Learning Module 1: Rate and Return

LOS 1a: interpret interest rates as required rates of return, discount rates, or opportunity costs and explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk

The time value of money is a concept that states that cash received today is more valuable than cash received in the future. If a person agrees to receive payment in the future, he foregoes the option of earning interest if he invests that amount of money today.

An interest rate or yield, usually denoted by *r;* is a rate of return that reflects the connection between cash flows dated at different times.

Assume you currently possess $100. Next, consider depositing this money into a savings account, expecting it to grow to $110 after one year. Intuitively, the compensation required for deferring the consumption of $100 now in favor of receiving $110 in one year is $10 (equal to 110 minus 100). This compensation is equivalent to a 10% rate of return (calculated as 10 divided by 100).

There are three ways to interpret interest rates:

1. **Required rate of return**: The minimum return an investor expects to earn to accept an investment.
2. **Discount rate**: The rate used to discount future cash flows to allow for the time value of money (to determine the present value **equivalent** of some money to be received sometime in the future). Discount rates and interest rates are used almost interchangeably.
3. **Opportunity cost**: The value of the **best-forgone alternative**; the most valuable alternative investors give up when they choose what to do with money.

Determinants of Interest Rates

Economics postulates that the forces of supply and demand determine interest rates. In this case, the investors (lenders) supply the money, and the borrowers demand money for their consumption.

As such, interest is a reward a borrower pays for using an asset, usually capital, belonging to a lender. It is compensation for the loss or value depreciation occasioned by the use of the asset.

Therefore, an interest rate is composed of a real risk-free interest rate plus a set of four premiums that represent compensation for bearing distinct types of risk:

$$\text{Interest (r)} = \begin{array}{l}\text{Real risk-free interest rate} \\ + \text{ Inflation Premium} \\ + \text{ Default risk premium} \\ + \text{ Liquidity premium} \\ + \text{ Maturity premium}\end{array}$$

The Real Risk-free Interest Rate

The real risk-free interest rate is the single-period interest rate for a completely risk-free security if no inflation is expected. According to economic theory, the real risk-free rate reflects people's preferences for current compared to real future consumption.

Types of Risk Premiums

Inflation Risk Premium

Inflation risk is the loss of purchasing power of money as a result of the increase in prices of consumer goods.

The inflation premium compensates investors for expected inflation. It represents the average inflation rate expected over the maturity of the debt. The risk of a decrease in purchasing power validates the inflation risk premium.

Liquidity Risk Premium

Liquidity refers to the ease with which an investment can be converted into cash without significantly sacrificing market value.

The liquidity premium compensates investors for the risk of loss relative to an investment's fair value if the investment needs to be converted to cash quickly.

Default Risk Premium

Default risk describes a situation where a borrower may fail to repay borrowed funds as a result of bankruptcy. This might result in significant losses on the side of the lender.

The **default risk premium** compensates investors for the possibility that the borrower **will fail to make a promised payment** at the contracted time and in the contracted amount.

Maturity Risk Premium

The maturity risk premium is the additional return an investor requires for assuming interest rate and reinvestment risk resulting from a longer investment maturity timeline. Maturity risk premium increases with an increase in the maturity timeline. In other words, the longer the maturity timeline of an investment, the higher the maturity risk premium.

Nominal Risk-free Interest Rate

The nominal risk-free interest rate is defined as the sum of the real risk-free interest rate and the inflation premium. In other words, the nominal risk-free interest rate can be seen as the combination of the real risk-free rate plus an inflation premium, as shown by the following equation:

$$(1+\text{Nominal risk-free rate})=(1+\text{Real risk-free rate })(1+\text{Inflation premium})$$

The above equation is generally approximated as follows:

$$\text{Nominal risk-free rate}=\text{Real risk-free rate}+\text{Inflation premium}$$

Most rates quoted on short-term government debts can be taken as nominal risk-free interest rates over the respective maturity.

Question

Which of the following is *most likely* an interpretation of interest rate as a benefit foregone when investors spend money on current consumption instead of saving or investing?

