Learning Module 6: Pricing and Valuation of Futures Contracts

LOS 6a: compare the value and price of forward and futures contracts

Recall that during the initiation of a forward commitment, no cash changes hands. Further, the forward commitment is neither a liability nor an asset to a buyer or the seller. As such, the value of both the forward contract and futures contract is zero:

$$V_0(T) = 0$$

Consider an underlying with no associated costs or benefits. Like forward contracts, the futures price is calculated by compounding the spot price of the underlying using the risk-free rate:

$$f_T(0) = S_0(1 + r)^T$$

Where:

$f_T(0)$ = Futures forward price.

S_0 = Spot price of the underlying at time $t = 0$.

r = Risk-free rate.

T = Time to maturity.

Note that, like forward contracts, we have used discrete compounding. However, continuous compounding is also preferred in futures contracts if the underlying assets comprise a portfolio, such as commodities, fixed income, and equity. Also, continuous compounding is preferred when the underlying is foreign exchange denominated in two currencies.

Using continuous compounding, the future price is given by:

$$f_T(0) = S_0 e^{rT}$$

Cost of Carry and Futures Price

Like forward contracts, the price of futures whose underlying has income (I) and costs (C) is adjusted as follows:

$$f_T(0) = [S_0 - PV_0(I) + PV_0(C)](1 + r)^T$$

Where:

$PV_0(I)$ = Present value of income or benefit associated with the underlying at time $t = 0$.

$PV_0(C)$ = Present value of costs associated with the underlying at time $t = 0$

Example: Futures Price Valuation

Minners Inc. enters a futures contract on an exchange via a financial intermediary to buy 80 kilos of gold. The current spot price is $52,950 per kilo.

If the risk-free rate of return is 3%, what is the no-arbitrage futures price for settlement in 95 days?

Solution

The futures price is equal to the compounded value of the spot price of the underlying at the risk-free rate for a period T:

$$\begin{aligned} f_T(T) &= S_0(1 + r)^T \\ &= \$52,950(1.03)^{\frac{95}{365}} \\ &= \$53,358.94 \end{aligned}$$

Mark-to-Market Valuation of a Future Contract Compared to a Forward Contract

As time passes, the value of futures and forward contracts changes. However, the forward contract price remains constant until maturity.

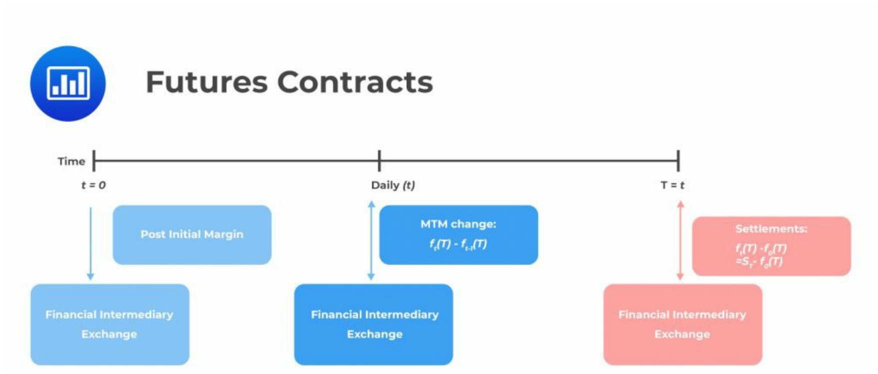
As seen previously, for the long position, the value of a forward contract during its life is calculated as the difference between the current spot price and the present value of the original forward price:

$$V_t(T) = S_t - F_0(T)(1 + r)^{-(T-t)}$$

The MTM value of the forward contract is not settled until its expiration date, which causes counterparty risk.

On the other hand, the futures price changes depending on market conditions. Moreover, the daily settlement resets the MTM value to zero. Besides, the variation margin is exchanged to cover the difference, decreasing counterparty risk.

Note that the cumulative MTM gain or loss is approximately equal to that of a comparable forward contract.



Interest Rate Futures and Forward Contracts

Remember that a forward rate agreement (FRA) uses implied forward rates as a no-arbitrage fixed rate. In this instance, the counterparties exchange fixed for floating payments at a specified time in the future.

