Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Q.1141 Arbitrage profit is the risk-free profit that is earned when two securities with different prices have:

     A. Low market liquidity.

     B. Identical cash flows.

     C. Identical term structures.

The correct answer is **B.**

The law of one price states that two securities with identical cash flows should have the same price. Any mispricing in such securities will earn an arbitrage return.

A is incorrect. Market liquidity refers to the ease with which an asset can be bought or sold in the market without affecting its price. While low liquidity can contribute to price discrepancies, it is not a fundamental condition for arbitrage. Arbitrage opportunities arise from price differences in securities with identical cash flows, regardless of the market's liquidity level. In fact, low liquidity can sometimes hinder arbitrage by making it difficult to execute trades quickly at the desired prices.

C is incorrect. The term structure of interest rates, which is the relationship between interest rates or bond yields and different terms or maturities, can influence the pricing of securities. However, the key condition for arbitrage is not identical term structures but identical cash flows. Securities can have the same term to maturity and yet offer different cash flows due to differences in coupon rates, credit quality, or other factors. Therefore, identical term structures alone do not guarantee the presence of an arbitrage opportunity.

CFA Level I, Derivatives, Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives. LOS (b): Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

Q.1147 In the context of commodity forwards and futures contracts, the benefit of holding the commodity and possibly selling it in the midst of a shortage is known as the:

    A. Cost of carry.

    B. Risk premium.

    C. Convenience yield.

The correct answer is **C.**

Convenience yield is the additional value that comes with holding an asset rather than a long forward or futures contract on the asset. For instance, the holder can take advantage of shortages. Jet fuel, for example, has a convenience yield because the holder can sell at higher prices during shortages.

A is incorrect. Cost of carry refers to the costs associated with holding an asset. These could be financial costs or even storage costs.

B is incorrect. The risk premium is an extra return expected by investors for bearing some specified risk.

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Q.1150 If the cost and benefit of holding a forward contract until expiration is zero, then which one of these is *most likely* the payoff of a long forward contract at expiration?

      A. Spot price at expiration minus forward price.

      B. Forward price minus spot price at expiration.

      C. Spot price at initiation minus the forward price.

The correct answer is **A.**

Recall that a forward contract is an agreement to buy or sell an asset at a predetermined future date for a price agreed upon today (the forward price). If you're long in a forward contract, you've agreed to buy the asset at this predetermined price.

Spot price at expiration minus forward price is the correct representation of the payoff for a long forward contract. At expiration, if you are long, you buy the asset at the forward price ($F_0(T)$), and the asset is worth the spot price at expiration. The difference between these two prices is your payoff.

B is incorrect. It is the reverse of what the payoff for a long forward contract would be. If you are long in a forward contract, your payoff would be the spot price at expiration minus the forward price you agreed to pay (not the other way around).

C is incorrect. This option represents the difference between the initial spot price (S_0) and the forward price at time T ($F_0(T)$). This does not accurately represent the payoff of a long forward contract at expiration, as the relevant prices for calculating payoff are the spot price at expiration, not the initial spot price, and the forward price agreed upon at the start of the contract.

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Q.3294 A hedge fund analyst is assessing a futures contract with the following characteristics:

Spot price	$80.96
Interest costs	$11.90
PV of Convenience yield	$5.50
PV of Storage costs	$7.80

The 'cost of carry' for the futures contract is *closest to*:

    A. -$2.3.

    B. $13.3.

    C. $14.2.

The correct answer is **A.**

The cost of carry (or carry) = PV(benefits of holding the asset)– PV(costs of holding the asset).

Hence,

$$\text{The cost of carry (or carry)} = \$5.5 - \$7.8 = \$-2.3.$$

B is incorrect. It mistakenly adds the interest costs to the calculation, which is not part of the correct formula for calculating the net cost of carry in this context. The interest costs are a separate component and should not be included in the calculation of the net benefits minus the net costs of holding the asset.

C is incorrect. It represents a misunderstanding of the cost of carry calculation. This does not accurately reflect the true cost of carry for the futures contract.

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Q.3369 Which of the following conditions will *least likely* make it difficult for the process of arbitrage to correct a mispricing?

     A. The asset class is illiquid.

     B. Transaction costs are high.

     C. Information on asset prices is easily available.

The correct answer is **C.**

If there is a free flow of information, arbitrageurs will know that different prices exist which will in turn facilitate the process of arbitrage. The asset can be purchased in the cheaper market and sold in the more expensive market earning a riskless profit. The combined actions of all parties will push the lower price up and the higher price down until the prices converge.

A is incorrect. Illiquidity will make it difficult to purchase and/or short sell an asset making arbitrage less feasible.

B is incorrect. High transaction costs will make the process of arbitrage, and hence the correction of mispricing, not worthwhile.

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Q.3379 When the convenience yield exceeds storage costs, the forward transaction will return:

A. Less than the spot transaction.

B. More than the spot transaction.

C. The present value of the forward price.

The correct answer is **A.**

When the convenience yield (benefits) exceeds storage costs, the forward transaction will return less than the spot transactions. The forward price will be adjusted downwards to reflect the net loss over the spot transaction. In other words, acquiring the asset in the forward market would be cheaper because it forgoes the benefits that exceed the costs.