A. Discount rate.

B. Opportunity cost.

C. Required rate of return.

Solution

The correct answer is **B**.

Opportunity cost is a key factor in interpreting interest rates. It refers to the interest foregone when investors opt for an alternate option, such as spending on current consumption instead of saving or investing.

A is incorrect. The discount rate is the interest rate used to discount future cash flows to reach the present value.

C is incorrect. The required rate of return is the minimum rate of return an investor would wish to earn to postpone current consumption.

LOS 1b: calculate and interpret different approaches to return measurement over time and describe their appropriate uses

Financial assets are primarily defined based on their return-risk characteristics. This helps when building a portfolio from all the assets available. Regarding returns, there are different ways of measuring returns.

Financial market assets generate two different streams of return: income through cash dividends or interest payments and capital gain or loss through financial asset price increases or decreases.

Some financial assets give only one stream of return. For instance, headline stock market indices typically report on price appreciation only. They do not include the dividend income unless the index specifies it is a "total return" series.

Holding Period Return

A holding period return is earned from holding an asset for a single specified period. The time period can be any specified period, such as a day, month, or ten years.

The general formula of the holding period return is given by:

$$R = \frac{(P_1 - P_0) + I_1}{P_0}$$

P_0 = Price of an asset at the beginning of the period (t=0).

P_1 = Price of an asset at the end of the period (t=1).

I_1 = Income received at the end of the period (t=1).

Example: Calculating Holding Period Return

An investor purchased 100 shares of a stock at $50 per share and held the investment for one year. During that period, the stock paid dividends of $2 per share. At the end of the year, the investor sold all the shares for $60 per share.

The holding period return is *closest to*:

Solution

In this case, we have:

$$\begin{aligned} P_0 &= 100 \text{ shares} \times \$50 \text{ per share} = \$5,000 \\ I_1 &= 100 \text{ shares} \times \$2 \text{ per share} = \$200 \\ P_1 &= 100 \text{ shares} \times \$60 \text{ per share} = \$6,000 \end{aligned}$$

Therefore,

$$R = \frac{(P_1 - P_0) + I_1}{P_0} = \frac{6,000 - 5,000 + 200}{5,000} = 24\%$$

Holding period returns can also be calculated for periods longer than a year. For instance, if we need to calculate the holding period return for a five-year period, we should compound the five annual returns as follows:

$$R = \frac{(P_5 - P_0) + I_{(1-5)}}{P_0}$$

Arithmetic Return

When we have assets for multiple holding periods, it is necessary to aggregate the returns into one overall return.

Denoted by $\bar{R}_i$ arithmetic mean for an asset i is a simple process of finding the average holding period returns. It is given by:

$$\bar{R}_i = \frac{R_{i,1} + R_{i,2} + \ldots + R_{i,T-1} + R_{iT}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

Where:

R_{it} = Return of asset i in period t.

T = Total number of periods.

For example, if a share has returned 15%, 10%, 12%, and 3% over the last four years, then the arithmetic mean is computed as follows:

$$\bar{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it} = \frac{1}{4}(15\% + 10\% + 12\% + 3\%) = 10\%$$

Geometric Return

Computing a geometric mean follows a principle similar to the one used to compute compound interest. It involves compounding returns from the previous year to the initial investment's value at the start of the new period, allowing you to earn returns on your returns.

A geometric return provides a more accurate representation of the portfolio value growth than an arithmetic return.

Denoted by $\bar{R}_{Gi}$ the geometric return for asset i is given by:

$$\begin{aligned}\bar{R}_{Gi} &= \sqrt[T]{(1 + R_{i,1}) \times (1 + R_{i2}) \times \ldots \times (1 + R_{i,T-1}) \times (1 + R_{iT})} - 1 \\ &= \sqrt[T]{\prod_{t=1}^{T} (1 + R_t)} - 1\end{aligned}$$

Using the same annual returns of 15%, 10%, 12%, and 3% as shown above, we compute the geometric mean as follows:

$$\begin{aligned}\text{Geometric mean} &= [(1 + 15\%) \times (1 + 10\%) \times (1 + 12\%) \times (1 + 3\%)]^{\frac{1}{4}} - 1 \\ &= 9.9\%\end{aligned}$$

Note that the geometric return is slightly less than the arithmetic return. Arithmetic returns tend to be biased upwards unless the holding period returns are all equal.

