Assignment 8

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The dynamic programming model I am interested in is the allocation of investment among different periods by an investor. The optimization problem for this investor is to decide her investment portfolio to maximize her utility which is decided by a return function. I assume that the investor will invest in a portfolio, which provides a return higher than the risk-free risk. The investment amount is denoted as, I. Based on this model, there are many variations that relax the assumptions and bring in more realistic complications what worth to incorporate in the future.

The Investment Decision Problem :

- discrete time t = 1, 2
- I_t is the total investment amount including both risk-free assets and risky assets in period t, and this investor will consume the remaining money after making the investment decision in the first period, and the available money to consume in the second period is the principal and return on the investment.
- I_t is the control variable in the model
- Preferences: $max(u(s_1 - I) + \beta\,\mathbb{E}(u(r * I)))$
  - $u'(\cdot) > 0 \rightarrow$ a higher return increases the utility
  - $0 \leq \beta \leq 1 \rightarrow$ a discount factor because the return has time value
  - $u'(0) = \infty \rightarrow$ Inada condition to eliminate corner solutions
  - r_t is the total return of the investment portfolio available to the investors
  - The Bellman equation $\rightarrow V(s_1, \epsilon) = \epsilon u(s_1 - I_1) + \beta\,\mathbb{E}_{\epsilon'|\epsilon}(u(r * I_1))$
  - The FOC $\rightarrow \epsilon u'(s_1 - I_1) = \beta\,\mathbb{E}_{\epsilon'|\epsilon}(u(r * I_1)) * r$
- Endowment:
  - $s_1 > 0 \rightarrow$ the initial positive saving of the investor
  - No extra capital/income in period 2
- Technology:
  - Storage technology: $s_2 = r_t * I_1$, as known as transition equation
  - s, r are the state variables in the model
- Market:

– There are asset markets which allow both long and short positions

- Information:
  – $eR(\cdot)$ is state-dependent and there are two states, R_H and R_L, representing a high return and a low return
  – The uncertainty follows a first-order Markov process, i.e., the return in this period is correlated with that in the last period $\rightarrow prob(r_{m,t+1} = j | r_t = i) = \pi_{i,j}$
  – This uncertainty exists only after period 1

Plots of the Results:

- Figure 1 shows the change of value function on the state variable in the model, the initial saving. It is a positively curved line showing that a higher level of the initial capital in the beginning is correlated with a higher level of utility
- Figure 2 shows the relationship between the state variable and the optimal investment amount. It shows that the optimal investment decision in this setting is to invest all capital in the first period and consume them all in the second period. It kind of makes sense because of the low discount rate, β

Figure 1: The Value Function Plot

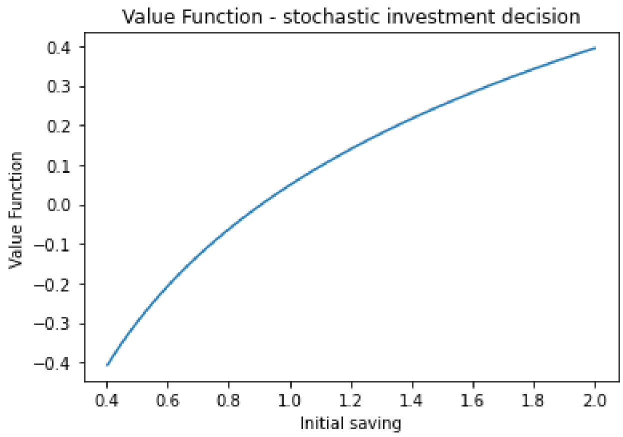
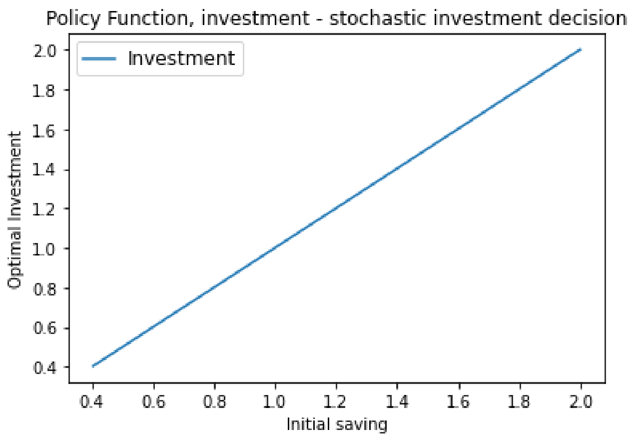


Figure 2: The Policy Function Plot



Potential Improvements:

- I am aware of many places where this model can be improved. For example, to derive a proper CAPM model, we need to include an income variable for each period, and this variable is correlated with the rate of return on securities. Moreover, it also helps to include a boundary condition for the portfolio, so that the investment decision can be more flexible, rather than extremely investing all or saving all.