Learning Module 5: Portfolio Mathematics

Q.301 Which of the following statements is *most accurate* ?

A. Correlation cannot be zero.

B. Covariance is always positive.

C. Correlation cannot be greater than 1.

The correct answer is **C.**

Correlation must be between -1 and +1.
A is incorrect. Correlation can be zero. Zero correlation occurs when two items are not correlated.

B is incorrect. Covariance is not always positive. A positive covariance implies that asset returns move in the same direction, whereas a negative covariance implies that asset returns move in opposite directions.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (a) Calculate and interpret the expected value, variance, standard deviation, covariances, and correlations of portfolio returns.

Q.308 Thirty percent of the stocks in your portfolio have a P/E ratio greater than 15, out of which 25% are in the technology industry. The probability that a randomly selected stock from the portfolio will have a P/E greater than 15 and be in the technology industry is *closest to*:

A. 0.075

B. 0.30

C. 0.475

The correct answer is **A.**

We know that:

- $P(P/E > 15) = 0.3$

- $P(\text{tech stock} \mid P/E > 15) = 0.25$

The joint probability formula is:

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

Where:

- A is the event that a stock has a P/E ratio greater than 15.

- B is the event that a stock is in the technology industry, given it has a P/E ratio greater than 15.

Therefore, the joint probability is:

$$\begin{aligned} P(P/E > 15 \text{ and tech stock}) &= P(P/E > 15) \times P(\text{tech stock} \mid P/E > 15) \\ &= 0.3 \times 0.25 \\ &= 0.075 \end{aligned}$$

CFA Level 1, Quantitative Methods, Learning Module 4:Probability Trees and Conditional Expectations, LOS 4c: calculate and interpret an updated probability in an investment setting using Bayes' formula

Q.2719 If the probability that Donald Trump will lose the Presidential elections is 52% and the probability that the USD will devalue given that Trump wins the election is 91%, then the joint probability of Trump winning the Presidential elections and the devaluation of the USD is *closest to*:

A. 0.4368

B. 0.4730

C. 0.9530

The correct answer is **A.**

p(T)=0.48 (the probability that Trump will win the election)
p(D|T)=0.91 (the probability that USD will devalue given the Trump wins the election)
The joint probability of the event:

$$p(DT) = p(D|T) \times p(T) = 0.91 \times 0.48 = 0.4368$$

B is incorrect. It considers the probability of Trump losing the election and that the USD will devalue as follows.

$$P(LT) = 0.52 \times 0.91 = 0.4730$$

C is incorrect. It assumes the probability of winning or the USD being devalued as follows;

$$P(D \text{ or } T) = (0.48 + 0.91) - (0.48 \times 0.91) = 0.9530$$

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.2728 Which of the following properties of covariance is *least likely* appropriate?

A. Covariance ranges from -1 to +1.

B. Covariance of (R,R) = Variance of R

C. Covariance measures how one random variable moves with another random variable.

The correct answer is **A.**

Covariance may range from negative infinity to positive infinity, whereas correlation ranges from -1 to +1. Options B) and C) are appropriate properties of covariance.
B is incorrect. It is true that covariance of (R, R)= Variance of R. for a random variable R

C is incorrect. The covariance measures how one random variable moves with another random variable.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.2730 Assuming that the covariance of returns of Stock X and Stock Y is Cov(RX, RY) = 0.093, the variance of RX = 0.69, and the variance of RY = 0.36, the correlation of returns of Stock X and Stock Y is *closest to*:

A. 0.112

B. 0.155

C. 0.187

The correct answer is **C.**

Recall that for random variables X and Y,

$$\mathrm{Corr}(R_X, R_Y) = \frac{\mathrm{Cov}(R_X, R_Y)}{\sigma_X, \sigma_Y} = \frac{0.093}{\sqrt{0.69} \times \sqrt{0.36}} = 0.1865$$

