Learning Module 9: Option Replication Using Put-Call Parity

Q.1149 In which of the following positions can an arbitrageur earn risk-free profits when the market is in contango?

A. Long forward contract and long underlying asset at the spot price.

B. Short forward contract and long underlying asset at the spot price.

C. Short forward contract and short underlying asset at the spot price.

The correct answer is **B.**

Contango is a situation where the futures price (or forward price) of a commodity is higher than the spot price.

When the forward price is higher than the spot price, an arbitrageur can (I) borrow funds at the risk-free rate, (II) buy the underlying asset at the spot price, and (III) short the asset at the higher forward price.

Example:

Assume that the spot price of oil is $50 a barrel. The market would be said to be in contango if the futures price of oil two months from now is $55. In these circumstances, a trader could take the following actions:

I. Borrow $50 at the risk-free rate.
II. Buy oil today at $50 and sit on it for two months.
III. Sell a futures contract for delivery two months out at $55.

By locking in that profit at the higher price, and then sitting on the physical oil for a couple of months, a trader makes substantial gains.

A is incorrect. Going long on both the forward contract and the underlying asset at the spot price does not exploit the price difference between the spot and forward prices in a contango market. This strategy would involve buying the asset now and agreeing to buy it again in the future at the forward price, which is higher than the spot price. This does not create an arbitrage opportunity but rather exposes the investor to the risk of paying more for the asset in the future than its current price.

C is incorrect. Shorting both the forward contract and the underlying asset at the spot price does not create an arbitrage opportunity in a contango market. This strategy would involve selling the asset now and agreeing to sell it again in the future at the forward price. Without owning the asset, the arbitrageur would have to purchase it in the future, potentially at a higher price, to fulfill the forward contract, which introduces significant risk and does not guarantee a profit.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (b): Explain put–call forward parity for European options.

Q.1155 Calculate the payoff of a fiduciary call option if the spot price is \$45, the strike price is \$37, and the payoff on the riskless bond is \$37.

A. \$8.

B. \$10.

C. \$45.

The correct answer is **C.**

A fiduciary call consists of a European call and a risk-free bond that matures on the option expiration date.

If the option is in the money, then the payoff of the fiduciary call is

$$X + (S - X) \text{or} 37 + (\$45 - \$37) = \$45$$

A is incorrect. Suggesting a payoff of \$8 overlooks the structure of a fiduciary call. The option alone, if exercised, would indeed result in a gain of $S - X = \$45 - \$37 = \$8$, but this does not account for the total value of the fiduciary call strategy, which also includes the payoff of the riskless bond.

B is incorrect. Suggesting a payoff of \$8 overlooks the structure of a fiduciary call. The option alone, if exercised, would indeed result in a gain of $S - X = \$45 - \$37 = \$8$, but this does not account for the total value of the fiduciary call strategy, which also includes the payoff of the riskless bond. The correct calculation combines the intrinsic value of the call option with the bond's payoff, leading to a total payoff of \$45, not just the option's intrinsic value.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.3351 Leslie Hower is a junior trader at a derivatives dealer firm. During her first week at the firm, Hower attempts to synthetically sell a risk-free bond using call and put options. She purchases call and put options with the same exercise price and time to maturity. She simultaneously buys the underlying. With respect to her attempts in creating a synthetic short position in a risk-free bond, Hower is *most accurate* regarding her decision to:

A. Purchase put options.

B. Purchase call options.

C. Buy the underlying short.

The correct answer is **B.**

Based on the rearranged put-call parity (see below), in order to synthetically short sell (issue) a risk-free bond, a call option should be purchased while short selling the underlying and a put option.

$$\frac{-X}{(1+r)^T} = c_0 - p_0 - S_0$$

Given the information in the question, Leslie Hower has purchased call and put options with the same exercise price and time to maturity and simultaneously bought the underlying asset. This creates a synthetic short position in a risk-free bond. According to put-call parity, the value of the call option and the put option should be equal. Therefore, the most accurate decision made by Hower is to purchase the call option, as this is consistent with creating a synthetic short position in a risk-free bond.

