Learning Module 8: Exchange Rate Calculations

LOS 8a: calculate and interpret currency cross-rates

It is possible to back out the cross rates given two exchange rates involving three currencies. Consider a foreign exchange market with the exchange rate between the South African rand and the Chinese yuan. This market can also quote the exchange rate between the South African rand and the Russian ruble (RUB). It is, therefore, possible to back out the cross-rates between the Chinese yuan and the Russian ruble, which is quoted as (RUB/CNY) according to market conventions and can be represented as follows:

$$\frac{\text{RUB}}{\text{ZAR}} \times \frac{\text{ZAR}}{\text{CNY}} = \frac{\text{RUB}}{\text{CNY}}$$

For example, the RUB/ZAR exchange rate is 1.4876, and the ZAR/CNY exchange rate is 1.6459. We can calculate the RUB/CNY exchange rate using sample spot exchange rates as follows:

$$\frac{\text{RUB}}{\text{ZAR}} \times \frac{\text{ZAR}}{\text{CNY}} = 1.4876 \times 1.6459 = 2.4484 \text{ Russian Rubble per Chinese Yuan}$$

Inversion

Sometimes, it is important to invert one of the quotes to get the intermediary currency to cancel out the equation and get the cross rate. For example, to get the Russian ruble–Japanese yen (JPY/RUB) quote, we first invert the South African rand–Russia ruble (RUB/ZAR) quote before multiplying it by the South African rand–Japanese yen (JPY/ZAR).

Example: Currency Cross-Rates

Let's assume we have spot exchange rates of RUB/ZAR =1.4876 and JPY/ZAR = 70.74. The South African rand–Russia ruble (RUB/ZAR) ruble exchange rate of 1.4876 inverts to:

$$\left(\frac{\text{RUB}}{\text{ZAR}}\right)^{-1} = \left(\frac{\text{ZAR}}{\text{RUB}}\right) = \frac{1}{1.4876} = 0.6722$$

Multiplying this figure with the JPY/ZAR quote of 70.74 gives us the JPY/RUB.

$$\left(\frac{\text{ZAR}}{\text{RUB}}\right) \times \left(\frac{\text{JPY}}{\text{ZAR}}\right) = 0.6722 \ \times \ 70.74 \ = \ 47.5531 \text{ JPY per RUB}$$

Triangular Arbitrage in Cross Rate Calculations

Market participants can access both cross-rate quotes (e.g., JPY/CAD for Japan yen–Canada) and the underlying component exchange rate quotes (e.g., CAD/USD for dollar–Canada and JPY/USD for dollar–yen). These cross rates must align with their respective calculations; if not, traders will exploit the discrepancy through arbitrage. This type of profit-seeking, termed **triangular arbitrage** (given its involvement with three currencies), would persist until the price imbalance is corrected.

Example: Illustrating Triangular Arbitrage

To illustrate, consider a JPY/CAD rate derived at 85.98 based on the underlying CAD/USD and JPY/USD rates of 1.3020 and 111.94, respectively:

$$\frac{\text{JPY}}{\text{CAD}} = \left(\frac{\text{CAD}}{\text{USD}}\right)^{-1} \times \left(\frac{\text{JPY}}{\text{USD}}\right) = (1.3020)^{-1} \times 111.94 = 85.98$$

If a misinformed dealer simultaneously offers a JPY/CAD rate of 86.20, it presents a different price for the same service, which, in this case, is converting yen to Canadian dollars. A savvy trader could purchase CAD1 for JPY85.98 and immediately sell it for JPY86.20, making a risk-free profit of JPY0.22 per CAD1.

In practice, such discrepancies in cross-rates are infrequent. Both human traders and automated trading algorithms vigilantly monitor the markets for any pricing inefficiencies, ensuring swift corrections.

Question

A forex trader noticed the USD/EUR spot rate is 1.3960. Similarly, the CHF/USD spot rate is 0.9587. Calculate the spot CHF/EUR cross-rate.

1. 1.7422.
2. 1.3383.
3. 1.4561.

Solution

The correct answer is **B**.

The spot rate is:

$$\frac{\text{CHF}}{\text{EUR}} = \frac{\text{CHF}}{\text{USD}} \times \frac{\text{USD}}{\text{EUR}} = 1.3960 \times 0.9587 = 1.3383$$

LOS 8b: explain the arbitrage relationship between spot and forward exchange rates and interest rates, calculate a forward rate using points or in percentage terms, and interpret a forward discount or premium

This section will consider the relationships between spot and forward rates, interest rates, and maturities based on market efficiencies.

