Learning Module 2: Portfolio Risk & Return: Part II

LOS 2i: calculate and interpret the Sharpe ratio, Treynor ratio, M2, and Jensen's alpha

Sharpe Ratio

The Sharpe Ratio is the portfolio risk premium divided by the portfolio risk.

$$\text{Sharpe ratio} = \frac{E(R_p) - R_f}{\sigma_p}$$

The Sharpe ratio, or reward-to-variability ratio, is the slope of the capital allocation line (CAL). The greater the slope (higher number), the better the asset. Note that the risk being used is the total risk of the portfolio, not its systematic risk, which is a limitation of the measure. The portfolio with the highest Sharpe ratio has the best performance, but the Sharpe ratio is not informative. In order to rank portfolios, the Sharpe ratio for each portfolio must be computed.

Further limitation occurs when the numerators are negative. In this instance, the Sharpe ratio will be less negative for a riskier portfolio, resulting in incorrect rankings.

Treynor Ratio

The Treynor ratio is an extension of the Sharpe ratio. Instead of using total risk, Treynor uses beta or systematic risk in the denominator.

$$\text{Treynor ratio} = \frac{E(R_p) - R_f}{\beta_p}$$

As with the Sharpe ratio, the Treynor ratio requires positive numerators to give meaningful comparative results. Apart from this, the Treynor ratio does not work for negative beta assets. Also, while both the Sharpe and Treynor ratios can rank portfolios, they do not provide information on whether the portfolios are better than the market portfolio. Similarly, they do not offer information about the degree of superiority of a higher ratio portfolio over a lower ratio portfolio.

M-Squared (M^2) Ratio

The concept behind the M^2 ratio is to create a portfolio P' that mimics the risk of the market portfolio by altering the weights of the actual portfolio P and the risk-free asset until portfolio P' has the same total risk as the market. The return on the mimicking portfolio P' is determined and compared with the market return.

The weight in portfolio P (w_p), which sets the portfolio risk equal to the market risk, can be written as:

$$w_p = \frac{\sigma_m}{\sigma_p}$$

With the balance (1 - w_p) invested in the risk-free asset.

The return for the mimicking portfolio P' is as follows:

$$R_{p'} = w_p R_p + (1 - w_p) R_f$$

Which we can reformulate as:

$$R_{p'} = \frac{\sigma_m}{\sigma_p} \times R_p + \left(1 - \frac{\sigma_m}{\sigma_p}\right) \times R_f$$

Therefore,

$$R_{p'} = R_f + \sigma_m \frac{[R_p - R_f]}{\sigma_p}$$

The difference in return between the mimicking portfolio and the market return is M^2 which is expressed as:

$$M^2 = [R_p - R_f] \frac{\sigma_m}{\sigma_p} + R_f = SR \times \sigma_m + R_f$$

A portfolio that matches the market's return will have an M^2 value equal to zero, while a portfolio that outperforms will have a positive value. By using the M^2 measure, it is possible to rank portfolios and also determine which portfolios beat the market on a risk-adjusted basis.

Jensen's Alpha

Jensen's alpha is based on systematic risk. The daily returns of the portfolio are regressed against the daily returns of the market. Essentially, this is done in order to compute a measure of this systematic risk in the same manner as the CAPM. The difference between the actual return of the portfolio and the calculated or modeled risk-adjusted return is a gauge of performance relative to the market.

$$\text{Jensen's alpha} = \alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$$

If \alpha$_p$ is positive, the portfolio has outperformed the market, while a negative value indicates underperformance. The alpha values can also be used to rank portfolios or the managers of those portfolios, with the alpha being a representation of the maximum amount an investor should pay for the active management of that portfolio.

Question

A client has three portfolio choices, each with the following characteristics:

	Expected Return	Volatility	Beta
Portfolio A	15%	12%	10%
Portfolio B	18%	14%	11%
Portfolio C	12%	9%	5%

The efficient market portfolio has an expected return of 20%, a standard deviation of 12%, and a risk-free interest rate of 5%.

Based on the Sharpe ratio for each portfolio, the client should choose:

1. Portfolio A.
2. Portfolio B.
3. Portfolio C.

Solution

The correct answer is portfolio **B**.

