Learning Module 9: The Term Structure of Interest Rates: Spot, Par and Forward Curves

Q.856 A 3-year bond offers a 7% coupon rate with interests paid annually. Assuming the following sequence of spot rates, the price of the bond is *closest to*:

Time to Maturity (Years)	Spot Rate (%)
1	4
2	5
3	5.5

A. 98.24

B. 104.05

C. 104.20

The correct answer is **C.**

The price of the bond is calculated using the general formula for calculating a bond price given the sequence of spot rates as follows;

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \frac{PMT+FV}{(1+Z_3)^3}$$

$$\Rightarrow \text{price} = \frac{7}{(1+0.04)^1} + \frac{7}{(1+0.05)^2} + \frac{107}{(1+0.055)^3} = 104.20$$

Tip: In an exam situation; this calculation can be easily done as one calculation by using brackets at each step.

$$(7/(1.04^1)) + (7/(1.05^2)) + (107/(1.055^3)) = 104.20$$

A is incorrect. It excludes the coupon payment in the last year:

$$PV = \frac{7}{1.04^1} + \frac{7}{1.05^2} + \frac{100}{1.055^3} = 98.24$$

B is incorrect. It uses the three-year spot rate only.

$$\text{price} = \frac{7}{(1+0.055)^1} + \frac{7}{(1+0.055)^2} + \frac{107}{(1+0.055)^3} = 104.05$$

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (a): Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.2505 An analyst has gathered the following estimated series of spot rates for a developing country:

Period (Years)	Spot rate (%)
0.5	2
1	3
1.5	3.55
2	4
2.5	4.5
3	5
3.5	5.45

Given that the information is accurate, the value of a 3-year, 5% annual coupon paying bond with a face value of $1,000 is *closest to*:

A. $907.03

B. $1,001.80

C. $1,041.82.

The correct answer is **B.**

The valuation of a bond involves discounting its future cash flows (coupon payments and the face value at maturity) back to the present using the appropriate spot rates for each period. In this case, the bond pays a 5% annual coupon on a face value of $1,000 for 3 years. Therefore, the bond will pay $50 (5% of $1,000) annually. The present value (PV) of these cash flows can be calculated using the given spot rates for each period as follows:

$$\text{PV} = \frac{50}{1.03^1} + \frac{50}{1.04^2} + \frac{1,050}{1.05^3} = 48.54 + 46.23 + 907.03 = \$1,001.80$$

A is incorrect. It only considers the present value of the bond's final payment (the face value plus the final coupon payment), which is $907.03. This calculation neglects the present value of the coupon payments received in the first and second years, which are crucial components of the bond's total present value.

$$\text{PV} = \frac{1,050}{1.05^3} = \$907.03$$

C is incorrect. It misapplies the spot rates, suggesting a calculation method that does not correspond to the correct application of spot rates for a bond's cash flows.

$$\text{PV} = \frac{50}{1.02^1} + \frac{50}{1.03^2} + \frac{1,050}{1.0355^3} = 49.02 + 47.13 + 945.67 = \$1,041.82$$

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (a): Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.2525 If the current 1-year spot rate is 3%, the 1-year forward rate one year from today (f_{11}) is 4%, the 1-year forward rate two years from today (f_{11}) is 5%, then the 3-year spot rate is *closest to*:

A. 3.997%.

B. 12%.

C. 12.5%.

The correct answer is **A.**

To calculate the 3-year spot rate, we need to understand how spot rates and forward rates interact within the context of the yield curve. Spot rates represent the yield of a zero-coupon bond (a bond that does not pay interest but is sold at a discount to its face value), while forward rates represent the expected future interest rates between specific periods. The relationship between spot rates and forward rates can be used to derive the yield for a specific period in the future. In this case, we are given the current 1-year spot rate, the 1-year forward rate one year from today, and the 1-year forward rate two years from today. Using these rates, we can calculate the 3-year spot rate using the formula for the geometric mean of the spot and forward rates:

$$\text{3-year spot rate} = \sqrt[3]{(1 + \text{1-year spot rate}) \times (1 + f_{1,1}) \times (1 + f_{2,1})} - 1$$

