

Learning Module 2: The Time Value of Money in Finance

Q.8 If you invest \$100,000 currently in a project paying an 8% interest rate compounded annually, the amount of the investment after three years is *closest to*:

A. \$108,000.00

B. \$108,215.23

C. \$125,971.20

The correct answer is **C**.

The question requires the calculation of the future value of a single-sum investment. Recall that future value is the total to which a present deposit will grow over time when placed in an account paying compound interest.

The formula for calculating FV is as follows:

$$FV = PV \left[1 + \frac{I}{Y} \right]^N$$

Where:

FV = Future Value,

PV = Present Value of the investment.

I/Y = Rate of return per compounding period.

N = Total number of compounding periods.

Therefore;

$$FV = \$100,000[1 + 0.08]^3 = \$125,971.20$$

A is incorrect. It represents the amount after one year using simple interest as follows:

$$FV = \$100,000 + (0.08 \times \$100,000) = \$108,000$$

B is incorrect. The calculation divides the interest (8%) by the number of compounding periods (3 years) as follows:

$$FV = \$100,000 \left[1 + \frac{0.08}{3} \right]^3 = \$108,215.23$$

Q.11 How much is an asset worth today, if it is supposed to pay \$7,000 per year for 10 years? The first payment is due one year from now and the required rate is 6% per year.

A. \$12,535.93

B. \$51,520.61

C. \$54,611.85

The correct answer is **B**.

The question requires the calculation of the Present Value of an ordinary annuity as follows:

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

Where;

A = Amount of the annuity.

r = Required rate of return.

N = Number of years of the investment.

Therefore;

$$PV = \$7,000 \left[\frac{1 - \frac{1}{(1+0.06)^{10}}}{0.06} \right] = \$7,000 \left[\frac{1 - \frac{1}{1.791}}{0.06} \right] = \$7,000 \left[\frac{1 - 0.558}{0.06} \right] = \$51,520.61$$

Using the BA II Plus Pro Calculator;

N=10; I/Y=6; PMT=\$7,000; FV=0; CPT=>PV=\$51,520.61

A is incorrect. The amount represents the future value, assuming that it was not an annuity as follows:

$$FV = PV(1+r)^N = \$7,000(1+0.06)^{10} = \$12,535.93$$

C is incorrect. The amount indicates the present value of the annuity, supposing it was an annuity in advance and not an ordinary annuity as follows;

$$PV = PV_{\text{Ordinary Annuity}}(1+r) = \$51,520.61(1+0.06) = \$54,611.85$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.

Q.19 Suppose a \$200,000 investment will earn 6% compounded continuously for 4 years. The future value of this asset is *closest to*:

A. \$252,495.39

B. \$253,354.02

C. \$254,249.83

The correct answer is **C**.

The question requires the calculation of the future value of an asset compounded continuously as follows;

$$FV_N = PV e^{rN}$$

Where;

PV = Present value of investment.

r = Interest rate.

N = Investment period.

Therefore;

$$FV_N = \$200,000 \times 2.7182818^{(0.06 \times 4)} = \$200,000 \times 1.271 = \$254,249.83$$

A is incorrect. The amount represents the future value of the amount, compounded annually, for 4four years and not continuously as follows;

$$FV_N = \$200,000(1 + 0.06)^4 = \$252,495.39$$

B is incorrect. The amount represents the future value compounded semi-annually instead of continuously as follows;

$$FV_N = \$200,000 \left[1 + \frac{0.06}{2} \right]^{(4 \times 2)} = \$253,354.02$$

CFA Level 1, Quantitative Methods, Learning Module 1: Rate and Return, LOS (d)
Calculate and interpret annualized return measures and continuously compounded returns, and describe their appropriate uses

Q.21 An asset will pay \$2,500 per year for seven years, with the first payment being made one year from today. If the required rate of return is 14% per year, the amount you will pay for this asset is *closest to*:

A. \$10,720.76

B. \$12,221.67

C. \$26,826.23

The correct answer is **A**.

The question requires the calculation of the Present Value of an ordinary annuity as follows;

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

Where;

A = Annuity payable per year.

r = Rate of return.

N = Investment period.

Therefore, in this case:

$$PV = \$2,500 \times \left[\frac{1 - \frac{1}{(1+0.14)^7}}{0.14} \right] = \$2,500 \times 4.288 = \$10,720.76$$

Using the Plus Pro- Calculator;

N=7; FV=0; I/Y=14; PMT=\$2,500; CPT => PV = \$10,720.76

B is incorrect. The amount represents the future value of an annuity in advance and not of an ordinary annuity as required by the question as follows;

$$FV_{\text{Annuity Advance}} = \$10,720.76(1 + 0.14) = \$12,221.67$$

C is incorrect. The amount represents the future value of the ordinary annuity and not its present value calculated using the BA II Plus Pro Calculator as follows;

N=7; PV=0; I/Y=14; PMT=\$2,500; CPT=>FV=\$26,826.23

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.

Q.22 The amount of money an investor has after ten years if they invest \$25,000 per year in an index fund that pays 7% annually is *closest to*:

- A. \$349,178.78
- B. \$349,744.72
- C. \$345,411.20

The correct answer is **C**.

