

Learning Module 12: Yield Based Bond Convexity and Portfolio Properties

LOS 12a: calculate and interpret convexity and describe the convexity adjustment.

Duration provides a linear approximation of the change in a bond's price with respect to changes in yield. On the other hand, convexity measures the non-linear, second-order effect of yield changes on a bond's price. It captures the curvature of the price-yield relationship.

While duration estimates price changes linearly, the true bond price-yield relationship is convex. Convexity becomes particularly crucial when considering significant yield changes and for bonds with longer maturities.

Calculating Convexity

Convexity can be calculated using the formula:

$$\% \Delta P_{V_{Full}} \approx (-\text{AnnModDur} \times \Delta \text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{Yield})^2 \right]$$

The first term captures the effect from modified duration. The second term represents the convexity adjustment.

Convexity can also be approximated using the following formula:

$$\text{ApproxCon} = \frac{(PV_-) + (PV_+) - [2 \times (PV_0)]}{(\Delta \text{Yield})^2 \times (PV_0)}$$

Factors Affecting Convexity

- i. Maturity: Longer maturity increases convexity.
- ii. Coupon rate: Lower coupon rate increases convexity.
- iii. YTM: Lower YTM increases convexity.
- iv. Cash Flow Dispersion: For two bonds with the same duration, the one with more

dispersed cash flows will have greater convexity.

Benefits of Convexity

Bonds with greater convexity perform better in both rising and falling yield scenarios, making them less risky for investors. This assumes that the difference in convexity is not reflected in the bond's price. For large yield changes, a bond's price will rise more with a decrease in yield and fall less with an increase in yield if it has higher convexity.

Example: Calculating Convexity

Consider a bond that has a term to maturity of 3 years, an annual coupon rate of 2%, a yield-to-maturity (YTM) of 2%, and is priced at 100 per 100 par value.

- i. Calculate the modified duration and convexity for the bond at issuance.
- ii. Calculate ApproxModDur and ApproxCon for the bond using a 10 bp increase and decrease in the yield-to-maturity.

Calculating Modified Duration and Convexity

Period	Time to Receipt	Cashflow Amount	Present Value	Weights	Time to Receipt*Weight
1	1.0000	2	1.9608	0.01960	0.02
2	2.0000	2	1.9223	0.01922	0.04
3	3.0000	102	96.1169	0.96118	2.88
Total			100.0000	1.0000	2.94

Annualized Macaulay Duration = 2.94

Annualized convexity = 11.46

Convexity for each period has been calculated as:

$$\text{Convexity} = \text{Time to receipt of cashflows} \times (\text{Time to receipt of cashflows} + 1) \times \text{Weight} \times \left(1 + \frac{\text{YTM}}{m}\right)$$

Where m is the periodicity.

Calculating ApproxModDur and ApproxCon

$$\text{ApproxCon} = \frac{(\text{PV}_-) + (\text{PV}_+) - [2 \times (\text{PV}_0)]}{(\Delta \text{Yield})^2 \times (\text{PV}_0)}$$

$$\text{PV}_0 = \frac{2}{1.02} + \frac{2}{1.02^2} + \frac{102}{1.02^3} = 100$$

$$\text{PV}_- = \frac{2}{1.019} + \frac{2}{1.019^2} + \frac{102}{1.019^3} = 100.288951$$

$$\text{PV}_+ = \frac{2}{1.021} + \frac{2}{1.021^2} + \frac{102}{1.021^3} = 99.71217249$$

$$\text{ApproxCon} = \frac{100.288951 + 99.71217249 - [2 \times 100]}{(0.001)^2 \times (100)} = 11.2349$$

Question

Which of the following factors *most likely* increases the convexity of a bond?

- A. Higher coupon rate
- B. Shorter maturity
- C. Lower yield-to-maturity (YTM)

Solution

The correct answer is C:

Lower YTM increases convexity.

A is incorrect: Lower coupon rates, not higher, increase convexity.

B is incorrect: Longer maturities, not shorter, increase convexity.

LOS 12b: calculate the percentage price change of a bond for a specified change in yield, given the bond's duration and convexity.

The percentage price change of a bond, given a specified change in yield, can be more accurately estimated using both the bond's duration and convexity compared to using duration alone. We will give an example to illustrate this.

Consider a 2-year, 4% semiannual coupon bond, settling on 10 June 2024, maturing on 10 June 2026, and yielding 4%—thus, priced at par. Suppose the investor has a position in the bond with a par value of USD50 million, and the yield-to-maturity increases by 100 bps.

