LOS 1a: describe characteristics of the major asset classes that investors consider in forming portfolios

All asset classes have risk and return characteristics. Historical returns are neither forward-looking nor expected returns. Nevertheless, it is noteworthy that by examining the performance of the historical returns, we can understand the likely characteristics of a particular asset class.

Returns of Major Asset Classes

The examination of an 83-year period, from 1926 to 2008, provides investors with annual return data. The data is often used to forecast the expected mean return for the asset classes. The major asset classes have produced the following annual nominal returns for the United States:

- US Large Company Stocks: 9.6%.
- US Small Company Stocks: 11.7%.
- US Long-term Corporate Bonds: 5.9%.
- US Long-term Government Bonds: 5.7%.
- US Treasury Bills: 4.0%.

Over this period of 83 years, the US inflation rate has averaged 3.0%. However, inflation has varied widely. Therefore, the use of real, inflation-adjusted returns is more appropriate, particularly when comparing asset class returns globally.

Using data from 1900 to 2008, an examination of nominal versus real returns global asset classes can be made. In nominal terms, world equities returned 8.4%, while world bonds returned 4.8%. The corresponding real returns are 5.2% and 1.8%, respectively.

Risk of Major Asset Classes

We cannot examine returns without examining the associated risk to each asset class. Risk, in this context, is measured by a standard deviation metric. By examining the United States' nominal returns over the period between 1926 to 2008, we observe the following standard deviations:

- US Large Company Stocks: 20.6%.
- US Small Company Stocks: 33.0%.
- US Long-term Corporate Bonds: 8.4%.
- US Long-term Government Bonds: 9.4%.
- US Treasury Bills: 3.1%.

Using nominal world data from 1900 to 2008, we note that the standard deviation for world equities is 17.3% and 8.6% for world bonds.

Risk-Return Tradeoff

A risk-return tradeoff refers to the relationship between risk and return. Ideally, if you want to achieve a higher return, you must accept a higher level of risk. Reviewing the US nominal asset class data, it can be noted that small company stocks delivered the highest return over the period (11.7%) despite bearing the highest risk (33.0%).

Risk Premium

The risk premium is the extra returns investors can expect for assuming additional risk after accounting for the nominal risk-free interest rate. The world equity risk premium over bonds is 3.4%. This is the additional return investors can hope to achieve from equities over bonds due to the additional equity risk.

Other Investment Characteristics

An assumption of a normal distribution of returns is made by making use of a mean and standard deviation when evaluating asset class characteristics. However, within a financial market context, an assumption of normality is flawed since returns are not normally distributed. The probability of extreme events is greater than a normal distribution suggests. An examination of the skewness and kurtosis of a distribution is required.

Skewness

Skewness is a measure of the asymmetry of a return distribution. If more returns are concentrated on the right end of the distribution, the returns are said to be negatively skewed and vice versa. Generally, stock returns tend to be negatively skewed.

Kurtosis

Kurtosis refers to the "fat tails" of the distribution. That is, the greater probability of extreme events than would ordinarily be assumed by a normal distribution.

Liquidity

Although not a function of the return distribution, liquidity is an important market factor that contributes to the risk of an investment. Liquidity tends to excite more concern in emerging markets than in developed markets. This trend is attributable to smaller trading volumes in those markets. It is equally a cause for concern for potentially more risky asset classes such as low-credit quality corporate bonds.

Question

Skewness is *most likely*:

A. A measure of the asymmetry of the probability distribution.

B. A measure of the "tailedness" of the probability distribution.

C. A measure that is used to quantify the amount of variation or dispersion of a set of data values.

Solution

The correct answer is **A**.

Skewness is a measure of the asymmetry of a return distribution.

Option B is incorrect. It is the definition of kurtosis.

Option C is incorrect. It is the definition of standard deviation.

LOS 1b: explain risk aversion and its implications for portfolio selection

Risk aversion is related to investor behavior. Some investors are more comfortable with uncertainty in the outcome than others and are prepared to tolerate more risk in the pursuit of greater portfolio returns.

Risk Seeking

Risk seekers actively pursue risk even when the potential outcome does not justify taking extra risk. This is a gambling instinct: choosing to place money at casinos, knowing the odds of winning are slim or that the expected return is actually negative.