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

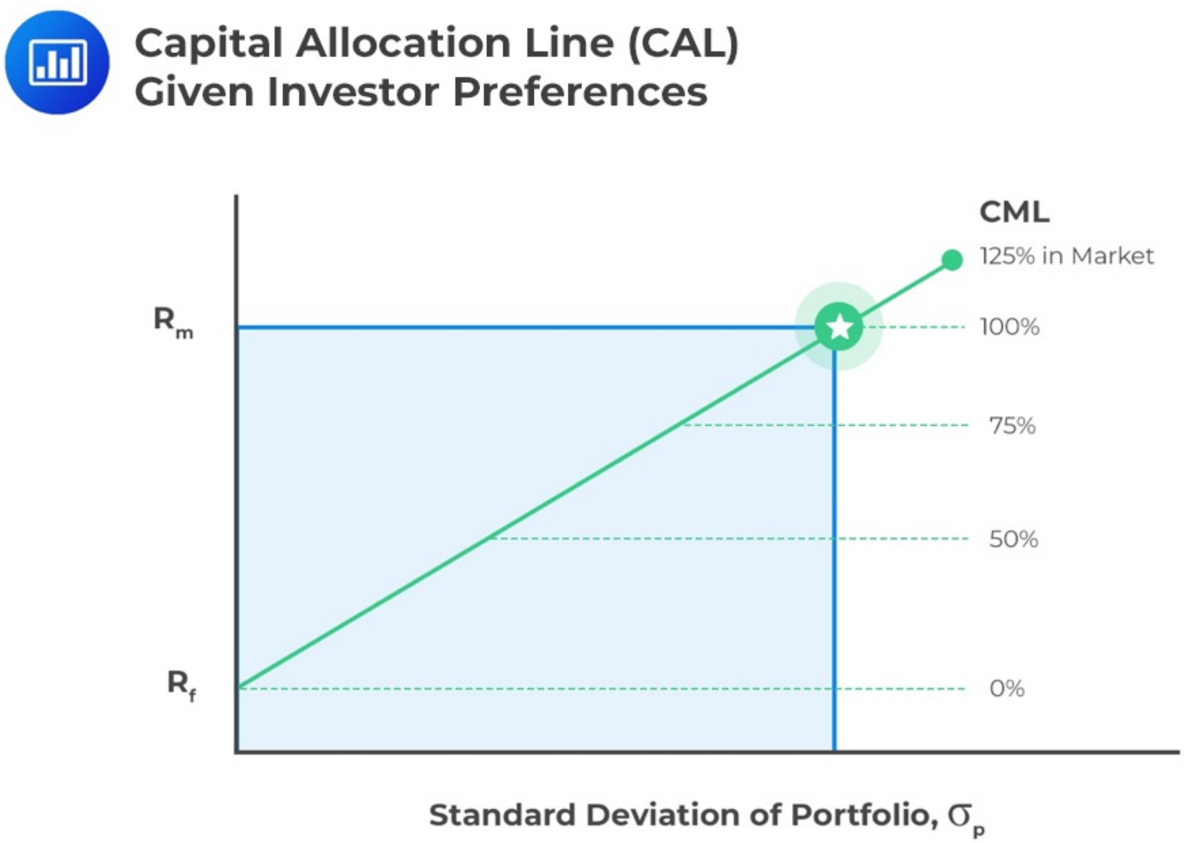
The portfolio with the highest Sharpe ratio has the best performance.

	Calculation	Sharpe Measure
Portfolio A	$(15\% - 5\%)/12\%$	0.83
Portfolio B	$(18\% - 5\%)/14\%$	0.93
Portfolio C	$(12\% - 5\%)/9\%$	0.77

Note: The Sharpe ratio uses total risk, not just the systematic risk of a portfolio (as represented by beta). Further, note that the information about the efficient market portfolio is useless in this case.

LOS 2a: describe the implications of combining a risk-free asset with a portfolio of risky assets

By combining a portfolio of risky assets with a risk-free asset, we can improve the return-risk characteristics of the portfolio and realize a better trade-off. This combination is called the capital allocation line (CAL). The proportion of allocation to risky assets versus allocation to risk-free assets will be dependent on the risk preferences of the investor.



Combined Portfolios

We calculate the expected return of a mixed portfolio by adding up the expected returns of its components. To assess the portfolio's risk, we need the allocation to each component, the standard deviation of each, and the correlation between them. When the assets aren't perfectly correlated, the portfolio's variance will be lower than that of its individual assets.

Each investor will have their own risky portfolio, dependent on the assumptions they make on the likely performance of the underlying assets. Since every investor makes their own unique assumptions about the underlying assets, there is no one optimal risk portfolio. We need to make a simplifying assumption that investor expectations are homogenous in order to derive what would be the optimal market portfolio.

Question

If two underlying assets are negatively correlated, combining them to form a portfolio will make the portfolio risk:

A. Increase.

B. Decrease.

C. Remain constant.

Solution

The correct answer is **B**.

When uncorrelated assets are combined to form a portfolio, a lack of correlation will reduce the overall portfolio volatility.

LOS 2b: explain the capital allocation line (CAL) and the capital market line (CML)

We form a capital allocation line when we combine a risky asset portfolio with a risk-free asset. This represents the allocation between risk-free and risky assets based on investor risk preferences. The capital market line is a special case of the CAL, where the portfolio of risky assets is the market portfolio.

Passive and Active Portfolios

A market can be informationally efficient. In such a case, the quoted security price in the market is an unbiased estimate of all the future discounted cash flows and reflects all publicly known information about the security. If all security prices reflect all publicly available information, then, in theory, there is no way to outperform the market. If this is the investor's belief, then investing in a passive portfolio is the simplest and most convenient approach. A passive portfolio will track and replicate the market.