Substituting the given values:

$$\text{3-year spot rate} = \sqrt[3]{[1.03 \times 1.04 \times 1.05]} - 1 = 0.03397 \text{ or } 3.997\%$$

B is incorrect. It suggests a simple sum of the rates, which is not how spot rates are calculated. Spot and forward rates must be compounded, not summed, to reflect the time value of money accurately.
C is incorrect. It misinterprets the calculation method for the 3-year spot rate and results from the following calculation;

$$[1.03 \times \times 1.04 \times 1.05] - 1 = 0.125 \text{ or } 12.5\%$$

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (a): Define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates

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Q.4754 The U.S. Treasury is evaluating issuing a new six-year bond with annual coupon payments. To estimate the pricing implications, you need to calculate the implied one-year forward rate starting in five years. Given the following annual spot rates:

- 5-year spot rate: 2.8%
- 6-year spot rate: 3.0%

The implied one-year forward rate starting in five years is *closest to*:

A. 4.01%

B. 5.00%

C. 3.24%

The correct answer is **A.**

Using the formula for forward rates:

$$((1 + Z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + Z_B)^B$$

Where:

- Z_A = 8% (5-year spot rate)
- Z_B = 3.0% (6-year spot rate)
- $IFR_{5,1}$= implied one-year forward rate from year 5 to year 6

Therefore,

$$\begin{aligned}(1.0028)^5 \times (1 + IFR_{5,1})^1 &= (1 + 0.030)^6 \\ IFR_{5,1} &= \frac{(1 + 0.030)^6}{(1 + 0.028)^5} - 1 \\ &= 0.04006 \approx 4.01\%\end{aligned}$$

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7b: Define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4755 An analyst is examining the yield curve of default-risk-free zero-coupon bonds. She refers to a graphical representation, which plots the yield-to-maturity against various maturities. This curve is *most likely* known as the:

A. spot curve.

B. forward curve.

C. par curve.

The correct answer is **A.**

The spot curve, or 'zero' curve, or 'strip' curve plots the yield-to-maturity of default-risk-free zero-coupon bonds against their maturities is commonly. It represents the pure yield of bonds that pay no intermediate coupons and are thus a fundamental tool for determining the time value of money for various terms.

B is incorrect. The forward curve represents anticipated future interest rates between different time periods, derived from current spot rates, but does not directly show yields of zero-coupon bonds.

C is incorrect. The par curve shows the coupon rates of bonds priced at par across different maturities. Unlike the spot curve, it reflects the yields of coupon-bearing bonds that would make them trade at their face value.

CFA Level I, Topic 7-Fixed Income , Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (a): Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.4756 Pius is reviewing the current economic conditions and notices that the yield curve of fixed income securities is becoming less steep, with yields on long-term bonds approaching those on short-term bonds. Which of the following is *most likely* the reason for this change in the yield curve?

A. Market participants anticipate an increase in inflation rates.

B. Market participants expect a decrease in inflation.

C. Market participants forecast a rapid increase in short-term interest rates.

The correct answer is **B.**

A flattening yield curve, where long-term yields are approaching short-term yields, often occurs when market participants anticipate a decrease in future inflation rates. This expectation reduces the demand for higher yields on longer-term bonds, leading to a convergence of long and short-term yields.

A is incorrect. An increase in expected inflation would typically steepen the yield curve, not flatten it, as investors would demand higher yields for longer maturities to compensate for the expected decrease in purchasing power.

C is incorrect. While an increase in short-term interest rates could influence the shape of the yield curve, it typically leads to a steepening effect if long-term rates do not rise as quickly. The described scenario of a flattening yield curve does not align with expectations of rapidly increasing short-term rates alone unless accompanied by similar or lesser adjustments in long-term rates.

CFA Level I, Topic 7-Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (a): Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.4757 An analyst observes a downward-sloping term structure in the current bond market. Based on this observation, which of the following relationships between par rates, spot rates, and forward rates is *most likely* correct?

