Learning Module 7: Yield and Yield Spread Measures for Fixed Rate Bonds

LOS 7a: calculate annual yield on a bond for varying compounding periods in a year

The yield on a bond is a measure of the return on investment, which depends on the interest rate and the frequency of compounding. Understanding how to calculate the annual yield for varying compounding periods is essential for investors to compare different investment options.

Periodicity and Annualized Yields

The periodicity of the annual rate refers to the number of interest periods in a year. It is a crucial factor in the calculation of the effective yield on a bond. The periodicity typically aligns with the frequency of coupon payments:

- **Annual**: Periodicity = 1.
- **Semiannual**: Periodicity = 2.
- **Quarterly**: Periodicity = 4

Calculating Future Value of Cash Flows

The future value (FV) of cash flows represents the total value of a series of payments at a future point in time. It is calculated using the formula:

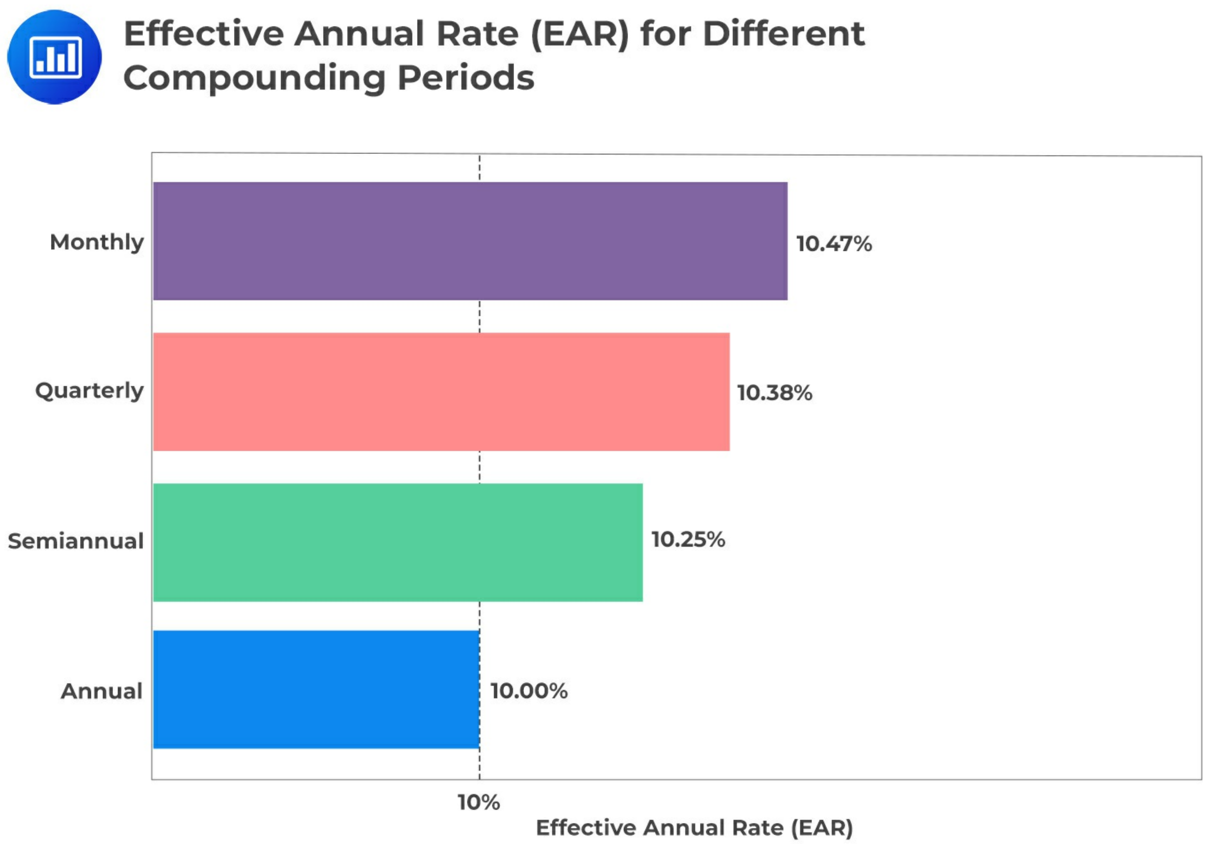
$$FV = FV(\text{rate}, \text{nper}, \text{pmt}, \text{pv}, \text{type})$$

Where:

- rate is the periodic reinvestment rate.
- nper is the number of periods in a year.

