

## **Learning Module 6: Fixed Income Bond Valuations: Prices and Yields**

**LOS 6a: calculate a bond's price given a yield-to-maturity on or between coupon dates**

### **Bond Price Calculation**

The price of a bond is influenced by various factors, including its cashflow features and market discount rate. The cash flow features are periodic payments made to bondholders, such as interest or coupon payments. On the other hand, the market discount rate is the required return based on the bond's risk. It reflects investors' expectations and the time value of money. At issuance, the bond price equals the present value (PV) of future interest and principal cash flows.

The price of a bond can be determined using mathematical formulas or spreadsheet functions as highlighted below:

$$\text{Bond price: } P = \sum_{t=1}^N \left( \frac{C_t}{(1+r)^t} \right) + \frac{FV_N}{(1+r)^N}$$

Where:

- $r$  = Market discount rate
- $C_t$  = Coupon payment at time  $t$
- $N$  = number of periods until maturity
- $FV$  = Face value of the bond
- $\text{PV of Bond coupon} = \left( \frac{C_t}{(1+r)^t} \right)$
- 

Spreadsheet Function for Bond Price:  $\text{Bond Price} = \text{PV}(\text{rate}, \text{nper}, \text{pmt}, \text{FV}, \text{type})$

Where:

- rate is the market discount rate per period.

- `nper` is the number of periods
- `pmt` is the coupon payment per period
- `FV` is the face value
- `type` refers to whether payments are made at the end (0) or beginning (1) of each period

## Types of Bonds

Bonds are categorized into three main types, each representing a specific relationship between price, coupon rate, and market discount rate:

- **Par bond:** Price equals future value; coupon rate equals the market discount rate.
- **Discount bond:** Price is less than future value; coupon rate is less than the market discount rate.
- **Premium bond:** Price is greater than future value; coupon rate is greater than the market discount rate.

## Yield-to-Maturity (YTM)

Yield-to-Maturity (YTM) represents the bond's internal rate of return (IRR), which is the single, uniform interest rate that, when applied to discount the bond's future cash flows, equals the current price of the bond. In essence, YTM is the implied or observed market discount rate. It is also known as the bond's "promised yield," assuming the issuer does not default. YTM ("yield") is used interchangeably with market discount rate or required yield. Instead of discussing bond prices, market participants might say "yields are rising" to mean "market discount rates are rising" or "bond prices are falling."

## Conditions for Earning YTM

An investor will achieve a return equal to the YTM if the following conditions are met:

- Holding the bond until maturity.
- The issuer making full coupon and principal payments on the scheduled dates.
- Reinvesting all coupon payments at the YTM.

## YTM Calculation

The formula for calculating YTM is as follows:

$$P = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \dots + \frac{C+F}{(1+r)^n}$$

Where:

- P = Price of the bond.
- C = Periodic coupon payment.
- r = Yield to maturity.
- F = Face value of the bond.
- n = Number of periods until maturity.

## Using Spreadsheet Functions

YTM can be calculated using specific functions in spreadsheet tools like Microsoft Excel or Google Sheets:

### YIELD Function:

`=YIELD(settlement, maturity, rate, pr, redemption, frequency, [basis])`

Where:

- settlement = Settlement date.
- maturity = Maturity date.

- rate = Semi-annual (or periodic) coupon.
- pr = Price per 100 face value.
- redemption = Future value at maturity.
- frequency = Number of coupons per year.
- [basis] = Day-count convention (optional).

**IRR Function:** YTM can also be calculated using the IRR function in these tools, as it represents an internal rate of return.

