第四周作业:

8.3.1 (2) 由 $y^2+z^2=1$ 绕 z 轴旋转

(4) 由 $z^2-\frac{y^2}{4}=1$ 绕 y 轴旋转

(6) 由 $x^2-y^2=1$ 绕 x 轴旋转

(8) 不是旋转面

8. $d^2=x^2+y^2+z^2=(z-4)^2$

$x^2+y^2=-8z+16$ 旋转抛物面

9. $4z^2=4x^2-4y^2 \Rightarrow 5x^2-3y^2=1$

柱面方程为 $5x^2-3y^2=1$

11. $$\frac{x^2}{16}+\frac{y^2}{4}-\frac{(x+3)^2}{20}=1$$

$$5x^2+20y^2-4(x+3)^2=80$$

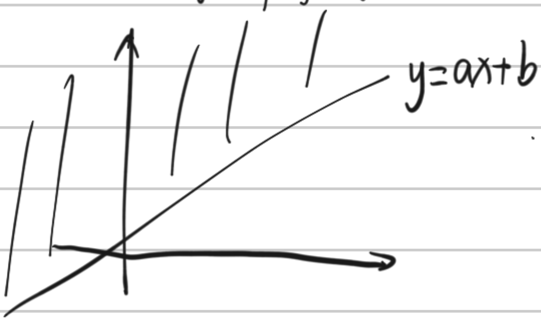
$$x^2-24x+20y^2-116=0$$

$$x^2-24x+144+20y^2=260$$

$$\begin{cases}(x-12)^2+20y^2=260\\ z=0\end{cases}$$

习题9.1

3. $\forall M(x_0,y_0)\in\{(x,y)\mid y>ax+b\}$, $d=\frac{|-ax_0+y_0-b|}{\sqrt{a^2+1}}>0$. $\exists r=\frac{d}{2}$, $B(M,r)\subset\{(x,y)\mid y>ax+b\}$



边界点 (x,y) 满足 $y=ax+b$

3.13

10. $f(1,1)=1$ $f(y,x)=\frac{2xy}{x^2+y^2}$ $f(1,\frac{y}{x})=\frac{2\frac{y}{x}}{1+\frac{y^2}{x^2}}=\frac{2xy}{x^2+y^2}$

$f(u,v)=\frac{2uv}{u^2+v^2}$ $f(\cos t,\sin t)=\frac{2\sin t\cos t}{1}=\sin 2t$

14. (1) $0<\frac{x^2+y^2}{|x|+|y|}<|x|+|y|$ 夹逼 $\to \lim=0.$

(4) $\lim\limits_{\substack{x\to\infty\\y\to a}}(1+\frac{1}{x})^{\frac{x^2}{x+y}}=\lim\limits_{\substack{x\to\infty\\y\to a}}e^{\frac{1}{x}\cdot\frac{x^2}{x+y}}=\lim\limits_{\substack{x\to\infty\\y\to a}}e^{\frac{x}{x+y}}=e$

(5) $\lim\limits_{\substack{x\to0\\y\to0}}\frac{x^3+y^3}{x^2+y^2}=\lim\limits_{\substack{x\to0\\y\to0}}x+y-\frac{xy^2+yx^2}{x^2+y^2}=0$

$0=x+y-\max\{x,y\}<$ $<x+y-\min\{x,y\}=0$

(8) $\lim\limits_{\substack{x\to1\\y\to0}}\frac{\ln(x+e^y)}{\sqrt{x^2+y^2}}=\frac{\ln 2}{1}=\ln 2$

15.