The question requires the calculation of the future value of an annuity as follows; **Step I:** Determine the Future Value annuity factor as follows;

$$FV_{\text{Annuity Factor}} = \frac{(1 + r)^N - 1}{r} = \frac{(1 + 0.07)^{10} - 1}{0.07} = 13.816$$

Step II: Determine the Future Value of the annuity as follows;

$$FV = \text{Annual investment} \times FV_{\text{Annuity Factor}} = \$25,000 \times 13.816 = \$345,411.20$$

Using the BA II Pro Plus Calculator;

N= 10; I/Y= 7; PMT=\$25,000; PV = 0; CPT=>FV = \$345,411.20

A is incorrect. The value of \$349,178.78 is higher than the correct future value of the annuity.

B is incorrect. The value of \$349,744.72 is also higher than the correct future value of the annuity.

CFA Level I, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance. LOS (b): Calculate and interpret the implied return of fixed-income instruments and the required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.23 An index fund is projected to pay an investor a 9% annual interest for eight years. The investor intends to invest \$15,000 every year, beginning next year. The total amount of money the investor will have at the end of the eight years is *closest to*:

- A. \$120,000.00
- B. \$165,427.11
- C. \$180,315.55

The correct answer is **B**.

The question requires the calculation of the Future Value of an annuity payment as follows;
Step I: Determine the Future Value annuity factor as follows;

$$FV_{\text{Annuity Factor}} = \left[\frac{(1 + r)^N - 1}{r} \right]$$

Where;

r = Annual interest rate

N = Investment period

Therefore;

$$FV_{\text{Annuity Factor}} = \left[\frac{(1 + 0.09)^8 - 1}{0.09} \right] = 11.028$$

Step II: Determine the total expected future amount as follows;

$$FV = \text{Annual Investment} \times FV_{\text{Annuity Factor}} = \$15,000 \times 11.028 = \$165,427.11$$

Using BA II Plus Calculator

N = 8; I/Y = 9 = 0.09; PV = 0; CPT = 15,000; CPT -> FV = \$165,427.11

A is incorrect. The amount represents the annuity after 8 years without considering the interest earned for the investment as follows:

$$FV = \$15,000 \times 8 \text{ years} = \$120,000.00$$

C is incorrect. The amount represents the future value of an annuity in advance and not ordinary as follows;

$$FV_{\text{Annuity Advance}} = \$165,427.11(1 + 0.09) = \$180,315.55$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.25 Assume an investment asset will pay \$75,000 after two years with an interest rate of 11%. The current value of the investment asset is *closest to*:

- A. \$60,871.70
- B. \$92,407.50
- C. \$128,439.25

The correct answer is **A**.

The question requires the calculation of the Present Value of future lumpsum payment as follows;

$$PV = FV_N(1 + r)^{-N}$$

Where;

FV_N = The expected future payment.

r = Applicable rate of interest.

N = Investment maturity period.

Therefore;

$$PV = \$75,000(1 + 0.11)^{-2} = \$75,000 \times 0.812 = \$60,871.68$$

Using the Plus Pro- Calculator:

$N = 2$; $I/Y = 11$; $PMT = 0$; $FV = \$75,000$; $CPT \Rightarrow PV = \$60,871.68$

B is incorrect. The amount represents the future value and not the present value as required in the question as follows;

$$FV = \$75,000(1 + 0.11)^2 = \$75,000 \times 1.232 = \$92,407.50$$

C is incorrect. The amount represents the present value of an ordinary annuity which is not stated in the question as follows;

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right] = \$75,000 \left[\frac{1 - \frac{1}{(1+0.11)^2}}{0.11} \right] = \$75,000 \times 1.713 = \$128,439.25$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows .

Q.34 You have a choice to take your retirement benefit either as a lump-sum or as an annuity. You can take a lump-sum of \$4.5 million or an annuity with 15 payments of \$400,000 a year with the first payment starting $t = 1$. The interest rate is 7% per year compounded annually. Which option is preferable, on the basis that it has the greater present value?

- A. The annuity payment option.
- B. The lump-sum payment option.
- C. There is no significant difference between the two options.

The correct answer is **B**.

The question requires calculating the present value of a series of equal cash flows compared to the lump sum payment. The annuity payment qualifies as an ordinary annuity since it has equal annuity payments, with the first payment starting at time $t=1$. Hence the formula is as follows:

$N = 15$; $I/Y = 7$; $PMT = 400,000$; $FV = 0$; $CPT \Rightarrow PV = 3,643,165.60$

The total annuity payment amount of \$3,643,165.60 is less than the lump sum payment of \$4.5 million; hence settle for a lump sum payment.

A is incorrect. The present value of the lump sum is greater than the present value of the annuity payments, as evidenced in Choice B.

C is incorrect. The present value of the lump sum is greater than the present value of the annuity payments.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows .

Q.35 XYZ Pension Fund plans to pay its retirees a total of \$29 million every year beginning in 2017. The payments will be made at the end of each year until 2031 for a total of 15 payments. Assuming a discount rate of 4% compounded annually, the present value of the pension payments to the retirees in 2017 is *closest to*:

- A. \$52,277,361.66
- B. \$322,433,235.50
- C. \$335,330,564.90

The correct answer is **B**.

The question requires calculating the Present Value of an ordinary annuity whereby the annuity makes N payments, with the first payment at $t = 1$ and the last at $t = N$.

The present value of an ordinary annuity can be expressed as the sum of the present values of each annuity payment as follows,

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

Where;

A = the annuity amount.

r = The interest rate per period corresponding to the frequency of annuity payments compounded annually.

N = The number of annuity payments.