Risk Neutrality

If an investor is indifferent to the outcome, they may be risk-neutral. This means they will likely pursue higher returns even if this comes with higher risk. This is often the case, particularly when the investment represents a small portion of their wealth or portfolio.

Risk Aversion

A risk-averse investor will gravitate towards a guaranteed outcome and shy away from risky investments. A lower, certain return will be preferable to a higher, less certain return. Market data typically represents risk-averse behavior on the part of investors, and risk aversion is, as such, a standard assumption.

Risk Tolerance

Risk tolerance refers to the amount of risk an investor is willing to take in order to achieve their investment goals and objectives. A higher risk tolerance shows a greater willingness to take risks. This implies that risk tolerance and risk aversion are negatively correlated.

Question

In a choice of a certain $45 versus a 50% chance of $100, an investor chooses the certain $45. What type of risk behavior does the following scenario represent?

A. Risk-seeking behavior.

B. Risk-neutral behavior.

C. Risk-averse behavior.

Solution

The correct answer is **C**.

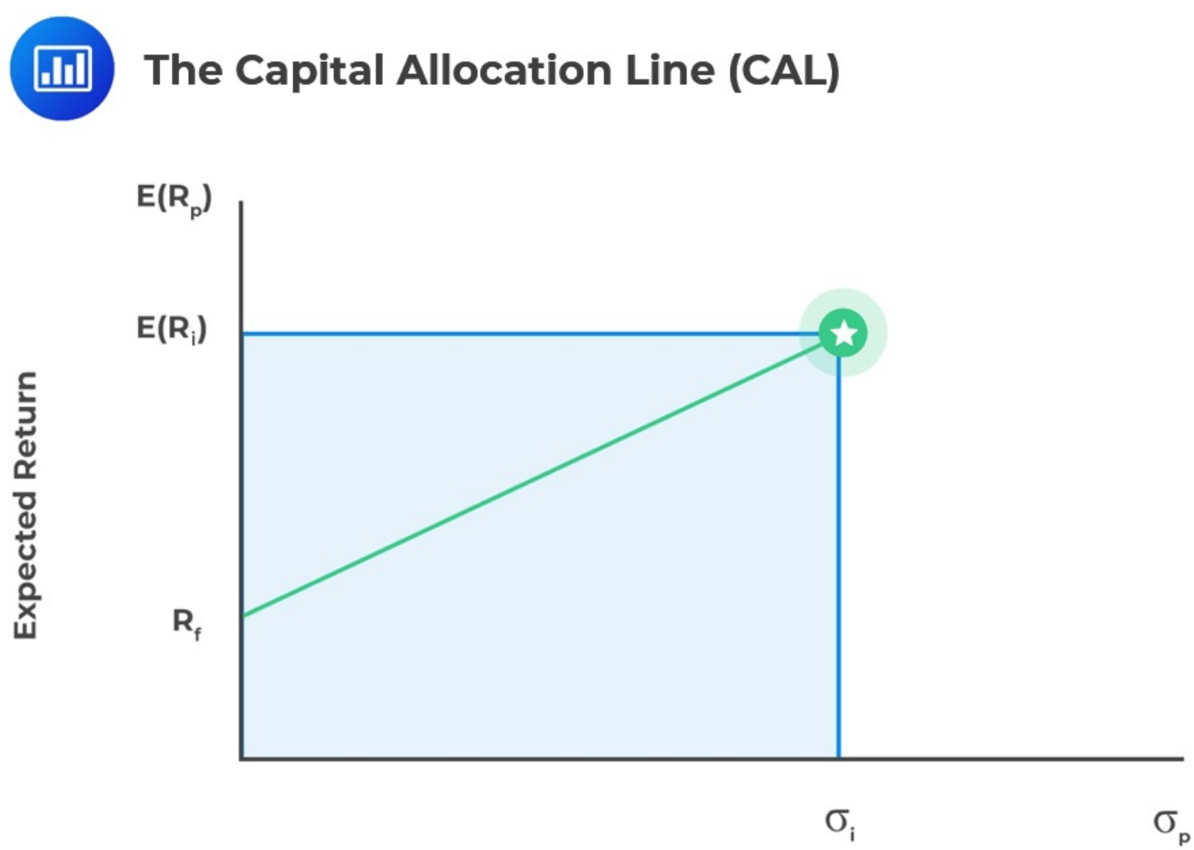
A risk-averse investor will likely select the guaranteed option instead of the uncertain outcome, even though the uncertain outcome has a higher expected return of $50.

