Learning Module 10: Interest Rate Risk and Return

LOS 10a: calculate and interpret the sources of return from investing in a fixed-rate bond.

Sources of Return

Investors in fixed-rate bonds achieve returns through the following:

- Receipt of anticipated coupon and principal payments.
- Reinvestment of coupon payments.
- Capital gains or losses are realized when the bond is sold prior to its maturity.

Discount bonds feature a coupon rate below the current market rate, while premium bonds have a coupon rate above the market rate. Over time, the book value of a bond is amortized to match its face value upon reaching maturity. The carrying value represents the bond's purchase price, adjusted for any amortized discount or premium. Rising interest rates decrease bond prices (and vice versa). This affects the total return, specifically if the bond is sold before maturity.

Yield to Maturity (YTM)

This metric is crucial for bond investors. If an investor holds a bond until maturity, avoids any bond defaults, and consistently reinvests coupons at the prevailing interest rate, the YTM accurately reflects the investor's actual rate of return.

Investment Horizon and Interest Rate Risk

The investment horizon is critical in assessing interest rate risks and returns. The interest rate risk comprises two offsetting risks:

- Coupon reinvestment risk.
- Market price risk.

Reinvestment Risk

Reinvestment risk pertains to the possibility that an investor may not be able to reinvest the cash flows from an investment at a rate matching the investment's existing rate of return (yield to maturity). Two factors affect the degree of reinvestment risk:

- **Maturity**: The longer the bond's maturity, the higher the reinvestment risk. This is because of the high possibility that interest rates will be lower than they were at the time the bond was purchased;
- **The coupon rate of the bond**: The higher the coupon rate, the higher the payments that have to be reinvested and, consequently, the higher the reinvestment risk. In fact, a bond selling at a premium is more dependent on reinvestment income than another bond selling at par. The only fixed-income instruments that do not have reinvestment risk are zero-coupon bonds since they have no interim coupon payments.

Market Risk (Price Risk)

Bond market prices will decrease in value when the prevailing interest rates rise. In other words, if an investor wishes to sell the bond prior to maturity, the sale price will be lower if rates are higher.

As noted earlier, these two risks offset each other to an extent. The dominant risk depends partially on the investment horizon. The lower the investment horizon, the lower the reinvestment risk, but the higher the market risk.

Horizon Yield (Realized Rate of Return)

This metric delves deeper, offering insight into an investor's internal rate of return (IRR). It considers the total return, which is composed of reinvested coupons and the sale/redemption amount divided by the purchase price of the bond.

$$r = \left(\frac{\text{FV} + \text{Sale/Redemption Amount}}{\text{PV}}\right)^{\frac{1}{n}} - 1$$

Where:

- FV = Future value of reinvested coupons.
- PV = Purchase price of the bond.
- T = Holding period.

Example: Sources of Return

An investor initially buys a 5-year, 8% annual coupon payment bond at the price of 85.00 per 100 of par value.

Case 1: Holding the Bond Until Maturity

The yield to maturity of the bond is calculated as follows.

$$85 = \frac{8}{(1+r)^1} + \frac{8}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{8}{(1+r)^4} + \frac{108}{(1+r)^5};\ r = 12.18$$

The bond's yield-to-maturity is 12.18%. The easiest way to determine the value of r is to use the financial calculator:

$$n = 5; PV = -85; PMT = 8; FV = 100; CPT => I/Y = 12.18$$

So the investor receives the series of 5 coupon payments of 8 (per 100 of par value), a total of 40, plus the redemption of principal (100) at maturity. Besides collecting the coupon interest and the principal, there is an opportunity to reinvest the cash flows. If the coupon payments are reinvested at 12.18% immediately after they are received, the coupon's future value on maturity date will amount to 51 per 100 par value, calculated as per the following table.

End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5
$\$8 \times (1.1218)^4$	$\$8 \times (1.1218)^3$	$\$8 \times (1.1218)^2$	$\$8 \times (1.1218)^1$	$\$8 \times (1.1218)^0$

The $1^{st\sim}$ coupon payment of $8 is reinvested at 12.18% for 4 years until the end of the $5^{th\sim}$ year, the $2^{nd\sim}$ is invested for 3 years, and so forth. The amount in excess of the coupons, 11 (= 51 – (5 × 8)), is called "interest-on-interest" gain from compounding.

The investor's total return is 151, the sum of reinvested coupons (51), and the redemption of principal at maturity (100). Therefore, the realized rate of return is 12.18%.

$$85 = \frac{151}{(1+r)^5};\ r = 12.18\%$$

As case 1 demonstrates, the yield-to-maturity at the time of purchase equals the investor's rate of return under the following three assumptions:

- The investor holds the bond to maturity.
- There is no default by the issuer.
- The coupon interest payments are reinvested at that same rate of interest.

