Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model

Q.3898 Consider a call option on a stock price that is currently at $50. The exercise price of the option is $52, and the risk-free rate is 5%. If the stock price can rise by 20% or fall by 25%, the value of the option using the one-period binomial model is *closest to:*

A. 0.67.

B. 2.11.

C. 5.10.

The correct answer is **C.**

Consider the following one-period binomial model;

$$u = 1.2 \quad S_1^+ = S_0 u = 50 \times 1.2 = 60 \quad c_1^+ = \max(0, S_1^+ - X) = 60 - 52 = 8$$

$$S_0 = \$50, \quad c_0 = ?$$

$$d = 0.75 \quad S_1^- = S_0 d = 50 \times 0.75 = 37.5 \quad c_1^- = \max(0, S_1^- - X) = 0$$

Using the binomial model, the value of the call option is given by:

$$c_0 = \frac{[\pi c_1^+ + (1-\pi) c_1^-]}{1+r}$$

where $\pi = \frac{1+r-d}{u-d} = \frac{1.05-0.75}{1.2-0.75} = 0.67$

$$c_0 = \frac{0.67 \times 8 + 0.33 \times 0}{1.05} = \$5.10$$

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (a): Explain how to value a derivative using a one-period binomial model.

Q.4189 An analyst is considering buying a one-year call option on a non-dividend-paying stock with an exercise price of $100. The current stock price is $95. The stock price is expected to go up or down by 18% in one year. Assume a risk-free rate of return is 4%. The number of units that the analyst needs to buy to create a risk-free portfolio is *closest to*:

A. 0.35

B. 0.45

C. 2.90

The correct answer is **A.**

This question requires us to calculate the hedge ratio. Denote the initial stock price by and consider the following diagram:

$$R^u = 1.18 \qquad S_1^u = S_0 R^u = \$95 \times 1.18 = \$112.10$$

$$c_1^u = \max(0, S_1^u - X)$$

$$\$112.10 - \$100 = \$12.10$$

$$\nearrow$$

$$S_0 = \$95$$

$$c_0 = ?$$

$$\searrow$$

$$R^d = 0.82 \qquad S_1^d = S_0 R^d = \$95 \times 0.82 = \$77.90$$

$$c_1^d = \max(0, 77.90 - 100) = \$0$$

The hedge ratio is given by:

$$h = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

$$= \frac{12.10 - 0}{112.10 - 77.90} = 0.353$$

Therefore, to create a risk-free portfolio, the analyst can buy a put option and 0.35 units of the underlying assets (or for each purchased unit of the underlying asset, we buy 2.9 units).

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (a): Explain how to value a derivative using a one-period binomial model.

Q.4190 Consider a one-year put option on a non-dividend paying stock with an exercise price of $50. The current stock price is $47. The stock price is expected to go up or down by 25%. The value of the hedged portfolio today, if the risk-free rate of return is 4%, is *closest to:*

A. $5.96

B. $35.46

C. $38.66

The correct answer is **B.**

Let the initial price be $S_0 = 47$ and the exercise price by $X = 50$. Therefore, when the stock price goes up, we have:

$$S_1^u = S_0R^u = \$47 \times 1.25 = \$58.75$$

And when the stock price goes down, we have:

$$S_1^d = S_0R^d = \$47 \times 0.75 = \$35.25$$

Consequently,

$$p_1^u = \max(0, X - S_1^u) = \max(50 - 58.5) = 0$$

And

$$p_1^d = \max(0, X - S_1^d) = \max(50 - 35.25) = 14.75$$

The hedge ratio of the option is:

$$h = \frac{p_1^u - p_1^d}{S_1^u - S_1^d} = \frac{0 - 14.75}{58.75 - 35.25} = -0.6277$$

At maturity, the value of a perfectly hedged portfolio is either,

$$V_1^u = hS_1^u + p_1^u = 0.6277 \times 58.75 + 0 = \$36.88$$

or

$$V_1^d = hS_1^d + p_1^d = 0.6277 \times 35.25 + 14.75 = \$36.88$$

You can either use the V_1^u or V_1^d to calculate the present value of the hedged position today as:

$$V_0 = V_1(1+r)^{-1} = 36.88(1.04)^{-1} = \$35.4615$$

A is incorrect. It gives the no-arbitrage put option price:

$$\begin{aligned} p_0 &= V_0 - hS_0 \\ &= \$35.4615 - 0.6277 \times 47 = \$5.96 \end{aligned}$$

C is incorrect. It is the value of the hedged portfolio at maturity.

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (a): Explain how to value a derivative using a one-period binomial model.

Q.4191 Which of the following is *most likely* true regarding the price of a call option if the up gross return is increased in a one-period binomial model? The price of a call option will:

A. increase

B. decrease

C. remain the same

The correct answer is **A.**

In a one-period binomial model, if the up gross return is increased, it is more likely that the stock price will increase in the future, making the call option more valuable.

Recall that in a one-period binomial model is that at maturity, an asset's spot price, S_o, can either increase to S_1^u, or decrease to S_1^d.

As a result, the call option price is expected to increase. The gross return, when the asset price increases, will be $R^u = \frac{S_1^u}{S_0}, > 1$, and when the asset price decreases, the gross return will be $R^d = \frac{S_1^d}{S_0} < 1$.

$$\begin{array}{lll} & \text{Prob} = q & S_1^u = S_0R^u \\ & \nearrow & \\ S_0 & & \\ & \searrow & \\ & \text{Prob} = 1 - q & S_1^d = S_0R^d \end{array}$$

After one year, the option expires. At this time, the value of the option will either be $c_1^u = \max(0, S_1^u - X) = S_1^u - X$ if the underlying price rises to S_1^u or $c_1^d = \max(0, S_1^u - X) = 0$ if the underlying price falls to S_1^d.