LOS 1c: explain the selection of an optimal portfolio, given an investor’s utility (or risk aversion) and the capital allocation line

Risk-free assets are usually government-issued with no risk. When you combine them with risky assets, you create a capital allocation line on a graph. This line connects the best risky portfolio to the risk-free asset.

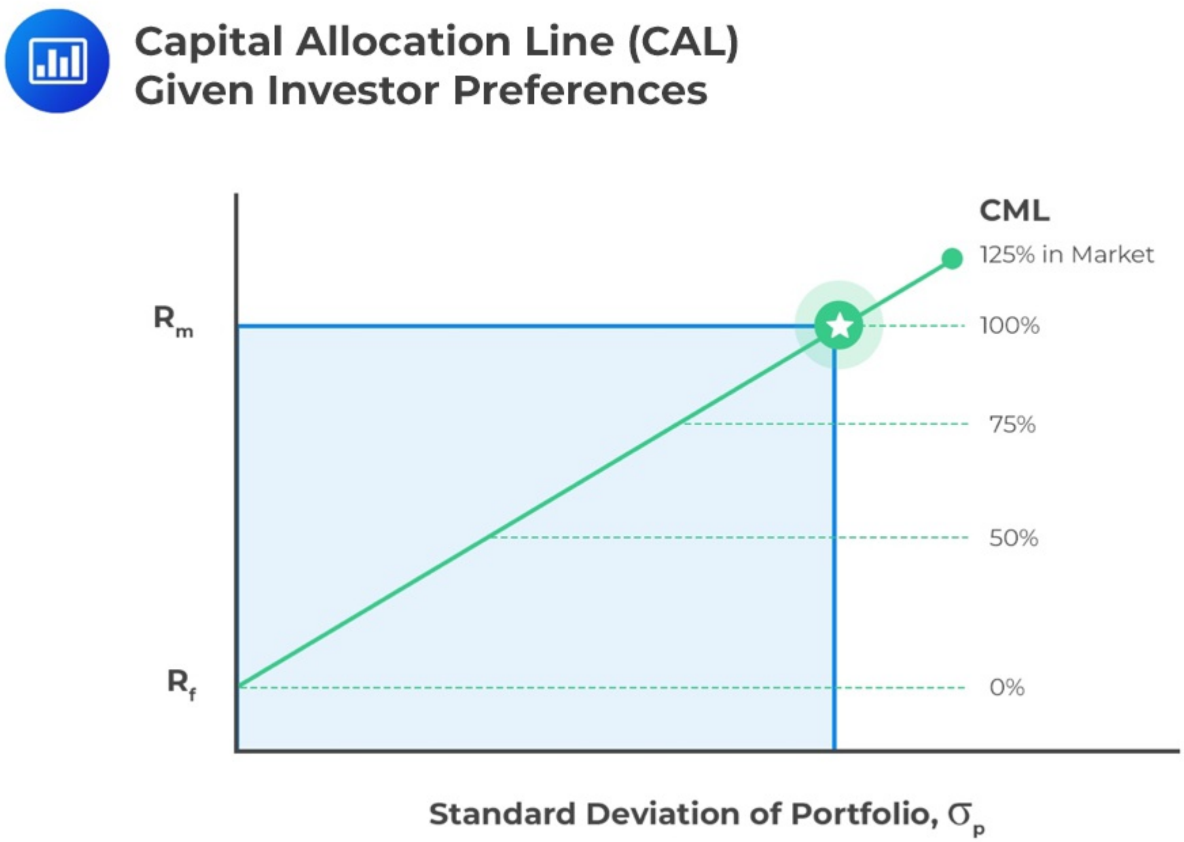
The Two-fund Separation Theorem

The two-fund separation theorem says all investors, no matter their preferences or wealth, use two funds: a risk-free one and a portfolio of risky assets. This splits portfolio building into two steps: first, we pick the best mix of risky assets based on their characteristics. Then, we decide how much to allocate to the risk-free asset based on the investor's risk preference. Combining the risk-free asset with the risky portfolio makes the capital allocation line (CAL) on a graph.