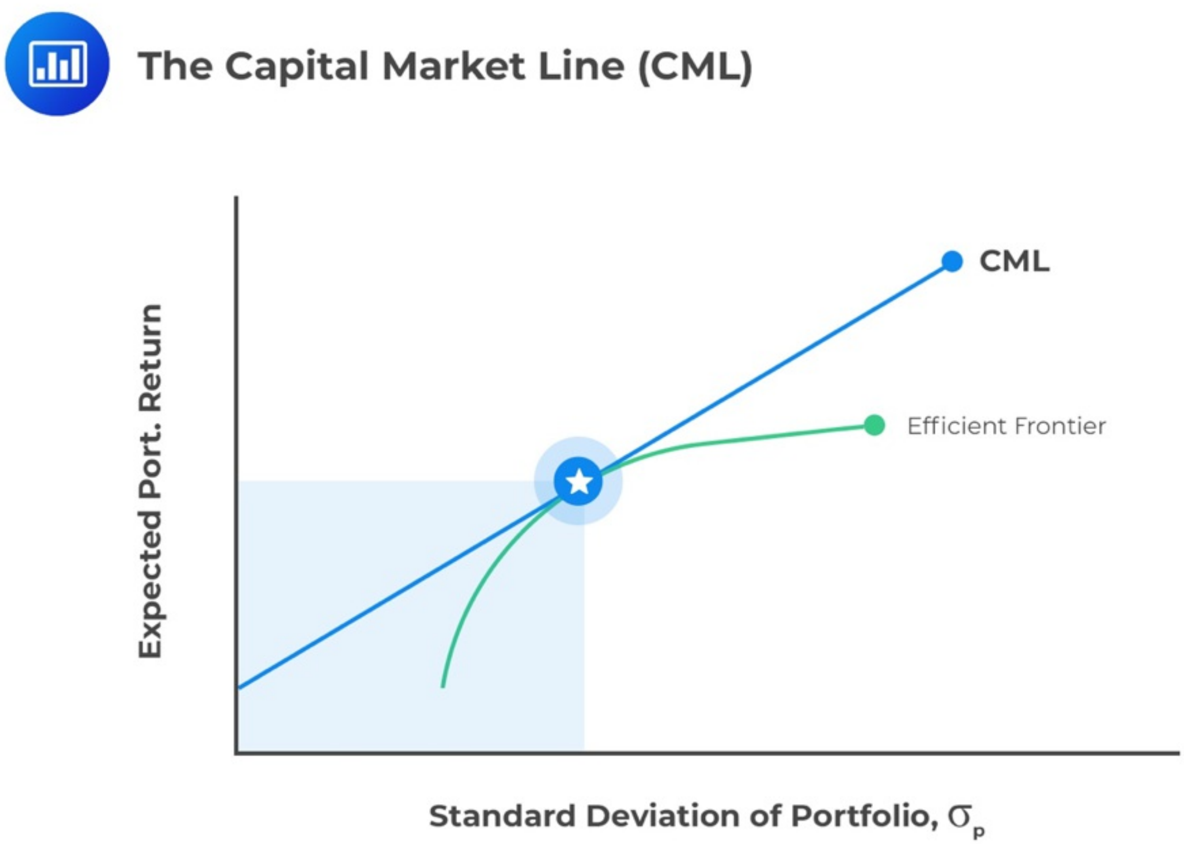
Many investors do not believe the market price accurately reflects valuations. They have confidence in their ability to determine these mispricings based on their evaluation models. Such investors take an active approach to investing and overweighting undervalued assets and underweighting (or shorting, if allowed) overvalued assets. This style of investing is called active management.

The Market

The market includes all risky assets or anything that has value – stocks, bonds, real estate, human capital, and commodities. These assets are all defined in "the market." Not all market assets are tradable or investable. If global assets are considered, hundreds of thousands of individual securities make up the market and are considered tradable and investable. A typical investor is likely to rely on their local or regional stock market as a measure of "the market".

The Capital Market Line (CML)

The Capital Market Line (CML) is a special case of the CAL, that is, the line that makes up the allocation between a risk-free asset and a risky portfolio for an investor. In the case of the CML, the risk portfolio is the market portfolio. Where an investor has defined "the market" to be their domestic stock market index, the expected return of the market is expressed as the expected return of that index. The risk-return characteristics for the potential risk asset portfolios can be plotted to generate a Markowitz efficient frontier. The point at which the line from the risk-free asset touches or is tangential to the Markowitz portfolio is defined as the market portfolio. The line connecting the risk-free asset with the market portfolio is the CML.



The expected return and variance for the portfolio can be represented as follows:

$$\text{Expected return} = E(R_P) = wR_f + (1 - w)E(R_m)$$

$$\text{Variance} = \sigma_P^2 = w^2\sigma_f^2 + (1 - w)^2\sigma_m^2 + 2w(1 - w)\text{Cov}(R_f, R_m)$$

Where:

R_f is the return on the risk-free asset.

R_m is the return on the market.

w is the weight of the risk-free asset in the portfolio.

$1 - w$ is the weight of the market asset in the portfolio.

Theoretically, the standard deviation of the risk-free asset is zero, and the term, $w^2\sigma_f^2$ falls out of the equation. Equally, the risk-free asset is assumed to have no covariance with the market portfolio. This means that the portfolio standard deviation is written as:

$$\sigma_p = (1 - w)\sigma_m$$

By substitution, we can write:

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m} \times \sigma_p$$

This is in the form of an equation of a straight line where the intercept is $R_{f,}$ and the slope is $\frac{E(R_m) - R_f}{\sigma_m}$. This is the CML line which has a positive slope as the market return is greater than the risk-free return.

Question

What happens to the portfolio risk and return, respectively, as an investor moves up the CML?

A. Risk decreases, return decreases.

B. Risk increases, return decreases.

C. Risk increases, return increases.

Solution

The correct answer is **C**.

The overall portfolio risk and return increase as an investor moves up the CML.

LOS 2c: explain systematic and nonsystematic risk, including why an investor should not expect to receive additional return for bearing nonsystematic risk

Systematic risk is inherent in the overall market and cannot be avoided. Non-systematic risk is limited to a particular asset class or security and can be avoided through appropriate portfolio diversification.

Systematic Risk

When you invest in a market, you face systematic risk. This risk is tied to market conditions like interest rates, inflation, and politics, among others. You can't escape systematic risk, but if you use leverage, it can make it even riskier.

Non-systematic Risk

Non-systematic risk is limited to a particular asset class or security and is a function of the "idiosyncrasies" of a particular asset. Investors can avoid non-systematic risk through portfolio diversification. A diversified portfolio reduces exposure or reliance on any one underlying security or asset class.

Pricing of Risk

When an asset has both systematic and non-systematic risk, and we expect to be compensated for both, it makes sense to diversify. Diversification means spreading your investments across different assets that don't move together. This way, you can reduce or eliminate the non-systematic risk, leaving you with only the systematic risk.

Even if an investor eliminates non-systematic risk, they wouldn't be compensated. If they kept adding more non-systematic risk, they'd eventually get zero expected return. So, we can't assume investors will be rewarded for non-systematic, diversifiable risk. In an efficient market, there's no extra reward for taking this kind of risk.

Therefore, only the systematic risk is priced and compensated for, while non-systematic risk does not generate any return. It is, therefore, an investor's interest to diversify the non-systematic risk element within a portfolio.

Question

Which statement *best* describes systematic risk?