Calculating the Actual Prices

$$PV_0 = \frac{2}{1.02} + \frac{2}{1.02^2} + \frac{2}{1.02^3} + \frac{102}{1.02^4} = 100$$

100bp decrease in yield (3%)

$$PV_- = \frac{2}{1.015} + \frac{2}{(1.015)^2} + \frac{2}{(1.015)^3} + \frac{102}{(1.015)^4} = 101.9271923$$

$$\% \Delta PV^{\text{Full}} = \frac{101.9271923}{100} - 1 = 1.9272\%$$

100bp increase in yield (5%)

$$PV_+ = \frac{2}{1.025} + \frac{2}{(1.025)^2} + \frac{2}{(1.025)^3} + \frac{102}{(1.025)^4} = 98.1190129$$

$$\% \Delta PV^{\text{Full}} = \frac{98.1190129}{100} - 1 = -1.8810\%$$

Calculating the Modified Duration

Period	Time to Receipt	Cashflow Amount	PV	Weights	Time to Receipt*Weight	Cor
1	1.0000	2.0000	1.9608	0.0196	0.0196	
2	2.0000	2.0000	1.9223	0.0192	0.0384	
3	3.0000	2.0000	1.8846	0.0188	0.0565	
4	4.0000	102.0000	94.2322	0.9423	3.7693	
			100.0000	1.0000	3.8839	

$$\text{Annualized Macaulay duration} = \frac{3.8839}{2} = 1.94195$$

$$\text{Annualized convexity} = \frac{18.4805}{2^2} = 4.620125$$

$$\text{Modified duration} = \frac{1.94195}{1 + \frac{0.04}{2}} = 1.9039$$

So, a 100bp increase (decrease) in yield-to-maturity results in % $\Delta P_{V_{Full}}$?-1.9039% (1.9039%)

Adding Convexity Adjustment to the Duration Estimate:

$$\% \Delta P_{V_{Full}} \approx (-\text{AnnModDur} \times \Delta \text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{Yield})^2 \right]$$

$$\% \Delta P_{V_{Full}} \approx (-1.9039 \times -0.01) + \left[\frac{1}{2} \times 4.6201 \times (-0.01)^2 \right] = 1.9270\%$$

$$\% \Delta P_{V_{Full}} \approx (-1.9039 \times 0.01) + \left[\frac{1}{2} \times 4.6201 \times (0.01)^2 \right] = -1.8808\%$$

The results can be summarized in the following table:

Change in yield	Actual % $\Delta P_{V_{Full}}$	Estimated using ModDur	Difference from actual change	Estimated using ModDur and Convexity	Difference
-100bps	1.9272%	1.9039%	-0.0233%	1.9270%	0.0002%
+100bps	-1.8810%	-1.9039%	-0.0229%	-1.8808%	-0.0002%

Notice the enhanced precision after adding the convexity adjustment, shown by the decreased difference from the actual change.

Money duration and money convexity capture the first-order and second-order effects on the full price of a bond in currency units, respectively. The money convexity is calculated using the

formula:

$$\text{MoneyCon} = \text{AnnConvexity} \times \text{PV}_{\text{Full}}$$

The change in the bond's full price using Money Duration and Money Convexity is calculated using the formula:

$$\Delta \text{PV}_{\text{Full}} \approx -(\text{MoneyDur} \times \Delta \text{Yield}) + \left[\frac{1}{2} \times \text{MoneyCon} \times (\Delta \text{Yield})^2 \right]$$

Question

An investor purchases a £5 million semi-annual 2.5% coupon bond with a yield-to-maturity of 1.75%, settling 01 July 2023 and maturing 01 July 2025. The bond's ApproxCon using a 1 bp increase and decrease in yield-to-maturity is *closest to*:

- A. 1.9464
- B. 4.9277
- C. 19.1756

Solution

The correct answer is B:

$$\text{ApproxCon} = \frac{(\text{PV}_-) + (\text{PV}_+) - [2 \times (\text{PV}_0)]}{(\Delta \text{Yield})^2 \times (\text{PV}_0)}$$

$$\text{PV}_0 = \frac{1.25}{1.00875} + \frac{1.25}{1.00875^2} + \frac{1.25}{1.00875^3} + \frac{101.25}{1.00875^4} = 101.467753$$

$$\text{PV}_- = \frac{1.25}{1.00870} + \frac{1.25}{(1.00870)^2} + \frac{1.25}{(1.00870)^3} + \frac{101.25}{(1.00870)^4} = 101.4875066$$

$$\text{PV}_+ = \frac{1.25}{1.00880} + \frac{1.25}{(1.00880)^2} + \frac{1.25}{(1.00880)^3} + \frac{101.25}{(1.00880)^4} = 101.4480044$$

$$\text{ApproxCon} = \frac{((101.4875066 + 101.4480044) - (2 \times 101.467753))}{(0.0001)^2 \times 101.467753} = 4.9277$$

LOS 12c: calculate portfolio duration and convexity and explain the limitations of these measures.