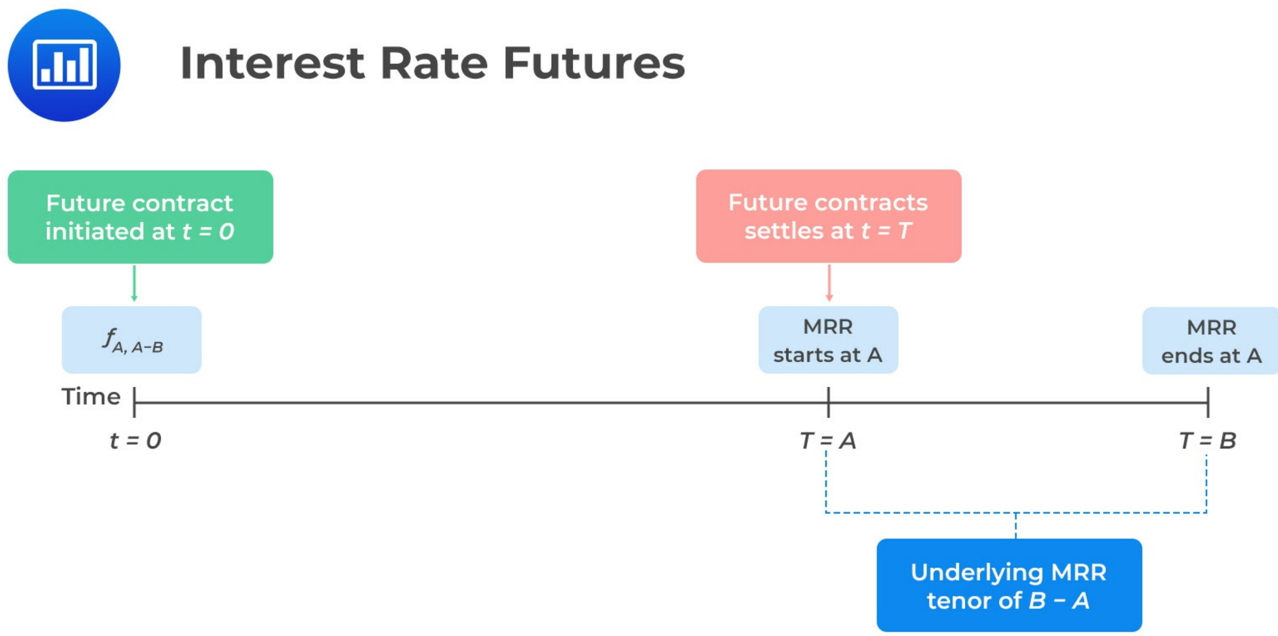
The futures contracts on short-term interest rates are more liquid and standardized than FRAs. These contracts are often available for monthly and quarterly market reference rates (MRRs).

Description of Interest Rate Futures

As is the case in FRA, the underlying of the interest rate futures is the market reference rate on a hypothetical amount of money at a future date. However, interest rate futures trade on a price basis, given by the following formula:

$$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})$$

Where: $f_{A,B-A} =$ futures price for the market reference rate for B–A periods that begin in A period ($MRR_{A,B-A}$).



Note that the formula $f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})$ can be written as $f_{A,B-A} = 100 - \text{yield}$.

Intuitively, the (100 – Yield) price convention leads to an inverse price versus yield relationship that differs from the price of a zero-coupon bond at a contract rate. As such, a long futures position receives MRR in A period while the short position pays MRR in A period.

In summary, the long position (lender) gains as prices rise and future MRR falls. In contrast, the short position (borrower) gains as prices fall and future MRR rises.

Interest Rate Futures Settlement

The **daily settlement** of the interest futures occurs depending on the price changes, regarded as **futures contract basis point value** (BPV) and calculated as follows:

$$\text{Futures contract BPV} = \text{Notional principlal} \times 0.01\% \times \text{Period}$$

For instance, consider USD 50 million for a 3-month MRR of 3% for 180/360 days. The futures contract BPV is:

$$\text{Futures contract BPV} = 50,000,000 \times 0.01\% \times \left(\frac{180}{360}\right) = \$2,500$$

Example: Calculating Futures Contracts Gains or Losses

A&M Bank has issued its clients a USD 10 million three-month loan at a fixed rate. To finance the loan, the bank has borrowed a one-month variable MRR. To hedge against interest rate risk, the bank sells futures contracts on two-month MRR. Assume that the bank agrees to sell the futures at \$97.75, but the actual settlement price is \$96.75.