A. Systematic risk can be diversified, and investors are not compensated for this risk.

B. Systematic risk cannot be diversified, and investors are compensated for this risk.

C. Systematic risk can be diversified, and investors are not compensated for this risk.

Solution

The correct answer is **B**.

You can't diversify systematic risk because it's market-related. Investors get rewards for taking systematic risks. However, they don't get compensated for non-systematic, diversifiable risk, which they should spread across various assets in their portfolios.

LOS 2d: explain return generating models (including the market model) and their uses

A return-generating model can provide investors with an estimate of the return of a particular security given certain input parameters. The most general form of a return-generating model is a multi-factor model. In its simplest form, the multi-factor model is the single index model, a common implementation that gives the market model.

Multi-factor Models

A multi-factor model is a financial model that employs multiple factors in its calculations to explain asset prices. These models introduce uncertainty stemming from multiple sources. CAPM, on the other hand, limits risk to one source – covariance with the market portfolio. Multi-factor models can be used to calculate the required rate of return for portfolios as well as individual stocks.

CAPM uses one factor, the market factor, to determine the required return. However, the market factor can further be split into different macroeconomic factors. These may include inflation, interest rates, business cycle uncertainty, etc.

A factor can be defined as a variable that explains the expected return of an asset.

A factor beta is a measure of the sensitivity of a given asset to a specific factor. The bigger the factor, the more sensitive the asset is to that factor.

A multi-factor appears as follows:

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{ik}F_k + e_i$$

Where:

R_i= Rate of return on stock i.

$E(R_i)$= Expected return on stock i.

β_{ik}= Sensitivity of the stock's return to a one unit change in factor k.

F_k= Macroeconomic factor k.

e_i= The firm-specific return or portion of the stock's return unexplained by macro factors.

The expected value of the firm-specific return is always zero.

Calculating the Expected Return of an Asset Using a Multi-factor Model

Assume that the common stock of BRL is examined using a multi-factor model based on two factors: unexpected percentage change in GDP and unexpected percentage change in interest rates. Further, assume that the following data is provided:

- Expected return for BRL = 10%.
- GDP factor beta = 1.50.
- Interest rate factor beta = 2.0.
- Expected GDP growth = 2%.
- Expected growth in interest rates = 1%.

Compute the required rate of return on BRL stock, assuming there’s no new information regarding firm-specific events.

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2$$

$$= 10\% + 1.5 \times 2\% + 2.0 \times 1\%$$

$$= 15\%$$

Three and Four-factor Models

One widely used multi-factor model that has been developed in recent times is the Fama and French three-factor model. A major weakness of the multi-factor model is that it is silent on the issue of the appropriate risk factors for use. The FF three-factor model puts three factors forward:

- Size of firms.
- Book-to-market values.
- Excess return on the market.

The firm size factor, also known as SMB (small minus big), is equal to the difference in returns between portfolios of small and big firms ($R_s - R_b$).

The book-to-market value factor, also known as HML (high minus low), is equal to the difference in returns between portfolios of high and low book-to-market firms ($R_H - R_L$).

Note: book-to-market value is book value per share divided by the stock price.

This begs the question: Why SMB and HML?

Fama and French put forth the argument that returns are higher on small versus big firms as well as on high versus low book-to-market firms. This argument has indeed been validated through historical analysis. Fama and French contend that small firms are inherently riskier than big ones, and high book-to-market firms are inherently riskier than low book-to-market firms.

The equation for the Fama-French three-factor model is:

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

The intercept term, α_i, equals the abnormal performance of the asset after controlling for its exposure to the market, firm size, and book-to-market factors. As long as the market is in equilibrium, the intercept should be equal to zero, assuming the three factors adequately capture all systematic risks.

Exam tip: SMB is a hedging strategy – long small firms, and short big firms. HML is also a hedging strategy – long high book-to-market firms and short low book-to-market firms.

The Single Index Model

The simplest return-generating model contains a single factor – the market factor. It looks much like the Capital Market Line.

$$E(R_i) - R_f = \frac{\sigma_i}{\sigma_m} \times [E(R_m - R_f]$$

The factor weight $\frac{\sigma_i}{\sigma_m}$ reflects the ratio of the security risk to the market risk.

The Market Model

The market model is a common implementation of the single index model. The market or index return is used as the single factor. The market model is constructed as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Where $\alpha_i = R_f(1 - \beta)$

The historical relationship between security returns and market returns is used to estimate the beta or slope coefficient.

Decomposition of Risk

The systematic and non-systematic risk can be decomposed using a single index model.

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f]$$

Instead of using the expected returns of the market, E(R_m), we can use realized returns. The difference between the expected return and the realized return is attributable to non-market changes and is represented as an error term e_i.

$$R_i - R_f = \beta_i[R_m - R_f] + e_i$$

The variance of realized returns is expressed as follows:

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_e^2 + 2\text{Cov}(R_m, e_i)$$

We can drop the term $2\text{Cov}(R_m, e_i)$ because any non-market return is by definition, uncorrelated with the market. In fact, this leaves:

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_e^2$$

Which states that total variance (σ_i^2) is equal to systematic variance ($\beta_i^2\sigma_m^2$) and non-systematic variance (σ_e^2).

Question

If the beta of a security is 1.3, the risk-free rate is 2%, and the market expected return is 8%, use the market model to calculate the expected return for the security. (Ignore error terms.)

A. 8.4%.

B. 12.4%.

C. 9.8%.

Solution

The correct answer is **C**.

The market model is given as follows:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Where: $\alpha_i = R_f(1 - \beta)$

$$\begin{aligned} R_i &= 2\%(1 - 1.3) + 1.3(8\%) = -0.6\% + 10.4\% \\ &= 9.8\% \end{aligned}$$

LOS 2e: calculate and interpret beta

Beta is a measure of systematic risk. Statistically, it depends on the degree of correlation between a security and the market.

