Part I

Q1.

1. $S = \frac{\log(1-P)}{\log(1-p^k)}$ where $P = 0.995$, $p = 0.7$ and $k = 4$ for homography

$= \frac{\log(1-0.995)}{\log(1-0.7^4)} \approx 20$ [round up]

2. It would require less iterations because the affine transformation only need 3 matches at random for RANSAC.

Q2.

1. From hint,

$$\vec{p} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = K\vec{P} = K\begin{bmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{bmatrix}$$

$$= \begin{bmatrix} f & 0 & P_x \\ 0 & f & P_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_0 + td_x \\ Y_0 + td_y \\ Z_0 + td_z \end{bmatrix}$$

$$= \begin{bmatrix} fX_0 + ftd_x + P_x Z_0 + tP_x d_z \\ fY_0 + ftd_y + P_y Z_0 + tP_y d_z \\ Z_0 + td_z \end{bmatrix}$$

$$\lim_{t \to \infty} x = \lim_{t \to \infty} \frac{wx}{w} = \frac{fd_x}{d_z} + P_x$$

$$\lim_{t \to \infty} y = \lim_{t \to \infty} \frac{wy}{w} = \frac{fd_y}{d_z} + P_y$$

coordinate is $\left(\frac{fd_x}{d_z} + P_x , \frac{fd_y}{d_z} + P_y \right)$

2. From 1, we define an arbitrary vanishing points

$$V_1 = \begin{bmatrix} \frac{f dx_1}{dz_1} + P_x \\ \frac{f dy_1}{dz_1} + P_y \end{bmatrix} \qquad V_2 = \begin{bmatrix} \frac{f dx_2}{dz_2} + P_x \\ \frac{f dy_2}{dz_2} + P_y \end{bmatrix}$$

By hint, $n_x dx + n_y dy + n_z dz = 0 \Rightarrow n_x \frac{dx}{dz} + n_y \frac{dy}{dz} + n_z = 0$

$$\Rightarrow V_1 - V_2 = f\left(\frac{dy_1}{dz_1} - \frac{dy_2}{dz_2}\right) \begin{bmatrix} -\frac{n_y}{n_x} \\ 1 \end{bmatrix}$$

$$= \frac{f\left(\frac{dy_1}{dz_1} - \frac{dy_2}{dz_2}\right)}{n_x} \begin{bmatrix} -n_y \\ n_x \end{bmatrix}$$

Since the coordinates in $\begin{bmatrix} -n_y \\ n_x \end{bmatrix}$ which does not depend on d, the vanishing points of all lines lying on the plane should form a line.

Q3.

1. WLOG, assume $l: ax+by+m=0$

$l': cx+dy+n=0$ that intersects,

$c(ax+by+m)-a(cx+dy+n)=0$

$$\Rightarrow \quad x=\frac{bn-dm}{ad-bc} \qquad y=\frac{an-cm}{bc-ad} \quad \Rightarrow \begin{bmatrix} \frac{bn-dm}{ad-bc} \\ \frac{an-cm}{bc-ad} \end{bmatrix}$$

Also, $p=l\times l'=\begin{bmatrix} a \\ b \\ m \end{bmatrix}\times\begin{bmatrix} c \\ d \\ n \end{bmatrix}$

$$=\begin{bmatrix} bn-dm \\ cm-an \\ ad-bc \end{bmatrix}$$

$$\text{in 2D} \Rightarrow \begin{bmatrix} \frac{bn-dm}{ad-bc} \\ \frac{an-cm}{bc-ad} \end{bmatrix}$$

$\therefore$ The intersection of the 2D line l and l' is the 2D point $p=l\times l'$.

2. WLOG, assume $p = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$, $p' = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

line $y = ax + b$ must satisfy $\begin{cases} ax_1 + b = y_1 \\ ax_2 + b = y_2 \end{cases}$ to pass p, p'

$$\Rightarrow a = \frac{y_1 - y_2}{x_1 - x_2} \quad b = y_1 - \frac{x_1 y_1 - x_1 y_2}{x_1 - x_2}$$

$$\Rightarrow l : (y_1 - y_2)x + (x_2 - x_1)y + (x_1 y_2 - x_2 y_1) = 0$$

$\Rightarrow$ line vector of l is $\begin{bmatrix} y_1 - y_2 \\ x_2 - x_1 \\ x_1 y_2 - x_2 y_1 \end{bmatrix}$

Also, $p \times p' = \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} y_1 - y_2 \\ x_2 - x_1 \\ x_1 y_2 - x_2 y_1 \end{bmatrix}$

$\therefore$ The line that goes through the 2D points p' and p' is $l = p \times p'$.

Part II

Q4 2.

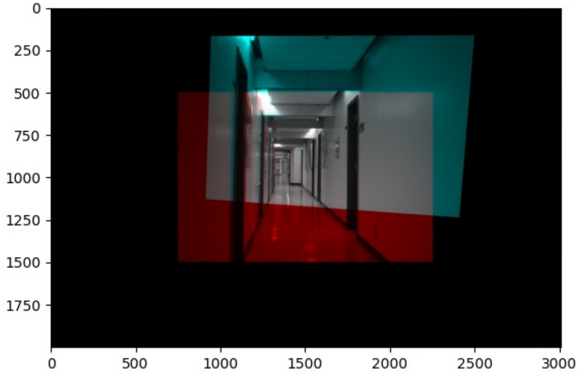
```
Case A Homography H:
[[ 8.29028521e-01  1.03732761e-02 -4.36445251e+01]
 [-1.05244870e-01  9.06523660e-01  4.00067971e+02]
 [-7.67788803e-05  1.49757924e-06  1.00000000e+00]]

Case B Homography H:
[[ 6.40828578e-01 -1.19466574e-01  3.29354146e+02]
 [-6.01852113e-02  8.83064922e-01  2.78186098e+02]
 [ 7.73752312e-05 -1.63061684e-04  1.00000000e+00]]

Case C Homography H:
[[ 1.87598459e+00 -5.52203039e-01  7.07182942e+00]
 [ 6.82422020e-01  1.13430165e+00 -6.87381704e+01]
 [ 7.64011873e-04 -2.63450129e-04  1.00000000e+00]]
```

Case A: Rotate in negative direction and have a slight translation.