Harmonic Mean

The harmonic mean is a measure of central tendency. It's especially useful for rates or ratios such as P/E ratios. Its formula is derived from the harmonic series, which is a specific mathematical sequence.

$$\bar{X}_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}},\ X_i > 0 \text{ for all } i = 1, 2, \ldots, n$$

The above formula is interpreted as the "harmonic mean of observations $X_1,\ X_2, \ldots,\ X_n$."

The harmonic mean is handy for averaging ratios when those ratios are consistently applied to a fixed quantity, resulting in varying unit numbers. For instance, it's applied in cost-averaging strategies where you invest a fixed amount of money at regular intervals.

Example: Calculating the Harmonic Mean

An investor is practicing cost averaging by investing in a particular stock over a period of three months. The investor decides to allocate different amounts of money each month. In the first month, the investor invests $2,000; in the second month, $3,000; and in the third month, $4,000. The share prices of the stock for these three months are $10, $12, and $15, respectively.

Calculate the average price paid per share for the three-month period.

Solution

Using the harmonic mean formula,

$$\bar{X}_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}} = \frac{3}{\frac{1}{10} + \frac{1}{12} + \frac{1}{15}} = 12$$

Trimmed and Winsorized Means

Trimmed and Winsorized means seek to lower the effect of outliers in a data set.

Trimmed Mean

The trimmed mean is a measure of central tendency in which we calculate the mean after excluding a small percentage of the lowest and highest values from the dataset.

For example, a data set consists of 10 observations: 12, 15, 18, 20, 22, 25, 27, 30, 35, and 40. We can calculate the trimmed mean after removing the highest and lowest values.

After removing these values, the remaining data set is: 15, 18, 20, 22, 25, 27, 30, and 35.

Now, let's calculate the trimmed mean by taking the average of these remaining values:

$$\frac{15+18+20+22+25+27+30+35}{8} = \frac{192}{8} = 24$$

Therefore, the trimmed mean of the given data set is 24.

Winsorized Mean

The Winsorized mean is a central tendency measure. It works by replacing extreme values at both ends of the data with the values of their closest observations. This process is similar to the trimmed mean. Essentially, it helps eliminate outliers in a dataset.

For example, consider a dataset of 12 observations: 8, 12, 15, 18, 20, 22, 25, 27, 30, 35, 40, and 50. We can calculate the Winsorized mean by replacing the lowest and highest values with those closest to the 10th and 90th percentiles, respectively. As such, the new values are **10**, 12, 15, 18, 20, 22, 25, 27, 30, 35, **37.5**, and 40, and the winsorized mean is:

$$\frac{10+12+15+18+20+22+25+27+30+35+37.5+40}{12} \approx 24.46$$

Question 2

What are the arithmetic mean and geometric mean, respectively, of an investment that returns 8%, -2%, and 6% each year for three years?

A. Arithmetic mean = 5.3%; Geometric mean = 5.2%.

B. Arithmetic mean = 4.0%; Geometric mean = 3.6%.

C. Arithmentic mean = 4.0%; Geometric mean = 3.9%.

Solution

The correct answer is **C**.

$$\text{Arithmetic mean} = \frac{8\% + (-2\%) + 6\%}{3} = 4\%$$

$$\text{Geometric mean} = [(1+8\%) \times (1+(-2\%)) \times (1+6\%)]^{1/3} - 1 = 3.9\%$$

LOS 1c: Compare the money-weighted and time-weighted rates of return and evaluate the performance of portfolios based on these measures

Money-weighted Rate of Return

The money-weighted return considers the money invested and gives the investor information on the actual investment return. Calculating money-weighted return is similar to calculating an investment's internal rate of return (IRR).

