第五周作业.

习题9.2

1.(2) $f'_x(1,\pi) = (\cos x^2y)\cdot 2xy \Big|_{x=1\ y=\pi} = -2\pi$

2. (2) $\frac{\partial z}{\partial x} = \ln 3\cdot 3^{-\frac{y}{x}}\cdot \frac{y}{x^2}$

$\frac{\partial z}{\partial y} = \ln 3\cdot(-\frac{1}{x})\cdot 3^{-\frac{y}{x}}$

(4) $\frac{\partial z}{\partial x} = \frac{1+\frac{x}{\sqrt{x^2+y^2}}}{x+\sqrt{x^2+y^2}}$ $\frac{\partial z}{\partial y} = \frac{\frac{y}{\sqrt{x^2+y^2}}}{x+\sqrt{x^2+y^2}}$

(6) $\frac{\partial u}{\partial x} = e^{x(x^2+y^2+z^2)}\cdot(3x^2+y^2+z^2)$ $\frac{\partial u}{\partial y} = e^{x(x^2+y^2+z^2)}\cdot 2xy$

$\frac{\partial u}{\partial z} = e^{x(x^2+y^2+z^2)}\cdot 2xz$

(8) $\frac{\partial u}{\partial x} = e^{-z} + \frac{1}{x+\ln y}$

$\frac{\partial u}{\partial y} = \frac{\frac{1}{y}}{x+\ln y}$

$\frac{\partial u}{\partial z} = -xe^{-z}+1$

3. $\frac{\partial f}{\partial x} = 2xy\cdot\frac{\sin x^2y}{x^2y}$

$\frac{\partial f}{\partial y} = x^2\cdot\frac{\sin x^2y}{x^2y}$

4. $f'_x = \lim\limits_{\Delta x\to 0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x} = \frac{0-0}{\Delta x} = 0$

7. $z=\sqrt{y^2+2}$ $\Rightarrow \begin{cases} x=1 \\ y=t \\ z=\sqrt{t^2+2}\end{cases}$

$\frac{dx}{dt}=0$

$\frac{dy}{dt}=1$

$\frac{dz}{dt}=\frac{t}{\sqrt{t^2+2}}$

切线方向向量 $\vec{t}=(0,1,\frac{t}{\sqrt{t^2+2}})$

在点$(1,1,\sqrt{3}) \rightarrow \vec{v}=(0,1,\frac{\sqrt{3}}{3})$

$\cos\theta_x=0$ $\cos\theta_y=1$ $\cos\theta_z=\frac{\sqrt{3}}{3}$

$\theta_x=\frac{\pi}{2}$ $\theta_y=\frac{\pi}{6}$ $\theta_z=\frac{\pi}{3}$

9.(2) $z=\arctan\frac{x+y}{1-xy}$ $\frac{\partial z}{\partial x}=\frac{1}{1+(\frac{x+y}{1-xy})^2}\cdot\frac{1-xy+y(x+y)}{(1-xy)^2}=\frac{1+y^2}{(1-xy)^2+(x+y)^2}=\frac{1+y^2}{x^2+y^2+x^2y^2+1}=\frac{1}{1+x^2}$

$\frac{\partial z}{\partial y}=\frac{1}{1+y^2}$

$\frac{\partial^2 z}{\partial x^2}=-\frac{2x}{1+x^2}$ $\frac{\partial^2 z}{\partial x\partial y}=0$ $\frac{\partial^2 z}{\partial y^2}=-\frac{2y}{1+y^2}$

(5) $\frac{\partial z}{\partial x}=\ln y\cdot y^{\ln x}\cdot\frac{1}{x}$

$\frac{\partial z}{\partial y}=\ln x\cdot y^{\ln x-1}$

$\frac{\partial^2 z}{\partial x^2}=-\frac{1}{x^2}\ln y\cdot y^{\ln x}+\ln y\cdot\frac{1}{x}(\ln y\cdot\frac{1}{x}\cdot y^{\ln x})$