Using the BA II Plus Pro Calculator;

N= 15; I/Y= 4; PMT=\$29,000,000; FV = 0; CPT => PV =\$322,433,235.50

A is incorrect. The amount represents the future value of a lump sum with annual compounding as follows;

$$FV_N = \$29,000,000 \left[1 + \frac{0.04}{1} \right]^{(15 \times 1)} = \$29,000,000 \times 1.801 = \$52,227,361.66$$

C is incorrect. The amount represents the future value of the annuity payment, assuming an annuity in advance and not an ordinary annuity as required by the question as follows;

$$FV_{\text{Annuity Advance}} = \$322,433,235.50 (1 + 0.04) = \$335,330,564.90$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows .

Q.37 Consider an annuity due with 20 payments of \$55,000 every year with a required rate of interest of 10% compounded annually. The present value of the annuity if the first payment is due at the end of the year is *closest to*:

A. \$370,012.50

B. \$468,246.00

C. \$566,577.67

The correct answer is **B**.

Hence the present value of an ordinary annuity is expressed as the sum of the present values of each annuity payment, as follows;

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

Where;

A = The annuity amount.

r = The interest rate per period corresponding to the frequency of annuity payments.

N = The number of annuity payments.

Therefore;

$$PV = \$55,000 \left[\frac{1 - \frac{1}{(1+0.1)^{20}}}{0.1} \right] = \$468,246$$

A is incorrect. The amount represents the projected future value of a lump sum with no interim Cash as follows;

$$FV_N = \$55,000(1 + 0.1)^{20} = \$370,012.50$$

C is incorrect. The amount represents the future value of the annuity payment, assuming an annuity in advance and not an ordinary annuity as required by the question as follows;

$$FV_{\text{Annuity Advance}} = \$515,070.60 (1 + 0.1) = \$566,577.67$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows .

Q.38 ABC Company Ltd has been in operation since 1966. In 2012 the company recorded revenue of \$15.8 million compared to \$11.4million in 2006. The revenue growth rate for the company for the six years is *closest to*:

- A. -5.30%
- B. 5.59%
- C. 38.60%

The correct answer is **B**.

The question requires the calculation of the growth rate in revenue as follows;

$$g = \left[\frac{FV_N}{PV} \right]^{\frac{1}{N}} - 1$$

Where;

FV_N = Future value at time N.

PV = Current present value.

N = Number of periods.

Therefore;

$$g = \left[\sqrt[6]{\frac{\$15,800,000}{\$11,400,000}} \right] - 1 = 5.59\%$$

The calculated growth rate of about 5.6 percent a year shows that ABC Company Ltd's revenue grew during the 2006-2012 period.

Using the BA II Pro Plus Calculator;

N=6; PV= -\$11.4; PMT=0; FV=\$15.8 CPT=> I/Y = 5.59%

NOTE: Remember to include a negative sign before the present value when calculating the financial calculator's interest rate.

A is incorrect. The rate is as a result of the inverse calculation as per choice B above as follows;

$$g = \left[\sqrt[6]{\frac{\$11,400,000}{\$15,800,000}} \right] - 1 = -5.30$$

C is incorrect. The rate ignores the applicable formula and has been determined as follows;

$$\text{Growth} = \$15.8 \text{ million} - \$11.4 \text{ million} = \$4.4 \text{ million}$$

And

$$g = \left[\frac{\$4,400,000}{\$11,400,000} \right] \times 100 = 38.60\%$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows

Q.39 A Government Bond pays \$2,500 per quarter year in perpetuity. If the required rate of return is 10%, price of the bond today is *closest to*:

A. \$11,000

B. \$25,000

C. \$100,000

The correct answer is **C**.

The question requires the calculation of the present value of a perpetuity with level payments for an indefinite period as follows;

$$PV = \frac{A}{r}$$

Where;

A = Quarterly perpetuity payments

r = Required rate of return

Therefore;

$$PV = \left[\frac{\$2,500}{0.1} \times 4 \right] = \$100,000$$

A is incorrect. The amount ignores the aspect of perpetuity calculation as follows;

$$PV = \$2,500 \times (1 + 0.1) \times 4 = \$11,000$$

B is incorrect. The resulting amount ignores the quarterly perpetuity payments as follows;

$$PV = \frac{\$2,500}{0.1} = \$25,000$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows

Q.40 Mr. Smith is planning to borrow \$150,000 from ABC Bank with a 20-year fixed-rate mortgage with monthly payments, and the first payment is due in exactly one month. Mr. Smith's monthly payment if the interest rate is 7% compounded monthly is *closest to*:

A. \$1,162.93

B. \$1,245.02

C. \$ 1,156.20

The correct answer is **A**.

The question requires the calculation of the size of payments on a Fixed-Rate Mortgage and ABC Bank will define the mortgage payments such that at the specified periodic interest rate, the present value of the payments will be equal to the amount borrowed, which is \$150,000.

The amount borrowed by Mr. Smith of \$150,000 is equivalent to approximately 129 monthly payments of \$1,163.56 with a stated interest rate of 7%.

Using the BA II Plus Pro Calculator;

N=12* 20=240; (12 months for 20 years)

I/Y=7/12=0.5833; (7% put into monthly interest)

PV=-\$150,000;

FV=0;

CPT => PMT = \$1,162.93

B is incorrect. The amount represents an annuity in advance, yet the first payment is due in exactly one month, not immediately as follows:

$$FV_{\text{Annuity Advance}} = \$1,163.56(1 + 0.07) = \$1,245.02$$

C is incorrect. The interest rate has not been divided by 12 to reflect monthly payments while using the BA II Plus Pro Calculator as follows;

N=12* 20=240; (12 months for 20 years)

I/Y=7;

PV=-\$150,000;

FV=0;

CPT =>PMT = \$10,500.00

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.42 Consider a level perpetuity of \$27,000 per year, with its first payment being at the end of year seven ($t = 7$). Its present value today (at $t = 0$), given an 8% discount rate is *closest to*:

A. \$124,817.75

B. \$212,682.25

C. \$337,500.00

The correct answer is **B**.

