Q1

1. To maximise the magnitude of the response of the Laplacian filter, the zeros need to convolve with all the negative portion of Laplacian.

$$\frac{x^2+y^2}{2\sigma^2} - 1 = 0 \Rightarrow x^2+y^2 = 2\sigma^2$$

$$\Rightarrow r^2 = 2\sigma^2$$

$$\Rightarrow \sigma^2 = \frac{r^2}{2}$$

$$\Rightarrow \sigma = \frac{D}{2\sqrt{2}}$$

2. Similar as previous one but we need to achieve a minimum this time.

$$\frac{x^2+y^2}{2\sigma^2} - 1 = 1 \Rightarrow r^2 = 4\sigma^2$$

$$\Rightarrow \sigma^2 = \frac{r^2}{4}$$

$$\Rightarrow \sigma = \frac{D}{4}$$

3. In ipynb.

Q2

1. $\det(N-\lambda I)=0$

$\Rightarrow \det\begin{pmatrix} I_x^2-\lambda & I_xI_y \\ I_xI_y & I_y^2-\lambda \end{pmatrix}=0$

$\Rightarrow I_x^2I_y^2-\lambda(I_x^2+I_y^2)+\lambda^2-I_x^2I_y^2=0$

$\lambda^2=\lambda(I_x^2+I_y^2)$

$\lambda_1=0$, $\lambda_2=I_x^2+I_y^2$

2. WTP M is positive semi-definite.

$u^TMu\geq 0$ Since λ_1 and $\lambda_2\geq 0$, N is positive semi-definite. and $w(x,y)\geq 0$

$u^TMu=\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\sum_x\sum_y w(x,y)N(u_1\ u_2)$

since $u^TNu\geq 0$ and $w(x,y)\geq 0$

$u^TMu\geq 0$

$\therefore M$ is positive semi-definite.

$\boxed{19}$