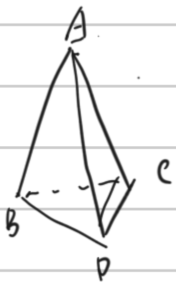
Hw 3

P15. 22. $\vec{a}=(3,-1,-2)$ $\quad$ $2\vec{a}-\vec{b}=(5,-4,-3)$

$\vec{b}=(1,2,-1)$ $\quad$ $2\vec{a}+\vec{b}=(7,0,-5)$

(1) $\vec{a}\times\vec{b}=(5,1,7)$ $\quad$ (2) $(2\vec{a}-\vec{b})\times(2\vec{a}+\vec{b})=(20,4,28)$

24.



$V=\frac{1}{6}|V_1|$

$\vec{AB}=(3,6,3)$

$\vec{AC}=(1,3,-2)$

$\vec{AD}=(2,2,2)$

$$\begin{vmatrix} 3 & 6 & 3 \\ 1 & 3 & -2 \\ 2 & 2 & 2 \end{vmatrix} = 6\begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$=6\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & -2 \end{vmatrix} = 6[-7+3+1]=-18$$

$V=3$

26. $\vec{AB}=(-1,-1,6)$

$\vec{AC}=(-2,0,2)$

$\vec{AD}=(1,-1,4)$

$$V=\begin{vmatrix} -2 & 0 & 2 \\ -1 & -1 & 6 \\ 1 & -1 & 4 \end{vmatrix} = -2\times(-4+6)+2(1+1)$$

$=0$

故共面

P24.

4. (1) 平行

(3) 重合

7. (1) $\vec{n_1}=(2,-1,1)$ $\quad$ $\vec{n_2}=(1,1,2)$

$\cos\theta=\frac{2-1+2}{\sqrt{6}\cdot\sqrt{6}}=\frac{3}{6}=\frac{1}{2}$ $\quad$ $\theta=\frac{\pi}{3}$

8. (1) $d=\frac{|16\times2+12-15-4|}{\sqrt{16^2+12^2+15^2}}=\frac{25}{25}=1$

9. (2) $4x-2y+4z+18=0$ $\quad$ $d=\frac{18+21}{\sqrt{4^2+2^2+4^2}}=\frac{39}{6}=\frac{13}{2}$

11. 等距离平面 $\Rightarrow$ 平行.

设 $x+2y-2z+m=0$.

$$D_1=\frac{|m+1|}{\sqrt{1^2+2^2+2^2}}=D_2=\frac{|m-3|}{\sqrt{1^2+2^2+2^2}}$$

$\Rightarrow |m+1|=|m-3| \quad m=1.$ 　平面为 $x+y-2z+1=0$

16. $\vec{n}_1=(2,3,-1) \quad \vec{n}_2=(3,-5,2)$ 　　$(2,3,-1)$ $(3,-5,2)$

$\vec{n}_1\times\vec{n}_2=(1,-7,-19)$.

令 $z=-2$ 　$\begin{cases}2x+3y=4+z\\3x-5y=-1-2z\end{cases} \Rightarrow \begin{cases}6x+9y=12+3z\\6x-10y=-2-4z\end{cases} \Rightarrow 19y=14+7z$

$\begin{cases}x=1\\y=0\end{cases}$

故参数方程为 $\begin{cases}x=1+t\\y=-7t\\z=-2-19t.\end{cases}$

3.6.

8.2.19.(2) $\vec{s}=(3,-1,4)$

$\vec{s_1}=(2,2,-1)$ $\vec{s_2}=(1,-1,-1)$

$\vec{s_1}\times\vec{s_2}=(-3,1,-4)\ //\ \vec{s}$

故两直线平行.