The question requires the calculation of the present value of projected perpetuity as follows;

Step I: Determine the present value of the perpetuity at $t = 6$ and then discount that amount back to $t = 0$.

It is important to note that perpetuity or an ordinary annuity has its first payment one period away, explaining the $t = 6$ index for our present value calculation.

$$PV = \frac{A}{r}$$

Where;

A = Annual perpetuity payment.

r = Discount rate.

$$\Rightarrow PV = \frac{\$27,000}{0.08} = \$337,500.00$$

Step II: Determine the present value of the future amount at $t = 6$.

Recall that from the perspective of $t = 0$, the present value of \$337,500.00 can be considered a future value.

Therefore, we need to find the present value of a lump sum as follows;

$$PV = FV_N(1 + r)^{-N} = \$337,500.00(1 + 0.08)^{-6} = \$212,682.25$$

A is incorrect. The amount represents the difference between the present value of the perpetuity at $t = 6$ and $t = 0$.

C is incorrect. The amount represents the present value at $t = 6$, not at $t = 0$, as calculated above.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows

Q.43 You are presented with 2 investment opportunities and must choose the one with the greater present value: A lump-sum of \$2.5 million or an annuity with 25 payments of \$250,000 a year with the first payment starting today. The interest rate is 9% per year compounded annually. Which one will you choose?

A. Annuity option.

B. Lump-sum.

C. Invest in both options since there's no difference between the two.

The correct answer is **A**.

The question requires the calculation of an annuity due as the present value of an immediate cash flow plus an ordinary annuity and comparing the outcome with the lump sum payment as follows;

Step I: Compare the present value of each investment option at time $t = 0$ and select the one with the larger value. For example, the first option's present value is \$2 million in this scenario, already stated in today's dollars, while the second option is an annuity due.

Step II: Since the first payment for the annuity due occurs at $t = 0$, we need to isolate the annuity benefits into two portions: an immediate \$250,000 to be paid today ($t = 0$) and an ordinary annuity of \$250,000 per year for 24 years.

Therefore;

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

Where:

A = Ordinary annuity payments.

r = Applicable rate of interest.

N = Number of payments.

Hence;

$$PV = \$250,000 + \$250,000 \left[\frac{1 - \frac{1}{(1+0.09)^{24}}}{0.09} \right] = \$2,676,652.94$$

Using the BA II Plus Pro Calculator as follows;

N = 24

$$I/Y = 9$$

$$PMT = \$250,000$$

$$FV = 0$$

$$CPT \Rightarrow PV = \$2,426,652.94,$$

and total annuity payment becomes;

$$PV = (\$250,000 + \$2,426,652.94) = \$2,676,652.94$$

Since the total value of the annuity option is \$2,676,652.94, the present value of the annuity is greater than the lump sum alternative of \$2.5 million.

B is incorrect. The above calculations show that the present value of the annuity is greater than the present value of the lump sum.

C is incorrect. As indicated in the above calculations, both options are quite different, with annuity being the superior option.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows

Q.44 Consider an investor with a Certificate of Deposit (CD) worth \$3 million and pays a 5% interest rate compounded annually. The number of years that it will take for the CD to triple in value in *closest to*:

- A. 2.9 years
- B. 3 years
- C. 22.5 years

The correct answer is **C**.

The question requires the calculation of the number of annual compounding periods needed for an investment to reach a specific value using the formula as follows;

$$FV_N = PV(1 + r)^N$$

To solve for the number of periods N, we readjust the formula and make N the subject as follows;

$$(1 + r)^N = \frac{FV_N}{PV} = \frac{\$9,000,000}{\$3,000,000} = 3 \Rightarrow N = \frac{\ln(3)}{\ln(1.05)} = 22.5 \text{ years}$$

Using the financial calculator, we use the following commands: I/Y = 5, PV = -3, PMT = 0, FV = 9
CPT -> N=22.5171

A is incorrect. The result assumes the natural logarithm in the formula as follows;

$$N = \frac{3}{1.05} = 2.9 \text{ years}$$

B is incorrect. The result assumes that by dividing the future value by the present value of the deposit certificate, it automatically gives the actual triple value, which is not the case.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows

Q.46 XYZ company's EPS at the beginning of each 4 consecutive years is as follows:

Year	EPS (\$)
Year 1	\$3.00
Year 2	\$4.20
Year 3	\$5.50
Year 4	\$7.60

The EPS compound annual growth rate during this period is *closest to*:

- A. 16.80%
- B. 20.40%
- C. 36.320%

The correct answer is **C**.

To calculate the EPS growth rate, we use the following formula;

$$g = \left[\frac{FV_N}{PV} \right]^{\frac{1}{N}} - 1$$

Where;

FV_N = Future EPS Value.

PV = Present EPS Value.

N = Number of periods.

Therefore;

$$g = \sqrt[4]{\frac{\$7.60}{\$3.00}} - 1 = 1.3632 - 1 = 36.32\%$$

Q.47 Consider a series of payments, each amounting to £6,500 is set to be received by an investor in perpetuity. Payments are to be made at the end of each year, with the first payment expected to start at the end of year 4. Suppose the discount rate is 9%, the present value of the perpetuity at $t = 0$ is *closest to*:

A. £51,164.03

B. £55,768.79

C. £72,222.20

The correct answer is **B**.