Investor Preferences

A highly risk-averse investor may choose to invest only in a risk-free asset. On the contrary, a less risk-averse investor may have a small portion of their wealth invested in the risk-free asset and a large portion invested in the risky portfolio. An investor with a high-risk tolerance may, in fact, choose to borrow from the risk-free asset and invest in a risky portfolio. This enables the investor to invest more than 100% of their assets and create a leveraged portfolio.



Utility and Indifference Curves

Utility is a measure of relative satisfaction that an investor derives from different portfolios. We can generate a mathematical function to represent this utility that is a function of the portfolio's expected return, the portfolio variance, and a measure of risk aversion.

$$U = E(r) - \frac{1}{2}A\sigma^2$$

Where:

U = Utility.

E(r) = Portfolio expected return.

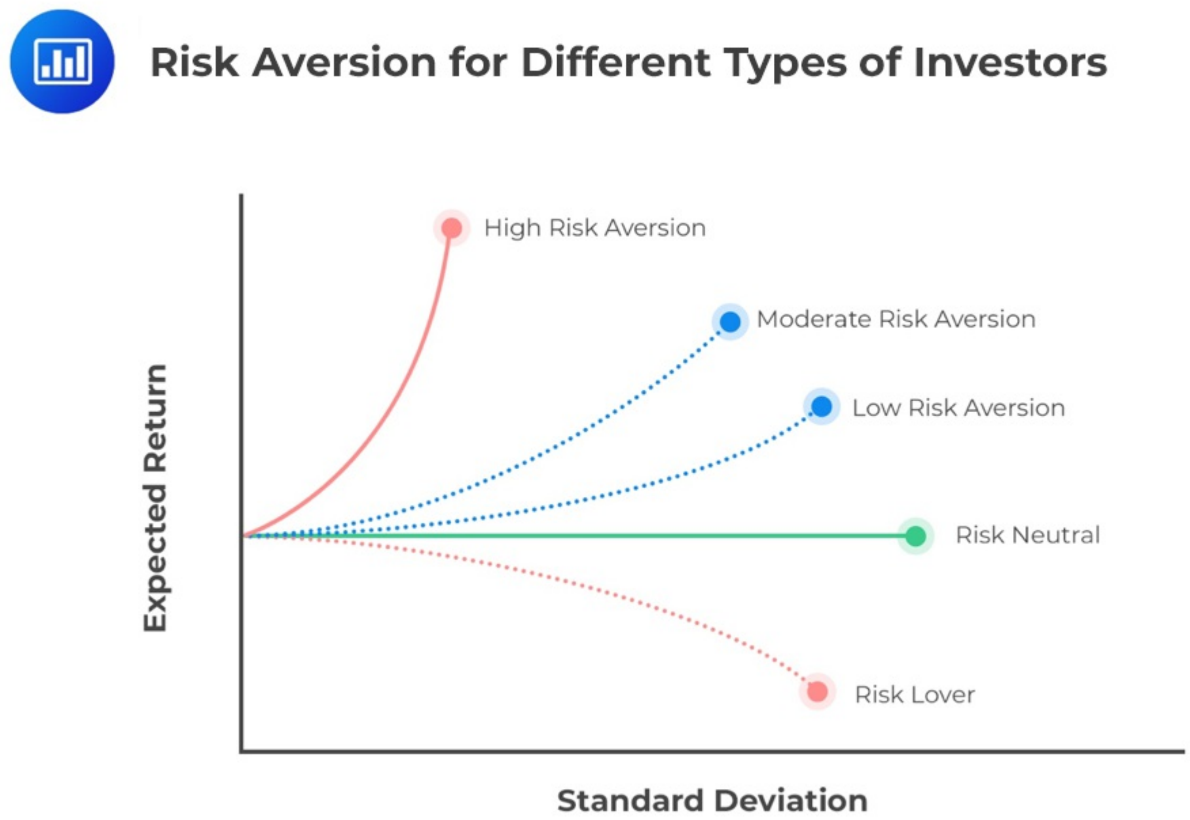
A = Risk aversion coefficient.

σ^2 = portfolio variance.