- pmt is the rate per period.
- pv is the present value (optional).
- type indicates when payments occur (optional).

The following figure illustrates the Effective Annual Rate (EAR) for different compounding periods, emphasizing the subtle differences in yield:



Though the differences between the EAR values may appear small, they reveal how the frequency of compounding affects the effective yield on an investment. These variations can have a significant impact over time, especially with large investments or longer investment horizons.

Investors can gain insights into the importance of compounding frequency when evaluating different investment options. It allows for a nuanced understanding and comparison between bonds with different payment frequencies, considering the subtle but meaningful variations in yield that arise from changes in compounding periodicity.

Periodicity Conversions

A crucial instrument in the study of fixed-income assets involves transforming an annualized yield from one compounding frequency to another. Such transformations are known as periodicity or compounding adjustments. The following equation is used to convert an annual percentage rate for periods per year, denoted, to an annual percentage rate for periods per year.

$$\left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n$$

Where:

- APR_m is the annual percentage rate for m periods per year.
- APR_n is the annual percentage rate for n periods per year.
- m is the number of periods per year in the original periodicity.
- n is the number of periods per year in the converted periodicity.

Example

An investor is considering a 3-year bond that pays a 6% annual coupon with a face value of $1,000. The bond is currently priced at $950. Calculate the effective annual rate of the bond at issuance assuming annual, semiannual, and quarterly compounding.

Solution

Parameters:

- Face Value (F): $1,000.
- Current Price (P): $950.

- Time to Maturity: 3 years.
- Coupon Rate: 6%.
- Coupon Payment: $60 (6% of $1,000).

Calculate the yield for Different Compounding Periods

Annual Compounding

$$950 = \frac{60}{(1+r)^1} + \frac{60}{(1+r)^2} + \frac{1000+60}{(1+r)^3}$$

r has been calculated using Excel as 7.938%

Semi-annual compounding (n=2)

Convert from a periodicity of m = 1 to periodicities of n = 2 and n = 4

This converts the effective annual rate to semiannual and quarterly bond equivalent yield.

$$(1 + \frac{0.07938}{1})^1 = (1 + \frac{APR_2}{2})^2$$

$$\sqrt{(1.07938)} = 1 + \frac{APR_2}{2}$$

$$APR_2 = 7.786\%$$

$$(1 + \frac{0.07938}{1})^1 = (1 + \frac{APR_2}{2})^2$$

Quarterly compounding (n=4)

$$(1+\frac{0.07938}{1})^1 = (1+\frac{APR_4}{4})^4$$

$$APR_4 = 7.712\%$$

Explanation

Annual Compounding: The bond yield is highest when compounded annually. This is because there is less frequent reinvestment of the coupon payments, leading to a lower accumulation of interest.

Semiannual and Quarterly Compounding: As the frequency of compounding increases, the YTM decreases slightly. This is due to the more frequent reinvestment of coupon payments, which compounds more often and thereby increases the effective yield of the bond.

Thus, varying compounding frequencies can affect the bond's yield, usually leading to a decrease in YTM as the frequency increases.

Question

A yield of 2.432% compounded quarterly is closest to an effective annual rate of:

A. 0.430%.

B. 2.447%.

C. 2.454%.

The correct answer is **C**.

The effective annual rate assumes a single compounding period in a year (periodicity of 1). To convert from quarterly to annual compounding, we need to change from a periodicity of 4 to 1:

$$\left(1+\frac{APR_m}{m}\right)^m=\left(1+\frac{APR_n}{n}\right)^n$$

$$\left(1+\frac{0.02432}{4}\right)^4=\left(1+\frac{APR_1}{1}\right)^1$$

$$APR_1 = 2.454\%$$

LOS 7b: compare, calculate, and interpret yield and yield spread measures for fixed-rate bonds

Yield Measures for Fixed-Rate Bonds

Understanding yield measures for fixed-rate bonds is essential for investors aiming to assess the potential return on a bond investment. The yield measures, including current yield, yield to maturity (YTM), yield to call (YTC), and yield to worst (YTW), offer different perspectives on the potential return, each taking into account various factors.