A is incorrect. Purchasing put options alone does not create a synthetic short position in a risk-free bond. While put options increase in value when the underlying asset's price decreases, which is a component of a synthetic short position, without the corresponding sale of a call option and the short sale of the underlying asset, the position does not fully replicate the desired payoff structure.

C is incorrect. Buying the underlying short is not a decision that can be made. The phrase likely intended to suggest short selling the underlying asset, which is indeed a part of creating a synthetic short position in a risk-free bond. Short-selling the underlying asset is necessary but not sufficient on its own to create the synthetic short bond position; it must be combined with the purchase of a call option and the sale of a put option.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.3362 A synthetic long position in a riskless bond is *most likely* created by combining:

A. A long position in a put, a long position in the underlying, and a short position in the call.

B. A long position in a call, a long position in the underlying, and a short position in the put.

C. A short position in a put, a short position in the underlying, and a long position in the call.

The correct answer is **A.**

A synthetic long position in a riskless bond is created by combining a long position in a put, a long position in the underlying, and a short position in the call.

$$X(1+r)^{-T} = p_0 + S_0 - c_0$$

Where:
c_0 = Price of the call option
p_0 = Price of the put option
S_0 = Current price of the underlying asset
X = Strike price of the options
r = Risk-free interest rate
T = Time to maturity

B is incorrect. A long position in a call, a long position in the underlying, and a short position in the put does not create a synthetic long position in a riskless bond. Instead, this combination leans more towards creating a leveraged position in the underlying asset, which does not guarantee a risk-free return equivalent to holding a bond. The risk profile of this combination is significantly different from that of a riskless bond, as it involves both the potential for unlimited gains (due to the long call) and losses (due to the short put).

C is incorrect. A short position in a put, a short position in the underlying, and a long position in the call is essentially a speculative strategy that does not replicate the payoff of a riskless bond. This combination could lead to unlimited losses due to the short positions in both the put and the underlying asset. The strategy's payoff is highly dependent on the movements of the underlying asset's price and does not provide the fixed, known return characteristic of a riskless bond.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.3376 Which of the following combinations is *most likely* equivalent to going long a bond?

A. Investing in a put option, purchasing the underlying, and at the same time selling a call.

B. Investing in a call option, selling the underlying, and at the same time purchasing a put.

C. Investing in a put option and a call option on the same underlying while going short the underlying.

The correct answer is **A.**

A long bond is equivalent to going long a put, long the underlying, and short a call. We could use the put-call parity to work this problem out.

$$C_o + \frac{X}{(1+r)^t} = P_o + S_o$$

. If we make the bond the subject:

$$\frac{X}{(1+r)^t} = P_o + S_o - C_o$$

An addition sign before a variable implies that that variable has been purchased (trader has taken a long position), whereas a subtraction sign implies that the variable has been sold (trader has taken a short position). A long bond is, therefore, equal to long a put, long the underlying and short a call.

B is incorrect. This option suggests investing in a call option, selling the underlying, and at the same time purchasing a put. This combination does not replicate the payoff of a long bond position. Instead, it represents a protective put strategy combined with a short position in the underlying asset, which does not align with the characteristics of holding a bond.

C is incorrect. The combination of investing in a put option and a call option on the same underlying while going short the underlying is known as a straddle. This strategy does not replicate the payoff of a long bond. A straddle aims to profit from significant movements in the price of the underlying asset in either direction, which is fundamentally different from the relatively stable and predictable returns associated with holding a bond.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.3377 A European put option is selling for $4.00 with an underlying priced at $52. The exercise price is $50, and the underlying makes no cash payments during the life of the option. The risk-free rate is 6.0%, and the option expires in 120 days. A call with the same exercise price and expiry sells for $8.50. This put is *most likely*:

A. overvalued by $0.33.

B. undervalued by $1.55.

C. undervalued by $0.58.

The correct answer is **B.**

$$P_0 = C_0 - S_0 + \frac{X}{(1+r)^T}$$
$$= 8.50 - 52 + \frac{50}{(1.06)^{\frac{120}{365}}} = \$5.55$$

Hence, the put is undervalued by $5.55 - $4 = $1.55.