## Example: Bond Price Calculation

A municipal bond that matures on 1 July 2040 pays semiannual coupons of 2.75% per year and has a face value of 100. The market discount rate is 3.5%. For a trade settlement date of 1 July 2035, the price of the bond as a percentage of par value, assuming a 30/360-day count, is closest to:

### Solution

The bond price is the sum of the coupon and principal payments discounted at the market discount rate.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_N + FV_N}{(1+r)^N}$$

- $C=1.375$ , i.e.,  $\frac{2.75\%}{2} \times 100$
- $r = \frac{3.5\%}{2} = 0.0175$
- $FV=100$
- $N=10$ , since payments are made twice a year for 5 years

1.375                      1.375                      1.375                      1.375                      1.375

$$PV = \frac{1.375}{(1 + 0.0175)^1} + \frac{1.375}{(1 + 0.0175)^2} + \frac{1.375}{(1 + 0.0175)^3} + \dots + \frac{101.375}{(1 + 0.0175)^{10}} = 96.587$$

## Flat Price, Accrued Interest, and the Full Price

In bond trading, especially when a bond is priced between coupon payment dates, three key components are considered: the flat price, accrued interest, and the full price.

1. **Flat Price** The flat price, also known as the quoted or "clean" price, represents the price of the bond without considering any accrued interest.
2. **Accrued Interest** Accrued interest is the interest that has accumulated since the last coupon payment but has not yet been paid. It is computed by considering the fraction of the coupon period that has elapsed. Formula for Accrued Interest:

$$AI = \frac{t}{T} \times PMT$$

Where:

- $t$  = Number of days from the prior coupon payment to the settlement date.
- $T$  = Number of days in the coupon period.
- $PMT$  = Coupon payment per period.

The graph below illustrates the relationship between the flat price, accrued interest, and full price of a bond over its entire lifetime, spanning multiple coupon periods.

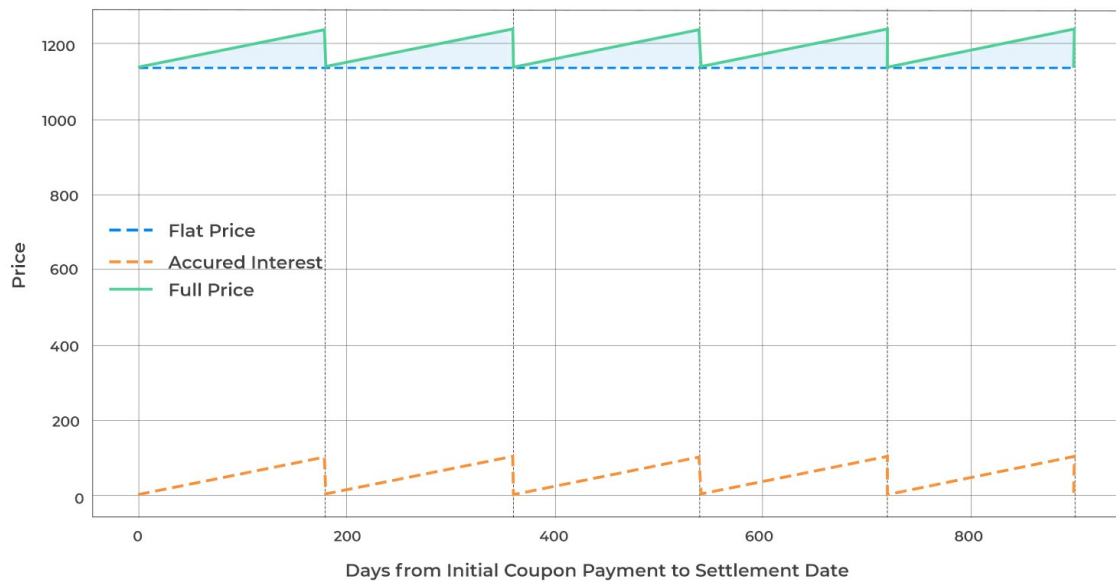
1. **Flat Price (Dashed Line):** Represents the present value of the bond's future cash flows without considering accrued interest. It remains constant throughout the bond's lifetime.
2. **Accrued Interest (Dash-Dot Line):** Depicts the interest that has accumulated since the last coupon payment. It starts at zero at the beginning of each coupon period and linearly increases until the next coupon payment.
3. **Full Price (Solid Line):** The sum of the flat price and accrued interest. It follows the same pattern as accrued interest but starts from the flat price level at the beginning of each coupon period.