The money-weighted rate of return (MWRR) is like the portfolio's internal rate of return (IRR). It's the rate at which the present value of cash flows equals zero. In simple terms, it's a way to measure how well a portfolio is performing.

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

Where:

T = Number of periods.

CF_t = Cash flow at time *t*.

IRR = Internal rate of return (or money-weighted rate of return).

The money-weighted rate of return (MWRR) looks at a fund's starting and ending values and all the cash flows in between. In an investment portfolio, cash inflows are a part of it. These inflows could be from deposits or investments made during a certain period. The MWRR considers these inflows and calculates the overall rate of return for the portfolio:

- The beginning value.
- Dividends/interest reinvested.
- Contributions made.

Cash outflows, on the other hand, refer to:

- Withdrawals made.
- Dividends or interest received.
- The final value of the fund.

Example 1: Calculating Money-weighted Rate of Return

An investor makes the following investments in a portfolio over a two-year period:

- At the beginning of year one, the investor invests $10,000.
- At the end of the first year, after the portfolio's value increases to $12,000, the investor adds $5,000, making the total portfolio value $17,000.
- At the end of the second year, the portfolio value further increases to $25,000.

The money-weighted rate of return for the investor’s portfolio is *closest* to:

Solution

We need to calculate the internal rate of return (IRR) considering the following cash flows:

- $\text{CF}_0 = -\$10,000$ (Initial investment)
- $\text{CF}_1 = -\$5,000$ (Additional investment at the end of year one)
- $\text{CF}_2 = +\$25,000$ (Final portfolio value at the end of year two)

To find the money-weighted rate of return, solve the equation for IRR:

$$\frac{\text{CF}_0}{(1+\text{IRR})^0} + \frac{\text{CF}_1}{(1+\text{IRR})^1} + \frac{\text{CF}_2}{(1+\text{IRR})^2} = \frac{-10,000}{1} + \frac{-5,000}{(1+\text{IRR})} + \frac{25,000}{(1+\text{IRR})^2} = 0$$

Using BA II Plus Calculator, $\text{IRR} \approx 35.08\%$.

Example: Calculating Money-weighted Return for a Dividend-paying Stock

Calvin Hair purchased a share of Superior Car Rental Company for \$85 at the beginning of the first year. He bought an additional unit for \$87 at the end of the first year. At the end of the second year, he sold both shares at \$90. During both years, Hair received a dividend of \$4 per share, which was not reinvested.

Calculate the money-weighted return.

Solution

To calculate the money-weighted return in this example, we need to consider the timing and amounts of cash flows and their respective investment periods.

Step 1: Calculate the total investment at the beginning (t=0):

$$\text{Initial investment} = -\$85$$

Step 2: Calculate the total investment at $t = 1$:

$$\begin{aligned}\text{Initial investment + Additional investment} &= \$87 - \$4(\text{ Dividend received at}\\ &\quad \text{the end of the first year, which is not reinvested})\\ &= -\$83\end{aligned}$$

Step 3: Calculate the final portfolio value at $t = 2$:

$$\begin{aligned}\text{Number of shares sold} \times \text{Selling price} &= 2 \text{ shares} \times \$90 = \$180 + 8(\text{ Dividend received for}\\ &\quad\text{both shares})\\ &= \$188\end{aligned}$$

As such, we have:

$CF_0 = -85$.

$CF_1 = -83$.

$CF_2 = 188$.

Using the BA II Plus calculator, you will get $\text{IRR} = 7.71\%$, which is equivalent to the money-weighted rate of return.

Shortcomings of the Money-weighted Rate of Return

The money-weighted rate of return (MWRR) considers all cash flows, such as withdrawals or contributions. If an investment spans multiple periods, MWRR gives more importance to the fund's performance when the account is at its largest. This can be a problem for fund managers because it might make their performance seem worse due to factors they can't control.

