Learning Module 11: Yield Based Bond Duration Measures and Properties

LOS 11b: explain how a bond’s maturity, coupon, and yield level affect its interest rate risk.

The sensitivity of a bond's price to changes in interest rates can be captured using Macaulay duration, modified duration, money duration, and the price value of a basis point (PVBP).

Relationships Between Bond Features and Duration

Coupon rate (c))

An increase in the coupon rate leads to a decrease in duration (inverse relationship). Bonds with lower coupon rates have higher durations. This implies more interest rate risk for lower-coupon bonds.

Yield to Maturity (r)

An increase in the yield to maturity results in a decrease in duration (inverse relationship). A bond with a lower yield-to-maturity will have a higher duration. This is because lower yields emphasize the weight of the bond's later cash flows, especially the maturity value.

Time-to-Maturity

An increase in the time to maturity leads to an increase in duration (direct relationship). Bonds with longer times to maturity will generally have higher durations, suggesting greater interest rate risk. However, a peculiarity arises with long-dated discount bonds: their Macaulay duration can decrease after reaching a certain time-to-maturity.

Fraction of current coupon period elapsed (t/T)

An increase in $\frac{t}{T}$ leads to a decrease in duration (inverse relationship). As more time passes within a coupon period, the Macaulay duration decreases. However, once a coupon is paid, the duration jumps slightly, creating a "saw-tooth" pattern.

Question

Which of the following bonds is most likely to have the highest duration?

A. A bond with a high coupon rate and short time to maturity.
B. A bond with a low coupon rate and long time to maturity.
C. A bond with a high coupon rate and long time to maturity.

Solution

The correct answer is **B.**

Bonds with lower coupon rates and longer times to maturity typically have higher durations. This indicates greater interest rate risk for such bonds.

A is incorrect: A high coupon rate would lead to a lower duration.

C is incorrect: While a long time to maturity increases duration, a high coupon rate decreases it.

LOS 11a: define, calculate, and interpret modified duration, money duration, and the price value of a basis point (PVBP).

Modified Duration

Modified duration captures the sensitivity of a bond's price to fluctuations in its yield-to-maturity (YTM). This relationship provides insight into how bond prices vary with shifts in yield. Specifically, bond prices and yields exhibit an inverse relationship: as yields rise, bond prices fall, and vice versa.

Relation to Macaulay Duration

Modified duration is an extension of the Macaulay duration, which conveys the weighted average time until a bond's cash flows are received. The link between these two measures is encapsulated by the formula:

$$\text{ModDur} = \frac{\text{Macaulay Duration}}{1 + \text{r}}$$

Where r represents the yield per period. To obtain the annual modified duration, divide the modified duration by the bond's number of coupon payments in a year. The larger the modified duration, the more pronounced the bond's price-yield curve becomes, leading to larger price swings for given changes in yield.

Approximating Modified Duration

In cases where the Macaulay duration is not available, the modified duration can be estimated by observing minute variations in bond prices as yields change. This approximation method is especially useful for bonds with embedded options or inherent default risks. The formula for this approximation is:

$$\text{AnnModDur} \approx \frac{(\text{PV}_{-} - \text{PV}_{+})}{2 \times \Delta\text{Yield} \times \text{PV}_0}$$

Where PV_- and PV_+ are bond prices corresponding to decreased and increased yields, respectively. Historically, this method has been highly accurate. To revert to the Macaulay duration, multiply the modified duration by $1 + r$.