B is incorrect. This misunderstanding might stem from not considering how the forward price is adjusted to reflect the net benefits of holding the asset. In reality, the forward price decreases to account for the higher convenience yield over storage costs, making the forward transaction less lucrative compared to the spot transaction.

C is incorrect. Suggesting that the forward transaction will yield the present value of the forward price does not directly address the impact of the convenience yield exceeding storage costs on the returns of forward versus spot transactions. While it is true that the forward price can be thought of in terms of its present value, this option does not capture the essence of how the excess of convenience yield over storage costs affects the relative attractiveness of forward transactions. The key issue is not the present value per se but how the net convenience yield influences the forward price and, consequently, the returns from engaging in forward transactions.

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Q.3398 Assuming all else is held constant, when a commodity stock is in short supply, investors can expect a:

    A. Positive cost of carry.

    B. High convenience yield.

    C. Positive expected return.

The correct answer is **B.**

Since the commodity is believed to be short in supply, investors should expect to earn a high convenience yield. In such a scenario, the holder of a commodity has the benefit of holding a commodity if market conditions suggest that the commodity should be sold.

A is incorrect. Assuming all else is held constant, high convenience yields should decrease the cost of carry. The cost of carry measures the net cost of carrying an asset and is equal to storage costs minus convenience yield.

C is incorrect. If the commodity is in short supply, the holders of the commodity may even expect to earn a price premium that is higher than otherwise justifiable in well-functioning financial markets. The spot price of the commodity can rise above the market's expectation of its futures price and result in a negative expected implied return.

CFA Level I, Derivatives, Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives. LOS (b): Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

Q.4143 The spot price of a barrel of oil is $1,000. A trader enters into a one-year forward contract to purchase oil at a forward price of $1,200 per barrel. Assuming a no-arbitrage opportunity and a risk-free rate of 10%, the cost of carry associated with the forward contract is *closest to*:

    A. $90.91

    B. $100

    C. $320

The correct answer is **A.**

Considering the cost of carry, the relationship between the spot price and futures price changes as follows:

$$F_0\,(T) = [S_0 - PV_0\,(I) + PV_0\,(C)]\,(1 + r)^T$$

This can be written as:

$$F_0\,(T) = [S_0 - (PV_0\,(I) - PV_0\,(C))]\,(1 + r)^T$$

Clearly,

$$\text{Cost of Carry} = PV_0\,(I) - PV_0\,(C)$$

Solve for the cost of carry to get:

$$\text{Cost pf Carry} = 1200(1.10)^{-1} - 1,000 = \$90.91$$

B is incorrect. It applies the following reasoning: the oil's forward price, assuming a no-arbitrage opportunity, would have been:

$$F_0\,(T) = S_0\,(T)\,[1 + r]^T = \$1000(1 + 0.1)^1 = \$1,100$$

However, the forward price is \$1,200. It assumes that the cost of carry of \$100 (-1,200 – 1,100).

C is incorrect. It calculates the cost of carry as:

$$\text{Cost of Carry} = 1200(1.10)^1 - 1,000 = \$320$$

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Q.4144 Two identical assets, assets A and B, have the same spot price. However, asset A has more associated costs than asset B. Assuming that there are no associated benefits in both assets which of the following statements is *most likely* correct regarding the forward prices of forward contracts on assets A and B?

      A. Asset A has a lower forward price than asset B.

      B. Asset A has a higher forward price than asset B.

      C. Assets A has a higher spot price than asset B at maturity.

The correct answer is **B.**

Costs increase an underlying's forward price, whereas benefits reduce an underlying's forward price. Costs are incurred by the underlying owner. These costs will have to be factored in (added) onto the forward price, thereby increasing the forward price, as seen in the formula below.

Denote the costs (**C**) and benefits/income (**I**). Considering the cost of carry, the relationship between the spot price and futures price changes as follows:

$$F_0(T) = [S_0 - PV_0(I) + PV_0(C)](1 + R)^T$$

Where

$F_0(T)$ = Forward price

S_0 = Spot price of the underlying at time t=0

$PV_0(I)$ = present value of the associated benefits.

$PV_0(C)$ = present value of the associated costs.

r = risk-free rate of interest.

A is incorrect. Contradicts option B.