Spot and Forward Rates

In the professional FX market, forward exchange rates are commonly quoted in terms of 'points' or 'pips.' These points represent the difference between the forward and spot exchange rate quotes. The scale is adjusted so that these points correspond to the last decimal place in the spot quote.

If the forward rate is higher than the spot rate, the points are positive, indicating that the base currency trades at a forward premium. On the other hand, when the forward rate is below the spot rate, the points are negative, suggesting the base currency trades at a forward discount. Notably, when the base currency is at a forward premium, the price currency will be at a forward discount, and vice versa.

To understand this argument, let's look at a scenario in 2023. The spot USD/CAD exchange rate was 1.3845, and the six-month forward rate was 1.38475. This suggests that the CAD (base currency) was trading at a premium compared to the spot rate. The six-month forward points were quoted as 2.5, and this can be calculated as follows:

$$1.38475 - 1.3845 = 0.00025$$

We multiply by 10,000 to reach the desired result. This scaling ensures the points align with the final digit of the spot exchange rate quote, which is usually the fourth decimal place. Additionally, it's noteworthy that while points are usually quoted to at least one decimal place, the forward rate might extend to five or even more decimal places.

Among major currencies, the yen stands as an exception. Its spot rates are typically quoted to just two decimal places. Therefore, the difference between its forward and spot rates is multiplied by 100 to adjust for its two-decimal-place precision.

Forward Points and Maturity

Forward rate quotes are typically presented as the number of forward points for each specific maturity, which refers to the time interval between the spot settlement and the forward contract settlement. Often, these forward points are also termed 'swap points' since an FX swap encompasses both a spot and a forward transaction executed simultaneously.

To convert forward quotations into an outright forward quotation, let's use an example with the RUB/CNY currency pair. We'll use a table that shows maturity and forward or spot rate points.

Maturity	Spot rate or forward points
Spot	1.6459
One week	−0.2
One month	−0.1
Three months	−5.6
Six months	−12.7
Twelve-month	−25.3

From the table above, notice that the absolute number of points increases with maturity. This trend emerges because the number of points is directly proportional to the yield differential between the two involved countries (in this case, Russia and China). This differential is then adjusted based on the term to maturity.

As the term to maturity extends, the absolute number of forward points increases. In the same vein, for a fixed term to maturity, a greater interest rate differential results in a higher absolute number of forward points.

Calculating Forward Exchange Rates

To calculate forward exchange rates using forward points, you divide the points by 10,000. This scales down the fourth decimal place found in the spot rate. Then, you add the result to the spot exchange rate quote.

We can take the case of the six-month forward rate in the above table. Here we have:

$$1.6459+\frac{-12.7}{10000}=1.6459-0.00127=1.64463$$

Often, the forward rate points are represented as a percentage of the spot and not as an absolute number of points. As such, the six-month forward rate for RUB/CNY can be shown as follows:

$$\frac{1.6459-0.00127}{1.6459}-1=-0.077\%$$

To convert this percentage into a forward rate, we simply need to multiply the spot rate by one plus the percentage forward premium or discount:

$$1.6459\times(1+(-0.00077))=1.6459\times0.99923=1.64463$$

Arbitrage Relationships between Spot and Forward Rates

To understand the arbitrage relations between spot and forward exchange rates, we need to consider interactions between spot rates, forward rates, and interest rates.

Note that forward exchange rates are derived from an arbitrage principle that ensures equal investment returns on two alternative yet equivalent investment opportunities.

Using a single-period analogy, an investor who has funds to invest in treasury securities has two alternatives:

1. Invest at the domestic risk-free rate (r_d).
2. Invest at the foreign risk-free rate (r_f).

Option i:

If the investor takes the first option, the fund held at the end of the period would be $(1+i_d)$.

Option ii:

Alternatively, the investor could convert the domestic currency to be invested in a foreign currency using the spot rate $S_{f/d}$. It is important to note that (f/d) is the currency quoting convention that expresses the number of foreign units per single domestic unit.

