第六周作业.

1. (2) $F(1,\frac{\pi}{2})=0$ $\quad F_y'=x^2(-\sin xy)$ $\quad F_y'\big|_{x=1\ y=\frac{\pi}{2}}=-1\neq 0.$

$F_x'=\cos xy-xy\sin xy$ 连续

$$y'(x)=\frac{-F_x'}{F_y'}=\frac{xy\sin xy-\cos xy}{x^2(-\sin xy)} \qquad y'(1)=\frac{\frac{\pi}{2}}{-1}=-\frac{\pi}{2}$$

$$\lim_{x\to 1} y''(x)=\pi$$

2.(2) $\ln\sqrt{x^2+y^2}=\arctan\frac{y}{x}$

$$d\ln\sqrt{x^2+y^2}=\frac{x}{x^2+y^2}dx+\frac{y}{x^2+y^2}dy$$

$$d\arctan\frac{y}{x}=\frac{-y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dy$$

$$\frac{x+y}{x^2+y^2}dx=\frac{x-y}{x^2+y^2}dy \Rightarrow \frac{dy}{dx}=\frac{x+y}{x-y}$$

$$\frac{d^2y}{dx^2}=\lim_{\Delta x\to 0}\frac{\frac{x+\Delta x+y}{x+\Delta x-y}-\frac{x+y}{x-y}}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{2y}{x+\Delta x-y}-\frac{2y}{x-y}}{\Delta x}=-2y$$

(5) $\frac{x}{z}=\ln\frac{z}{y}$ $\qquad d\frac{x}{z}=\frac{1}{z}dx-\frac{x}{z^2}dz$

$$d\ln\frac{z}{y}=\frac{y}{z}\cdot\left(-\frac{z}{y^2}\right)dy+\frac{y}{z}\cdot\frac{1}{y}dz$$

故 $-\frac{1}{y}dy+\frac{1}{z}dz=\frac{1}{z}dx-\frac{x}{z^2}dz$ $\qquad \left(\frac{1}{z}+\frac{x}{z^2}\right)dz=\frac{1}{z}dx+\frac{1}{y}dy.$

$$\Rightarrow F_x'=\frac{\frac{1}{z}}{\frac{1}{z}+\frac{x}{z^2}}=\frac{z}{z+x} \qquad F_y'=\frac{\frac{1}{y}}{\frac{1}{z}+\frac{x}{z^2}}=\frac{z^2}{y(x+z)}$$

(6) $F(x,x+y,x+y+z)=0.$ $\qquad F(m,n,k)=0$

$$0=(F_m'\cdot 1+F_n'\cdot 1+F_k'\cdot 1)dx+(F_n'+F_k')dy+F_k'dz$$

$$\frac{\partial z}{\partial x}=-\left(\frac{F_m'}{F_k'}+\frac{F_n'}{F_n'}+1\right) \qquad \frac{\partial z}{\partial y}=-\left(\frac{F_n'}{F_k'}+1\right)$$

3. $x^2+xy+y^2-27=0$

$$0=(2x+y)dx+(x+2y)dy$$

$\frac{dy}{dx}=-\frac{2x+y}{x+2y}$ $\quad$ $2x+y=0.$ $\quad$ $4x^2+4xy+y^2=0.$

$x^2+xy+y^2=27$ $\quad$ $4x^2+4xy+4y^2=27\times4 \Rightarrow 3y^2=27\times4\ y^2=9\times4$

$y_1=6 \quad y_2=-6$ $\quad$ 检验. y_1时 二阶导<0

y_2时 二阶导>0

4. (3) $\left(3u^2+6(x+y)u\right)du+3z^2dz+3u^2dx+3u^2dy=0$

$$du=\frac{-1}{3u^2+6(x+y)u}\cdot(3z^2dz+3u^2dx+3u^2dy)$$

(4) $0=(F'_{x-y}-F'_{z-x})dx+(F'_{y-z}-F'_{x-y})dy+(-F'_{y-z}+F_{z-x})dz$

$=0.$

$$dz=\frac{(F'_{x-y}-F'_{z-x})dx+(F'_{y-z}-F'_{x-y})dy}{F'_{y-z}-F'_{z-x}}$$

6. $d\,LHS=2\cos(x+2y-3z)dx+4\cos(x+2y-3z)dy-6\cos(x+2y-3z)dz$

$d\,RHS=dx+2dy-3dz$

$$dz=\frac{2\cos(x+2y-3z)-1}{6\cos(x+2y-3z)-3}dx+\frac{4\cos(x+2y-3z)-2}{6\cos(x+2y-3z)-3}dy$$