4. **Vertical Lines (Dotted Grey Lines):** Indicate the end of each coupon period, marking the moments when coupon payments are made.

The shaded area between the flat price and full price lines visually represents the accrued interest at any given point in time. This graph provides a clear understanding of how these key components of bond pricing interact and evolve over time.



### Flat Price, Accrued Interest, and Full Price of a Bond



1. **Full Price** The full price, or "dirty" price, is the sum of the flat price and the accrued interest. Full Price of a Fixed-Rate Bond is expressed as:

$$PV_{\text{Full}} = \frac{C}{(1+r)^{1-t/T}} + \frac{C}{(1+r)^{2-t/T}} + \dots + \frac{C+FV}{(1+r)^{N-t/T}}$$

Where:

- $C$  = Coupon payment per period.
- $r$  = Market discount rate per period.

- $t$  = Number of days from the prior coupon payment to the settlement date.
- $T$  = Number of days in the coupon period.
- $N$  = Total number of periods.
- $FV$  = Face value of the bond.

Day count conventions specify how days are counted within a period. 30/360 assumes 30 days in a month and 360 days in a year. On the other hand, Actual/Actual uses the actual number of days in a month/year.

### **Example: Calculating the Flat Price**

A certain bond pays semiannual coupons of 2.0% per year on 30 June and 31 December each year, with a face value of 100. The YTM is 2.5%. The bond is purchased and will settle on 15 September, when there will be four coupons remaining until maturity. The flat price of the bond as a percentage of par value, assuming an actual/actual day count, is closest to:

### **Solution**

- $PMT = 1.00$ , i.e.,  $2\% \times \frac{100}{2}$
- $r = 0.0125$  (2.5% annual market discount rate, divided by 2 for semiannual)
- $t = 77$  (days from 30 June to 15 September)
- $T = 184$  (days from 30 June to the next coupon on 31 December)
- $FV = 100$
- $N = 4$  (remaining coupons)

$$PV^{\text{Flat}} = PV^{\text{Full}} - AI$$

$$PV^{\text{Full}} = \left[ \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \dots + \frac{PMT + FV}{(1+r)^N} \right] \times (1+r)^{\left(\frac{t}{T}\right)}$$

$$PV^{\text{Full}} = \left[ \frac{1}{(1.0125)^1} + \frac{1}{(1.0125)^2} + \frac{1}{(1.0125)^3} + \frac{1+100}{(1.0125)^4} \right] \times (1.0125)^{\left(\frac{77}{184}\right)} = 99.547$$

$$AI = \frac{77}{184} \times 1.00 = 0.418$$

$$PV^{\text{Flat}} = 99.547 - 0.418 = 99.129$$

## Question

A bond that matures on 1 July 2040 pays semiannual coupons of 2.5% per year and has a face value of 100. The market discount rate is 4.0%. For a trade settlement date of 1 July 2038, the price of the bond as a percentage of par value is closest to:

- A. 94.555
- B. 90.018
- C. 97.144

## Solution

**The correct answer is C:**

Using the given values:

$$\text{PMT} = 1.25, \text{ i.e., } \frac{(2.5\%)}{2} \times 100$$

$r = 0.020$  (4% annual market discount rate, divided by 2 for semiannual)

$$\text{FV} = 100$$

$N = 4$  (Since payments are made twice a year for 2 years)

Formula for bond price:

$$\text{PV} = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \frac{(C + \text{FV})}{(1+r)^4}$$

$$\text{PV} = \frac{1.25}{(1.020)^1} + \frac{1.25}{(1.02)^2} + \dots + \frac{101.25}{(1.020)^4} = 97.144$$

