Learning Module 13: Curve Based and Empirical Fixed Income Risk Measures

LOS 13a: explain why effective duration and effective convexity are the most appropriate measures of interest rate risk for bonds with embedded options.

Yield duration and convexity assume predictable bond cash flows. However, bonds with embedded options, e.g., callable or puttable bonds, have future cash flows which are uncertain. The option is exercised based on market interest rates relative to the coupon interest paid or received. Bonds with embedded options do not have clear-cut yield-to-maturity. For such bonds, traditional measures like Macaulay and modified durations do not accurately capture interest rate risk. Instead, "effective duration" is used, which focuses on how the bond's price reacts to changes in a benchmark yield curve, like the government par curve. Finally, while the interest rate risk estimates of yield and convexity are only useful for small changes in yields, the estimates from effective duration and convexity are useful for both small and large yield changes.

Effective Duration

Effective duration serves as an essential tool in measuring the bond's price sensitivity to changes in the benchmark yield curve. The formula for calculating effective duration (EffDur), which is very similar to the one for determining the approximate modified duration, is expressed as:

$$\text{EffDur} = \frac{(PV_{-} - PV_{+})}{2 \times (\Delta\text{Curve}) \times (PV_0)}$$

Where:

- PV_{-} and PV_{+}: The present values are calculated using option pricing models when the interest rate is decreased and increased, respectively.

- ΔCurve: The change in the benchmark yield curve.

- PV_0: The present value at the current interest rate.

Effective Duration for Callable and Non-Callable Bonds

Non-callable bonds are consistently priced higher than callable bonds due to the value of the embedded call option held by the issuer. When interest rates are low, the value of the call option in callable bonds rises; this limits the bond's price appreciation. Effective durations of callable and non-callable bonds are similar when yields are high. However, for low interest rates, callable bonds have a lower effective duration because of the presence of the call option.

Effective Duration for Putable and Non-Putable Bonds

Putable bonds allow investors to sell them back to the issuer before maturity at face value. This provision protects investors from benchmark yield increases that could lower the bond's price below par. A putable bond's price is always greater than its non-putable counterpart due to the value of the embedded put option. Put options minimize bond sensitivity, especially during rising interest rates, in terms of effective duration. When rates are lower than the bond's coupon rate, the put option's value is limited, hence the bond's market reaction to yield changes resembles non-putable bonds. As benchmark rates rise, the put option's value rises, protecting investors from price declines.

Effective Convexity

Effective convexity analyzes the second-order effects of shifts in the benchmark yield curve. It helps in understanding the potential changes in curve shape and the transition into negative regions, particularly when the value of the embedded call option increases. The formula for calculating effective convexity (EffCon) is:

$$\text{EffCon} = \frac{[(PV_{-} + PV_{+}) - 2 \times PV_0]}{(\Delta \text{Curve})^2 \times PV_0}$$

When the benchmark yield falls, non-callable bonds' price-yield curve steepens, suggesting positive convexity. The curve of callable bonds flattens and even turn negative when the benchmark yield falls. Both bond types have positive convexity at high benchmark yields. As yields fall, the callable bond may become negative convexity due to the embedded call option's value.

On the other hand, putable bonds are characterized by positive convexity. The embedded put option protects investors, especially during rising interest rates. This option lets investors sell the bond back to the issuer at par, limiting price losses.

Effective duration and convexity are also relevant for mortgage-backed securities (MBSs). MBSs cash flows depend on homeowners' refinancing decisions, especially prevalent in areas where refinancing is common during low-interest-rate scenarios.

Example: Interest Rate Sensitivity of a Callable Bond

A portfolio manager is contemplating investing in a callable bond and approaches you, the lead analyst in the fixed-income team, to assess its interest rate sensitivity. The manager has a preference for bonds with a duration between 6 and 7 years and a positive convexity. The full price of the callable bond is 105.50 per 100 of par value. When the government par curve shifts by 30bps, your option valuation model indicates the new full prices for this callable bond are 103.40 when raised and 107.350 when lowered. Therefore:

- $PV_0 = 105.500$
- $PV_+ = 103.400$
- $PV_- = 107.350$
- $\Delta \text{Curve} = 0.0030$

