Learning Module 8: Pricing and Valuation of Options

LOS 8a: Explain the exercise value, moneyness, and time value of an option

Options are derivative instruments that give the option buyer the right, but not the obligation, to buy (call option) or sell (put option) an asset from (or to) the option seller at a fixed price on or before expiration.

In other words, options are **contingent claims** that give the option buyer the right but not the obligation to transact the underlying, and the option seller is obligated to meet the obligation chosen by the buyer. The **payoff** of an option is either positive or zero. The **profit**, on the other hand, can be negative because of the option premium.

Exercise Value, Moneyness, and Time Value of an Option

An option's time value and exercise value constitute the value of an option. The **exercise value** of an option would be the value if it were immediately exercisable. On the other hand, an option's **time value** reflects the passage of time and variability of the underlying.

The **moneyness** of an option describes the relationship between the underlying price and the exercise price.

Now we look into these factors, considering the European options with no associated costs or benefits of owning an underlying asset.

Exercise Value of Options at Maturity

Call Options

Remember that in call options, the buyer has the right but not the obligation to buy the underlying. Moreover, the call option will only be exercised if the payoff is positive. Otherwise, the option expires worthless, and the option buyer incurs a loss equal to the option premium paid to the seller.

That is, the buyer would only exercise (buy the underlying) the option if (S_T>X). As such, the payoff (exercise value) to the buyer at expiration is given by:

$$c_T = \max(0, S_T - X)$$

Conversely, the exercise to the seller at expiration is:

$$-c_T = -(\max(0, S_T - X))$$

In other words, the exercise value of a call option is greater than either zero or the underlying price at expiration minus the exercise price.

Put Options

On the other hand, in a put option, the buyer has the right but not an obligation to exercise the option at expiry. Exercising the option means that the buyer sells the underlying S_T at the exercise price X at expiration. As such, the put option is only exercisable if $S_T < X$.

Therefore, the payoff to the buyer is given by:

$$p_T = \max(0, X - S_T)$$

Conversely, the payoff to the put option seller is

$$-p_T = -(\max(0, X - S_T))$$

Exercise Value of Options before Maturity

At any time before maturity ($t < T$), the investors estimate the value of options based on the underlying spot price S_t and the exercise price, X.

The exercise value of a call option is the value of an option contract at any time $t < T$, and it is calculated as spot price (S_t) minus the present value of the exercise price:

$$c_t = \max(0, S_t - X(1+r)^{-(T-t)}$$

Conversely, for the put option, the exercise value at any time $t < T$ is given by;

$$p_t = \max((0, X(1+r)^{-(T-t)} - S_t)$$

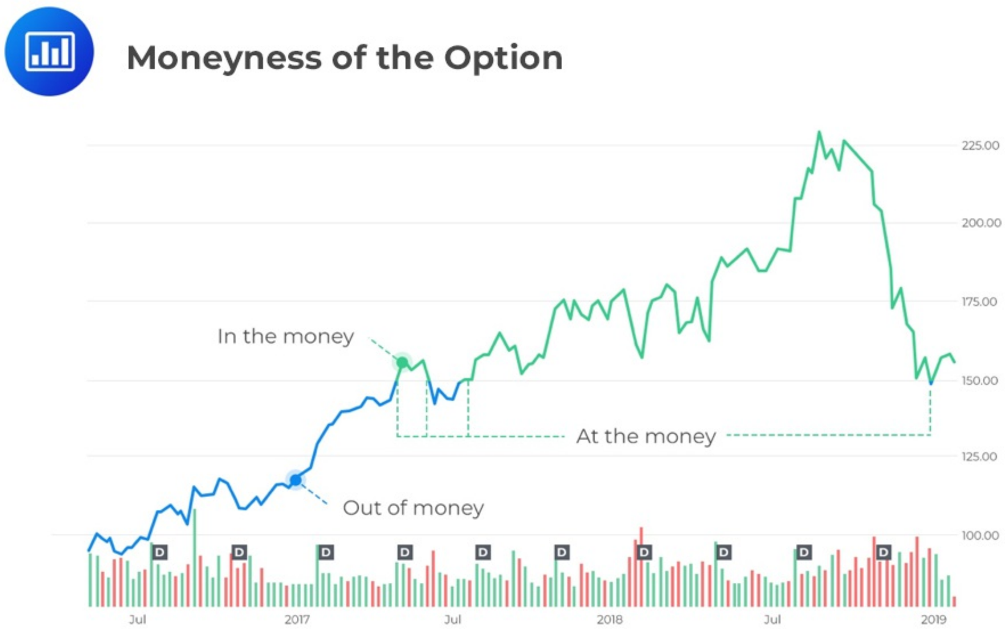
Assuming that $X = F_0(T)$ and ignoring the upfront option premium, the exercise value of a call option is equivalent to the value of the long forward commitment at time t only if the spot price is greater than the present value of X.

