INTRODUCTION TO CALCULUS

Calculus is a branch of mathematics that studies continuous change and motion. It provides a framework for modeling systems where change is fundamental and for deriving predictions about the behavior of these systems. Calculus consists of two major branches: differential calculus and integral calculus.

Differential Calculus:

Differential calculus concerns rates of change and slopes of curves. Its central concept is the derivative.

The Derivative:
- Represents the instantaneous rate of change of a function with respect to one of its variables
- Geometrically, it's the slope of the tangent line to the function's graph at a specific point
- Notation: $f'(x)$, dy/dx, or $d/dx[f(x)]$

Basic Differentiation Rules:
1. Power Rule: $d/dx[x^n] = n \cdot x^{(n-1)}$
2. Product Rule: $d/dx[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
3. Quotient Rule: $d/dx[f(x)/g(x)] = [f'(x) \cdot g(x) - f(x) \cdot g'(x)]/[g(x)]^2$
4. Chain Rule: $d/dx[f(g(x))] = f'(g(x)) \cdot g'(x)$

Applications:
- Finding maximum and minimum values
- Related rates problems
- Motion analysis (velocity and acceleration)
- Optimization problems

Integral Calculus:

Integral calculus concerns accumulation of quantities and areas under curves. Its central concept is the integral.

The Integral:
- Definite integral: $\int[a,b]f(x)dx$ represents the area under the curve $f(x)$ from $x=a$ to $x=b$
- Indefinite integral: $\int f(x)dx$ represents the antiderivative of $f(x)$
- Fundamental Theorem of Calculus: $\int[a,b]f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$

Basic Integration Rules:
1. Power Rule: $\int x^n \, dx = x^{(n+1)}/(n+1) + C$ (where $n \neq -1$)
2. Integration by Parts: $\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx$
3. Substitution Method: $\int f(g(x))g'(x)dx = \int f(u)du$, where $u = g(x)$

Applications:
- Area calculation
- Volume calculation
- Work done by a force
- Total change from rate of change
- Probability distributions

Calculus is foundational to many disciplines, including physics, engineering, economics, statistics, and computer science, providing tools to model and analyze systems that change continuously.