To determine risk aversion (A), we measure the marginal reward an investor needs in order to take more risk. A risk-averse investor will need a high-margin reward for taking more risks. The utility equation shows the following:

- Utility can be positive or negative – it is unbounded.
- High returns add to utility.
- High variance reduces utility.
- Utility does not measure satisfaction but can be used to rank portfolios.

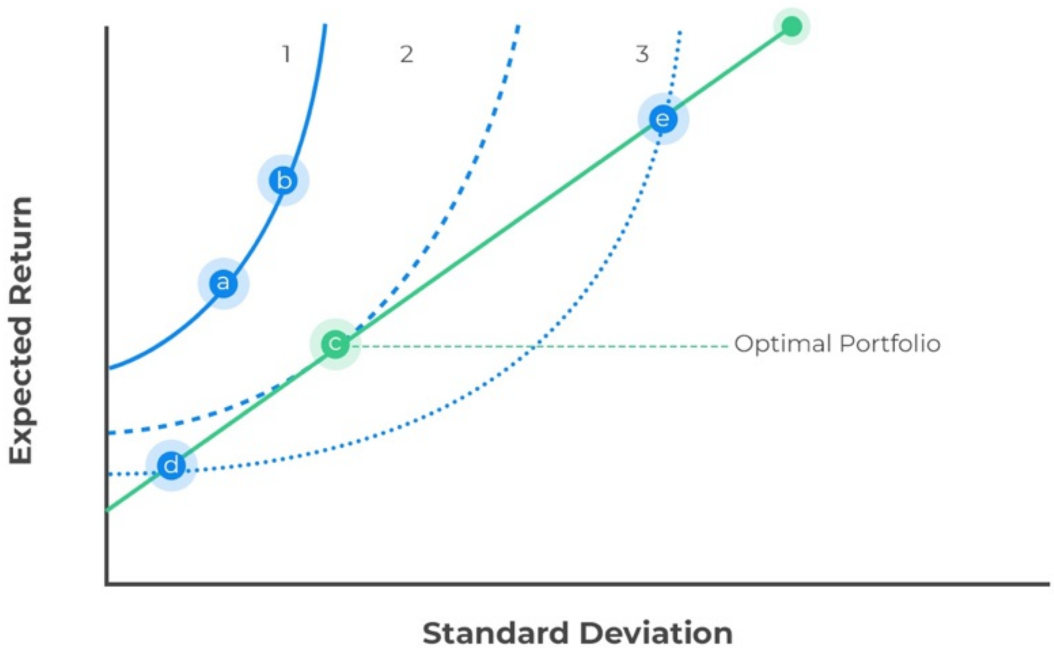
The risk aversion coefficient, A, is positive for risk-averse investors (any increase in risk reduces utility). It is 0 for risk-neutral investors (changes in risk do not affect utility) and negative for risk-seeking investors (additional risk increases utility).



An indifference curve plots the combination of risk and returns that an investor would accept for a given level of utility. For risk-averse investors, indifference curves run "northeast" since an investor must be compensated with higher returns for increasing risk. It has the steepest slope. A more risk-seeking investor has a much flatter indifference curve as their demand for increased returns as risk increases is much less acute.

We can overlay an investor's indifference curve with the capital allocation line to determine their optimal portfolio.

Optimal Portfolio Given Different Utility Functions



Question

Using the utility function $U = E(r) - \frac{1}{2}A\sigma^2$ and assuming A = -4, which of the following statements best describes the investor's attitude to risk?

A. The investor is risk-neutral.

B. The investor is risk-averse.

C. The investor is risk-seeking.

Solution

The correct answer is **C**.

A negative risk aversion coefficient (A = -4) means the investor receives a higher utility (more satisfaction) for taking more portfolio risk. A risk-averse investor would have a risk aversion coefficient greater than 0, while a risk-neutral investor would have a risk aversion coefficient equal to 0.

LOS 1d: calculate and interpret the mean, variance, and covariance (or correlation) of asset returns based on historical data

Investors seek to manage portfolio risk while maintaining returns. This involves understanding portfolio risk components. Diversification, particularly with assets having low correlations, can mitigate risk without necessarily lowering returns. Portfolio return is the weighted average of individual asset returns. Portfolio risk, however, isn't simply a weighted average due to correlations among assets. Covariance and correlation measures help quantify portfolio risk, simplifying the computation of variance for a two-asset portfolio.