The question requires the calculation of the Present Value of projected perpetuity as follows;

Step I: Determine the present value of the perpetuity at $t = 3$ and then discount that amount back to $t = 0$.

It's important to note that perpetuity or an ordinary annuity has its first payment one period away, explaining the $t = 3$ indexes for our present value calculation.

$$PV = \frac{A}{r}$$

Where;

A = Annuity amount.

r = Discount rate.

Therefore;

$$PV = \frac{£6,500}{0.09} = £72,222.20$$

Step II: Determine the present value of the future amount at $t = 3$. Note that from the perspective of $t = 0$, the present value of £72,222.20 can be considered a future value. So now we need to find the present value of a lump sum:

$$PV = FV_N (1 + r)^{-N} = £72,222.20(1 + 0.09)^{-3} = £55,768.79$$

A is incorrect. At year 4 ($t=3$), the present value has been discounted by 4 instead of by 3 years as follows;

$$PV = £72,222.20(1 + 0.09)^{-4} = £51,164.03$$

C is incorrect. The amount represents the present value at time $t=3$ and not at time $t=0$.

Q.49 Consider a homeowner who wants to purchase a £230,000 home by making a down payment of £60,000 and borrowing the remainder with a 25 year fixed rate mortgage with monthly payments and the stated annual interest rate of 9% with monthly compounding. The monthly rate of mortgage repayment is *closest to*:

A. £1,267.50

B. £1,426.63

C. £1,901.30

The correct answer is **B**.

The question requires the calculation of the size of payments on a Fixed-Rate Mortgage repayment is as follows;

Note that the lending institution will determine the mortgage payments. At the stated periodic interest rate, the present value of the payments will be equal to the amount borrowed (in this case, £170,000).

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

Determine the annuity amount (A) as the present value divided by the present value annuity factor as follows;

$$PV_{\text{Annuity Factor}} = \left[\frac{1 - \frac{1}{\left(\frac{1+r_s}{m}\right)^{mN}}}{\frac{r_s}{m}} \right]$$

Where;

r_s = Monthly compounding interest rate.

m = Monthly compounding.

N = Mortgage remainder period.

Therefore;

$$PV_{\text{Annuity Factor}} = \frac{1 - \frac{1}{\left(\frac{1+0.09}{12}\right)^{12 \times 25}}}{\frac{0.09}{12}} = 0.894 \times \frac{12}{0.09} = 119.162$$

The annuity amount (A) is thus calculated as follows;

$$A = \frac{PV}{PV_{\text{Annuity Factor}}} = \frac{£170,000}{119.62} = £1,426.63$$

Using the financial calculator:

$N=25 \times 12=300$; $I/Y=9/12=0.75$ (make sure to use 0.75 and not 0.0075); $PV=230,000-60,000=170,000$; $FV=0$;

$CPT \Rightarrow PMT = 1,426.63$

A is incorrect. The amount represents the mortgage repayments payable for annual and not for monthly compounding.

C is incorrect. The calculation has not subtracted the down payment of £60,000 from the initial cost of the house to obtain the amount borrowed to finance the purchase.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows

Q.50 Suppose a Certificate of Deposit (CD) pays a 10% annual interest rate, the Effective Annual Rate (EAR) if the CD compounded monthly is *closest to*:

- A. 10.30%
- B. 10.40%
- C. 10.50%

The correct answer is **C**.

The question requires the calculation of EAR given a stated annual interest rate that does not give a future value directly as follows;

$$\text{EAR} = (1 + \text{periodic interest rate})^m - 1$$

Where;

m = The number of compounding periods in one year.

Periodic interest rate = The stated annual interest rate divided by m.

Therefore;

$$\text{EAR} = \left[1 + \frac{0.1}{12}\right]^{12} - 1 = 0.105 \text{ or } 10.50\%$$

A is incorrect. It has equated the compounding periods to semi-annually as opposed to monthly compounding as follows;

$$\text{EAR} = \left[1 + \frac{0.1}{2}\right]^2 - 1 = 0.103 \text{ or } 10.30\%$$

B is incorrect. It has equated the compounding periods to quarterly as opposed to monthly compounding as follows;

$$\text{EAR} = \left[1 + \frac{0.1}{4}\right]^4 - 1 = 0.104 \text{ or } 10.40\%$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.

Q.52 Consider an investor who wants to double his £1,500,000 worth of investments. If the interest rate is 9% compounded annually, the time it will take for the value of the investment to double is *closest to*:

- A. 1 year.
- B. 8 years.
- C. 12 years.

The correct answer is **B**.

The question requires the calculation of the number of annual compounding periods required for an investment to reach a specific value as follows;

$$FV_N = PV(1 + r)^N$$

Where;

FV_N = The future value of the investment amount.

PV = The present value of the investment amount.

r = Interest rate compounded annually.

N = Required number of periods for the investment to double.

Therefore;

$$(1 + 0.09)^N = \frac{FV_N}{PV} = \frac{£3,000,000}{£1,500,000} = 2 = N = \frac{\ln(2)}{\ln(1.09)} = \frac{0.693}{0.086} = 8.042 \text{ years}$$

Note that you can solve this question easily using a financial calculator by letting $I/Y=9$, $PV=-1,500,000$, $PMT=0$ and $FV = 3,000,000$ then hit the CBT button followed by N. You will get $N = 8.0432$ (4 sf).