## **LOS 6b: identify the relationships among a bond's price, coupon rate, maturity, and yield-to-maturity**

### **Inverse Relationship - Bond Price and YTM**

The price of a bond and the yield-to-maturity have an inverse relationship. The same is shown in the blue line in the figure below for a bond with a maturity of 5 years and a coupon rate of 4%. A higher discount rate (or yield) lowers the present value of the fixed future cash flows, thereby reducing the bond's price (moving right along the blue line). Conversely, a lower discount rate (or yield) increases the present value of the fixed future cash flows, resulting in a higher bond price (moving left along the blue line).

This inverse relationship reflects the natural fluctuation in the value of money over time and is central to understanding bond pricing and valuation.

### **The Coupon Effect**

The size of the bond coupon affects the price change for a given yield change. The coupon effect is illustrated by the red dashed line in the figure, representing the fixed coupon rate of 4%. The lower a bond's coupon, the higher the proportion of total cash flow that occurs at maturity. This makes the bond more susceptible to changes in the yield-to-maturity, as the final cash flow is magnified by the discount factor of  $(1 + r)^N$ . On the other hand, a higher coupon bond provides more periodic interest payments, reducing the impact of yield changes on the bond's price.

### **Maturity Effect**

Longer-term bonds experience greater percentage price change than shorter-term bonds for the same change in market discount rates. This heightened sensitivity is due to the higher number of periods ( $N$ ) until maturity in the bond pricing equation for the longer-maturity bond. The maturity effect is shown by the green dashed line in the figure, representing the yield to maturity at which the bond is priced at par value.

Although the maturity effect applies broadly, exceptions do exist and usually apply to low-coupon

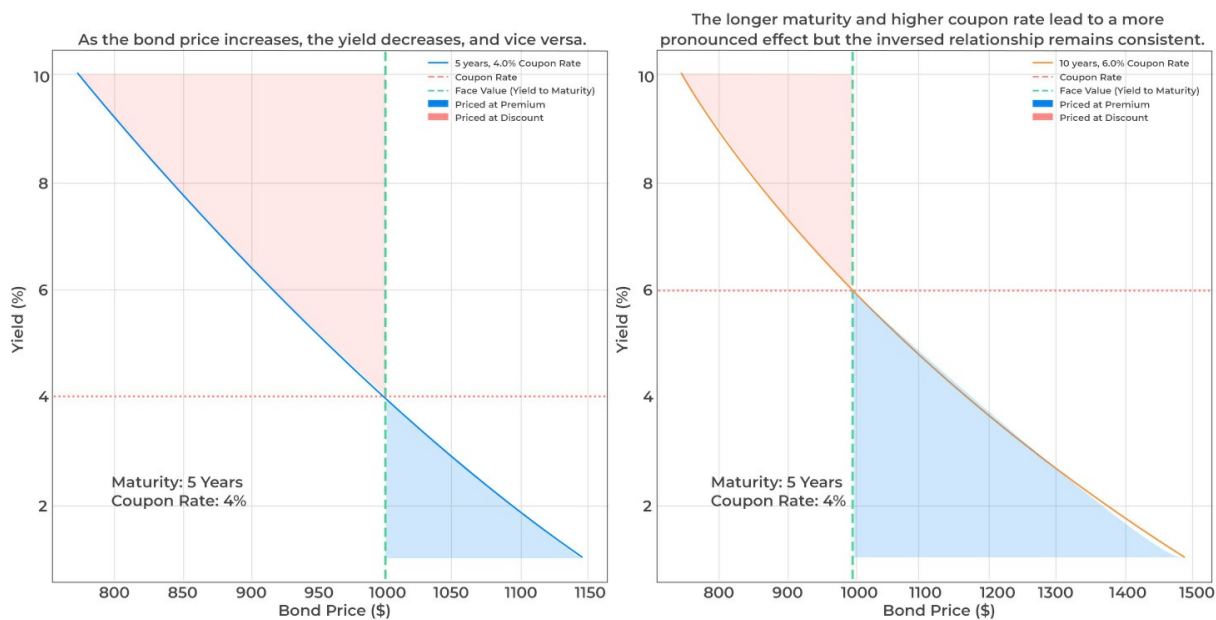
long-term bonds trading at a discount. However, it consistently holds for zero-coupon bonds and for bonds priced at or above par value, making it a valuable tool in bond analysis.