At the end of the investment period, $S_{f/d}\,(1 + r_f)$ units of foreign currency would be held by the investor. Then, the funds would have to be converted back into the domestic currency using the initial (pre-agreed) forward rate.

Note that the two investment alternatives are risk-free because they are invested in risk-free assets.

Since these investment alternatives are equal by considering the risk characteristics, the returns must also be equal. As such, we have the following relationship:

$$1 + r_d = S_{f/d}(1 + r_f)\left(\frac{1}{F_{f/d}}\right)$$

This formula above outlines two alternative investments (represented on either side of an equation) expected to yield equal returns. Should the returns differ, a risk-free arbitrage opportunity arises. An investor can capitalize on this by short-selling the lower-yield investment and directing those funds into the higher-yield one.

Note that is the number of units of domestic currency for each unit of foreign sold forward. The relationship above can be rearranged to get the formula for a forward rate as illustrated below:

$$F_{f/d} = S_{f/d}\left(\frac{1 + r_f}{1 + r_d}\right)$$

Where:

$S_{f/d}$ = Current spot exchange rate.

$F_{f/d}$ = Current forward exchange rate.

r_d = Domestic risk-free rate.

r_f = Foreign risk-free rate.

Example: Calculating the Forward Exchange Rate

Given that the spot exchange $S_{f/d}$ is 1.502, the domestic risk-free rate for 12 months is 4%, and the 12-month foreign risk-free rate is 6.2%, the forward rate $F_{f/d}$ is:

$$\begin{aligned} F_{f/d} &= S_{f/d}\left(\frac{1+r_f}{1+r_d}\right) \\ &= 1.502\left(\frac{1+0.062}{1+0.04}\right) = 1.5338 \end{aligned}$$

Forward Rate as a Percentage of the Spot Rate

We can rearrange the no-arbitrage equation between the spot and exchange rates as follows:

$$\frac{F_{f/d}}{S_{f/d}} = \frac{1+r_f}{1+r_d}$$

Intuitively, from the above equation, under an f/d quoting convention, if foreign interest rates exceed domestic rates, the forward rate will be priced at a premium relative to the spot rate.

Generally speaking, without considering the quoting convention, the currency with the higher (lower) interest rate will always trade at a discount (premium) in the forward market.

We can interpret the forward exchange rate as the expected future spot rates. If we let $F_t = F_{f/d}$, $S_t = S_{f/d}$ and finally $F_t = S_{t+1}$ then the above equation can be written as:

$$\begin{aligned} \frac{S_{t+1}}{S_t} &= \frac{1+r_f}{1+r_d} \\ \Rightarrow \%\Delta S_{t+1} &= \frac{S_{t+1}}{S_t} - 1 = \left(\frac{r_f - r_d}{1+r_d}\right) \end{aligned}$$

As such, if we interpret forward rates as expected future spot rates, the expected percentage change in the spot rate is proportional to the interest rate differential $(r_f - r_d)$.

Interpreting forward rates are interpreted as expected future spot rates have some setbacks. For instance, the relationship between spot and forward exchange rates is influenced by any factor affecting the yield curve in either the domestic or foreign market. Consequently, FX markets are interconnected with global events, reflecting influences from other global markets.

As such, in real-world trading, currency traders and strategists do not rely solely on forward rates for their currency expectations or strategies.

However, it is useful to understand forward exchange rates simply as a product of the arbitrage equation and forward points as being related to the interest rate differential between the two countries.

Forward Discount Premiums based on Spot and Interest Rates

Recall that a forward discount is when the domestic current spot exchange rate is traded at a higher level than the current domestic future spot rates. On the other hand, A forward premium is a situation when the forward exchange rate is higher than the spot exchange rate. Conversely, a forward discount is when the forward exchange rate is lower than the spot exchange rate.

The analysis of the expectations from the market depends mostly on discounts and premiums. Also, they enable one to know the currencies that should appreciate and those that will depreciate in the near future.

Calculation

Recall the arbitrage formula:

$$F_{f/d} = S_{f/d}\left(\frac{1 + i_f}{1 + i_d}\right)$$

Where:

$S_{f/d}$ = Current spot exchange rate.

$F_{f/d}$ = Current forward exchange rate.

r_d = Domestic risk-free rate.

r_f = Foreign risk-free rate.

