Learning Module 7: Pricing and Valuation of Interest Rate and Other Swaps

LOS 7a: describe how swap contracts are similar to but different from a series of forward contracts

Recall that a swap is a derivative contract between two counterparties to exchange a series of future cash flows. In comparison, a forward contract is also an agreement between two counterparties to exchange a single cash flow at a later date. A single-period swap can, therefore, be considered a single-forward contract.

Swaps and forward contracts are similar in that both are forward commitments with symmetric payoff profiles. Besides, in both interest swaps and forward contracts, no cash exchanges hand at initiation.

A distinguishing factor, however, is that the fixed swap rate is constant, while a series of forward contracts have different forward rates at each expiration.

Forward Rate Agreement and Interest Rate Swaps

A forward rate agreement (FRA) is a cash-settled over-the-counter (OTC) contract between two counterparties. In this contract, the buyer (long position) is borrowing a notional sum (underlying) at a fixed interest rate (the FRA rate) and for a specified period starting at an agreed-upon date.

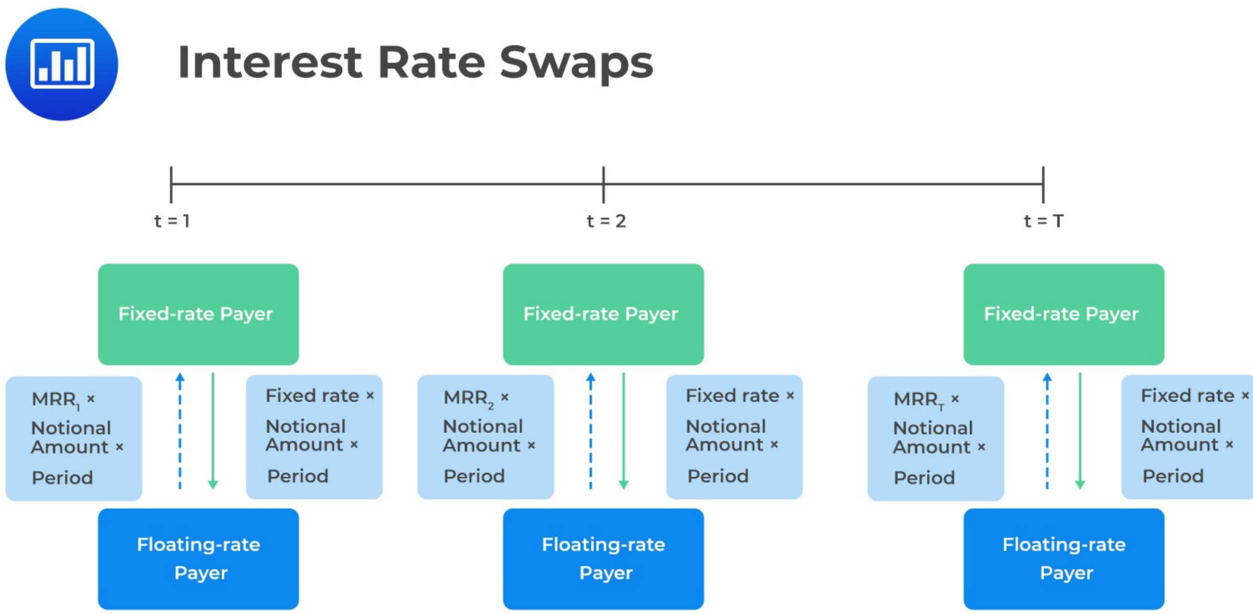
The seller deposits interest based on the market reference rate (MRR), where the MMR is established before the settlement dates, at time $t - A$.

The FRA settlement amount is a function of the difference between the forward interest rate $(IFR_{A,B-A})$ and the market reference rate (MRR_{B-A}) or the MMR for $B - A$ periods.