$M_1=(-7,5,9)$ $\overrightarrow{M_1M_2}=(22,-14,-7)$

$M_2=(15,-9,2)$ $d=\dfrac{|\overrightarrow{M_1M_2}\times\vec{s}|}{|\vec{s}|}=25$

22. (1) $M_1=(0,1,0)$

$\overrightarrow{P_0M_1}=(-1,1,1)$ $d=\dfrac{|\overrightarrow{P_0M_1}\times\vec{s}|}{|\vec{s}|}=\dfrac{3\sqrt{2}}{\sqrt{6}}=\sqrt{3}$

$\vec{s}=(1,2,-1)$

23. (1) $\vec{s_1}=(4,-3,1)\ \cancel{//}\ \vec{s_2}=(-2,9,2)$

154

$M_1=(9,-2,0)$ $M_2=(0,-7,2)$

11 2 42

$\overrightarrow{M_1M_2}=(-9,-5,2)$ $\begin{vmatrix}4&-3&1\\-2&9&2\\-9&-5&2\end{vmatrix}=4\times28+3\times14+91=245$

$\vec{s_1}\times\vec{s_2}=(-15,-10,30)=5(-3,-2,6)$

4 -3 1

-2 9 2

$|\vec{s_1}\times\vec{s_2}|=5\times\sqrt{9+4+36}=35$

$d=\dfrac{245}{35}=\dfrac{49}{7}=7$

24. $3x+2y-z-1+\lambda(2x-3y+2z+2)=0$

$(3+2\lambda)x+(2-3\lambda)y+(2\lambda-1)z+2\lambda-1=0.$

法向量 $(3+2\lambda,2-3\lambda,2\lambda-1)$

另一个平面法向量 $(1,2,3)$

故 $3+2\lambda+4-6\lambda+6\lambda-3=0 \Rightarrow \lambda=-2$

方程为 $-x+8y-5z-5=0$

25. $\vec{s_1} = (2,-1,1)$ $\quad \vec{s_1}\times\vec{s_2} = (-1,3,5) = \vec{s_4}$

$\vec{s_2} = (1,2,-1)$

$\frac{x-1}{3} = \frac{y-3}{2} = \frac{z+2}{-1}$ $\quad \vec{s_3} = (3,2,-1)$

$(-1,3,5)$

$\vec{s_3}\times\vec{s_4} = (13,-14,11)$ 过点 $(1,3,-2)$

故方程为 $13(x-1) - 14(y-3) + 11(z+2) = 0$

28 $\frac{x+7}{1} = \frac{y+2}{2} = \frac{z+2}{3}$ $\quad \vec{s} = (1,2,3)$

$M_0 = (x_0, y_0, z_0)$ $\quad (2-x_0, 3-y_0, 1-z_0)\cdot(1,2,3) = 0$

$2-x_0+6-2y_0+3-3z_0 = 0$

$2(7+x_0) = (y_0+2)$

$3(7+x_0) = z_0+2$

$y_0 = 12+2x_0$

$z_0 = 19+3x_0$

$2-x_0+6-24-4x_0+3-57-9x_0 = 0$

$14x_0 = -70$ $\quad x_0 = -5$

$y_0 = 2$

$z_0 = 4$

$M(-5,2,4)$

31. $M_1 = (1,-4,3)$ $\quad M_2 = (-3,9,-14)$

68 - 91

$\overrightarrow{M_1M_2} = (-4,13,-17)$

$\vec{s_1} = (\lambda,5,-3)$

$\vec{s_2} = (3,-4,7)$

$$\begin{vmatrix} \lambda & 5 & -3 \\ 3 & -4 & 7 \\ -4 & 13 & -17 \end{vmatrix} = -23\lambda + 5\times 23 - 3\times 23 = 0$$

$\lambda+5-3 = 0$ $\quad \lambda = 2$

35-12

$\vec{s_1} = (2,5,-3)$

$\vec{s_2} = (3,-4,7)$

$\vec{s_1}\times\vec{s_2} = (23,-23,-23) \parallel (1,-1,-1)$

平面 S 为 $(x-1) - (y+4) - (z-3) = 0$

$x+y+3 = z+14$

$5x-5 = 2y+8$

$-4x-12 = 3y-27$

$15x-15 = 6y+24$

$-8x-24 = 6y-54$

$78 = 23x+9$

$69 = 23x$

交点 $(3,-1,0)$

33. $\vec{s_1}=(1,1,-1)$ $\quad \vec{s}\parallel\vec{s_1}\times\vec{s_2}=(0,-2,-2)\parallel(0,1,1)$

