Розрахункова робота

Варіант 6

1) $2AB-3C$ $A=\begin{pmatrix}2\\0\\-2\end{pmatrix}$ $B\begin{pmatrix}0 & -2 & 3\end{pmatrix}$ $C\begin{pmatrix}-1 & 1 & 1\\4 & -2 & 0\\-2 & 2 & -1\end{pmatrix}$

$$AB=\begin{pmatrix}2\\0\\-2\end{pmatrix}\cdot\begin{pmatrix}0 & -2 & 3\end{pmatrix}=\begin{pmatrix}2\cdot0 & 2(-2) & 2\cdot3\\0\cdot0 & 0\cdot(-2) & 0\cdot3\\-2\cdot0 & -2\cdot(-2) & (-2)\cdot3\end{pmatrix}=\begin{pmatrix}0 & -4 & 6\\0 & 0 & 0\\0 & 4 & -6\end{pmatrix}$$

$$2AB=\begin{pmatrix}0 & -8 & 12\\0 & 0 & 0\\0 & 8 & -12\end{pmatrix}$$

$$3C=\begin{pmatrix}-3 & 3 & 3\\12 & -6 & 0\\-6 & 6 & -3\end{pmatrix}$$

$$2AB-3C=\begin{pmatrix}0 & -8 & 12\\0 & 0 & 0\\0 & 8 & -12\end{pmatrix}-\begin{pmatrix}-3 & 3 & 3\\12 & -6 & 0\\-6 & 6 & -3\end{pmatrix}=$$

$$=\begin{pmatrix}3 & -11 & 9\\-12 & 6 & 0\\6 & 2 & -9\end{pmatrix}$$

2) $$\begin{cases}3x_1+5x_2-x_3+x_4=3\\13x_1+4x_2-3x_3+x_4=0\\-3x_1+8x_2+2x_3-x_4=16\\2x_1+6x_2-x_3+2x_4=4\end{cases}$$

Метод Гаусса

$$\bar{A}=\left(\begin{array}{cccc|c}3 & 5 & -1 & 1 & 3\\13 & 4 & -3 & 1 & 0\\-3 & 8 & 2 & -1 & 16\\2 & 6 & -1 & 2 & 4\end{array}\right)\begin{array}{l}13\,\mathrm{I}-3\,\mathrm{II}\\\mathrm{I}+\mathrm{III}\\2\,\mathrm{I}-3\,\mathrm{IV}\end{array}$$

$$\sim \left(\begin{array}{cccc|c} 3 & 5 & -1 & 1 & 3 \\ 0 & 53 & -4 & 10 & 39 \\ 0 & 13 & 1 & 0 & 19 \\ 0 & -8 & 1 & -4 & -6 \end{array}\right) \underset{8\,\mathrm{II} + 53\,\mathrm{III}}{\xrightarrow{13\,\mathrm{II} - 53\,\mathrm{III}}} \left(\begin{array}{cccc|c} 3 & 5 & -1 & 1 & [illegible] \\ 0 & 53 & -4 & 10 & [illegible] \\ 0 & 0 & -105 & 130 & -500 \\ 0 & 0 & 21 & -132 & [illegible] \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 3 & 5 & -1 & 1 & 3 \\ 0 & 53 & -4 & 10 & 39 \\ 0 & 0 & -21 & 26 & -100 \\ 0 & 0 & 21 & -132 & -6 \end{array}\right) \xrightarrow{\mathrm{III} + \mathrm{IV}} \left(\begin{array}{cccc|c} 3 & 5 & -1 & 1 & 3 \\ 0 & 53 & -4 & 10 & 39 \\ 0 & 0 & -21 & 26 & -100 \\ 0 & 0 & 0 & -106 & -106 \end{array}\right) \to$$