Using the provided data, you calculate the effective duration and effective convexity for this callable bond as:

$$\text{EffDur} = \frac{(PV_- - PV_+)}{2 \times \Delta\text{Curve} \times PV_0}$$

$$\text{EffDur} = \frac{(107.350 - 103.400)}{2 \times 0.0030 \times 105.500} = 6.2401$$

$$\text{EffCon} = \frac{(PV_{-} + PV_{+}) - 2 \times PV_0}{\Delta\text{Curve}^2 \times PV_0}$$

$$\text{EffCon} = \frac{(107.350 + 103.400) - 2 \times 105.500}{(0.0030)^2 \times 105.500} = -263.296$$

Based on your analysis, you advise the portfolio manager to exercise caution when considering this callable bond. Although its effective duration is within the desired range, the bond exhibits negative effective convexity. A bond displaying negative effective convexity will experience a more pronounced decrease in its price due to a rise in the benchmark yield compared to the price increase resulting from a decrease in the benchmark yield.

Question

An investor plans to allocate $500,000 in two-year Mega-Corp bonds. One of the bonds, Bond X, is standard, while Bond Y has an embedded put option. Which duration metric is best suited to assess the interest rate risk for these bonds?

A. Money duration
B. Effective duration
C. Macaulay duration

Solution

The correct answer is **B.**

Bond Y has an embedded put option, introducing optionality to the bond. Bonds with embedded options, such as callable or putable bonds, have uncertain future cash flows because the exercise of the option depends on market interest rates relative to the bond's coupon interest. Since these bonds do not have clearly defined yields-to-maturity, traditional measures like Macaulay and modified durations are not suitable. Instead, effective duration, which measures a bond's price sensitivity to changes in a benchmark yield curve (like the government par curve), is the appropriate measure for bonds with embedded options.Therefore, effective duration is the best measure to use. Effective duration is also applicable for bonds without embedded options, enabling the investment advisor to compare the interest rate risks of both Bond X and Bond Y.

A is incorrect. While money duration measures the sensitivity of the bond's price to interest rate changes, it does not specifically cater to bonds with embedded options.

C is incorrect. Macaulay duration provides the weighted average time until a bond's cash flows are received, but it does not adjust for embedded options.

LOS 13b: calculate the percentage price change of a bond for a specified change in benchmark yield, given the bond's effective duration and convexity.

Effective duration and effective convexity are curve-based metrics that are crucial for assessing the interest rate risk of complex instruments, such as those with embedded contingency provisions. These metrics are typically determined from bond prices derived using an option valuation model, given specific changes in the underlying benchmark government yield curve. Effective duration and effective convexity can be used to estimate the percentage change in a bond's full price for a given shift in the benchmark yield curve (?Curve). It is calculated as:

$$\%\Delta PV^{Full} \approx (-\text{EffDur} \times \Delta\text{Curve}) + [\frac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2]$$

Example: Bond Price Impact from Par Curve Shifts

Consider the effective duration and effective convexity data for Bond X and Bond Y. We will examine the effects of a 200bps shift in the benchmark government par curve.

Bond	EffDur	EffCon
Bond X	7.425	−295.0
Bond Y	6.891	−278.310

Upward Shift by 200 bps:

$$\%\Delta PV^{Full} \approx (-\text{EffDur} \times \Delta\text{Curve}) + [\frac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2]$$

Where $\Delta\text{Curve} = 0.02$,

Bond X: $\%\Delta PV^{Full} = (-7.425 \times 0.02) + (0.5 \times -295.0 \times (0.02)^2) = -20.75\%$

Bond Y: $\%\Delta PV^{Full} = (-6.891 \times 0.02) + (0.5 \times -278.310 \times (0.02)^2) = -19.35\%$

The significant decline in bond prices in response to increasing interest rates can be attributed to the high effective durations of Bond X and Bond Y, indicating their increased vulnerability to changes in interest rates. The situation is compounded by the negative convexities, which amplify the declines in bond prices. Investors who hold these bonds are, therefore, at risk of incurring losses in the event of rising interest rates. The negative convexity also indicates that bond prices may not experience significant increases even if interest rates subsequently decrease, as we will see below.