## Constant-Yield Price Trajectory

The constant-yield Price Trajectory is shown by the shaded regions in the figure. As the bond's maturity date approaches, the bond's price gradually converges toward the par value (green dashed line). The light blue region shows where the bond is priced at a premium, and the light red region shows where it's priced at a discount. This phenomenon, known as being "pulled to par," illustrates the dynamic nature of bond pricing.



### Bond "Pulled to Par"



## Convexity Effect

The convexity effect explores the non-linear relationship between a bond's price and its yield. This relationship is clearly visible in the figure above, where the blue line representing the bond price versus yield curve is not a straight line but curved and convex.

The Convexity Effect can be understood as follows:

- **Positive Convexity:** The curve's shape means that the percentage price increase for a bond (when yields fall) is greater, in absolute value than the percentage price decrease for an equivalent change in yield (when yields rise). This is beneficial to bondholders as they gain more when yields fall and lose less when yields rise.
- **Non-Linear Relationship:** The curve's convexity illustrates that the price-yield relationship is not directly proportional. As yields decrease, the price increases at an increasing rate, and as yields increase, the price decreases at a decreasing rate.

The convexity effect plays a vital role in bond portfolio management, as it helps in assessing the sensitivity of bond prices to changes in interest rates. It provides an added layer of understanding beyond simple duration measures, capturing the complex and non-linear dynamics of bond pricing.

## Question #1

In terms of the maturity effect, how does a 30-year bond's percentage price change compare to a 5-year bond when market discount rates change by the same amount?

- A. The 30-year bond experiences a lesser percentage price change.
- B. The 30-year bond experiences the same percentage price change.
- C. The 30-year bond experiences a greater percentage price change.

## Solution

The correct answer is **C**.

In terms of the maturity effect, a 30-year bond will experience a greater percentage price change compared to a 5-year bond when market discount rates change by the same amount. This is because longer-maturity bonds have a greater duration, making them more sensitive to interest rate changes. The longer the time until maturity, the more significant the impact of a change in discount rates on the bond's present value.

**A is incorrect.** The 30-year bond will experience a greater, not lesser, percentage price change.

**B is incorrect.** The 30-year bond will not experience the same percentage price change as the 5-year bond; it will be more affected by changes in discount rates.

## Question #2

What does the constant-yield price trajectory illustrate as the bond's maturity date approaches?

- A. The bond's price moves away from par value.
- B. The bond's price fluctuates randomly.
- C. The bond's price gradually converges towards the par value.

### Solution

The correct answer is **C**.

According to the coupon effect, a higher coupon rate reduces the impact of yield changes on a bond's price. With a higher coupon rate, a greater proportion of the bond's cash flow is realized earlier in the form of periodic interest payments, reducing the sensitivity of the bond's price to changes in yields.

**A is incorrect.** A higher coupon rate decreases, not increases, the impact of yield changes.

**B is incorrect.** The coupon rate does have an effect on the impact of yield changes; it is not unaffected.

### Question #3

How does the coupon effect relate to the proportion of total cash flow realized at maturity for a higher coupon bond?

- A. The proportion of total cash flow realized at maturity is higher.
- B. The proportion of total cash flow realized at maturity is unchanged.
- C. The proportion of total cash flow realized at maturity is lower.

### Solution

The correct answer is **C**.