A is incorrect. It's the outcome of dividing the $\mathrm{Cov}(R_X, R_Y)$ by the resulting σ_X.
B is incorrect. It's the outcome of dividing the $\mathrm{Cov}(R_X, R_Y)$ by the resulting σ_Y.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.2743 A young investor consults an investment manager to advise him regarding a certain type of the portfolios which give him at least 7% of return on his investment (threshold return). The investment manager presents three portfolios exhibited in the following table. Assuming that the investor invests in portfolio B, then the probability of the portfolio return falling below the threshold return of 7% according to the Safety-First ratio is *closest to*:

	Portfolio A	Portfolio B	Portfolio C
Expected Return	19%	23%	36%
Standard Deviation	14%	26%	39%

(See Z-table)

A. 27%.

B. 61.5%.

C. 73%.

The correct answer is **A.**

$$\text{The Safety-First Ratio} = \frac{\text{(Expected Return - Threshold return)}}{\text{Standard Deviation of returns}} = \frac{(0.23 - 0.07)}{0.26} = 0.6153$$

Since the question asks for the probability of Portfolio B's return falling below 7%:
$p(R_B - 7\%) = N(-0.6153) = 1 - N(0.6153) = 1 - 0.73 = 0.27$

Note: The value of N is estimated using the cumulative probabilities from the normal distribution table.

B is incorrect. It indicates the Safety-First Ratio.

C is incorrect. It's the resulting figure from N(0.6153) from the normal distribution tables.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (c) Define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.

Q.2744 An investor consults an investment manager to advise him regarding a certain type of the portfolios which would give him at least a 7% return on his investment (threshold return). The investment manager presents three portfolios exhibited in the following table:

	Portfolio A	Portfolio B	Portfolio C
Expected Return	19%	23%	36%
Standard Deviation	14%	26%	39%

Using the Safety-First ratio assumption, the portfolio that is the *most appropriate* for the investor is:
(See Z-table)

A. Portfolio A.

B. Portfolio B.

C. Portfolio C.

The correct answer is **A.**

As provided in the following table, the Safety-First ratio of Portfolio A is the highest so it has the lowest probability of the portfolio returns falling below the investor's threshold level of 7%.
The probability of N(-0.8571) is calculated using the cumulative probabilities table.

	Portfolio A	Portfolio B	Portfolio C
Expected Return	19%	23%	36%
Standard Deviation	14%	26%	39%
Safety First Ratio	$\frac{(0.19-0.07)}{0.14} = 0.8571$	$\frac{(0.23-0.07)}{0.26} = 0.6153$	$\frac{(0.36-0.07)}{0.39} = 0.7435$

B is incorrect. Portfolio B has the lowest Safety-First Ratio.

C is incorrect. Portfolio C has an average Safety-First Ratio compared to Portfolios A and B.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (c) Define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.

Q.3307 The exhibit below summarizes risk, return, and fee data for three market-neutral hedge funds:

Exhibit: Risk, Return and Fee Data

	Fund A	Fund B	Fund C
Risk-free rate	2%	2%	2%
Annualized return	15%	22%	9%
Annualized standard deviation	20%	26%	15%
Fees	1.0 and 10	1.5 and 15	2.0 and 20

Which of the following funds is most suitable for investments?

A. Fund A

B. Fund B

C. Fund C

The correct answer is **B.**

To determine which fund is most appropriate for investment, the Sharpe ratio of the three funds is calculated as follows:

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

$$\text{Fund A} = \frac{(15\% - 2\%)}{20\%} = 0.65$$

$$\text{Fund B} = \frac{(22\% - 2\%)}{26\%} = 0.77$$

$$\text{Fund C} = \frac{(9\% - 2\%)}{15\%} = 0.47$$

Fund B has the highest Sharpe ratio which means that it will enhance risk-adjusted performance. This fund is the most suitable from an investment perspective.

Q.3455 Rohan Chatterjee is planning to invest in mutual funds. His sole instruction to his portfolio manager is to generate a minimum return of 5%. The mutual funds in which the portfolio manager can invest are given in the following exhibit.