Case 2: Selling the Bond Before Maturity

If another investor buys the same bond but chooses to sell it after four years and reinvests all coupon payments at 12.18%, the future value of these reinvested coupons will be 38.3356% of the bond's face value at the end of the fourth year. This is calculated as follows:

End of Year 1	End of Year 2	End of Year 3	End of Year 4
$\$8 \times (1.1218)^4$	$\$8 \times (1.1218)^3$	$\$8 \times (1.1218)^2$	$\$8 \times (1.1218)^1$

Total = $38.3356

The interest-on-interest gain from compounding is $6.3356(= 38.3356 - 32)$.

At the time the bond is sold, it has one year remaining until maturity. If the yield-to-maturity remains 12.18%, the sale price of the bond (calculated as the PV of anticipated cash flows) is:

$$\text{Price}_{t=4} = \frac{108}{1.1218} = 96.2738$$

Therefore, the total return is $134.6094(= 38.3356 + 96.2738)$, and the realized rate of return is 12.18%.

$$85 = \frac{134.6094}{(1+r)^4};\ r = 12.18\%$$

Case 2 demonstrates that the realized horizon yield matches the original yield-to-maturity provided two conditions are met:

- Coupon payments are reinvested at the same interest rate as the original yield-to-maturity.
- The bond is sold at a price on the constant-yield price trajectory, i.e., the investor does not have any capital gains or losses when the bond is sold. The price trajectory is the time series of a bond's prices from some date (usually the date on which the bond is purchased) until its maturity.

Question

For a fixed-rate bond, what will most likely happen to its market price if interest rates rise?

A. The market price will rise.
B. The market price will remain unchanged.
C. The market price will fall.

Solution

The correct answer is **C**.

Bond prices and interest rates have an inverse relationship. Thus, when interest rates increase, bond prices tend to fall.

A is incorrect: As mentioned, bond prices and interest rates have an inverse relationship. So, the market price won't rise with rising interest rates.

B is incorrect: Bond prices are sensitive to changes in interest rates, so they will not remain unchanged.

LOS 10b: describe the relationships among a bond’s holding period return, its Macaulay duration, and the investment horizon.

Holding Period Return (Horizon Yield)

This represents the total return an investor anticipates from holding a bond over a specific duration. It's influenced by both the coupon payments received and any change in the bond's price due to interest rate movements.

Formula:

$$r = \left(\frac{FV + F}{PV}\right)^{\frac{1}{T}} - 1$$

Where:

- r = Realized rate of return or Horizon Yield.
- FV = Future value of the reinvested coupons.
- F = Face value of the bond (often considered as 100).
- PV = Present value or the bond's current price.
- T = Investment horizon.

An example of calculating the horizon yield was provided in the previous learning objective.

Macaulay Duration

Introduced by Frederick Macaulay in 1938, the Macaulay duration provides a measure of the weighted average time until a bond's cash flows are received. It serves as an indicator of the bond's price sensitivity to interest rate changes. When the investment horizon matches the Macaulay duration of a bond, the bond is nearly hedged against interest rate risk. Any loss from price risk due to rising rates is approximately offset by gains from reinvestment risk and vice versa. We will delve deeper on the Macaulay Duration on the next learning objective.

Investment Horizon

This is the period an investor plans to hold onto a bond. The relationship between the investment horizon and the Macaulay duration determines the bond's dominant source of interest rate risk:

- **Investment Horizon > Macaulay Duration**: The bond faces a dominant risk from reinvestment, arising due to the reinvestment of coupons, possibly at a less favorable rate.
- **Investment Horizon < Macaulay Duration**: Price risk is the prevailing concern. The bond's price might be adversely affected by rising interest rates.
- **Investment Horizon = Macaulay Duration**: The bond is nearly immune to interest rate risk. Here, price risk and reinvestment risk balance out.

Duration Gap

This represents the difference between a bond's Macaulay duration and the investor's investment horizon. Duration gap=Macaulay duration?Investment horizon

A negative duration gap indicates that the bond's Macaulay duration is lower than the investment horizon of the investor. In this scenario, the primary concern is reinvestment risk, predominantly arising due to falling interest rates. On the other hand, a positive duration gap suggests that the bond's Macaulay duration is higher than the investor's investment horizon. Here, the main risk stems from potential price fluctuations, mainly driven by increasing interest rates.

Overall, the duration gap helps in identifying the primary source of interest rate risk a bond faces, be it from reinvestment or price changes.

Question

Which of the following statements about a bond with a Macaulay duration higher than the investment horizon is MOST accurate?

A. The bond predominantly faces reinvestment risk.
B. The bond is primarily exposed to price risk.
C. The bond is nearly hedged against interest rate risk.

Solution

The correct answer is B:

When the Macaulay duration of a bond is higher than the investment horizon, the bond is primarily exposed to price risk, especially from rising interest rates.

A is incorrect: This would be the case when the investment horizon is higher than the Macaulay duration.

C is incorrect: The bond is nearly hedged against interest rate risk when the investment horizon matches the Macaulay duration.

LOS 10c: define, calculate, and interpret Macaulay duration.