Time-Weighted Rate of Return

The time-weighted rate of return (TWRR) calculates the compound growth of an investment. Unlike the money-weighted rate, it doesn't care about withdrawals or contributions. TWRR is like finding the average return of different time periods within your investment.

Steps of Calculating Time-weighted Rate of Return

Step 1: Value the portfolio immediately before any significant cash inflow or outflow of funds. Divide the evaluation period into subperiods based on dates of significant additions or withdrawals of funds.

Step 2: Compute the holding period return on the portfolio for each period.

Step 3: Compound or link the holding period returns to the annual rate of return, which is the time-weighted rate of return.

$$\text{TWRR} = (1+\text{HPR}_1 \times (1+\text{HPR}_2) \times (1+\text{HPR}_3) \ldots \times (1+\text{HPR}_{n-1}) \times (1+\text{HPR}_n) - 1$$

If the evaluation period is more than one year, compute the geometric mean of the annual returns to get the time-weighted return for the investment period.

$$\begin{aligned}\bar{R}_{Gi} &= \sqrt[n]{(1+\text{HPR}_1) \times (1+\text{HPR}_1) \ldots \times (1+\text{HPR}_n)} - 1 \\ &= [(1+\text{HPR}_1) \times (1+\text{HPR}_1) \ldots \times (1+\text{HPR}_n)]^{\frac{1}{n}} - 1\end{aligned}$$

Example: Calculating the Time-Weighted Rate of Return (Period More than one year)

An investor purchases a share of stock at t = 0 for $200. At the end of the year (at t = 1), the investor purchases an additional share of the same stock, this time for $220. She then sells both shares at the end of the second year for $230 each. She also received annual dividends of $3 per share at the end of each year. Calculate the annual time-weighted rate of return on her investment.

Solution

First, we break down the two years into two one-year periods.

Holding period 1:

Beginning value = 200.

Dividends paid = 3.

Ending value = 220.

Holding period 2:

Beginning value = 440 (2 shares × 220)

Dividends paid = 6 (2 shares × 3)

Ending value = 460 (2 shares × 230)

Secondly, we calculate the HPR for each period:

$$HPR_1 = \frac{(220 - 200 + 3)}{200} = 11.5\%$$

$$HPR_2 = \frac{(460 - 440 + 6)}{440} = 5.9\%$$

Lastly, we need to find the geometric mean of the HPRs since we are dealing with a period of more than a year.

$$\begin{aligned} TWRR &= [(1 + HPR_1) \times (1 + HPR_1) \ldots \times (1 + HPR_n)]^{\frac{1}{n}} - 1 \\ &= (1.115 \times 1.059)^{0.5} - 1 = 8.7\% \end{aligned}$$

Example: Calculating the Time-weighted Rate of Return (Period Less

than One Year)

The beginning value of a portfolio as of January 1, 2020, was $1,000,000. On February 10, the portfolio's value was $1,100,000, including an additional contribution of the $50,000 injected into the portfolio on this date. The portfolio's ending value at the beginning of April was $1,350,000.

The time-weighted rate of return is *closest to:*

Solution

The time-weighted return is calculated as follows:

$$\begin{aligned}
HPR_1 &= \frac{V_1 - V_0}{V_0} = \frac{(1,100,000 - 50,000) - 1,000,000}{1,000,000} = 5\% \\
HPR_2 &= \frac{V_2 - V_1}{V_1} = \frac{1,350,000 - 1,100,000}{1,100,000} = 22.73\% \\
\Rightarrow TWRR &= (1 + HPR_1) \times (1 + HPR_1) - 1 \\
&= 1.05 \times 1.2273 - 1 = 28.87\%
\end{aligned}$$

Question

A chartered analyst buys a share of stock at time t = 0 for $50. At t = 1, he purchases an extra share of the same stock for $53. The share gives a dividend of $0.50 per share for the first year and $0.60 per share for the second year. He sells the shares at the end of the second year for $55 per share. Calculate the annual time-weighted rate of return.