$$\to \begin{cases} 3x_1 + 5x_2 - x_3 + x_4 = 3 \\ 0x_1 + 53x_2 - 4x_3 + 10x_4 = 39 \\ 0x_1 + 0x_2 - 21x_3 + 26x_4 = -100 \\ -106x_4 = -106 \end{cases} \to \begin{cases} 3x_1 + 5x_2 - x_3 = 2 \\ 53x_2 - 4x_3 = 29 \\ -21x_3 = -126 \\ x_4 = 1 \end{cases} \to$$

$$\to \begin{cases} 3x_1 + 5x_2 = 8 \\ 53x_2 = 53 \\ x_3 = 6 \\ x_4 = 1 \end{cases} \to \begin{matrix} x_1 = 1 \\ x_2 = 1 \\ x_3 = 6 \\ x_4 = 1 \end{matrix}$$

Матричний метод

$$\begin{pmatrix} 3 & 5 & -1 & 1 \\ 13 & 4 & -3 & 1 \\ -3 & 8 & 2 & -1 \\ 2 & 6 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 16 \\ 4 \end{pmatrix}$$

$$\underset{A}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}} = \underset{C^{-1}}{\begin{pmatrix} 3 & 5 & -1 & 1 \\ 13 & 4 & -3 & 1 \\ -3 & 8 & 2 & -1 \\ 2 & 6 & -1 & 2 \end{pmatrix}^{-1}} \cdot \underset{B}{\begin{pmatrix} 3 \\ 0 \\ 16 \\ 4 \end{pmatrix}}$$

$$C^{-1} = 1 : \det C \cdot C^{d}$$

$$C^{d} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{pmatrix}^{T}$$

$$\det C = \begin{vmatrix} 3 & 5 & -1 & 1 \\ 13 & 4 & -3 & 1 \\ -3 & 8 & 2 & -1 \\ 2 & 6 & -1 & 2 \end{vmatrix} = 3 \cdot \begin{vmatrix} 4 & -3 & 1 \\ 8 & 2 & -1 \\ 6 & -1 & 2 \end{vmatrix} -$$

$$-5 \begin{vmatrix} 13 & -3 & 1 \\ -3 & 2 & -1 \\ 2 & -1 & 2 \end{vmatrix} + (-1) \begin{vmatrix} 13 & 4 & 1 \\ -3 & 8 & -1 \\ 2 & 6 & 2 \end{vmatrix} -$$

$$-1 \begin{vmatrix} 13 & 4 & -3 \\ -3 & 8 & 2 \\ 2 & 6 & -1 \end{vmatrix} = 3 \cdot \overset{58}{(16-8+18-12-4+48)} -$$

$$-5 \overset{26}{(52+3+6-4-13-18)} - \overset{268}{(208-18-8-16+78+24)} -$$

$$- \overset{-154}{(-104+54+16+48-156-12)} = 174-130-268+154 =$$

$$= -70$$

$$C_{11} = \begin{vmatrix} 4 & -3 & 1 \\ 8 & 2 & -1 \\ 6 & -1 & 2 \end{vmatrix} = 16-8+18-12-4+48 = 58$$

$$C_{12} = -\begin{vmatrix} 13 & -3 & 1 \\ -3 & 2 & -1 \\ 2 & -1 & 2 \end{vmatrix} = -(52+3+6-4-13-18) = -26$$

$$C_{13} = \begin{vmatrix} 13 & 4 & 1 \\ -3 & 8 & -1 \\ 2 & 6 & 2 \end{vmatrix} = (208-18-8-16+78+24) = 268$$

$$C_{14} = -\begin{vmatrix} 13 & 4 & -3 \\ -3 & 8 & 2 \\ 2 & 6 & -1 \end{vmatrix} = -(-154) = 154$$

$$C_{21} = -\begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & -1 \\ 6 & -1 & 2 \end{vmatrix} = -(20-8+6-12-5+16) = -17$$

$$C_{22} = \begin{vmatrix} 3 & -1 & 1 \\ -3 & 2 & -1 \\ 2 & -1 & 2 \end{vmatrix} = (12+3+2-4-3-6) = 4$$