A is incorrect. As per the calculation in A above, it's practical that the investment will only double in 8 years.

C is incorrect. The number of years indicated only applies if the amount of investment is tripled, but rather the question specifically required the investment to double.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.

Q.53 GHG Corp.'s net profit increased from £2,300,000 in 2012 to £4,800,000 in 2016. The net profit growth rate is *closest to*:

A. 15.90%

B. 20.20%

C. 52.10%

The correct answer is **B**.

To determine the growth rate, we use the following formula;

$$= \left[\frac{FV_N}{PV} \right]^{\frac{1}{N}} - 1 = \left(\frac{£4,800,000}{£2,300,000} \right)^{\frac{1}{4}} - 1 = 20.20\%$$

Or we can use the financial calculator with the following inputs:

N=4; PV=-2,300,000; PMT=0; FV=4,800,000;

CPT => I/Y = 20.19%

Note: When computing 1/Y, a negative sign has to be included before the Present Value.

A is incorrect. The indicated growth rate assumes that N=5 instead of 4 as follows;

$$g = \left(\frac{£4,800,000}{£2,300,000} \right)^{\frac{1}{5}} - 1 = 0.159 \text{ or } 15.90\%$$

C is incorrect. The indicated growth rate has been arrived at as follows;

$$= \left(\frac{£4,800,000 - £2,300,000}{£2,300,000} \right)^{\frac{1}{4}} - 1 = 52.10\%$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.398 The Effective Annual Rate (EAR) of an investment with a stated annual interest rate of 33%, if the rate is compounded daily is closest to:

A. 35.70%

B. 37.30%

C. 39.10%

The correct answer is **C**.

The EAR is calculated using the following formula;

$$\text{EAR} = (1 + \text{periodic interest rate})^m - 1$$

Note that the periodic interest rate is the stated annual interest rate divided by m, where m is the number of compounding periods in one year.

$$\text{EAR} = \left[1 + \frac{0.33}{365}\right]^{365} - 1 = 39.10\%$$

A is incorrect. It represents the EAR of the investment compounded half-yearly as follows;

$$\text{EAR} = \left[1 + \frac{0.33}{2}\right]^2 - 1 = 35.70\%$$

B is incorrect. It represents the EAR of the investment compounded quarterly as follows;

$$\text{EAR} = \left[1 + \frac{0.33}{4}\right]^4 - 1 = 37.30\%$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.

Q.401 The current valuation of Genius Corporation confirms a networth of AUD 2.5 million. Three years ago, the firm was sold to its new investors for AUD 800,000. The growth rate for the Genius Corporation within the three years is *closest to*:

- A. 32.00%
- B. 46.20%
- C. 68.00%

The correct answer is **B**.

The growth rate is calculated using the formula as follows;

$$= \left[\frac{FV_N}{PV} \right]^{\frac{1}{N}} - 1 = \sqrt[3]{\frac{AUD2,500.00}{AUD800,000}} - 1 = 46.20\%$$

Where;

FV_N = Future value after the three years.

PV = Present value at three years.

N = The period during the investment growth.

A is incorrect. The growth rate indicates the erroneous calculation as follows;

$$g = \frac{AUD800,00}{AUD2,500,00} \times 100\% = 32\%$$

C is incorrect. The growth rate indicates the erroneous calculation as follows;

$$g = \left[\frac{AUD2,500,000 - AUD800,000}{AUD2,500,00} \right] \times 100\% = 68\%$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (a) Calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.

Q.402 Mr. Thomas Newborn wants to purchase a \$150,000 home and has already made a cash deposit of \$20,000. The balance is financed through a 25-year mortgage borrowing with an annual interest rate of 6% compounded monthly. The first monthly mortgage payment is due at $t = 1$. The monthly mortgage repayment rate is *closest to*:

- A. \$837.59
- B. \$875.92

C. \$966.49

The correct answer is **A**.

The question requires the calculation of the size of payments on a Fixed-Rate Mortgage, and it's important to note that the mortgage lender will determine the mortgage payments such that at the stated periodic interest rate, the present value of the payments will be equal to the amount borrowed (in this case, \$130,000).

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$$

To solve for the annuity amount, A, as the present value divided by the present value annuity factor as follows;

$$PV_{\text{Annuity Factor}} = \left[\frac{1 - \frac{1}{(1 + \frac{rs}{m})^{mN}}}{\frac{rs}{m}} \right]$$

Where;

PV= Present value of the mortgage.

rs = Annual interest rate.

m = Monthly compounding periods

N = Amount of mortgage repayment period

Therefore;

$$PV_{\text{Annuity Factor}} = \left[\frac{1 - \frac{1}{(1 + \frac{0.06}{12})^{12 \times 25}}}{\frac{0.06}{12}} \right] = 0.776 \times \frac{12}{0.06} = 155.207$$

And;

$$A = \frac{PV}{PV_{\text{Annuity Factor}}} = \frac{\$130,000}{155.207} = \$837.59$$

We could also use the financial calculator:

N = 300; I/Y = 6/12 = 0.5; PV = -130,000; FV = 0; CPT -> PMT = 837.59

In conclusion, the amount borrowed, \$130,000, is equivalent to 155 monthly payments of \$837.59 with a stated interest rate of 6%.

B is incorrect. It represents the monthly payment that would have been made if the payments were to be expected at the beginning (annuity in advance) and not at the end of the year

(ordinary annuity).