$\frac{\partial^2 z}{\partial y^2}=\ln x\cdot(\ln x-1)\cdot y^{\ln x-2}$

$\frac{\partial^2 z}{\partial x\partial y}=\frac{1}{x}y^{\ln x-1}+\ln y\cdot y^{\ln x-1}\cdot\frac{1}{x}\cdot\ln x$

12. $f''_{xy}(0,0)=\lim_{y\to0}\frac{f'_x(0,y)-f'_x(0,0)}{y}=-1$

$f'_x(0,y)=\lim_{x\to0}\frac{f(x,y)-f(0,y)}{x}=-y$

$f'_x(0,0)=\lim_{x\to0}\frac{f(x,0)-f(0,0)}{x}=0$

$f''_{yx}(0,0)=\lim_{x\to0}\frac{f'_y(x,0)-f'_y(0,0)}{x}=1$

$f'_y(x,0)=\lim_{y\to0}\frac{f(x,y)-f(x,0)}{y}=1$

$f'_y(0,0)=\lim_{y\to0}\frac{f(0,y)-f(0,0)}{y}=0$

13.(2)

$\frac{\partial z}{\partial x}=\frac{x(x^2+y^2)-2x^2y}{(x^2+y^2)^2}$

$\frac{\partial z}{\partial y}=\frac{y(x^2+y^2)-2y^2x}{(x^2+y^2)^2}$

(6) $\frac{\partial z}{\partial x} = 4x^3 - 8y^2x \Big|_{x=0,\ y=0} = 0$

$\Big|_{x=1\ y=1} = -4$

17.

$\left| (x^2+y^2) \cdot \sin\frac{1}{\sqrt{x^2+y^2}} \right| \le \left| \sqrt{x^2+y^2} \right| < \delta$

$f'_x = 2x \sin\frac{1}{\sqrt{x^2+y^2}} + (x^2+y^2)\cos\frac{1}{\sqrt{x^2+y^2}} \cdot \frac{x}{\sqrt{x^2+y^2}}$

存在,不连续.

19. (3)

$$\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r}=\frac{2x}{x^2+y^2}\cdot e^{t+s+r}$$

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial s}=\frac{2x}{x^2+y^2}\cdot e^{t+s+r}+\frac{2y}{x^2+y^2}\cdot 8s$$

$$\frac{\partial u}{\partial t}=\frac{2x}{x^2+y^2}e^{t+s+r}+\frac{2y}{x^2+y^2}\cdot 8t$$

(4)

$$\frac{du}{dx}=\frac{d}{dx}\frac{e^{ax}(a\sin x-\cos x)}{a^2+1}=\frac{ae^{ax}}{a^2+1}(a\sin x-\cos x)+(a\cos x+\sin x)\cdot\frac{e^{ax}}{a^2+1}$$

20. (3)

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial(x^2-y^2)}\cdot 2x+\frac{\partial u}{\partial e^{xy}}\cdot y\cdot e^{xy}$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial(x^2-y^2)}\cdot 2y+\frac{\partial u}{\partial e^{xy}}\cdot x\cdot e^{xy}$$

$$\frac{\partial^2 u}{\partial x\partial y}=2y\frac{\partial\frac{\partial u}{\partial(x^2-y^2)}}{\partial x}+\frac{\partial\frac{\partial u}{\partial e^{xy}}}{\partial x}\cdot x\cdot e^{xy}+(1+xy)e^{xy}\cdot\frac{\partial u}{\partial e^{xy}}$$

21. $\frac{l}{|l|}=\left(\frac{3}{\sqrt{11}},-\frac{1}{\sqrt{11}},\frac{1}{\sqrt{11}}\right)$

$u_x'=yz\quad u_y'=xz\quad u_z'=xy$

$$\frac{\partial u}{\partial l}=\frac{3}{\sqrt{11}}yz-\frac{1}{\sqrt{11}}xz+\frac{1}{\sqrt{11}}xy=\frac{-3}{\sqrt{11}}$$