That is, if $S_t > PV(X)$, then:

$$S_t - PV(F_0(T)) = \max(0, S_t - PV(X))$$

Moneyness of an Option

Recall that the moneyness of an option is the relationship between the total value of an option and its exercise price. The best way to understand the concept of moneyness is via a graphical example. Below, you can see an example for a USD 150 call option. The green line represents when the option is in the money, and the blue represents times when the option is out of the money. The inverse would be true for a USD 150 put option. For both call and put options, the option is at the money when the line switches from blue to green.



In-the-Money Options

A call is said to be in the money if the underlying spot price is above the exercise price. That is, $S_t > X$.

On the other hand, a put is in the money if the spot price is less than the exercise price. That is, $S_t < X$.

In addition, an option is said to be **deep in the money** if it is highly exercisable, meaning the stock price is much higher than the exercise price.

Out-of-the-Money Options

A call is out of the money if the spot price falls below the current exercise price. That is, $S_T < X$.

On the other hand, a put is out of the money if the spot price is higher than the current exercise price $S_T > X$.

An option is said to be **deep out of the money** if it is unlikely to be exercised, for example, if the stock price is USD 1 and the call option's strike price is USD 50.

At-the-Money Options

A call or a put is called at the money if the option's exercise price equals the current underlying spot price. That is, $S_t = X$.

The moneyness of an option can be summarized in the table below:

Moneyness	Call Options	Put Options
In the Money (ITM)	$S_t > X$	$S_t < X$
At the Money (ATM)	$S_T = X$	$S_T = X$
Out of the Money (OTM)	$S_t < X$	$S_t > X$

Example: Exercise Value of an Option

Consider a 2-year call option with an exercise price of USD 100 and a risk-free rate of 2.31%. If, in six months, the underlying spot price is USD 98, the exercise value is *closest* to:

Solution

$$\begin{aligned} c_t &= \max(0, S_t - X(1+r)^{-(T-t)}) \\ &= \max(0, 98 - 100(1.0231)^{-1.5}) \\ &= \max(0, 98 - 96.63) \\ &= \text{USD } 1.37 \end{aligned}$$

Time Value of an Option

European options are only exercised at the expiration date. However, they can be purchased or sold before maturity at a price that captures the option's future expected payoff.

Denote the price by c_t for the call option and p_t for a put option. The time value of an option is defined as the difference between the current option price and the option's current payoff (or exercise value).

For a call option, the time value is given by:

$$\text{Time value} = c_t - \max\left(0, S_t - X(1+r)^{-(T-t)}\right)$$

We can rearrange this so that:

$$c_t = \max(0, S_t - X(1+r)^{-(T-t)}) + \text{Time value}$$

For a put option, the time value is given by:

$$p_t = \max(0, X(1+r)^{-(T-t)} - S_t) + \text{Time value}$$

From the formulas above, it is easy to see that the current option price equals the exercise value plus the time value for both call and put options:

$$c_t = \text{Exercise value} + \text{Time value}$$

$$p_t = \text{Exercise value} + \text{Time value}$$

Example: Time Value of an Option

Consider a 2-year call option with an exercise price of USD 100 and a risk-free rate of 2.31%. If, in six months, the spot price of the underlying is USD 98 and the price of the option is USD 1.88, the option's time value is *closest* to:

Solution

$$\begin{aligned}
c_t &= \text{Exercise value} + \text{Time value} \\
\text{Time value} &= c_t - \text{Exercise value} \\
&= c_t - \max(0, S_t - X(1+r)^{T-t}) \\
&= 1.88 - 1.37 \\
&= \text{USD } 0.51
\end{aligned}$$

Question

A European put option on an underlying stock has four months to maturity. The option's exercise price is USD 60. At option's maturity, the underlying price is USD 53. The underlying has no associated cost of carry.

If the risk-free rate is 1.5% and the current option price is USD 3, the time value of the option is *closest to*:

A. \$0.

B. -\$3.703.

C. \$3.00.

Solution

The correct answer is **B**.

The time value of a put option is given by:

$$\begin{aligned} p_t &= \text{Exercise value} + \text{Time value} \\ \text{Time value} &= p_t - \text{Exercise value} \\ &= p_t - \max(0, X(1+r)^{-(T-t)} - S_t) \\ &= 3 - \max((0, 60(1.015)^{-\frac{4}{12}} - 53) \\ &= 3 - \max(0, 6.703) \\ &= 3 - 6.703 \\ &= -3.703 \end{aligned}$$

A is incorrect. It is the payoff of the put option, four months to maturity:

$$\begin{aligned} \text{Payoff} &= \max(0, X(1+r)^{-(T-t)} - S_t) \\ &= \max(0, 53(1.015)^{-\frac{4}{12}} - 60) \\ &= 0 \end{aligned}$$

C is incorrect.Calculates the payoff of the put option as:

$$
\begin{aligned}
\text{Time value} &= p_t - \max(0, X(1+r)^{-(T-t)} - S_t) \\
&= 3 - \max(0.53(1.015)^{-\frac{4}{12}} - 60) \\
&= 3 - \max(0, -7.2624) \\
&= 3 + 0 \\
&= 3.00
\end{aligned}
$$

LOS 8b: Contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims

Recall that arbitrage opportunities occur if the law of one price does not hold. The no-arbitrage conditions in options are based on the payoff at maturity.

Unlike forward commitments with symmetric profiles (as presented earlier), contingent claims have asymmetric payoff profiles. That is:

$$c_T = \max(0, S_T - X)$$
$$p_T = \max(0, X - S_T)$$

Moreover, in contrast to forward commitments with an initial value of zero at initiation, the option buyer pays the seller a **premium** c_0) for call options and p_0 for a put options. Profits at maturity are:

$$\Pi_{call} = \max(0, S_T - X) - c_0$$
$$\Pi_{put} = \max(0, X - S_T) - p_0$$

An option is only exercised when it is in the money. As such, this condition calls for upper and lower no-arbitrage price bounds at any time t.

Upper and Lower Arbitrage Bounds

Call Option

A call option is exercisable if the underlying price exceeds the exercise price. That is $S_t > X$. As such, the lower bound of a call price is the underlying price minus the present value of the exercise price or zero, whichever is greater.

$$\text{Lower bound} = \max(0, S_t - X(1 + r)^{-(T-t)})$$

A call buyer will not pay more than the underlying price for the right to buy the underlying. As such, the upper bound is the current underlying price.

$$\text{Upper bound} = S_t$$

Generally, the no-arbitrage bounds of a call option are stated as follows:

$$\max(0, S_t - X(1+r)^{-(T-t)} < c_t \leq S_t)$$

Put Options

A call option buyer exercises a put option only if $S_T < X$. As such, the upper bound on the put value is thus the exercise price.

