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Optimization of cycle paths with mathematical programming

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Abstract

The recent evolution and development of urban areas has dramatically transformed the layout of cities and has had a significant impact on mobility. Decision makers have become aware of the problem and have begun to take measures to manage the changing demand for mobility through the diversification and promotion of less aggressive and more efficient transport modes: walking, cycling and using public transport. New standards in sustainable mobility are being incorporated into this new scenario to encourage a reduction in car use. These standards form the basis for the design of planning tools and more efficient management systems, among them, encouraging the use of bicycles as an everyday mode of mobility in urban areas.
There are a number of programs aimed at the promotion of cycling in cities. One in particular is for the planning and design of cycle paths through the establishment of networks that allow the use of bicycles in preferential paths with high safety guarantees. This paper presents a mathematical programming model for the optimal design of a network intended for cyclists. Specifically, the model determines which type of infrastructure (type of bike lane) is most appropriate on each link of a road network, based on criteria of cost to users and the investment cost of the infrastructure itself.
As an application of the proposed model, several experiments are presented on a testing network based on the known Sioux Falls network. As a result of these experiments a number of useful conclusions are obtained for the design of cycle networks from a social and operational perspective within a pre-defined cost.
The model has been developed to be highly versatile and to allow any type of change (different network, different levels of demand , etc ...) and to assure the least consumption of computational resources.

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Keywords: Sustainable mobility; mathematical programming; cycle pahts; optimization

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1. Introduction

During the 1990s, science started to propose alternatives to existing mobility models. The society at the time had doubts about the validity of the existing model, the pollution, management, and the loss of public space are subjects that society needs to address through urban planning and environmental policies. Attempts are being made to recuperate part of these lost public spaces, the most obvious being the pedestrianization of historic town centers and increasing investment in public transport. The increasing presence of the bicycle in urban areas has meant that people friendly spaces are being created to circulate either on foot or by bicycle, creating safe zones with fewer or no vehicles and a changing mentality is beginning to spread among the population. The current challenge facing cities is to plan a mobility systems aimed at the citizen, to plan the city on a more local scale, where the concept of nearness has greater relevance (Pozueta, Daudén, & Schettino, 2009).

Although the bicycle is not the only solution to traffic and environmental problems in urban areas, it does constitute a response which can easily be inserted into any urban renewal legislation and policy at a relatively low economic outlay. The individual evolution of different towns and cities has led to great diversity in urban morphologies over the years and the cyclist has always looked for the shortest available route to optimize their trip which, depending on the particular urban morphology of the town in question, will be longer or shorter. In this sense, the cyclist must consider such restriction.

Urban architecture is a key element in the promotion of cycling and the type of infrastructure, the cycle lane, has a great influence on speed, physical exertion and safety, which are all important factors when considering whether to travel by bicycle. A positive correlation between the number of bicycle journeys and the density of the bicycle lanes has been observed in a study of the largest cities in the United States of America (Dill y Carr, 2003). Cyclists make variable choices, they adjust their routes in order to use the infrastructure prepared specifically for their use (Howard and Burns, 2001).

Nevertheless, cycling infrastructure does come at a cost, which depends on the characteristics of the dedicated cycle lane: segregated or not, width, type of surface, etc. This infrastructure is also limited by the space available for it in urban areas.

According to Kauffman (Kauffman, 1972), the Network Theory is a branch of Set Theory based on the work of Köning, which is helpful for modeling and solving certain problems of a combinatory nature which appear in diverse domains. The mathematical support of graph theory is special for each type of problem, which are normally very complex combinatory problems, and allows the problem to be solved in a more simple way than using conventional mathematical programming.

The aim of this research is, therefore, to propose a cycling network optimization model by establishing the type of infrastructure required on each link, which will minimize the cost to the user (cyclist) and is subject to budgetary constraints. The methodology is applied to the Sioux Falls network, which allows a sensitivity analysis to be performed as well as an evaluation of the cost increases resulting from improvements made to the network to provide a quality sustainable transport system. The resulting network is capable of reducing the conflict between motorized traffic and bicycles.

The optimization methodology is presented below along with the considered specifications, which will later be applied to the Sioux Falls network. In section 3 this application will produce the resulting structure for each link within the applied budgetary constraints and the suitability of each link will be evaluated. Alternative scenarios will then be generated along with their impact on costs. Finally, the conclusions derived from this study will be presented along with considerations for future research.