Calculating Beta

We begin with the single index model using realized returns constructed as follows:

$$R_i - R_f = \beta_i[R_i - R_f] + e_i$$

Which we can also formulate as:

$$R_i = (1-\beta_i)R_f + \beta_i \times R_m + e_i$$

Systematic risk depends on the correlation between the asset and the market. Therefore, beta can be measured by examining the covariance between R_i and R_m:

$$\text{Cov}(R_i, R_m) = \text{Cov}(\beta_i \times R_m + e_i, R_m)$$

$$\text{Cov}(R_i, R_m) = \beta_i \text{Cov}(R_m, R_m) + \text{Cov}(e_i, R_m)$$

$$\text{Cov}(R_i, R_m) = \beta_i \sigma_m^2 + 0$$

Note: $Cov(e_i, R_m) = 0$ because the error term is uncorrelated with the market. By rearranging the equation to solve beta, we have:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2}$$

Where $\text{Cov}(R_i, R_m) = \rho_{i,m}\sigma_i\sigma_m$ which, when substituted into the equation, simplifies it to $\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$.

Beta provides a measurement of the sensitivity of the asset returns to the market as a whole. Aside from this, it captures the portion of the asset risk that cannot be diversified.

Estimating Beta

The variances and correlations required to calculate beta are usually determined using the historical returns for the asset and market. A regression analysis can be performed. The analysis essentially plots the market returns on the x-axis and the security returns on the y-axis and then finds the "best fit" straight line through these points. The slope of the regression line is the measure of beta. Using return data over the prior 12 months tends to represent the security's current level of systematic risk. However, this approach may be less accurate than a beta measured over 3 to 5 years, given that a short-term event may impact the data.

It is important to recognize that irrespective of the data period, beta is an estimate of systematic risk based on historical data and may not represent future systematic risk.

Interpreting Beta

A positive beta indicates that the asset moves in the same direction as the market, whereas a negative beta indicates the opposite.

The beta of a risk-free asset is zero because the covariance of the risk-free asset and the market is zero. The market's beta is, by definition, 1, and most developed market stocks tend to exhibit high, positive betas.

Question

If the correlation between an asset and the market is 0.6, the standard deviation of the asset is 18%, and the standard deviation of the market is 14%, what is the asset beta?

A. 0.77.

B. 0.47.

C. 0.99.

Solution

The correct answer is **A**.

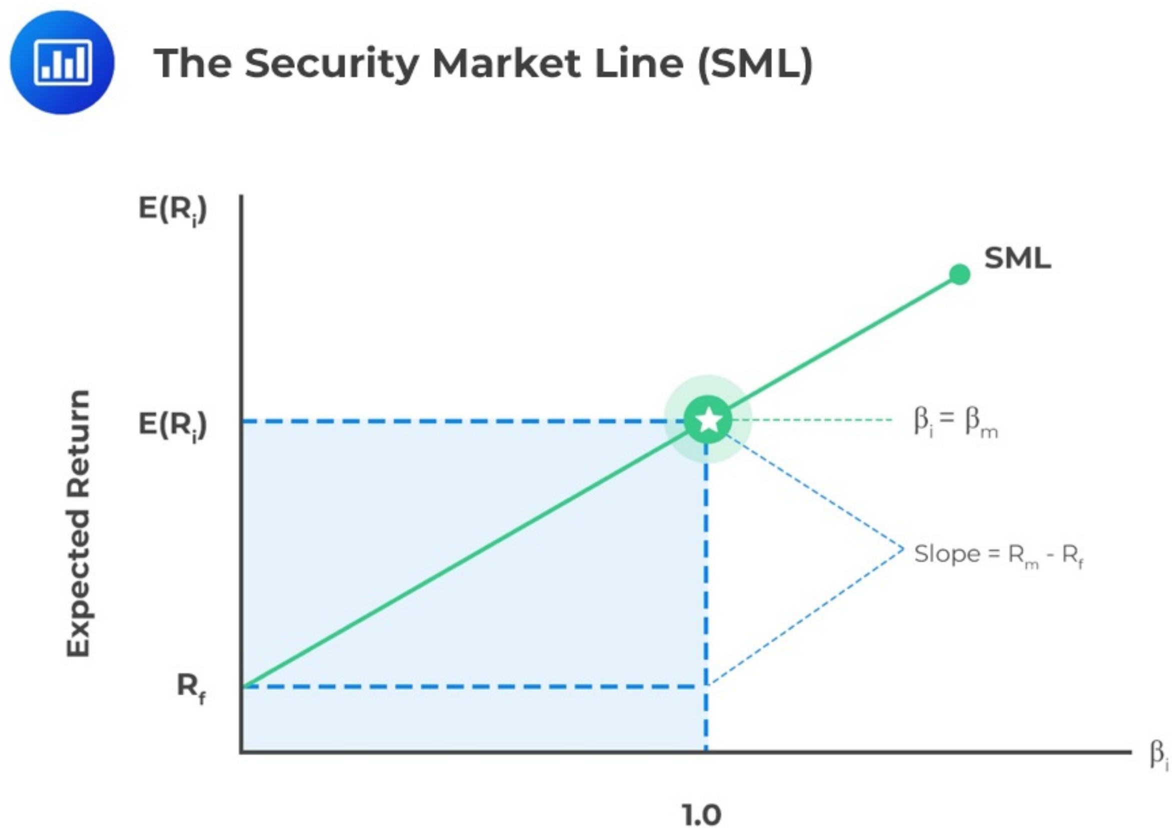
$\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$

$\beta_i = \frac{0.6 \times 0.18}{0.14}$

$\beta_i = 0.77$

LOS 2f: explain the capital asset pricing model (CAPM), including its assumptions, and the security market line (SML)

The Capital Asset Pricing Model (CAPM) provides a linear relationship between the expected return for an asset and the beta. The Security Market Line (SML) represents CAPM on a graph. As opposed to the Capital Market Line (CML), where the X-axis was the standard deviation, we're now using beta – systematic risk – to approximate the expected return.