Exhibit: Potential Mutual Funds

Mutual Fund	Mean Return	Std. Dev. of Return
X	10%	3%
Y	12%	4%
Z	9%	2%

The portfolio manager will *most likely* invest in:

A. X

B. Y

C. Z

The correct answer is **C.**

The threshold level of return = 5%
We must find the Safety-First ratio:

Mutual Fund	Mean Return	Std. Dev. of Return	SF Ratio
X	10%	3%	(10% - 5%)/3% = 1.67
Y	12%	4%	(12% - 5%) / 4% = 1.75
Z	9%	2%	(9% -5%) / 2% = 2

Fund X has the highest Safety-First Ratio. Therefore, the portfolio manager must invest in Mutual Fund Z.

Additional Explanation:
Roy's Safety First Ratio represents the excess return earned over and above the threshold return per unit of risk. As a matter of fact, it is calculated as the difference between the expected return and the threshold return divided by the standard deviation of the portfolio.
Intuitively, the manager should go for the portfolio with the highest SF ratio. A higher ratio implies more excess return (earnings) per unit of risk.

A is incorrect. Fund X has the medium average Safety-First Ratio.

B is incorrect. Fund Y has the lowest Safety-First Ratio.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (c) Define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.

Q.3457 The covariance matrix of two stocks is given in the following exhibit.

Exhibit: Covariance Matrix

Stock	X	Y
X	650	120
Y	120	450

The correlation of returns for stocks X and Y is *closest* to:

A. 0.22

B. 0.45

C. 0.83

The correct answer is **A.**

$\sigma(X) = (650)^{0.5} = 25.50$
$\sigma(Y) = (450)^{0.5} = 21.21$

Covariance(X,Y) = 120

$$\text{Correlation}(X, Y) = \frac{120}{25.50 \times 21.21} = 0.22$$

B is incorrect. It assumes that 450 is the covariance between X and Y.

C is incorrect. It assumes that 650 is the covariance between X and Y.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.3458 A portfolio consists of two funds A and B. The weights of the two funds in the portfolio and the covariance matrix of the two funds are given in the following two exhibits.

Exhibit 1: Weight of the Funds in the Portfolio

Fund	A	B
Weights	60%	40%

Exhibit 2: Covariance Matrix

Fund	A	B
A	700	200
B	200	500

The portfolio variance is *closest* to:

A. 200.00

B. 428.04

C. 500.00

The correct answer is **B.**

Based on the covariance matrix:

Covariance (A,B) = 200

$$\begin{aligned}\text{Variance}_{\text{Portfolio}} &= \sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \text{Cov}(A,B)\\ &= 0.60^2 \times 700 + 0.40^2 \times 500 + 2 \times 0.60 \times 0.40 \times 200\\ &= 428.04\end{aligned}$$

A is incorrect. It indicates the Covariance (A, B).

C is incorrect. It indicates the variance of Fund B.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.3714 The probability of an increase in the annual dividend paid out to shareholders of ABC Limited is 0.4. The probability of an increase in share price given an increase in dividends is 0.7. The joint probability of an increase in dividends and an increase in share price is *closest to*:

A. 0.28.

B. 0.70.

C. 0.82.

The correct answer is **A.**

Let:
A be the event that the dividend is increased and,
B be the event that the share price increases
Therefore, P(A) = 0.4 and P(B | A) = 0.7
The joint probability of an increase in dividends and an increase in share price is P(B ∩ A)
The multiplication rule of probability states that:
P(B | A) = P(B ∩ A)/P(A)
Hence P(B ∩ A) = P(B | A) * P(A)
= 0.7 * 0.4
= 0.28 or 28% (Note that P(A ∩ B) = P(B ∩ A))

B is incorrect. It represents probability of an increase in share price given an increase in dividends.

C is incorrect. It assumes the addition rule of probability is used to determine the probability that at least one of two events will occur as follows;

$$P\,(\text{A or B}) = (0.4 + 0.7) - (0.4 \times 0.7) = 0.82$$

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (c) Define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.

Q.3718 A renowned economist has calculated that the Canadian economy will be in one of 3 possible states in the coming year: Boom, Normal, or Slow. The following table gives the returns of stocks A and B under each economic state.