A. 5.90%.

B.12.24%.

C. 7.00%.

The correct answer is **A.**

We have two one-year holding periods:

$$\begin{array}{cc} HP_1 & HP_2 \\ P_0 = 50 & P_0 = 106 \\ D = 0.5 & D = 1.2 \\ P_1 = 53 & P_1 = 110 \end{array}$$

We now calculate the holding period returns:

$$\begin{gathered} HPR_1 = \frac{(53 - 50 + 0.5)}{50} = 7\% \\ HPR_2 = \frac{(110 - 106 + 1.2)}{106} = 4.9\% \\ \Rightarrow TWRR = 1.07 \times 1.049 - 1 = 12.24\% \end{gathered}$$

Therefore,

$$\text{Annual TWRR} = (1 + 0.1224)^{0.5} - 1 = 5.9\%$$

LOS 1d: Calculate and interpret annualized return measures and continuously compounded returns and describe their appropriate uses

To compare returns over different timeframes, we need to annualize them. This means converting daily, weekly, monthly, or quarterly returns into annual figures.

Non-Annual Compounding

Interest may be paid semiannually, quarterly, monthly, or even daily – interest payments can be made more than once a year. Consequently, the present value formula can be expressed as follows when there are multiple compounding periods in a year:

$$PV = FV_N \left(1 + \frac{R_S}{m}\right)^{-mN}$$

Where:

m = Number of compounding periods in a year.

R_s = Quoted annual interest rate.

N = Number of years.

Example: Calculating the Present Value of a Lump Sum (More than One Compounding Period)

Jane Doe wants to invest money today and have it become $500,000 in five years. The annual interest rate is 8%, and it's compounded quarterly. How much should Jane invest right now?

Using the formula above:

$FV_N = \$500,000.$

$R_S = 8\%.$

$m = 4.$

$R_S/m = \frac{8\%}{4} = 2\% = 0.02.$

$N = 5.$

$mN = 4 \times 5 = 20.$

Therefore,

$$PV = FV_N\left(1 + \frac{R_S}{m}\right)^{-mN} = \$500,000 \times (1.02)^{-20} = \$336,485.67$$

Using BA II *Plus* Calculator:

- Press the [2nd] button, then the [FV] button to clear the financial registers. The display should show "CLR TVM."
- Enter the future value (FV). This is the amount Jane wants to have in five years, which is $500,000. To do this, type "500000" and press the [FV] button.
- Enter the interest rate (I/Y). This is the annual interest rate, which is 8%. However, since interest is compounded quarterly, we need to divide this by 4. To do this, type "8", press the [÷] button, type "4", then press the [ENTER] button, and finally press the [I/Y] button.
- Enter the number of periods (N). This is the number of quarters in five years, which is 5*4 = 20. To do this, type "20" and press the [N] button.
- Compute the present value (PV). To do this, press the [CPT] and then the [PV] buttons. The display should show the amount Jane needs to invest today, approximately $336,485.49.

Annualized Returns

To annualize a return for a period shorter than a year, you need to account for how many times that period fits into a year. For example, if you have a weekly return, you would compound it 52 times because there are 52 weeks in a year.

Generally, we can annualize the returns using the following formula:

$$\text{Return}_{\text{annual}} = (1 + \text{Return}_{\text{period}})^{c} - 1$$

Where:

$\text{Return}_{\text{period}}$ = Quoted return for the period.

c = Number of periods in a year.

Example: Annualizing Returns

If the monthly return is 0.7%, then the compound annual return is:

$$\begin{aligned}\text{Return}_{\text{annual}} &= (1 + \text{Return}_{\text{monthly}})^{12} - 1\\ &= (1.007)^{12} - 1 = 0.0873 = 8.73\%\end{aligned}$$

For a period of more than one year, for example, a 15-month return of 16% can be annualized as:

$$\begin{aligned}\text{Return}_{\text{annual}} &= (1 + \text{Return}_{15\text{ month}})^{\frac{12}{15}} - 1\\ &= (1.16)^{\frac{4}{5}} - 1 = 12.61\%\end{aligned}$$

We may apply the same procedure to convert weekly returns to annual returns for comparison with weekly returns.

$$\text{Return}_{\text{annual}} = (1 + \text{Return}_{\text{weekly}})^{52} - 1$$

For comparison with weekly returns, we can convert annual returns to weekly returns by making $(\text{Return}_{\text{weekly}})^{52}$ the subject of the formula.