Overview of Yield Conventions

Actual/Actual

This convention counts the real number of days from the previous coupon payment to the settlement date and divides it by the actual number of days in a coupon period based on the actual number of days in a year. It is generally applied to government bonds.

30/360

It assumes 30 days in a month and 360 days in a year while calculating the number of days from the last coupon payment to the settlement date. It is most commonly used for corporate bonds.

Street Convention

A yield calculation that does not take weekends and bank holidays into account, assuming that cash flows are received on scheduled dates.

True Yield

True yield adjusts for weekends and bank holidays, assuming cash flows will be received after their scheduled dates. It is never greater than the street convention yield due to the time delay in payment.

Government Equivalent Yield

This converts the Yield-to-Maturity from a 30/360 day count to an actual/actual day count. It is used to adjust the Yield-to-Maturity (YTM) of a corporate bond to determine its spread above the corresponding government bond YTM.

Simple Yield

This is a yield metric calculated as the sum of coupon payments plus the straight-line amortized portion of any gain or loss, all divided by the flat price of the bond. It is mainly used to quote Japanese government bonds (JGBs).

Current Yield

The **current yield** is a simple measure that represents the annual income (interest) as a proportion of the bond's current market price. This measure only focuses on interest income and ignores the time value of money, frequency of coupon payments, and accrued interest. It is calculated as:

$$\text{Current Yield} = \frac{\text{Annual Income}}{\text{Current Market Price}}$$

Yield to maturity (YTM)

The **Yield to Maturity (YTM)** is a comprehensive measure, representing the total anticipated return if the bond is held to maturity. It takes into account both the current market price and the total amount of interest payments.

To solve for YTM using the bond pricing equation:

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \cdots + \frac{PMT_N + FV_N}{(1+r)^N}$$

Where:

- PV = Current market price
- PMT = Periodic Coupon payments
- r = Yield-to-maturity
- FV = Face value of the bond
- N = the remaining number of periods until maturity

Yield Measures for Callable Bonds

Special consideration is required for fixed-rate bonds with embedded options, such as callable bonds. These bonds have features that allow the issuer to buy back the bond at specific prices on predetermined dates, requiring alternative yield measures.

Yield to Call (YTC)

The Yield to Call (YTC) considers the possibility of the bond being called prior to maturity. This yield is valid only if the bond is called and is calculated as

$$PV = \frac{PMT}{(1+r)} + \frac{PMT}{(1+r)^2} + \ldots + \frac{PMT + \text{Call Price}}{(1+r)^N}$$

Yield to Worst (YTW)

The Yield to Worst (YTW) is the most conservative measure, representing the minimum yield that can be received without the issuer defaulting. It is calculated by considering the worst-case scenario, including provisions like prepayments or callback, and is the minimum of YTM and YTC.

$$\text{Yield to Worst} = \text{Minimum}(\text{Yield to Maturity}, \text{Yield to Call}).$$

Yield Spread Measures for Fixed-Rate Bonds

Yield spread measures are used to compare bonds by evaluating the differences between their yields. These measures are crucial for understanding the relative value and risks associated with different bonds.

Absolute Yield Spread

This measure represents the difference in yield between two bonds, usually measured in basis points (bps), and is calculated as

$$\text{Absolute Yield Spread} = \text{Yield of Bond A} - \text{Yield of Bond B}.$$

Relative Yield Spread

The relative yield spread normalizes the absolute yield spread by dividing it by the yield of the benchmark bond, expressing it as a percentage. It is calculated as:

$$\text{Relative Yield Spread} = \frac{\text{Absolute Yield Spread}}{\text{Yield of Benchmark Bond}}$$

Yield Ratio

The yield ratio provides a comparison between the yield of a bond and a benchmark bond, expressed as a ratio, and is calculated as

$$\text{Yield Ratio} = \frac{\text{Yield of Bond}}{\text{Yield of Benchmark Bond}}$$

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Example: Yield Measures for Fixed-Rate Bonds

Consider a 5-year, semiannual-pay 3% callable bond with a face value of $1,000. The bond can be called each year after 3 years at a call price of $1,050. An investor is considering purchasing the bond at a current market price of $980. Calculate the Current Yield, Yield to Maturity (YTM), Yield to Call (YTC), and Yield to Worst (YTW) for this bond. Additionally, if there is a similar non-callable bond in the market with a yield of 5.8%, calculate the Absolute Yield Spread, Relative Yield Spread, and Yield Ratio between the callable bond and the benchmark non-callable bond.