Example: Approximating Modified Duration

A 4.5% semiannual-pay fixed-coupon bond is issued at par on 1 June 2026 and matures on 1 June 2030. For a 50bps increase and decrease in yield-to-maturity, PV_+ and PV_- are 98.207 and 101.831, respectively. The approximate modified duration can be determined as follows:

Formula:

$$\text{AnnModDur} \approx \frac{(PV_- - PV_+)}{2 \times \Delta\text{Yield} \times PV_0}$$

$PV_- = 101.831$

$PV_+ = 98.207$

$\Delta\text{Yield} = 50/10000 = 0.005$

$$\text{AnnModDur} \approx \frac{101.831 - 98.207}{2 \times 0.005 \times 100} = 3.624$$

Predicting Price Changes Based on Modified Duration

Modified duration unveils the bond price-yield relationship, allowing predictions of the bond's percentage price alteration in relation to shifts in its YTM. The formula to determine this is:

$$\%\Delta PV^{\text{Full}} \approx -\text{AnnModDur} \times \Delta\text{AnnYield}$$

As an illustration, a bond with a modified duration of 5 would likely experience a 5% price drop if its yield surges by 100 basis points. Hence, bonds with higher modified durations exhibit steeper price-yield curves, making them more susceptible to yield variations. It's crucial to note that this formula offers a linear approximation for the inherently nonlinear price-yield relationship. The inclusion of the negative sign emphasizes the inverse correlation between bond prices and their yields-to-maturity.

Money Duration

While modified duration gauges the percentage price change of a bond given variations in its yield-to-maturity (YTM), money duration provides insights into the price change in terms of currency units. In the U.S., it is also referred to as "dollar duration."

Money duration is calculated using the formula:

$$\text{MoneyDur} = \text{AnnModDur} \times \text{PV}^{\text{Full}}$$

$\text{PV}^{\text{Full~}}$ can be either the bond price as a percent of par value or the currency value of the bond holding.

Using Money Duration, one can estimate the bond price change in currency units for a given change in YTM:

$$\%\Delta \text{PV}^{\text{Full}} \approx -\text{MoneyDur} \times \Delta \text{Yield}$$

Example: Calculating Money Duration

Consider a bond with an annualized modified duration of 5.5, a coupon of 4% and a price of 102. The money duration is closest to:

$$\text{MoneyDur} = \text{AnnModDur} \times \text{PV}^{\text{Full}}$$

$$\text{Money Duration} = 5.5 \times 102$$

This means that for a 1% (or 100 basis points) change in yield, the bond's price will change by $561.

Price Value of a Basis Point (PVBP)

PVBP provides an estimate of the change in the full price of a bond for a minuscule 1bp change in its YTM. PVBP can be determined using the formula:

$$PVBP = \frac{(PV_{-}) - (PV_{+})}{2}$$

This measure is often termed as "PV01" or in the U.S., "DV01" (Dollar Value of 1bp). PVBP is especially handy for bonds where future cash flows are unpredictable, like callable bonds.

Basis Point Value (BPV) is a close relative to PVBP, and it is the product of Money Duration and 0.0001 (1bp).

Question

An investment analyst is reviewing a 4-year bond, issued on 1 January 2024, set to mature on 1 January 2028. This bond features a 4% coupon rate, paid semi-annually, and carries a yield-to-maturity of 6%. The bond's annualized Macaulay duration and Modified duration, respectively, are *closest to:*

A. 3.46 and 3.26
B. 3.69 and 3.48
C. 3.72 and 3.62

Solution

The correct answer is C:

The Macaulay duration is 7.4481. This can be annualized by dividing by the number of coupon payments in a year.

Period	Time to receipt	Cashflow amount	PV	Weights	Time to Receipt*Weight
1	1.0000	2	1.9417	0.0209	0.0209
2	2.0000	2	1.8852	0.0203	0.0406
3	3.0000	2	1.8303	0.0197	0.0591
4	4.0000	2	1.7770	0.0191	0.0764
5	5.0000	2	1.7252	0.0186	0.0928
6	6.0000	2	1.6750	0.0180	0.1081
7	7.0000	2	1.6262	0.0175	0.1224
8	8.0000	102	80.5197	0.8660	6.9279
Total			92.9803	1.0000	7.4481

$$\text{Annualized Macaulay duration} = \frac{7.4481}{2} = 3.72405$$

$$\text{ModDur} = \frac{3.72405}{1.03} = 3.6156$$