Example: Comparing Investments by Annualizing Returns

An investor is evaluating the returns of two recently formed bonds. Selected return information on the bonds is presented below:

Bond	Time Since Issuance	Return Since Issuance (%)
A	120 days	2.50
B	8 months	6.00

Annualized Return Calculation

To compare the annualized rate of return for both bonds, you can use the formula for annualizing returns based on different time periods:

Annualized Return = $(1 + \frac{\text{Return Since Issuance}}{100})^{\frac{365}{\text{Time Since Issuance}}} - 1$

Let's calculate the annualized returns for both bonds:

For Bond A:

Time Since Issuance = 120 days

Return Since Issuance = 2.50%

Annualized Return for Bond A = $(1 + \frac{2.50}{100})^{\frac{365}{120}} - 1$

Annualized Return for Bond A = $(1 + 0.025)^{3.0417} - 1$

Annualized Return for Bond A = 1.079847 - 1 = 0.079847 or 7.98%

For Bond B:

Time Since Issuance = 8 months = 240 days

Return Since Issuance = 6.00%

Annualized Return for Bond B = $(1 + \frac{6.00}{100})^{\frac{365}{240}} - 1$

Annualized Return for Bond B = $(1 + 0.06)^{1.5208} - 1$

Annualized Return for Bond B = 1.092751 - 1 = 0.092751 or 9.28%

Comparing the annualized returns:

Bond A has an annualized return of approximately 7.98%.

Bond B has an annualized return of approximately 9.28%.

Therefore, Bond B has a higher annualized rate of return compared to Bond A.

Continuously Compounded Returns

The continuously compounded return is calculated by taking the natural logarithm of one plus the holding period return. For example, if the monthly return is 1.2%, you'd calculate it as ln(1.012), which equals approximately 0.01192.

Generally, continuously compounded from t to $t+1$ is given by:

$$r_{t,t+1} = \ln\left(\frac{P_{t+1}}{P_t}\right) = \ln(1 + R_{t,t+1})$$

Assume now that the investment horizon is from time $t = 0$ to time $t = T$ then the continuously compounded return is given by:

$$r_{0,T} = \ln\left(\frac{P_T}{P_0}\right)$$

If we apply the exponential function on both sides of the equation, we have the following:

$$P_T = P_0 e^{r_{0,T}}$$

Note that $\frac{P_T}{P_0}$ can be written as:

$$\frac{P_T}{P_0} = \left(\frac{P_T}{P_{T-1}}\right)\left(\frac{P_{T-1}}{P_{T-2}}\right)\ldots\left(\frac{P_1}{P_0}\right)$$

If we take natural logarithm on both sides of the above equation:

$$\ln\left(\frac{P_T}{P_0}\right) = \ln\left(\frac{P_T}{P_{T-1}}\right) + \ln\left(\frac{P_{T-1}}{P_{T-2}}\right) + \ldots + \ln\left(\frac{P_1}{P_0}\right)$$
$$\Rightarrow r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \ldots + r_{0,1}$$

Therefore, the continuously compounded return to time T is equivalent to the sum of one-period continuously compounded returns.

Question

The weekly return of an investment that produces an annual compounded return of 23% is *closest to*:

A. 0.40%.

B. 0.92%.

C. 0.41%.

The correct answer is A.