Note that in the above formula, we assumed a single-period analogy. Suppose the investment term is a fraction, τ, of the period for which the interest rates are quoted. Consequently, the interest earned in the domestic and foreign markets would be $r_d\tau$ and $r_f\tau$ respectively.

As such, the arbitrage formula transforms into:

$$F_{f/d} = S_{f/d}\left(\frac{1 + r_f\tau}{1 + r_d\tau}\right)$$

Intuitively, if we wish to calculate the forward premium or discount, we find the difference between the forward and spot exchange rates:

$$\begin{aligned}
F_{f/d} - S_{f/d} &= S_{f/d}\left(\frac{1 + r_f\tau}{1 + r_d\tau}\right) - S_{f/d} \\
&= S_{f/d}\left[\left(\frac{1 + r_f\tau}{1 + r_d\tau}\right) - 1\right] \\
&= S_{f/d}\left(\frac{r_f - r_d}{1 + r_d\tau}\right)\tau \\
\therefore F_{f/d} - S_{f/d} &= S_{f/d}\left(\frac{r_f - r_d}{1 + r_d\tau}\right)\tau
\end{aligned}$$

Therefore, forward points, when appropriately scaled, are proportional to the spot exchange rate and the interest rate differential. They are approximately, but not exactly, proportional to the horizon of the forward contract.

The common day count (for example, LIBOR deposits) is actual/360. Consider the following example.

Example: Forward Discount or Premium

Let's say we have a 31-day forward exchange rate. The domestic 31-day risk-free interest rate is 2.5% per year, and the foreign 31-day risk-free interest rate is 3.5%. The spot exchange rate is 1.5630. We can calculate the forward premium or discount using two methods:

Method 1:

We first calculate the forward exchange rate, and then we subtract the spot exchange rate from it:

$$
\begin{aligned}
F_{f/d} &= S_{f/d}\left(\frac{1+r_f\tau}{1+r_d\tau}\right) \\
&= 1.5630\left(\frac{1+0.035\times\frac{31}{360}}{1+0.025\times\frac{31}{360}}\right) = 1.56434
\end{aligned}
$$

Hence, the forward trading premium is:

$$F_{f/d} - S_{f/d} = 1.56434 - 1.5630 = 0.00134$$

Since forward premiums or discounts are usually quoted in pips or points, multiplying the result by 10,000 will give us $0.00134 \times 10,000 = 13.4$ pips, which is the forward trading premium quoted in pips or points.

Method 2:

Inserting the data directly into the formula:

$$
\begin{aligned}
F_{f/d} - S_{f/d} &= S_{f/d}\left(\frac{r_f - r_d}{1+r_d\tau}\right)\tau \\
&= 1.5630\left(\frac{0.035-0.025}{1+0.025\times\frac{31}{360}}\right)\times\frac{31}{360} \\
&= 0.00134 \approx 13.4 \text{ pips}
\end{aligned}
$$

Question #1

An Italian company has secured a contract with a US client, expecting a payment of USD 40 million in 45 days. The finance manager of the Italian firm wishes to hedge the FX risk of this deal and gets the following rates from a broker:

USD/EUR spot rate: 0.9220

One-month forward points: +2.0

According to the exchange rate information provided, the finance manager could hedge the FX risk by:

1. Buying euro (selling US dollars) at a forward rate of 0.9222.
2. Buying euro (selling US dollars) at a forward rate of 0.9200.
3. Selling euro (buying US dollars) at a forward rate of 0.9200.

Solution

The correct answer is **A.**

The Italian company would aim to change the US dollar to its home currency, the euro (it intends to sell US dollars and buy euros), using a forward rate calculated as 0.9220 + (+2.0/10,000) = 0.9222.

By doing this, the company is able to protect itself against any unfavorable movements in the foreign exchange rate over the next 45 days (before payment is received). In other words, the company wants locks in the exchange rate of USD/EUR = 0.9222 now. This can be achieved using a forward contract, which allows the company to exchange currencies at a predetermined rate in the future.

Question #2

When is a foreign currency *most likely* trading at a forward premium?

1. When the forward rate expressed in the domestic currency is below the spot rate.
2. When the forward rate expressed in the domestic currency is above the spot rate.
3. When the forward rate expressed in the foreign/domestic currency is at equilibrium.

Solution

The correct answer is **B**.

A foreign currency is at a forward premium if the forward rate expressed in domestic currency is above the spot rate.