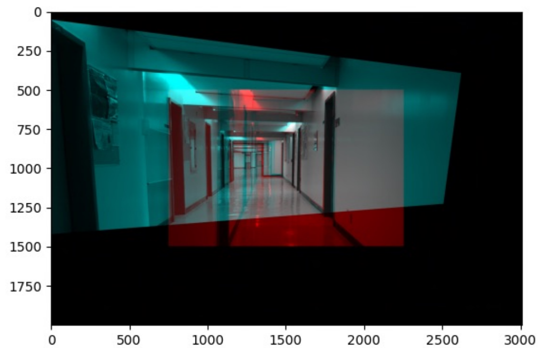
Case B: Shears in vertical direction and have a translation

Case C: Shears in horizontal direction and have a slight translation.

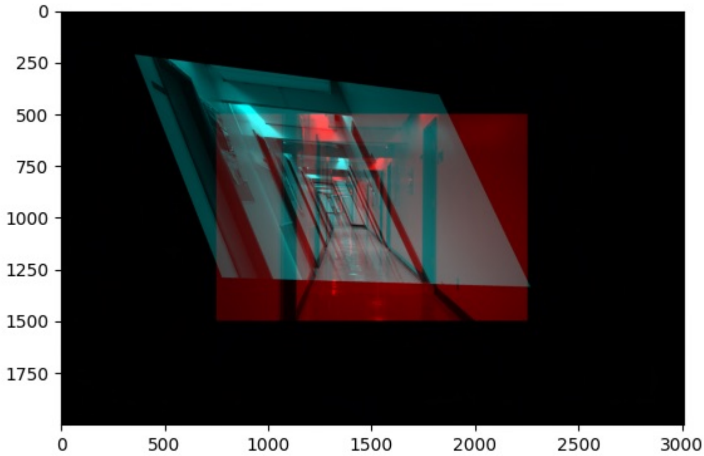
4.



From Case A, the camera is tilted toward the up right direction



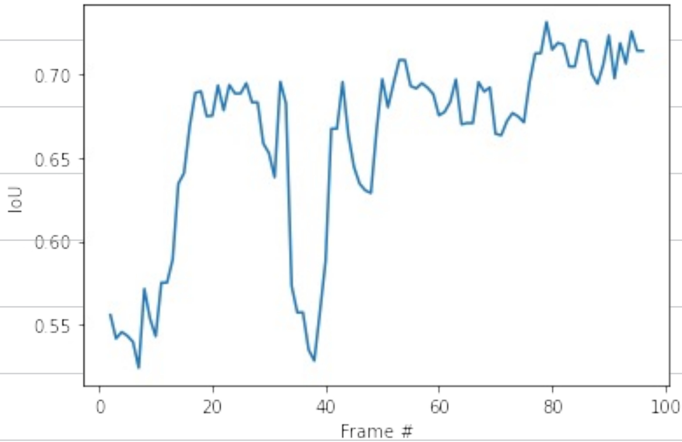
From Case B, the right wall is more Lambertian and the camera is tiled toward up left



From Case C, the camera is tiled toward up left direction. The floor is less Lambertian.

Q5

1.



Small IoU Frame

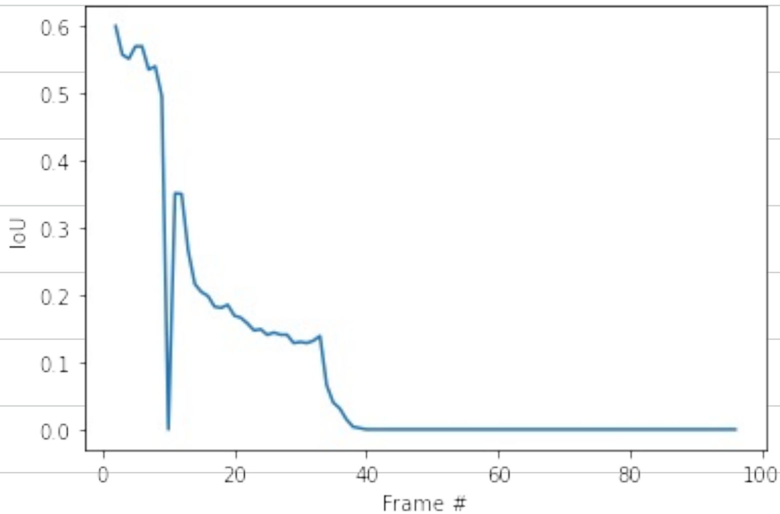
Green for Face Detector

Large IoU Frame

The percentage of frames in which the IoU is larger than 50% is 100%.

The Face detector seems to be more accurate since the tracked bounding box does not include bottom part of the face. The green box seems to be more complete.

2.



Small IoU Frame

Large IoU Frame

The percentage of frames in which the IoU is larger than 50% is 8.33%