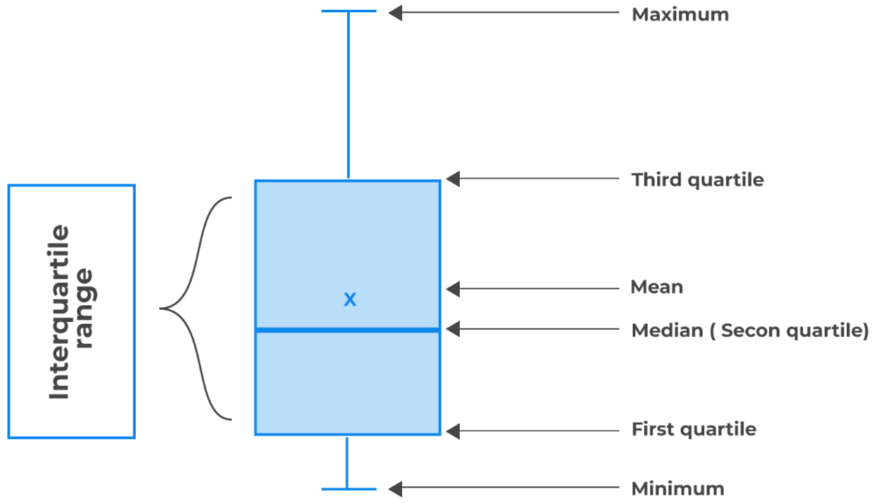
TOPIC			VARIABLES
QUANTITATIVE ANALYSIS	Nominal Risk-free Rate	(1 + Nominal risk-free rate) = (1 + Real risk-free rate) (1 + Inflation premium)	
	Approximation of Nominal Risk-free Rate	Nominal risk-free rate = Real risk-free rate + Inflation premium	
	Holding Period Return (R)	$R = \frac{(P_1 - P_0) + I_1}{P_0}$	P_0 – Price at time 0 P_1 – Price at time 1 I_1 – Interest earned during period
	Geometric mean return	$\bar{R}_{Gi} = ((1 + R_{i1} \times (1 + R_{i2}) \times \ldots (1 + R_{i,T-1}) \times (1 + R_{i,T}))^{\frac{1}{T}} - 1$ $= \sqrt[T]{\prod_{t=1}^{T}(1 + R_t)} - 1$	R_{it} – Return in period t T – Total number of periods
	Harmonic Mean	$\bar{X}_H = \frac{n}{\sum_{i=1}^{n}\frac{1}{X_i}}, X_i > 0 \text{ for } i = 1, 2, \ldots, n.$	X_i – Individual observations n – Number of observed values
	Present Value (PV)	$PV = FV_N \left(1 + \frac{R_s}{m}\right)^{-mN}$	FV – Future Value R_s – Stated annual interest rate m – Number of compounding periods per year N – Number of years

	Annualizing Returns	$R_{annual} = \left(1 + R_{period}\right)^{c} - 1$	c – Number of periods in a year R_{period} – return for a period less than a year.
	Continuously Compounded Return ($r_{0,T}$)	$r_{0,T} = \ln\left(\frac{P_T}{P_0}\right)$	P_0 – Price of an asset at time t = 0 P_T – Price of an asset at time t = T
	Real Return	$(1 + \text{real retun}) = \frac{(1 + \text{Real risk free rate})(1 + \text{Risk premium})}{(1 + \text{Inflation premium})}$	
	Internal Rate of Return (IRR)	$\sum_{t=0}^{T} \frac{CF_t}{(1 + IRR)^t} = 0$	CF_t – Cash flows at period t. T – The number of periods
	Return on a Leveraged Portfolio (R_L)	$R_L = \frac{\text{Portfolio Return}}{\text{Portfolio Equity}}$ $= \frac{[R_P \times (V_E + V_B) - (V_B \times r_D)]}{V_E}$ $= R_P + \frac{V_B}{V_E}(R_P - r_D)$	V_E – The equity of the portfolio V_B – Borrowed funds R_P – Total return on leveraged portfolio r_D – Cost of debt
	Future Value (FV) of a Cash Flow	$FV_t = PV(1 + r)^t$	FV_t – Future value of a cashflow at time t. PV – Present value of a cashflow