$$C_{23} = -\begin{vmatrix} 3 & 5 & 1 \\ -3 & 8 & -1 \\ 2 & 6 & 2 \end{vmatrix} = -(48-18-10-16+18+30) = -52$$

$$C_{24} = \begin{vmatrix} 3 & 5 & -1 \\ -3 & 8 & 2 \\ 2 & 6 & -1 \end{vmatrix} = (-24+18+20+16-36-15) = -21$$

$$C_{31} = \begin{vmatrix} 5 & -1 & 1 \\ 4 & -3 & 1 \\ 6 & -1 & 2 \end{vmatrix} = (-30-4-6+18+5+8) = -9$$

$$C_{32} = -\begin{vmatrix} 3 & -1 & 1 \\ 13 & -3 & 1 \\ 2 & -1 & 2 \end{vmatrix} = -(-18-2-13+6+3+26) = -2$$

$$C_{33} = +\begin{vmatrix} 3 & 5 & 1 \\ 13 & 4 & 1 \\ 2 & 6 & 2 \end{vmatrix} = +(24+78+10-8-18-130) = -44$$

$$C_{34} = -\begin{vmatrix} 3 & 5 & -1 \\ 13 & 4 & -3 \\ 2 & 6 & -1 \end{vmatrix} = -(-12-78-30+8+65+54) = -7$$

$$C_{41} = -\begin{vmatrix} 5 & -1 & 1 \\ 4 & -3 & 1 \\ 8 & 2 & -1 \end{vmatrix} = -(15+8-8+24-10-4) = -25$$

$$C_{42} = \begin{vmatrix} 3 & -1 & 1 \\ 13 & -3 & 1 \\ -3 & 2 & -1 \end{vmatrix} = (9+26+3-9-6-13) \quad 10$$

$$C_{43} = -\begin{vmatrix} 3 & 5 & 1 \\ 13 & 4 & 1 \\ -3 & 8 & -1 \end{vmatrix} = -(-12+104-15+12-24+65) = -130$$

$$C_{44} = \begin{vmatrix} 3 & 5 & -1 \\ 13 & 4 & -3 \\ -3 & 8 & 2 \end{vmatrix} = (24-104+45-12+72-130) = -105$$

в матрицу

$(-1)^{i+j}$

$$C^{*}=\begin{pmatrix}58 & -26 & 268 & 154\\ -17 & 4 & -52 & -21\\ -9 & -2 & -44 & -7\\ -25 & 10 & -130 & -105\end{pmatrix}^T$$

$$C^{*}=\begin{pmatrix}58 & -17 & -9 & -25\\ -26 & 4 & -2 & 10\\ 268 & -52 & -44 & -130\\ 154 & -21 & -7 & -105\end{pmatrix}$$

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}-\frac{58}{-70} & \frac{17}{70} & \frac{9}{70} & \frac{25}{70}\\ \frac{26}{70} & -\frac{4}{70} & \frac{2}{70} & -\frac{10}{70}\\ -\frac{268}{70} & \frac{52}{70} & \frac{44}{70} & \frac{130}{70}\\ -\frac{154}{70} & \frac{21}{70} & \frac{7}{70} & \frac{105}{70}\end{pmatrix}\cdot\begin{pmatrix}3\\0\\16\\4\end{pmatrix}$$

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}\frac{-58\cdot3+17\cdot0+9\cdot16+25\cdot4}{70}\\ \frac{26\cdot3-4\cdot0+2\cdot16-10\cdot4}{70}\\ \frac{-268\cdot3+52\cdot0+44\cdot16+130\cdot4}{70}\\ \frac{-154\cdot3+21\cdot0+7\cdot16+105\cdot4}{70}\end{pmatrix}$$

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}\frac{70}{70}\\ \frac{70}{70}\\ \frac{420}{70}\\ \frac{70}{70}\end{pmatrix}\rightarrow\begin{pmatrix}1\\1\\6\\1\end{pmatrix}\rightarrow$$