The cumulative gain/loss to the contract from the bank's perspective is *closest* to:

Solution

We need to start by calculating the contract's BPV:

$$\begin{aligned} \text{Futures contract BPV} &= \text{Notional principal} \times 0.01\% \times \text{Period} \\ &= 10,000,000 \times 0.01\% \times \frac{2}{12} \\ &= \$166.67 \end{aligned}$$

We need to calculate corresponding market reference rates (MRRs) for both prices. Note that

$$f_{A,B-A} = 100 - (100 \times \text{MRR}_{A,B-A})$$

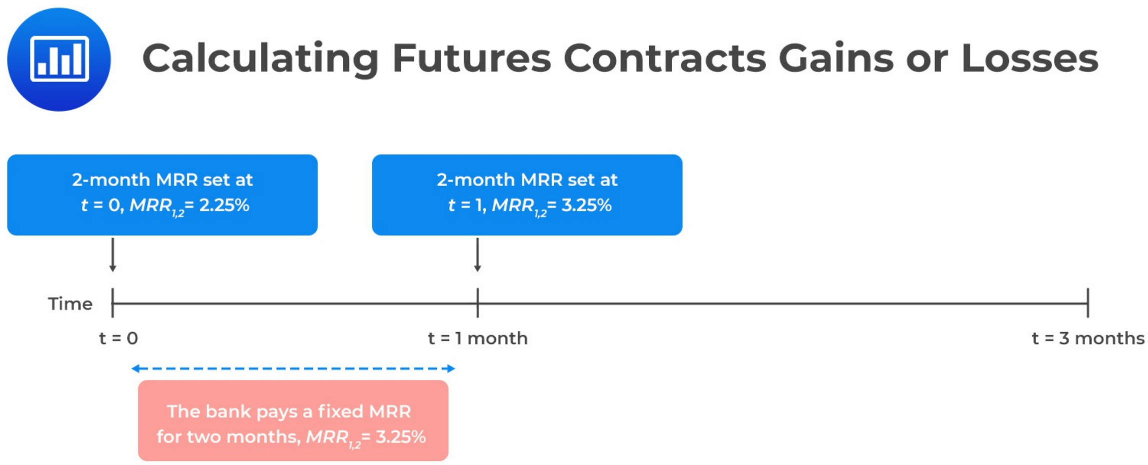
Therefore;

$$\begin{aligned} 96.75 &= 100 - (100 \times \text{MRR}_{A,B-A}) \rightarrow \text{MRR}_{1,2} = 3.25\% \\ 97.75 &= 100 - (100 \times \text{MRR}_{A,B-A}) \rightarrow \text{MRR}_{1,2} = 2.25\% \end{aligned}$$

Now, we calculate the cumulative gain/loss:

- Cumulative gain/loss = Change in interest rates × Contract size
- Cumulative gain/loss = 1% × USD 10,000,000 = USD 100,000

So, the cumulative gain/loss to the contract from the bank's perspective is a loss of USD 100,000.



Question

Which of the following *best describes* the difference between the price of a futures contract and its value?

A. The price determines the profit to the buyer, and the value determines the profit to the seller.

B. The futures price is fixed at the start, and the value starts at zero and changes throughout the contract’s life.

C. The futures contract value is a benchmark against which the price is compared to determine whether a trade is advisable.

Solution

The correct answer is **B**.

The futures price is fixed at the start, whereas the value starts at zero and then changes, either positively or negatively, throughout the contract’s life.

LOS 6b: explain why forward and futures prices differ

Forward and futures contracts share similar features; however, how they are traded and the resulting cash flows mean forward and futures contracts with the same underlying asset may trade at a different price.