A is incorrect. Since assets A and B have the same spot prices, they must have similar spot prices at maturity, assuming no-arbitrage conditions hold. Assume that the spot of the assets is equal to $S_0 = S_A = S_B$ then, based on no-arbitrage conditions, the spot price at maturity S_T is given by:

$$S_T = S_0(1 + r)^T$$

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Q.4146 Miles discovers an arbitrage opportunity in a market. She notices that the spot share price of a company share is $70, and a 3-month forward contract on the same shares sells at a forward price of $74. In order to take advantage of the apparent arbitrage opportunity, Miles borrows money at a risk-free rate of 5% and buys the shares at a price of $70 per share. At the same time, she takes a short position in the forward contract. Assuming that the asset has no associated costs or benefits, the arbitrage profit per share that Miles earns from the trade at the contract maturity is *closest* to:

    A. $0.50

    B. $3.14

    C. $4.00

The correct answer is **B.**

Rhoda borrows EUR70 at the risk-free rate to purchase the shares today. After three months, the amount Rhoda returns to the lender, i.e., principal plus interest, is:

$$S_0(1 + r)^T = 70(1 + 0.05)^{0.25} = \$70.86$$

She then delivers the shares and receives $74 per share, earning her an arbitrage profit of;

$$(74 - 70.86) = \$3.14 \text{ per share}$$

A is incorrect. 0.5 has been incorrectly obtained by assuming the maturity of the forward contract is one year and not three months as stated in the question, i.e.,

$$74 - 70\,(1 + 0.05)^1) = \$0.50$$

C is incorrect. The value 4 has been obtained by directly subtracting the asset's spot price from its forward price without factoring in borrowing costs.

CFA Level I, Derivatives, Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives. LOS (b): Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

Q.4147 Given a stock index that pays a dividend yield of 2% and is trading at a spot price of $65, and assuming a risk-free rate of 5%, the stock's six-month forward price is *closest to:*

    A. $65.98

B. $66.64

C. $68.25

The correct answer is **A.**

Since the dividend has been given as a percentage (dividend yield), we will calculate the forward price using the equation;

$$F_0(T) = S_0 e^{(r+c-i)T}$$

Where,

$F_0(T) =$ forward price,

$S_0 =$ spot price,

r = risk-free rate,

i = benefits expressed as a rate of return

c = costs expressed as a rate of return

T = time to maturity.

$$F_0(T) = 65e^{(0.05+0-0.02)0.5} = 65.98$$

The forward price is slightly greater than the spot price since the cost (the risk-free rate) is higher than the benefit (the dividend yield).

B is incorrect. The option suggests a forward price of $66.64, which does not correctly apply the formula for calculating the forward price of an asset that pays a continuous dividend yield. This value might result from a miscalculation or misunderstanding of the formula's components, such as an incorrect adjustment for the dividend yield or the risk-free rate over the specified time period.

C is incorrect. The option suggests a forward price of $68.25, which significantly overestimates the impact of the risk-free rate and the dividend yield on the forward price. It might result from an overestimation of the risk-free rate, the dividend yield, or both, or from a misunderstanding of how these factors influence the forward price over the specified time period.

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Q.4148 A trader borrows \$1,000 at a risk-free rate of 5% to purchase an asset whose spot price is \$1,000. He then sells the asset after two years at a spot price of \$1,200 and uses some of the proceeds from the sale to repay his loan plus interest. The trader has *most likely* used which of the following replication strategies?

    A. Long-forward replication.

    B. Short-forward replication.

    C. Risk-free trade replication.

The correct answer is **A.**

From the trade, the trader makes a profit of

$$S_T - F_o(T) = 1,200 - 1,000 \times (1 + 0.05)^2 = 97.5$$

which is equivalent to a long-forward commitment. The strategy presented is, therefore, a long-forward replication.

B is incorrect. A short-forward replication is the opposite of a long-forward replication. It results in a return equal to $F_0(T) - S_T$. A trader can replicate a short forward by short-selling an asset at a spot price of S_0 at time $t = 0$, then lend the proceeds from the sale at the risk-free rate, r, and buy back the asset at time $t = T$, at a spot price of S_T.

C is incorrect. A risk-free trade replication would have earned the trader a return equal to the risk-free rate. A trader can replicate a risk-free trade by buying an asset at a spot price of S_0, at time $t = 0$, then simultaneously enter into a forward contract to sell the asset at the forward price, $F_0(T)$.

CFA Level I, Derivatives, Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives. LOS (b): Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset.

Q.4149 The present value of the benefits of holding a barrel of oil is $1,000. If the barrel of oil is trading at a spot price of $5,000 and assuming that the present value of the storage cost is $2,000 and the risk-free rate is 5%, the one-year forward price of a forward contract on a barrel of oil is *closest to*:

        A. $4,200

        B. $6,000

        C. $6,300

The correct answer is **C.**

Recall that,

$$F_0\,(T) = [S_0 - PV_0\,(I) + PV_0\,(C)](1 + r)^T$$

Where;

$F_0\,(T)$ = forward price.

S_0 = Spot price.

$PV_0\,(I)$ = Present value of benefits.

$PV_0\,(C)$ = Present value of costs.

$$F_0\,(T) = [5,000 - 1,000 + 2,000](1 + 0.05)^1 = 6,300$$

A is incorrect. $4,200 has been incorrectly obtained by adding instead of subtracting the present value of benefits (and subtracting instead of adding the present value of costs) to the spot price.

B is incorrect. $6,000 has been incorrectly obtained by failing to compound the sum of the spot price, the present value of costs, and the present value of benefits, for one year at the risk-free rate.

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