The computation of mean, variance, and covariance statistics allows portfolio managers to compare the underlying securities' return-risk characteristics and potential portfolio impact. These metrics are quantitatively determined and rely on historical price or return data. While we can compute the historical profile, this does not necessarily mean the relationship between assets or their return-risk profile will remain the same in the future.

Mean

The mean of a set of values or measurements is the sum of all the measurements divided by the sum of all the measurements in the set:

$$\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n}$$

If we compute the population's mean, we call it the parametric or population mean, denoted by ? (read “mu”). If we get the mean of the sample, we call it the sample mean, denoted by the x bar.

Population vs. Sample

A population refers to the summation of all the elements of interest to the researcher.

- Examples: The number of people in a country, the number of hedge funds in the U.S., or even the total number of CFA candidates in a given year.

A sample is just a set of elements that represent the population as a whole. By analyzing sample data, we are able to make conclusions about the entire population.

- For example, if we sample the returns of 30 hedge funds spread across the U.S., we can use the results to make reasonable conclusions about the market as a whole (well over 10,000 hedge funds).

Variance

Variance is a measure of dispersion around the mean and is statistically defined as the average squared deviation from the mean. It is noted using the symbol ?².

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

Where ? is the population mean, and N is the population size.

The standard deviation, ?**,** is the square root of the variance and is commonly referred to as the volatility of the asset. Essentially, it is a measure of how far, on average, the observations are from the mean. A population's variance is given by:

The population standard deviation equals the square root of the population variance. The sample variance is given by:

$$S^2 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{n - 1}$$

Where X-bar is the sample mean, and n is the sample size.

Note that the sample standard deviation equals the square root of the sample variance.

Covariance

Covariance is a measure of how closely two assets move together. In covariance, we focus on the relationship between the deviations of some two variables rather than the deviation from the mean of one variable.

If the means of random variables X and Y are known, then the covariance between the two random variables can be determined as follows:

$$\hat{\sigma}_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)$$

If we do not know the means, then the equation changes to:

$$\hat{\sigma}_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)$$

Correlation

Correlation is a concept that is closely related to covariance in the following way:

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Correlation ranges between +1 and -1 and is much easier to interpret than covariance. Two variables are perfectly correlated if their correlation is equal to +1. Note that they are uncorrelated if their correlation equals 0 and move in perfectly opposite directions if their correlation equals -1.

Question

In a two-asset portfolio, which combination of assets would result in the most diversified portfolio?

A. Correlation coefficient = 0.75.

B. Correlation coefficient = -0.2.

C. Correlation coefficient = 0.

Solution

The correct answer is **B**.

A diversified portfolio is produced, and portfolio risk is lowered within a two-asset portfolio by combining negatively correlated assets.

LOS 1e: calculate and interpret portfolio standard deviation

The standard deviation of a portfolio of assets, or portfolio risk, is simply not the sum of the risk of the underlying securities. Due to the correlation between securities, the computation of portfolio risk must incorporate this correlation relationship.

Computing Portfolio Standard Deviation

The portfolio standard deviation and variance are important. They involve the variance of the assets and the covariance between asset pairs. For a portfolio with assets X and Y, the portfolio variance can be calculated as follows:

$$\text{Portfolio variance} = w_X^2\sigma_X^2 + w_Y^2\sigma_Y^2 + 2w_X w_Y \sigma_X \sigma_Y \rho_{XY}$$

Therefore,

$$\text{Portfolio standard deviaton} = \sqrt{w_X^2\sigma_X^2 + w_Y^2\sigma_Y^2 + 2w_X w_Y \sigma_X \sigma_Y \rho_{XY}}$$

Where:

w = Weight of the asset within the portfolio.

σ = Standard deviation.

ρ = Correlation coefficient.

Note that $\sigma_X \sigma_Y \rho_{XY} = \text{Covariance}_{XY}$

Question

Consider two assets in a portfolio. Asset A has an allocation of 80% and a standard deviation of 16%. Asset B has an allocation of 20% and a standard deviation of 25%. The correlation coefficient between asset A and asset B is 0.6. In this case, the portfolio standard deviation is *closest to*:

A. 16.3%.

B. 2.7%.

C. 22%.

Solution

The correct answer is **A**.