Duration and convexity can be used to measure the interest rate risk of a portfolio of bonds, similar to a single bond. There are two methods to calculate the duration and convexity of a bond portfolio:

- i. Using the weighted average of time to receipt of the aggregate cash flows.
- ii. Using the weighted averages of the durations and convexities of the individual bonds in the portfolio.

The first technique is theoretically more precise. However, its practical application proves challenging. Consequently, the emphasis tends to lean towards the second approach, largely due to its common usage among fixed-income portfolio managers. However, it operates under the assumption that yield changes are uniform across all maturities, leading to a parallel shift in the yield curve. Contrary to this assumption, yield curve shifts are typically observed as steepening, flattening, or even twisting yield curves, making pure parallel shifts rare.

Example: Calculating Weighted Average Duration and Convexity

An investor purchases \$5 million par value of a 4-year, zero-coupon bond and a 5-year, fixed-rate semi-annual coupon bond. Details of the bonds are shown below.

Bond	Maturity (Years)	Coupon (%)	Price	YTM (%)	Duration	Convexity
Zero	4	0	87.1442228	3.5	3.8647	19.32367
Semi-annual	5	4.5	101.115515	4.25	4.441605	23.12742

- i. Calculate the weighted-average modified duration for the portfolio.
- ii. Calculate the weighted-average convexity for the portfolio.
- iii. Calculate the estimated percentage price change of the portfolio given a 100 bp increase in yield-to-maturity on each of the bonds.

Calculating the weighted-average modified duration for the portfolio.

To compute the weighted-average modified duration:

Determine the market value for each bond.

Zero-coupon bond: $87.1442228 \times \$5,000,000 = 435,721,114$

Semi-Annual Bond: $101.115515 \times 5,000,000 = 505,577,575$

Calculate the weight for each bond.

Total market value = $435,721,114 + 505,577,575 = 941,298,689$

Weight of Zero – coupon bond : $435,721,114/941,298,689 = 0.46289357$

Weight of Semi – Annual Bond : $505,577,575/941,298,689 = 0.537106426$

Multiply the weight of each bond by its duration and sum the results.

Weighted – average modified duration = $(0.46289357 \times 3.8647 + (0.537106426 \times 4.441605) = 4.1$

Calculating the weighted-average convexity for the portfolio.

Similar to the duration calculation above:

1. Determine the market value for each bond (which we have already done in step i).
2. Calculate the weight for each bond (which we have also done in step ii).
3. Multiply the weight of each bond by its convexity and sum the results.

Weighted – average convexity = $(0.46289357 \times 19.32367 + (0.537106426 \times 23.12742) = 21.366$

Calculating the estimated percentage price change of the portfolio given a 100 bp increase in yield-to-maturity on each of the bonds.

$$\% \Delta PV_{\text{Full}} \approx (-\text{Duration} \times \Delta y) + \left[\frac{1}{2} \times \text{Convexity} \times (\Delta y)^2 \right]$$

Where Δy is the change in yield (in decimal form).

For a 100bp change, $\Delta y = 0.0100$.

$$\% \Delta PV_{\text{Full}} \approx (-4.174559367 \times 0.01) + \left[\frac{1}{2} \times 21.36668849 \times (0.01)^2 \right] = -0.040677259 \approx -4.0677\%$$

Question

Given that a bond portfolio has a duration of 5 years and a convexity of 50, estimate the percentage change in the portfolio's value if there is an increase of 50 basis points in the yield-to-maturity.

- A. -2.438%
- B. -2.500%
- C. 2.563%

Solution

The correct answer is A.

Formula:

$$\% \Delta PV_{\text{Full}} \approx (-\text{Duration} \times \Delta y) + \left[\frac{1}{2} \times \text{Convexity} \times (\Delta y)^2 \right]$$

Where Δy for 50 basis points is 0.005

$$\% \Delta PV_{\text{Full}} \approx (-5 \times 0.005) + \left[\frac{1}{2} \times 50 \times (0.005)^2 \right] = -2.438\%$$