A. Par rates are greater than spot rates and forward rates are lower than spot rates.

B. Par rates are lower than spot rates and forward rates are greater than spot rates.

C. Par rates and forward rates are equal to spot rates.

The correct answer is **A.**

In a downward-sloping term structure, the typical relationship observed is that par rates exceed spot rates, reflecting higher long-term borrowing costs relative to immediate borrowing costs. Similarly, forward rates are lower than spot rates, indicating expectations of decreasing rates in the future.

B is incorrect. It describes the conditions typical of an upward-sloping term structure, not a downward-sloping one.

C is incorrect. It does not typically occur in standard market conditions and does not reflect the relationships that emerge in either upward or downward-sloping term structures.

CFA Level I, Topic 7-Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (a): Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.4758 A change in the underlying spot rates for bond maturities will *most likely* cause:

A. a decrease in the corresponding par rates.

B. an increase in the volatility of bond prices.

C. a decrease in the liquidity of the bond market

The correct answer is **B.**

Changes in spot rates impact bond prices. As spot rates fluctuate, bond prices become more volatile. Longer-maturity bonds are particularly sensitive to rate changes.

A is incorrect. When spot rates change, par rates (also known as coupon rates) are not directly affected. Par rates are fixed at issuance and remain constant throughout the bond's life. However, the par rates for new bond issuances can vary based on the current interest rate environment, as they reflect the coupon rate at which a new bond would be issued at par.

C is incorrect. Spot rate changes do not necessarily affect bond market liquidity. Liquidity depends on factors like trading volume, market participants, and overall economic conditions.

CFA Level I, Topic 7-Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7a: Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.4759 Consider the term structure of spot rates for government bonds as follows:

- 1-Year Spot Rate: 2.00%
- 2-Year Spot Rate: 1.75%
- 3-Year Spot Rate: 1.50%
- 4-Year Spot Rate: 1.25%

Assume the bond pays annual coupons and has a face value of $100. The price of a 0.90% coupon, four-year government bond is *closest to*:

A. $97.15

B. $98.62

C. $99.45

The correct answer is **B.**

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT + FV}{(1+Z_N)^N}$$

Recall that,

$$\text{Coupon Payment (PMT)} = \text{Face Value} \times \text{Coupon Rate}$$

Plugging in the numbers:

$$\text{Coupon Payment} = \$100 \times 0.90\% = \$100 \times 0.009 = \$0.90$$

Therefore;

$$\begin{aligned} PV &= \frac{0.90}{(1+2.00\%)^1} + \frac{0.90}{(1+1.75\%)^2} + \frac{0.90}{(1+1.50\%)^3} + \frac{0.90+100}{(1+1.25\%)^4} \\ &= \$98.62115 \end{aligned}$$

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7a: Define spot rates and the spot curve, and calculate the price of a bond using spot rates.

Q.4760 A forward rate of 3y1y indicates:

A. Denotes the current borrowing cost for a four-year loan period.

B. Guarantees the return on rolling over a two-year investment into a subsequent one-year term.

C. The expected one-year interest rate three years from now, reflecting market forecasts for future rates.

The correct answer is **C.**

The 3y1y forward rate is an indicator of what the market expects the one-year interest rate to be, beginning three years from the present. It reflects a collective forecast based on current economic conditions and expectations about future rates.

A is incorrect: It confuses forward rates with spot rates. Spot rates represent the interest rates for immediate borrowing over a fixed period, not expectations of future rates.

B is incorrect: Forward rates are predictive and do not offer guaranteed returns; they reflect expectations and are not binding.

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7b: define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4761 Which one of the following *best* describes a spot curve of government bonds?

A. Upward sloping.

B. Downward sloping.

C. Approximates flat line across all maturities due to generally stable short-term interest rate expectations.

The correct answer is **A.**

Typically, a spot curve slopes upward, suggesting that longer-term government bonds yield more than shorter-term bonds, but it may flatten as times to maturity extend.

