③ for F_0

$CD \backslash AB$	00	01	10	11
00		1	1	
01	1			1
10	1			1
11		1	1	

$$F_0 = \overline{ABC}D + \overline{AB}C\bar{D} + \bar{A}B\overline{CD} + \bar{A}BCD + A\overline{BCD} + A\bar{B}CD +$$
$$AB\bar{C}D + ABC\bar{D}$$

4.3 Full-adders and half-adders

We can add three 1-bit numbers by using a 1-bit full-adder

Hence we can use one 1-bit FA add A_0, A_1, A_2 and one 1-bit FA add A_3, A_4, A_5.

After this, we have another two FA, one takes sum of previous two FAs and one takes carry-out of previous two FAs. We treat A_6 as carry-in, $F_0 F_1 F_2$ are bits represent number of 1's.