	Future Value (FV) of a Cash Flow under Continuous Compounding	$FV_t = PVe^{rt}$	
	Present Value (PV) of a Cash Flow	$PV = FV_t(1+r)^{-t}$	FV_t – Future value of a cashflow at time t. PV – Present value of a cashflow
	Present Value (PV) of a Cash Flow under Continuous Compounding	$PV = FV_t e^{-t}$	
	PV (Discount Bond)	$PV = FV_t(1+r)^{-t}$	PV – Price paid FV – Full principal (FV) received at maturity.
	PV (Coupon Bond)	$PV(\text{Coupon Bond}) = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{(PMT_N + FV_N)}{(1+r)^N}$	PMT = Coupon payment. FV = Future value. r = Market discount rate (YTM). N = Number of periods.
	PV (Perpetual Bond)	$PV(\text{Perpetual Bond}) = \frac{PMT}{r}$	PMT = Coupon payment. r = Market discount rate (YTM).

	Future value of an ordinary annuity	$FV_N = A\left[\frac{(1+r)^N - 1}{r}\right]$	A – Annuity amount N – Number of years r – Rate of return
	Periodic annuity cash flow (A) of an ordinary annuity	$A = \frac{r(PV)}{1-(1+r)^{-t}}$	r – Market interest rate for period PV – Present value/principal amount of loan t – number of payment periods
	Present value of an ordinary annuity	$PV = A\left[\frac{1-\left(\frac{1}{(1+r)^N}\right)}{r}\right]$	A – Annuity amount N – Number of years r – Rate of return
	Present value of a perpetuity	$\frac{A}{r}$	A – Annuity amount r – Required rate of return
	Price of a preferred or common share (paying a constant periodic dividend)	$PV_t = \frac{D_t}{r}$	PV_t – Present value at time t. D_t – Dividend payment at time t. r – Discount rate.
	Price of a preferred or common share (assume a constant dividend growth rate (g) into perpetuity)	$PV_t = \frac{D_{t+1}}{r-g}$, where $(r-g) > 0$	PV_t – Present value at time t. D_{t+1} – Expected Dividend in the next period. r – Required rate of return. g – Constant growth rate.

	Sample Mean	$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$	x_i – Individual observations n – Number of observed values
	Median of odd-numbered sample of n observations	Occupies the $\frac{n+1}{2}$ position.	
	Median of Even-Numbered Sample of n Observations	mean of the values of the observations occupying the $\frac{n}{2}$ and $\frac{n+2}{2}$ positions (the two middle observations).	
	Trimmed Mean	Computing arithmetic mean after excluding a stated small percentage of the lowest and highest values.	
	Winsorized Mean	Computing arithmetic mean after assigning one specified low value to a stated percentage of the lowest values in the dataset and one specified high value to a stated percentage of the highest values in the dataset.	
	Interquartile Range (IQR)	$IQR = Q_3 - Q_1$	Q_1– First quartile Q_3– Third quartile

	Box and Whisker Plot		
	Position of observation at a given percentile	$L_y = (n+1) \times \frac{y}{100}$	L_y – position of percentile n – Number of observations y – Percentiles
	Range	Max value – Min Value	
	Mean Absolute Deviation	$\frac{\sum_{1=1}^{N} \lvert X_i - \overline{X} \rvert}{n}$	X_i – Individual observed value $\overline{X}$ – Individual values n – Number of observed values

	Population Variance	$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$	X_i – Individual observed value μ – Population mean N – Number of observations
	Sample Variance	$$s^2 = \frac{\sum_{i=1}^{N}(X_i - \overline{X})^2}{n-1}$$	X_i – Individual observed value μ – Sample mean N – Number of observations
	Sample target semi–deviation	$$S_{Target} = \sqrt{\sum_{\text{for all } X_i \le B}^{n} \frac{(X_i - B)^2}{n-1}}$$	X_i – Individual observations n – Number of observed values B – Target value
	Coefficient of Variation	$$CV = \frac{s}{\overline{X}}$$	s – Sample standard deviation $\overline{X}$ – Sample mean
	Sample covariance of X and Y	$$s_{XY} = \frac{\sum_{i=1}^{N}(X_i - \overline{X})(Y_i - \overline{Y})}{n-1}$$	x_i – individual observed value $\overline{x}$ – Individual values y_i – Individual observed value $\overline{y}$ – Individual values n – Number of observed values