B is incorrect: It describes an inverted yield curve, which is not a typical characteristic of spot curves. While spot curves can invert, they are not in their constant state.

C is incorrect: It describes a flat yield curve, which does not capture the dynamic nature of spot curves that can slope upward or become inverted based on market conditions and expectations.

CFA Level I, Topic 7-Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7c: compare the spot curve, par curve, and forward curve.

Q.4762 In a normal interest rate environment where the yield curve is upward-sloping, how do forward rates typically compare to their corresponding spot rates?

A. Forward rates are below spot rates.

B. Forward rates are above spot rates.

C. Forward rates are equal to spot rates.

The correct answer is **B.**

An environment with an upward-sloping yield curve, forward rates are typically higher than spot rates. This reflects market expectations of higher future interest rates, often due to anticipated economic growth and inflation.

A is incorrect: This describes an environment where the yield curve is inverted, indicating a downward-sloping maturity structure of interest rates. In such conditions, the spot rates start higher and decrease for longer maturities, reflecting a pessimistic outlook on the economy's future, such as anticipating a slowdown or recession. Consequently, forward rates are set below the current spot rates as the market expects even lower rates in the future.

C is incorrect: This describes an environment where the yield curve is flat, indicating a constant maturity structure of interest rates. In such scenarios, spot rates remain consistent across different maturities, reflecting market expectations that future interest rates will remain stable. Therefore, in a flat term structure environment, forward rates naturally align with spot rates, as there are no anticipated changes in interest rates over time.

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS7c: compare the spot curve, par curve, and forward curve.

Q.4763 Which of the following statements *accurately* describes the nature of spot and forward rates in the context of interest rate investments?

A. Forward rates are today's rates for immediate funding, similar to spot rates but generally higher due to risk premiums.

B. Spot rates are for immediate borrowing or investing, while forward rates are for future-dated transactions.

C. Spot rates apply to long-term borrowing agreements, whereas forward rates are used for short-term financial planning.

The correct answer is **B.**

Spot rates reflect the current borrowing and investing rates for funds that are transacted immediately or very soon, covering the specified tenor. Forward rates, in contrast, are used for agreements to borrow or invest starting on a future date, also for a specified tenor.

A is incorrect. Forward rates do not apply to immediate funding but to future financial commitments and are not necessarily higher than spot rates; they reflect expectations of future interest rates rather than current conditions.

C is incorrect. Spot rates are not exclusively for long-term agreements but apply to immediate or near-immediate transactions for any tenor. Forward rates are used for future agreements, not necessarily short-term, and can cover any length of time as specified in the contract.

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7b: define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4764 Which of the following *best* describes why forward rates would be positive while spot rates are negative in an environment with an upward-sloping yield curve?

A. This is likely a result of calculation errors in the financial models used.

B. Positive forward rates indicate market expectations that spot rates will rise.

C. The central bank sets forward rates to be positive as a deliberate strategy to counteract the negative spot rates and boost economic confidence.

The correct answer is **B.**

Forward rates are influenced by market expectations about the future trajectory of interest rates. An upward-sloping yield curve, suggests that investors expect future spot rates to increase, potentially moving from negative to positive.

A is incorrect: Positive forward rates in a setting of negative spot rates are not necessarily indicative of calculation errors. Instead, they reflect rational market anticipations based on the current and expected future economic conditions.

C is incorrect: While central banks influence overall monetary conditions, forward rates are determined by market conditions and investor expectations, not directly set by central banks.

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (c): compare the spot curve, par curve, and forward curve.

Q.4765 An investor is comparing two bond investment strategies. The first involves purchasing a five-year zero-coupon bond. The second strategy consists of buying a four-year bond and planning to reinvest the proceeds into a one-year bond upon maturity. This decision centers around the "4y1y" implied forward rate, which denotes the rate required for both investment strategies to yield equivalent returns. The current "4y1y" implied forward rate is 1.8%. If the investor anticipates that the one-year yield after four years will rise to 2.3%, which option should the investor select to potentially maximize returns?