Downward Shift by 200 bps:

$\Delta \text{Curve} = -0.02.$

Bond X: $\%\Delta PV^{\text{Full}} = (-7.425 \times -0.02) + (0.5 \times -295.0 \times (-0.02)^2) = 8.95\%$

Bond Y: $\%\Delta PV^{\text{Full}} = (-6.891 \times -0.02) + (0.5 \times -278.310 \times (-0.02)^2) = 8.22\%$

Despite declining interest rates, the price appreciation of both bonds is moderate. This seemingly counterintuitive behavior can be attributed back to the negative convexities of the bonds. In rate-declining scenarios, one would anticipate bond prices to rise more significantly. However, the negative convexity lowers this increase, causing the price appreciation to be less than would be predicted based solely on duration.

Question

The effective duration and effective convexity of a bond are 4.816 and 26.723, respectively, The percentage changes in the bond's full price for ±100bp shifts in the benchmark government par curve are closest to?

A. -4.68% and 4.95%
B. -4.47% and 4.71%
C. -4.816% and 26.72%

Solution

The correct answer is A.

$\%\Delta PV^{Full} \approx (-\text{EffDur} \times \Delta\text{Curve}) + [\frac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2]$

Upward shift:

$\Delta\text{Curve} = 0.01$

$\%\Delta PV^{Full} = (-4.816 \times 0.01) + (0.5 \times 26.723 \times (0.01)^2) = -4.68\%$

Downward shift:

$\Delta\text{Curve} = -0.01$

$\%\Delta PV^{Full} = (-4.816 \times -0.01) + (0.5 \times 26.723 \times (-0.01)^2) = 4.95\%$

LOS 13c: define key rate duration and describe its use to measure price sensitivity of fixed-income instruments to benchmark yield curve changes.

Key rate duration (partial duration) is a financial metric that measures the sensitivity of a bond's price to changes in interest rates at specific points along the yield curve. On the other hand, effective duration gauges sensitivity to overall parallel shifts in the benchmark curve. Key rate durations sum up to the effective duration. To compute key rate durations, only specific points on the yield curve (e.g., 2-year, 5-year rates) are adjusted rather than the entire curve.

Key rate duration helps identify shaping risk — a bond's reaction to changes in the shape of the yield curve. For bonds with embedded options (e.g., callable bonds), the shape of the curve matters. A downward shift can impact the bond's price due to its negative convexity.

Key rate duration can be calculated using the following formula:

$$\text{KeyRateDur}_k = -\frac{1}{\text{PV}} \times \frac{\Delta \text{PV}}{\Delta r_k}$$

Where:

Δr_k represents the change in the kth key rate.

$\triangle \text{PV}$ is the change in the bond's price.

PV is the bond's initial price.

The sum of key rate durations results to effective duration as per the following formula.

$$\sum_{k=1}^{n} \text{KeyRateDur}_k = \text{EffDur}$$

The percentage change in bond price is expressed mathematically as:

$$\frac{\Delta \text{PV}}{\text{PV}} (\% \Delta \text{PV}) = -\text{KeyRateDur}_k \times \Delta r_k$$

Example: Analyzing Bond Price Sensitivity to Non-Parallel Shifts in the

Benchmark Government Par Curve

Consider a scenario with non-parallel shifts in the benchmark government par curve, characterized by a pronounced steepening at longer maturities. We will determine the expected price change for each of the two bonds, Bond C and Bond D, as a result of these shifts.

Shifts in Benchmark Government Par Curve:

Maturity	Expected Change
1 year	+80 bps
5 years	+120 bps
10 years	+180 bps
20 years	+230 bps
30 years	+280 bps

Bond Details

Bond	Tenor	Key Rate Duration
Bond C	5 years	2.30
Bond D	10 years	3.60

We can compute the expected percentage price change for each bond using the following formula:

$$\frac{\Delta PV}{PV}(\%\Delta PV) = -\text{KeyRateDur}_k \times \Delta r_k$$

For Bond C:

$$\%\Delta PV_{\text{Bond C}} = -2.30 \times 1.20\% = -2.76\%$$

For Bond D:

$$\%\Delta PV_{\text{Bond D}} = -3.60 \times 1.80\% = -6.48\%$$

Bond D's price is more sensitive to the shifts, decreasing by roughly 6.48%, compared to Bond C's decrease of 2.76%. This is attributed to Bond D's higher key rate duration.