2. Methodology

The present research proposes an optimization model for the design of a network of infrastructure for bicycles. The developed model minimizes the costs of the users whilst considering the costs of creating each type of bike lane and imposing a maximum overall budgetary constraint, which cannot be passed in constructing a network scenario generated by the model.

Nomenclature	
Z	objective function
i,j	network nodes
t	type of bike lane
$T_{i,j}^{t}$	travel time on link 'i'-'j' according to link type 't'
$X_{i,j}^{t}$	flow on link 'i'-'j' according to link type 't'
o,d	origin node and destination node
$M^{o,d}$	elements of the origin-destination matrix
$Cost^{t}$	economic cost per kilometer of constructing link type't'
$L_{i,j}$	length of link 'i'-'j'
B_{max}	maximum budget
$Xod_{i,j}^{o,d,t}$	flow on link 'i'-'j' according to link type 't' for the O-D pair 'o'-'d'
$p_{i,j}$	slope of the link 'i'-'j' in %
$V_{i,j}$	average bicycle speed on the link 'i'-'j'

Each type of lane, *t*, is defined by its own surface, its width and the surface of the area on which it is built, either the road or the sidewalk, being segregated in a greater or lesser way from the movement of the other modes of transport. Each kind of lane *t* supposes a specific cost, but also has a time implication $T_{i,j}^{t}$ required to travel from one node *i* to another *j*. Therefore, the flow on each link $X_{i,j}^{t}$ is influenced by the characteristics of the lane.

The objective function minimizes the cost for the users through the following expression:

$$Z = Min \sum_{i,j,t} T_{i,j}^{t} \cdot X_{i,j}^{t} \tag{1}$$

As indicated by equation 1, the proposed model minimizes the travel time of all the users where the time spent on each link, which depends on the kind of infrastructure, is multiplied by the flow on the link, which will also vary according to the infrastructure. Therefore, we are dealing with a global equilibrium situation that also yields the user's equilibrium since the network is assumed non-congested.

The constraints imposed on the objective function are presented below.

$$\sum_{j,t} Xod_{i,j}^{o,d,t} - \sum_{j,t} Xod_{j,i}^{o,d,t} = \delta_i^o \cdot M^{o,d} - \delta_i^d \cdot M^{o,d}; \forall i,o,d \tag{2}$$

Equation 2 assures the continuity, at each node, of the flow of each origin-destination route. This constraint demands that for each O-D pair, the sum of all the journeys that enter the node is equal to the sum of all the journeys that leave the same node.

$$X_{i,j}^{t} = \sum_{o,d} Xod_{i,j}^{o,d,t}; \forall i,j,t \tag{3}$$

The constraint imposed by expression 3 assures that the flow on each link is the sum of the flows of all the routes with different O-D pairs passing along that link.

$$B_{\max} \geq \sum_{i,j,t} L_{i,j} \cdot Cost^{t} \cdot \left(\frac{X_{i,j}^{t}}{X_{i,j}^{t} + \varepsilon} - 0.5 \cdot \frac{X_{i,j}^{t}}{X_{i,j}^{t} + \varepsilon} \cdot \frac{X_{j,i}^{t}}{X_{j,i}^{t} + \varepsilon} \right) \tag{4}$$

Equation 4 introduces the overall budgetary constraint for building the entire infrastructure network, B_{max}, which must be the same as the sum of the length of each link multiplied by the cost per built linear meter according to the resulting type of lane being built on each link. This calculation is only applied to those links with a flow that is not null. It should also be indicated that bidirectional bike lanes are assumed in all cases. As expressed by equation 4, if a link has a flow in both directions, in other words, from *i* to *j* and from *j* to *i*, half of the cost is assessed for each direction. On the contrary, if two different types of lane were obtained for each direction of the same link, the last multiplication of equation 4 would be zero since the flows involved in such multiplication are assumed of the same type of infrastructure *t*. Therefore, two different types of infrastructure would be assessed for the same link, one for each direction. Consequently, this restriction pushes the lane of both directions to be of the same type since this solution always results in a lower cost. The term ε takes a value of 0.00001 to avoid the denominator being '0' in the case of the flow being null.

$$\sum_{t,t'} X_{i,j}^{t} \cdot X_{i,j}^{t'} = 0; \forall i, j, t \neq t' \qquad (5)$$

Expression 5 avoids more than one type of bike lane for the same direction existing on the same link. This is achieved by demanding that the sum of the multiplication of all combinations of flows concerning different types of infrastructure results zero. Thereby guaranteeing that only one term is different from zero, or, only one type of bike lane is associated to the flow on the link.