Definition

Macaulay duration was introduced in the previous learning objective. It provides an understanding of the bond's sensitivity to interest rate fluctuations. At its core, Macaulay duration is the weighted average time until a bond's cash flows are received. It signifies the holding period for a bond that balances both reinvestment and price risk.

Calculation

The calculation for Macaulay Duration is derived from the bond's cash flows. Each cash flow is weighted by its share of the bond's full price, which is its present value. The following steps outline the calculation:

i. **Time to Receipt of Cash Flow**: Determine the time until each cash flow is received.
ii. **Cash Flow**: Identify the cash flow amount for each period.
iii. **Present Value of Cash Flow**: Calculate the present value of each cash flow using the bond's yield-to-maturity.
iv. **Weight of Cash Flow**: Compute the weight of each cash flow by dividing its present value by the total present value of all cash flows.
v. **Weighted Average Time**: Multiply the time to receipt of each cash flow by its respective weight.
vi. **Macaulay Duration**: Sum up the results from step 5 to obtain the Macaulay Duration.

Formula

The general formula to calculate Macaulay duration, represented as MacDur, is:

$$\text{MacDur} = \frac{\sum_{i=1}^{N} \frac{t \times CF_t}{(1+r)^t}}{\sum_{i=1}^{N} \frac{CF_t}{(1+r)^t}}$$

Where:

- t is the time (in periods) until the cash flow is received.
- CF_t is the cash flow at time t.
- r is the yield-to-maturity per period.
- N is the total number of periods.

Example: Calculating the Macaulay Duration

Think about a bond with five years left to maturity, a 1% annual coupon, and a yield-to-maturity of 0.10%. Assume it's 120 days into the first coupon period and follows a 30/360 day-count basis. What's the closest estimate for the bond's annualized Macaulay duration?

Considerations:

1. The bond has a 1% annual coupon, which means a cash flow of 1 per year for the next 4 years and 101 ($1 + 100$ par value) in the 5th year.
2. The yield-to-maturity is 0.10% or 0.0010 in decimal form.
3. It is 120 days into the first coupon period, so the first cash flow will be received in $1 - \frac{120}{360}$ years or 0.6667 years.
4. Macaulay duration is calculated as:

$$\text{MacDur} = \frac{\sum_{i=1}^{N} \frac{t \times CF_t}{(1+r)^t}}{\sum_{i=1}^{N} \frac{CF_t}{(1+r)^t}}$$

Where:

- CF_t = Cash Flow at time t.
- r = Yield to Maturity.
- t = Time to receipt of the cash flow.

Period	Time to Receipt	Cashflow Amount	PV	Weight	Time to Receipt*Weight
1	0.6667	1	0.9993	0.0096	0.01
2	1.6667	1	0.9983	0.0096	0.02
3	2.6667	1	0.9973	0.0095	0.03
4	3.6667	1	0.9963	0.0095	0.03
5	4.6667	101	100.5300	0.9618	4.49
Total			104.5213	1	4.5712

This means that an investor would, on average, wait **4.5712** years to receive the bond's cash flows, weighted by their present value.

Interpretation of Macaulay Duration

The Macaulay Duration provides insights into the bond's interest rate risk. A bond with a higher Macaulay Duration has greater sensitivity to interest rate changes.

For instance, if the investment horizon matches the Macaulay Duration, the bond is nearly hedged against interest rate risk. Any losses due to rising interest rates (price risk) would approximately be offset by gains from the reinvestment of coupons (reinvestment risk) and vice versa.

Furthermore, the Macaulay Duration is often annualized. For bonds with semiannual coupons, the Macaulay Duration is divided by 2 to get the annualized figure.

It's also noteworthy that the Macaulay Duration is typically less than the bond's time-to-maturity because it's a present value-weighted average of the time until cash flows are received.

Question

Consider a bond that has three years remaining to maturity, a coupon of 3.5% paid semiannually, and a yield-to-maturity of 3.80%. Assuming it is 18 days into the first coupon period and using a 30/360 basis, the bond's annualized Macaulay duration is *closest to*:

A. 2.81 years.
B. 2.82 years.
C. 2.84 years.

Solution

The correct answer is **C**.

Given:

- The bond has three years remaining to maturity.
- A coupon of 3.5% is paid semiannually.
- Yield-to-maturity is 3.80%.
- 18 days into the first coupon period.
- A 30/360 basis.

This means the bond pays 1.75% every 6 months. The yield per period is 1.90% (3.80% divided by 2).

Let's compute MacDur and then divide by 2 to annualize it since the bond pays semiannually.

Period	Time to Receipt	Cashflow Amount	Present Value (PV)	Weight	Time to Receipt *Weight
	0.95	1.75	1.718987	0.01732	0.016454
	1.95	1.75	1.686935	0.016997	0.033144
	2.95	1.75	1.655481	0.01668	0.049206
	3.95	1.75	1.624613	0.016369	0.064657
	4.95	1.75	1.594321	0.016064	0.079515
	5.95	101.75	90.96996	0.916571	5.453598
			99.2503	1	5.696573

The annualized MacDur is **5.6966/2 =** 2.8483