2D+ Challenge – Modeling the Systems World

Systems World Data Sheet

Last edited: March 19, 2015

1 Problem description

The Systems World part of the 2D+ challenge, as described in the general description document, is as follows:

> On Saturday of Week 11, each team must submit a report consisting of two parts. The first part will contain a mathematical model describing bacterial growth on a plate in an environment possibly affected by radiation. You should try to capture the most important aspects of the process, and write a model that is as general as possible. Explain your notation clearly. The second part of the report should contain mathematical optimization models that can be used to find the values of the parameters that minimize the regression error when trying to t fit a growth curve to experimental data, according to the specifications detailed in the companion 10.007 Systems World Data Sheet that will be provided to you on Week 8.

We will now describe the growth curve fitting problem and the corresponding data.

2 Curve fitting

In order to understand how the presence of radiation affects the growth of E. Coli, we first want to have a clear picture of how the bacterial population size evolves with time in a non-radioactive environment. For this purpose, we have recorded a measure of E. Coli population density on a culture plate; the measure we used is the light absorbance at 600nm (A_{600}), because E. Coli population is directly related to how much light is absorbed by the colony. Table 1 provides the values of A_{600} over time.

From this data, we want to infer a growth curve that relates the population size to time. If we let y be the value of A_{600}, and x the time, we want to compute a function of the form $y = f(x)$. Denote the data samples in Table 1 as (x_i, y_i) for $i = 1, \ldots, 11$, and denote the i-th *absolute prediction error* as $r_i = |f(x_i) - y_i|$. Proceed as follows:

(a) Guess a functional form $f(x)$ (i.e. a "type of function" that depends on some parameters) that in your opinion will fit the data in Table 1 well.

(b) Write down a *linear* optimization model to compute the value of the parameters for the function $f(x)$ that minimizes some measure of the total prediction error with the goal of obtaining a curve that fits the data points well. This measure should be obtained starting from the absolute prediction errors r_i, and it must be modeled using a *linear* program. Justify your choice. Note: it is often possible to obtain a linear optimization model, even if the function $f(x)$ you choose is nonlinear. In this case, a linear optimization model is *required*.

(c) Implement the linear model of part (b) on a spreadsheet using Solver. Which parameters do you obtain? What is the largest absolute prediction error?

The above questions are required to be answered in the report. But the 2D+ project is an open ended problem, any additional work/analysis to be done is up to you.

Table 1: Time (in minutes) and population density (measured by the light absorbance at 600nm) for an E. Coli colony.

Time	A_{600}
0	0.090
18	0.102
78	0.124
142	0.253
205	0.487
255	1.020
322	1.980
378	3.950
446	5.880
504	6.760
564	7.200

3 Submission details

Your submission should contain:

- A report (in PDF format) consisting of two parts: 1) your mathematical model to describe bacterial growth on a plate in an environment possibly affected by radiation; 2) the mathematical optimization model that you wrote for the curve fitting problem, with its solution. Justify your models clearly and explain your notation.
- A spreadsheet file containing the model for curve fitting, and its solution saved in the appropriate cells of the spreadsheet (in other words, save the file with the solution after running Solver).

The submission deadline is Saturday 11 April, at 7PM. Send the two files indicated above to both your cohort instructors. Late submission will not be accepted and the date of receipt of the email determines the submission time.