故 $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$

10. $\begin{cases} dx+dy+dz=0 \\ 2xdx+2ydy+2zdz=0 \end{cases}$ $\quad$ $dx+dy=-dz$ $\quad$ $(y-x)dx=(z-y)dz$

$xdx+ydy=-zdz$

$(x-y)dy=(z-x)dz$

$\frac{dy}{dz}=\frac{z-x}{x-y} \quad \frac{dx}{dz}=\frac{z-y}{y-x}$

11. (2)

$$x\,du - y\,dv + u\,dx - v\,dy = 0 \qquad u\,dx - v\,dy = y\,dv - x\,du$$

$$y\,du + u\,dy + x\,dv + v\,dx = 0 \qquad v\,dx + u\,dy = -x\,dv - y\,du$$

$$du = \frac{\begin{vmatrix} u\,dx - v\,dy & -x \\ v\,dx + u\,dy & -y \end{vmatrix}}{\begin{vmatrix} y & -x \\ -x & y \end{vmatrix}} = \frac{\begin{vmatrix} u & -x \\ v & -y \end{vmatrix}}{y^2 - x^2} dx + \frac{\begin{vmatrix} -v & -x \\ u & -y \end{vmatrix}}{y^2 - x^2} dy$$

① ②

$$dv = \frac{\begin{vmatrix} y & u\,dx - v\,dy \\ -x & v\,dx + u\,dy \end{vmatrix}}{y^2 - x^2} = \frac{\begin{vmatrix} y & u \\ -x & v \end{vmatrix}}{y^2 - x^2} dx + \frac{\begin{vmatrix} y & -v \\ -x & u \end{vmatrix}}{y^2 - x^2} dy$$

③ ④

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{pmatrix} ① & ② \\ ③ & ④ \end{pmatrix}$$

12. (2)

$$dx = (\sin v + e^u)\,du + u\cos v\,dv$$

$$dy = (-\cos v + e^u)\,du + u\sin v\,dv$$

$$du = \frac{\begin{vmatrix} dx & u\cos v \\ dy & u\sin v \end{vmatrix}}{\begin{vmatrix} \sin v + e^u & u\cos v \\ -\cos v + e^u & u\sin v \end{vmatrix}} = \frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{u + u\sin v e^u - u\cos v e^u} dx + \frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{u + u\sin v e^u - u\cos v e^u} dy$$

$$= \frac{u\sin v}{u + u\sin v e^u - u\cos v e^u} dx - \frac{u\cos v}{u + u\sin v e^u - u\cos v e^u} dy$$

$$dv = \frac{\begin{vmatrix} \sin v + e^u & 1 \\ -\cos v + e^u & 0 \end{vmatrix} dx}{u + u\sin v e^u - u\cos v e^u} + \frac{\begin{vmatrix} \sin v + e^u & 0 \\ -\cos v + e^u & 1 \end{vmatrix} dy}{u + u\sin v e^u - u\cos v e^u}$$

$$= \frac{\cos v - e^u}{u + u\sin v e^u - u\cos e^u} dx + \frac{\sin v + e^u}{u + u\sin v e^u - u\cos v e^u} dy$$

P.10.9

1. $r'(t) = (a\cos t,\ a\sin t,\ 2bt)$

$r''(t) = (-a\sin t,\ a\cos t,\ 2b)$

$\frac{1}{2}b\sin 2t$

5. (1) $\begin{cases} x = a\sin^2 t \\ y = b\sin t\cos t \\ z = \cos^2 t\, c \end{cases}$ $\quad r'(t) = (2a\sin t\cos t,\ b\cos 2t,\ 2\cos t(-\sin t)c)$

$x_0 = \frac{1}{2}a \quad y_0 = \frac{1}{2}b \quad z_0 = \frac{1}{2}c$ $\quad = (a\sin 2t,\ b\cos 2t,\ -\sin 2t\, c)$

$= (a, 0, -c)$

切线方程 $\dfrac{x-\frac{1}{2}a}{a} = \dfrac{y-\frac{1}{2}b}{0} = \dfrac{z-\frac{1}{2}c}{-c}$ 法平面: $a(x-\frac{a}{2}) - c(z-\frac{c}{2}) = 0$

6.(2)