	Sample correlation coefficient	$$r_{XY} = \frac{s_{XY}}{s_X \times s_Y}$$	s_{XY} – sample covraince s_X – Standard deviation of x s_Y – Standard deviation of y
	Sample Skewness	$$\text{Skewness} = \frac{1}{n} \times \frac{\sum_{i=1}^{n}(X_i - \overline{X})^3}{s^3}$$	n – Number of observations X_i – Individual observed values $\overline{X}$ – Sample mean s – Sample standard deviation
	Sample Kurtosis	$$\frac{1}{n} \times \frac{\sum_{i=1}^{n}(X_i - \overline{X})^4}{s^4}$$	n – Number of observations X_i – Individual observed values $\overline{X}$ – Sample mean s – Sample standard deviation
	Sample Excess Kurtosis	$$K_E = \left[\frac{1}{n} \times \frac{\sum_{i=1}^{n}(X_i - \overline{X})^4}{s^4}\right] - 3$$	n – Number of observations X_i – Individual observed values $\overline{X}$ – Sample mean s – Sample standard deviation

	Expected Value of a Discrete Random Variable	$E(X) = P(X_1)X_1 + P(X_2)X_2 + \cdots + P(X_n)X_n$ $= \sum_{i=1}^{n} P(X_i)X_i$	Where, X_i – One of n possible outcomes of the discrete random variable X. $P(X_i) = P(X_i = x_i)$ – Probability of X taking the value x.
	Variance of a Discrete Random Variable	$\sigma^2(X) = E[X - E(X)]^2$ $= P(X_1)[X_1 - E(X)]^2 + P(X_2)[X_2 - E(X)]^2 + \cdots + P(X_n)[X_n - E(X)]^2$ $= \sum_{i=1}^{n} P(X_i)[X_i - E(X)]^2$	
	Conditional Expectation	$E(X \mid S) = X_1 \cdot P(X_1 \mid S) + X_2 \cdot P(X_2 \mid S) + \cdots + X_n \cdot P(X_n \mid S)$ $= \sum_{i=1}^{n} X_i \cdot P(X_i \mid S)$	$X_1, X_2, \ldots X_n$ – Outcomes form a set of mutually exclusive and exhaustive events. S – A scenario or event
	Total Probability Rule for Expected Value	$E(X) = P(S_1) \cdot E(X \mid S_1) + P(S_1) \cdot E(X \mid S_1) + \cdots + P(S_n) \cdot E(X \mid S_n)$ $= \sum_{i=1}^{n} P(S_i) \cdot E(X \mid S_i)$	$S_1, S_2, \ldots S_n$ – Mutually exclusive and exhaustive scenarios or events.
	Bayes' Formula (Updated Probability)	$\text{Updated Probability} = \frac{\text{Probability of new information for an event}}{\text{Unconditional probability of new information}} \times \text{Prior Probability}$	

	Bayes' Formula (Probability Notation)	$P(\text{Event} \mid \text{Information}) = \frac{P(\text{Information} \mid \text{Event})}{P(\text{Information})} \times P(\text{Event})$	
	Expected Return on the Portfolio $(E(R_p))$	$E(R_p) = w_1E(R_1) + w_2E(R_2) + \cdots w_nE(R_n)$	$w_1, w_2, \ldots, w_n$ – Weights (market value of asset/market value of the portfolio) attached to assets $1,2,\ldots,n$. $R_1, R_2, \ldots, R_n$ – Expected returns for assets $1,2,\ldots,n$.
	Portfolio Variance (with two assets A and B)	$\text{Portfolio Variance} = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2(w_A)(w_B)\text{Cov}(R_A, R_B)$	w_A – Weight of assets A in the portfolio. w_B – Weight of assets B in the portfolio $\sigma^2(R_A)$ – Variance of the returns on assets A. $\sigma^2(R_B)$ –Variance of the returns on assets B.