A. Choose the five-year zero-coupon bond because it secures a fixed return now, which is safer than betting on uncertain future rates.

B. Invest in the four-year bond and reinvest the proceeds at the anticipated higher rate, as it might offer better returns than the current implied forward rate.

C. Both strategies are equally viable since the expected future yield difference is too small to impact the investment decision significantly.

The correct answer is **B.**

The investor expects the future one-year yield after four years to be 2.3%, which is higher than the "4y1y" implied forward rate of 1.8%. Investing in the four-year bond and then reinvesting the proceeds at the higher expected future rate can potentially yield greater returns than simply holding a five-year zero-coupon bond, where the yield is fixed and lower than the expected future rate.

A is incorrect. This choice might seem safe because it locks in a return with less risk related to future rate changes. However, it is incorrect in the context of maximizing returns, which is the investor's goal. Given the expected increase in rates, choosing a strategy that benefit from higher future rates would be more advantageous.

C is incorrect. This option incorrectly assumes that the difference in the forward rate and the expected future rate is negligible. However, the forward rate and the expected future rate do differ significantly (1.8% vs. 2.3%), which can impact total returns over the investment horizon. This difference can be meaningful in bond investment decisions, especially in a low-rate environment where even small differences in interest rates can lead to substantial differences in compounded returns over time.

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS b: define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4766 An investor is evaluating two potential strategies for a five-year investment horizon:

- Acquiring a four-year government bond with a current annual spot rate of 3.5%, followed by reinvesting the proceeds in a one-year bond at the expected future rate

after four years

- Investing in a five-year government bond that yields an annual spot rate of 4.5%.

Given this information, which of the following strategies should the investor select?

A. Purchase the five-year government bond with a steady rate.

B. Invest in the four-year bond and reinvest the proceeds at the anticipated higher rate after four years.

C. Both strategies will yield approximately the same returns because the effective yield of reinvesting the four-year bond's proceeds is likely to compensate for the lower initial rate when compared to the steady 4.5% offered by the five-year bond.

The correct answer is **B.**

To make an informed decision, the investor calculates the implied one-year forward rate that will be applicable after four years, using the following formula:

$$(1 + Z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + Z_B)^B$$

Where:

Z_A= short-term spot rate, with tenorA

Z_B = longer-term spot rate, with tenor B.

$IFR_{A,B-A}IFR_{5,1}$ = implied forward rate, for a security, begins at t = A and matures at t = B (tenor B – A).

In this case,

A=4

B=5

Z_A =3.5%

Z_B=4.5%

$IFR_{4,1}$=?

Therefore,

$$(1 + 0.035)^4 \times (1 + IFR_{4,1}) = (1 + 0.045)^5$$
$$1.035^4 (1 + IFR_{4,1}) = 1.045^5 1.147523 (1 + IFR_{4,1}) = 1.246182$$
$$\Rightarrow IFR_{4,1} = 1.085976 - 1 = 0.085976 \approx 8.6\%$$

Therefore, the future one-year rate expected after four years is 8.6%. The strategy of buying the four-year bond at 3.5% and reinvesting at a higher future rate will likely yield increased returns compared to a constant 4.5% over five years. This choice leverages the anticipated increase in rates to potentially enhance returns beyond what the five-year bond can offer.

A is incorrect. The continuous rate of 4.5% for five years is lower than the potential gains from reinvesting at an expected higher rate after the initial four-year bond maturity.

C is incorrect. It assumes without justification that the reinvestment rate after four years will be high enough to offset the initial lower rate compared to the five-year bond's constant 4.5%..

