习题4.2

1.

(5) $\int \frac{x}{(x+1)^2(x^2+x+1)}dx$

$=\int \frac{-1}{(x+1)^2}+\frac{1}{x^2+x+1}dx$

$=\frac{1}{x+1}+\int\frac{1}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}dx$

$=\frac{1}{x+1}+\frac{2}{\sqrt{3}}\arctan\frac{2(x+\frac{1}{2})}{\sqrt{3}}+C$

(8)

$\int\frac{x^{15}}{(x^8+1)^2}dx=\int\frac{x^7\cdot x^8}{(x^8+1)^2}dt$

$=\frac{1}{8}\int\frac{x^8}{(x^8+1)^2}d\,x^8=\frac{1}{8}\int\frac{t}{(1+t)^2}dt$

$=\frac{1}{8}\left[\ln(x^8+1)+\frac{1}{x^8+1}\right]+C$

2.

(4) $\int \frac{\sin^2 x\cos x}{\sin x+\cos x}dx = \int \frac{\sin^2 x}{\tan x+1}dx = \int \frac{\frac{1}{1+\tan^2 x}}{\tan x+1}dx$

$= \int \frac{t^2}{(1+t)(1+t^2)^2}dt = \frac{1}{4}\int \left(\frac{1}{1+t} - \frac{t-1}{1+t^2} + \frac{2t-2}{(1+t^2)^2}\right)dt$

$= \frac{1}{4}\left[\ln|1+t| - \ln\sqrt{1+t^2} - \frac{1+t}{4(1+t^2)}\right] + C$

(6) $\int 1 - \frac{1}{1+\sin^2 x}dx = x - \int \frac{1}{1+\sin^2 x}dx$ $t = \tan\frac{x}{2}$, $x = 2\arctan t$

$= x - 2\int \frac{1}{1+\frac{2t}{1+t^2}}\cdot\frac{1}{1+t^2}dt = x - 2\int\frac{1}{(1+t)^2}dt$

$= x + 2\cdot\frac{1}{1+t} + C$

(7) $\int \frac{\sin x\cos x}{\sin x+\cos x}dx = \frac{1}{2}\int \sin x+\cos x - \frac{1}{\sin x+\cos x}dx$

$= \frac{1}{2}\left[-\cos x+\sin x\right] - \frac{1}{2\sqrt{2}}\int\frac{d(x+\frac{\pi}{4})}{\sin(x+\frac{\pi}{4})}$

$= \frac{1}{2}(-\cos x+\sin x) - \frac{1}{2\sqrt{2}}\ln\left|\frac{1}{\sin(x+\frac{\pi}{4})} - \cot(x+\frac{\pi}{4})\right| + C$

(8) $\int \frac{1}{\sin^4 x\cos^6 x}dx = \int\frac{1}{(\frac{1}{2}\sin 2x)^4}dx$

$u = \sin^2 2x$

$= 16\int\frac{1}{(\sin^2 2x)^2}dx = 16\int\frac{1}{u^2}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{1-u}}du$ $\frac{\arcsin\sqrt{u}}{2} = x$

$= 8\int\frac{1}{u^2\sqrt{1-u}}du = 8\int\frac{1}{(m^2-1)^2 m}\cdot 2m\,dm$ $dx = \frac{1}{2}\cdot\frac{1}{\sqrt{1-u}}$

$m = \sqrt{1-u}$

$= 16\int\frac{1}{(m^2-1)^2}dm$ $u = m^2-1$

$du = 2m\,dm$

$= 4\int -\frac{m-1}{(m-1)^2} + \frac{1}{(m-1)^2} + \frac{m+1}{(m+1)^2} + \frac{1}{(m+1)^2}dm = 4\left[-\ln(m-1) + \ln(m+1) - \frac{1}{m-1} - \frac{1}{m+1}\right] + C$

(9) $\int \frac{1}{2\sin x+\sin 2x}dx = \frac{1}{2}\int \frac{1}{\sin x(1+\cos x)}dx$ $\quad t=\tan\frac{x}{2}$

$x=2\arctan t$

$= \frac{1}{2}\int \frac{1}{\frac{2t}{1+t^2}\left(1+\frac{1-t^2}{1+t^2}\right)}\cdot 2\cdot\frac{1}{1+t^2}\,dt$

$= \int \frac{1}{\frac{2t}{1+t^2}\cdot\frac{2}{1+t^2}}\cdot\frac{1}{1+t^2}\,dt$

$= \frac{1}{4}\int \frac{1+t^2}{t}\,dt = \frac{1}{4}\int \frac{1}{t}+t\,dt$

$= \frac{1}{4}\ln|t| + \frac{1}{8}t^2 + C.$