$x_1=1$

$x_2=1$

$x_3=6$

$x_4=1$

③ $A(-2;1;0)$ $B(3;2;7)$ $C(4;-1;2)$ $D(-6;1;5)$

1) $\overline{AB} = (3+2;\ 2-1;\ 7-0) = (5;1;7)$

$|\overline{AB}| = \sqrt{5^2+1^2+7^2} = \sqrt{75} = 5\sqrt{3}$

2) $\overline{BC} = (4-3;\ -1-2;\ 2-7) = (1;-3;-5)$

$\text{Пр}_{\overline{BC}}\overline{AB} = \dfrac{(\overline{AB}, \overline{BC})}{|\overline{BC}|} = \dfrac{5\cdot1-1\cdot3-7\cdot5}{\sqrt{1^2+(-3)^2+(-5)^2}} = \dfrac{-33}{\sqrt{35}} =$

$= \dfrac{-33\sqrt{35}}{35}$

3) $\overline{AC} = (4+2;\ -1-1;\ 2-0) = (6;-2;2)$

$\cos(\widehat{\overline{AB}\,\overline{AC}}) = \dfrac{(\overline{AB}, \overline{AC})}{|\overline{AB}|\,|\overline{AC}|} = \dfrac{5\cdot6-1\cdot2+7\cdot2}{\sqrt{5^2+1^2+7^2}\ \sqrt{6^2+(-2)^2+2^2}} =$

$= \dfrac{42}{\sqrt{75}\cdot\sqrt{44}} = \dfrac{7\cdot\sqrt{33}}{55}$

$(\widehat{\overline{AB}\,\overline{AC}}) = \arccos\dfrac{7\cdot\sqrt{33}}{55}$

4) $S_{\Delta ABC} = \dfrac{1}{2}\,|\overline{AB}\times\overline{BC}|$

$\overline{AB}\times\overline{BC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 5 & 1 & 7 \\ 1 & -3 & -5 \end{vmatrix} = \bar{i}\begin{vmatrix} 1 & 7 \\ -3 & -5 \end{vmatrix} - \bar{j}\begin{vmatrix} 5 & 7 \\ 1 & -5 \end{vmatrix} +$

$+\bar{k}\begin{vmatrix} 5 & 1 \\ 1 & -3 \end{vmatrix} = 16i + 32j - 16k$

$S_{\Delta ABC} = \dfrac{1}{2}\sqrt{16^2+32^2+16^2} = \dfrac{1\cdot16}{2}\sqrt{1^2+2^2+1^2} = 8\sqrt{6}$ кв.од.

5) $\overline{AD} = (-6+2;\ 1-1;\ 5-0) = (-4;0;5)$

$$(\overline{AB},\overline{AC},\overline{AD}) = \begin{vmatrix}5 & 1 & 7\\ 6 & -2 & 2\\ -4 & 0 & 5\end{vmatrix} = -50+0-8-56-0-30 = -144$$

$$V_{ABCD} = \frac{1}{6}\left|(\overline{AB},\overline{AC},\overline{AD})\right| = 24 \text{ куб. один.}$$

6) $\overline{CD} = (-6-4;\ 1+1;\ 5-2) = (-10;2;3)$

$$(\overline{AD},\overline{BC},\overline{CD}) = \begin{vmatrix}-4 & 0 & 5\\ 1 & -3 & -5\\ -10 & 2 & 3\end{vmatrix} = 36+10+0-150-40-0-\cancel{144}$$

$$= -144 < 0$$

Висновок: трійка векторів ~~[illegible]~~ є лівою ~~[illegible]~~

④ $$V = \begin{pmatrix}3 & 5\\ 4 & 2\\ 2 & 3\\ 6 & 3\end{pmatrix} \cdot \begin{pmatrix}100\\ 130\end{pmatrix} = \begin{pmatrix}3\cdot 100+5\cdot 130\\ 4\cdot 100+2\cdot 130\\ 2\cdot 100+3\cdot 130\\ 6\cdot 100+3\cdot 130\end{pmatrix} = \begin{pmatrix}950\\ 660\\ 590\\ 990\end{pmatrix}$$