CFA Level I, Topic 7-Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7b: Define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4767 Consider the following annual effective spot rates for government bonds:

Term	Sport Rate
1-Year	3.00%
2-Year	3.50%
3-Year	4.00%

The par rate for a three-year government bond assuming annual payments and compounding is *closest to*:

A. 2.974%

B. 3.451%

C. 3.974%

The correct answer is **C.**

To find the par rate, use the equation where the present value of the bond's cash flows equals its face value, typically assumed to be $100. The cash flows include the annual coupon payments (PMT) and the face value returned at maturity:

$$100 = \frac{\text{PMT}}{(1+\text{Z}_1)^1} + \frac{\text{PMT}}{(1+\text{Z}_2)^2} + \cdots + \frac{\text{PMT}+100}{(1+\text{Z}_\text{N})^\text{N}}$$

Where:

$\text{Z}_1, \text{Z}_2, \ldots, \text{Z}_\text{N}$ are the spot rates for the 1-year, 2-year, and 3-year terms respectively.

Plugging in the spot rates:

$$\begin{aligned} 100 &= \frac{\text{PMT}}{(1.0300)^1} + \frac{\text{PMT}}{(1.0350)^2} + \frac{\text{PMT}+100}{(1+0.0400)^3} \\ 100 &= \frac{\text{PMT}}{1.0300} + \frac{\text{PMT}}{1.071225} + \frac{(\text{PMT}+100)}{1.124864} \\ \text{PMT} &= 3.973776 \approx 3.974\% \end{aligned}$$

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7b: define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4768 An investment analyst needs to estimate the implied three-year spot rate using the forward rate curve for one-year segments given in the following table:

Time Period	Forward Rate
0y1y	1.75%
1y1y	2.45%
2y1y	3.10%

Assume all rates are annualized and are effective yearly rates. The three-year implied spot rate is *closest to*:

A. 2.43%

B. 2.55%

C. 2.70%

The correct answer is **A.**

Using the formula:

$$(1 + z_{0y1y}) \times (1 + z_{1y1y}) \times (1 + z_{2y1y}) = (1 + z_3)^3$$

Then,

$$\begin{aligned} 1.0175 \times 1.0245 \times 1.0310 &= (1 + z_3)^3 \\ 1.074744 &= (1 + z_3)^3 \\ 1.074744^{\frac{1}{3}} - 1 &= z_3 \\ \Rightarrow z_3 &= 0.024318 \approx 2.43\% \end{aligned}$$

CFA Level I, Topic 7 - Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS 7b: define par and forward rates and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.

Q.4769 An analyst is examining the yields-to-maturity of a three-year and a six-year zero-coupon bond, which are 3.20% and 3.90%, respectively. The yields are quoted on a semi-annual bond basis. The analyst is interested in determining the "3y3y" implied forward rate.The annualized "3y3y" implied forward rate is *closest to*:

A. 2.30%

B. 4.10%

C. 4.60%

The correct answer is **C.**

The general formula for implied forward rate is given as

$$(1 + Z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + Z_B)^B$$

Where:

A = 6 periods (since 3 years × 2 semi-annual periods per year)

B = 12 periods (since 6 years × 2 semi-annual periods per year)

B − A = 6 periods (duration of the forward period in semi-annual terms)

$z_6 = \frac{0.0320}{2} = 0.016$ (Semi-annual yield for the three-year bond)

$z_12 = \frac{0.0390}{2} = 0.0195$ (Semi-annual yield for the six-year bond)

Substitute the given yields into the formula:

$$\begin{aligned}(1 + 0.016)^6 \times (1 + IFR_{6,6})^6 &= (1 + 0.0195)^12 \\ 1.099923(1 + 1FR_{6,6})^6 &= 1.260802 \\ (1 + IFR_{6,6})^6 &= \frac{1.260802}{1.099923} = 1.146264 \\ (1 + IFR_{6,6}) &= 1.146264^{\frac{1}{6}} = 1.023012 \\ IFR_{6,6} &= 0.23012 \approx 2.30\%\end{aligned}$$

The “3y6y” implies forward rate is approximately 2.30% on a semi-annual basis. Annualized, the “3y6y” implied forward yield is 2.30% × 2 = 4.6%.

CFA Level I, Fixed Income, Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves. LOS (b): define par and forward rates, and calculate par rates, forward rates from spot rates, spot rates from forward rates, and the price of a bond using forward rates.