The coupon effect relates to the proportion of total cash flow realized at maturity. For a higher coupon bond, the proportion of total cash flow realized at maturity is lower. This is because a higher coupon bond provides more of its cash flow earlier through periodic interest payments, reducing the proportion of cash flow realized at maturity.

**A is incorrect.** The proportion of total cash flow realized at maturity is lower for a higher coupon bond, not higher.

**B is incorrect.** The proportion is not unchanged; it is affected by the coupon rate.



## **LOS 6c: describe matrix pricing**

### **Matrix Pricing Process**

Matrix pricing is a valuation method widely utilized by financial institutions to estimate the fair value of a security that is not actively traded. This process is especially significant for bonds and other fixed-income securities, which may not have regular market quotations. Unlike securities traded on major exchanges, many bonds are traded over the counter (OTC), leading to less transparent pricing. Matrix pricing addresses this challenge by leveraging observable market data and statistical techniques.

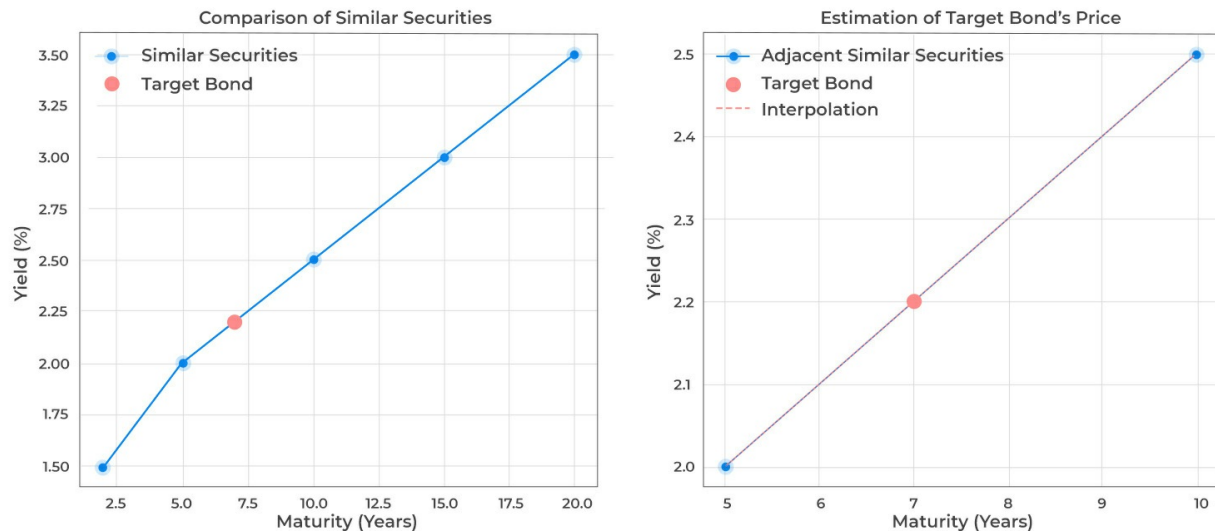
Matrix pricing revolves around pinpointing comparable securities with established market prices and leveraging them as benchmarks to evaluate the value of the target security. Taking into account parameters like credit ratings, maturities, and coupon rates, matrix pricing establishes a link between securities with akin features. This method facilitates a more comprehensive and data-driven valuation, especially for securities without a vibrant trading background.

Investment companies, mutual funds, and portfolio managers often rely on matrix pricing to maintain an accurate and up-to-date valuation of their holdings. The method aligns with fair value accounting principles and helps in achieving compliance with regulatory requirements. Furthermore, it contributes to a more transparent and realistic portrayal of a portfolio's value, enhancing investor trust and confidence.

The diagram below offers an intuitive view of the matrix pricing process. It shows how we identify similar securities, gather market data, and use interpolation or statistical techniques to estimate the price of the desired security. This process helps determine security prices efficiently.