$$\text{Upper bound} = X$$

The lower bound is the present value of the exercise price minus the spot price or zero, whichever is greater:

$$\text{Lower bound} = \max(0, X(1+r)^{-(T-t)} - S_t)$$

Generally, the no-arbitrage bounds of a put option are stated as follows:

$$\max(0, X(1+r)^{-(T-t)} - S_t) < p_t \leq X$$

Example: No-arbitrage Bounds of a Call Option

Consider a 3-year call option with an exercise price of USD 100 and a risk-free rate of 1.5%. If, after six months, the spot price of the underlying is USD 105, the no-arbitrage upper and lower bounds are *closest to*:

Solution

For a call option,

$$\begin{aligned}
\text{Lower bound} &= \max(0, S_t - X(1+r)^{-(T-t)}) \\
&= \max(0, 105 - 100(1.015)^{-2.5}) \\
&= \text{USD } 8.65 \\
\text{Upper bound} &= S_t = \text{USD } 105
\end{aligned}$$

Replication in Contingent Claims

Note that replication refers to a strategy in which a derivative's cash flow stream may be recreated using a combination of long or short positions in an underlying asset and borrowing or lending cash.

Replication mirrors or offsets a derivative position, given that the law of one price holds and arbitrage does not exist. A trader can take opposing positions in a derivative and the underlying, creating a default risk-free hedge portfolio and replicating the payoff to a risk-free asset.

Replicating Call Options

Replication of a call option at the contract initiation involves borrowing at a risk-free rate, r, and then utilizing the proceeds to buy the underlying asset at a price of S_0.

At the expiration date $t = T$, there exist two replication outcomes:

- If $S_T > X$, exercise the option: Sell the underlying at S_T and use the proceeds to repay the risk-free loan.
- If $S_T < X$, no exercise: No settlement is needed.

If the exercise of the option is certain, we will borrow $X(1+r)^{-T}$ just like in forwards. However, the exercisability of the option is not certain. As such, a proportion of $X(1+r)^{-T}$ is borrowed depending on the likelihood of exercise at time T and linked to the moneyness of an option.

The **non-linear nature of option payoff** requires replicating transactions to be adjusted based on the likelihood of exercise.

Replicating Put Options

Replication of a put option at the contract initiation involves selling the underlying short at a price of S_0 and lending the proceeds at the risk-free rate, r.

At the expiration date ($t = T$), there exist two replication outcomes:

- If $S_T < X$, exercise the option: Buy the underlying at S_T from the proceeds of the risk-free loan.

- If $S_T > X$, no exercise: No settlement is needed.

As with call options, a proportion of $X(1 + r)^{-T}$ is borrowed depending on the likelihood of exercise at time T and linked to the moneyness of an option.

Question

A 6-month put option on an underlying stock with no associated costs or benefits has an exercise price of \$50. The underlying price at the contract inception is \$47, and the risk-free rate is 1.5%. After three months, the underlying stock price is \$45.75.

The lower bound of the put option price is *closest* to:

A. \$4.06.

B. \$50.

C. \$45.75.

Solution

The correct answer is **A**.

The lower bound of a put option is given by:

$$\begin{aligned}\text{Lower bound} &= \max(0, X(1+r)^{-(T-t)} - S_t)\\ &= \max(0, 50(1.015)^{-(0.5-0.25)} - 45.75)\\ &= \max(0, 4.064)\\ &= \$4.064\end{aligned}$$

LOS 8c: Identify the factors that determine the value of an option and describe how each factor affects the value of an option

The factors that affect the value of an option include the value of the underlying, exercise price, time to maturity, risk-free rate, volatility, and income or cost associated with the underlying.

Value of the Underlying

The value of the underlying has a direct impact on the right to exercise an option. For a call option, it is exercisable if $S_T > X$. As such, the value of the call option (and long forward) appreciates when the **spot price** of the underlying increases.

In contrast, the put option (and short forward) appreciates when the spot price of the underlying declines. Recall that the put option is in the money if $S_T < X$.