$\vec{s_2}=(1,-1,1)$

$\vec{n}=(1,1,1)$ $\quad \vec{s}=(0,1,1)$ $\quad$ 过 $(0,0,-1)$

$(0,1,1)$

$\vec{n}\times\vec{s}=(0,-1,1)$ $\quad$ 故 $\begin{cases} x+y+z=0 \\ -y+z+1=0 \end{cases}$、

35. 垂足 $\vec{M}=(x_0,y_0,z_0)$ $\quad \vec{s}=(4,3,-2)$

$\vec{M}\cdot\vec{s}=4x_0+3y_0-2z_0=0$

$9x_0-21=12y_0$

$3(x_0-5)=4(y_0-2)$ $\quad 3x_0-15=4y_0-8$ $\quad 3x_0=4y_0+7$

$-2(x_0-5)=4(z_0+1)$ $\quad -x_0+5=2z_0+2$ $\quad 2z_0=-x_0+3$

$4x_0+3y_0+x_0-3=0$ $\quad 5x_0+3y_0=3$

$20x_0+12y_0=12$ $\quad 29x_0=33$

$x_0=\frac{33}{29}$ $\quad y_0=-\frac{26}{29}$ $\quad z_0=\frac{27}{29}$ $\quad$ 故直线方程为 $\frac{x}{33}=\frac{y}{-26}=\frac{z}{27}$

习题 8.3

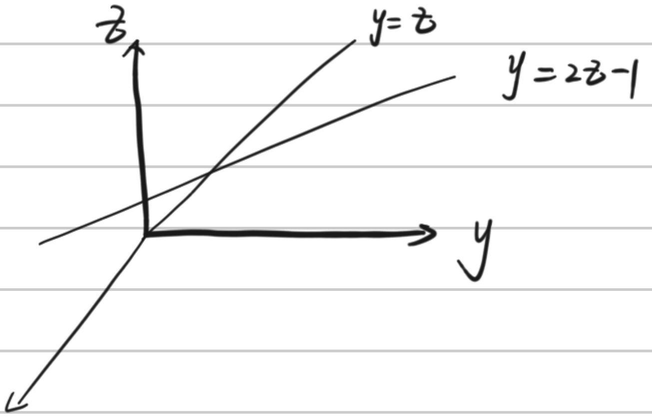
3. (1) $\begin{cases} F(y,z)=0 \\ x=0. \end{cases}$ $\quad x_1^2+y_1^2=y^2$ $\quad F(\pm\sqrt{x^2+y^2},z)=0.$