C is incorrect. The amount has not considered the cash payment of \$20,000, which should be deducted from \$150,000.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.3403 A bank advertises that it pays an annual interest of 10% with semi-annual compounding on its savings account. The effective annual rate is *closest* to:

A. 10.25%.

B. 10.38%.

C. 10.47%.

The correct answer is **A**.

Recall that:

$$\text{Effective annual rate (EAR)} = \left(1 + \frac{\text{Annual rate}}{\text{Compounding frequency}}\right)^{(\text{Compounding frequency})} - 1 = \left[1 + \frac{10\%}{2}\right]^2 - 1 =$$

B is incorrect. It is the effective annual interest rate with quarterly compounding.

$$\text{EAR} = \left(1 + \frac{0.10}{4}\right)^4 - 1 = 0.1038 = 10.38\%$$

C is incorrect. It is the effective annual interest rate with monthly compounding.

$$\text{EAR} = \left(1 + \frac{0.10}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.3405 Norman Smith is considering an investment opportunity presented by his portfolio

manager, which involves making an annual investment of \$1,000 at the end of each year for a duration of 10 years. Assuming the investment yields an annual return of 10%, the amount that Smith can expect to receive at the end of the 10th year is *closest to*:

- A. \$10,000.00.
- B. \$15,937.42.
- C. \$17,531.17

The correct answer is **B**.

The investment pattern above is similar to an ordinary annuity. In an ordinary annuity, the cash flows occur at the end of the compounding period. Therefore, we can consider this problem as involving ordinary annuity.

To understand the principle behind the question, let's construct the cash flow pattern of the investment.

Investment at the end of the...	Sum	Withdrawal of investment	Period until which the money is invested	Maturity amount of the money at the end of 10 years
1st year	\$1,000	End of 10 years	9 years	$\$1,000 \times (1 + 10\%)^9 = \$2,357.95$
2nd year	\$1,000	End of 10 years	8 years	$\$1,000 \times (1 + 10\%)^8 = \$2,143.95$
3rd year	\$1,000	End of 10 years	7 years	$\$1,000 \times (1 + 10\%)^7 = \$1,948.72$
4th year	\$1,000	End of 10 years	6 years	$\$1,000 \times (1 + 10\%)^6 = \$1,771.56$
5th year	\$1,000	End of 10 years	5 years	$\$1,000 \times (1 + 10\%)^5 = \$1,610.51$
6th year	\$1,000	End of 10 years	4 years	$\$1,000 \times (1 + 10\%)^4 = \$1,464.10$
7th year	\$1,000	End of 10 years	3 years	$\$1,000 \times (1 + 10\%)^3 = \$1,331.00$
8th year	\$1,000	End of 10 years	2 years	$\$1,000 \times (1 + 10\%)^2 = \$1,210.00$
9st year	\$1,000	End of 10 years	1 years	$\$1,000 \times (1 + 10\%)^1 = \$1,100.00$
10th year	\$1,000	End of 10 years	0 years	$\$1,000 \times (1 + 10\%)^0 = \$1,000.00$
Total				\$15,937.42

The above illustration shows the main basis of the calculation. However, it can also be calculated by using the calculator as:

$N=10$; $I/Y=10$; $PMT= -1,000$; $PV=0$

$CPT = FV = 15,937.42$

A is incorrect. It is simply the amount after ten years, excluding the interest earned on the amount.

C is incorrect. It is the future value of an annuity in advance, yet this is an ordinary annuity. To arrive at this answer using a financial calculator, set the calculator to BGN mode and proceed as above. i.e., $N=10$; $I/Y=10$; $PMT= -1,000$; $PV=0$ $CPT = FV=17,531.17$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the

present value (PV) and cash flows.

Q.3406 An investor wants to invest \$1,000 at the beginning of each year for the next 10 years, after which he can redeem his investment at the beginning of the 10th year. The amount received by the investor at the end of the 10th year, if the investment generates a yearly return of 10%, is *closest* to:

- A. \$2,593.74
- B. \$15,937.42
- C. \$17,531.17.

The correct answer is **C**.

$$\text{Annuity due} = \text{Ordinary Annuity} \times (1 + \text{Rate of compounding})$$

We can first calculate the ordinary annuity using the financial calculator as:

$$N=10; I/Y=10; PMT= -1,000; PV=0$$

$$CPT \Rightarrow FV = \$15,937.42$$

$$FV (\text{annuity due}) = 15,937.42 \times 1.1 = 17,531.17$$

To calculate annuity due directly using the financial calculator: First, set the calculator to BGN mode. This is done by pressing 2ND PMT, then 2ND ENTER, then 2ND CPT. Then input values: PMT=1,000; N=10; 1/Y=10; CPT=>FV=17,531.17

A is incorrect. It is the future value of a lump sum amount and not an annuity.

$$1,000(1 + 0.10)^{10} = 2,593.74$$

B is incorrect. It is the future value of an ordinary annuity and not that of an annuity in advance as required by the question.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.3407 Nathan Lewis is planning to subscribe to an investment plan which will generate a return of 5% and provide him with \$2,000 at the end of each year for the next 5 years. However, due to financial constraints, he plans to subscribe to the investment plan in 2 years. The present value of the investment plan today is *closest* to:

A. \$4,942.26.

B. \$8,246.62

C. \$8,658.95

The correct answer is **A**.