Exercise Price

The exercise price determines whether an option buyer will exercise the option at the expiration. Remember that the payoff of a call option at maturity is $\max(0, S_T - X)$. Intuitively, a lower exercise price will increase both the likelihood of exercise and settlement value if it is in the money.

For the put option, the exercise price is the upper bound of the option price. Moreover, the payoff of a put option is $\max(0, X - S_T)$. As such, a high exercise price increases the value of the put option.

Time to Expiration

The time value of an option represents the likelihood that favorable changes to the underlying price will increase the profitability of the exercise. For both call and put options, a longer time to maturity increases the likelihood of the option finishing in the money, thus increasing the option's value

Risk-Free Rate

A risk-free rate can be seen as the opportunity cost of holding an asset. A risk-free rate is used in the no-arbitrage valuation of derivatives. Note that the value of a call option at any time before maturity $(t < T)$ is given by:

$$c_t = \max(0, S_t - X(1+r)^{-(T-t)})$$

It is easy to see that a higher risk-free rate increases the value of the call option. This is because a higher risk-free rate lowers the present value of the exercise price, provided the option is in the money. For a put option, its value at any time before maturity $(t < T)$ is given by:

$$p_t = \max(0, X(1+r)^{-(T-t)} - S_t)$$

Intuitively, a higher risk-free rate decreases the exercise value of a put option due to the same explanation in the call option.

The Volatility of the Underlying

Volatility measures the expected dispersion of future movements of an underlying asset. Higher volatility of the underlying asset increases the chances of call and put options finishing in the money without affecting the downside case – the option expires worthless. For instance, as volatility increases, a broader possibility of underlying prices increases the time value of an option and the likelihood of being in the money.

In contrast, lower volatility decreases the time value of both put and call options.

Income or Cost Associated with Owning Underlying Asset

Income (or other non-cash benefits) accrue to the underlying asset owner, not the derivative owner. In other words, the present value of the income or benefits is subtracted from the underlying price. As such, income decreases the value of a call option and increases the value of a put option.

If the asset owner incurs costs (in addition to opportunity cost), compensation is done to cover the added costs. As such, the present value of the costs is added to the underlying price. Therefore, cost increases the value of the call option and decreases the value of the put option.

The table below summarizes the factors that affect the value of an option.

Factor	Value of European Call option	Value of European Put option
Value of the Underlying	Directly proportional	Inversely proportional
Exercise price	Inversely proportional (as the exercise price increases, value decreases)	Directly proportional (as exercise price increases, value increases)
Time to Maturity	Directly proportional	Directly proportional
Risk-free rate	Directly proportional	Inversely proportional
Volatility	Directly proportional	Directly proportional
Benefits	Inversely proportional	Directly proportional
Costs	Directly proportional	Inversely proportional

Question

Which of the following is *most likely to have* the same effect on the value of a call option?

A. High risk-free rate and negative cost of carry.

B. Low exercise price and positive cost of carry.

C. Longer time to maturity and low volatility.

Solution

The correct answer is **A**.

Both a high risk-free rate and low cost of carry increase the value of a call option.

The risk-free rate increases the value of the call option because a higher risk-free rate lowers the present value of the exercise price, provided the option is in the money.

Recall that cost of carry is the net of the costs and benefits associated with owning an underlying asset for a period. Therefore, the negative cost of carry implies that the cost associated with the underlying is higher than the benefits. The present value of the costs is added to the underlying price. Therefore, cost increases the value of the call option and decreases the value of the put option.

B is incorrect. A low exercise price will increase both the likelihood of exercise and settlement value if it is in the money.

A positive cost of carry implies that the present value of the benefits associated with the underlying is higher than the present value of the costs. The present value of the income or benefits is subtracted from the underlying price. As such, income or other non-cash decreases the value of a call option and increases the value of a put option.

C is incorrect. The time value of an option represents the likelihood that favorable changes to the underlying price will increase the profitability of the exercise.

Therefore, a longer time to maturity for a call option increases the option’s value.

Lower volatility decreases the time value of both put and call options.