Recall that:

$$\text{Return}_{\text{annual}} = (1 + \text{Return}_{\text{weekly}})^{52} - 1$$

We can rewrite the above equation as follows:

$$\begin{aligned}\text{Return}_{\text{weekly}} &= (1 + \text{Return}_{\text{annual}})^{\frac{1}{52}} - 1\\ &= (1 + 0.23)^{\frac{1}{52}} - 1\\ &\approx 0.40\%\end{aligned}$$

LOS 1e: calculate and interpret major return measures and describe their appropriate uses

Other Return Measures

Gross and Net Return

The gross return is what an asset manager earns before subtracting various costs such as management fees, custody fees, taxes, and other administrative expenses. However, it does account for trading costs such as commissions.

Gross return does not consider management or administrative costs. For this reason, it is a suitable metric for assessing and comparing the investment expertise of asset managers.

Net return is a metric for how much an investment has earned for the investor. It considers all administrative and management costs that reduce an investor's return.

Pre-tax and After-tax Nominal Return

Unless otherwise stated, all returns are nominal pre-tax returns in general. Depending on the jurisdiction, different rates apply to capital gains and income. Long-term and short-term taxes may also be applied to capital gains.

The after-tax nominal return is determined by subtracting any tax deductions applied to dividends, interest, and realized gains from the total return.

Real Returns

Returns are typically presented in nominal terms, which consist of three components: the real risk-free return as compensation for postponing consumption, inflation as compensation for the loss of purchasing power, and a risk premium. Real returns are useful in comparing returns over different periods, given that inflation rates vary over time.

Recall the relationship between the nominal rate and the real rate:

$$(1+\text{ Nominal Risk-free rate})=(1+\text{Real risk free rate})(1+\text{Inflation premium})$$

We can find the connection between nominal and real returns by considering the real risk-free rate of return and the inflation premium. This relationship can be expressed as:

$$(1+\text{Real Return})=\frac{(1+\text{Real risk-free rate})(1+\text{Risk premium})}{1+\text{Inflation premium}}$$

Real returns become particularly useful when you want to compare returns across various time periods and different countries. This is especially important when returns are shown in local currencies and when inflation rates vary from one country to another.

After-tax real return is the amount the investor receives as payment for delaying consumption and taking on risk after paying taxes on investment.

Leveraged Returns

If an investor uses derivative instruments within a portfolio or borrows money to invest, then leverage is introduced into the portfolio. The leverage amplifies the returns on the investor's capital, both upwards and downwards.

The leveraged return considers the actual return on the investment and the cost of the borrowed money. The cost of borrowing and financing fees are subtracted from the overall return produced by the investment to determine the leveraged return.

Using the borrowed capital (debt) increases the size of the leveraged position by the additional borrowed capital.

Intuitively, the leveraged return is given by:

$$\begin{aligned}R_L &= \frac{\text{Portfolio return}}{\text{Portfolio equity}}\\ &= \frac{[R_P\times(V_E+V_B)-(V_B\times r_D)]}{V_E}\\ &= R_P+\frac{V_B}{V_E}(R_P-r_D)\end{aligned}$$

Where:

R_L = Return earned on the leveraged portfolio.

R_P = Total investment return earned on the leveraged portfolio.

V_B = Value of debt in the portfolio.

V_E = Value of equity in the portfolio.

r_D = Borrowing cost on debt.

Example: Calculating Leveraged Return

For a $250,000 equity portfolio with an annual 9% total investment return, 40% financed by debt at 6%, the leveraged return would be:

$$R_L = R_P + \frac{V_B}{V_E}(R_P - r_D) = 9\% + \frac{\$100,000}{\$150,000}(9\% - 6\%) = 11\%$$

Question

A \$7,500,000 equity portfolio is 35% financed by debt at a cost of 5% per annum. If the equity portfolio generates a 9% annual total investment return, the leverage return is *closest* to:

A. 11.15%.

B. 14.00%.

C. 8. 25%.

The correct answer is **A**.

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\begin{aligned}
R_L &= R_P + \frac{V_B}{V_E}(R_P - r_D) \\
&= 9\% + \frac{\$2,625,000}{\$4,875,000}(9\% - 5\%) = 11.15\%
\end{aligned}
$$