We determine the portfolio variance as follows:

$$\text{Portfolio variance} = (0.8)^2 \times (0.16)^2 + (0.2)^2 \times (0.25)^2 + 2(0.8)(0.2)(0.16)(0.25)(0.6)$$

Then, we use the square root of the variance to get the standard deviation:

$$\text{Portfolio standard deviation} = \sqrt{2.66\%} = 16.3\%$$

LOS 1f: describe the effect on a portfolio’s risk of investing in assets that are less than perfectly correlated

The portfolio standard deviation, or risk, is not simply the addition of the risk of each portfolio holding. The interaction between portfolio holdings contributes to the overall portfolio risk.

Correlation

Correlation is a statistical measure of the relationship between two series. The series need not pertain to financial assets. In the context of a portfolio, the series will consist of the historical returns of two potential portfolio constituents.

When the returns move in "lockstep" with one another, they are said to be perfectly correlated and have a correlation coefficient of +1. The converse implies a correlation coefficient of -1.

When you put assets together in a portfolio with correlation coefficients less than +1 (they don't have to be negatively correlated), it reduces the overall risk of the portfolio. Having uncorrelated assets means they don't move together in the same direction all the time. This risk diversification leads to a portfolio with less volatility, and different assets contribute to the portfolio's return at various times.

Correlation plays a crucial role in risk diversification. Assets with negative correlations, like Beachwear and DVD rental, move in opposite directions, providing a hedge against risk. Lower correlations generally mean lower risk, but finding assets with significantly low correlations can be challenging.

Historical returns may not always predict future returns accurately, but historical risk tends to remain relatively stable. Correlations among assets within the same country are consistent, while intercountry correlations have risen due to globalization.

Diversification across asset classes, countries, and industries is key to mitigating risk. Index funds offer a cost-effective way to diversify, especially for small portfolios. Investing in foreign countries and avoiding over-investment in one's employer's stock are also recommended diversification strategies.

Insurance and investments with negative correlations, like gold, can further reduce portfolio risk. Options such as put options provide protection against significant losses, albeit with associated costs. Overall, diversification is vital for a resilient portfolio that can weather market fluctuations.

Question

Given the following correlation coefficients, which two-asset portfolio combination is likely to exhibit the lowest risk?

- Asset A - Asset B correlation = 0.7.
- Asset A - Asset C correlation = 0.3.
- Asset B - Asset C correlation = 0.5.

A. Portfolio AB.

B. Portfolio AC.

C. Portfolio BC.

Solution

The correct answer is **B**.

The portfolio with the lowest correlation between underlying assets is likely to have the lowest portfolio risk. An understanding of the standard deviations of the underlying assets, as well as the allocation to those assets, would be required to give a definite answer.

LOS 1g: describe and interpret the minimum-variance and efficient frontiers of risky assets and the global minimum-variance portfolio

In theory, we could form a portfolio made up of all investable assets. However, this is not practical, and we must find a way to filter the investable universe. A risk-averse investor wants to find a combination of portfolio assets that minimizes risk for a given level of return.

Minimum-variance Frontier

When constructing a portfolio, it's important to consider both the expected return and the level of risk involved. These portfolio characteristics depend on the assets included and how they interact with each other, which is measured through correlation. To explore various investment opportunities, we adjust the allocation to each asset. Different allocations create portfolios with distinct risk and return profiles. These profiles can be visually represented on a graph, with the expected return on one axis and the standard deviation on the other. This visualization helps investors make informed decisions about their portfolios.