Solution

- Face Value: $1,000
- Coupon Rate: 6%
- Current Market Price: $980
- Maturity: 5 years (10 periods)
- Callable after 3 years, with subsequent calls every year
- Yield of Benchmark Non-Callable Bond: 5.8%

Current Yield:

Since the coupon rate is 6%, the annual coupon payment is calculated as:

$$\text{Annual Income} = 6\% \times \$1,000 = \$60.$$

The current yield is a measure that represents the annual income as a proportion of the bond's current market price. It can be calculated using the formula:

$$\text{Current Yield} = \frac{\text{Annual Income}}{\text{Current Market Price}}$$

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By substituting the given values into the formula:

$$\text{Current Yield} = \frac{\$60}{\$980} \approx 6.12\%$$

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Yield to Maturity (YTM):

Since the coupon rate is 6%, the annual coupon payment is $60. The bond pays coupons semi-annually, so the semi-annual coupon payment is:

$$\text{Coupon Payment} = \frac{\$60}{2} = \$30.$$

The Yield to Maturity is the interest rate that equates the present value of the bond's future cash flows to its current market price. It can be found by solving the following equation:

$$PV = \frac{\text{Coupon Payment}}{(1+r)} + \frac{\text{Coupon Payment}}{(1+r)^2} + \ldots + \frac{\text{Coupon Payment} + \text{Face Value}}{(1+r)^{10}}$$

where r is the yield per period (YTM/2).

Yield to Maturity ≈ 6.475%.

The Yield to Maturity (YTM) for the given callable bond is approximately 6.475%, representing the total anticipated return if the bond is held to maturity.

Yields to Call:

To calculate the yield-to-first call, we calculate the yield-to-maturity using the number of semiannual periods until the first call date of 10 and the call price of 1,050.

$$PV = \frac{\text{Coupon Payment}}{(1+r)} + \frac{\text{Coupon Payment}}{(1+r)^2} + \ldots + \frac{\text{Coupon Payment} + \text{Call Price}}{(1+r)^N}$$

where r is the yield per period (Yield to Call/2), and N is the number of periods to the call date.

Assuming that the bond can be called every year after the 3rd year, we will calculate the yield to the first, second, and third call. Yields to Calls:

Yield to First Call: 8.269%

Yield to Second Call: 7.682%

Yield to Third Call: 7.331%

For the given callable bond, the decreasing pattern in the yield to calls is observed due to the fixed call price of $1,050 and the semi-annual coupon payments. The yield to call accounts for both the call price and the remaining interest payments. As we move closer to maturity, the present value of the fixed call price becomes a larger proportion of the overall yield calculation, and the effect of the remaining interest payments diminishes. Since the call price is greater than the current market price but less than the face value, the yields to calls lie between the current yield and the YTM. This specific pattern leads to decreasing yields to calls, with the YTM being the lowest of all, reflecting the structure and pricing of the bond.

Yield to Worst Call:

The yield to worst call is the minimum yield among all the possible call dates, and the yield to maturity:

Yield to Worst Call = 6.475%.

The purpose of yield-to-worst is to provide to the investor with the most conservative assumption for the rate of return.