State	Probability (state)	Return for stock A	Return for stock B
Boom	40%	12%	18%
Normal	35%	10%	15%
Slow	25%	8%	12%

The covariance of the returns for stocks A and B is *closest to*:

A. 0.0003765

B. 0.103

C. 0.1545

The correct answer is **A.**

$\text{Cov}(A, B) = \sum P(s) * [R_A - E(R_A)] * [R_B - E(R_B)]$

First, you have to determine the expected return for every stock:

$E(R_A) = 0.4 * 0.12 + 0.35 * 0.1 + 0.25 * 0.08 = 0.103$

$E(R_B) = 0.4 * 0.18 + 0.35 * 0.15 + 0.25 * 0.12 = 0.1545$

State	P(S)	RA	RB	P(S) * [RA - E(RA)] * [RB - E(RB)]
Boom	0.4	0.12	0.18	0.4 * [0.12 - 0.103] * [0.18 - 0.1545] = 0.0001734
Normal	0.35	0.10	0.15	0.35 * [0.1 - 0.103] * [0.15 - 0.1545] = 0.000004725
Slow	0.25	0.08	0.12	0.25 * [0.08 - 0.103] * [0.12 - 0.1545] = 0.0001984

Cov(A,B) = 0.0001734 + 0.000004725 + 0.0001984 = 0.0003765

B is incorrect. It indicates the expected return of stock A=$E(R_A)$.

C is incorrect. It indicates the expected return of stock B= $E(R_B)$.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.3719 Which of the following statements is *least likely* true regarding the correlation coefficient?

A. The correlation coefficient has no units.

B. The correlation coefficient ranges from 0 to +1.

C. The correlation coefficient measures the strength of the linear relationship between two random variables.

The correct answer is **B.**

In finance, the correlation coefficient attempts to measure the degree to which two random variables, say, returns for different stocks, move in relation to each other. The correlation coefficient always lies between -1 and +1. A positive value indicates that the random variables move in the same direction, i.e., if an increase (decrease) is recorded in one variable, we expect an increase (decrease) in the other variable, which can either be proportionate or disproportionate depending on the value of the correlation. On the other hand, a negative value usually indicates that the random variables move in opposite directions, i.e., if there is an increase in one variable, then there will be a decrease in the other variable.
A is incorrect. B is a true statement. The correlation coefficient has no units. It is simply a number between -1.0 and 1.0

C is incorrect. A is a true statement. The correlation coefficient is used to measure the strength of the relationship between two random variables. The closer to 1 (for positive correlation) and to -1 (for negative correlation), the stronger the relationship.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.3724 Two stocks, X and Y, have a correlation of 0.50. Stock Y's return has a standard deviation of 0.26. Given that the covariance between X and Y is 0.005, the variance of returns for stock X is *closest to*:

A. 0.00148

B. 0.0385

C. 0.26

The correct answer is **A.**

Correlation between X and Y,

$$\text{Corr(X,Y)} = \frac{\text{Cov(X,Y)}}{(\sigma_X \times \sigma_Y)} \Rightarrow 0.50 = \frac{0.005}{(\sigma_X \times 0.26)}$$

$$\therefore \sigma_X = 0.0385$$

Therefore,

$$\text{Variance (X)} = \sigma^2 = 0.03852^2 = 0.00148$$

B is incorrect. It is the result of $\sigma_X = 0.0385$.
C is incorrect. It indicates the standard deviation of stock Y's return.

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.

Q.4022 Consider the following two stocks from different portfolios;

	Stock A	Stock B
Expected Return	6%	10%
Standard Deviation	7%	14%
Current Portfolio weights	0.3	0.7

Given the correlation between the two stocks returns is 0.40, the covariance between the returns of Stock A and B is *closest to*:

A. 0.0024

B. 0.0039

C. 0.0088

The correct answer is **B.**

Correlation between two stocks is determined as follows;

$$\rho_{(R_A R_B)} = \frac{\text{Cov}_{(R_A R_B)}}{\sigma_{R_A}\sigma_{R_B}}$$

Hence, to calculate the covariance, the calculation becomes;

$$\text{Cov}_{(R_A R_B)} = \rho_{(R_A R_B)} \times \sigma_{R_A}\sigma_{R_B} = 0.07 \times 0.14 \times 0.4 = 0.0039$$

CFA Level 1, Quantitative Methods, Learning Module 5: Portfolio Mathematics, LOS (b) Calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.