(1) $\lim\limits_{r\to0^+}e^{\frac{1}{x^2-y^2}}=\lim\limits_{r\to0^+}e^{\frac{1}{r^2}\cdot\frac{1}{\cos2\theta}}$ $\cos2\theta<0$ 时，$\lim=0.$ $\cos2\theta\geq0$ 显然无极限

$\frac{\pi}{2}<2\theta<\frac{3\pi}{2}$ $\frac{5\pi}{2}<2\theta<\frac{7\pi}{2}.$

(2) $\lim\limits_{r\to+\infty}e^{r^2\cos2\theta}\cdot\sin r^2\sin2\theta$ $\cos2\theta<0$ 时 $\lim=0.$ $\cos2\theta>0$ 显然无极限

当 $\cos2\theta=0$ 时 $\sin2\theta=\pm1.$ $\lim\limits_{r\to+\infty}\sin r^2$ 无极限.

16. $\forall \varepsilon>0,\ \exists \delta.\ \forall\ 0<|x-x_0|<\delta_2 \quad |f(x,y)-B_y|<\varepsilon/2$

$|x-x_0|<\delta_1\ \ |y-y_0|<\delta_1 \quad |f(x,y)-A|<\varepsilon/2$

要证 $\lim\limits_{y\to y_0} B=A$ 即

$\forall \varepsilon>0 \quad \exists \delta_3=\min\{\delta_1,\delta_2\} \quad 0<|y-y_0|<\delta_3,\ 0<|x-x_0|<\delta_3$

$$|B_y-A|=|B_y-f(x,y)-A+f(x,y)|<|f-B|+|f-A|<\varepsilon$$

$x\neq x_0$时同理. □

17. (1) $\dfrac{xy}{x-y}$ 令 $x=y+t$ 原式$=\dfrac{y^2+ty}{t}=\dfrac{y^2}{t}+y$ $t\to 0$ 极限不存在

(3) $\dfrac{x^2y}{x^2+y^2}$ 令 $x=r\cos\theta \quad y=r\sin\theta$

$LHS=\dfrac{r^3\cos^2\theta\sin\theta}{r^2}=r\cos^2\theta\sin\theta \qquad \lim\limits_{r\to 0^+} r\cos^2\theta\sin\theta=0$ 故极限为0.

19.

$\forall \varepsilon>0 \quad \exists \delta \qquad |x-x_0|<\delta \quad |y-y_0|<\delta$

$$|f(x,y)-f(x_0,y_0)|=|f(x,y)-f(x_0,y)+f(x_0,y)-f(x_0,y_0)|$$

$$\le |f(x,y)-f(x_0,y)|+|f(x_0,y)-f(x_0,y_0)| \quad \text{--- ①}$$

$|y-y_0|<\delta$.

考虑 $|f(x_0,y)-f(x_0,y-\frac{\delta}{2})|+|f(x_0,y-\frac{\delta}{2})-f(x_0,y_0)|<\frac{\varepsilon}{4}$

$|f(x_0,y)-f(x_0,y+\frac{\delta}{2})|+|f(x_0,y+\frac{\delta}{2})-f(x_0,y_0)|<\frac{\varepsilon}{4}$

上式①$<\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon$.

20. $\forall \varepsilon>0$, $\exists \delta = \varepsilon$. $|x-x_0|<\delta$ $|y-y_0|<\delta$

$x_0-\delta<x<\delta+x_0$

$\delta+x_0 < f(x,y)-x_0 < \delta+x_0 \Rightarrow |f(x,y)-x_0|<\varepsilon$

故连续

不一定将闭集映成闭集 如 $\{(x,y)\in R^2 \mid y=\frac{1}{x}\}$

$\downarrow$

$R\setminus\{0\}$ $\{0\}$为闭集

故 $R\setminus\{0\}$为开集

23. $f(x,y)-f(x_0,y_0) = \frac{1}{1-xy} - \frac{1}{1-x_0y_0}$

$$\left|\left(1-\frac{1}{2n}, 1-\frac{1}{2n}\right) - \left(1-\frac{1}{n}, 1-\frac{1}{n}\right)\right| \longrightarrow 0$$

$$|f(\quad) - f(\quad)| \longrightarrow \infty$$