## Matrix Pricing



### Key Aspects of Matrix Pricing

- **Comparison with similar securities:** Matrix pricing involves using the prices of similar securities to determine the value of a security that is not actively traded. The left plot in the figure illustrates the yield curve of similar securities, and the target bond is highlighted in red.
- **Use of observable market data:** As shown in both plots, the process requires observable market data, such as interest rates and yield curves. The blue points represent similar securities with known yields and maturities.
- **Application in investment portfolios:** Mutual funds and other investment companies often use matrix pricing to value fixed-income securities, such as bonds, that may not be frequently traded.

### Steps in Matrix Pricing

- **Identify similar securities:** The first step, as shown in the left plot, is to identify securities with similar credit ratings, maturities, and coupon rates.

- **Observe market data:** The next step, represented by the blue points, is to observe market data for the identified similar securities, including credit quality, yields, and maturities.
- **Estimate the price:** The right plot demonstrates how the price of the target bond is estimated. The price is estimated using observed market data and interpolation (dashed red line) between the yields of adjacent similar securities.

The figure above elucidates how matrix pricing leverages information from similar securities to estimate the price of a bond that may not be actively traded. The comparison with similar securities and the interpolation process is central to understanding this valuation method.

## Example: Estimating Illiquid Bond Price Using Matrix Pricing

An investment manager is determining the price of an illiquid six-year, 3.5% annual coupon corporate bond. They find two similar-quality bonds: a five-year, 4.00% coupon bond priced at 105.25 per 100 of par value and a seven-year, 3.25% coupon bond priced at 103.50 per 100 of par value. With matrix pricing, the estimated price of the illiquid bond per 100 of par value is closest to:

### Solution

The first step is to determine the yields-to-maturity on the observed bonds.

The required yield on the five-year, 4.00% bond priced at 105.250 is 2.858%.

$$105.250 = \frac{4.00}{(1+r)^1} + \frac{4.00}{(1+r)^2} + \frac{4.00}{(1+r)^3} + \frac{4.00}{(1+r)^4} + \frac{104.00}{(1+r)^5}$$

$$r = 2.858\%$$

The required yield on the seven-year, 3.25% bond priced at 103.500 is 2.695%

3.25%      3.25%      3.25%      3.25%      3.25%      3.25%      4.00%

$$103.500 = \frac{3.25}{(1+r)^1} + \frac{3.25}{(1+r)^2} + \frac{3.25}{(1+r)^3} + \frac{3.25}{(1+r)^4} + \frac{3.25}{(1+r)^5} + \frac{3.25}{(1+r)^6} + \frac{103.25}{(1+r)^7}$$

$$r = 2.695\%$$

The estimated yield for a six-year bond having the same credit quality is the average of two required yields:

$$\text{Average yield} = (2.858\% + 2.695\%)/2 = 2.777\%$$

Given an estimated yield-to-maturity of 2.777%, the estimated price of the illiquid six-year, 3.5% annual coupon payment corporate bond is 103.95 per 100 of par value:

$$\text{Price} = \frac{3.50 \cdot (1 - v^6)}{r} + 100v^6$$

$$\text{Price} = \frac{3.50(1 - (1.02777)^{(-6)})}{0.02777} + 100(1.02777)^{(-6)} = 103.95$$

## Matrix Pricing Variation for New Bond Issues

Another variation for matrix pricing primarily focuses on spreads, specifically examining the differences between bond yields and benchmark bond yields. Often, default risk-free bonds, such as U.S. Treasury bonds, are used as benchmarks for U.S. dollar-denominated corporate bonds. To estimate the Yield-to-Maturity (YTM) for a new bond, one must identify the appropriate spread to the yield of a Treasury bond with similar maturity. By adding this spread to the yield of the benchmark issue, the YTM for the new bond can be accurately estimated.

## Example: Estimating the Yield for a New 8-Year, BBB-Rated Bond Issue

Consider the following market yields:

- 6-year U.S. Treasury bond, YTM 1.95%.
- 6-year BBB-rated corporate bond, YTM 3.25%.