As with many financial models, it relies on a number of assumptions in order to simplify some of the complexities of the financial markets.

CAPM Assumptions

As with many financial models, not all the complexities of the financial markets are accounted for. CAPM makes the following assumptions:

Investors are Risk-averse, Utility-maximizing, Rational Individuals

This assumption does not require all investors to have the same degree of risk aversion. Instead, it requires investors to be risk-averse as opposed to risk-neutral or risk-seeking. Investors are assumed to be rational if they correctly evaluate all available information to arrive at well-informed decisions. The rationality of investors has been criticized because personal bias can result in irrational decision-making. However, this behavior does not affect the model outcome.

Markets are Frictionless, Devoid of Transaction Costs and Taxes

In addition to indifference to transaction costs, the model also assumes that investors can borrow and lend at a risk-free rate. The transaction costs of many large institutions are negligible, and many investors do not pay taxes. The practical inability to borrow or lend at risk does not materially affect the CAPM results. In spite of this, costs and restrictions on short-selling can introduce an upward bias on asset prices. It is noteworthy that the prices do not affect the CAPM conclusions.

Investors Plan for the Same, Single Holding Period

The assumption of a single holding period is convenient since multi-period models have become very difficult. A single-period assumption has shortcomings. It, however, does not severely limit the applicability of the CAPM.

Investors have Homogenous Expectations or Beliefs

This is the assumption that all investors analyze securities the same way, and using the same probability distributions and inputs for future cash flows. This then means that all asset valuations are identical, and the same optimal portfolio of risky assets is generated – the market portfolio. This assumption can be relaxed as long as the generated optimal risky portfolios are not significantly different.

All Investments are Infinitely Divisible

This is the assumption that investors can hold fractions of assets. It is deemed convenient from a modeling perspective, considering that it allows for continuous rather than discrete jump functions.

Investors are Price Takers

If investors are price takers, no investor can single-handedly influence prices through their trades. This assumption is generally true in practice.

The Security Market Line (SML)

The Security Market Line (SML) is the graphical representation of the CAPM with beta reflecting systematic risk on the x-axis and expected return on the y-axis. The SML intersects the y-axis at the risk-free rate, and the slope of the line is the market risk premium, R_m - R_f.

The SML is formulated as follows:

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Where $\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$

The Capital Market Line (CML) and SML are similar. Despite their similarity, the CML only applies to portfolios on the efficient frontier providing optimal combinations of risk and return. The SML, on the other hand, applies to any security, regardless of whether it is efficient or not. Total risk and systematic risk are equal for an efficient portfolio because the non-systematic risk has been diversified.

Question

Which statement does not identify assumptions of the capital asset pricing model?

A. Investors are price takers, investors are rational, and transaction costs are ignored.

B. Investors are risk-seeking, fractional ownership is possible, and investors are price takers.

C. Investors have the same holding period, investors value securities identically, and taxes can be ignored.

Solution

The correct answer is **B**.

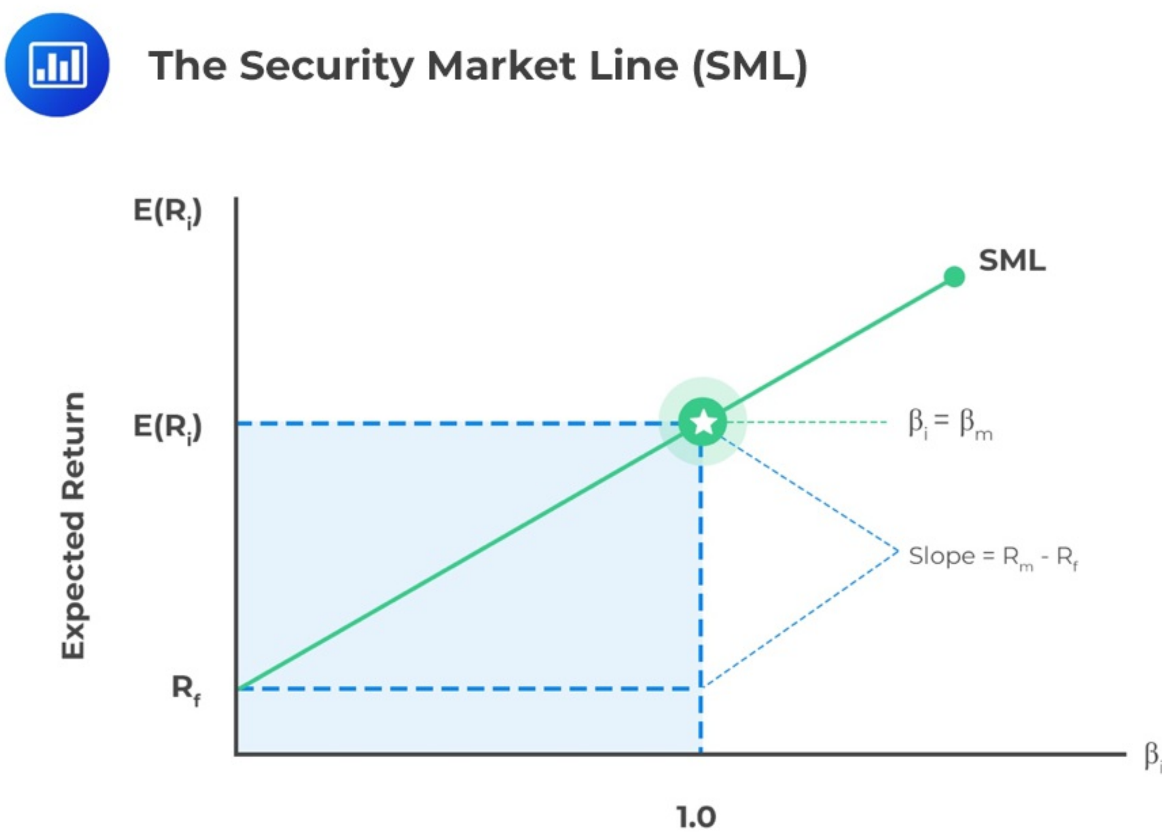
CAPM assumes all investors are risk-averse, utility-maximizing, and rational individuals.