$$C = \begin{pmatrix}8 & 10 & 12 & 15\end{pmatrix} \cdot \begin{pmatrix}950\\ 660\\ 590\\ 990\end{pmatrix} = (8\cdot 950+10\cdot 660+12\cdot 590+$$

$$+15\cdot 990) = 7600+6600+$$

$$+7080+14850 = \cancel{36130}$$

$$= 36130$$

(5) $p: 4x - y + 5z - 3 = 0;$ $M(1, 3, -1)$

$n = (4; -1; 5)$

$p_1: 4(x-1) - 1(y-3) + 5(z+1) = 0$

$4x - 4 - y + 3 + 5z + 5 = 0$

$4x - y + 5z + 4 = 0$

$$d = \frac{|4 \cdot 1 + (-1) \cdot 3 + 5(-1) - 3|}{\sqrt{4^2 + (-1)^2 + 5^2}} = \frac{|4 - 3 - 5 - 3|}{\sqrt{42}} = \frac{7}{\sqrt{42}} = \frac{7\sqrt{42}}{42} =$$

$$= \frac{\sqrt{42}}{6}$$

(6) $L_1: -x + 2y - 7 = 0$ $L_2: -2x - 2y + 7 = 0$

$n_1 = (-1; 2)$ $n_2(-2; -2)$

$\frac{-1}{2} \neq \frac{-2}{2} \rightarrow L_2 \nparallel L_2$

$$\cos\varphi = \frac{-1 \cdot 2 + (-2) \cdot (-2)}{\sqrt{1+4} \cdot \sqrt{4+4}} = \frac{2}{\sqrt{40}} = \frac{\sqrt{10}}{10}$$

$\varphi = \arccos\left(\frac{\sqrt{10}}{10}\right)$

$$\begin{cases} -x + 2 \cdot y - 7 = 0 \\ -2x - 2 \cdot y - 7 = 0 \end{cases}$$

$-3x - 14 = 0$

$-3x = 14$

$x = -4\frac{2}{3}$

$2y = -4\frac{2}{3} + 7$

$2y = 2\frac{1}{3}$

$y = \frac{7}{6}$

$M\left(-4\frac{2}{3}; \frac{7}{6}\right)$

(7)

(8)

$\frac{7\sqrt{42}}{42} =$

(7) $\frac{x-2}{2} = \frac{y+8}{1} = \frac{z+1}{-1}$ $\quad 2x+3y-5z+2=0$

$\frac{x-2}{2} = \frac{y+8}{1} = \frac{z+1}{-1} = t$

$x = 2+2t$

$y = -8+1t$

$z = -1-1t$

$2(2+2t)+3(-8+1t)-5(-1-1t)+2=0$

$4+4t-24+3t+5+5t+2=0$

$12t-13=0$

$t = \frac{13}{12}$

$x = \frac{25}{6} \quad y = \frac{-83}{12} \quad z = \frac{-25}{12}$

$\left(\frac{25}{6}; \frac{-83}{12}; \frac{-25}{12}\right)$ – точка перетину

(8) $\frac{x^2}{36} - \frac{y^2}{9} = 1$

$c^2 - a^2 = b^2$

$c = \sqrt{a^2+b^2} = \sqrt{45} = 3\sqrt{5}$

координати фокусів $F_1(3\sqrt{5}; 0)$

$F_2(-3\sqrt{5}; 0)$

$e = \frac{\sqrt{a^2+b^2}}{a} = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$

$$y = \pm\frac{b}{a}x = \pm\frac{3}{6}x = \pm 2x$$

$$d_1: \quad x = \frac{a}{e} = \frac{6}{\frac{\sqrt{5}}{2}} = \frac{12}{\sqrt{5}} = \frac{12\sqrt{5}}{5}$$

$$d_2: \quad x = \frac{-12\sqrt{5}}{5}$$