- 10-year U.S. Treasury bond, YTM 2.70%.
- 10-year BBB-rated corporate bond, YTM 4.30%.
- 8-year U.S. Treasury bond, YTM 2.30%.

Estimate the required yield on a newly issued 8-year, BBB-rated corporate bond.

**Calculate the spreads to the benchmark (Treasury) yields:** The spread on the 6-year corporate bond is  $3.25 - 1.95 = 1.30\%$ . The spread on the 10-year corporate bond is  $4.30 - 2.70 = 1.60\%$ .

**Calculate the average spread because the 8-year bond is the midpoint of six and ten years:**

$$\text{Average spread} = \frac{(1.30 + 1.60)}{2} = 1.45$$

Add the average spread to the YTM of the 8-year Treasury (benchmark) bond:  $2.30 + 1.45 = 3.75\%$ , which is our estimate of the YTM on the newly issued 8-year BBB-rated bond.

## Question #1

Consider the following market yields:

- 5-year US. Treasury bond, YTM 2.10%.
- 5-year A-rated corporate bond, YTM 3.60%.
- 9-year US. Treasury bond, YTM 2.80%.
- 9-year A-rated corporate bond, YTM 4.60%.
- 7-year US. Treasury bond, YTM 2.40%.

Estimate the required year on a newly issued 7-year, A-rated corporate bond.

- A. 1.65%.
- B. 4.05%.
- C. 4.10%.

## Solution

The correct answer is **B**.

Calculate the spreads to the benchmark (Treasury) yields:

The spread on the 5-year corporate bond is  $3.60 - 2.10 = 1.50\%$ .

The spread on the 9-year corporate bond is  $4.60 - 2.80 = 1.80\%$

Calculate the average spread because the 7-year bond is the midpoint of five and nine years:

$$\text{Average Spread} = \frac{(1.50 + 1.90)}{2} = 1.65$$

Add the average spread to the YTM of the 7-year Treasury (benchmark) bond:

$2.40 + 1.65 = 4.05\%$ , which is our estimate of the YTM on the newly issued 7-year.

## Question #2

In matrix pricing, what type of market data is typically observed for the identified similar securities?

- A. The company's earnings reports.
- B. Price-to-earnings ratios.
- C. Prices, yields, and other relevant data.

## Solution

The correct answer is **C**.

In matrix pricing, the market data observed for similar securities includes prices, yields, and other relevant data. This information provides the necessary inputs to interpolate or extrapolate the price or yield for the bond being valued. By analyzing the relationship between price and yield among similar bonds, matrix pricing can accurately estimate the value of a bond that may not have recent or regular trading activity.

**A is incorrect.** The company's earnings reports are related to equity valuation and are not typically relevant to the valuation of fixed-income securities.

**B is incorrect.** Price-to-earnings ratios are used in equity analysis and are not relevant to matrix pricing for bonds.

## Question #3

Which of the following best describes the reason for using matrix pricing in the valuation of portfolio securities?

- A. To provide speculative price targets.
- B. To estimate the fair value of securities not actively traded.
- C. To analyze market trends for active trading.

## Solution

The correct answer is **B**.

Matrix pricing is used to estimate the fair value of securities not actively traded, especially in the context of bonds. For securities that do not have a liquid market, it becomes difficult to determine a fair market price. Matrix pricing overcomes this challenge by utilizing data from similar, more liquid securities to derive an estimated value for the illiquid security. This approach is systematic and grounded in observable market data, making it a reliable method for valuing non-actively traded bonds.

**A is incorrect.** Speculative price targets are not the aim of matrix pricing. The method strives for an accurate and reasonable estimate of value based on observable data, not speculation.

**C is incorrect.** While market trends may be of interest in other contexts, matrix pricing is not a tool designed for active trading strategies or trend analysis.