LOS 2g: calculate and interpret the expected return of an asset using the CAPM

Given as asset systematic risk, the expected return can be computed using the capital asset pricing model. The CAPM result is usually used as a first estimate of return. In addition, it is used in capital budgeting and the determination of economic feasibility. Besides providing security expected returns, CAPM can be used for estimating the cost of capital and setting insurance premiums.

Calculating Expected Returns

The Security Market Line (SML) is the graphical representation of the CAPM with beta reflecting systematic risk on the x-axis and expected return on the y-axis. The SML intersects the y-axis at the risk-free rate, and the slope of the line is the market risk premium, R_m - R_f.



The SML is formulated as follows:

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Where $\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$

With data for the risk-free rate, the market expected return, the beta of a security (or its correlation with the market), and the standard deviations of the security and the market, we can calculate the expected return using CAPM.

Question

Assume the risk-free rate is 2%, a security has a correlation of 0.8 with the market index and a standard deviation of 16%, while the standard deviation of the market is 12%. If the market expected return is 8%, what is the security's expected return?

A. 10.56%.

B. 5.60%.

C. 8.42%.

Solution

The correct answer is **C**.

$E(R_i) = R_f + \beta_i[E(R_m) – R_f]$

Where $\beta_i = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$

Step 1: Find the Beta

\beta$_i$ = (0.8 × 0.16) / 0.12 = 1.07

Step 2: Find the expected return

E(R_i) = 2% + 1.07 × (8% - 2%) = 8.42%

LOS 2h: describe and demonstrate applications of the CAPM and the SML

CAPM can be extended to a number of areas. It provides additional applications beyond the estimation of security returns. A key area is in performance evaluation, where a number of commonly used metrics are employed.

Performance Evaluation

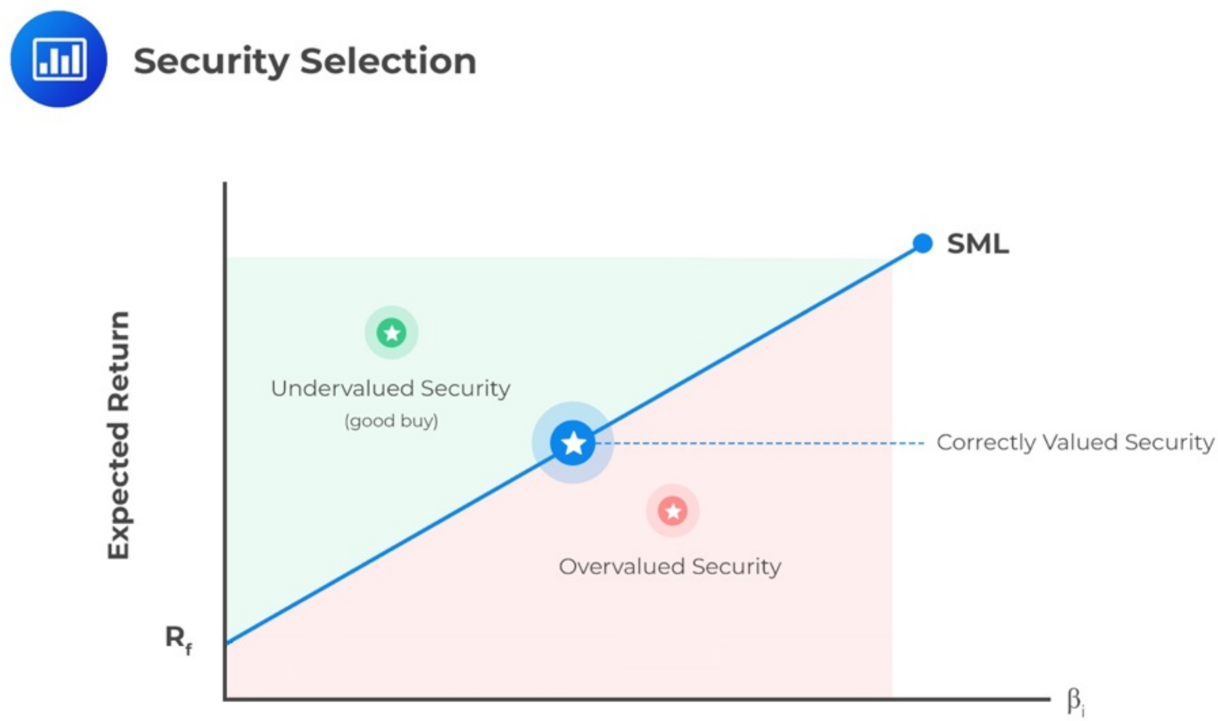
Various computable performance metrics are extensions of CAPM. These metrics allow for the assessment of portfolio performance and evaluation. Active managers are expected to perform better than their passive counterparts or to, at least, cover the costs of active management. There are four ratios commonly used in performance evaluation. All measures assume that the benchmark market portfolio is the correct one, and if it is not, it may make results inaccurate. The benchmark should be appropriate for the portfolio being measured. Besides, it should exhibit similar characteristics. We shall see these performance measures in detail in the next LOS.

Security Selection

The CAPM assumes that investors have homogeneous expectations, are rational and risk-averse, and assign the same value to all assets to create the same risky market portfolio. If investors are heterogeneous, their different beliefs could result in a valuation or price for a security that is different from the CAPM-calculated price. The CAPM-calculated price is the current market price because it reflects the beliefs of all other investors in the market. An investor's estimated price can sometimes be higher than the current market price. Such a circumstance should inform the investor's decision to buy the asset because it is considered undervalued by the market.