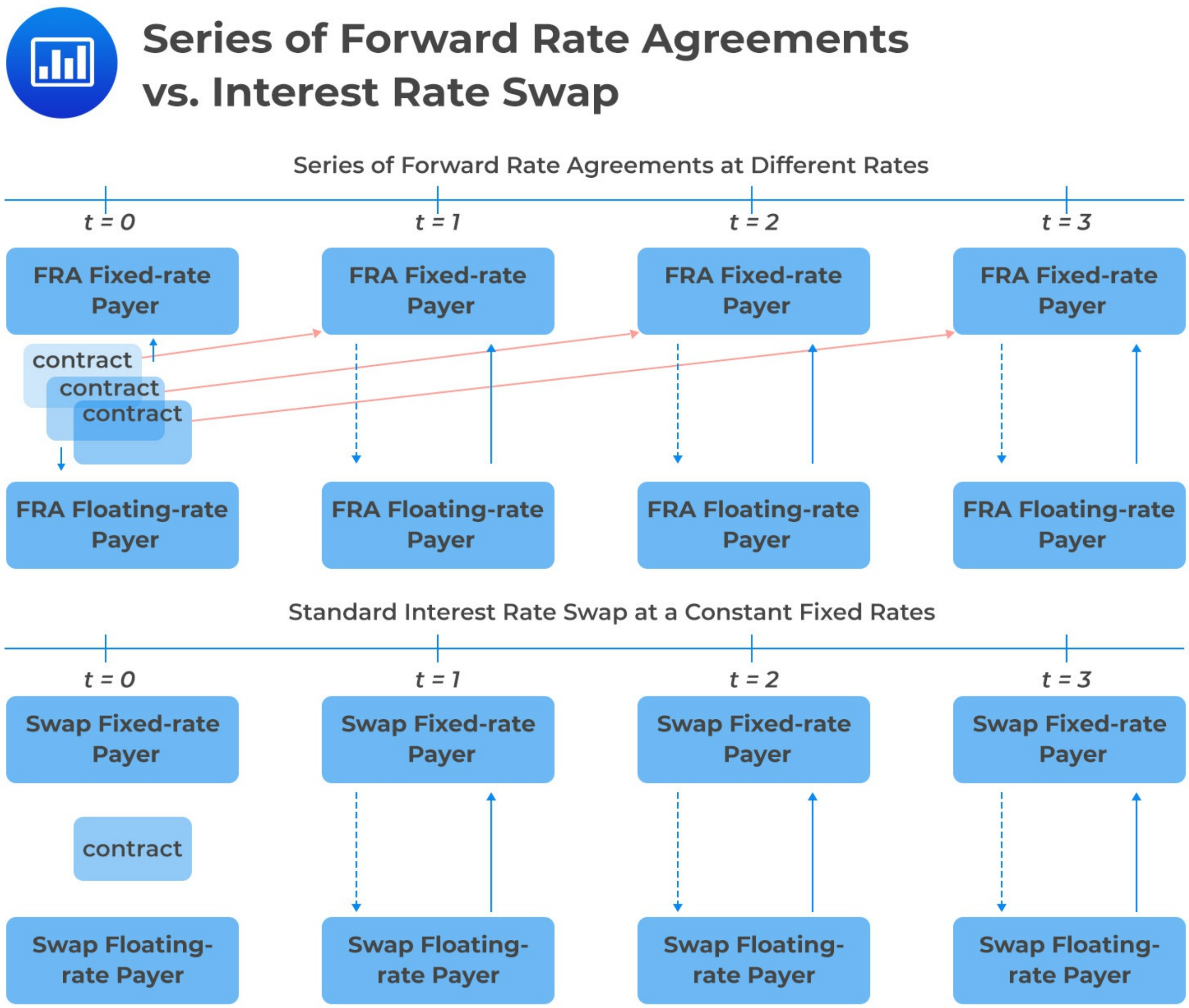
$$\text{Net payment} = (MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional principal} \times \text{Period}$$

Interest Rate Swaps

In a swap contract, two parties agree to exchange a series of cash flows. In the agreement, one party pays a variable (floating) series of cash flows that will be determined by a market reference rate (MRR) that resets every period. The other party pays either (1) a variable series based on a different underlying asset or rate or (2) a fixed series.



Note that, like a single-period swap, a forward rate agreement (FRA) consists of a single cash flow. As such, a multi-period swap can be viewed as a series of forward rate agreements.



In single-period swaps and FRAs, the net difference between the fixed rate agreed on at inception and the market reference rate set in the future is used as the basis for determining cash settlement. Further, note that both FRAs and interest swaps have symmetric payoff profiles, no upfront cash at contract initiation, and counterparty credit exposure.