The PV of the ordinary annuity after 2 years, when Lewis starts the investment plan, can be calculated as under:

$N=3$; $I/Y=5$; $PMT= -2,000$; $FV=0$

$CPT \Rightarrow PV = \$5,446.52$

However, this plan will start in 2 years. Assuming the rate of interest in these two years is 5%, the PV of the annuity today can be computed as:

$$PV = \frac{\$5,446.52}{(1.05)^2} = \$4,942.26$$

A is incorrect. It is the present value of the annuity at time $t = 2$ and not at time $t = 0$.

C is incorrect. It is the present value of an annuity in advance, not of an ordinary annuity as required by the question, at time $t = 0$.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.3408 A construction company is bidding for a new project. The projected cash flows of the project for the next 2 years are given in the following exhibit.

Exhibit: Projected Cash Flows

Year	Cash Flow
End of the 1st year	\$100,000
End of the 2nd year	\$600,000

If the company intends to generate a return of 10%, then the present value of the projected cash flows today is *closest* to:

A. \$525,920.36

B. \$578,512.40.

C. \$586,776.86.

The correct answer is **C**.

The cash flows and the corresponding details are as given below:

Year	Cash Flow	Time from Today
End of the 1st year	\$100,000	1 year
End of the 2nd year	\$600,000	2 years

$$\text{PV of \$100,000 today} = \frac{\$100,000}{1.1} = \$90,909.09$$

$$\text{PV of \$600,000} = \frac{\$600,000}{1.1^2} = \$495,867.77$$

$$\text{PV of combined cash flows} = \$90,909.09 + \$495,867.77 = \$586,776.86$$

We can also use the CF button of the BA II Plus Pro calculator to determine the present value of the cash flow.

CF0=0 ↓, CF1 = 100,000 ENTER ↓ ↓; CF2= 600,000 ENTER

Then press "CPT NPV" after inputting all CFs, then input I/Y=10 ENTER ↓, press the "down arrow button", then finally press "CPT" to get the NPV.

A is incorrect. It assumes the calculation as follows;

$$\text{PV of \$600,000} = \frac{\$700,000}{1.1^3} = \$525,920.36$$

B is incorrect. It assumes the calculation as follows:

$$\text{PV of \$100,000 today} = \frac{\$100,000}{1.1^2} = \$82,644.63$$

$$\text{PV of \$600,000} = \frac{\$600,000}{1.1^2} = \$495,867.77$$

$$\text{PV of combined cash flows} = \$82,644.63 + \$495,867.77 = \$578,512.40$$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income

instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.3409 A project manager is looking to fund his new project through bank borrowings. The new project requires a funding of \$1,000,000, so the manager approaches a commercial bank. The bank is willing to fund the project at an interest rate of 3% and wants the firm to pay back the entire loan in 10 years in 10 equal payments. The yearly payment required to completely pay off the loan is *closest* to:

A. \$101,380.15

B. \$104,171.23.

C. \$117,230.51.

The correct answer is **C**.

Using the financial calculator:

$N=10$; $I/Y=3$; $PV=-1,000,000$, $FV=0$

$CPT = PMT = \$117,230.51$

A is incorrect. The amount assumes a monthly interest rate and not annual as follows.

$N=10$; $I/Y=3/12=0.25$; $PV=-1,000,000$, $FV=0$ $CPT \Rightarrow PMT = \$101,380.15$

B is incorrect. The amount assumes a Quarterly interest rate and not annual as follows.

$N=10$; $I/Y=3/4=0.75$; $PV=-1,000,000$, $FV=0$ $CPT \Rightarrow PMT = \$104,171.23$

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.

Q.3410 Veronica Rose borrowed \$5,000 from GRF Bank. The terms and conditions of the loan are given in the following exhibit.

Exhibit: GRF Bank - \$5,000 Loan

Loan	Short Term
Amount	\$5,000
Tenure	3 years
Payment	3 equal payments
Rate	3%
Prepayment Penalty	Nil

If Rose decided to make a payment of \$2,000 at the end of the 1st year, then the payments required for the remaining 2 years would be *closest* to:

A. \$1,646.22

B. \$1,676.22

C. \$1,686.22

The correct answer is **A**.

The payments required to be made so that the loan is completely paid off in three years can be calculated as:

$N=3$; $I/Y=3$; $FV=0$; $PV=5000$;

$CPT = PMT = \$1,767.65$

The amortization schedule of the loan is:

Year	Amount	Rate	Payment	Interest	Principal Payment	Principal Left
1	\$5,000.00	3%	\$1,767.65	\$150.00	\$1,617.65	\$3,382.35
2	\$3,382.35	3%	\$1,767.65	\$101.47	\$1,666.18	\$1,716.17
1	\$1,716.17	3%	\$1,767.65	\$51.49	\$1,716.17	\$0.00

However, since Rose makes a payment of \$2,000 at the end of the 1st year:

Year	Amount	Rate	Payment	Interest	Principal Payment	Principal Left
1	\$5,000.00	3%	\$2,000.00	\$150.00	\$1,850.00	\$3,150.00

Therefore, at the end of the 1st year, Rose needs to pay \$3,150 in two years. The payment required to clear the remaining part of the loan can be computed as:

$N = 2$; $I/Y = 3$; $FV = 0$; $PV = 3,150$;

$CPT = PMT = \$1,646.22$

B is incorrect. It is the yearly mortgage payment that would have been made if she had not paid the 2,000 at the end of the first year.

C is incorrect. It is basically what remains after paying the 2,000, divided by 2, without factoring in interest payments.

CFA Level 1, Quantitative Methods, Learning Module 2: The Time Value of Money in Finance, LOS (b) Calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.