	Covariance between Random Variables X and Y	$\text{Cov}(X,Y) = \sigma(X,Y) = E[(X - E[X])(Y - E[Y])$	Cov(X, Y) – Covariance of X and Y E[X] – Expected value of the random variable X. E[Y] – Expected values of the random variable Y.
	Sample Covariance	$\text{Cov}(X,Y) = \sum_{i=1}^{n} \frac{(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$	$\bar{X}$ – Sample mean of X. $\bar{Y}$ – Sample mean of Y. X_i and Y_i – i-th data points of X and Y, respectively.
	Correlation between Random Variables X and Y	$\text{Corr}(X,Y) = \rho(X,Y) = \frac{\text{Cov}(X,Y)}{\text{Standard deviation}(X) \times \text{Standard deviation}(Y)} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$	
	Safety First Ratio (SF)	$\frac{E(R_p) - R_L}{\sigma_p}$	$E(R_p)$ – Expected Portfolio return R_L – Threshold level of return σ_p– Standard deviation of the portfolio
	Mean and Variance of Lognormal Distribution	Mean: $\mu_L = e^{\left(\mu + \frac{1}{2}\sigma^2\right)}$ Variance: $\sigma_L^2 = e^{2u+\sigma^2}\left(e^{\sigma^2} - 1\right)$	μ and σ – Mean and variance of the associated normal distribution

	Standard Error of Sample Mean	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$	σ – Population standard deviation n – Number of observations
	Standard Error of Sample Mean when Population Standard deviation is Unknown	$s_{\bar{X}} = \frac{s}{\sqrt{n}}$	s – Sample standard deviation n – Number of observations
	Central Limit Theorem (CLT)	Given a population described by any probability distribution having mean μ and finite variance σ^2, the sampling distribution of the sample mean $\bar{X}$ computed from random samples of size n from this population will be approximately normal with mean μ (the population mean) and variance $\frac{\sigma^2}{n}$ (the population variance divided by n) when the sample size n is large	
	Test of a single mean (t–distributed)	$t_{n-1} = \frac{\bar{X} - \mu_o}{s/\sqrt{n}}$	$\bar{x}$ – Sample mean μ_o – Hypothesized value of the population mean s – Sample standard deviation n – Sample size
	Test of Differences in Means with Independent Samples (t-distributed)	$t_{n_1+n_2-2} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)^{\frac{1}{2}}}$ Where: $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$	$\bar{X}_1$ – Mean of the first sample μ_1 – Population of the first population μ_2 – Population of the second population $\bar{X}_2$ – Mean of the second sample

			s_1^2 – Variance of the first sample s_2^2– Variance of the second sample n_1 – Number of observations of the first sample n_2 – Number of observations of the second sample
	Test of the Mean of Differences (t-distributed)	$t_{n-1} = \dfrac{\bar{d} - u_{d0}}{s_{\bar{d}}}$	$\bar{d}$ – Sample mean difference u_{d0} – hypothesized value of population mean differences $s_{\bar{d}}$ – standard error of $\bar{d}$.
	Test of a Single Variance (Chi-square Distributed)	$\chi^2_{n-1} = \dfrac{(n-1)s^2}{\sigma_o^2}$	χ^2_{n-1} - The chi-square statistic with (n−1) degrees of freedom n – Sample size s^2 – Sample variance σ_o^2 – Known population variance
	Test of Difference in Variances (F-distributed)	$F_{n-1,n-2} = \dfrac{s^2_{Before}}{s^2_{After}}$	s^2_{Before} – Sample variance before an event s^2_{After} – Sample variance after an event