A Jensen's alpha for individual securities can also be computed with positive values. This indicates that the security is likely to outperform the market on a risk-adjusted basis.

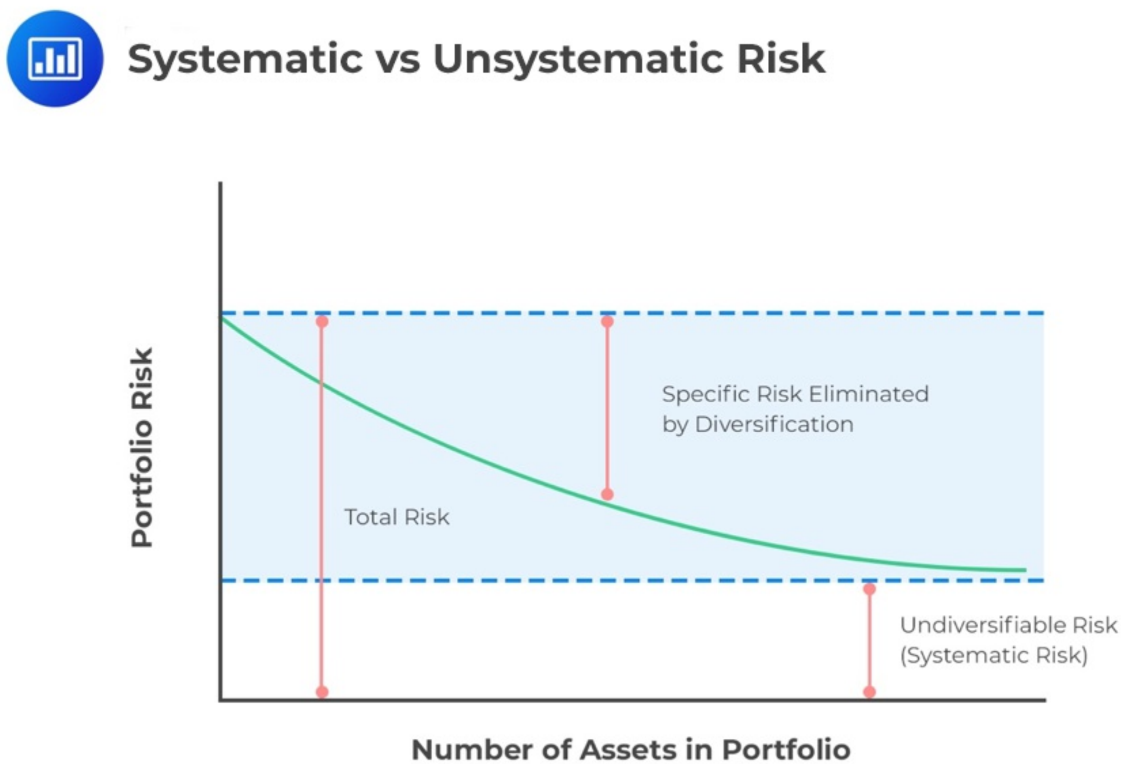
A Security Market Line (SML) can present similar information on a graph. The expected return and beta for security can be assessed against the SML with undervalued securities relative to market consensus appearing above the SML. The securities overvalued relative to market consensus will appear below the SML.



Portfolio Construction

CAPM suggests that investors should hold a market portfolio and a risk-free asset. The true market portfolio consists of a large number of securities, and it may not be practical for an investor to own them all. Much of the non-systematic risk can be diversified by holding 30 or more individual securities. However, these securities should be randomly selected from multiple asset classes. An index may serve as the best method of creating diversification.

It is important to note that only non-systematic risk can be eliminated through the addition of different securities into the portfolio. Systematic risk – the risk inherent to the entire market – cannot be diversified.



Securities not included within the index can be evaluated relative to the index to determine their suitability for portfolio inclusion. The alpha and beta of the security can be estimated relative to the index, and those with a positive alpha should be included. The same exercise can be conducted for securities within the index – those with negative alphas relative to the index should be excluded or sold short.

To determine the weight of each security within a portfolio, those securities with higher alpha should be given more weight. Nonetheless, this weight should be proportional to the alpha divided by the security's non-systematic variance (risk).

Limitations of CAPM

The CAPM is subject to theoretical and practical implications. From a theoretical perspective, it is both a single-factor and single-period model. There may be other factors over multi-time periods that would be more appropriate in modeling expected returns. Practically, the following are the limitations:

- **Market portfolio**: The true market portfolio includes all assets, financial and non-financial, which may not be investable or tradeable.
- **Proxy for market portfolio**: A proxy for the market portfolio is used, but different analysts tend to use different proxies.
- **Estimation of beta risk**: A long history is required to estimate beta. However, the history may not be an accurate representation of the future beta. Indeed, different historical periods (3 years versus 5 years) and different data frequencies (daily versus monthly) are likely to produce different betas.
- **Poor returns prediction**: The empirical support for CAPM is weak – the model is not good at predicting future returns. This, in turn, indicates that asset returns cannot be determined solely by systematic risk.
- **Homogeneity in investor expectations**: In reality, investors are unlikely to have homogeneous expectations. There will be many optimal risky portfolios and numerous Security Market Lines (SMLs).

Question

An overvalued security would *most likely* plot:

A. Below the Security Market Line (SML).

B. On the Security Market Line (SML).

C. Above the Security Market Line (SML).

Solution

The correct answer is **A**.

Securities overvalued, relative to market consensus, will appear below the SML. On the other hand, securities undervalued, relative to market consensus, will appear above the SML. Securities correctly priced will appear directly on the SML.