However, an FRA's single settlement is done at the beginning of the period, while in interest swap rate, periodic settlements occur at the end of the respective period. In addition, looking at a swap as a series of FRAs, we would have a different fixed rate for each future time period. In the case of an interest rate swap, the fixed rate set today would apply throughout the period of the contract.

Par Swap Rate

Note that a standard interest rate swap involves an exchange of fixed payments at a constant rate for a series of floating-rate cash represented by the implied forward rates (IFR) at time $t = 0$.

The **par swap rate** is a fixed rate that equates the present value of all future expected floating cash flows to the present value of fixed cash flows. That is,

$$\sum_{i=1}^{N} \frac{IFR}{(1+Z_i)^i} = \sum_{i=1}^{N} \frac{S_i}{(1+Z_i)^i}$$

Where:

IFR = Implied forward rates.

s_i = Par swap rate for period i.

z_i = Spot rates for period i.

Example: Solving Par Swap (Fixed) Rate

A three-year bond has the following characteristics:

Years to Maturity	Annual Coupon	PV (per 100 FV)	Zero Rates
1	1.25%	99.016	2.2565%
2	2.5%	98.634	3.2282%
3	3.0%	97.222	4.0354%

Determine the par swap (fixed) rate for a three-year contract and the fixed rate for each of the three one-year FRAs that would match the single three-year swap.

We already have the one-year forward rate at $t = 0$ i.e., $IFR_{0,1} = 2.2565\%$. We need to determine the **implied one-year forward rates (IFR)** at $t = 1$ and $t = 2$.

$$(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B$$

Therefore, solving for $IFR_{1,1}$

$$\begin{aligned}(1+0.022565)^1 \times (1+IFR_{1,1})^1 &= (1+0.032282)^2 \\ \rightarrow IFR_{1,1} &= \frac{1.032282^2}{1.022565} - 1 \\ &= 0.042091\end{aligned}$$

and $(1+IFR_{2,1})^1$.

$$\begin{aligned}(1+0.032282)^2 \times (1+IFR_{2,1})^1 &= (1+0.040354)^3 \\ \rightarrow IFR_{2,1} &= \frac{1.040354^3}{1.032282^2} - 1 \\ &= 0.056688\end{aligned}$$

Consider the following table:

Years to Maturity	Annual Coupon	PV (per 100 FV)	Zero Rates	IFR
1	1.25%	99.016	2.2565%	2.2565%
2	2.5%	98.634	3.2282%	4.2091%
3	3.0%	97.222	4.0354%	5.6688%

In this case, the par swap rate, is the fixed rate that equates the present value of all future expected floating cash flows to the present value of fixed cash flows:

$$\sum_{i=1}^{N} \frac{IFR}{(1+Z_i)^i} = \sum_{i=1}^{N} \frac{S_i}{(1+Z_i)^i}$$

$$\begin{aligned}\rightarrow \frac{2.2565\%}{1.022565} + \frac{4.2091\%}{1.032282^2} + \frac{5.6688\%}{1.040354^3} &= \frac{s_3}{1.022565} + \frac{s_3}{1.032282^2} + \frac{s_3}{1.040354^3} \\ 0.11191 &= 2.80445 s_3 \\ \therefore s_3 &= \frac{0.11191}{2.80445} \\ &= 0.03990 \approx 3.99\%\end{aligned}$$

The three-year swap rate of 3.99% may be interpreted as a multi-period breakeven rate, or the rate at which an investor would be indifferent to:

- Paying the fixed swap rate and receiving the respective forward rates.
- Receiving the fixed swap rate and paying the respective forward rates.

Question

Which of the following *most likely* distinguishes forward rate agreements and interest rate swaps?

A. Fixed rate at each period.

B. Symmetric payoff profiles.

C. Netting of payments.

Solution

The correct answer is **A.**

Considering a swap as a series of FRAs, we would have a different fixed rate for each future time period. On the other hand, in an interest rate swap, the fixed rate set today would apply throughout the period of the contract.

Remember that interest rates are characterized by term structure, and, as such, we would expect FRAs to have fixed rates for different times to maturity.