Causes of Differences

1. Mark-to-Market (MTM), Margining, Settlement of Gains and Losses, and Risks

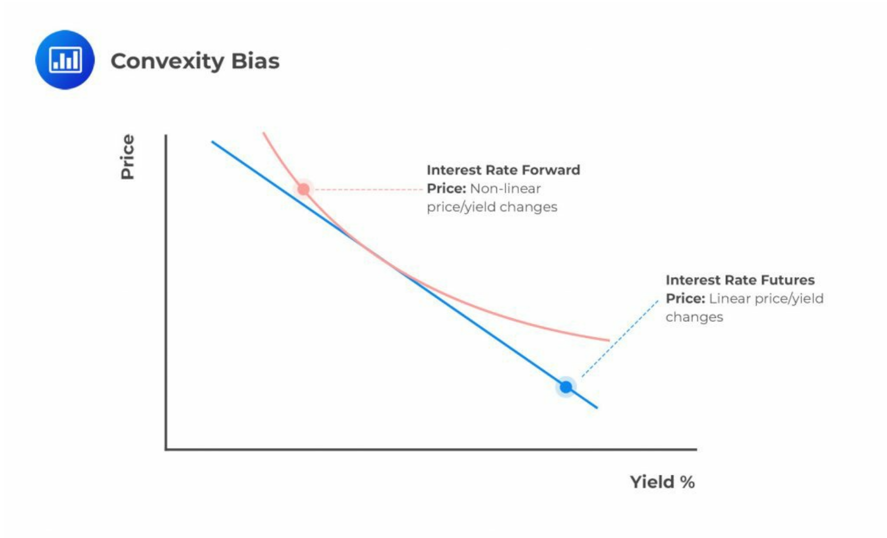
Comparable forwards and futures have symmetric payoff profiles at expiration. However, the pricing and valuation differ over the life of the comparable contracts. Remember that futures are exchange-traded derivative contracts. As such, the distinguishing features of future contracts include posting initial margin, daily mark-to-market, and settlement of gains and losses.

On the other hand, a forward contract is an OTC contract where the credit terms are privately negotiated between the counterparties, and there are no daily mark-to-market (MTM) settlements. Consequently, forward contracts are riskier than futures contracts.

One specific risk is counterparty risk, which is the risk that one party will default on the agreement. A forward contract is more prone to counterparty risk, particularly because settlement only occurs at maturity as a one-time cash settlement.

2. Convexity Bias

Convexity bias occurs when there are different price changes between interest rate futures and forward prices. Interest rate futures have a fixed linear payoff profile for a given change in basis point. On the other hand, interest rate forward (for example, FRAs) have non-linear relation with the basis point change (convexity property).



Consequently, convexity bias causes the percentage price change to be greater in absolute value when MRR decreases than when it increases for a forward contract compared to a futures contract.

Causes of Similarities

1. Relationship between Interest Rates and Futures Prices

Despite the differences in (FRAs) pricing and valuation of futures and forwards, there are instances where their prices are equal. The following assumptions must hold for the futures and forward prices to be identical:

- The interest rates must be constant.
- The futures prices and interest rates are uncorrelated.

What happens when the above assumptions do not hold? For instance, if there is a positive correlation between futures prices and interest rates, the long futures contract is more profitable than the comparable long forward contract. Rising futures prices generate futures profits that are reinvested in periods of rising interest rates. Falling futures prices, on the other hand, attract losses incurred during periods of falling interest rates.

When there is a negative correlation between the futures prices and interest rates, short futures contracts are more attractive than comparable short forward positions. This is because falling futures prices result in profits that are reinvested in periods of high-interest rates. Rising futures prices result in losses incurred during periods of falling interest rates.

2. Central Clearing on OTC Derivatives

The emergence of central clearing of derivatives has resulted in futures-like margining requirements for over-the-counter (OTC) derivative dealers. For instance, dealers are required to post cash or highly liquid securities to a central counterparty. The dealers then impose the same requirements on the derivative end-users.

As such, such a clearing structure on OTC derivatives has reduced the differences in prices between exchange-traded futures and OTC forward contracts.

Question

Which of the following statements is *most likely* true?

A. If there is a positive correlation between futures prices and interest rates, a long futures contract is more profitable than comparable long forward contracts.

B. If futures prices and interest rates are negatively correlated, short forward positions are more attractive than a comparable short futures contract.

C. Central clearing of derivatives increases the difference in futures and forward prices.

Solution

The correct answer is **A**.

When futures prices rise with interest rates, the profits from the long futures position can be reinvested during periods of high interest. On the other hand, losses incurred when futures prices fall occur during decreasing interest rates.

B is incorrect. When there is a negative correlation between the futures prices and interest rates, short futures positions are more attractive than comparable short-forward positions. This is because falling futures prices result in profits that are reinvested in periods of high-interest rates. Rising futures prices result in losses that are incurred during periods of falling interest rates.

C is incorrect. A clearing structure on OTC derivatives has reduced the differences in prices between exchange-traded futures and OTC forward contracts.