A is incorrect. Suggesting the put is overvalued by $0.33 does not align with the calculation based on the put-call parity. The calculation clearly shows that the put option is undervalued, not overvalued.

C is incorrect. While this option also suggests that the put is undervalued, the amount of $0.58 does not match the correct calculation. The accurate undervaluation amount, as derived from the put-call parity, is $1.55.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.3397 A three-month call option with an exercise price of $55 is being sold for $8. A three-month Treasury bond is being sold in the marketplace with the same face value as the option's exercise price. The underlying is currently worth $60, and the risk-free rate is 4.30%. Assuming the put-call parity holds, a put option is being sold for:

A. $0.73.

B. $2.42.

C. $12.34.

The correct answer is **B.**

$$\text{Put-call parity} = c_0 + \frac{X}{(1 + r)^T} = S_0 + p_0$$

$$p_0 = \$8 + \left[\frac{\$55}{(1.043)^{0.25}}\right] - \$60 = \$2.42$$

A is incorrect. The calculation of $0.73 does not align with the put-call parity formula given the provided values.

C is incorrect. The calculation of $12.34 significantly overestimates the price of the put option.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.4176 Assume a two-year put on a stock of Lakeview Inc. has a price of $9 and an exercise price of $89. A forward contract expiring in two years has a forward price of $92. If the risk-free interest rate is 8%, the price of the call option using put-call forward parity is *closest to:*

A. $10.57

B. $11.57

C. $15.00

The correct answer is **B.**

Using the put-call forward parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

Making c_0 the subject of the formula, we get:

$$c_0 = p_0 - \frac{X - F}{(1+r)^T}$$

As such,

$$c_0 = \$9 - \frac{(89 - 92)}{(1 + 8\%)^2} = \$11.57$$

A is incorrect. It underestimates the value of the call option by not accurately accounting for the present value of the exercise price in the context of the put-call forward parity.

C is incorrect. It significantly overestimates the value of the call option.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (b): Explain put–call forward parity for European options.

Q.4177 Which of the following positions *most likely* has the same no-arbitrage value as the long put option?

A. Long underlying and short call option.

B. Short put options, short underlying, and long risk-free bonds.

C. Long risk-free bond, long call option, and short underlying.

The correct answer is **C.**

The put-call parity is given by:

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}$$

Making p_0 the subject we get:

$$p_0 = c_0 + X(1 + r)^{-T} - S_0$$

As such, long put positions can be represented with a long risk-free bond, long call option, and short underlying.

A is incorrect. A long position in the underlying asset combined with a short call option does not replicate the payoff of a long put option. This combination represents a covered call strategy, which has a different risk and return profile compared to a long put option. The covered call strategy involves holding the underlying asset while selling a call option on the same asset, aiming to generate income from the option premium, which does not align with the payoff structure of a long put option.

B is incorrect. Shorting put options, shorting the underlying, and holding long risk-free bonds does not replicate the payoff of a long put option. This combination suggests a strategy that involves taking on obligations to potentially buy the underlying asset if the put options are exercised, while also betting on the decline in the asset's price through the short position. The inclusion of long risk-free bonds adds a fixed income component to the strategy. However, this combination does not provide the same payoff as holding a long put option, which offers the right but not the obligation to sell the underlying asset at a predetermined price, providing downside protection.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.4178 Which of the following *most likely* represent the no-arbitrage value of a fiduciary call position?