B is incorrect. Both FRAs and interest rate swaps have symmetric payoff profiles.

C is incorrect. In single-period swaps and FRAs, the net difference between a fixed rate agreed on at inception, and an MRR (market reference rate) set in the future is used to determine cash settlement on a given notional principal over the specified time period.

LOS 7b: contrast the value and price of swaps

Remember that a swap contract involves a series of periodic settlements with a final settlement at maturity. **Swap price** (or **par swap rate)** is a periodic fixed rate that equates the present value (PV) of all future expected floating cash flows to the PV of fixed cash flows.

The swap rate is equivalent to the forward rate, $F_0(T)$; it satisfies no-arbitrage conditions. On the other hand, the current market reference rate (MRR) is the "spot" price. Therefore, from the fixed-rate payer perspective, the periodic value is given by:

$$\text{Periodic settlement value} = (\text{MRR} - S_N) \times \text{Notional amount} \times \text{Period}$$

The swap value on any settlement date is calculated as the current settlement value using the above formula plus the present value of all the remaining future swap settlements.

Like all other forward commitments, the value of a swap contract at initiation is zero.

Note that it's our assumption that MRR is set at the beginning of each interest period and has the same periodicity and day count as the swap rate. In addition, the net of fixed and floating differences is exchanged at the end of each period.

Examples: Calculating the Swap Value and Effect of Varying MRRs

FinnLay LTD has entered a 4-year interest rate swap with a financial institution with a notional amount of USD 100 million. The contract states that FinnLay signed to receive a semiannual USD fixed rate of 2.5% and, in turn, pay a semiannual market reference rate (MRR).

The MRR is expected to equal the respective implied forward rates (IFRs).

Scenario 1

If at the beginning of the sixth month, the MRR is 0.85%, the first swap settlement value from Finnlay's perspective is *closest to*:

Solution

$$
\begin{aligned}
\text{Periodic settlement value} &= (\text{MRR} - \text{S}_\text{N}) \times \text{Notional Amount} \times \text{Period} \\
&= (2.5\% - 0.85\%) \times \text{USD } 100\text{m} \times 0.5 \\
&= \text{USD } 0.825\text{m}
\end{aligned}
$$

Scenario 2

If implied forward rates **remain constant** as set at trade inception, how will this affect the MTM value from Finnlay's perspective immediately after the first settlement?

Solution

The swap price (or fixed swap rate) of 2.5% is set at the initiation of the trade, which equates to the PV of fixed versus floating payments.

If there is no change in interest rate expectations, the PV of remaining floating payments rises above the PV of fixed payments.

As such, Finnlay, as a fixed receiver, realizes an MTM loss on the swap because:

$$\sum \text{PV(Floating payments paid)} > \sum \text{PV(Fixed payments received)}$$

Scenario 2

If implied forward rates **decline** just after initiation, how will this affect the MTM value from Finnlay's perspective ?

Solution

A decrease in expected forward rates just after initiation will reduce the PV of floating payments while the fixed swap rate will remain constant.

Since FinnLay is the fixed-rate receiver, it will realize an MTM gain because:

$$\sum \text{PV(Floating payments paid)} < \sum \text{PV(Fixed payments received)}$$

Question

Invest Capital Inc has signed a three-year swap contract to receive a fixed interest rate of 2.5% on a semiannual basis and pay a 6-month USD MRR. The notional amount of the swap contract is USD 100,000.

Assume that the initial 6-month MRR sets at 0.56%, and MRR is expected to be upward sloping. The first settlement value in six months from Invest Capital is *closest* to:

A. $970.

B. $1,940.

C. $2,500.

Solution

The correct answer is **A**.

From the fixed-rate payer perspective, the periodic value is given by:

$$\begin{aligned}\text{Periodic settlement value} &= (S_N - \text{MRR}) \times \text{Notional Amount} \times \text{Period}\\ &= (2.5\% - 0.56\%) \times \text{USD } 100{,}000 \times 0.5\\ &= \$970\end{aligned}$$

B is incorrect. It is calculated as $= 2.5\% - 0.56\% \times$ USD 100,000. It omits the period in the formula.

C is incorrect. It is the amount of the fixed interest amount after six months.