3. Application

Most documents and plans referring to bicycle use in urban areas mention different types of bike lanes. An example of this is the description found in the Spanish DGT (Dirección General de Tráfico de España) document, which distinguishes the following kinds of lanes (Traffic law 19/2001, of 19 December): bike lane, protected bike lane, bike lane on sidewalk, bike track and cyclable track.

The DGT typologies mentioned above have been generalized into three kinds of lanes for the calculations made in this research, as presented in Table 1: bike lane on sidewalk, the segregated bike lane on asphalt and an absence of any bike lane. The latter case assumes that the bicycles are ridden on the road with no separation from motorized traffic. Clearly, this alternative does not require any construction or funding.

Table 1. Characterization of the types of bike lanes considered (Liñán et al., 2013)

Type of infrastructure ID	Type of bike lane	Recommended width	Applied width	Required budget (€/m)
1	Road	3 m.	-	-
2	Bike lane on sidewalk	2.5 m.	2.5	200
3	Segregated bike lane on asphalt	2.5 – 2.8 m.	3	250

The bike lane on the sidewalk supposes signage for the space reserved for cyclists, but there is nothing to physically stop pedestrians from using the space, so it is open to possible crossings as well as the use of pedestrians. The bike lane must be able to physically fit onto the sidewalk, so the minimum width of the sidewalk should be greater than the sum of 2.5m of a two-way bike lane, plus the 1.5 m required as the minimum width of a pedestrian sidewalk, in other words, the sidewalk needs to be at least 4 m wide to accept the possible inclusion of a bike lane. One linear meter of bike lane on a sidewalk is assumed to cost 200 €.

The option of a bike lane on asphalt is the only one of the three considered that implies the complete segregation of the bicycles from the other modes of transport. The width of road surface required to build a segregated bike lane would be 3m for a two way bike lane, apart from the 6m minimum required for a two way road, so a total of 9m. This will obviously be the most expensive option and supposes a cost of 250 €/m.

The values that have been assumed for the different characteristics have been defined from various Spanish studies. The widths of the bike lanes in each case are based on the compendium of design criteria for bicycle infrastructure proposed by Santos (2008), Liñán, Merino and Martínez (2013), and the public administrations of Zaragoza (2006), Cataluña (2008) and Madrid (2007).

The mentioned specifications have an influence on the demand of a particular link or another depending on which type of infrastructure has been installed. As has been mentioned already, speed varies as a function of the space available for bicycles mobility and flows are seen to vary with the availability of different alternatives.

Table 2 shows the characteristics of the Sioux Falls network on which the optimization will be evaluated. The model assumes two-way bike lanes in all cases and, as in a real case, the widths of the sidewalks and roads have been considered to be as those presented above. This physical restriction could implicate the impossibility of installing a bike lane on certain links due to the lack of space.