23. $u_x'=2x+y+3$

$u_y'=4y+x-2$

$u_z'=6z-6$

在点(1,1,-1)梯度为 $\overline{(6,3,-12)\cdot\vec{e}}$

$$=(6,3,-12)\cdot\frac{(6,3,-12)}{|(6,3,-12)|}$$

$$=\sqrt{189}$$

24. $r=\sqrt{x^2+y^2+z^2}$

$f=\frac{1}{r^2}=\frac{1}{x^2+y^2+z^2}$ $\quad f_x'=-(x^2+y^2+z^2)^{-2}\cdot 2x$ $\quad \operatorname{grad}\frac{1}{r^2}=-\frac{2}{r^4}\vec{r}$

$f_y'=-(x^2+y^2+z^2)^{-2}\cdot 2y$

$f_z'=-(x^2+y^2+z^2)^{-2}\cdot 2z$

$f=\ln r=\ln\sqrt{x^2+y^2+z^2}$ $\quad f_x'=\frac{x}{\sqrt{x^2+y^2+z^2}}\cdot(x^2+y^2+z^2)^{-\frac{1}{2}}=\frac{x}{x^2+y^2+z^2}$

$f_y'=\frac{y}{x^2+y^2+z^2}$ $\quad f_z'=\frac{z}{x^2+y^2+z^2}$ $\quad \operatorname{grad}\ln r=\frac{1}{r^2}\vec{r}$

29. $\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\cdot\cos\varphi+\frac{\partial u}{\partial y}\cdot\sin\varphi$

$\frac{\partial u}{\partial \varphi}=\frac{\partial u}{\partial x}\cdot(-r\sin\varphi)+\frac{\partial u}{\partial y}\cdot r\cos\varphi$

$(\frac{\partial u}{\partial r})^2+(\frac{1}{r}\frac{\partial u}{\partial\varphi})^2=(\frac{\partial u}{\partial x})^2(\cos^2\varphi+\sin^2\varphi)+(\frac{\partial u}{\partial y})^2(\sin^2\varphi+\cos^2\varphi)$

$=(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2$ $\quad\square$

32. $(3\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y})(\frac{2\partial z}{\partial x}+\frac{\partial z}{\partial y})=0$

且 $\frac{\partial z}{\partial u}\cdot\frac{\partial z}{\partial v}=0$.

又∵ $\frac{2\partial z}{\partial x}+\frac{\partial z}{\partial y}=2(\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v})+(\frac{-2\partial z}{\partial u})+\frac{a\partial z}{\partial v}$

$=(2+a)\frac{\partial z}{\partial v}$

$3\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=3\frac{\partial z}{\partial u}+3\frac{\partial z}{\partial v}+2\frac{\partial z}{\partial u}-\frac{a\partial z}{\partial v}$

$=5\frac{\partial z}{\partial v}+(3-a)\frac{\partial z}{\partial v}$.

故有 $(2+a)\cdot(3-a)=0$ 且 $2+a\neq0$. $\Rightarrow a=3$

35. $u(x,2x)=x$

$1=\frac{\partial u}{\partial x}(x,2x)=u_x'(x,2x)+2\cdot u_y'(x,2x)=x^2+2u_y'(x,2x)$

$\Rightarrow u_y'(x,2x)=\frac{1}{2}(1-x^2)$ --①

又 $u_x'(x,2x)=x^2$

$u_{xx}''(x,2x)+u_{xy}''(x,2x)\cdot2=2x$

对① $u_{yy}''(x,2x)\cdot2+u_{yx}''(x,2x)=-x$

$\Rightarrow u_{xy}''(x,2x)=\frac{5x}{3}$

$u_{xx}''(x,2x)=-\frac{4x}{3}$

$=u_{yy}''(x,2x)$