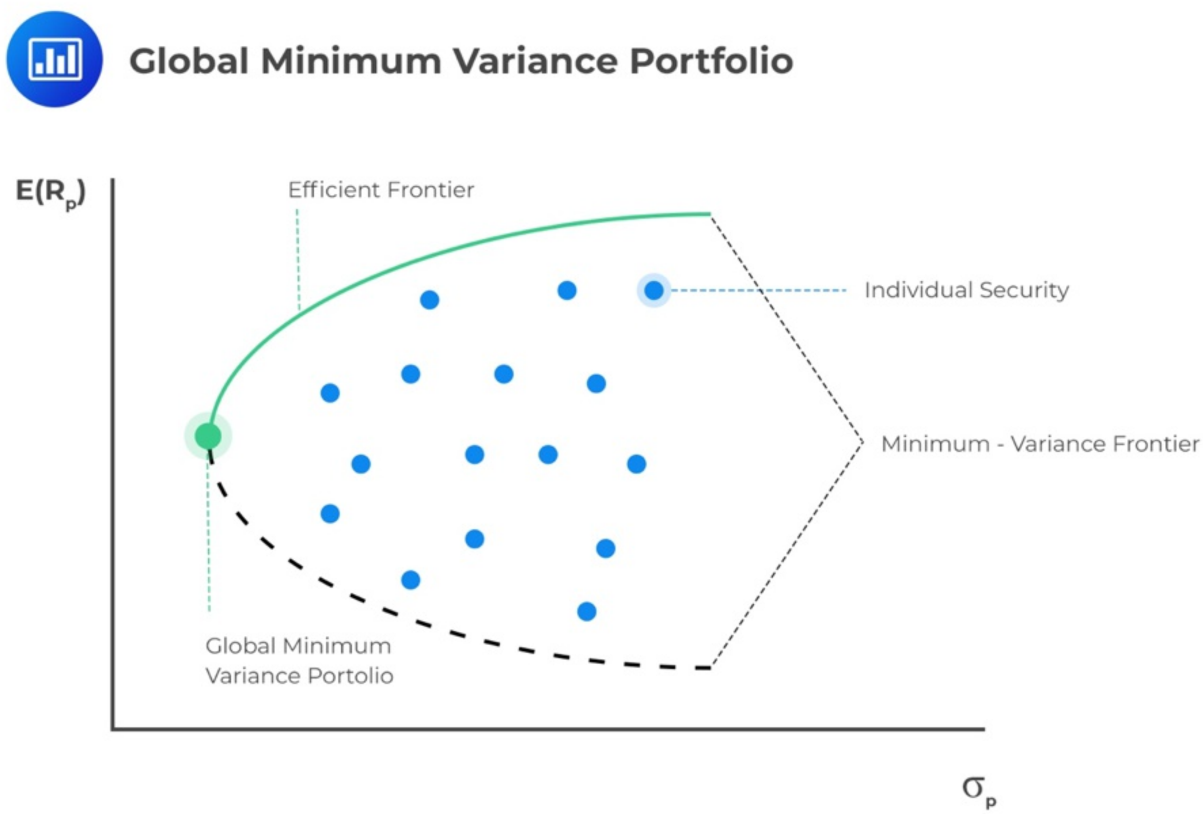
For each level of return, the portfolio with the minimum risk will be selected by a risk-averse investor. This minimization of risk for each level of return creates a minimum-variance frontier – a collection of all the minimum-variance (minimum-standard deviation) portfolios. At a point along this minimum-variance frontier curve, there exists a minimum-variance portfolio that produces the highest returns per unit of risk.

Global Minimum-variance Portfolio

Along the minimum-variance frontier, the left-most point is a portfolio with minimum variance when compared to all possible portfolios of risky assets. This is known as the global minimum-variance portfolio. An investor cannot hold a portfolio of risky (note: risk-free assets are excluded at this point) assets with a lower risk than the global minimum-variance portfolio.

Efficient Frontier

The portion of the minimum-variance curve that lies above and to the right of the global minimum variance portfolio is known as the Markowitz efficient frontier. It contains all portfolios that rational, risk-averse investors would choose. We can also monitor the slope of the efficient frontier, the change in units of return per unit of risk. As we move to higher levels of risk, the resulting increase in return begins to diminish. The slope begins to flatten. This means we cannot achieve ever-increasing returns as we take on more risk, quite the opposite. Investors experience a diminishing increase in potential returns as portfolio risk increases.



Question

Which statement *best describes* the global minimum-variance portfolio?

A. The global minimum variance portfolio gives investors the highest levels of returns.

B. The global minimum variance portfolio gives investors the lowest risk portfolio made up of risky assets.

C. The global minimum variance portfolio lies to the right of the efficient frontier.

Solution

The correct answer is **B**.

The global minimum variance portfolio lies to the far left of the efficient frontier. It is made up of a portfolio of risky assets that produces the minimum risk for an investor.