Absolute Yield Spread:

The Absolute Yield Spread is the difference in yield between the callable bond and the benchmark bond. It is calculated as:

$$\text{Absolute Yield Spread} = \text{Yield of Callable Bond} - \text{Yield of Benchmark Bond} = 6.475\% - 5.8\% = 0.6$$

Relative Yield Spread:

The Relative Yield Spread normalizes the absolute yield spread by dividing it by the yield of the benchmark bond. It is calculated as:

$$\text{Relative Yield Spread} = \frac{\text{Absolute Yield Spread}}{\text{Yield of Benchmark Bond}} = \frac{0.675\%}{5.8\%} \approx 11.64\%.$$

Yield Ratio:

The Yield Ratio provides a comparison between the yield of the callable bond and the benchmark bond, expressed as a ratio. It is calculated as:

$$\text{Yield Ratio} = \frac{\text{Yield of Callable Bond}}{\text{Yield of Benchmark Bond}} \approx 1.116.$$

The Absolute Yield Spread, Relative Yield Spread, and Yield Ratio between the callable bond and the benchmark non-callable bond provide insights into the relative value and risks associated with the bonds.

Yield Spreads over Benchmark Rates

Understanding yield spreads is crucial for bond investors. Yield spreads enable analysts to compare the current spread against historical averages and other bonds. This helps in identifying whether a bond is underpriced or overpriced relative to its risk.

Types of Spreads

G-Spread

This is the difference between the yield-to-maturity of a specific bond and a benchmark bond, often a government bond. It represents the return for bearing risks relative to the sovereign bond.

G-Spread = YTM of Bond - YTM of Benchmark Bond

Z-Spread

This represents the constant yield spread over a benchmark yield curve that makes the present value of a bond's cash flows equal to its price. It is used to derive the term structure of credit spreads for an issuer.

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \cdots + \frac{PMT+FV}{(1+z_N+Z)^N}$$

$z_1, z_2, \ldots, z_N$ are the benchmark spot or zero rates obtained from the government yield curve or from fixed rates on interest rate swaps. On the other hand, Z is the Z-spread per period and is the same for all time periods.

Option-Adjusted Spread (OAS)

This is the Z-spread adjusted for the value of any embedded options like call or put options. It is based on an option-pricing model and an assumption about future interest rate volatility.

$\text{OAS} = \text{Z} - \text{Spread} - \text{Option value in basis points per year}$

I-Spread

This is the yield spread of a bond over the standard swap rate in the same currency and with the same tenor. Commonly used for euro-denominated corporate bonds.

Example: Calculating G-spread, I-spread and Z-spread

An analyst is evaluating a 4-year, 3% annual coupon bond issued by XYZ Corp. Currently, the bond's yield-to-maturity (YTM) is 3.5%. The 4-year swap rate is 2.2%. The government spot rates are as follows:

Maturity (Years)	Government Spot Rate (%)
	1.0
2	1.4
3	2.0
4	2.5

The bond's current price is 97.50% of its par value.

The G-spread, I-spread, and Z-spread (in basis points) for the XYZ Corp bond are *closest to*:

G-spread

The G-spread is a yield spread above that of a government bond with the same maturity date. The yield-to-maturity for the corporate bond is 3.5%. The yield-to-maturity for the government benchmark bond is 2.5%.

G-spread =3.5%?2.5%=1%=100 bps.

I-spread

This is the yield spread of a bond over the standard swap rate in the same currency and with the same tenor. The yield-to-maturity for the corporate bond is 3.5%, and the swap rate for the same maturity is 2.2%.

I-spread = 3.5% – 2.2% = 1.30% = 130bps

Z-spread

To calculate the -spread, we must solve for in the following equation, given the spot rates and price of the bond:

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \cdots + \frac{PMT+FV}{(1+z_N+Z)^N}$$

The Solver add-in for Microsoft Excel finds Z = 1.22, or 122bp, by setting the price (sum of present values of cash flows) equal to 97.50 as the objective and Z as the change variable.

Question

A 2-year sovereign non-callable bond is priced at 104.50 per 100. The bond pays a 1.5% semiannual coupon. The annual yield-to-maturity for the bond is closest to:

A. -0.365%

B. -0.730%

C. 1.435%

Solution

The correct answer is B.

To find the yield-to-maturity, we can solve for *r* in the equation for the present value of the bond's cash flows.

$$104.50 = \frac{0.75}{(1+r)^1} + \frac{0.75}{(1+r)^2} + \frac{0.75}{(1+r)^3} + \frac{100.75}{(1+r)^4}$$

$$r = -0.730\%$$