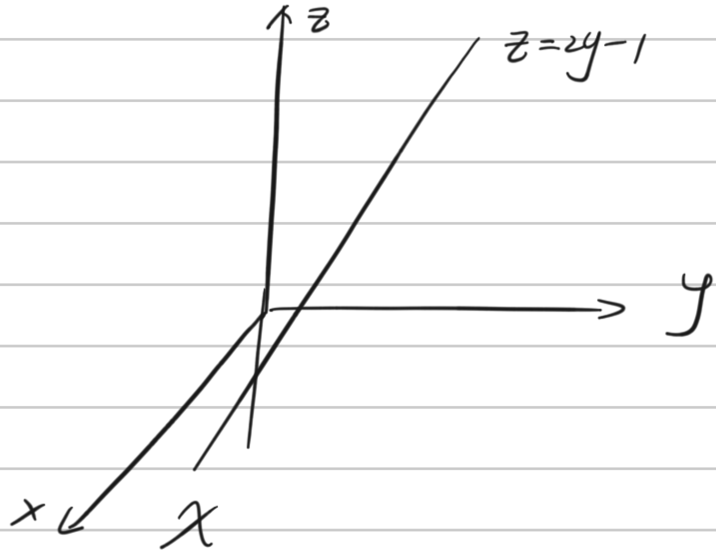
即 $x^2+y^2-\frac{z^2}{4}=1$

(2) $F(x,y)=0.$ $\quad y_0^2+z_0^2\to y^2$ $\quad$ 故 $F(x,\pm\sqrt{y^2+z^2})=0$

即 $\pm\sqrt{y^2+z^2}=\sin x$

即 $y^2+z^2=\sin^2 x$





$F(y,z)=0.$

z不变 $y^2 \longrightarrow x^2+y^2$

$F(\pm\sqrt{x^2+y^2},\ z)=0.$

法一：旋转思想

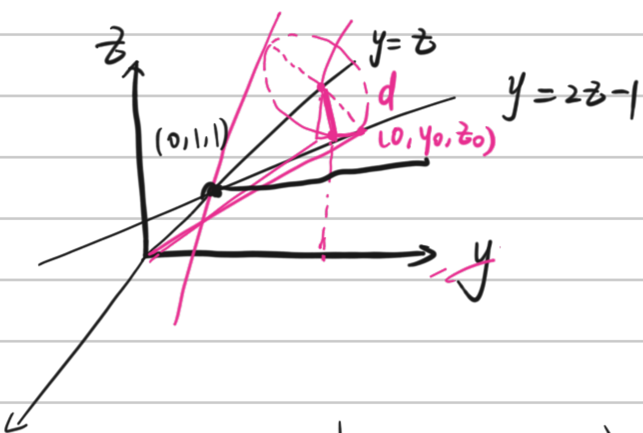
$$z+1=2(\pm\sqrt{x^2+y^2})$$

$$(z+1)^2=4(x^2+y^2)$$

$$(y,z)\begin{pmatrix}\cos 45^\circ & -\sin 45^\circ\\ \sin 45^\circ & \cos 45^\circ\end{pmatrix}=\left(\frac{\sqrt2}{2}y+\frac{\sqrt2}{2}z,\ -\frac{\sqrt2}{2}y+\frac{\sqrt2}{2}z\right).$$

故原方程为 $\left(-\frac{\sqrt2}{2}y+\frac{\sqrt2}{2}z+1\right)^2=4\left(x^2+\frac{1}{2}y^2+\frac{1}{2}z^2+yz\right)$

法二：



$\cos\theta_1=\frac{\sqrt2}{2}$

$\sin\theta_1=\frac{\sqrt2}{2}$

$\cos\theta_2=\frac{2}{\sqrt5}$

$\sin\theta_2=\frac{1}{\sqrt5}$

$$\cos(\theta_1-\theta_2)=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2=\frac{3}{\sqrt{10}}$$

即 $$\frac{|(x,y-1,z-1)\cdot(0,1,1)|}{\sqrt{x^2+(y-1)^2+(z-1)^2}\cdot\sqrt2}=\frac{3}{\sqrt{10}}$$

$$10(y-1+z-1)^2=9[x^2+(y-1)^2+(z-1)^2]$$

5. $\vec{s_1}=(1,1,-1)$ $(1,-1,2)$

$\vec{n}=(1,-1,2)$ $(1,1,-1)$

$\vec{n_1}\times\vec{s_1}=(-1,3,2)$. 平面过 $(1,0,1)$

$L_0:\begin{cases}-1(x-1)+3y+2(z-1)=0\\ x-y+2z-1=0.\end{cases}$

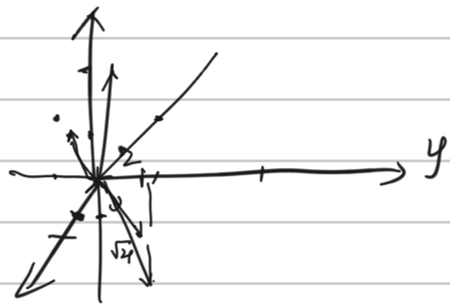
$\vec{s_2}=(-1,3,2)$

$\vec{s_3}=(1,-1,2)$

$\vec{s_2}\times\vec{s_3}=(8,4,-2)$

$\parallel(4,2,-1)$

即 $\frac{x-1}{4}=\frac{y}{2}=\frac{z-1}{-1}$



$$x^2+z^2=x_1^2+z_1^2 \quad y=y_1$$

$P_1(x_1,y_1,z_1)$ 在 L_0 上.

故 $\begin{cases}x_1=2y_1\\ z_1=-\frac{y_1-1}{2}\end{cases}$

$$x^2+z^2=4y^2+\frac{(y-1)^2}{4}$$

即 $x^2+z^2=4y^2+\frac{(y-1)^2}{4}$