Clearly, the call option price is expected to increase if the upward gross return increases.

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (a): Explain how to value a derivative using a one-period binomial model.

Q.4192 Consider a one-year call option on a non-dividend stock. The current stoke price is $70. The value of the perfectly hedged portfolio (combination of call option and underlying asset) after one year is $16.50. If the hedge ratio is 0.65 and the risk-free rate is 1.5%, the no-arbitrage price of the call option is *closest to:*

A. $16.26

B. $29.24

C. $45.50

The correct answer is **B.**

Recall that the no-arbitrage price of a call option is given by:

$$c_0 = hS_0 - V_0$$

Where

c_0 = non-arbitrage call option price.

S_0 = Initial stock price

V_0 = Present value of the hedged position.

As such, we have:

$$\begin{aligned} c_0 &= hS_0 - V_0 \\ &= 0.65 \times 70 - 16.50(1.015)^{-1} = \$29.24 \end{aligned}$$

A is incorrect. It gives the present value of the hedged position at maturity as:

$$V_0 = V_1(1 + r)^{-1} = 16.50(1.015)^{-1} = \$16.26$$

C is incorrect. It gives the value of $hS_0 = 0.65 \times 70 = 45.5$

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (a): Explain how to value a derivative using a one-period binomial model.

Q.4193 Rose Associates is holding non-dividend stocks of Xerox limited. The stock's current value is USD 100, and in one year, the price can go up by 11% or down by 8%. The current risk-free rate is 3 percent, and the exercise price is USD 99. The risk-neutral probability of an up-move and down-move for a 1-year European call option on the stock is *closest to*:

A. Up-move = 0.58; Down-move = 0.42

B. Up-move = 0.11; Down-move = 0.89

C. Up-move = 0.68; Down-move = 0.32

The correct answer is **A.**

Risk neutral probability of an upward move is given by:

$$\pi = \frac{1 + r - R^d}{R^u - R^d}$$

Where,

R^d = gross return when the asset price decreases.

R^u = gross return when the asset price increases

r = risk-free rate.

Therefore, the risk-neutral probability of the up-move is

$$\pi = \frac{1 + r - R^d}{R^u - R^d}$$
$$= \frac{1.03 - 0.92}{1.11 - 0.92} = 0.58$$

This implies that the risk-neutral probability of a down-move is:

$$1 - \pi = 1 - 0.58 = 0.42$$

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (b): Describe the concept of risk neutrality in derivatives pricing.

Q.4194 Which of the following *best* describes the idea behind the risk-neutral pricing formula in the one-period binomial model? The option price is equal to the:

A. Discounted value of the perfectly hedged position at maturity.

B. Expected value using the real-world probabilities discounted at a risk-free rate.

C. Expected value using the risk-neutral probabilities discounted at a risk-free rate.

The correct answer is **C.**

Recall that the value of a call option is the discounted expected value at expiration at the risk-free rate, as summarized in the equation below.

$$c_0 = \frac{(\pi c_1^u + (1 - \pi)c_1^d}{(1 + r)^T}$$

Similarly, the value of the put option is given by:

$$p_0 = \frac{(\pi p_1^u + (1 - \pi)p_1^d}{(1 + r)^T}$$

Clearly, the risk-neutral price of an option is the risk-neutral expected payoff discounted at the risk-free rate of interest.

A is incorrect. This is applied in the no-arbitrage pricing of the option before calculating the no-arbitrage the price of an option.

B is incorrect. Real-world probabilities are not required when calculating the value of an underlying asset.

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model. LOS (b): Describe the concept of risk neutrality in derivatives pricing.

Q.4195 John Crewe is an analyst at Predict Inc. Some of Crewe's clients have significant non-dividend holdings at Finlay, a tea processing company in Kenya. Crewe anticipates Finlays' stock price will rise next year and advises his clients to buy one-year call options at an exercise price of $80. Finlay's spot price is $80, and the risk-free interest is 0.45%. Crewe estimates that there is an equal chance that the stock price will rise or fall by 10%. The risk-neutral price of the option is *closest to*:

A. $4.16.

B. $4.78

C. $8.00.

The correct answer is **A.**

If Finlay's stock price rise by 10%, then:

$$c_1^u = \max(S_0 \times R^u - X) = \max(0, 80 \times 1.10 - 80) = 8.00$$

And

$$c_1^d = \max(S_0 \times R^d - X) = \max(0, 80 \times 0.90 - 80) = 0$$

The risk-neutral probability of upward movement is given by:

$$\pi = \frac{1 + r - R^d}{R^u - R^d} = \frac{1.0045 - 0.90}{1.10 - 0.90} = 0.5225$$

Thus, the risk-neutral pricing is, therefore,

$$c_o = \frac{(\pi c_1^u + (1 - \pi)c_1^d}{(1 + r)^T} = \frac{(0.5225 \times 8 + (1 - 0.5225)0}{(1 + 0.0045)^1} = 4.16$$

A is incorrect. Uses the real-world probability to calculate the risk-neutral price of the call option.

$$c_o = \frac{(\pi c_1^u + (1 - \pi)c_1^d}{(1 + r)^T} = \frac{(0.6 \times 8 + (1 - 0.40)0}{(1 + 0.0045)^1} = 4.78$$

B is incorrect. State the payoff of the call option at maturity if the stock price rises as the risk-neutral price.

CFA Level I, Derivatives, Learning Module 10: Valuing a Derivative Using a One-Period

Binomial Model. LOS (b): Describe the concept of risk neutrality in derivatives pricing.