$(a\cos\theta\cos\varphi,\ b\cos\theta\sin\varphi,\ c\sin\theta)$

$(-a\sin\theta\sin\varphi,\ b\sin\theta\cos\varphi,\ 0)$

叉乘得 $(\underset{a_1}{cb\sin^2\theta\cos\varphi},\ \underset{a_2}{ac\sin^2\theta\sin\varphi},\ \underset{a_3}{ab\sin\theta\cos\theta})$

切平面方程为 $cb\sin^2\theta\cos\varphi(x - a\sin\theta_0\cos\varphi_0) + ac\sin^2\theta\sin\varphi(y - b\sin\theta_0\sin\varphi_0)$
$+ ab\sin\theta\cos\theta(z - \cos\theta_0)$

法线: $\dfrac{x-x_0}{a_1} = \dfrac{y-y_0}{a_2} = \dfrac{z-z_0}{a_3}$

8.

(3) $F(x,y,z) = e^z - z + xy - 3 = 0$

$F'_x = y \quad F'_y = x \quad F'_z = e^z - 1$ $\quad (1,2,0)$ $\quad$ 应. $(2,1,0)$

切平面: $1(x-2) + 2(y-1) = 0$

法线 $\dfrac{x-2}{1} = \dfrac{y-1}{2} = \dfrac{z-0}{0}$

(4) $F'_x = 1 + \dfrac{x}{\sqrt{x^2+y^2+z^2}} = 1 + \dfrac{2}{7} = \dfrac{9}{7}$ $\quad F'_y = 1 + \dfrac{3}{7} = \dfrac{10}{7}$ $\quad F'_z = 1 + \dfrac{6}{7} = \dfrac{13}{7}$

$4 + 9 + 36 = 49 = 7^2$

切平面 $\frac{9}{7}(x-2) + \frac{10}{7}(y-3) + \frac{13}{7}(z-6) = 0$

法线: $\dfrac{x-2}{\frac{9}{7}} = \dfrac{y-3}{\frac{12}{7}} = \dfrac{z-6}{\frac{13}{7}}$

11. 平面 // $(2,1,-2)$

$F_x' = 2x \quad F_y' = 4y \quad F_z' = 6z$

点 $M(x_0, y_0, z_0)$

平面方程为：$2x_0(x-x_0) + 4y_0(y-y_0) + 6z_0(z-z_0) = 0 \Rightarrow x_0x + 2y_0y + 3z_0z = 21$

法向量 $(2x_0, 4y_0, 6z_0) \parallel (x_0, 2y_0, 3z_0)$ 　　点积为 0.

$A(6,3,\frac{1}{2})$ $B(0,0,\frac{7}{2})$ 在平面上

$\Rightarrow z_0 = 2 \quad 6x_0 + 6y_0 + \frac{3}{2}z_0 = 21 \quad x_0^2 + 2y_0^2 + 3z_0^2 = 21$

$\Rightarrow x_0 + y_0 = 3 \quad x_0^2 + 2y_0^2 = 9 \Rightarrow \begin{cases} x_0 = 3 \\ y_0 = 0 \\ z_0 = 2 \end{cases}$ 或 $\begin{cases} x_0 = 1 \\ y_0 = 2 \\ z_0 = 2 \end{cases}$

故切平面方程为 $x + 2y = 7$ / $x + 4y + 6z = 21$

13. $F_x' = 2x - a, \quad F_y' = 2y \quad F_z' = 2z$

$(2x-a, 2y, 2z)$

另一个 $(2x, 2y-b, 2z)$

点积 $4x^2 - 2ax + 4y^2 - 2by + 4z^2 = 0$. 故正交

16.(2). $F_x' = -1 - y\sin xy = -1$

$F_y' = 2 - x\sin xy = 2$

$\frac{x-1}{-1} = \frac{y-0}{2}$ 法线

切线：$-1(x-1) + 2(y-0) = 0$.

17.(2).

$F_x' = 4x = -8 \quad F_y' = 6y = 6 \quad F_z' = 2z = 12$

$G_x' = 2x = 8 \quad G_y' = 4y = 4 \quad G_z' = -1$

$(-4, 3, 6)$

$(8 \quad 4 \quad -1)$

叉乘. 切向量 $= (-27, -44, -40)$

切线: $\dfrac{x+2}{-27} = \dfrac{y-1}{-44} = \dfrac{z-6}{-40}$

法平面: $-27(x+2) + (-44(y-1)) + 40(z-6) = 0$.