While understanding the portfolio duration and the general shift of the benchmark yield curve offers a rapid assessment of potential profits or losses, employing key rate durations enables a portfolio manager to adjust weights in specific tenors to optimize the risk-adjusted return.

Question #1

Which of the following best describes the key rate duration of a bond?

A. The bond's sensitivity to a uniform change in all yields of the benchmark yield curve.
B. The bond's sensitivity to a change in the benchmark yield at a specific maturity.
C. The bond's sensitivity to changes only in the short-term rates of the benchmark yield curve.

Solution:

The correct answer is B:

Key rate duration (or partial duration) measures a bond's sensitivity to a change in the benchmark yield at a specific maturity.

A is incorrect: This description aligns with effective duration, which measures a bond’s sensitivity when all yields of the benchmark change uniformly.

C is incorrect: Key rate duration refers to sensitivity at a specific maturity, not just short-term rates.

Question #2

In the context of key rate durations, "shaping risk"for a bond most likely refers to:

A. The risk that all yields on the benchmark curve will change by the same amount.
B. The risk associated with changes in the shape of the benchmark yield curve, such as steepening, flattening, or twisting.
C. The risk that only short-term rates on the benchmark curve will change.

Solution:

The correct answer is B:

"Shaping risk" refers to a bond's sensitivity to changes in the shape of the benchmark yield curve, such as it becoming steeper, flatter, or undergoing a twist.

A is incorrect: This is a description of a parallel shift, not shaping risk.

C is incorrect: Shaping risk refers to changes in the entire shape of the curve, not just short-term rates.

LOS 13d: describe the difference between empirical duration and analytical duration.

Analytical duration utilizes mathematical models, assuming credit spreads and government bond yields are uncorrelated and independent. It is a solid method for estimating the bond's price-yield relationship in numerous situations. On the other hand, empirical duration relies on historical data within statistical frameworks, factoring in elements influencing bond prices. It recognizes the correlations between credit spreads and benchmark yields, enhancing its precision in specific contexts. It becomes particularly relevant during periods of economic unrest, like financial crises when the relationship between bond yields and spreads becomes more complex.

Summary of the Important Duration Measures and their Significance

- Approximate Modified Duration: It is an approach based on yield that estimates the slope of a bond's price-yield curve. It estimates the modified duration of a bond.

- Effective Duration: It is a curve-based method used to estimate a bond's price sensitivity to a change in a benchmark yield curve. It estimates the modified duration for complex bonds whose cashflows are uncertain.

- Key Rate: It measures the sensitivity of a bond's price to changes in interest rates at specific points along the yield curve. It measures the sensitivity of a bond’s price to non-parallel shifts in the benchmark yield curve.

- Empirical Duration: Statistical estimate that considers the correlation between yield spreads and benchmark yield changes across different economic scenarios.

Flight to Quality

In times of economic uncertainty, investors often shift from risky assets to secure ones, like government bonds. This can lead to a decline in government benchmark yields and widening credit spreads. During crises like COVID-19, "flight to quality" can cause government benchmark yields to fall while credit spreads widen.

Empirical vs. Analytical Duration: Practical Implications

For government bonds with minimal credit risk, analytical and empirical durations are similar. On the other hand, for corporate bonds, especially during market downturns, empirical duration is more relevant because of the negative correlation between benchmark yields and credit spreads.

Analysts must weigh the interplay between benchmark yields and credit spreads when determining which duration type to employ, ensuring accurate bond price predictions in varied economic contexts.

Question

In which scenario would empirical duration *most likely* provide more accurate estimates than analytical duration?

A. For government bonds with low credit risk during stable economic conditions.
B. For corporate bonds during periods of economic volatility where there's a "flight to quality."
C. For bonds when, government bond yields and credit spreads are perfectly positively correlated.

Solution

The correct answer is **B.**

For corporate bonds during periods of economic volatility where there's a "flight to quality."

A is incorrect. For government bonds with low credit risk, analytical and empirical durations are expected to provide similar estimates, especially in stable economic conditions.

C is incorrect. The situation described is not typical; during periods like economic crises, credit spreads and benchmark yields tend to be negatively correlated.