Table 2. Characterization of Sioux Falls network

Origin node	Destination node	Length (km)	Slope (%)	Road width (m)	Sidewalk width (m)	Origin node	Destination node	Length (km)	Slope (%)	Road width (m)	Sidewalk width (m)
1	2	1.1	0	15	5	17	18	0.08	0	15	5
1	3	0.2	5	15	5	18	19	0.35	-2.86	15	5
2	6	0.2	2.5	15	5	19	37	0.3	0	15	5
3	4	0.3	0	15	5	20	21	0.2	-5.00	15	5
3	14	0.25	2	15	5	21	22	0.2	0	15	5
4	5	0.4	-1.25	15	5	21	36	0.2	0	15	5
4	15	0.2	5	15	5	22	23	0.2	0	15	5
5	6	0.4	0	15	5	23	24	0.2	0	7	0
5	9	0.2	0	15	5	23	28	0.4	1.25	15	5
6	8	0.2	5	15	5	23	33	0.2	0	15	5
7	8	0.4	3.75	15	5	25	26	0.2	-2.50	15	5
7	27	0.2	0	15	5	25	27	0.4	-5.00	15	5
8	9	0.4	-2.50	15	5	26	28	0.2	-2.50	15	5
8	25	0.2	2.5	15	5	27	31	0.94	0	15	5
9	10	0.2	0	15	5	28	29	0.14	-3.53	15	5
10	11	0.2	5	15	5	29	30	0.06	0	15	5
10	24	0.2	0	7	0	30	31	0.25	-2.00	15	5
10	25	0.4	3.75	15	5	31	32	0.4	0	15	5
10	26	0.45	2.24	15	5	31	34	0.25	2	15	5
11	12	0.2	5	15	5	32	33	0.25	2	15	5
12	13	0.3	0	15	5	32	38	0.2	0	15	5
12	15	0.2	-2.50	15	5	33	34	0.22	0	15	5
12	20	0.2	-5.00	15	5	33	35	0.2	0	15	5
13	14	0.15	-6.67	15	5	35	36	0.2	0	15	5
13	16	0.25	-4.00	15	5	36	37	0.25	-2.00	15	5
16	17	0.17	-2.96	15	5	37	38	0.2	0	15	5

The influence of the slope on the speed is based on the calibration of the bicycle free flow speed and the slope on the links (Romero, 2013), distinguishing the following expressions based on the steepness of the slopes:

$$V_{i,j} = \begin{cases} 27.296 \cdot e^{0.1072 \cdot p_{i,j}}; if\ p_{i,j} \le -0.92 km/h \\ 20.832 \cdot e^{-0.188 \cdot p_{i,j}}; if\ -0.92 < p_{i,j} \le 6 km/h \\ 3; if\ 6 < p_{i,j} \le 10 km/h \\ 0; if\ p_{i,j} > 10 km/h \end{cases} \tag{6}$$

Table 3 specifies the demand for each origin-destination pair presenting trips.

Table 3. Origin – destination matrix considered.

Origin	Destination	Demand	Origin	Destination	Demand	Origin	Destination	Demand	Origin	Destination	Demand
1	8	6	5	30	14	19	1	23	31	2	24
1	12	14	5	36	9	19	6	17	31	18	7
1	22	5	6	3	8	20	5	14	31	27	16
1	27	15	6	36	8	20	6	23	32	11	8
1	31	20	7	9	14	22	1	8	34	13	13
1	38	7	9	1	13	24	35	11	35	1	17
2	1	16	11	5	8	26	1	3	35	3	16
2	31	14	11	33	25	26	16	19	35	31	8
3	8	12	14	16	5	27	20	12	36	15	9
3	25	4	14	30	34	27	31	12	36	23	14
4	13	3	15	1	4	28	38	13	38	4	14
5	19	16	18	23	8	30	8	13			

Finally, given the above characterizations and the demand presented in Table 3, along with the set of the assumptions described in the previous section, and a maximum budget of 600,000 € for the whole work, the following result is obtained:

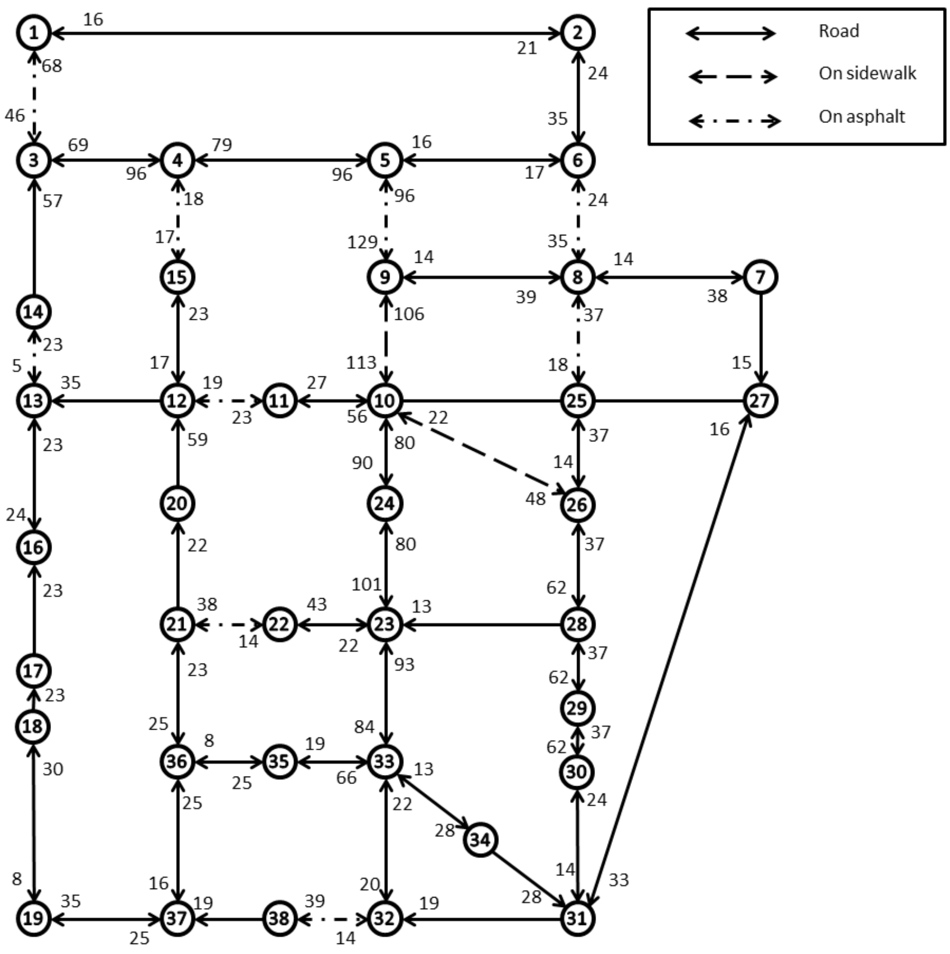

Figure 1.Optimized solution

The optimal solution results in a cost of 567,500 €, making it 32,500 € under the total budget for construction. Obviously, such excess amount does not allow the construction of any length of cycle path on the sidewalk or on asphalt. Figure 1 also shows the resulting flows on each link.

Two links were assumed to be 7 meters wide and to have no sidewalk, therefore bike lanes could not be constructed on such links. As a result of the effect of the slope on the speed, various links do not experiment flow. Such is the case for the links 12 to 20, 20 to 21. On the other hand, the link connecting nodes 10 and 26 captures the flow from the surrounding nodes since it has resulted in a bike lane on the sidewalk, whereas the other links in the same area do not show bike lane. The combination of slope, type of bike lane and demand has lead the previous fact, thereby resulting in zero flow for the link from 10 to 25.

As it can be seen in Figure 1, the model yields different types of infrastructure for consecutive links. Therefore, the application of the proposed model to a real case study would need to consider this aspect according to the morphology of the city.

4. Conclusions

The limited space currently available in urban areas needs to be shared by all modes of transport. Correct mobility management will facilitate the introduction of the bicycle as a normal everyday urban mode of transport. To achieve this goal, towns and cities must provide a cyclable infrastructure as part of their transport network, however, this comes at a cost which is directly linked to user demand and network mobility patterns. These aspects have been considered in the network optimization model proposed in this article which minimizes the cost to the system users, subject to urban and budgetary constraints related to the different kinds of pre-defined possible infrastructure.

The realism of the model is noteworthy in that is guarantees the feasibility of the project whilst considering necessary constraints such as the current legal minimum widths of roads and sidewalks, along with budgetary restrictions which can be set before designing the resulting scenario. A future line of research in this field could be the calibration of improvements in speed depending on the type of infrastructure.

The proposed optimization model provides a real and tangible solution which is necessary to promote the use of bicycles as a sustainable mode of transport making use of the existent network in the metropolitan area of study. The application for the Sioux Falls network approves the general use of the algorithm, which can be applied to any case study. Furthermore, the analysis made from the optimal solution has quantified the effect that a change in the infrastructure has on construction cost, but also on the users' costs, confirming the optimal character of the solution obtained with the algorithm.

Finally, this research leaves open future lines such as the calibration of the improvement on the speed of each type of bike lane compared to the absence of infrastructure dedicated to cycling. Another interesting approach could be based on assuring a certain degree of continuity in the network, this is, prioritizing which consecutive links present the same type of bike lane. Such policy would make the city more attractive for cycling and would contribute to the promotion of sustainable mobility in the city.

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