A. Long put, Long stock, Short bond.

B. Long put, Long Stock.

C. Short put, Short stock, Long bond.

The correct answer is **B.**

A fiduciary call combines a long call position and a long position in the risk-free bond. Essentially, it is the left-hand side of the put-call parity equation written as:

$$\underbrace{\boxed{S_0 + p_0}}_{\text{Fiduciary Call}} = c_0 + X(1 + r)^{-T}$$

A is incorrect. It suggests a combination of a long put, long stock, and short bond. This combination does not accurately represent a fiduciary call. A fiduciary call is specifically about creating a position that mimics the payoff of a call option through the combination of a risk-free bond and the underlying stock. The inclusion of a long put and short bond in this option introduces elements that are not part of the fiduciary call structure and thus does not align with the no-arbitrage value representation of a fiduciary call.

C is incorrect. It describes a position that is essentially the opposite of a fiduciary call. A short put, short stock, and long bond position would not replicate the payoff of a call option, which is what a fiduciary call aims to achieve. The fiduciary call strategy is designed to mimic the upside potential of owning a call option while protecting against downside risk through a risk-free bond. The combination suggested in option C would not achieve this objective, as shorting the stock and put would expose the investor to unlimited risk, which is contrary to the protective nature of a fiduciary call.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (a): Explain put–call parity for European options.

Q.4180 Consider options and forward the contract expiring in 70 days. The exercise price of options is AUD 87, and the risk-free rate is 4.5%. If the call price is AUD 18.5 and the forward price is AUD 95, the put premium is *closest to*:

A. AUD 8.95.

B. AUD 10.57.

C. AUD 11.34.

The correct answer is **B.**

Using the put-call forward parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

Making p_0 the subject of the formula, we get:

$$p_0 = c_0 + (X - F_0(T))^{-T}$$

As such,

$$\begin{aligned} p_0 &= 18.5 + \frac{87-95}{(1.045)^{\frac{70}{365}}} \\ &= \text{AUD } 10.56725 \approx \text{AUD } 10.57 \end{aligned}$$

is incorrect. The calculation for a put premium of AUD 8.95 does not align with the put-call parity for forward contracts. The put-call parity relationship provides a direct method to calculate the put premium based on the call premium, the exercise price, the risk-free rate, the time to expiration, and the forward price, which leads to a different result.

C is incorrect. A put premium of AUD 11.34 suggests a calculation that does not accurately apply the put-call parity for forward contracts. This value could be the result of overestimating the impact of the risk-free rate or the time to expiration on the put premium. The put-call parity ensures that the relationship between the call price, put price, forward price, and exercise price is maintained, preventing arbitrage opportunities and ensuring market efficiency.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (b): Explain put–call forward parity for European options.

Q.4181 Tanya Glen is an investor who wants to take a position in a six-month forward contract. The put price value exceeds the call value by $12, with both having the exercise price of $90, and the risk-free rate is 6%. Assuming that options expire in six months, the forward price is *most likely*:

A. less than the exercise price.

B. more than the exercise price.

C. not known due to the lack of sufficient information.

The correct answer is **A.**

Recall the put-call forward parity:

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

We can rearrange the above equation as follows:

$$p_0 - c_0 = X(1+r)^{-T} - F_0(T)(1+r)^{-T}$$

Intuitively, if the left side of the equation is positive $(p_0 - c_0 \geq 0)$, then $F_0(T) < X$. Since, $p_0 - c_0 = \$12$ then, it is true that $F_0(T) < X$.

B is incorrect. It suggests that the forward price is more than the exercise price. This contradicts the put-call parity and the given condition that the put price exceeds the call price. In a scenario where the put price is higher than the call price, it generally indicates a market expectation of a decrease in the asset's price, leading to a forward price that is less than the exercise price.

C is incorrect. It states that the forward price cannot be determined due to the lack of sufficient information. However, using the put-call parity and the given information about the put and call prices, as well as the risk-free rate, we can indeed infer the relationship between the forward price and the exercise price. The information provided is sufficient to conclude that the forward price is less than the exercise price.

CFA Level I, Derivatives, Learning Module 9: Option Replication Using Put-Call Parity. LOS (b): Explain put–call forward parity for European options.