	Test of a Correlation (t-Distributed)	$t_{n-2} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$	r – Correlation coefficient n – Sample size
	Type I error	Results from falsely rejecting the null hypothesis when it is true.	
	Type II error	Results from failing to reject the null hypothesis when it is false.	Type II error
	Spearman Rank Correlation Coefficient (r_S)	$r_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$	d_i^2 – Squared differences in ranks n – Sample size
	Test of independence (categorical data)	$X^2 = \sum_{i=1}^{m} \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$ $df = (r-1)(c-1)$ $E_{ij} = \frac{(\text{Total row i}) \times (\text{Total Column j})}{\text{Overall Total}}$	m – Number of cells in the table, which is the number of groups in the first class multiplied by the number of groups in the second class O_{ij} – Observed frequency in the cell at the i-th row and j-th column of the contingency table. E_{ij} – Expected frequency in the same cell, which is calculated under the assumption of independence between the row and column variables. r – Number of rows

			c – Number of columns.
	Slope coefficient	$$\hat{b}_1 = \frac{\text{Covairance of Yand X}}{\text{Variance of X}} = \frac{\frac{\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})}{n-1}}{\frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{n-1}} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{N}(X_i - \bar{X})^2}$$	Y_i – Particular observation of variable Y $\bar{Y}$ – Mean value of Y X_i – Particular observation of variable X $\bar{X}$ – Mean value of X n – Sample size
	Intercept	$$\hat{b} = \bar{Y} - \hat{b}_1\bar{X}$$	$\bar{Y}$ – Mean value of Y $\hat{b}_1$ –Slope Coefficient $\bar{X}$ – Mean value of X
	Total sum of squares (SST)	$$SST = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$	n - The number of observations or data points. Y_i –The actual or observed values of the dependent variable $\bar{Y}$ –The mean of the observed values of the dependent variable.

	Sum of Squares Regression (SSR)	$$SSR = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2$$	n – Number of observations or data points. $\hat{Y}_i$ – Predicted values of the dependent variable based on the regression model. $\bar{Y}$ – Mean of the observed values of the dependent variable.
	Sum of squared errors or residuals (SSE)	$$SSE = \sum_{i=1}^{N} \left(Y_i - \hat{Y}_i\right)^2$$	n – Number of observations or data points. Y_i – Actual or observed values of the dependent variable. $\hat{Y}_i$ – Predicted values of the dependent variable based on the regression model.
	Coefficient of determination (R^2)	$$R^2 = \frac{\text{Explained variation}}{\text{Total variation}} = \frac{\text{Sum of Squares Regression(SSR)}}{\text{Sum of squares total(SST)}}$$	
	F–statistic	$$F = \frac{\frac{SSR}{k}}{\frac{SSE}{n-(k+1)}} = \frac{\text{Mean square regression}}{\text{Mean square error}} = \frac{MSR}{MSE}$$	n – Total number of observations (n) k – Total number of independent variables RSS – Regression sum of squares SSE – Sum of squared errors or residuals

	Standard error of estimate (SEE)	$\text{SEE} = \sqrt{\text{MSE}}$	MSE – Mean Square Error
	Standard error of the slope coefficient ($S_{\hat{b}_1}$)	$S_{\hat{b}_1} = \dfrac{\text{SEE}}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$ $= \dfrac{\text{model's standard error of the estimate}}{\text{square root of the variation of the independent variable}}$	X_i – Individual values of the independent variable. $\bar{X}$ – Mean of the independent variable.
	Standard Error of the Intercept	$S_{\hat{b}_0} = \sqrt{\dfrac{1}{n} + \dfrac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$	n – Number of observations in the sample. X_i – Individual values of the independent variable. $\bar{X}$ – Mean of the independent variable.
	Standard Error of the Forecast	$s_f^2 = s_e{}^2\left[1 + \dfrac{1}{n} + \dfrac{(X_f - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right]$	$s_e{}^2$ – The squared standard error of estimate n – Number of observations X_f – Value of the independent variable $\bar{X}$ – Estimated mean s^2 – Variance of the independent variable

	Log–lin model	$\ln Y_i = b_0 + b_1X_i$	Y_i – Dependent variable b_0 – Intercept b_1 – Slope coefficient X_i – Independent variable
	Lin–log model	$Y_i = b_0 + b_1 \ln X_i$	Y_i – Dependent variable b_0 – Intercept b_1 – Slope coefficient X_i – Independent variable
	Log–log model	$\ln Y_i = b_0 + b_1 \ln X_i$	Y_i – Dependent variable b_0 – Intercept b_1 – Slope coefficient X_i – Independent variable